

# Forecasted Learning

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# What is the paper about?

Decision-makers often learn in an environment where they:

- Choose between different biases/distorted sources of information (e.g. news they read)
- Know they'll receive info beyond their control in the future (e.g. news others share)

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**How does the expectation of additional info affect individual learning decisions?**

**How does news sharing affect agents' choice of media bias?**

## Results Preview

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**The expectation of additional information affects the optimal choice of bias**

This Project:

- Focuses on rational Bayesian DMs and independent sources (Independence conditional on the state)
- Rationalizes phenomenon such as Opposite-Biased-Learning and Coordination
- Characterizes optimal choice of bias with regard to binary signals
- Applies results to strategic setting with social interaction and information sharing

# The Model

- A Bayesian decision-maker (DM) chooses  $a \in \{L, R\}$
- Unknown state of the world  $\theta \in \{L, R\}$
- Objective: choose the correct action, *support the correct policy*

$$u(a, \theta) = \begin{cases} 1 & a = \theta \\ 0 & a \neq \theta \end{cases}$$

# The Model

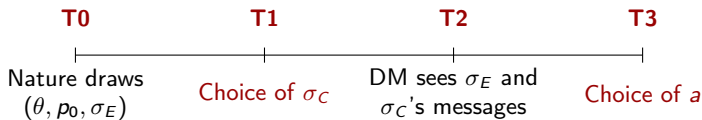
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- Prior belief  $p_0 \in [0, 1]$  is the probability that  $\theta = R$
- Before choosing  $a$ , DM obtains info from two sources (ind. signals):
  - A **chosen source**  $\sigma_C$ , *news she reads*
  - An **exogenous source**  $\sigma_E$ , *news shared by others*

# The Timing

**T0.** Fix a state  $\theta$ , a prior  $p_0$  and an exogenous source  $\sigma_E$

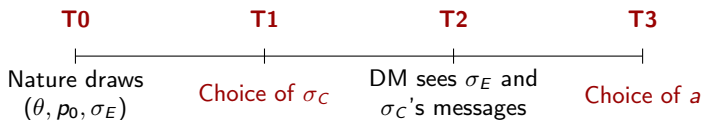




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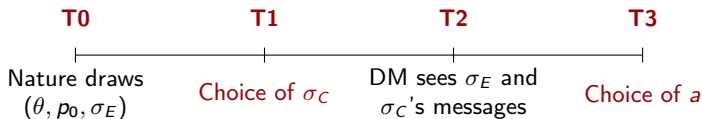
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**T1.** Knowing  $\sigma_E$  but not its message, the DM chooses a source  $\sigma_C$



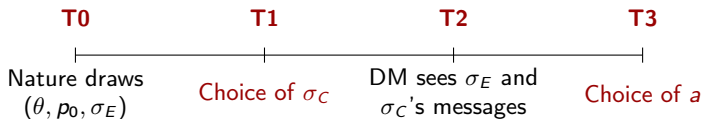
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- T2.** Messages from  $\sigma_C, \sigma_E$  are realized and observed by the DM



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- T2.** Messages from  $\sigma_C, \sigma_E$  are realized and observed by the DM
- T3.** DM chooses an action  $a$  and obtains utility  $u(a, \theta)$



# The Sources – A Binary Signal Structure

Sources (signals) can send two possible messages (realizations):  $l$  or  $r$

A source  $\sigma$  is characterized by the prob. of each message conditional on  $\theta$

- Prob. of message  $l$  in state  $L$ :  $\mathbb{P}(l|L)$
- Prob. of message  $r$  in state  $R$ :  $\mathbb{P}(r|R)$

	$m = l$	$m = r$
$\theta = L$	$\mathbb{P}(l L)$	$1 - \mathbb{P}(l L)$
$\theta = R$	$1 - \mathbb{P}(r R)$	$\mathbb{P}(r R)$

where  $\mathbb{P}(l|L) > 1 - \mathbb{P}(r|R)$

# The Sources – Assumptions for this Talk

Two sources of information:  $\sigma_L$  and  $\sigma_R$

- DM chooses between the two  $\sigma_C \in \{\sigma_L, \sigma_R\}$
- On top, expects  $\sigma_E \in \{\sigma_L, \sigma_R\}$ , *everyone can access the same media*

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Each source **biased** towards one state:  $\sigma_L$  Left-biased and  $\sigma_R$  Right-biased

## Definition

For every source  $\sigma$  characterized by  $\mathbb{P}(l|L)$ ,  $\mathbb{P}(r|R)$  as above:

- the source is *Left-biased* if  $\mathbb{P}(l|L) > \mathbb{P}(r|R)$
- the source is *Right-biased* if  $\mathbb{P}(r|R) > \mathbb{P}(l|L)$

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The two sources,  $\sigma_L$  and  $\sigma_R$ , are **symmetric**

## Definition

Two sources  $\sigma^L, \sigma^R$  are *symmetric* if:

$$\mathbb{P}(m_L = l|L) = \mathbb{P}(m_R = r|R) \text{ and } \mathbb{P}(m_L = r|R) = \mathbb{P}(m_R = l|L)$$

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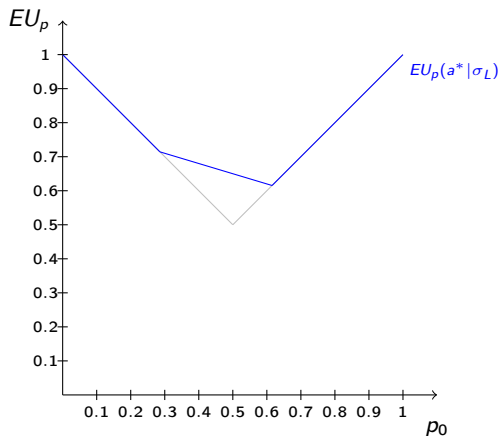
Messages from  $\sigma_C, \sigma_E$  are **independent** conditional on  $\theta$

- If DM (mis)matches  $\sigma_C$  with  $\sigma_E$  is not due to correlation
- Note,  $\sigma_C$  and  $\sigma_E$  can send different messages even if  $\sigma_C = \sigma_E$



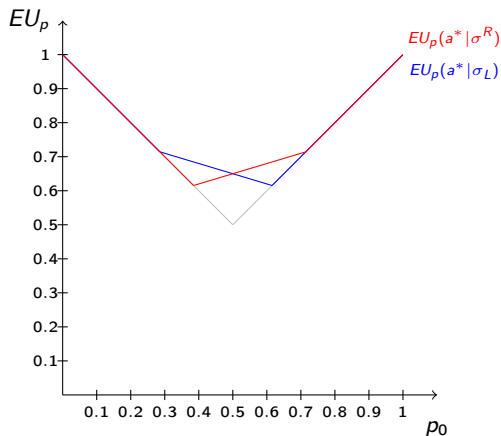
# Optimality of Own-Biased Learning w/o Additional Info

The expected value of each source *in isolation* depends on the prior belief



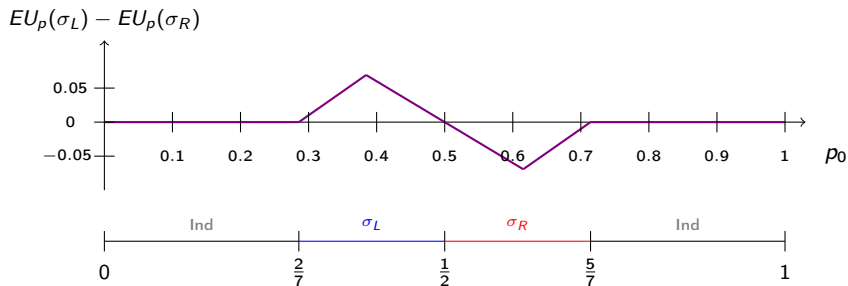
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# Optimality of Own-Biased Learning w/o Additional Info

The difference in expected value of sources also depends on the prior belief



- Own-biased learning is weakly optimal
- Strictly for sufficiently uncertain DMs

## Solving for the Optimal Strategy w/ Additional Info

Optimal strategy can be found:

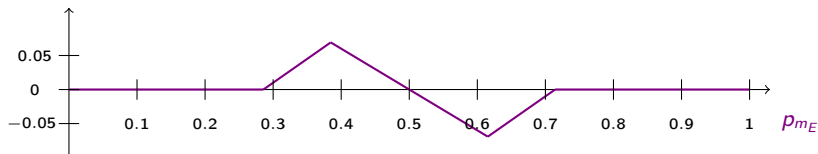
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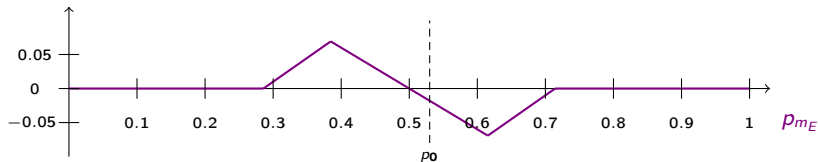


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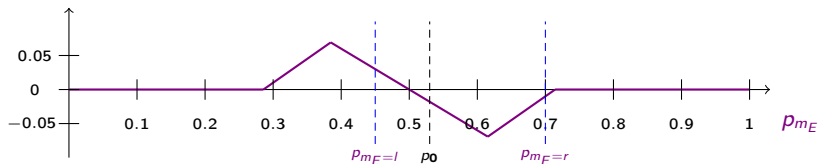


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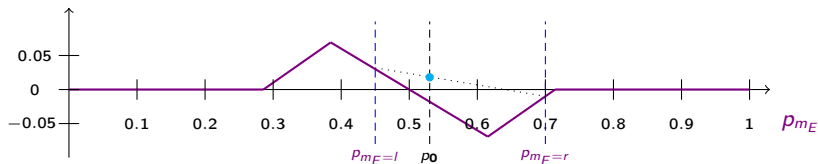


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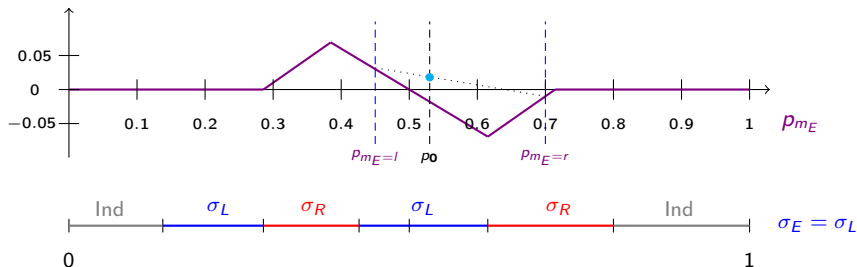


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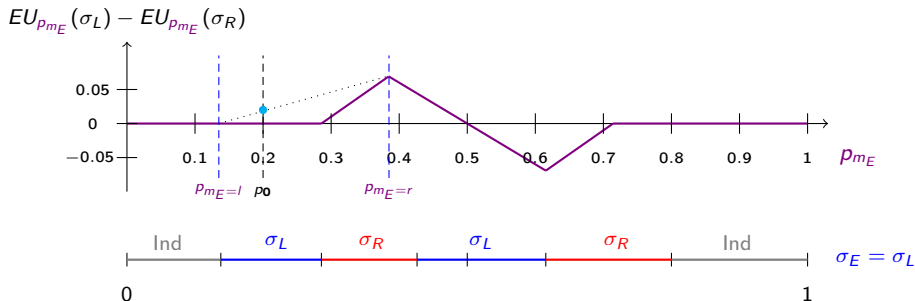


Example of a very certain Left-biased DM who chooses own-biased learning

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Example of a Right-biased DM who chooses opposite-biased learning

# Structure of the Optimal Strategy

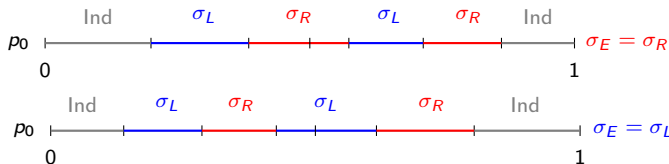
CS

Thresholds

## Proposition

For a given  $\sigma_L, \sigma_R$  and  $\sigma_E \in \{\sigma_L, \sigma_R\}$ , there exist five thresholds  $0 \leq p_1 < p_2 \leq p_3 \leq p_4 < p_5 \leq 1$  (all strict when  $\mathbb{P}_L(l|L), \mathbb{P}_L(r|R) \neq 1$ ) s.t.

- If  $p_0 \in [0, p_1] \cup [p_5, 1]$ , a DM is **indifferent** between any choice of  $\sigma_C$
- If  $p_0 \in [p_1, p_2] \cup [p_4, p_5]$ , DM chooses **own-biased learning**
- If  $p_0 \in [p_2, \min\{p_3, \frac{1}{2}\}] \cup [\max\{p_3, \frac{1}{2}\}, p_4]$ , DM chooses **opp-biased learning**
- If  $p_0 \in [\min\{p_3, \frac{1}{2}\}, \max\{p_3, \frac{1}{2}\}]$ , DM chooses to **match**  $\sigma_E$



# General Features of the Optimal Strategy

Many features extend to any binary exogenous source  $\sigma_E$ , change with informativeness of  $\sigma_E$

- Extremely certain priors: Indifference
- Very certain priors: Own-BL
- Opposite-BL for moderately certain priors (if  $\sigma_E$  suff. info/bias)
- Uncertain priors: Own-BL, Indifference or Matching

# Review & Additional Results

## Project Key Points

- Adds **expectation of future, independent information** to standard Bayesian learning model
- Characterize the preference for source bias/distortion for binary signals.
- Rationalize opposite-biased learning and consuming varied bias.

## Generality of Mechanism [▶ More](#)

- Additional ind. info can change optimal info choice in more general contexts
- Sources with more messages, multiple states, other utility functions

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## Strategic interaction leads to rich predictions [▶ More](#)

- Game of news choice, where both players want to learn
- Rationalize (mis)coordinating, (mis)matching news choices..

Thanks for listening!

## Related Literature

### Individual learning from biased sources for a Bayesian agent:

- **Optimal to consume an own-biased medium:** Calvert, 1985; Suen, 2004; Burke, 2008; Gentzkow and Shapiro, 2006; Meyer, 1991; Mullainathan and Shleifer, 2005; Oliveros and Várdy, 2015.
- **Optimal to multi-home and opposite bias learning:** Che and Mierendorff, 2019; Nikandrova and Pancs, 2018; Mayskaya, 2020; Liang et al., 2022.

**(Bayesian) learning from others:** Sethi and Yildiz, 2016; Sethi and Yildiz, 2012; Bowen et al., 2021;...

### Exogenous manipulation of beliefs:

- **Manipulation of attention:** Gossner et al., 2021; Liang et al., 2022.
- **Persuasion with unknown beliefs:** Dworzak and Pavan, 2022; Laclau, Renou, et al., 2017; Kolotilin et al., 2017.

**Complementarity btw signals:** Börgers et al., 2013; Brooks et al., 2022



## Thresholds of the Optimal Strategy: Likelihood Ratios

$$p_1 = 1 - \frac{\mathbb{P}_L(r|R)\mathbb{P}_E(r|R)}{\mathbb{P}_L(r|L)\mathbb{P}_E(r|L) + \mathbb{P}_L(r|R)\mathbb{P}_E(r|R)}$$

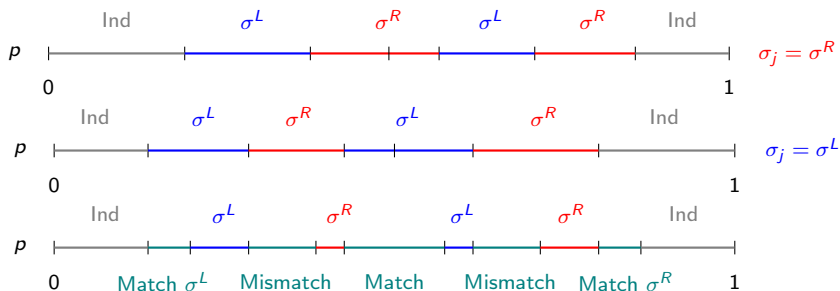
$$p_2 = 1 - \frac{\mathbb{P}_E(r|R)}{\mathbb{P}_E(r|L) + \mathbb{P}_E(r|R)}$$

$$p_3 = \frac{\mathbb{P}_L(r|L)\mathbb{P}_E(I|L) + \mathbb{P}_R(I|L)\mathbb{P}_E(r|L)}{\mathbb{P}_L(r|L)\mathbb{P}_E(I|L) + \mathbb{P}_R(I|L)\mathbb{P}_E(r|L) + \mathbb{P}_L(r|R)\mathbb{P}_E(I|R) + \mathbb{P}_R(I|R)\mathbb{P}_E(r|R)}$$

$$p_4 = \frac{\mathbb{P}_E(I|L)}{\mathbb{P}_E(I|L) + \mathbb{P}_E(I|R)}$$

$$p_5 = \frac{\mathbb{P}_R(I|L)\mathbb{P}_E(I|L)}{\mathbb{P}_R(I|L)\mathbb{P}_E(I|L) + \mathbb{P}_R(I|R)\mathbb{P}_E(I|R)}$$

# How does this translate into equilibria?



## Many different types of equilibria

- **Dominance-solvable** e.g. dominant strategy and best-reply
- **Coordination:** both want to match or mismatch
- **Miscoordination:** one wants to match, one to mismatch

## General Importance of Additional Information

Often there exists  $\sigma^e$  that changes the DM's optimal choice of info

- If no source of info is better than the other for all priors (in isolation)
- Some independent signal can change the DM's optimal info choice

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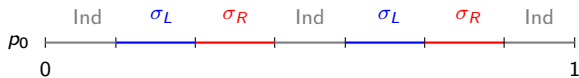
### Proposition

*For any two sources  $\sigma^x, \sigma^y$ , and any finite, discrete action, utility space: If there exists at least one belief,  $\hat{p}$ , s.t.  $EU(\sigma^x|\hat{p}) > EU(\sigma^y|\hat{p})$ , then for all beliefs  $p$  there exists  $\sigma_e$ , s.t.  $EU(\sigma^x, \sigma_e|p) > EU(\sigma^y, \sigma_e|p)$*

▶ Back

# Three Types of Optimal Strategy

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If  $\sigma_E$  is substantially less informative than  $\sigma_L, \sigma_R$



If  $\sigma_E$  is equally or relatively more informative than  $\sigma_L, \sigma_R$

