Identifying Dynamic LATEs with a Static Instrument

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Motivation

- How can we identify dynamic effects with a time-invariant binary IV?
- Examples:
  - Dynamic effects of training programs.
    Schochet et al. (2008), Alzúa et al. (2016), Hirshleifer et al. (2016), Das (2021), and others.
  - Effects of having children on parents' labor supply.
    Bronars and Grogger (1994), Angelov and Karimi (2012), Silles (2015), Lundborg et al. (2017), and others.
- Main challenge: we can have individuals starting treatment at different time periods ("dynamic compliance").

Why Bother?

- Common approach: period by period comparisons.
- Problem: individuals who are not offered training in the first period but that are trained afterward "contaminate" these estimates.

$\Rightarrow$ Negative weights for some causal effects in the usual estimands.

  - Estimands may be negative even when the treatment makes everyone better off.

This Paper

- Decomposition results for the usual estimands (period by period comparisons).
  - Understand when there is contamination and which groups drive it.
- Point identification of dynamic effects for first-period compliers.
  - Achieved with some types of homogeneity assumptions on causal effects.
  - No additional source of exogenous variation is required.
- Partial identification when treatment effects are bounded.
  - Bounds valid without any homogeneity on causal effects.
  - Tighter bounds as a middle ground.

Two-Period Setting

Treatment and Lottery

- For example, consider a two-period evaluation of a training program.
  - See paper for the T-period setting.
- A lottery determines who gets a training offer in the first period.
- Z_i is the lottery outcome and $D_{i,t}$ indicates whether i is treated at $t \in \{1, 2\}$.
- We assume that treatment is irreversible.

Assumption 1

$D_{i,1} = 1 \implies D_{i,2} = 1.$

Potential Outcomes

- $Y_{i,t}(0)$ if i is not treated at t.
- $Y_{i,t}(1,\tau)$ if i started to be treated τ periods before t.
  - At $t = 1$:
    0: not treated at $t = 1$.
    $(1,0)$: treated at $t = 1$.
  - At $t = 2$:
    0: not treated at $t = 2$.
    $(1,0)$: treated at $t = 2$.
    $(1,1)$: treated at $t = 1$.
- Exclusion restriction: the lottery does not affect potential outcomes directly.
- $Y_{i,t}$ is the observable outcome.

Latent Groups

- Potential treatment statuses for observation i at t are denoted by $D_{i,t}(z)$, $z \in \{0, 1\}$.
- We define the four usual IV latent types by period.

	$D_{i,t}(1) = 1$	$D_{i,t}(1) = 0$
$D_{i,t}(0) = 1$	AT_t	F_t
$D_{i,t}(0) = 0$	C_t	NT_t

- We say we have "dynamic compliance" if latent groups may change over time.
  - Example: (C_1, AT_2) is someone who is only treated at $t = 1$ if received $Z_i = 1$, but who will be treated at $t = 2$ regardless of Z_i.

Target Parameters

- Treatment effects of interest compare treated and untreated potential outcomes:

$$\Delta_t^{\tau}(g) := \mathbb{E}\left[Y_{i,t}(1,\tau) - Y_{i,t}(0)|g\right],$$

  where g is a history of IV types.

- We focus on:
  - $\Delta_1^0(C_1)$: effect in the first period for C_1 units.
  - $\Delta_2^1(C_1)$: effect at $t = 2$ of being treated in the first period for C_1 units.

- Focus on the dynamic effects over time for a fixed population (compliers from $t = 1$).

Basic Assumptions

- As a basic requirement, we assume that Z_i satisfies the following.

Assumption 2

1. The instrument is excluded from potential outcomes given time period and treatment length.
2. Potential outcomes and potential treatment statuses are drawn independently of Z_i.
3. $FS_1 \neq 0$.
4. $\mathbb{P}(F_1) = 0$.

- Notes:
  1. For our solutions, we only need relevance of the instrument at $t = 1$;
  2. Also, we only need no defiers at $t = 1$.

Period by Period Estimators

Estimands (Period by Period Estimators)

- Reduced form and first stage estimands are

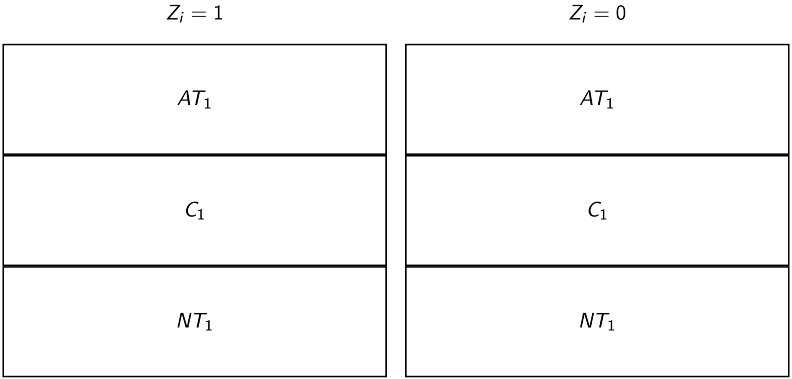
$$RF_t := \mathbb{E}\left[Y_{i,t}|Z_i = 1\right] - \mathbb{E}\left[Y_{i,t}|Z_i = 0\right],$$
$$FS_t := \mathbb{E}\left[D_{i,t}|Z_i = 1\right] - \mathbb{E}\left[D_{i,t}|Z_i = 0\right].$$

- Moreover,

$$\text{IV estimand at } t := \frac{RF_t}{FS_t}.$$

IV comparisons at $t = 1$

- $\Delta_1^0(C_1)$ is identified by RF_1/FS_1 (Imbens and Angrist 1994).
  - At $t = 1$ we don't have dynamic compliance problems.



IV comparisons at $t = 1$

- $\Delta_1^0(C_1)$ is identified by RF_1/FS_1 (Imbens and Angrist 1994).
  - At $t = 1$ we don't have dynamic compliance problems.

$Z_i = 1$	$Z_i = 0$
AT_1 $Y_{i,1}(1,0)$	AT_1 $Y_{i,1}(1,0)$
C_1 $Y_{i,1}(1,0)$	C_1 $Y_{i,1}(0)$
NT_1 $Y_{i,1}(0)$	NT_1 $Y_{i,1}(0)$

IV comparisons at $t = 1$

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$Z_i = 1$	$Z_i = 0$
AT_1 ~~$Y_{i,1}(1,0)$~~	AT_1 ~~$Y_{i,1}(1,0)$~~
C_1 $Y_{i,1}(1,0)$	C_1 $Y_{i,1}(0)$
NT_1 ~~$Y_{i,1}(0)$~~	NT_1 ~~$Y_{i,1}(0)$~~

IV comparisons at $t = 1$

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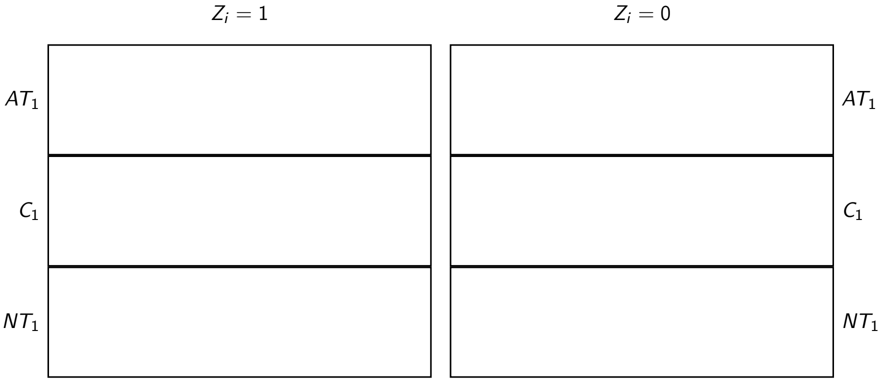
$Z_i = 1$	$Z_i = 0$
AT_1 ~~$Y_{i,1}(1,0)$~~	AT_1 ~~$Y_{i,1}(1,0)$~~
C_1 $Y_{i,1}(1,0)$	C_1 $Y_{i,1}(0)$
NT_1 ~~$Y_{i,1}(0)$~~	NT_1 ~~$Y_{i,1}(0)$~~

$$RF_1 = \mathbb{P}(C_1)\Delta_1^0(C_1),$$

$$FS_1 = \mathbb{P}(C_1),$$

$$\frac{RF_1}{FS_1} = \Delta_1^0(C_1).$$

IV comparisons at $t = 2$



IV comparisons at $t = 2$

$Z_i = 1$

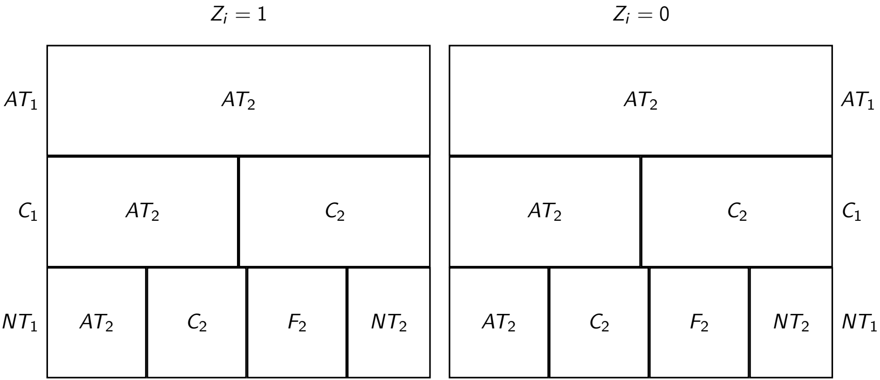
AT_1	AT_2	C_2	F_2	NT_2
C_1	AT_2	C_2	F_2	NT_2
NT_1	AT_2	C_2	F_2	NT_2

$Z_i = 0$

AT_2	C_2	F_2	NT_2	AT_1
AT_2	C_2	F_2	NT_2	C_1
AT_2	C_2	F_2	NT_2	AT_1

IV comparisons at $t = 2$

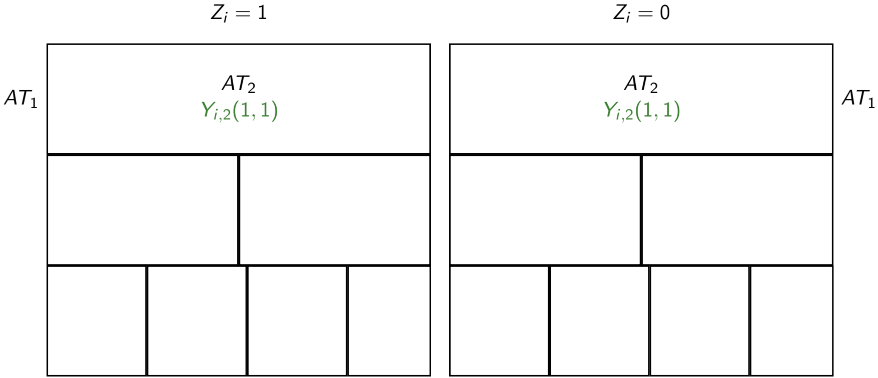
$$RF_2 = \mathbb{E}[Y_{i,2}|Z_i = 1] - \mathbb{E}[Y_{i,2}|Z_i = 0].$$



IV comparisons at $t = 2$

$Y_{i,2}(0)$: not treated at $t = 2$
$Y_{i,2}(1,1)$: treated at $t = 1$
$Y_{i,2}(1,0)$: treated at $t = 2$

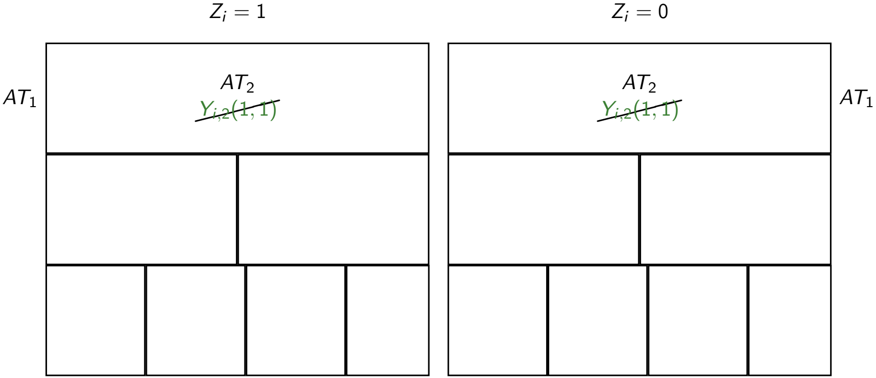
$$RF_2 = \mathbb{E}[Y_{i,2}|Z_i = 1] - \mathbb{E}[Y_{i,2}|Z_i = 0].$$



IV comparisons at $t = 2$

$Y_{i,2}(0)$: not treated at $t = 2$
$Y_{i,2}(1,1)$: treated at $t = 1$
$Y_{i,2}(1,0)$: treated at $t = 2$

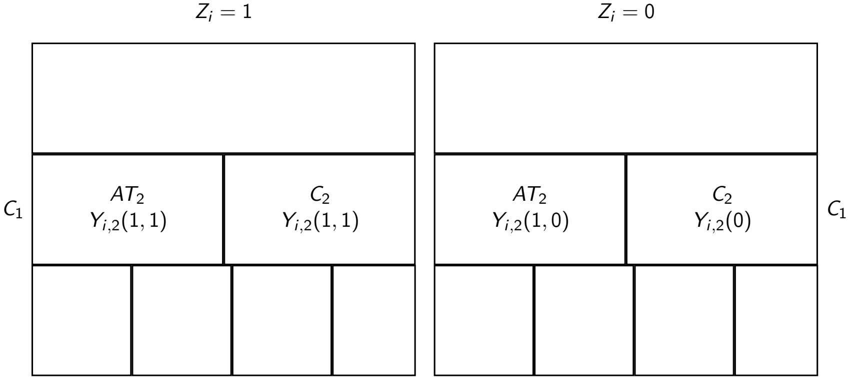
$$RF_2 = \mathbb{P}(AT_2) \cdot 0 + ...$$



IV comparisons at $t = 2$

$Y_{i,2}(0)$: not treated at $t = 2$
$Y_{i,2}(1,1)$: treated at $t = 1$
$Y_{i,2}(1,0)$: treated at $t = 2$

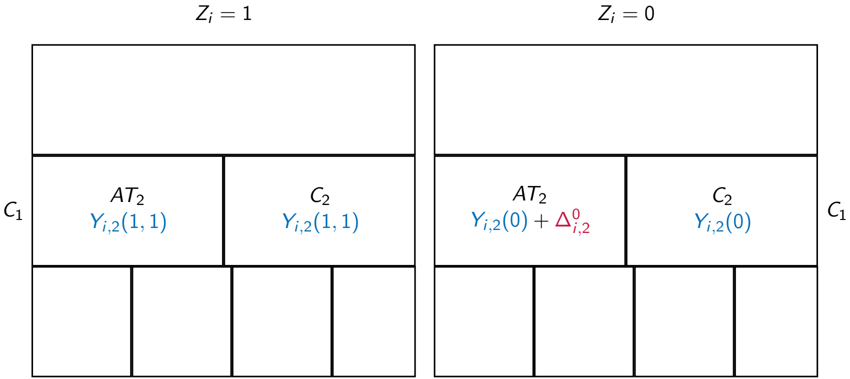
$$RF_2 = \mathbb{P}(AT_2) \cdot 0 + \ldots$$



IV comparisons at $t = 2$

$Y_{i,2}(0)$: not treated at $t = 2$
$Y_{i,2}(1,1)$: treated at $t = 1$
$Y_{i,2}(1,0)$: treated at $t = 2$

$$RF_2 = \mathbb{P}(C_1)\Delta_2^1(C_1) - \mathbb{P}(C_1, AT_2)\Delta_2^0(C_1, AT_2) + \ldots$$

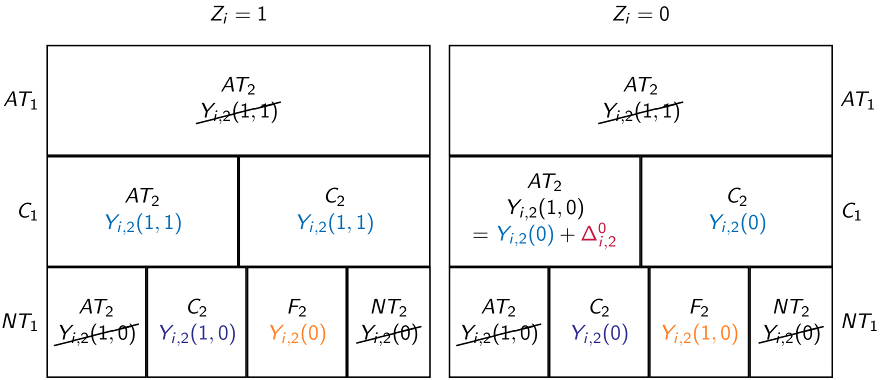


- **Problem**: potential outcomes of (C_1, AT_2) for $Z_i = 1$ and $Z_i = 0$ are not comparable, even though this group is treated at $t = 2$ regardless of Z_i.

IV comparisons at $t = 2$

$Y_{i,2}(0)$: not treated at $t = 2$
$Y_{i,2}(1,1)$: treated at $t = 1$
$Y_{i,2}(1,0)$: treated at $t = 2$

$$RF_2 = \mathbb{P}(C_1)\Delta_2^1(C_1) - \mathbb{P}(C_1, AT_2)\Delta_2^0(C_1, AT_2) + \mathbb{P}(NT_1, C_2)\Delta_2^0(NT_1, C_2) - \mathbb{P}(NT_1, F_2)\Delta_2^0(NT_1, F_2).$$



IV comparisons at $t = 2$

Δ_2^0: effect at $t = 2$, treated at $t = 2$
Δ_2^1: effect at $t = 2$, treated at $t = 1$

$$RF_2 = \mathbb{P}(C_1)\Delta_2^1(C_1) - \mathbb{P}(C_1, AT_2)\Delta_2^0(C_1, AT_2) + \mathbb{P}(NT_1, C_2)\Delta_2^0(NT_1, C_2) - \mathbb{P}(NT_1, F_2)\Delta_2^0(NT_1, F_2)$$

and

$$FS_2 = \mathbb{P}(C_1) - \mathbb{P}(C_1, AT_2) + \mathbb{P}(NT_1, C_2) - \mathbb{P}(NT_1, F_2).$$

- Notes:
  - Weights in the IV estimand sum to one; but some may be negative.
  - If $FS_2 < FS_1$, then there must be negative weights.
  - No defiance in all periods does not eliminate all negative weights:

$$RF_2 = \mathbb{P}(C_1)\Delta_2^1(C_1) - \mathbb{P}(C_1, AT_2)\Delta_2^0(C_1, AT_2) + \mathbb{P}(NT_1, C_2)\Delta_2^0(NT_1, C_2) - \mathbb{P}(NT_1, F_2)\Delta_2^0(NT_1, F_2).$$

When Does the IV Estimand Work?

Compliance is static

Δ_2^0: effect at $t = 2$, treated at $t = 2$
Δ_2^1: effect at $t = 2$, treated at $t = 1$

- Compliance is static $\Rightarrow \mathbb{P}(C_1, AT_2) = \mathbb{P}(NT_1, C_2) = \mathbb{P}(NT_1, F_2) = 0$.

$$RF_2 = \mathbb{P}(C_1)\Delta_2^1(C_1) - \cancel{\mathbb{P}(C_1, AT_2)\Delta_2^0(C_1, AT_2)} + \cancel{\mathbb{P}(NT_1, C_2)\Delta_2^0(NT_1, C_2)} - \cancel{\mathbb{P}(NT_1, F_2)\Delta_2^0(NT_1, F_2)},$$

$$FS_2 = \mathbb{P}(C_1) - \cancel{\mathbb{P}(C_1, AT_2)} + \cancel{\mathbb{P}(NT_1, C_2)} - \cancel{\mathbb{P}(NT_1, F_2)}.$$

When Does the IV Estimand Work?

Treatment length homogeneity

Assumption 3

For any latent group $g \in \{(C_1, AT_2), (NT_1, C_2), (NT_1, F_2)\}$ such that $\mathbb{P}(g) > 0$, $\Delta_2^1(C_1) = \Delta_2^0(g)$.

- Treatment effect may vary with calendar time, but does not depend on length of exposure to the treatment.
  - Also needs homogeneity assumptions wrt latent groups.

$$RF_2 = \Big[\mathbb{P}(C_1) - \mathbb{P}(C_1, AT_2) + \mathbb{P}(NT_1, C_2) - \mathbb{P}(NT_1, F_2)\Big]\Delta_2^1(C_1)$$

$$FS_2 = \Big[\mathbb{P}(C_1) - \mathbb{P}(C_1, AT_2) + \mathbb{P}(NT_1, C_2) - \mathbb{P}(NT_1, F_2)\Big]$$

Point Identification Under Alternative Assumptions

Calendar Time Homogeneity

Δ_2^0: effect at $t = 2$, treated at $t = 2$
Δ_2^1: effect at $t = 2$, treated at $t = 1$

Assumption 4

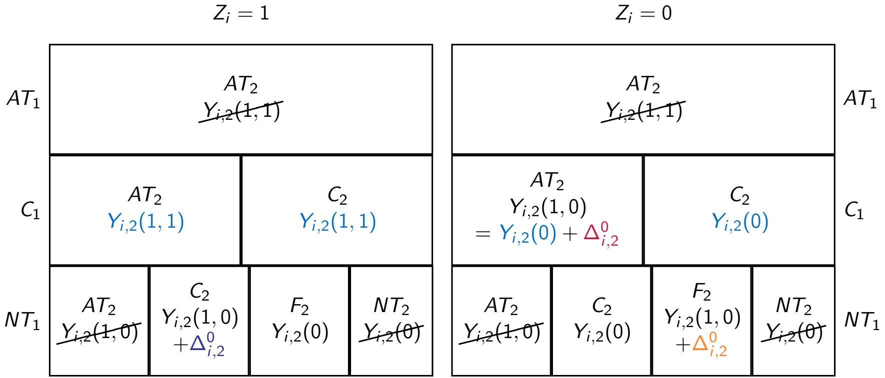
For any latent group $g \in \{(C_1, AT_2), (NT_1, C_2), (NT_1, F_2)\}$ such that $\mathbb{P}(g) > 0$,
$\Delta_1^0(C_1) = \Delta_2^0(g)$.

- Treatment effect can vary with length of exposure to treatment (but does not depend on calendar time).
  - More reasonable than previous assumptions if we consider a time range in which economy is stable.
  - Also need homogeneity assumptions wrt latent groups.

Identification Under Calendar Time Homogeneity

Assumption 4

For any latent group $g \in \{(C_1, AT_2), (NT_1, C_2), (NT_1, F_2)\}$ such that $\mathbb{P}(g) > 0$, $\Delta_1^0(C_1) = \Delta_2^0(g)$.



Identification Under Calendar Time Homogeneity

- $\Delta^0(C_1)$ is identified by the first-period IV estimand.
- Use it to correct the contamination term.

$Z_i = 1$

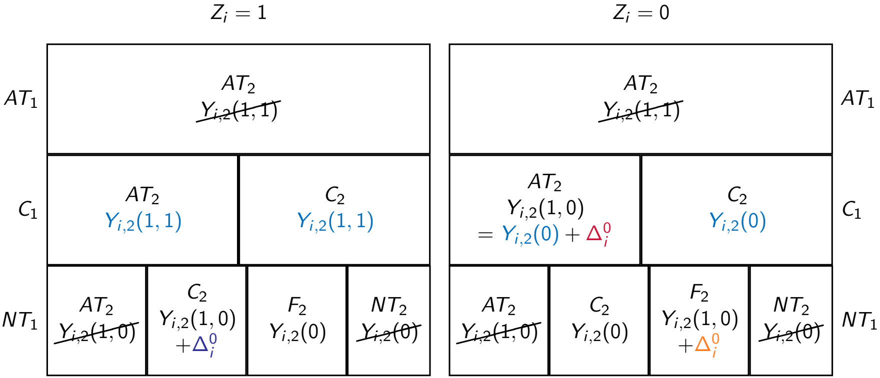
AT_1	AT_2 ~~$Y_{i,2}(1,1)$~~			
C_1	AT_2 $Y_{i,2}(1,1)$	C_2 $Y_{i,2}(1,1)$		
NT_1	AT_2 ~~$Y_{i,2}(1,0)$~~	C_2 $Y_{i,2}(1,0)$ $+\Delta_i^0$	F_2 $Y_{i,2}(0)$	NT_2 ~~$Y_{i,2}(0)$~~

$Z_i = 0$

AT_2 ~~$Y_{i,2}(1,1)$~~				AT_1
AT_2 $Y_{i,2}(1,0)$ $= Y_{i,2}(0) + \Delta_i^0$	C_2 $Y_{i,2}(0)$			C_1
AT_2 ~~$Y_{i,2}(1,0)$~~	C_2 $Y_{i,2}(0)$	F_2 $Y_{i,2}(1,0)$ $+\Delta_i^0$	NT_2 ~~$Y_{i,2}(0)$~~	NT_1

Identification Under Calendar Time Homogeneity

$$\Delta_2^1(C_1) = \Delta^1(C_1) = \frac{RF_2}{FS_1} + \frac{FS_1 - FS_2}{FS_1}\frac{RF_1}{FS_1}$$



Comparison with the IV Estimand

- IV estimand: calendar time heterogeneity but no time-since-treatment heterogeneity.
- Correction: calendar time homogeneity but unrestricted time-since-treatment heterogeneity.
- Both assume homogeneous effects for the other groups or that they do not exist (or a combination in between).
- Our correction only requires relevance in the first period.

Estimation

- We can define a just-identified linear IV model in which the solution of the moment condition is the corrected IV estimand.

- Estimate

$$Y_{i,t} = \gamma_1 1(t=1) + \gamma_2 1(t=2) + \beta_1 (D_{i,t} - D_{i,t-1}) + \beta_2 1(2 \leq t)(D_{i,t-1} - D_{i,t-2}) + \varepsilon_{i,t}$$

  treating the time dummies as exogenous and using $1(t=1)Z_i$ and $1(t=2)Z_i$ as instruments for the endogenous variables.

- Estimation using standard GMM or two-stage least squares software gives valid inference.

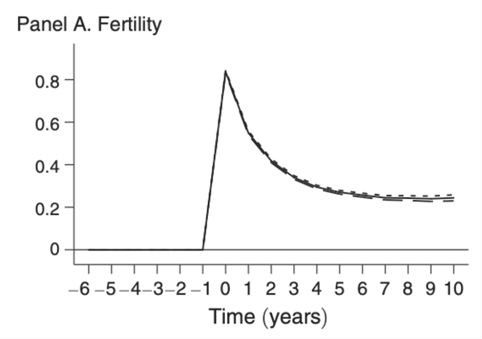
Empirical Illustration

Lundborg et al. 2017

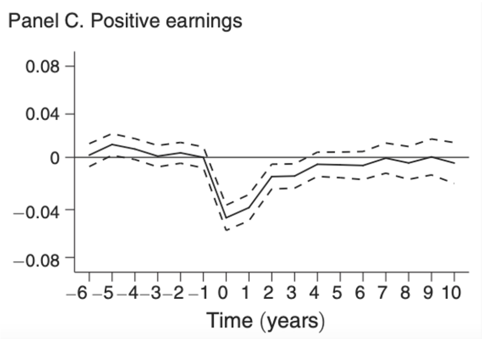
"Can Women Have Children and a Career? IV Evidence from IVF Treatments"

- $D_{i,t}$: fertility;
- Z_i: in vitro fertilization (IVF) treatment success;
- $Y_{i,t}$: labor force participation.

First stage



Reduced form



Figures from Lundborg et al. 2017

Lundborg et al. 2017

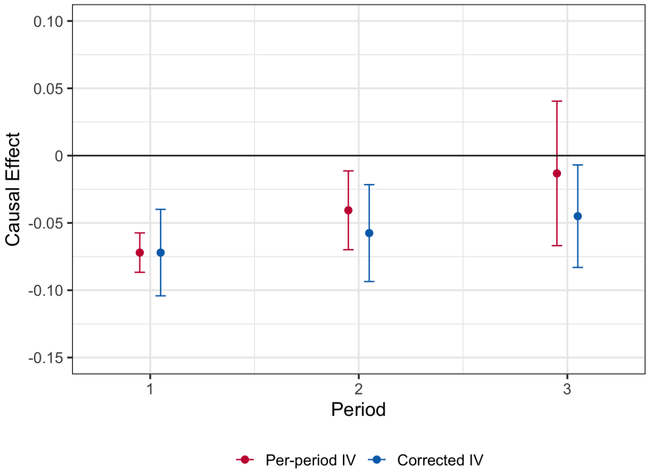
"Can Women Have Children and a Career? IV Evidence from IVF Treatments"

- Challenge: we don't have access to the microdata.
- Construct a dataset that matches the FS and the RF for all periods.
- Limitation 1: we cannot get the exact trajectories of $Y_{i,t}$ for each observation.
  - Trajectory does not affect our point estimates;
  - Very minor differences on se's; we use worst case over 1000 possible datasets.
- Limitation 2: we don't have information on $\mathbb{E}[Y_{i,t}|Z_i = 0]$.
  - se's are maximized when $\mathbb{E}[Y_{i,t}|Z_i = 0] = 0.5$;
  - We use $\mathbb{E}[Y_{i,t}|Z_i = 0] = 0.8$ as a conservative estimate.
- Limitation 3: we don't have information on covariates.
  - se's for our correction are higher than they would be if we had covariates.

Lundborg et al. 2017

"Can Women Have Children and a Career? IV Evidence from IVF Treatments"

- Per-period vs Corrected IV.



Partial Identification

Partial Identification

- Under Assumptions 1 and 2 only,

$$FS_1 \Delta_2^1(C_1) = RF_2 - \mathbb{P}(NT_1, C_2)\Delta_2^0(NT_1, C_2) + \mathbb{P}(C_1, AT_2)\Delta_2^0(C_1, AT_2) + \mathbb{P}(NT_1, F_2)\Delta_2^0(NT_1, F_2).$$

  - We want to identify the target parameter $\Delta_2^1(C_1)$.

Partial Identification

- Under Assumptions 1 and 2 only,

$$
\begin{aligned}
FS_1\Delta_2^1(C_1) = RF_2 &- \mathbb{P}(NT_1, C_2)\Delta_2^0(NT_1, C_2) \\
&+ \mathbb{P}(C_1, AT_2)\Delta_2^0(C_1, AT_2) + \mathbb{P}(NT_1, F_2)\Delta_2^0(NT_1, F_2).
\end{aligned}
$$

  - We want to identify the target parameter $\Delta_2^1(C_1)$.
  - We observe directly the first stage and the reduced form.

Partial Identification

- Under Assumptions 1 and 2 only,

$$FS_1 \Delta_2^1(C_1) = RF_2 - \mathbb{P}(NT_1, C_2)\Delta_2^0(NT_1, C_2) + \mathbb{P}(C_1, AT_2)\Delta_2^0(C_1, AT_2) + \mathbb{P}(NT_1, F_2)\Delta_2^0(NT_1, F_2).$$

  - We want to identify the target parameter $\Delta_2^1(C_1)$.
  - We observe directly the first stage and the reduced form.
  - Assume bounds on the treatment effects, $\overline{\Delta}$ and $\underline{\Delta}$ (natural if Y is bounded).

Partial Identification

- Under Assumptions 1 and 2 only,

$$FS_1 \Delta_2^1(C_1) = RF_2 - \mathbb{P}(NT_1, C_2)\Delta_2^0(NT_1, C_2) + \mathbb{P}(C_1, AT_2)\Delta_2^0(C_1, AT_2) + \mathbb{P}(NT_1, F_2)\Delta_2^0(NT_1, F_2).$$

  - We want to identify the target parameter $\Delta_2^1(C_1)$.
  - We observe directly the first stage and the reduced form.
  - Assume bounds on the treatment effects, $\overline{\Delta}$ and $\underline{\Delta}$ (natural if Y is bounded).
  - Bounds on the probabilities:
    - $\mathbb{P}(NT_1, C_2)$: at most, prob of switching into treatment at $t = 2$ given $Z_i = 1$;
    - $\mathbb{P}(C_1, AT_2) + \mathbb{P}(NT_1, F_2)$: at most, prob of switching into treatment at $t = 2$ for $Z_i = 0$.

Partial Identification

- Given that,

$$\frac{RF_2}{FS_1} - \frac{\mathbb{P}\left(D_{i,2} > D_{i,1} | Z_i = 1\right)}{FS_1}\overline{\Delta} + \frac{\mathbb{P}\left(D_{i,2} > D_{i,1} | Z_i = 0\right)}{FS_1}\underline{\Delta}$$

is a lower bound for $\Delta_2^1(C_1)$, and

$$\frac{RF_2}{FS_1} - \frac{\mathbb{P}\left(D_{i,2} > D_{i,1} | Z_i = 1\right)}{FS_1}\underline{\Delta} + \frac{\mathbb{P}\left(D_{i,2} > D_{i,1} | Z_i = 0\right)}{FS_1}\overline{\Delta}$$

is an upper bound.

Partial Identification: Tighter Bounds

Assumption 5

For all $g, g' \in \{(C_1, AT_2), (NT_1, C_2), (NT_1, F_2)\}$ with $\mathbb{P}(g) > 0$ and $\mathbb{P}(g') > 0$, $\Delta_2^0(g) = \Delta_2^0(g')$.

- If Assumption 5 holds,

$$\frac{RF_2}{FS_1} + \left[1(FS_2 \leq FS_1)\underline{\Delta} + 1(FS_2 > FS_1)\overline{\Delta}\right]\frac{FS_1 - FS_2}{FS_1},$$

  is a lower bound for $\Delta_2^1(C_1)$ and

$$\frac{RF_2}{FS_1} + \left[1(FS_2 \leq FS_1)\overline{\Delta} + 1(FS_2 > FS_1)\underline{\Delta}\right]\frac{FS_1 - FS_2}{FS_1}$$

  is an upper bound.

- These bounds are (weakly) tighter than the previous ones.
  - Intuition: under these assumptions, we don't need bounds on the probabilities.

Conclusion

- Setting:
  - Static instrument and dynamic compliance.
- Results:
  - Decomposition: possibility of negative weights.
  - Identification: require a different type of treatment effect homogeneity assumption.
  - Partial identification: relax these assumptions.

Related Literature

- IV:
  - Static IV: Lundborg et al. (2017) and Miquel (2002).
  - IV in a general dynamic setting: Han (2021).
  - Lower Dimensional IV: Angrist and Imbens (1995), Torgovitsky (2015), D'Haultfœuille and Février (2015), Masten and Torgovitsky (2016), Caetano and Escanciano (2021) and Hull (2018).
  - IV with covariates: Kolesár (2013), Blandhol et al. (2022) and Słoczyński (2022).
- Dynamic Causal Effects: Ding and Lehrer (2010), Heckman et al. (2016), Bojinov et al. (2021), van den Berg and Vikström (2022).
- DiD:
  - Fuzzy DiD: Hudson et al. (2017), de Chaisemartin and D'Haultfœuille (2017) and Picchetti and Pinto (2022).
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