Endogenous Clustering and Analogy-Based Expectations Equilibrium

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Contributed Session - Game Theory and Behavioural

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Introduction

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- We propose using insights from psychology about categorization to put structure on those.

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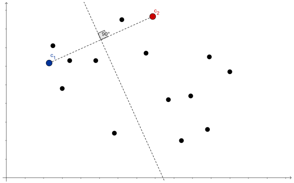
- Analogy-Based Expectation Equilibrium in which analogy classes represent how players cluster contingencies.
- But, how are analogy partitions picked?
- We propose using insights from psychology about categorization to put structure on those.
- Can also be seen as using insights from Machine Learning

Categorization and Prototypes

- Psychology: Categories and Prototypes, Eleanor Rosch, 1978
- Machine Learning: K-means clustering

Categorization and Prototypes: K-means Algorithm



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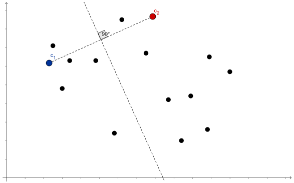
Random Initialization c_1, c_2

Categorization and Prototypes: K-means Algorithm



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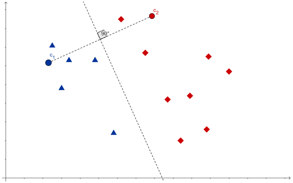

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Assigning data to nearest c

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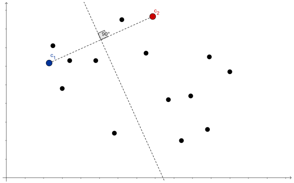

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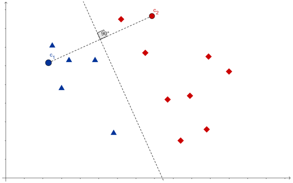

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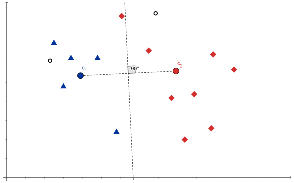

Iteration 1 - Step 2

Recomputing c_1, c_2

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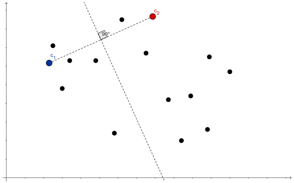

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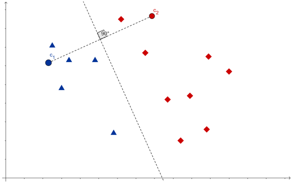

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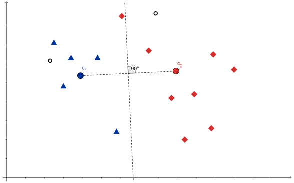

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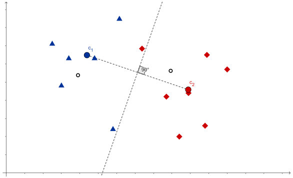

Iteration 2: Step 1 and 2

1) assign data, 2) recompute c

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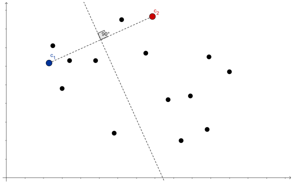

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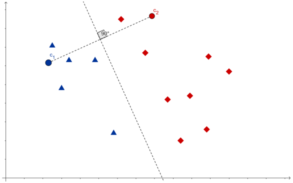

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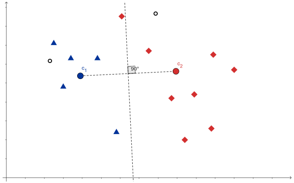

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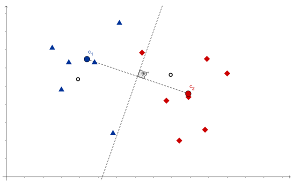

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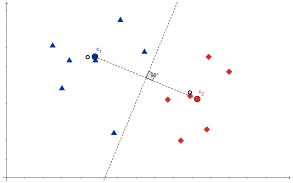

Iteration 3: Step 1 and 2

1) assign data, 2) recompute c. STOP

Categorization and Prototypes

Dataset: **distribution of behaviors** for various contingencies

↓

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$\downarrow$

Clustering to minimize prediction error: **analogy partition**

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Representative objects in each cluster: **Expectations**

$\downarrow$

New datapoints via best-reponses

$\downarrow$

...

- We study **steady states**

Contributions I

- Provide a conceptual framework (different notions of proximity, mixing, etc)
  $\rightarrow$ **Clustered ABEE**.
- Observe that steady states involving a single analogy partition may not exist
  $\rightarrow$ **Mixed extension**.
- Learning foundation

Contributions II

- Environments with **Self-Repelling** Categorizations
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  $\rightarrow$ **Heterogeneous** processing of same data
- Environments with **Self-Attractive** Categorizations
  $\rightarrow$ novel source of **multiplicity**
- Application to families of games with linear best-responses

Literature

Bounded Rationality: Jehiel 2005, Eyster and Rabin 2005, Jehiel and Koessler 2008, Spiegler 2016, Esponda and Pouzo 2016, Osborne and Rubinstein 1998, Jehiel and Mohlin 2024

Categories: Rosch 1978, Anderson 1991, Reed 1972, Fryer and Jackson 2008, Peski 2011, Mohlin 2014, Al-Najjar and Pai 2014, Mengel 2012, Gibbons Licalzi Warglien 2021
+ Miller 1956 (nb of items individuals can remember)

K-means: MacQueen 1967, Steinhaus 1957, Lloyd 1957, Banerjee Guo and Wang 2005, Banerjee Merugu Dhillon and Ghosh 2005

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- $u_i(a_i, a_j; \omega)$: utility of i in ω
- $\sigma_i = (\sigma_i(\omega))_{\omega \in \Omega}$: player i's strategy, where $\sigma_i(\omega) \in \Delta A_i$ describes i's behavior in ω.

ABEE (Jehiel 2005)

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A) σ_i is a **best-response** to β_i iff for all $\omega \in \Omega$, and all $a_i \in A_i$,

$$u_i(\sigma_i(\omega), \beta_i(\alpha_i(\omega)); \omega) \geq u_i(a_i, \beta_i(\alpha_i(\omega)); \omega)$$

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Definition

σ is an ABEE given An iff there is some β such that, for each i,

1. σ_i is a best-response to β_i
2. β_i is consistent with σ_j

Endogenous Clustering

- Data points for player i: $\sigma_j(\omega) \in \Delta A_j$ for all $\omega \in \Omega$

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  - $d(x, y) = \sum_{a_j} x(a_j) \ln \frac{x(a_j)}{y(a_j)}$: Kullback-Leibler divergence of x to y
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Observation: K-means algorithm converges for each above d (only **local** minimum):

$$\tilde{\beta}(c) \equiv \sum_{\omega \in c} p(\omega | c) \sigma_j(\omega) = \arg\min_q \sum_{\omega \in c} p(\omega | c) d(\sigma_j(\omega), q)$$

Clustered ABEE

Definition

An_i is **locally clustered** w.r.t. σ_j iff for all ω, all $\alpha'_i \in An_i$,

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(σ, An) is a locally (resp. globally) **Clustered ABEE** iff

1. σ is an ABEE given An
2. An is locally (resp. globally) clustered wrt σ

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Example

Three (equally likely) matching-pennies games: ω_x, $x = a, b, c$, where $0 < a < b < c < 2$

ω_x	L	R
U	$(1+x, 0)$	$(0, 1)$
D	$(0, 1)$	$(1, 0)$

Claim. *When $K_1 = 2$ and $K_2 = 3$ there is no Clustered ABEE*

In the paper: distributional ABEE, existence and learning foundations

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  - Stage 2: game ω is picked for each player i who has to make a choice in A_i (*Best response to representative point (ABEE)*)

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Self-Attractive Categorization: A Beauty Contest Illustration

- Fundamental value $\theta \in \Theta \subseteq \mathbb{R}$
- $U_i(a_i, a_{-i}; \theta) = -(1-r)(a_i - \theta)^2 - r(a_i - a_{-i})^2$
- Hence, $BR_\theta = (1-r)\theta + r\mathbb{E}(a_{-i})$
- Players observe θ and use K categories

Similar to Morris and Shin (2002) except that no private information (but Categorization approach to form expectation)

- Let $(\Theta_k)_{k=1}^K$ be an analogy partition used by players and let

$$\bar{\theta}_k = \mathbb{E}[\theta|\theta \in \Theta_k]$$

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Proposition

Take any partition $(\Theta_k)_{k=1}^K$ such that $\bar{\theta}_k$ are all different. Then for r sufficiently close to 1, any such partition together with the corresponding ABEE is a CABEE

Proof. When r is close to 1, actions in Θ_k are all close to $\bar{\theta}_k$ and the clustering of such datapoints lead to $(\Theta_k)_{k=1}^K$

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- In other words, categorizations are self-attractive in this environment
- Can possibly be related to echo chambers?
- Very different insight from Morris and Shin (uniqueness)

Conclusion

- Endogenized categorization in a simple framework that combines Psychology/Machine Learning and ABEE
- We also related (in the paper) self-attractive (*resp.* self-repelling) partitions to strategic complementarity (*resp.* substitutability) in a class of games with linear best-replies.
- Endogenize number K of clusters (e.g. requiring variance is below some threshold)?
- Study learning convergence?

THANK YOU!