UNIVERSITÄT BIELEFELD
Faculty of Business Administration and Economics

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# The Texas Shoot-Out under Knightian Uncertainty

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Center for Mathematical Economics
Bielefeld University

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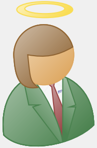
Rotterdam

# All good things . . .

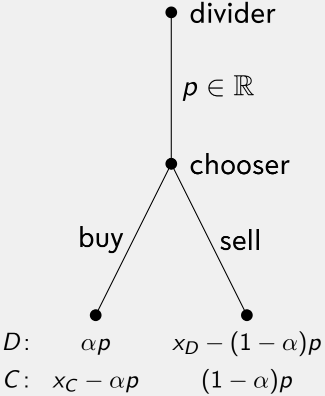


# All good things … (might) come to an end.

# Setting

- allocation of an indivisible object
- two agents:
  - a divider with shares $\alpha \in [0, 1]$
  - a chooser with shares $1 - \alpha$
- private valuations $x_D, x_C \in [0, 1]$

# The Texas Shoot-Out



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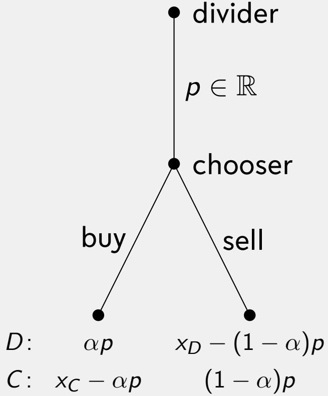
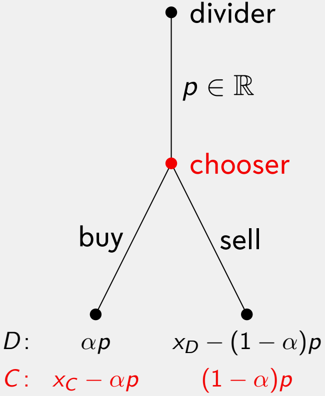



# The Texas Shoot-Out — The chooser's best reply




The chooser sells the company if and only if

$$x_C - \alpha p \leq (1-\alpha)p$$

$$\iff \quad x_C \leq p$$

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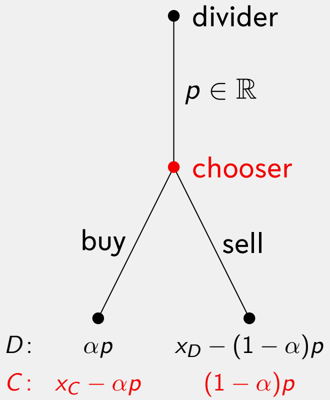


The chooser sells the company
if and only if

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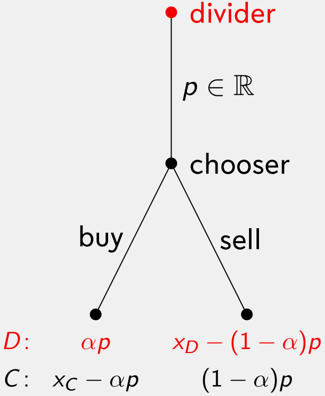
$$\iff \quad x_C \leq p$$

- dominant strategy
  — independent of $x_D$

# The Texas Shoot-Out — The divider's problem



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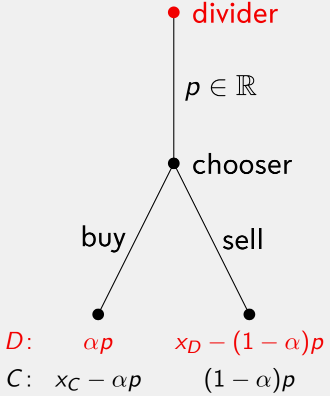


- favorable $p$ depends on the expected chooser's action
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  ▸ Bayes ▸ adversarial

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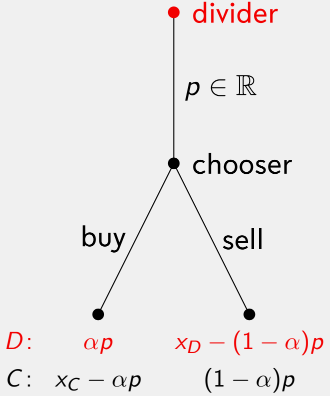


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- $\overline{p} = x_D$ (truth-telling) guarantees

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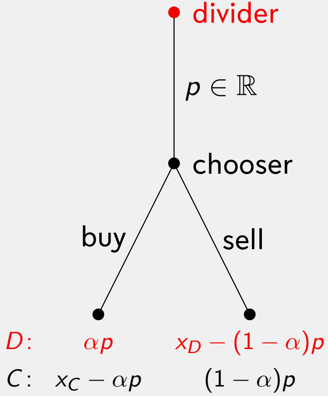


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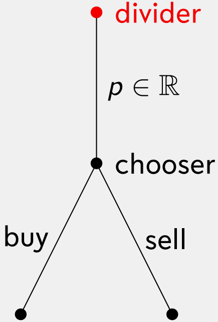


$D$: $\alpha p$ $x_D - (1-\alpha)p$

$C$: $x_C - \alpha p$ $(1-\alpha)p$

- favorable $p$ depends on the expected chooser's action
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- $\overline{p} = x_D$ (truth-telling) guarantees
  - a safe payoff $\alpha x_D$,
  - efficiency.

    (highest valuation gets company)

▸ puzzle

# Knightian Uncertainty

The divider wants to know $\mathbb{P}(x_C \leq p)$ - the probability that $p$ is accepted.

We consider the more general case in which only bounds for this CDF are known.

- ➤ robustness
- ➤ lack of information

# Knightian Uncertainty (distribution bands)

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- robustness
- lack of information

Fix two CDFs $G_0 \leq G_1$ and let divider consider the set of CDFs

$$\mathcal{G} = \{G \text{ CDF on } [0,1] \mid G_0(p) \leq G(p) \leq G_1(p) \text{ for all } p\}.$$

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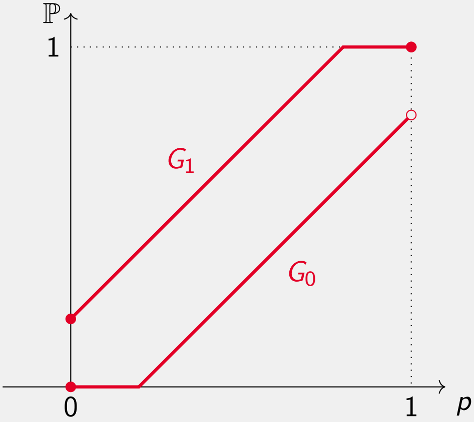
Maxmin expected utility (Gilboa and Schmeidler, 1989)

$$\pi(p \mid x_D) := \min_{G \in \mathcal{G}} \pi_G(p \mid x_D) \quad \text{with optimal prices} \quad m(x_D) := \arg\max_p \pi(p \mid x_D),$$

where $\pi_G(p \mid x_D) := (x_D - (1-\alpha)p) \cdot G(p) + \alpha p \cdot (1 - G(p))$.

# Illustration of $\mathcal{G}$



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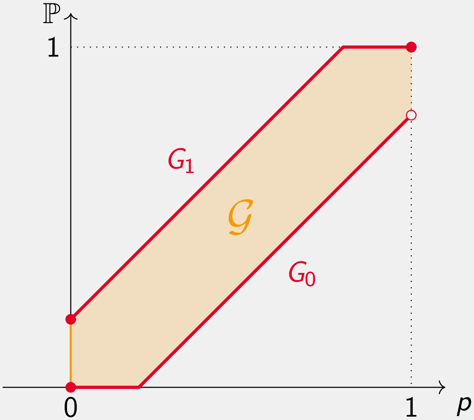
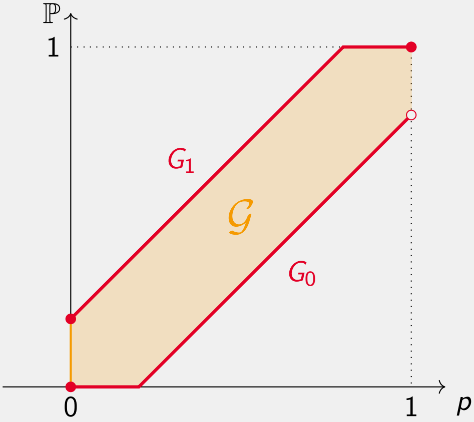


# A path from Bayes Nash to adversarial maxmin prices




$G_0 = G_1$

$\rightsquigarrow$ *Bayesian prices* (McAfee, 1992)

▸ Bayes

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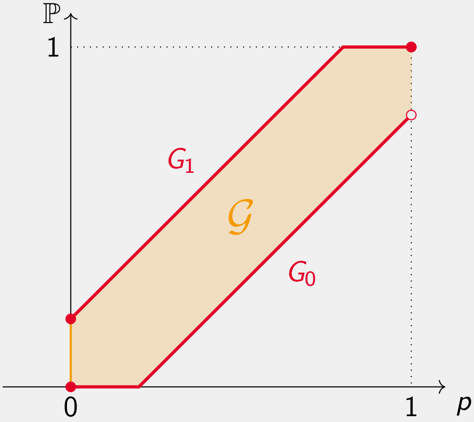



$G_0 = G_1$

⇝ *Bayesian prices* (McAfee, 1992)

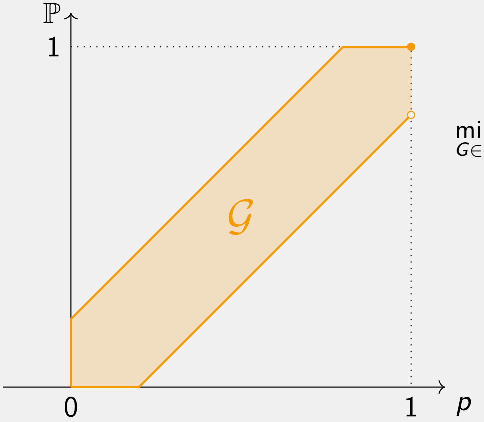
▸ Bayes

$G_1 - G_0 \equiv 1$

⇝ *adversarial maxmin prices* (van Essen and Wooders, 2020)

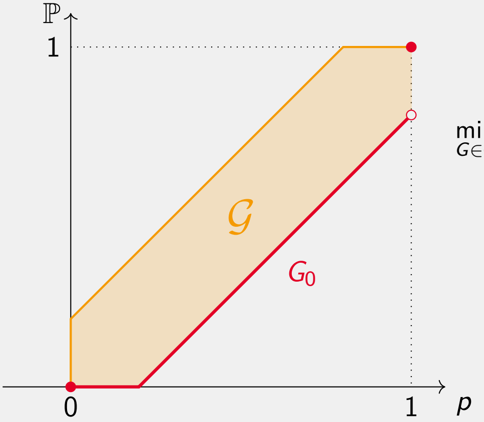
▸ adversarial

# Stochastic Dominance




$$\min_{G \in \mathcal{G}} \begin{array}{l} (x_D - (1-\alpha)p) \cdot G(p) \\ +\alpha p \cdot (1 - G(p)) \end{array}$$

# Stochastic Dominance

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1. $x_D > p$
$\rightsquigarrow G = G_0$

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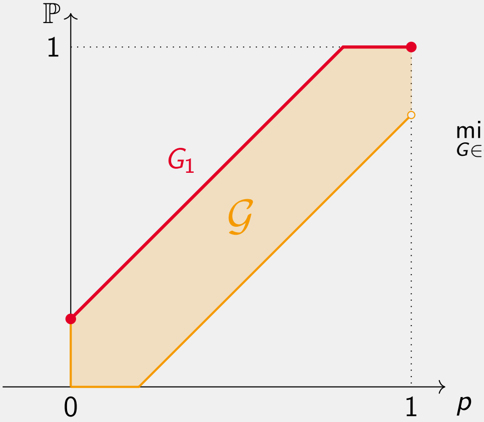
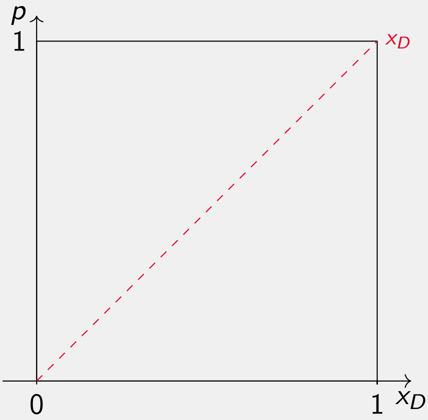



$$\min_{G \in \mathcal{G}} \begin{array}{l}(x_D - (1-\alpha)p) \cdot G(p) \\ +\alpha p \cdot (1 - G(p))\end{array}$$

1. $x_D > p$
   $\rightsquigarrow G = G_0$

2. $x_D < p$
   $\rightsquigarrow G = G_1$

# Prices under uncertainty - graphically



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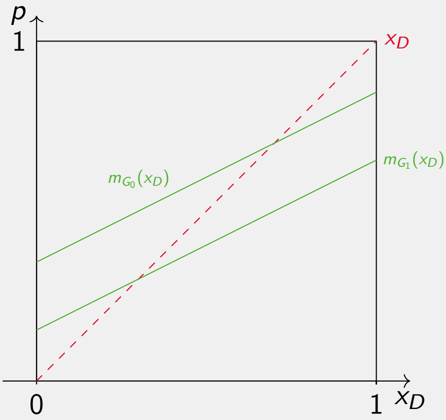

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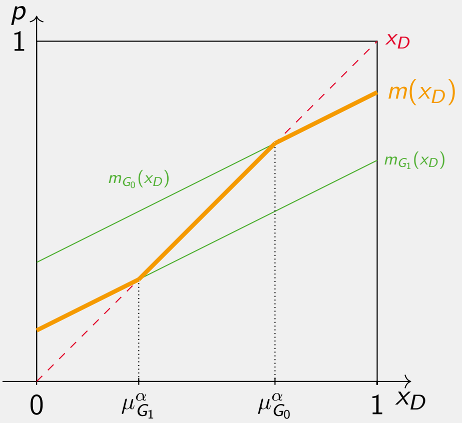


Figure 1: Optimal price announcement $m(x_D)$ for $\mathcal{G}$. $\mu^{\alpha}_{G}$ are the resp. $\alpha$-quantiles.

# Prices under uncertainty - analytically

## *Theorem*

*If* $G_0 \leq G_1$ *are piecewise continuously differentiable and* $\pi_{G_0}(\cdot \mid x_D), \pi_{G_1}(\cdot \mid x_D)$ *strictly quasi-concave, we have*

$$m(x_D) = \begin{cases} m_{G_1}(x_D) & \text{, if } \quad x_D < \mu^{\alpha}_{G_1}, \\ x_D & \text{, if } \mu^{\alpha}_{G_1} \leq x_D \leq \mu^{\alpha}_{G_0}, \\ m_{G_0}(x_D) & \text{, if } \mu^{\alpha}_{G_0} < x_D. \end{cases}$$

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▸ efficiency ▸ interim utility ▸ trigger game ▸ correlation ▸ assumptions ▸ proof sketch

# Allocative efficiency



Is the good given to the agent with the highest valuation?

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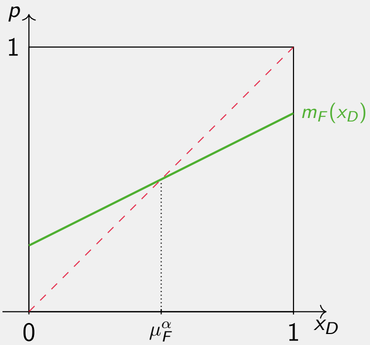
$\varepsilon = 0$




Fix a CDF $F$ and consider

$$\mathcal{G}(F, \varepsilon) = \{G \mid F(p - \varepsilon) \leq G(p) \leq F(p + \varepsilon)\}$$

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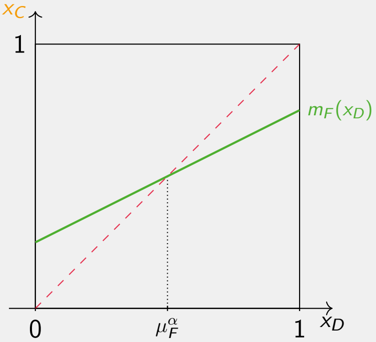


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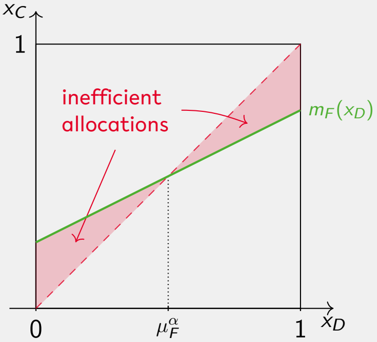


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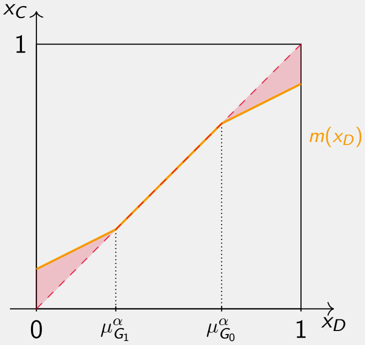
$\varepsilon > 0$

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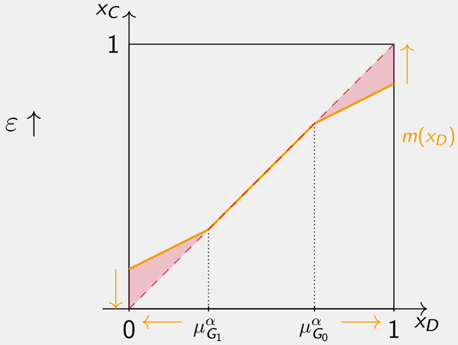


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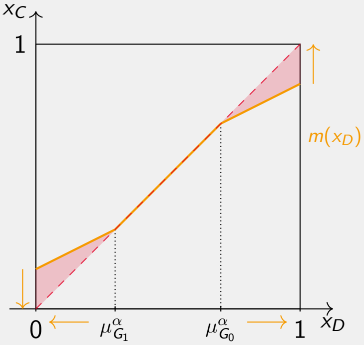
$\varepsilon \uparrow$




Fix a CDF $F$ and consider

$$\mathcal{G}(F,\varepsilon) = \{G \mid F(p-\varepsilon) \le G(p) \le F(p+\varepsilon)\}$$

**Proposition**

*The set of inefficient allocations is shrinking in uncertainty.*

For any valuation $x$ and facing $\mathcal{G}$ one can define the worst-case EU of being the divider resp. chooser $\Phi_D(x)$ resp. $\Phi_C(x)$.

# You better not cut the cake.

▸ register ▸ end

For any valuation $x$ and facing $\mathcal{G}$ one can define the worst-case EU of being the divider resp. chooser $\Phi_D(x)$ resp. $\Phi_C(x)$.

**Theorem**

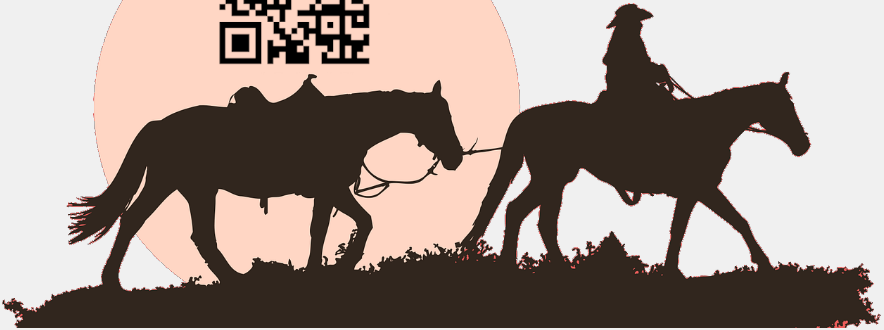
*Let $\alpha = \frac{1}{2}$. For all $x$*

$$\Phi_D(x) \leq \Phi_C(x),$$

*with strict inequality if and only if $G_1(x) - G_0(x) < 1$.*

▸ details ▸ pics ▸ $\alpha \neq 0.5$

# Thanks for your attention!



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# Bibliography I


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- McAfee, R Preston (1992). "Amicable divorce: Dissolving a partnership with simple mechanisms". In: *Journal of Economic Theory* 56.2, pp. 266–293.
- van Essen, Matt and John Wooders (2020). "Dissolving a partnership securely". In: *Economic Theory* 69.2, pp. 415–434.

# Bayesian Nash equilibrium

If the chooser's valuation is (believed to be) drawn from a CDF $F$

$$\pi_F(p \mid x_D) := (x_D - (1-\alpha)p) \cdot F(p) + \alpha p \cdot (1 - F(p)).$$

**Theorem** (McAfee, 1992): *SHRCs* on $F \implies \exists!\, m_F(x_D) \in \arg\max_p \pi_F(p \mid x_D)$

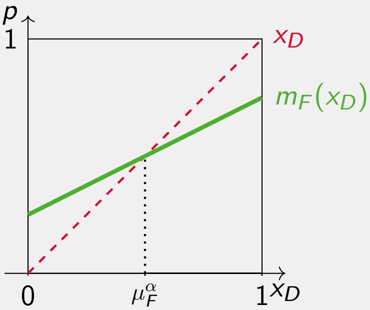


Figure 2: Bayesian prices for $F \sim \mathcal{U}([0,1])$, $\mu_F^\alpha$ the $\alpha$-quantile of $F$. ▸ calc.

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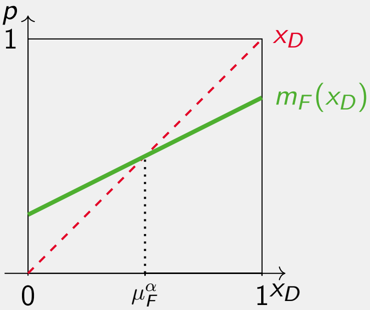


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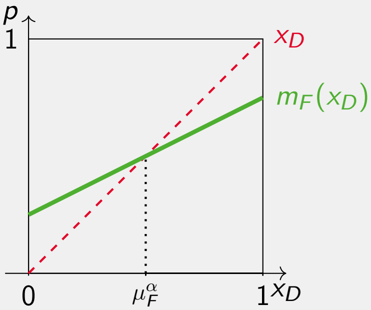


Figure 2: Bayesian prices for $F \sim \mathcal{U}([0,1])$, $\mu_F^\alpha$ the $\alpha$-quantile of $F$.

calc.

Let $F$ with strictly pos. density fulfill the *standard hazard rate conditions* (SHRCs)

$$\frac{\partial}{\partial x}\left(x + \frac{F(x)}{F'(x)}\right) \geq 0 \quad \text{and} \quad \frac{\partial}{\partial x}\left(x - \frac{1 - F(x)}{F'(x)}\right) \geq 0.$$

Then, there is a unique $m_f(x_D) \in \arg\max_p \pi_F(p \mid x_D)$. Furthermore,

$$m(x_D) \gtreqqless x_D \quad \text{if and only if} \quad x_D \lesseqqgtr \mu_F^\alpha,$$

where $\mu_F^\alpha$ is the $\alpha$-quantile of $F$, i.e., $F(\mu_F^\alpha) = \mathbb{P}_F(x_C \leq \mu_F^\alpha) = \alpha$.

Idea: The chooser manages to play always that action that hurts the divider the most – irrespective of their own losses.

- $\overline{p} = x_D$

# Adversarial maxmin price van Essen and Wooders, 2020

tree ill register

Idea: The chooser manages to play always that action that hurts the divider the most – irrespective of their own losses.

- $\overline{p} = x_D$

For all $(x_D, p)$ there is $x_C$ leading to the worst action for the divider, e.g., $x_C = x_D$:

$$\text{'Sell' is bad for divider} \iff x_D \leq p$$
$$\text{Sell' is played by chooser} \iff x_C \leq p.$$

If $\mathcal{G} = \{G \mid G \text{ is a CDF on } [0,1]\}$ (full uncertainty): $\rightsquigarrow \delta_{x_D} \in \mathcal{G}$

- maxmin price is full uncertainty price

back register

Optimal price announcement for $F \sim \mathcal{U}([0,1])$.

$$m(x_D) = \begin{cases} \frac{x_D+\alpha-\varepsilon}{2} & \text{, if } 0 \leq x_D < \alpha - \varepsilon, \\ x_D & \text{, if } \alpha - \varepsilon \leq x_D \leq \alpha + \varepsilon, \\ \frac{x_D+\alpha+\varepsilon}{2} & \text{, if } \alpha + \varepsilon < x_D \leq 1. \end{cases}$$

back

# Strict quasi-concavity

register

## Assumption

- $G_0$, $G_1$ piecewise continuously differentiable,
- $\pi_{G_0}(. \mid x_D)$, $\pi_{G_1}(. \mid x_D)$ strictly quasi-concave

thm

# Strict quasi-concavity

▸ register

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▸ thm

## *Lemma*

*The assumption is satisfied for $\mathcal{G}(F, \varepsilon)$ if $F$ fulfills the SHRCs*

$$\frac{\partial}{\partial x}\left(x + \frac{F(x)}{F'(x)}\right) \geq 0 \quad \textit{and} \quad \frac{\partial}{\partial x}\left(x - \frac{1 - F(x)}{F'(x)}\right) \geq 0.$$

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## Example

P.w. linear, truncated normal, triangular, classes of Beta distributions. ▸ pics

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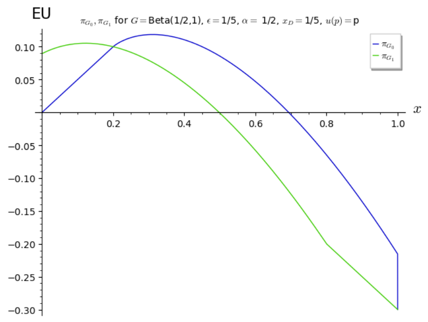
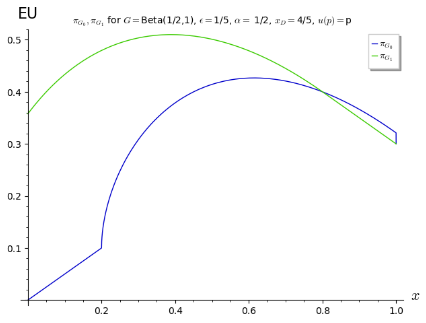
# Examples of $\pi_{G_0}, \pi_{G_1}$ for different distributions

register




back

# A puzzle?



“The possibility that the person naming the price can be forced either to buy or to sell keeps the first mover honest.”

Circuit Chief Judge Easterbrook

*Valinote v. Ballis, 295 F.3d 666 (7th Cir. 2002)*

“The cake-cutting mechanism has a disappointing performance, as it fails to reach ex post efficiency.”

McAfee, 1992

# Sketch of the proof

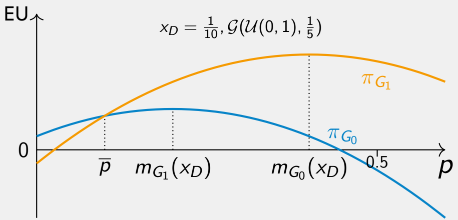
▸ register

Recall that $\pi_{G_0}(\cdot \mid x_D), \pi_{G_1}(\cdot \mid x_D)$ are strictly quasi-concave and intersect in $\overline{p} = x_D$. There are two main cases, depending on $x_D$:

# Sketch of the proof



Recall that $\pi_{G_0}(\cdot \mid x_D), \pi_{G_1}(\cdot \mid x_D)$ are strictly quasi-concave and intersect in $\overline{p} = x_D$. There are two main cases, depending on $x_D$:




- $x_D < m_{G_1}(x_D) < m_{G_0}(x_D)$

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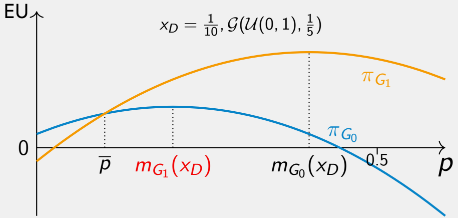


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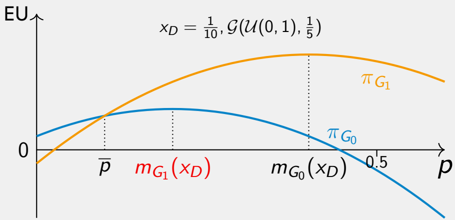


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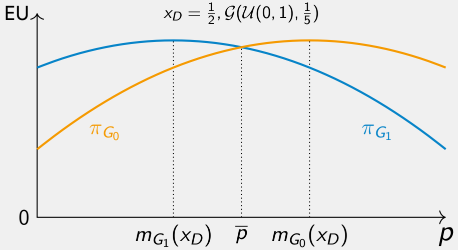


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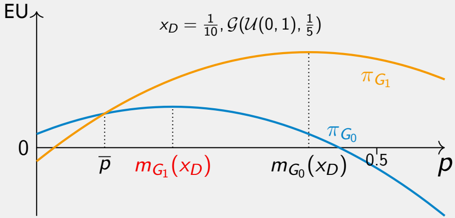


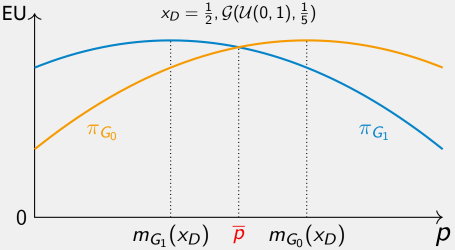


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- $m_{G_1}(x_D) < x_D < m_{G_0}(x_D)$

# Interim utility



$$\Phi_D(x) := \pi(m^\alpha(x) \mid x),$$
$$\Phi_C(x) := \min_{G \in \mathcal{G}} \mathbb{E}_G \left[\max\left\{x - (1-\alpha)m^{1-\alpha}(z), \alpha m^{1-\alpha}(z)\right\}\right].$$

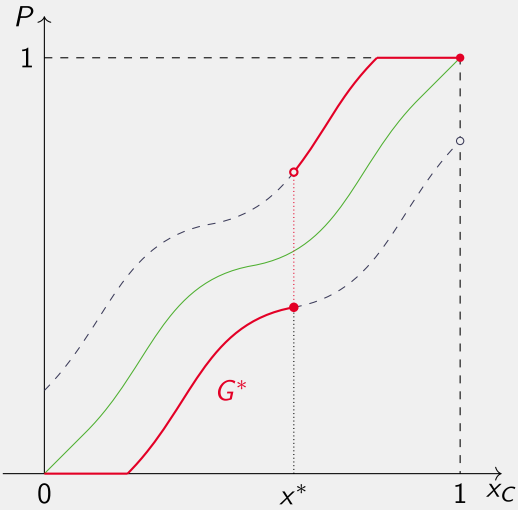
A chooser with valuation $x_C$ has worst case utility

$$\Phi_C(x) = \begin{cases} \mathbb{E}_{G_1}[m^{1-\alpha}(z)] & \text{, if } x_C < \min m^{1-\alpha}(z), \\ \mathbb{E}_{G^*(x_C)}[\max\{x - (1-\alpha)m^{1-\alpha}(z), \alpha m^{1-\alpha}(z)\}] & \text{, if } x_C \in \text{range}(m^{1-\alpha}), \\ \mathbb{E}_{G_0}[x - m^{1-\alpha}(z)] & \text{, if } \max m^{1-\alpha}(z) < x_C, \end{cases}$$

where $G^*(x)$ is the distribution function that switches from $G_0$ to $G_1$ at $x^* = \left(m^{1-\alpha}\right)^{-1}(x)$.

# $G^*$ illustration





# You better not cut the cake.



For any valuation $x$ and facing $\mathcal{G}$ one can define the worst-case EU of being the divider resp. chooser $\Phi_D(x)$ resp. $\Phi_C(x)$.

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**Theorem**

*Let $\alpha = \frac{1}{2}$. For all $x$*

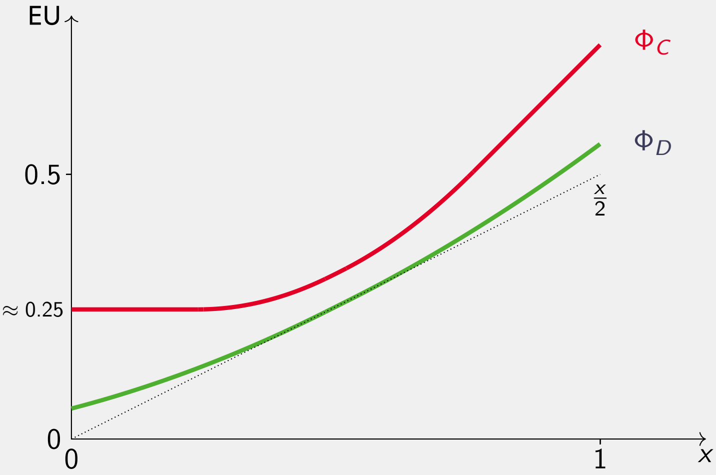
$$\Phi_D(x) \leq \Phi_C(x),$$

*with strict inequality if and only if $G_1(x) - G_0(x) < 1$.*
*($F \sim \mathcal{U}([0,1]) \rightsquigarrow x \in [1-\varepsilon, \varepsilon]$)*

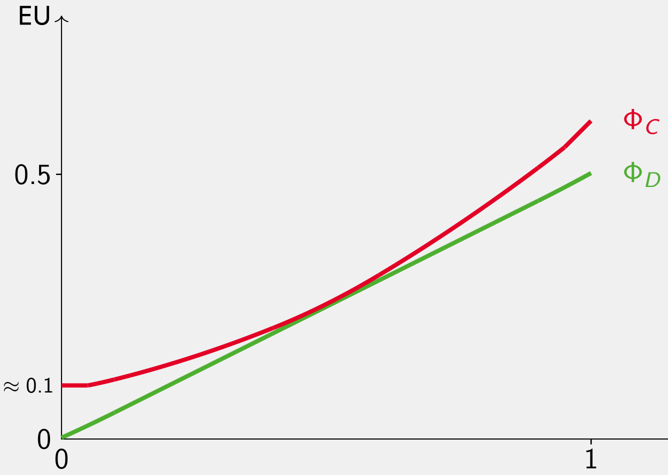
▸ $\alpha \neq 0.5$ ▸ pics

# Comparison for $\varepsilon = 0.02$



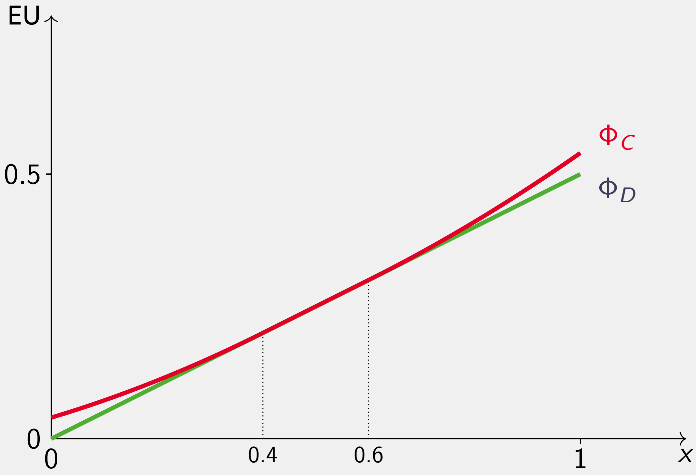


# Comparison for $\varepsilon = 0.4$



# Comparison for $\varepsilon = 0.6$





# Better let me cut the cake!

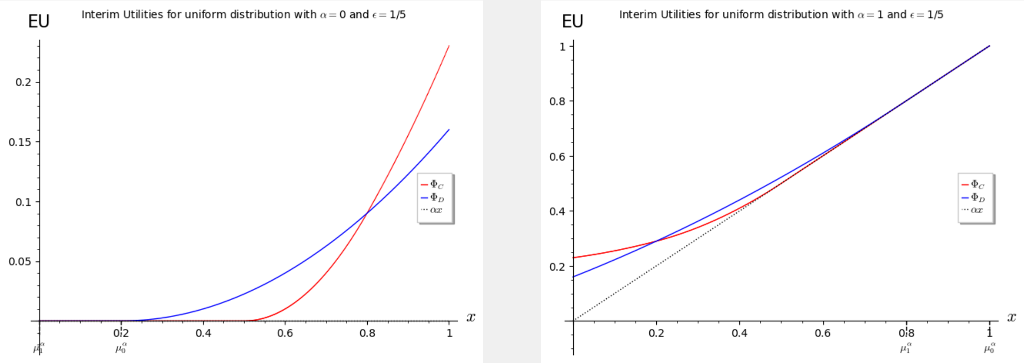



Figure 3: Not for all valuations an agent prefers to be the chooser if $\alpha \neq \frac{1}{2}$.

# A trigger game



Agent 1 with valuation $x_1$ and shares $\alpha = \alpha_1$ etc.

| $1 \setminus 2$ | T | DT |
|---|---|---|
| T | $\frac{1}{2}\Phi_D^{\alpha}(x_1) + \frac{1}{2}\Phi_C^{\alpha}(x_1), \frac{1}{2}\Phi_D^{1-\alpha}(x_2) + \frac{1}{2}\Phi_C^{1-\alpha}(x_2)$ | $\Phi_D^{\alpha}(x_1), \Phi_C^{1-\alpha}(x_2)$ |
| DT | $\Phi_C^{\alpha}(x_1), \Phi_D^{1-\alpha}(x_2)$ | $\alpha x_1, (1-\alpha)x_2$ |

# A trigger game



Agent 1 with valuation $x_1$ and shares $\alpha = \alpha_1$ etc.

| $1 \setminus 2$ | T | DT |
|---|---|---|
| T | $\frac{1}{2}\Phi_D^{\alpha}(x_1) + \frac{1}{2}\Phi_C^{\alpha}(x_1), \frac{1}{2}\Phi_D^{1-\alpha}(x_2) + \frac{1}{2}\Phi_C^{1-\alpha}(x_2)$ | $\Phi_D^{\alpha}(x_1), \Phi_C^{1-\alpha}(x_2)$ |
| DT | $\Phi_C^{\alpha}(x_1), \Phi_D^{1-\alpha}(x_2)$ | $\alpha x_1, (1-\alpha)x_2$ |

*DT* dominates *T* for *i* iff both

- $\Phi_C^{\alpha_i}(x_i) \geq \Phi_D^{\alpha_i}(x_i)$
- $\alpha_i x_i \geq \Phi_D^{\alpha_i}(x_i)$

# A trigger game

register end

Agent 1 with valuation $x_1$ and shares $\alpha = \alpha_1$ etc.

| $1 \setminus 2$ | T | DT |
|---|---|---|
| T | $\frac{1}{2}\Phi_D^{\alpha}(x_1) + \frac{1}{2}\Phi_C^{\alpha}(x_1), \frac{1}{2}\Phi_D^{1-\alpha}(x_2) + \frac{1}{2}\Phi_C^{1-\alpha}(x_2)$ | $\Phi_D^{\alpha}(x_1), \Phi_C^{1-\alpha}(x_2)$ |
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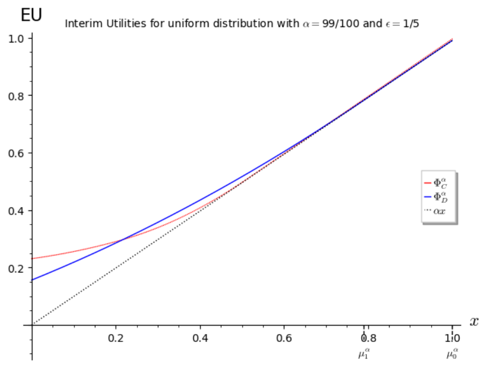
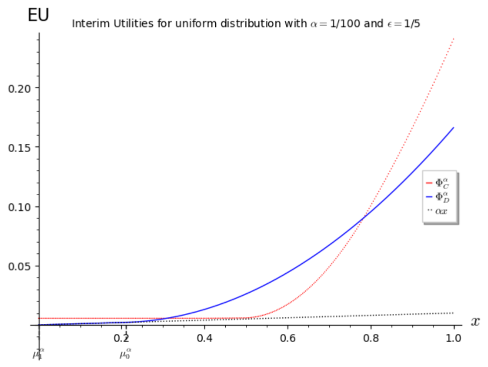
➤ If $\alpha = \frac{1}{2}$ then $(DT, DT)$ is an equilibrium for $x_i \in [\mu_{G_1}^{\alpha_i}, \mu_{G_0}^{\alpha_i}]$.

cut the cake

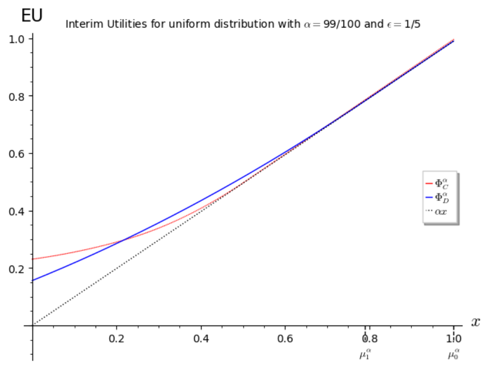
$(T, T)$ is an equilibrium iff $\Phi_D^{\alpha_i}(x_i) \geq \max\{\alpha_i x_i, \Phi_C^{\alpha_i}(x_i)\}$

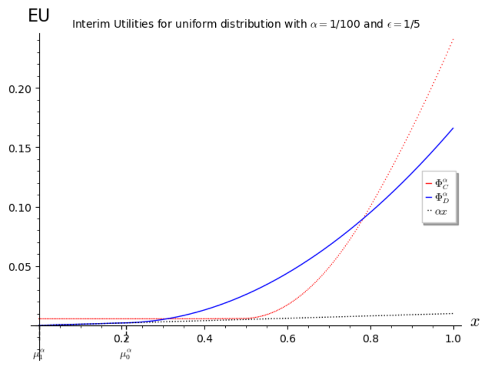



$\alpha_1 = 99\%, x_1 = 0.3, x_2 = 0.7, \varepsilon = \frac{1}{5}$.

$(T, D)$ is an equilibrium iff
$\Phi_D^{\alpha_1}(x_1) \geq \max\{\alpha_1 x_1, \Phi_C^{\alpha_1}(x_1)\}, \min\{(1-\alpha_1)x_2, \Phi_C^{1-\alpha_1}(x_2)\} \geq \Phi_D^{1-\alpha_1}(x_2)$




$\alpha_1 = 99\%, x_1 = 0.3, x_2 = 0.1, \varepsilon = \frac{1}{5}$.

# Correlation



In a partnership one might expect $x_C \approx x_D$.

register end

In a partnership one might expect $x_C \approx x_D$.

E.g., if $x_C$ is drawn from the triangular distribution with mode $x_D$.


Figure 4: PDF of $\text{Tri}^{x_D}$ for $x_D = 0.3$

# Prices under Correlation



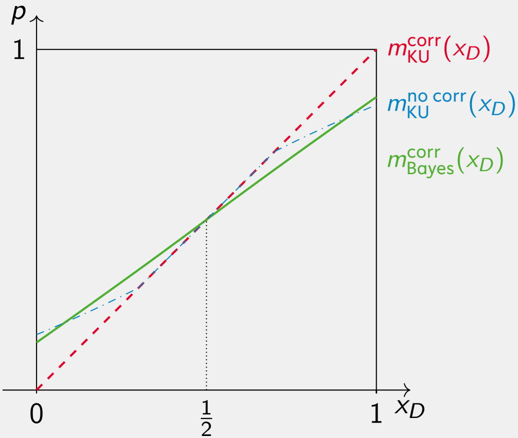



Figure 5: Prices for the cases of correlated and uncertain $\mathcal{G}(\text{Tri}^{x_D}, \frac{1}{5})$, the correlated Bayesian $\mathcal{G}(\text{Tri}^{x_D}, 0)$ and uncertain case without correlation $\mathcal{G}(\text{Tri}^{0.5}, \frac{1}{5})$.