Earnings Dynamics with On-the-job Learning

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Motivation

- Life-cycle earnings display significant deviation from normality. Capturing these features in quantitative applications is important as they affect models' ability to produce realistic consumption and wealth dynamics.
- Potential solution: increase statistical flexibility of a standard permanent/transitory earnings decomposition and fit to a broader set of moments for earnings
- Alternative: Use firm information in standard earnings decomposition to capture a part of the non-gaussian features

Our Paper

Research Question

How does latent firm heterogeneity contribute to earnings dynamics?

- We develop an earnings decomposition that allows for latent firm heterogeneity
- In Finnish data, mean and variance of permanent income shocks vary with latent firm heterogeneity, labelled as ”on-the-job learning opportunities” by Arellano-Bover and Saltiel (2024)
- Our model produces approx. 25% of skewness and excess kurtosis in earnings without specifically targeting these moments
- Persistent firm status influences individuals’ risk premiums, savings behavior, and wealth distribution

Related Literature

- Idiosyncratic earnings risk: Lilliard & Willis (1978), Lilliard & Weiss (1979), MaCurdy (1982), Storesletten, Telmer & Yaron (2004), Guvenen (2009), Hryshko(2012), Daly, Hryshko & Manovskii (2022) and Crawley, Blomhoff Holm & Tretvoll (2022)
- Non-gaussian dynamics: Geweke & Keane (2000), Druedahl & Munk-Nielsen (2020), De Nardi, Fella & Paz-Pardo (2020) and Guvenen, Karahan, Oskan & Song (2021)
- Learning environment heterogeneity: Gregory (2023) and Arellano-Bover and Saltiel (2024)

Extending the Earnings Decomposition

Individual's earnings (Baseline):

$$y_{it} = p_{it} + z_{it} \tag{1}$$

$$p_{it} = p_{it-1} + \xi_{it} \tag{2}$$

$$\phi(L)z_{it} = \theta(L)\varepsilon_{it}, \tag{3}$$

where

$$\xi_{it} \sim F, \quad \varepsilon_{it} \sim H \tag{4}$$

Extending the Earnings Decomposition

Individual's earnings (Extended):

$$y_{it} = p_{it} + z_{it} \tag{1}$$

$$p_{it} = p_{it-1} + \sum_{k=1}^{K} w_{k(j),it}\xi_{k(j),it} \tag{2}$$

$$\phi(L)z_{it} = \theta(L)\varepsilon_{it}, \tag{3}$$

where

$$\xi_{k(j),it} \sim F_{k(j)}, \quad \varepsilon_{it} \sim H \tag{4}$$

and

$$w_{k(j),it} = \begin{cases} 1, & \text{current firm } \boldsymbol{j} \text{ in group } \boldsymbol{k} \\ 0, & \text{otherwise} \end{cases} \tag{5}$$

Variance of the Permanent Income Shocks

The variance of the permanent income shocks in the extended version:

$$\sigma_\xi^2 = \sigma_\mu^2 + \sigma_\upsilon^2, \tag{6}$$

where

$$\sigma_\mu^2 = \sum_{k=1}^{K} \pi_k \mu_k^2 \qquad \text{(Variation from the group-specific means)} \tag{7}$$

$$\sigma_\upsilon^2 = \sum_{k=1}^{K} \pi_k \sigma_k^2 \qquad \text{(Variation from the group-specific variances)} \tag{8}$$

$$\pi_k = \Pr(w_{k(j),it} = 1) \qquad \text{(Probablity of belonging to a certain group)} \tag{9}$$

Estimating Model Parameters

- We decompose by latent firm characteristic (on-the-job learning opportunities) forming groups K from firm indexes J, with $|J| > |K|$
- Data needed: individual i's firm j at time t
- Two-step procedure:
  1. Identify firm groups from income differences using a version k-means method (Bonhomme, Lamadon & Manresa, 2019)
  2. Recover parameters of the extended income process with moment matching

Finding the Firm Groups with K-means Clustering

- Goal: Identify groups $k(j)$ for all firms ($obs > 15$) to get data on $w_{k(j),it}$ and π_k
- The change in earnings for an individual Δy_{it} is determined by a group specific $\xi_{k(j),it}$ plus some noise

$$\implies \quad \Delta y_{it} \text{ is informative of } k(j)$$

- Method uses firm's distribution of wage differences to find grouping K where:
  1. Centroid distributions for each group k are far apart
  2. Earnings growth distributions of firms in group k are close to the centroid distribution

$\implies$ Wage growth appears similar within groups, different across groups

Data

- Finnish employer-employee data 1997-2018

1st stage:

- Firm groups are determined from a sample containing all firms with $n > 15$, which results to a sample size of $5.7M$

2nd stage:

- Population subsample selection criteria:
  - 35-50 year old male
  - work for private sector
  - work for companies we can group
- Sample size for income process estimation is around $960k$

Results with MA(1) Shock and Three Groups

Benchmark Income Process					
Transitory Component, z_t		Permanent Component, p_t			
θ	σ_ε^2	σ_ξ^2			
0.207	0.0100	0.0064			
(0.004)	(0.0001)	(0.0001)			
Extended Income Process					
Transitory Component, z_t		Permanent Component, p_t			
θ	σ_ε^2	σ_ξ^2	=	σ_μ^2 +	σ_υ^2
0.207	0.0101	0.0064		0.0000	0.0064
(0.004)	(0.0001)	(0.0001)		(0.0000)	(0.0001)

Further Decomposition of σ_ξ^2

Permanent Component, p_t			
σ_ξ^2	=	σ_μ^2 +	σ_υ^2
0.0064		0.0000	0.0064
(0.0001)		(0.0000)	(0.0001)
k	π_k	μ_k	σ_k^2
1	0.183	−0.0076	0.0098
	(0.001)	(0.0002)	(0.0004)
2	0.656	0.0001	0.0027
	(0.002)	(0.0001)	(0.0001)
3	0.161	0.0082	0.0174
	(0.001)	(0.0003)	(0.0004)

▸ More Groups ▸ Transitory component as AR(1)

Potential Quantitative Impact

- Simulating the income paths reveals that the extended process can generate skewness and kurtosis when the firm group is persistent. ▸ details
- Individuals' risk premium under the extended income process is higher. ▸ details
- Evolution of assets over the life-cycle change significantly under extended income process. ▸ details
- Wealth distribution in illustrative OLG setting has thicker right tail. ▸ details

Conclusion

- A method to incorporate latent firm heterogeneity in standard earnings decomposition.
- In Finnish data, variances of permanent income shocks differ across firm groups and the method captures non-Gaussian features.
- The extended income process likely has impact on key quantitative dimensions.
- The persistence of firm groups play a role for the quantitative implications.

Appendix

Earnings Realizations for a Single Cohort

- Assumptions:
  - Normality of shocks (previous results didn't require this)
  - Starting value: $\mathbf{0}$
  - Group changes with prob. $\mathbf{0.05}$ (when $K = 3$; consistent with data)
  - Group redrawn from stationary distribution
- Implication: Individuals firm group status is persistent.

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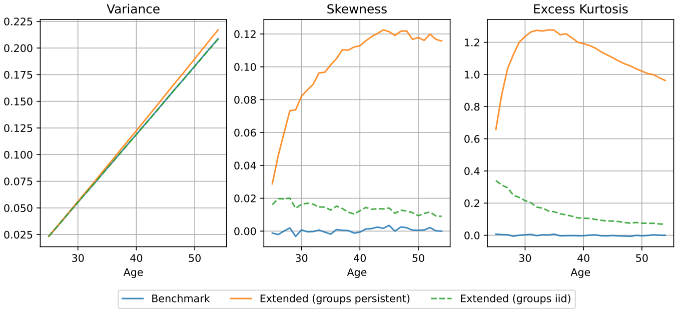
Key Simulation results

- Extended process with persistent groups increases the number of ”extreme paths” for earnings and this happens only when groups are persistent

- Majority of action relates to third and fourth central moments in levels:



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Stochasticity Aversion and Risk Premium

- Risk premium: how much lower consumption, w.r.t. average, individual would accept to get rid of earnings risk
- To get simple Arrow-Pratt type of measure, we assume $c_t = y_t$ and use fourth order approximation to get:

$$\begin{aligned}\text{risk premium} &\approx \text{variance aversion} \\ &+ \text{neg. skewness aversion} \\ &+ \text{kurtosis aversion}\end{aligned}$$

- Key parameters for the illustrative calculation: risk aversion γ, discount factor β, and length of working age T

▸ equations ▸ back

Risk Premium

Component	Benchmark	Extended (groups persistent)	Extended (groups iid)
Variance	8.7	9.0	8.7
Neg. Skewness	0.0	−0.3	0.0
Excess Kurtosis	0.0	1.2	0.1
Risk Premium	8.7	9.9	8.8

Risk premium calculation, with $\gamma = 2$, $\beta = 0.95$ and $T = 30$. The number are presented in percents.

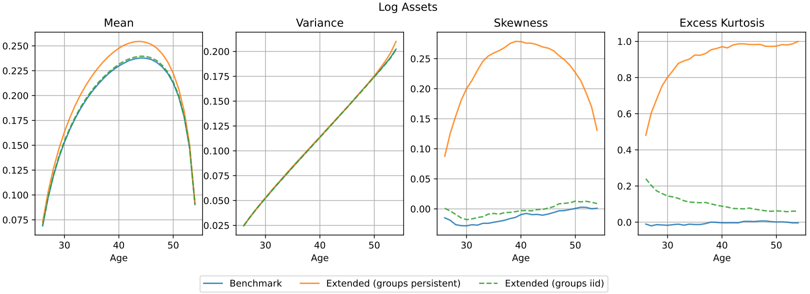
▸ $\gamma = 10$ ▸ full model ▸ back

Consumption-Savings Problem: Preliminaries

- Standard consumption-savings problem to analyze the effects of the extended process
- Main features:
  - CRRA preferences, $r = 0.04$ and $\beta = 0.95$
  - finite lifetime of 30 years
  - group redrawn with prob. $\phi = 0.098$ from stationary distribution
  - credit constraint $a \geq 0$

▸ back

Assets over Life-cycle



▸ Consumption ▸ Assets ▸ Log Consumption ▸ back

Wealth Distribution

- In OLG setting with constant generation size the right tail of asset distribution widens

Ratio	Benchmark	Extended (groups persistent)	Extended (groups iid)
P90/P50	1.56	1.55	1.56
P99/P50	2.37	2.64	2.39

▸ back

Additional Results: MA(1)

	Benchmark	Three Groups	Five Groups	Ten Groups
σ_ε^2	0.0100	0.0101	0.0101	0.0101
	(0.0001)	(0.0001)	(0.0001)	(0.0001)
σ_ξ^2	0.0064	0.0064	0.0064	0.0064
	(0.0001)	(0.0001)	(0.0001)	(0.0001)
θ	0.207	0.207	0.207	0.208
	(0.004)	(0.004)	(0.004)	(0.004)
σ_μ^2	-	0.0000	0.0000	0.0000
		(0.0000)	(0.0000)	(0.0000)
σ_υ^2	-	0.0064	0.0064	0.0063
		(0.0001)	(0.0001)	(0.0001)

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Additional Results: MA(1), Decomposition of σ_ξ^2

	Three Groups			Five Groups			Ten Groups		
Group	σ_k^2	μ_k	π_k	σ_k^2	μ_k	π_k	σ_k^2	μ_k	π_k
1	0.0098	-0.0076	0.183	0.0299	-0.0112	0.042	0.0169	-0.0181	0.010
	(0.0004)	(0.0002)	(0.001)	(0.0011)	(0.0007)	(0.001)	(0.0018)	(0.0014)	(0.000)
2	0.0027	0.0001	0.656	0.0045	-0.0064	0.171	0.0029	-0.0064	0.061
	(0.0001)	(0.0001)	(0.002)	(0.0003)	(0.0002)	(0.001)	(0.0005)	(0.0004)	(0.001)
3	0.0174	0.0082	0.161	0.0012	0.0012	0.393	0.0367	-0.0085	0.025
	(0.0004)	(0.0003)	(0.001)	(0.0002)	(0.0001)	(0.002)	(0.0014)	(0.0009)	(0.001)
4	-	-	-	0.0084	0.0011	0.352	0.0098	-0.0067	0.108
				(0.0002)	(0.0002)	(0.002)	(0.0005)	(0.0003)	(0.001)
5	-	-	-	0.0215	0.0168	0.041	0.0021	-0.0038	0.165
				(0.0009)	(0.0007)	(0.001)	(0.0003)	(0.0002)	(0.001)
6	-	-	-	-	-	-	0.0049	0.0001	0.284
							(0.0003)	(0.0002)	(0.002)
7	-	-	-	-	-	-	0.0005	0.0023	0.167
							(0.0003)	(0.0002)	(0.001)
8	-	-	-	-	-	-	0.0158	0.0075	0.097
							(0.0006)	(0.0004)	(0.001)
9	-	-	-	-	-	-	0.0039	0.0099	0.072
							(0.0005)	(0.0004)	(0.001)
10	-	-	-	-	-	-	0.0345	0.0262	0.011
							(0.0022)	(0.0018)	(0.000)

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Additional Results: AR(1)

	Benchmark	Three Groups	Five Groups	Ten Groups
σ_ε^2	0.0118	0.0118	0.0118	0.0119
	(0.0001)	(0.0001)	(0.0001)	(0.0001)
σ_ξ^2	0.0051	0.0050	0.0050	0.0050
	(0.0001)	(0.0001)	(0.0001)	(0.0001)
ϕ	0.300	0.302	0.301	0.303
	(0.005)	(0.006)	(0.005)	(0.006)
σ_μ^2	-	0.0000	0.0000	0.0000
		(0.0000)	(0.0000)	(0.0000)
σ_υ^2	-	0.0050	0.0050	0.0049
		(0.0001)	(0.0001)	(0.0001)

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Additional Results: AR(1), Decomposition of σ_ξ^2

	Three Groups			Five Groups			Ten Groups		
Group	σ_k^2	μ_k	π_k	σ_k^2	μ_k	π_k	σ_k^2	μ_k	π_k
1	0.0084	-0.0076	0.183	0.0286	-0.0112	0.042	0.0155	-0.0181	0.01
	(0.0004)	(0.0002)	(0.001)	(0.0011)	(0.0007)	(0.001)	(0.0018)	(0.0015)	(0.000)
2	0.0014	0.0001	0.656	0.0031	-0.0064	0.171	0.0015	-0.0064	0.061
	(0.0001)	(0.0001)	(0.002)	(0.0003)	(0.0003)	(0.001)	(0.0005)	(0.0004)	(0.001)
3	0.0160	0.0082	0.161	-0.0001	0.0012	0.393	0.0354	-0.0085	0.025
	(0.0004)	(0.0003)	(0.001)	(0.0002)	(0.0001)	(0.002)	(0.0015)	(0.0009)	(0.001)
4	-	-	-	0.0070	0.0011	0.352	0.0084	-0.0067	0.108
				(0.0002)	(0.0002)	(0.002)	(0.0005)	(0.0003)	(0.001)
5	-	-	-	0.0201	0.0168	0.041	0.0007	-0.0038	0.165
				(0.0009)	(0.0007)	(0.001)	(0.0004)	(0.0002)	(0.001)
6	-	-	-	-	-	-	0.0035	0.0001	0.284
							(0.0003)	(0.0002)	(0.002)
7	-	-	-	-	-	-	-0.0009	0.0023	0.167
							(0.0003)	(0.0002)	(0.001)
8	-	-	-	-	-	-	0.0144	0.0075	0.097
							(0.0005)	(0.0004)	(0.001)
9	-	-	-	-	-	-	0.0025	0.0099	0.072
							(0.0005)	(0.0004)	(0.001)
10	-	-	-	-	-	-	0.0331	0.0262	0.011
							(0.0022)	(0.0018)	(0.000)

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Stochasticity Aversion and Risk Premium

- Risk premium π can be solved from

$$\sum_{t=0}^{T} \beta^t u(\bar{c}_t(1-\pi)) = \mathrm{E}_0\left[\sum_{t=0}^{T} \beta^t u(\bar{c}_t(1+\delta_t))\right], \tag{10}$$

  where $\delta_t = \frac{c_t - \bar{c}_t}{\bar{c}_t}$.

- By applying Taylor approximation around reference level this gives the following approximation for the premium:

$$\pi \approx \left(\frac{\gamma}{2}\left(\sum_{t=0}^{T} \beta^t \mathrm{E}_0[\delta_t^2]\right) - \frac{(\gamma+1)\gamma}{6}\left(\sum_{t=0}^{T} \beta^t \mathrm{E}_0[\delta_t^3]\right) + \frac{(\gamma+2)(\gamma+1)\gamma}{24}\left(\sum_{t=0}^{T} \beta^t \mathrm{E}_0[\delta_t^4]\right)\right)\left(\frac{1-\beta}{1-\beta^T}\right) \tag{11}$$

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Risk Premium

Component	Benchmark	Extended (groups persistent)	Extended (groups iid)
Variance	43.7	44.9	43.7
Skewness	0.0	−6.2	−0.7
Kurtosis	−0.1	68.4	6.1
Risk Premium	43.6	107.1	49.1

Risk premium calculation, with $\gamma = 10$, $\beta = 0.95$ and $T = 30$. The number are presented in percents.

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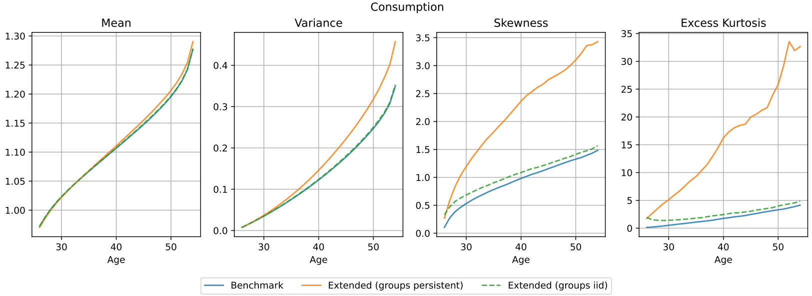
Risk Premium from Consumption-Savings Problem

- Risk premium: How much lower consumption w.r.t. average individual would accept to get rid of earnings risk, given that they hold average assets and income

Age Span	Benchmark	Extended (groups persistent)	Extended (groups iid)
25-55	5.9	6.1	5.9
35-55	4.4	4.8	4.5
45-55	2.4	2.9	2.4

▸ back

Consumption over Life-cycle



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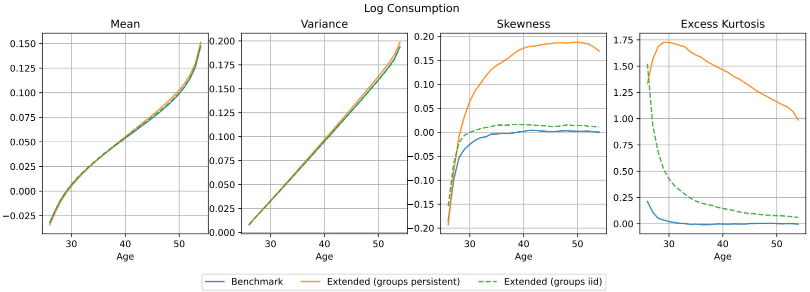
Assets over Life-cycle

▸ back

Log Consumption over Life-cycle



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Can We Predict Firm Group from Observables?

- We use multinomial logit and random forest to predict firm's group using observables
- Using average wages, share of educated workers, turnover, number of employees and industry codes we get success rate of 47% with logit and 49% with random forest

How Realistic is the Exogeneity Assumption of the Transitions?

		Class in the next period		
		1	2	3
Class	1	0.864	0.111	0.024
in this	2	0.010	0.979	0.011
period	3	0.020	0.067	0.913

Table 1: Empirical transition probability matrix for three classes

		35, university			50, university		
		Class in the next period					
		1	2	3	1	2	3
Class	1	0.850	0.126	0.024	0.826	0.141	0.033
in this	2	0.009	0.982	0.009	0.008	0.981	0.011
period	3	0.021	0.079	0.900	0.015	0.064	0.921
		35, non-university			50, non-university		
		Class in the next period					
		1	2	3	1	2	3
Class	1	0.886	0.094	0.020	0.866	0.106	0.028
in this	2	0.013	0.978	0.010	0.011	0.977	0.012
period	3	0.026	0.071	0.903	0.019	0.058	0.923

Table 2: Conditional empirical transition probability matrix for three classes