

# Credit Rationing in Unsecured Debt Markets

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# Outline

- 1 Motivation and Results
- 2 Competitive Equilibrium
- 3 Social Planner
- 4 Tightening Credit Constraints

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# Financial Constraints and Inefficiencies

- Lack of commitment
- Are competitive markets (constrained) inefficient?
- Rationale for policy interventions
- Two strands of the literature provide different answers

# Debt Secured by Collateral

- Borrowing constraints depend directly on relative prices of assets/goods
- Private agents fail to internalize the GE effects of their individual decisions on market prices, and therefore on debt constraints
- This may lead to excessive borrowing in equilibrium
- Kiyotaki and Moore (1997), Aiyagari and Gertler (1999), Caballero and Krishnamurthy (2001, 2003), Lorenzoni (2008), Farhi, Golosov and Tsyvinski (2009), Jeanne and Korinek (2010), Bianchi (2011), Bianchi and Mendoza (2011), and Dávila and Korinek (2018)

# Unsecured Debt

- Debt levels self-enforced by the threat of default punishment
- Borrowing subject to the largest debt limits compatible with repayment incentives
  - ▶ financial constraints depend on market prices
- Contingent bonds + single commodity + default-punishment=autarky  $\implies$  constrained efficiency
- Kehoe and Levine (1993), Alvarez and Jermann (2000, 2001), Bloise and Reichlin (2011)

# Unsecured Debt with Weaker Punishment

- Bulow and Rogoff (1989) assume defaulting agents cannot borrow but can save
- Krueger and Uhlig (2006) provide a micro-foundation for this default punishment
  - ▶ dynamic equilibrium risk-sharing contracts between profit-maximizing intermediaries and agents facing idiosyncratic income uncertainty

# Equilibrium with Bubbly Debt Limits

- Hellwig and Lorenzoni (2009)
  - ▶ GE: competitive debt markets
  - ▶ not-too-tight debt limits with Bulow and Rogoff's default punishment
- Equilibrium with positive levels of debt can be sustained
- Largest self-enforcing debt limits must form a bubble



# Asset Price Bubbles

- Asset bubbles can be harmful
  - ▶ distort price signals
  - ▶ cause a misallocation of resources
- Dot-com boom (late 1990s) and the housing boom (mid 2000s)
- Call for policy intervention to stem asset bubbles from arising

# Bubbles under Financial Frictions

- Asset price bubbles can help smooth financial frictions and mitigate liquidity problems
  - ▶ Caballero and Krishnamurthy (2006), Farhi and Tirole (2021), Miao and Wang (2012, 2015, 2018), Martin and Ventura (2012, 2016, 2018), Xavier (JEEA 2023)
- Debt bubbles
  - ▶ Domeij and Ellingsen (2018), Hellwig and Lorenzoni (2019), Brunnermeier, Merkel, and Sannikov (2022), Kocherlakota (2022, 2023, 2023)

# Questions

- Are bubbly debt limits the most efficient way to mitigate the lack of commitment?
- Why should we allow borrowers to take on maximum self-enforcing debt?
- What happens if we impose self-enforcing but possibly too tight debt limits?
- What would a social planner do?

# Contributions

- Construct a social planner program
- Show it is equivalent to a Ramsey program where the social planner maximizes among competitive equilibrium outcomes with self-enforcing debt limits (**possibly too-tight**)
- Analyze FOCs in a baseline economy and show that *laissez-faire* equilibria cannot be the outcome of a social planner
- Illustrate how tightening debt constraints can help Pareto improve the *laissez-faire* equilibrium

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# Income Risks

- Finite set  $I$  of agents
- Stochastic income  $y^i(s^t) > 0$  available at every event  $s^t$
- Life-time continuation utility

$$U^i(c|s^t) := u^i(c(s^t)) + \sum_{\tau \geq 1} \beta^\tau \sum_{s^{t+\tau} \succ s^t} \pi(s^{t+\tau}|s^t) u^i(c(s^{t+\tau}))$$

# Dynamic Trading with Financial Constraints

- Dynamic trading of contingent claims
- Fix a date- $t$  event  $s^t$
- Solvency constraint

$$c^i(s^t) + \sum_{s^{t+1} \succ s^t} q(s^{t+1}) a^i(s^{t+1}) \leq y^i(s^t) + a^i(s^t)$$

- Financial constraints

$$a^i(s^{t+1}) \geq -D^i(s^{t+1}), \quad \text{for every } s^{t+1} \succ s^t$$

# Self-Enforcing Debt Limits

- Default punishment
  - ▶ assets seized upon default
  - ▶ no access to credit
  - ▶ agents retain the ability to save
- self-enforcing: when  $a^i(s^t) \geq -D^i(s^t)$ , the agent prefers to repay debt
- not-too-tight: when  $a^i(s^t) = -D^i(s^t)$ , the agent is indifferent between repaying or defaulting



# Competitive Equilibrium

## Definition

A **self-enforcing** equilibrium is a collection  $(q, (c^i, a^i, D^i)_{i \in I})$  such that

- $(c^i, a^i)$  is optimal in  $B^i(D^i, a^i(s^0) | s^0)$
- $D^i$  is self-enforcing
- markets clear:  $\sum_{i \in I} c^i = \sum_{i \in I} y^i$  and  $\sum_{i \in I} a^i = 0$
- We use the term **laissez-faire** equilibrium when  $D^i$  is self-enforcing and **not-too-tight**
- In a laissez-faire eq. debt limits satisfy exact roll-over:

$$D^i(s^t) = \sum_{s^{t+1} \succ s^t} q(s^{t+1}) D^i(s^{t+1})$$

# Laissez-Faire: First Order Conditions

- Principle of optimality implies

$$\underbrace{a^i(s^t) > -D^i(s^t)}_{\text{debt limit}} \iff \underbrace{U^i(c^i|s^t) > V_{\text{def}}^i(q|s^t)}_{\text{participation constraint}}$$

- Euler equations

$$q(s^t) \geq q^i(s^t) := \beta \pi(s^t | \sigma(s^t)) \frac{u'(c^i(s^t))}{u'(c^i(\sigma(s^t)))}$$

with equality if  $U^i(c^i|s^t) > V_{\text{def}}^i(q|s^t)$

- Without any loss of generality,

$$q(s^t) = \max_{i \in I} q^i(s^t)$$

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# Efficiency when Autarky is the Default Punishment

- Alvarez and Jermann (2000,2001)
- Assume  $V_{\text{def}}^i(q|s^t) = U^i(y^i|s^t)$ 
  - ▶ every laissez-faire equilibrium cannot be Pareto dominated by a competitive equilibrium with self-enforcing debt limits
  - ▶ every laissez-faire equilibrium is the solution of a social planner problem
  - ▶ every solution to a social planner problem is a laissez-faire equilibrium
- Do these results remain valid when agents can save after default?

# Constrained Efficiency: Social Planner Problem

- A social planner seeks for efficient (Pareto optimal) consumption allocations among those that are **socially feasible**

- $(c^i)_{i \in I}$  is socially feasible when

Ⓐ markets clear, that is,  $\sum_{i \in I} c^i = \sum_{i \in I} y^i$

- Ⓑ participation constraints are satisfied

$$U^i(c^i | s^t) \geq V_{\text{def}}^i(q | s^t), \quad \text{for all } s^t \succeq s^0$$

- Ⓒ the price  $q$  is given by

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# Implementability

## Proposition

Every socially feasible consumption allocation can be implemented as a competitive equilibrium with self-enforcing debt limits

- Debt limits can be too tight



# Generalized KKT Conditions

FOC for  $c^i(s^t)$ <sup>1</sup>

$$\mu(s^t) = \beta^t \pi(s^t) u'(c^i(s^t)) \left[ \lambda^i + \xi^i(s^0) + \xi^i(s^1) + \dots + \xi^i(s^t) \right] \\ + A(c^i(s^t)) \left[ \chi^i(s^t) q(s^t) - \sum_{s^{t+1} \succ s^t} \chi^i(s^{t+1}) q(s^{t+1}) \right]$$

FOC for  $q(s^t)$

$$\sum_{i \in I} \chi^i(s^t) = \sum_{i \in I} \sum_{r=0}^t \beta^r \pi(s^r) \xi^i(s^r) \frac{\partial V^i(\cdot | s^r)}{\partial q(s^t)}(q)$$

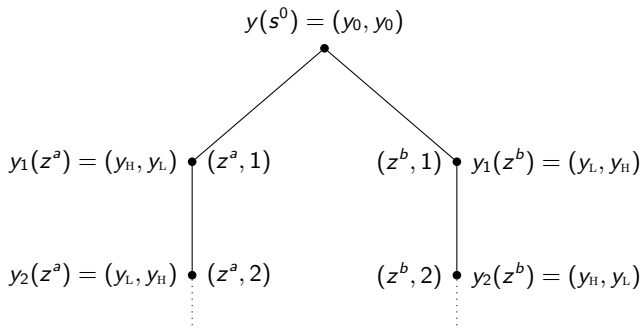
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<sup>1</sup> $A(x) = -u''(x)/u'(x)$

# Baseline Economy

- If  $\partial V^i / \partial q = 0$  then laissez-faire is constrained efficient
- FOCs not satisfied  $\implies$  room for Pareto improvement
  - ▶ Simple baseline economy
  - ▶ two agents
  - ▶ uncertainty only at the first period  $t = 0$
  - ▶ no aggregate uncertainty
  - ▶ for every  $t \geq 1$ : deterministic economy where endowments alternate between a high  $y_H$  and a low value  $y_L$

# Baseline Economy



## Symmetric First-Best Allocation

$$c_0^i = y_0 \quad \text{and} \quad c_t^i = c^{\text{fb}} = \frac{y_H + y_L}{2}, \quad \forall t \geq 1$$

# Laissez-Faire Equilibrium: Zero Interest Rate

- Assume enough gains to trade

$$\beta u'(y_L) > u'(y_H)$$

- Symmetric Markov equilibrium:  $c_H^{lf} > c_L^{lf}$  such that

$$1 = \beta \frac{u'(c_L^{lf})}{u'(c_H^{lf})} \quad \text{and} \quad c_L^{lf} + c_H^{lf} = y_L + y_H$$

- Some but imperfect risk-sharing

$$c_L^{lf} < c^{fb} < c_H^{lf} \quad \text{or} \quad 0 < x^{lf} < x^{fb}$$

# Laissez-faire versus Social Planner: FOCs

- at  $t = 0$

$$u'(y_0)\lambda^i = \mu_0$$

- at  $t = 1$

$$\beta(A_H x^{\text{lf}} + 1)\xi_1 = \pi_L - \beta\pi_H$$

- at  $t = 2$

$$(A_L x^{\text{lf}} - 1)\xi_1 + \beta(A_H x^{\text{lf}} + 1)\xi_2 = \pi_H - \beta\pi_L$$

- at  $t = 3$

$$\beta(A_H x^{\text{lf}} + 1)\xi_1 + (A_L x^{\text{lf}} - 1)\xi_2 + \beta(A_H x^{\text{lf}} + 1)\xi_3 = \pi_L - \beta\pi_H$$

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- at  $t = 0$

$$u'(y_0)\lambda^i = \mu_0$$

- at  $t = 1$

$$\beta(A_H x^{\text{lf}} + 1)\xi_1 = \pi_L - \beta\pi_H$$

- at  $t = 2$

$$(A_L x^{\text{lf}} - 1)\xi_1 + \beta(A_H x^{\text{lf}} + 1)\xi_2 = \pi_H - \beta\pi_L$$

- at  $t = 3$

$$\beta(A_H x^{\text{lf}} + 1)\xi_1 + (A_L x^{\text{lf}} - 1)\xi_2 + \beta(A_H x^{\text{lf}} + 1)\xi_3 = \pi_L - \beta\pi_H$$

# Laissez-faire versus Social Planner

## Theorem

*Assume that*

$$0 < A_L x^{\text{lf}} - 1 \quad \text{and} \quad A_L x^{\text{lf}} - 1 \neq \beta(A_H x^{\text{lf}} + 1),$$

*then the laissez-faire equilibrium outcome can be Pareto dominated by another equilibrium with self-enforcing debt limits*

# Room for Pareto Improvement

Assume  $u'(c) = c^{-\gamma}$  with  $\gamma > 1$  and pose  $\zeta := y_H/y_L$

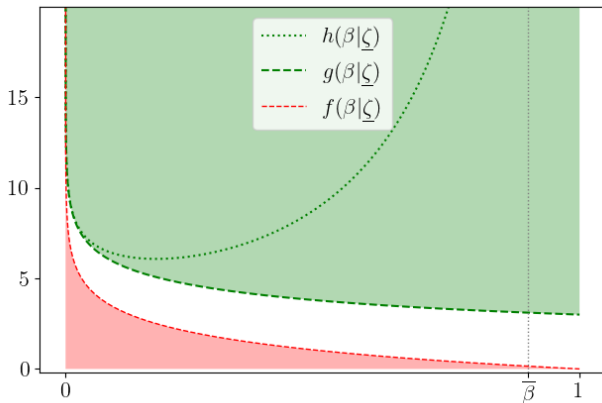


Figure: Set of  $(\beta, \gamma)$  for  $\zeta = 2$  and  $\bar{\beta} = 0.9$



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# Tightening Debt Limits Can Improve Welfare

Tightening debt constraints at date  $\tau$   $\implies$  Net-trade decreases



Increase of asset prices at some dates  $t \leq \tau$



Decrease the default value at those dates



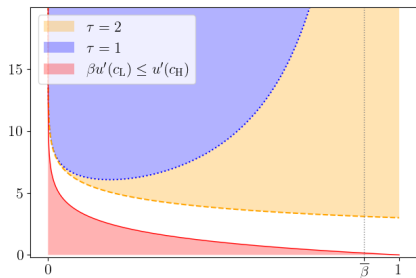
Increase the not-too-tight debt limits  
contingent to those dates  $\implies$  Net-trade increases

Pareto improvement

Intertemporal tradeoff

# Intervention at $\tau = 1$ and $\tau = 2$

(a) Explicit Interventions



(b) Implicit Interventions

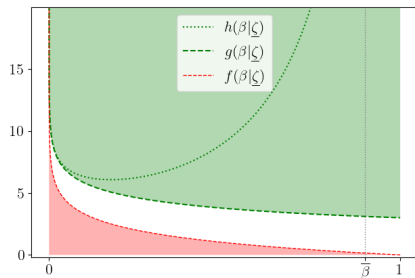


Figure: Laissez-faire can be Pareto improved.

# Conclusion

- Challenge traditional models that focused on maximizing permissible debt limits
- Imposing tighter debt constraints can paradoxically lead to Pareto improvements
- Debt bubbles are not the most efficient way to provide liquidity
- Robustness: endowment loss in case of default  $\implies$  positive interest rates