# Misspecification Averse Preferences

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# Motivation

- Agents often use stylized models to help guide their decisions.
- A growing literature studies the implications of subjective EU agents using misspecified models (e.g., Esponda and Pouzo (2016), Fudenberg et al. (2021)).
- A key finding is that misspecification matters in shaping agents' behavior and can persist even asymptotically.
- Aware that their models are only approximations, agents might, therefore, develop a concern for misspecification.

# Agenda

- In this paper, I provide an axiomatic foundation of general preferences that are averse to the possibility of misspecification.
- The representation allows the disentangling of misspecification aversion from aversion to model ambiguity.
- In particular, I show that comparative statics on the degree of misspecification and model ambiguity aversion are independently captured by two different elements of the representation.
- I also show that two misspecification averse decision criteria recently introduced in the literature can be recovered as special cases of this representation.

# Literature

- **Decision Criteria incorporating Misspecification Concern:**

  Cerreia-Vioglio et al. (2020); Hansen and Sargent (2022); Lanzani (2024)

- **Preferences and Sufficient Statistics:**

  Al-Najjar and De Castro (2014); Cerreia-Vioglio et al. (2013); Epstein and Seo (2010); Klibanoff et al. (2014).

- **Ambiguity Models:**

  Cerreia-Vioglio et al. (2011); Denti and Pomatto (2022); Hansen and Sargent (2001); Maccheroni et al. (2006).

# Decision Environment

- $\Omega$ is the set of *states of the world* endowed with a $\sigma$-algebra of *events* $\mathcal{G}$.
- $X$ is a convex set of possible *consequences* (e.g. set of monetary lotteries).
- $\mathcal{F}$ is the set of all simple, measurable functions $f : \Omega \to X$, called *acts*.
- DM's preferences are a binary relation $\succsim$ over $\mathcal{F}$.
- **Notation**:
    - For all events $E$ and acts $f, g \in \mathcal{F}$, denote by $fEg$ the act equal to $f(\omega)$ if $\omega \in E$ and equal to $g(\omega)$ if $\omega \in \Omega \setminus E$.

# Exogenous Probability Models

- To help guide her decision, the DM employs a set of probability models $\mathcal{M} \subseteq \Delta(\Omega)$.
- This is a family of distributions the DM believes are plausible descriptions of state uncertainty.
- Two different layers of uncertainty:
  - *model ambiguity:* the DM lacks information to determine what model $\mathcal{M}$ is the best approximation of the environment;
  - *model misspecification:* the DM is concerned that the set of models $\mathcal{M}$ does not contain the true probability distribution.

# Best-fit Map

- The DM has at her disposal a *best-fit map* $\mathfrak{q} : \Omega \to \mathcal{M}$ that identifies the best approximation in $\mathcal{M}$ given different realizations of $\omega \in \Omega$.
- Interpret the set $E^m = \{\omega \in \Omega : \mathfrak{q}(\omega) = m\}$ as the event that $m \in \mathcal{M}$ is the best-fit model.
- The statistical procedure is "point-identified" in the sense that each model $m \in \mathcal{M}$ assigns probability one to the event $E^m$ that $m$ is the best-fit model.
- $\mathcal{A} = \{E^m : m \in \mathcal{M}\}$ represents the *missing information* the DM would need to observe to determine what model in $\mathcal{M}$ is the best-fit model.

# Main Axioms

- Other than standard basic conditions UA MC, three main axioms:
  i. *$\mathcal{M}$-Coherence.* The preference $\succsim_m$ conditional on $m$ being the best-fit model (defined as $f \succsim_m g$ iff $fE^m h \succsim gE^m h$ for all $h \in \mathcal{F}$) is well-defined for all $m \in \mathcal{M}$. Coherence
  ii. *Consistency.* If $f \succsim_m g$ for all $m \in \mathcal{M}$, then $f \succsim g$. Consistency
  iii. *Misspecification Aversion.* For each $m \in \mathcal{M}$, $\succsim_m$ does not necessarily satisfy full-fledged independence, but only a weaker version of it, namely weak C-Independence. MA

# Intuition

- $\succsim_m$ can be interpreted as the conditional preferences of the DM if she were to observe sufficient information to determine that $m$ is the best-fit model in $\mathcal{M}$.
- Consistency means that the DM takes into account the statistical framework in her decisions. If $f$ is ranked unanimously better than $g$ by each model $m \in \mathcal{M}$, then DM prefers $f$ to $g$.
- The fact that $\succsim_m$ still doesn't satisfy full-fledged independence - even after all ambiguity about the identity of the best-fit model is resolved- reflects the DM's concern that the set $\mathcal{M}$ is misspecified.

# Main Result. Misspecification Averse Representation

- **Theorem 1.** DM's preferences $\succsim$ satisfy *Main Axioms* iff there exist an affine function $u : X \to \mathbb{R}$, a convex statistical distance $c : \Delta(\Omega) \times \mathcal{M} \to [0, \infty]$, and a monotone, normalized, quasiconcave, and continuous aggregator $I : B(\mathcal{M}) \to \mathbb{R}$[1], such that
  - for each $m \in \mathcal{M}$, $\succsim_m$ is represented by

$$V_m(f) = \min_{p \in \Delta(\Omega)} \left\{ \mathbb{E}_p[u(f)] + c(p, m) \right\};$$

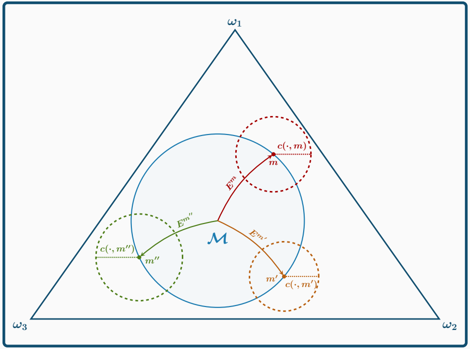
  - $\succsim$ is represented by

$$V(f) = I\left[ \min_{p \in \Delta(\Omega)} \left\{ \mathbb{E}_p[u(f)] + c(p, \cdot) \right\} \right] = I\left[ (V_m(f))_{m \in \mathcal{M}} \right].$$

[1] $B(\mathcal{M})$ is the space of bounded, measurable functions mapping the set of models to the real line.

# Comments

- The representation separates misspecification aversion from aversion to model ambiguity.
- Conditional on each $m \in \mathcal{M}$, the DM forms a *robust* evaluation $V_m(f)$ of act $f$ due to misspecification concerns:




  - $c(\cdot, m)$ captures the DM's misspecification aversion;
  - when $c(\cdot, m)$ is lower, the robust evaluation $V^m(f)$ takes into account more models $p \in \Delta(\Omega)$ around $m$, reflecting a higher concern for misspecification.
- The aggregator $I$ is a *certainty equivalent* capturing aversion to model ambiguity.

# Special Case. Cautious Aggregator

- Suppose the DM displays a form of caution over the ambiguity regarding the identity of the best-fit model. Caution
- **Theorem 2.** DM's preferences $\succsim$ satisfy *Main Axioms* and Caution iff they are represented by

$$V(f) = \min_{m \in \mathcal{M}} \left[ \min_{p \in \Delta(\Omega)} \{\mathbb{E}_p[u(f)] + c(p, m)\} \right].$$

- Caution allows us to recover the criterion introduced by Cerreia-Vioglio et al. (2020):

$$\min_{p \in \Delta(\Omega)} \left\{ \mathbb{E}_p[u(f)] + \min_{m \in \mathcal{M}} c(p, m) \right\}.$$

# Special Case. Bayesian Aggregator

- Suppose the DM evaluates the ambiguity regarding what model is the best approximation using a Bayesian approach. BA

- **Theorem 3.** DM's preferences satisfy $\succsim$ *Main Axioms* and BA iff they are represented by

$$V(f) = \int_{\mathcal{M}} \phi\left(\min_{p\in\Delta(\Omega)} \{\mathbb{E}_p[u(f)] + c(p,m)\}\right) d\mu(m)$$

  where:

  - $\mu$ is a prior over the set of models $\mathcal{M}$;
  - $\phi$ captures model ambiguity attitudes.

- When $\phi$ is affine and $c$ is proportional to the KL divergence, this becomes the average robust control criterion axiomatized by Lanzani (2024).

# Comparative Statics. Definition

- Consider two DMs with preferences $\succsim^1$ and $\succsim^2$.
- DM 1 is *more averse to misspecification* than DM 2 if for all $m \in \mathcal{M}$, $f \in \mathcal{F}$, $x \in X$,

$$f \succsim_m^1 x \implies f \succsim_m^2 x.$$

- DM 1 is *more averse to model ambiguity* than DM 2 if for all $f$ measurable wrt $\mathcal{A}^2$ and $x \in X$,

$$f \succsim^1 x \implies f \succsim^2 x.$$

[2] Recall $\mathcal{A} = \{E^m : m \in \mathcal{M}\}$ is the missing information to determine the best-fit model.

# Comparative Statics. Characterization

- Suppose that $\succsim_1$ and $\succsim_2$ both have a misspecification averse representation.
- **Proposition 1.** DM 1 is more *averse to misspecification* than DM 2 iff $u^1$ is a positive affine transformation of $u^2$ and (after normalization) $c^1(\cdot, m) \leq c^2(\cdot, m)$ for all $m \in \mathcal{M}$.
- **Proposition 2.** DM 1 is more *averse to model ambiguity* than DM 2 iff $u^1$ is a positive affine transformation of $u^2$ and (after normalization) $I^1(\cdot) \leq I^2(\cdot)$.

# Conclusions

- I have provided an axiomatization of general preferences that are averse to misspecification.
- This representation allows us to meaningfully distinguish aversion to model misspecification from the usual aversion to model ambiguity.
- The comparative statics show that we can rank agents in terms of their misspecification aversion independently of their attitudes towards model ambiguity, and viceversa.
- Currently working on:
  - extend this approach to a forward-looking agent facing a dynamic decision problem;
  - study the implications of misspecification aversion in strategic contexts.

# Thank You! Questions?

# Appendix

# Axiom. Uncertainty Averse Preferences

Back

i. *Weak Order.* $\succsim$ is complete and transitive.

ii. *Monotonicity.* For all $f, f' \in \mathcal{F}$, if $f(\omega) \succsim f'(\omega)$ for all $\omega \in \Omega$, then $f \succsim f'$.

iii. *Mixture Continuity.* If $f, f', f'' \in \mathcal{F}$, the sets $\{\alpha \in [0,1] : \alpha f' + (1-\alpha) f'' \succsim f\}$ and $\{\alpha \in [0,1] : f \succsim \alpha f' + (1-\alpha) f''\}$ are both closed.

iv. *Risk Independence.* For all $x, y, z \in X$ and $\alpha \in [0,1]$,

$$x \succsim y \iff \alpha x + (1-\alpha) z \succsim \alpha y + (1-\alpha) z \ .$$

v. *Uncertainty Aversion.* For all $f, f' \in \mathcal{F}$ and $\alpha \in (0,1)$,

$$f \sim f' \implies \alpha f' + (1-\alpha) f \succsim f \ .$$

vi. *Unboundedness.* There exist $x, y \in X$ such that $x \succ y$ and for all $\alpha \in (0,1)$, there are $z, z' \in X$ such that $\alpha z + (1-\alpha) y \succ x \succ y \succ \alpha x + (1-\alpha) z'$.

# Axiom. Monotone Continuity

Back

*Monotone Continuity.* For all $f, f' \in \mathcal{F}$ and $x \in X$, for all $(E_n)_{n\in\mathbb{N}} \subseteq \mathcal{G}$ such that $E_1 \supseteq E_2 \supseteq \cdots$ and $\bigcap_{n\in\mathbb{N}} E_n = \emptyset$, if $f \succ f'$, then, there exists $n_0 \in \mathbb{N}$ such that $xE_{n_0}f \succ f'$.

- Intuition: perturbations of acts on vanishing events do not affect strict preferences.
- Implication: countable additivity of the probabilities involved in the representation.

# Axiom. $\mathcal{M}$-coherence

Back

i. For all models $m \in \mathcal{M}$, $E^m$ is nonnull[3] and $fE^m h \succsim gE^m h$ if and only if $fE^m h' \succsim gE^m h'$ for all $f, g, h, h' \in \mathcal{F}$.

ii. For all $m \in \mathcal{M}$ and $f, g, h \in \mathcal{F}$,

$$f = g \;\text{ a.s.-}m \implies fE^m h \sim gE^m h\ .$$

iii. For all $x \in X$ and $f \in \mathcal{F}$, the set $\{m \in \mathcal{M} : fE^m x \succsim x\}$ is measurable.

iv. For all $m \in \mathcal{M}$, if $p \ll m$ but $p \neq m$, then there exist $f \in \mathcal{F}$ and $x \in X$ such that $fE^m x \succsim x$ but $x \succ \mathbb{E}_p[f]$.

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[3] An event $E$ is *null* if $fEg \sim f'Eg$ for all $f, f', g \in \mathcal{F}$, and it is *nonnull* if it is not null.

# Implication of Coherence

Back

- For each model $m \in \mathcal{M}$, define $f \succsim_m g$ iff $fE^m h \succsim gE^m h$ for all $h \in \mathcal{F}$.
- $\succsim_m$ is complete and can be interpreted as the conditional preferences of the DM if she were to observe sufficient information to determine that $m$ is the best-fit model in $\mathcal{M}$.
- The second requirement implies that if two acts are equal a.s. according to $m$, they are then ranked as indifferent conditional on $m$ being the best-fit model.

# Axiom. Consistency

Back

*Consistency.* For all $f, g \in \mathcal{F}$,

$$f \succsim_m g \text{ for all } m \in \mathcal{M} \implies f \succsim g.$$

- The DM takes into account the models in her decisions.
- If $f$ is ranked unanimously better than $g$ by each model $m \in \mathcal{M}$, then then DM prefers $f$ to $g$.

# Axiom. Misspecification Aversion

Back

*$\mathcal{M}$-weak C-Independence.* For all $m \in \mathcal{M}$, for all $f, g \in \mathcal{F}$, $x, y \in X$, and $\alpha \in (0,1)$,

$$\alpha f + (1-\alpha)x \succsim_m \alpha g + (1-\alpha)x \implies \alpha f + (1-\alpha)y \succsim_m \alpha g + (1-\alpha)y$$

- After being told $E^m$, all ambiguity about what is the best-fit model is resolved.
- If the DM were certain that $\mathcal{M}$ contains the true probability law, conditioning on $E^m$ she should infer that $m$ is, in fact, the true model.
- The fact that $\succsim_m$ still doesn't satisfy full-fledged independence reflects the DM's concern that the set $\mathcal{M}$ is misspecified.

# Axiom. $\mathcal{M}$-Caution

Back

- *$\mathcal{M}$-Caution.* For all $f \in \mathcal{F}$ and $x \in X$, if $\exists m \in \mathcal{M}$ s.t. $x \succ_m f$, then $x \succsim f$.

# Axiom. $\mathcal{M}$-SEU

Back

- *$\mathcal{M}$-SEU.* The restriction of $\succsim$ to $\mathcal{A}$-measurable acts satisfies Savage's P2-P6.

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