Frustration and Personal Motivation

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July 16, 2024

Abstract

Understanding how emotions stemming from previous failures affect investment decisions is critical for the study of choice under uncertainty. I translate insights from the psychological literature into two simple principles to integrate the effect of emotions on investment decisions: frustration stemming from past failures lingers, and success brings emotional relief. Frustration's negative impact on utility creates a trade-off. On one side, the agent wants to limit her exposure to future frustration and decrease investment. Conversely, if her success probability increases with investment, frustration creates an incentive to eliminate its negative effect. In general, the framework allows me to (i) precisely characterise how frustrating events interact with an agent's preference, environment and past frustration to influence her investment level, (ii) rationalise data patterns, that are hard to reconcile with a single theory. On the empirical side, I investigate the impact of frustration in baseball and find that it increases the speed of pitches and affect their quality. I show that frustration's impact on quality can have impact on a pitchers' career level performance.

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1 Introduction

Emotional reactions to unfavourable outcomes can impact economic decisions in heterogeneous ways. They prompt individuals to take more risks in various environments (Post et al., 2008; Foellmi et al., 2016; Callen et al., 2014; Passarelli and Tabellini, 2017), while reducing risk tolerance in many others (Cohn et al., 2015; Guiso et al., 2018; Meier, 2022). Understanding these effects and the source of this heterogeneity is essential. For example, if they influence an agent's effort level, then the principal setting an optimal contract should consider them. An unexpected bear market can trigger negative emotions, affecting investors' willingness to invest. In turn, these reactions can accelerate or dampen market trends. Similarly, emotions stemming from lay-offs might affect labour searches and occupational sorting. However, economists largely abstain from considering the role of emotions after adverse outcomes.

Economic theory generally posits that negative outcomes can change beliefs, prompt learning-by-doing, or change wealth. While these dynamics are essential in many applications, they do not apply to all contexts. In these cases, previous events should not affect future investment. However, they do. Negative outcomes still impact investment even when these traditional dynamics are absent, and they do so in heterogeneous ways. (Heath, 1995; Augenblick, 2016; Dalmia and Filiz-Ozbay, 2021; Martens and Orzen, 2021; Negrini et al., 2022). Moreover, negative outcomes influence investment behaviour even when standard behavioural economics explanations, such as loss aversion, do not apply or are controlled for (Heath, 1995; Malmendier and Nagel, 2011; Guiso et al., 2018; Dalmia and Filiz-Ozbay, 2021).[1]

This paper aims to understand the dynamic effect of emotions stemming from below-expectation outcomes (frustrating events hereafter) on investment decisions.[2] The main conceptual novelty of this paper is to define the circumstances that trigger the emotional reaction while keeping the effect of emotions on utility general. As such, I use the umbrella term **frustration** to name all emotions stemming from such frustrating events.[3]

[1] To be more precise, previous sunk costs influence behaviour insofar they influence wealth/endowment in loss-aversion models such as cumulative prospect theory (Tversky and Kahneman, 1992). Heath (1995); Dalmia and Filiz-Ozbay (2021) show that an effort sunk cost affects subsequent behaviour when there are no changes in wealth, while Malmendier and Nagel (2011); Guiso et al. (2018) control for wealth changes.

[2] The focus on frustrating event is natural, and has a long tradition in psychology (Dollard et al., 1939). In general, modern emotion theories would take the evaluation of an event as frustrating as one of the first steps that could lead to many negative emotions and reactions (Keltner and Lerner, 2010).

[3] Dollard et al. (1939) use the term frustration to denote frustrating events. I find this rather counter-intuitive so I use the term frustrating event to denote the event triggering frustration. See Breuer and

In contrast, most of the current economic literature usually studies the effect of specific emotions on behaviour. An event-based approach to emotions brings several advantages. (1) Frustrating events provoke various negative emotions, such as disappointment, sadness, and anger, which can affect behaviour differently (Shirai et al., 2021; Bell, 1985; Loomes and Sugden, 1986; Keltner and Lerner, 2010; Breuer and Elson, 2017; Battigalli et al., 2019). Restricting the analysis to a single emotion would therefore paint an inaccurate picture of how people react to such situations. (2) It is less culturally dependent than a definition-based approach. Emotions are neuro-biological and socio-cultural in nature. As such, culture will influence the way people react to frustrating events and even what type of emotion they are capable of having (Mesquita and Ellsworth, 2001; Mesquita and Walker, 2003; Mesquita and Markus, 2004). (3) The approach also sheds light on why one type of emotional reaction - as revealed by the agent's action - arises instead of another and how this reaction depends on the environment and the agent's preferences.

The second distinctive feature of the approach is to consider emotions as dynamic processes. I translate insights from the psychological literature into two simple and general principles to answer this question. First, emotions stemming from successive frustrating events accumulate (Pe and Kuppens, 2012). Frustrating events are the source of our decision-maker's frustration. As such, I say that success, defined as an above-expectation outcome, brings emotional relief (Goldberg et al., 1999; Han et al., 2007). I use this conceptual framework to develop to study the dynamics of investment provision of frustration-prone decision-makers that suffers a string of frustrating events.

I consider a forward-looking decision-maker who invests resources in projects until she succeeds, which is given by the first event of a Poisson law. Her investment either increases her return or her probability of success. When she fails, her frustration increases by the amount of wasted investment: it accumulates. When she succeeds, she gets some utility, called **success utility**, the decision problem stops, and frustration's effect on utility disappears: success brings emotional relief. Frustration exerts an **emotional cost** and impacts utility negatively.[4] Second, frustration creates **appraisal tendencies** (Lerner and Keltner, 2000, 2001), which impact the agent's evaluation of her environment and preferences for the choices at hand. In my setting, this translates into a change (positive or negative) in the marginal utility of investment in the same way habit formation would.

The main theoretical contribution is to provide a closed-form solution that represent

Elson (2017) for a modern summary of the theory.

[4]The negative impact on utility can be considered as the valence of the emotion, or in the cognitive appraisal framework, the "pleasantness"(Smith and Ellsworth, 1985).

the effect of frustration on an agent's investment level.[5] When does frustration increase or decrease the agent's investment? On the one hand, the agent wants to limit her exposure to future emotional costs and decrease investment. On the other hand, if her success probability increases with investment, frustration also creates an incentive to eliminate its negative effect. I characterize the result of this trade-off, which shows the influence of the agent's belief of control on frustration's effect on investment. This result is important. First, the controllability of a success probability can be measured and used to predict responses to frustration. Second, the result fits stylised fact extremely well. For example, psychologists usually consider that anger is associated with high feelings of control over their environment. Moreover, angry individual tend to increase their involvement in a task (investment here). I get that result endogenously as the result of the agent's maximisation problem. If one believes that one major difference between sadness and anger relies in the agent's evaluation of her action effectiveness at solving the problem that was causing her distress, than, the solutions of the model provides the action tendencies[6] of these emotions endogenously.

The second contribution clarifies the role of appraisal tendencies in emotional responses. While the previous results show that positive appraisal tendencies are not necessary to get a positive effect of frustration on investment, I also show that they are not sufficient either. Indeed, if the emotional cost is too high, positive appraisal tendencies might not be enough to tip the balance towards a positive effect of frustration on investment. As such, while important, appraisal tendencies are not the only force driving agent's reactions. Moreover, I show that trying to infer emotions through action is not straightforward. There is no one-to-one revealed preference (or emotions) argument one can use to infer the action from emotion or emotion to action. For example, sadness and anger are behaviourally equivalent in some instances, even though their appraisal tendencies go in opposite directions. Similarly, I show that angry individuals might decrease their investment level, even though the stereotypical action associated to anger would be an increase in investment.

From an empirical perspective, I provide evidence for the principles on which I build the model. I use a large pitch-by-pitch Major League Baseball Data with a detailed set of controls to do this. These allow to control for most, if not all possible confounders. I exploit the expected score variations due to the pitch outcome to measure the frustration triggered

[5]For psychologists, this closed-form expression can be considered as the result of the second (or other higher order) evaluation in Lazarus and Folkman (1984); Scherer (2009). In other words, the model endogenously links valence, cognitive appraisal, and appraisal tendencies to action tendencies.

[6]Whether angry or sad individuals tend to increase or decrease investment.

by failed pitches. Frustration is measured as the difference between the expected numbers of runs (points) before and after the pitch. I find signs of frustration accumulation in pitchers' behaviour. Success seems to dissipate its effect. Frustration always increases pitches' speed and tends to decrease their quality when its effect is significant. The magnitude of the effect on pitchers' behaviour is important. One standard deviation increase in frustration[7] accounts for 45.72% to 64.70 % of the magnitude of the effect of one standard deviation increase in fatigue during an inning (5.16 throws). Finally, I show that frustration also has important implications in terms of productivity. Pitchers whose pitch quality is more negatively affected by frustration tend to have lower career average skills measures.

In section 3, I study the determinant of emotional reactions stemming from frustrating events. I first focus on the impact of appraisal tendencies. I then study how frustration interacts with the agent's control over her success probability through investment. Section 4 investigates the effect of frustration accumulation using pitch-by-pitch Major League Baseball data. I focus on pitchers' emotional reactions to frustrating events. Section 5 concludes and discusses the forward-looking and time consistency assumption used in the model.

This paper broadly relates to three strands of literature.

Psychological Game Theory: An active literature has been modelling emotions in interpersonal contexts using psychological game theory tools (Geanakoplos et al., 1989; Battigalli and Dufwenberg, 2009). The literature focuses on the role of beliefs in emotion and their strategic implication. Different emotions such as guilt (Battigalli and Dufwenberg, 2007) and anger (Battigalli et al., 2019) have been modelled. Specifically, Battigalli et al. (2019) studies how frustration and blame can lead to aggressive behaviour. My definition of frustration is similar, as frustration in their paper is also triggered when other players (or nature) reduce the maximum expected pay-off the agent could have gotten. However, I do not specifically focus on anger and do not consider strategic situations.

Emotions in decision theory: Over the years, many contributions have modelled the effect of particular emotions such as regret (Loomes and Sugden, 1982; Bell, 1982), disappointment (Bell, 1985; Loomes and Sugden, 1986) anxiety or excitement (Caplin and Leahy, 2001). One key feature of these models is that emotions only affect decision-making through the expectation of what can happen, in a forward-looking manner, but past emotions do not influence current decisions. Loewenstein (2000) develops a framework to show

[7] A standard deviation change in frustration equals 0.27 expected runs the opposing team could score because of the pitch's failure.

the effect of visceral factors[8] on current decision but does not consider the visceral factors' dynamics as the decision-maker do no anticipate them.[9] Here, both past and anticipated emotions affect decisions, allowing novel theoretical predictions to be developed. One exception is Wälde (2018), who studies the effect of stress and coping strategies on working memories. In this model, stochastic stress shocks decrease working memory and one's capacity to work. The model then studies the various coping strategies an individual might choose to reduce stress to an acceptable level.

Temporal risk aversion: Many factors might affect the risk aversion and, in turn investment provision. For example, time, wealth variations and consumption habit formation can affect risk aversion, see (Kreps and Porteus, 1978; Segal, 1990; Tversky and Kahneman, 1992; Campbell and Cochrane, 1999; Rozen, 2010; Imas, 2016). Here, frustrating events affects behaviour independently of these effects. Köszegi and Rabin's reference-dependent model (Kőszegi and Rabin, 2006, 2009) is also similar in the sense that frustrating events are defined relative to an expectation. Specifically, these types of preference can be applied to an inter-temporal framework, but the dynamics stemming from changes in beliefs about future consumption are very different from those studied here.[10] Finally, Dillenberger and Rozen (2015) axiomatise a model of history-dependent risk-attitude. Their paper bears important similarities with this ones as risk aversion depends on the history of realization of lotteries. In particular, frustrating events (disappointing in their terminology) yield higher risk aversion. One important difference between our two approaches is that in (Dillenberger and Rozen, 2015), intensity of past frustrating events plays no role in determining risk aversion. Here, intensity and its endogenous relation with the investment level plays a central role in the trade-off I study.

2 General Framework

Decision environment

I study an agent who is investing resources $x_t \in [0, I]$ at every point in time $t \in \mathbb{R}_+$, to possibly get a successful investment.[11] Investment might increase utility in case of success,

[8]Visceral factors are drive states that have a hedonic impact on utility, such as hunger and possibly emotions.

[9]This is natural as it is apparently difficulty to anticipate visceral states (Loewenstein, 2000).

[10]In this setting, a failure always carries the same information because investment decisions are identical. As such, (Kőszegi and Rabin, 2009) would predict a constant investment level after the first failed attempt.

[11]Resources can represent many different things: money, capital, time, force, space, effort etc..

or increase the probability of success, depending on the application in the following sections. Success is determined by a Poisson process q, where $q = 0$ if the investment failed. The probability that q increases by 1 during dt, i.e. $dq = 1$, is given by $\bar{\pi}(x_t)dt$, where $\bar{\pi}(x_t)$ is the arrival rate of the Poisson process during the time interval dt. In other words, $\bar{\pi}(x_t)dt$ denotes the probability of getting a success during the time interval dt. As such, the probability of failing from 0 to time t, denoted p_t, is given by an inhomogeneous Poisson point process with:

$$p_t = \frac{\left(\int_0^t \bar{\pi}(x_s)ds\right)^0}{0!} e^{-\int_0^t \bar{\pi}(x_s)ds} = e^{-\int_0^t \bar{\pi}(x_s)ds}. \tag{1}$$

As such, p_t takes a very convenient form as it will act as an endogenous discount rate to the inter-temporal maximisation problem.

Psychological dynamics

Emotions are dynamic by nature. The central part of the framework is that frustration f_t is modelled as a stock that lingers if the investor continues to suffer failures. Specifically, frustration increases by the amount of wasted investment when she fails but decays at a rate $\delta \in (0, 1)$:

$$\dot{f}_t = x_t - \delta f_t \tag{2}$$

Where $\dot{f}_t$ represents the derivative of frustration with regards to time.[12] The decision problem stops if the investor succeeds at time τ and all future utilities equal 0. These modelling choices capture several critical dynamic features of emotions. First, the decay rate δ represents that emotional response tends to fade when time passes and revert to a neutral state (Loewenstein, 2000; Lerner et al., 2015).

Second, the effect of frustration disappears after achieving success. This represents what psychologists call the "*goal-attainment hypothesis*", whereby the impact of emotion on behaviour ceases when the problem at its source is solved (Han et al., 2007; Goldberg et al., 1999).

Finally, frustration accumulates. In general, emotions of the same sign (positive or negative) that are triggered consecutively tend to overlap and create a phenomenon called

[12] Continuous time dynamics are not standard in decision theory or economic psychology. However, it makes sense to consider them here as emotions' dynamics are continuous by nature. A discrete-time formulation of the same dynamics would be $f_{t+1} = x_t + (1 - \delta)f_t$.

"*emotion augmentation*" (Pe and Kuppens, 2012; Kuppens and Verduyn, 2017). In these cases, the effect of one emotional experience tends to increase the following one. The model represents this with a time dependence between current and past frustration levels.

Material preferences

Investment is costly. I represent the investment cost as an increasing and (weakly) convex cost function $c(x)$. If she succeeds, the agent gets utility $u(x_t, f_t)$, called **success utility**, which is increasing in investment and jointly concave in investment and frustration. The impact of frustration on success utility is described in the following subsection.

The impact of frustration on preferences

This section aims to provide the essential ingredients necessary to model any emotion stemming from frustrating events. I consider that frustration impacts utility in two ways.

First, one feature that any emotions stemming from frustrating events will share is that they will be negative emotions. As such, if she is still investing at time t, the agent suffers from an emotional cost stemming from her frustration stock. I represent the emotional cost as an increasing and convex function $v(f)$. Psychologists would call the negative effect of $v(f)$ on utility the *valence* of the emotion. In other words, $v(f)$ construction reflects that ceteris paribus, people prefer to be less sad or less angry and prefer situations with lower f.

Second, frustration triggers *appraisal tendencies* that affect the agent's judgement. As such, some options can become more appealing when in an emotional state than in a neutral state. This is important as emotions with the same valence can have different appraisal tendencies (e.g. fear and anger). Similarly, emotions with opposite valence can have the same appraisal tendencies (e.g., anger and happiness) (Lerner and Keltner, 2001, 2000). Typically, angry individuals might want to increase their effort because their goal becomes more important; they want "bigger" successes, while sadness could decrease the project's appeal. Similarly, angry (sad) individuals tend to become over-(under) estimate the effectiveness of their actions.

To represent this, I say that frustration exhibits **negative (positive) appraisal tendencies** if $u_{xf} < (>) 0$.[13] In other words, negative (positive) appraisal tendencies

[13] In what follows, I represent derivatives by subscripts. For a function $f(x,y)$, I call $f_x = \frac{\partial f(x,y)}{\partial x}$, $f_{xy} = \frac{\partial^2 f(x,y)}{\partial x \partial y}$, $f_{xx} = \frac{\partial^2 f(x,y)}{\partial x^2}$, and so on.

decrease (increase) the marginal utility of investment one gets in case of success. As such, positive appraisal tendencies characterise a situation where the decision-maker believes her actions are becoming more efficient at increasing her success utility or simply because she marginally values this success more.

My objective is to understand how appraisal tendencies, wherever they come from, interact with the environment and the emotional cost. Indeed, this model is the first to simultaneously study emotional costs (valence) and appraisal tendencies in economics. As such, given the research question, I feel it is appropriate to keep the source of these appraisal tendencies exogenous.[14]

Appraisal tendencies could affect motivation through another channel: the decision maker's actual or assessment of the Poisson law arrival rate, with for example, some function $\bar{\pi}(x, f)$. This channel is essential, as emotions influence risk assessment (Lerner and Keltner, 2000, 2001) and can impair performance (Wei et al., 2016).[15] This framework can easily be adapted to model these effects. Still, it is also somewhat analytically redundant because it would create similar trade-offs as the one analysed in Section 3.1 and 3.2.

The inter-temporal decision problem

Let me now give the general form of the instantaneous expected utility representing the decision maker's preferences:

$$U(x_t, f_t) = \bar{\pi}(x_t)u(x_t, f_t) - v(f_t) - c(x_t) \tag{3}$$

During a time interval dt, a probability $\bar{\pi}(x_t)$ of success exists. In case of success, the agent gets her success utility $u(x_t, f_t)$. Note that in this general formulation, the probability of success and utility depend on x_t. While I do not focus on this general case in the next sections, it is not improbable.[16] Finally, the two last terms of equation (3) represent the emotional cost $v(f_t)$ the agent has to endure because she is still investing at time t and the

[14] Note that it is, in theory, possible to endogenise them, but one would need to be more specific about which emotion is triggered and what the socio-cultural and personal context is. For example, Battigalli et al. (2019) endogenise appraisal tendencies (action tendency in their lingo) to reduce other players' payoff depending on how they blame them in leader-follower games.

[15] In fact, most of the literature studying appraisal tendencies focus on the effect of these tendencies on the evaluation of decision environment but not the choices at hand.

[16] For example, a researcher deciding to go for a more ambitious project would see the higher effort level impacting the probability (higher $\bar{\pi}$) and the quality of the publication (higher u). Similarly, an entrepreneur launching a larger marketing campaign for a product will affect the probability of being picked up by a large distributor, leading to higher sales.

investment cost $c(x_t)$. Let $\rho > 0$ be the agent's subjective discount rate. As the decision-making process only continues until time t with probability p_t, the agent must also discount these situations accordingly. As such, let me denote by $\phi(x_t, t) = \int_0^t \bar{\pi}(x_s)ds + \rho t$ be the effective discount rate. The inter-temporal decision problem is given by:

$$
\begin{aligned}
V(t_0, f_{t_0}) &= \max_{x_t \in [0,I]} \int_{t_0}^{\infty} e^{-\phi(x_t,t)} U(x_t, f_t) dt \\
\dot{f}_t &= x_t - \delta f_t \\
f_{t_0} &\text{ given}
\end{aligned}
\tag{G.O}
$$

As such, the decision maker, who stands at t_0 decides on optimal investment levels x_t for all t. As such, x_t is a function $x_t : [t_0, \infty) \to [0, I]$ that gives, for every t, the optimal investment level the agent is willing to put in if she is still failing at time t. To ensure sufficiency of the first order conditions, I assume joint concavity of $e^{-\phi(x_t,t)}U(x_t, f_t)$. This assumption is not trivial, as the endogenous probability of success can create important convexities in the inter-temporal maximisation problem.

Note that I focus on situations where frustration accumulation is the only dynamic. This assumption allows me to get more straightforward results, but it is also possible to enrich the model. For example, in Appendix 8, I explore the combined effect of learning-by-doing and frustration accumulation and characterise the conditions under which the optimal investment path can become non-monotonic.

Before delving into the analysis, one should also note that without frustration, the optimal investment path is constant. In such cases, all investment decisions are independent and identical; as such, the decision-maker should always choose the same optimal investment level. I focus the analysis on two distinct set-ups. First, I explore the inter-temporal effects of appraisal tendencies, because they are reminiscent of previous works in the literature (Loewenstein, 2000; Laibson, 2001). Indeed, appraisal tendencies affect the agent's marginal utility in case of success, as do Laibson (2001) 's cues (which create craving/disgust and then habits) or Loewenstein (2000)'s visceral factors. While no cost is associated with the cues in Laibson (2001), one can see this model as an inter-temporal version of the visceral factor model Loewenstein (2000). As we will see, it is essential to consider the temporal dimension to understand the effect of emotional cost (or valence) on behaviour.

In Section 3.2, I go beyond current models of emotions and consider an environment where investment affects the probability of success with $\bar{\pi}(x) = \gamma \cdot x$, $\gamma > 0$, but frustration does not directly influence the preferences for the choices at hand i.e. there are no appraisal tendencies, which are the driving forces in Loewenstein (2000) and Laibson (2001).

3 The formation of emotional reactions

3.1 Appraisal tendencies

To get cleaner dynamics and isolate the role of appraisal tendencies, I focus on situations where the success probability is exogenous, i.e. the arrival of the Poisson process will be $\bar{\pi}(x_t) = \pi > 0$. As such, the investor invests because her success utility increases with investment. The instantaneous expected utility at time t for a period dt is:

$$U(x_t, f_t) = \pi u(x_t, f_t) - v(f_t) - c(x_t) \tag{4}$$

The overall optimisation problem is as in (G.O) presented before, with the specification given here.[17] After linearising the system obtained by solving the optimisation problem around the steady state, I get the following system:[18]

$$\dot{\hat{x}}_t = (\rho + \pi + \delta)\hat{x}_t + \Omega_A^* \cdot \hat{f}_t \tag{5a}$$

$$\dot{\hat{f}}_t = \hat{x}_t - \delta \hat{f}_t \tag{5b}$$

$$\text{Where} \qquad \Omega_A^* = \frac{1}{U_{xx}^*}\left(U_{ff}^* + (\rho + \pi + 2\delta)U_{xf}^*\right) \tag{6}$$

Where U_{ff}^*, U_{xf}^* and U_{xx}^* indicate the value of the second-order derivatives at the steady state. The first line in (5a) is the Euler equation; it indicates how investment varies for a given level of investment $\hat{x}_t$ and $\hat{f}_t$.(5b) reiterates the frustration law of motion.

The variable of interest is Ω_A^*, which captures the temporal complementarity of invest-

[17] One can ensure that the solutions are in the interior of the control variable set by setting Inada type conditions $\lim_{x \to 0} U_x(x, f) = \infty$ and $\lim_{x \to I} U_x(x, f) = -\infty$ for all f. To have sufficiency, the Hamiltonian must be jointly concave in x and f. In other words, the appraisal tendencies of frustration cannot be too strong. Finally, note that these conditions also ensure the uniqueness of the steady-state

[18] I introduce a variable transformation such that the steady-state value of frustration and investment equals zero. As such, values below the steady state level are strictly negative. The scaled variables are denoted with a hat.

ment and frustration when there are appraisal tendencies.[19] Its sign indicates whether an increase in frustration today increases or decreases the optimal investment level tomorrow. Let us examine the phase diagrams representing the linearised system in Figure 1 to see this. Given that they are not common in behavioural economics, I will spend some time on their interpretation.

The $\dot{\hat{x}}_t = 0$ and $\dot{\hat{f}}_t = 0$ loci respectively represent the set of points $(\hat{x}_t, \hat{f}_t)$, for which the investment and frustration level do not change over time. One can compute their equation by equating the differential equations 5a and 5b to 0. Note that by doing this, one can infer that if Ω_A^* is positive (negative), the $\dot{\hat{x}}_t = 0$ will be downward (upward) sloping. The intersection of the two loci naturally gives the steady state which is normalised at $(0, 0)$.

Let f_{t_0} be below its steady-state value. Then, the system will only reach an interior steady state if it is in quadrant A in both graphs.[20] While quadrant A in both graphs imply that frustration will increase to reach its steady-state level, this is not the case for the investment level. Indeed, the optimal investment level will decrease to reach its steady-state level, as on the left graph of Figure 1. On the other hand, if $\Omega_A^* < 0$, investment increases to reach the steady state, as in the right graph of Figure 1.

Proposition 1 formally shows the dependence of the system's dynamics on Ω_A^* and clarifies when the maximisation problem is well-behaved and has a stable steady-state.

Proposition 1. 1. *An increase in f leads to an increase (decrease) in investment provision x if and only if $\Omega_A^* < (>)0$,*

2. *The system exhibits saddle path stability as long as $\Omega_A^* > -\delta(\rho + \pi + \delta)$.*

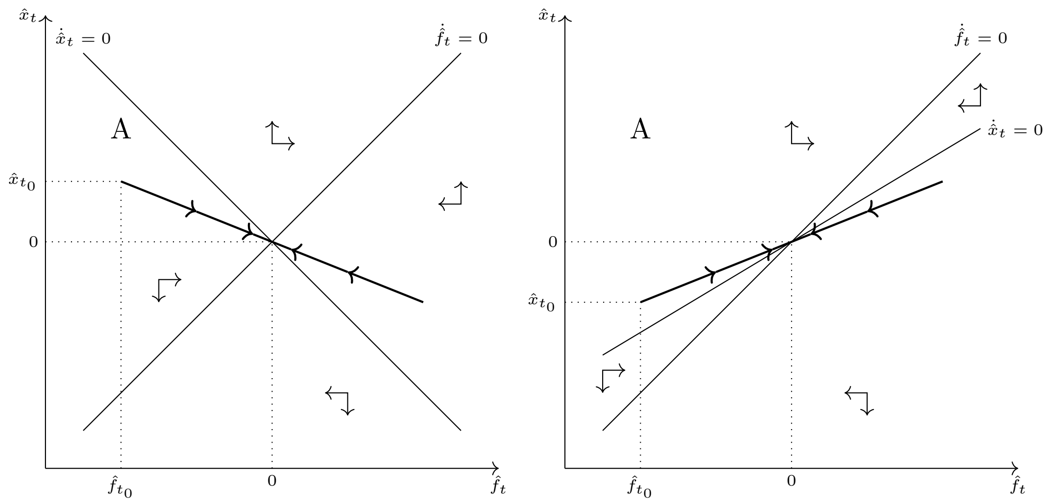
3.1.1 Clarifying the role of appraisal tendencies in behaviour

Let us now study the determinant of the value of Ω_A^* described in equation (6) and interpret the model. Its sign will be determined by the relative size of $(\rho + \pi + 2\delta)U_{xf}^*$ and U_{ff}^*. The second term, $U_{ff}^* < 0$, represents the frustration cost: a more concave utility leads to steeper utility losses if frustration increases. On the other hand, U_{xf}^* represents the

[19] To be precise, Ω_A^* is the value of the Volterra derivative of the optimal investment path functional $X^*(t)$ at the steady state, see Ryder Harl E. and Heal (1973), for more details. It is also easy to notice the similarity with Becker and Murphy (1988) Rational Addiction model because the appraisal tendencies U_{xf}^* are akin to addictive tendencies. In the next section, I set these appraisal tendencies to zero to study different dynamics.

[20] One can guess the dynamics of frustration and investment by getting the sign of the differential equations 5a and 5b. For example, the frustration stock will decrease if $\hat{f}_t$ is too large relative to $\hat{x}_t$. In this case, we are at the right of the $\dot{\hat{f}}_t = 0$ locus.

Figure 1: Phase diagrams, with increasing and decreasing optimal investment path.



appraisal tendencies triggered by frustration. If they are positive, they can mitigate the negative effect of the emotional cost. If strong enough, they can trigger an increase in investment.

Also note that the parameters ρ, π and δ play an essential role. Suppose the investor does not discount future failures heavily because she thinks that failures are likely (low π). In that case, the effects of appraisal tendencies are relatively discounted compared to long-term frustration costs because they only affect success utility. Similarly, lower discounting (low ρ) would put relatively more weight on U^*_{ff}. The opposite effect happens for the decay rate of frustration, which amplifies the effects of appraisal tendencies. If the decay rate increases, frustration cannot reach higher levels and is potentially less emotionally costly in the long run.

However, one should not interpret having positive appraisal tendencies as directly leading to an investment increase. Indeed, appraisal tendencies are only one part of the trade-off the decision maker is considering. The following example shows how using emotion labels or inferring emotions from behaviour can sometimes lead to misleading conclusions. Let us focus on the case of anger and sadness.

Example. *Let us consider the case of Emmanuel, who needs to buy clothes at every point in time. Spending $x = 0$ would buy Emmanuel regular clothes, but buying more expensive clothes might bring some success utility that increases in x as he likes it when people praise* *his wealth. Emmanuel has impeccable taste. As such, the probability of getting praise does not depend on the price of his clothes. When Emmanuel does not get any praise when spending x_t, he gets frustrated, and his frustration follows the law of motion described in 2. If he gets at least one praise, the decision problem stops.*[21]

Emmanuel's example showcases how difficult it can be to infer what kind of emotion will be triggered by frustrating events outside a lab. Will Emmanuel be sad that nobody notices how rich he is despite his best efforts? Or will he be angry because of his entourage's lack of interest. [22]. However, what interests economists is whether Emmanuel spends more or less money on clothes and why. In other words, I ultimately want to characterise the emotional trade-off described in Ω_A^*.

Example. *Now let us consider the appraisal tendencies typically associated with sadness for Emmanuel.*

Sadness*: Psychologists associate sadness with irrevocable feelings of loss and a desire to change circumstances (Lerner et al., 2004). On the one hand, the sense of loss can be linked to the frustration accumulated because of the financial sunk cost.*

On the other hand, $U_{xf}^* \leq 0$ *can represent Emmanuel's general disinterest in buying expensive clothes. Alternatively, sadness has also been shown to influence people's judgement on how effective they are at changing their circumstances. When sad, Emmanuel believed that the same amount of money spent on clothing would translate into a lower level of wealth praises. The marginal utility of getting praise for his wealth is lower for greater frustration levels. Intuitively, sadness decreases the interest in cloth investment at the margin. As such,* $\Omega_A^* > 0$. *In this case, the frustration stock always leads to lower investment as both the appraisal tendencies and the sensitivity to frustration impact are negative.*

So far, so good. The appraisal tendencies associated with sadness amplify its action tendency to decrease investment. Emmanuel is decreasing his clothing expenditures even more when they are present. But what happens to Emanuel if the situation makes him somewhat angry? His resentment of not getting any praise might be such that showing signs of wealth becomes more important to him. Angry individuals typically try to move against the problem and approach it directly. Similarly, he could also believe that people would get him more praise for the same amount of money. In our context, this can be interpreted

[21]He could start repurchasing cloth in a separate maximisation programme.

[22]These are, of course, only two emotions out of many that could be triggered in this situation.

as Emmanuel being willing to double down on the wealth signalling and getting even more utility for higher levels of wealth demonstration. That is, $U^*_{xf} \geq 0$. In this situation, the cloth investment marginal utility in case of praise increases with frustration.

Example. *In the case of anger, two situations can emerge.*
Contained anger: $U^*_{xf} \geq 0$ *and* $(\rho + \pi + 2\delta)U^*_{xf} < -U^*_{ff}$. *Emmanuel would marginally want more expensive clothes. However, anger would still decrease his cloth expenditure as the possible future emotional cost as measured by* U^*_{ff} *is higher than* $(\rho + \pi + 2\delta)U^*_{xf}$.

All out anger: *If* $U^*_{xf} > 0$ *and* $\Omega^*_A < 0$ *an increase in anger today will increase investment tomorrow in case of failure. In this case, the effects of anger's appraisal tendencies are so powerful that they dominate Emmanuel's emotional trade-off, and he increases her effort whenever frustration rises.*

As such, one cannot make a one-to-one revealed preference argument on emotions. If cloth expenditure increases, one can be sure Emmanuel was angry. However, if it decreases, one can only say that appraisal tendencies are either negative or positive and strong enough to counterbalance the emotional cost, as contained anger and sadness are behaviourally equivalent.

This point is essential, as economic applied works are starting to cite the appraisal-tendency framework to build and guide their analysis, for example, John Griffith and Shen (2020); Brooks et al. (2023); Meier (2022). This analysis shows that one must be cautious when inferring the emotional consequences on actions.

On the one hand, an emotional state might not always lead to the stereotypical reaction to the emotion. Similarly, one cannot always infer what emotion was triggered by an event by looking at how the agent reacted. This might explain why the literature has sometimes produced mixed results when trying to understand the effect of emotions on action outside the lab, see Brooks et al. (2023) for a review.

3.2 The effect of the controllability of the environment

This section shows how the perceived control of an environment influences emotional reactions stemming from frustrating events. Perceived control is generally seen as a critical driver in emotional responses. People who believe they control their environment more also tend to be more proactive in solving the problem causing their distress (Smith and Ellsworth, 1985; Lerner and Keltner, 2001). It is also one of the main features differentiating individuals who state they are angry or sad, even without considering appraisal tendencies. One appealing feature of this framework is that its solution provides action tendencies (increase or decrease investment) that resemble the reaction of anger and sadness, depending on how they control their environment.

To show this, let us adapt the general framework. I equate control over the environment with the control over the probability of success during each time interval dt, given by $\bar{\pi}(x_t) = \gamma \cdot x_t dt$, with $\gamma > 0$. The higher the γ, the higher the control. These situations are essential in many critical economic situations. Principal-agent models almost exclusively deal with agents whose effort increases the probability of success. In general, many real-life situations outside finance involve people increasing their efforts to increase their chances of success, like student training for an exam. Let $\phi(x_t, t) = \int_0^t \gamma x_s ds + \rho t$. The fact that the probability of success depends on investment is crucial. Investment increases the chances of a successful investment. However, more importantly, it increases the chance of getting rid of the current frustration stock. This means that even though it is a negative emotion, frustration can create an emotional incentive to invest more. Let us use the following specification to fix ideas.

$$U(x_t, f_t) = \gamma \cdot x_t u - v(f_t) - c(x_t) \tag{7}$$

Success utility $u > 0$ is fixed, and the probability of having a success during dt, $\gamma \cdot x_t \cdot dt$, depends on x. Notice that $U_{xf} = 0$: there are no appraisal tendencies, and increasing f_t does not change the marginal value of investment x_t at time t. However, this does not mean that frustration does not affect the inter-temporal marginal value of investment. Indeed, the relevant objective function to consider at time t is $F(x_t, f_t) = e^{-\phi(x_t,t)} U(x_t, f_t)$. The effect of x_t is inter-temporal by nature as an increase in x_t today changes the probability of continuing tomorrow. Even though today's frustration cost is sunk, as x_t does not affect $v(f_t)$, it affects the probability of still being frustrated tomorrow. Frustration impacts the marginal utility of investment because $F_{xf} > 0$. Solving the optimization problem G.O given the specification described here, (see Appendix 6.1) yields the following Proposition:

Proposition 2. *Let* $\Omega^*_C = \frac{1}{U_{xx}} \left(U^*_{ff} - (\rho + \gamma \cdot x^* + 2\delta) \frac{\gamma U^*_f}{\rho + \gamma \cdot x^* + \delta} \right)$.

1. *An increase in frustration leads to an increase (decrease) in investment provision if and only if* $\Omega^*_C < (>) 0$*,*

2. *The system exhibits saddle path stability as long as* $\Omega^*_C > -\delta(\rho + \gamma \cdot x^* + \delta)$*.*

Ω^*_C and Ω^*_A share the same structure. On the one hand, the potential future emotional cost U^*_{ff} remains. On the other hand, frustration is now accompanied by another factor multiplied by $(\rho + \gamma \cdot x^* + 2\delta)$. This second factor represents the shadow cost of frustration at the steady state multiplied by γ, the marginal effect of investment on the arrival rate of success. It represents how effective additional effort is at avoiding the additional emotional cost a new failure would entail. Naturally, this depends on how effective investment is at increasing the odds of success and how much the potential increase in frustration will decrease utility.

The sign of Ω^*_C will determine the system's dynamics, and the decision-maker will face a similar trade-off as with Ω^*_A. As such, I will not spend too much time on it. However, contrary to the previous section, where appraisal tendencies could influence investment either way, environment controllability can only increase the investment level when frustration rises. This is in line with the psychological literature (Smith and Ellsworth, 1985; Lerner and Keltner, 2001).

4 Illustration

This part aims to provide an illustration that fits the model's set-up and test its building blocks. It also shows how one can use an event-based approach to emotion to get meaningful information about the emotional process to which the individuals respond. To do so, I use Major League Baseball pitching data to study the effect of frustration accumulation on pitcher behaviour.

4.1 The Dataset and Baseball Rules

I first give an extremely summarised version of Baseball rules. Readers acquainted with baseball can skip the following three paragraphs. The basic unit of play is called an inning and works like the half-time in other sports, except there are typically nine innings (or more) in a baseball game. An inning comprises two half-inning periods where attacking and defending teams switch roles. I focus on players called *pitchers*, who are in the defending team, throwing the baseball at another teammate, called the *catcher*. Between these two players is a third player called the *batter*, who is in the offensive team and tries to score points.

The batter aims to hit the ball to advance as far as possible around four bases while the defending team tries to get the ball back to the catcher.[23] If the batter does not reach

[23]The four bases create a square, with each side measuring 27.43 meters (90 feet) in length.

the fourth base (home plate), he waits on one of the bases for the next opportunity to advance during the following plays. A base with a player on it is "loaded". Once a batter is out or on a base, one of his teammates replaces him, and the process starts again. If one member of the offensive team goes around the four bases, the team scores a run (a point). The team with the most runs at the end of the game wins.

A pitcher's primary goal is to throw pitches that are difficult for the batter to hit. His first objective is to prevent the batter from scoring runs. The second is to get three batters out (which ends the current half-inning), thus preventing the other team from scoring further runs. The four more important pitch outcomes are (1) balls: if the pitch is not thrown correctly, it is considered a ball, and four balls grant first base to the batter. (2) strikes, if the batter did not hit a valid pitch in a valid area, the batter is out with three strikes;[24] (3) the batter bats the pitch in a valid area: it is in game.

The data set consists of measurements for every pitch thrown during the 2010-2019 period retrieved on a Major League Baseball (MLB) run website for more than 7 million pitches. Physical information about the pitches is gathered through the PITCHf/x system for the 2010-2015 period and Statcast for 2015-2019. These two systems are automated camera systems developed to analyse player's movements. Physical information about the pitches includes speed at release, location of the pitch at different points in time, spin rate, spin angle, acceleration etc... The dataset includes detailed game information, including game, pitcher and batter identifiers, base occupancy and strike, and ball and out counts. The MLB also provides information about the batter team's change in run-expectancy before and after each pitch. The information was retrieved using the python package pybasebeball.[25]

4.2 How to measure investment for pitchers

The primary dependent variable for the analysis is pitch velocity. This is a natural choice as it is arguably the variable over which the pitcher has the most control. Moreover, it has a natural interpretation. Faster pitches should increase the odds of success until the pitcher starts to lose precision. As I argue below, pitchers should always throw their pitches at an optimum speed, given a pitch type and a strategic environment. As such, one can interpret any significant frustration coefficient as over- or under-investment relative to the optimal level with the appropriate control strategy.

[24] A pitch batted in a non-valid area is called a foul.

[25] See `https://pypi.org/project/pybaseball/`

Isn't Baseball a strategic game? It is true that changing the speed of the pitch might be a strategy to surprise the batter. However, baseball culture has developed so that pitches with significant differences in speed are also categorised differently. Each category is called a pitch type and differs in terms of speed, variations in release point, spin, and mechanics that affect the ball's velocity.[26] For example, the slow version of a (four-seam) fast-ball is called a change-up and is, as such, categorised differently. Much of a pitcher's strategy is expressed through the strategic randomisation of the type of pitch he is throwing to catch the batter off-guard. From a game theoretic point of view, a pitcher's optimal strategy should be a probability distribution over the pitches in his repertoire, which is a mixed Nash equilibrium. As my model does not consider such strategic considerations, I control for the different pitch types and eliminate this strategic component of the game from my analysis. In other words, my analysis exploits the fact that, within pitch type, speed should not be affected by strategic consideration.[27]

4.2.1 Objective of the analysis

I structure this analysis around three axes. (1) Does frustration impact pitch velocity? I will not focus on the sign of the impact but on whether it is statistically and economically significant. If it is, then building a model of frustration accumulation is warranted. (2) The second objective is to focus on the most essential building block of the model: frustration dynamics. First, frustration decays with time in the framework. Second, the model's assumption that success brings emotional relief is crucial. As such, I expect to observe these two features in the data. (3) The final objective will be to determine through which channel frustration affects pitch velocity: frustration's expected emotional cost, frustration's appraisal tendencies or/and the controllability of the environment. The result section 4.6 is organised around these three axes.

4.3 Frustration and pitcher's utility for speed

To understand how speed affects the pitcher's expected utility for speed, given a frustration level, let me introduce a functional form to guide the analysis. The catcher and pitcher

[26] Pitcher and catcher must communicate using a secret and limited sign language. Using signs, the catcher proposes a pitch type that the pitcher should throw and prepares accordingly to catch it. Note that this specificity evolved because all MLB players understand English. In contrast, in cricket, this is only sometimes the case as pitchers and catchers can communicate more precisely using local dialects that the other team cannot understand.

[27] Although I carefully control for possible additional confounders.

first choose the pitch they want to play. I focus on the pitcher's expected utility once this decision is made. Let x_t be the pitch speed, with $x_t = 0$ being the optimal speed given the pitch type and the strategic environment. Let s_t denote the game's strategic state at time t and let u_{s_t} measure the success utility of a pitch and a state of the game s_t. Finally, I represent the possible appraisal tendency effect of frustration on speed by $\mu(x_t, f_t)$. I assume that $\mu(x_t, 0) = 0$ and $\mu_{xf} > (<) 0$ if there are positive or negative appraisal tendencies. The expected utility is:

$$U(x_t, f_t, s_t) = \gamma x_t u_{s_t} + \mu(x_t, f_t) - v(f_t) - c(x_t) \tag{8}$$

The fact that the probability of success only increases linearly reflects the fact that the estimation will be a local one. Even though the effect of speed on the probability of success is probably hump-shaped, the dataset only comprises the best pitchers in the world. As such, their training must be such that we will not observe significant deviation from the optimum. In that case, considering a linear approximation of the effect of speed on the probability of success seems appropriate. Equation (8) shows the challenges of this illustration: the success utility is game-state dependent. As such, it will be capital to control for such game-state effects as they would naturally affect speed.[28] Finally, note that the expected utility exhibits an endogenous success probability and appraisal tendencies. As such, the analysis here will be derived from the general form of the problem shown in (G.O) and solved in the Appendix.

4.4 Frustration definition and measurement

In this section, I present the definition of my variable of interest, frustration. At every point in time, it is possible to compute the expected number of runs the opposing team can score in the rest of the half-inning. Let R_t be the run expectancy of the opposing team during this half-inning before throw number t, where $t \leq T$ and T are the number of consecutive throws the pitcher does during this half-inning. The pitcher's objective is to prevent the other team from scoring runs, that is, to always have $R_{t+1} - R_t + runs_t < 0$, where $runs_t$ indicates the number of runs the other team scored at time t. Fortunately, this difference in expectation is provided by the MLB.[29] I can then classify pitches' outcomes

[28] Notice that success utility is not frustration dependent, as it was before when I introduced appraisal tendencies. I explain in section 4.6 why this would not affect the analysis.

[29] Computing the expected number of runs for any game state is relatively straightforward as it is simply the historical average number of runs that are scored until the end of the innings given the current game

by their effect on the expected score. This yields the following definition of failure:

Definition. *A pitch a time t is considered a failure if it increases the run expectancy of the opposing team, that is, if $R_{t+1} - R_t + runs_t \geq 0$.*

Now, I need to measure the level of frustration associated with any failure in the game. I consider the difference in run expectancy before and after a failure as my frustration proxy. Doing this allows me to get two desirable properties. First, the measure is in runs and is easily interpretable. Second, it objectively measures the frustration intensity, as different failures can have other repercussions. For example, a ball at the beginning of an inning will have a negligible impact on the expected game score[30], while a home run or a fourth ball with three bases loaded can be decisive. Accordingly, the measure reflects that more minor mistakes are less frustrating and that the same event can trigger different levels of frustration depending on the state of the game[31].

Definition. *The frustration triggered by a failure at time t, Δf_t is:*

$$\Delta f_t = R_{t+1} - R_t \geq 0 \tag{9}$$

Next,[32] following the model's set-up, the frustration level increases by the (positive) difference in run expectancy in case of a frustrating event and goes to 0 following success. Let C_t represent the set of time period s that are in a string of consecutive failure before t, that is:

$$C_t = \{s < t : \Delta f_s > 0 \,, \text{and} \,, \forall s' :, t > s' \geq s \, : \Delta f_{s'} > 0\}$$

Definition. *The stock of frustration at the beginning of period $t \leq T$, F_t is:*

$$F_t = \sum_{s \in C_t} \Delta f_s$$

state. The game state description mirrors the Game State FE presented in Section 4.5.

[30]Indeed, some baseball fans would even be reluctant to consider a ball at the beginning of an inning as frustrating. A ball at the beginning of an inning would increase frustration by 0,034. This is negligible but still worse than any other non-frustrating outcome (strikes, fouls, ...)

[31]Notice that the measure can sometimes miss subtleties specific to the throw. For example, it is possible to have a positive difference in run expectancy after flawlessly executed pitches where the rest of the team did not live up to the pitcher's performance or for failures where the pitcher was at fault. In other words, this measure only takes the effect of the pitch on the game's outcome into account without characterising who is responsible for the poor performance. However, regardless of the source of the emotional reaction, the measure should be a good proxy for its intensity.

[32]The previous definition also considers a foul ball after two strikes as a failure, but with a frustration value of 0. As such, it is a neutral event that does not break strings of consecutive failures.

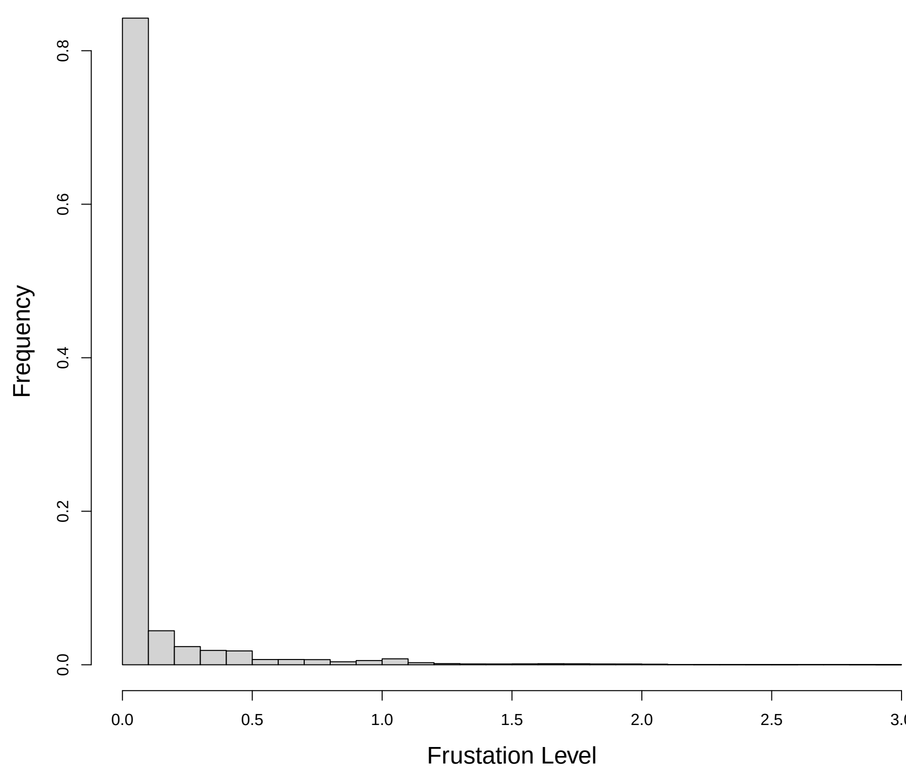
As such, in case of consecutive failures, the measured frustration law of motion is:

$$F_{t+1} = \Delta f_t + F_t,$$

while $F_{t+1} = 0$ if there is a success at time t, as C_{t+1} is empty. Note that this means that the frustration does not directly depend on the pitcher's action but only on the consequences of those actions and the state of the game. In other words, frustration increases will not depend on the pitcher's investment (speed) in the pitch. Moreover, this *measured* law of motion does not feature a decay rate δ, while the *actual* law of motion does. I get back to this when studying frustration's temporal effect in section 4.6 to study the value of δ.

Figure 2 gives insight into how the frustration stock is distributed in the dataset.

Figure 2: Frustration level histogram

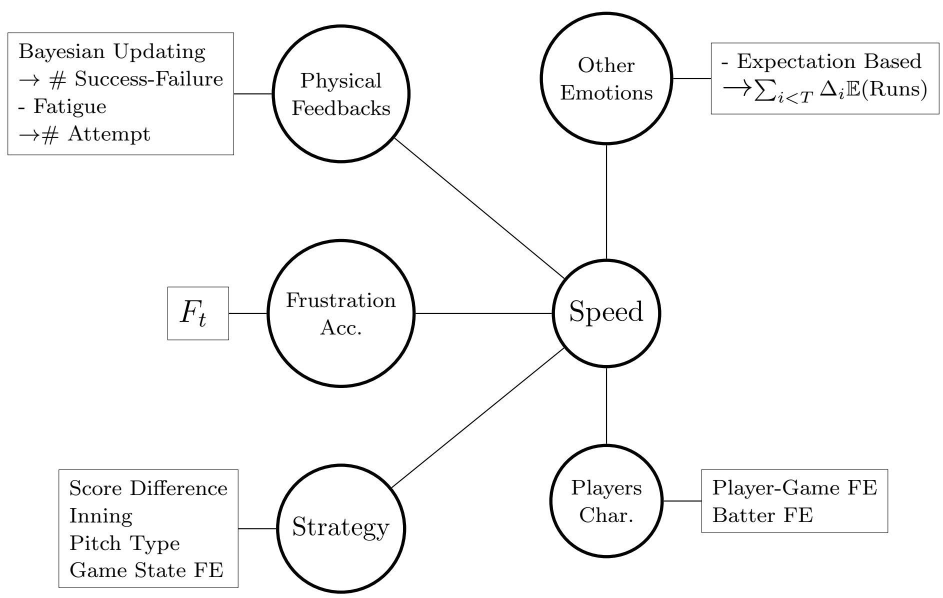


Having a high frustration stock is not common. This is natural. First, the definition chosen is somewhat extreme, as frustration goes to 0 after any success. An alternative could be to have only a share of F_t remaining after a successful pitch. However, determining the level of the share seems arbitrary. I prefer to show that the results hold under the most extreme assumption. Second, remember that given the frustration is measured in runs. As such, it seems natural that professional pitchers usually do not concede high numbers of consecutive runs without being replaced.

4.5 Estimation Strategy

Even though it is impossible to develop a quasi-experiment in this dataset, the sheer amount of information available for every pitch makes it possible to control for most confounders that could bias the analysis results. Before investigating the potential effect of frustration on speed, one must review all the possible reasons why a pitcher might vary the speed of a given pitch. Figure 3 gives a schematic and simplified view of the possible influence one must control and Table 1 describes the controls used in the regressions.

Figure 3: Schematics of factors that can influence pitch velocity



Note: Schematic view of the possible confounders of the analysis. Note that some elements could also be in other categories, and the links between categories are not represented. I mention these overlaps between categories in the discussion below. Δ Exp., represents the difference in run expectancy between the current throw and the beginning of the half-inning.

Strategy: As mentioned in section 4.2, much of a pitcher's strategy is expressed through the strategic randomisation of the type of pitch he is throwing to catch the batter off-guard. To control for this, I use MLB's pitch type classification.[33] Note that even though strategy affects pitch selection, pitch speed given a game state and a pitch type should not vary.

Other strategic interactions might also affect the pitches' speed. Specifically, the value of the success utility u_{s_t} in the pitcher's expected utility (8) depends on s_t. For example, suppose a game or plate appearance is not particularly competitive. In that case, the pitcher might decide to reduce the speed of his pitches to save his arm. The *Game State* fixed effect is a categorical variable that tries to describe the current state of an inning comprehensively. The objective is to capture Game-State dependent strategic considerations affecting the speed of a given throw. Each category is a 6-digit number, representing the number of outs, strikes and balls, whether a player is on first, second or third base.[34] Finally, I also control for Batter and Pitcher $\times$ Game fixed effect, inning number and difference in score to account for the plausible strategic impact on player's behaviours.

Physical feedback: Belief updating might play an essential role in a pitcher's pitch velocity decision. Here, I focus on belief updating, although these fixed effects could capture strategic incentives to some extent. First, a player might update his beliefs about his current ability by looking at the performance of his throws. As such, I create a variable counting the number of successes and failures. Assuming that the agents consistently update failures and successes, this should proxy the effect of Bayesian updating. It also allows for controlling for asymmetric updating where the agent puts relatively more weight on bad news, for example. I also control for the number of throws the pitcher already threw in an inning to consider fatigue.

Other emotions: Emotional triggers are usually belief dependent Keltner and Lerner (2010). To control for other possible emotions, I include Δ Exp., which represents the difference in run expectancy between the current throw and the beginning of the half-inning. Many emotions that frustrating events might not trigger are rooted in changes in beliefs and expectations. As such, this should, together with Bayesian updating, capture the effect of other belief-based emotions. Finally, one crucial feature I want to test is the relief the pitcher is supposed to feel once he succeeds after a string of failures. Then, if there was no relief effect, this variable should capture the effect of the frustration coefficient.

Player Characteristics: Sometimes, physical characteristics can influence pitches'

[33] They use a player-specific neural network called PitchNet to classify pitches. The accuracy reaches 78.30% for rookies and 96,25% for regular players. See Sharpe and Schwartz (2020).

[34] For example, a Game State variable being equal to 213101 describes an inning with two outs, one strike, three balls, one player on first base, no player on second and one on the third.

velocity. Temperature, weather, or a pitcher's current condition can significantly impact a pitcher's performance. However, it should also be easily controlled with the Pitcher × Games fixed effect.

Table 1: Controls for regressions

Possible factors	Controls
Fatigue	*# Attempt*: Number of pitches in Inning
Player's current ability	*Game × Player FE*
Strategy on a given game	*Game × Player FE*
Strategy in a Game State	*Game State FE*: #Out × #Strikes × #Balls × Bases load
Batter characteristics	*Batter FE*
Pitch type	*Pitch type FE*
Score	*Difference in Score*
Score in inning	Δ *Inscore*: Difference in score in the inning.
Inning	*Inning FE*
Bayesian Updating	*# Failures and Successes*
Other expectation-based emotions	Δ *Exp.*: Net Difference in exp. since start of Inning

Given this, I estimate the following regression using a multi-way fixed effect estimator:

$$\begin{aligned}Speed_{gpti} = {} & \beta_0 + \beta_1 F_{gpti} + \beta_2 \Delta Exp._{gpti} \\ & + \beta_3 GameState_{gpti} + \beta_4 PitchType_{gpti} + \beta_5 Batter_{gpti} + \beta_6 \Delta Score_{gpti} + \beta_7 \Delta InScore_{gpti} \\ & + \beta_8 \#Attempt_{gpti} + \beta_9 \#Failures_{gpti} + \beta_{10} \#Successes_{gpti} \\ & + \beta_{11}\alpha_{gp} + \beta_{12}\alpha_i\end{aligned}$$

Where the subscripts g denote game, p denotes pitcher, t denotes the throw number, and i is the inning, α_{gp} is the $Player \times Game$ fixed effect, α_i is the inning fixed effect.

4.6 Results

4.6.1 Frustration's effect on pitchers' behaviour

Table 2 presents the main result of this section. One additional unit of frustration $\mathbf{F_t}$ (measured in runs) increases the speed of pitches by 0.12 mph (0.19km/h). Depending on the type of pitch thrown, this accounts for an increase in speed between 0.08% to 0.17%.[35]

[35] See table 8, in the Appendix, for more detailed results

The careful strategy to control for cofounders also seems to work with an adjusted R^2 of 0.912.[36].

To better understand the magnitude of the effect, let us compare it to the impact of fatigue, as defined in Table 1. The effect of one standard deviation increase in frustration (0.27 runs) accounts for 45.72% of the magnitude of one standard deviation increase in the fatigue gained during an inning by a pitcher (5.16 throws). Suppose one considers the standard deviation of frustration for positive frustration values only. In that case, the effect rises to 64.70 % of one standard deviation increase in fatigue[37]. As such, frustration seems to have a non-negligible impact on pitcher's behaviour.

Emotional Relief

One crucial building block of the model is that success brings emotional relief, in that frustration's effect disappears after success. Notice the frustration coefficient is positive and significant while controlling for Δ Exp.. In other words, the impact of frustration cannot be captured by a net effect of an elation/frustration function that Δ Exp. (the net difference in run expectancy since the beginning of the inning) should capture. One can interpret this as evidence of an asymmetric drop in F_t after a success. There must be a relief and accumulation effect, as in F_t's construction.

Temporal Effects

Finally, the central insight from this paper is that emotions are inter-temporal processes: frustration accumulates and decays with time. Table 1 analysis shows that frustration, under the form of the variable F_t, affects pitchers' behaviour. However, it does not explore the temporal effect of frustration accumulation. F_t can increase by the same amount over one or several failures. If frustration has a temporal effect, then the marginal frustration Δf_t, as defined in equation (9), gained several failures ago should still affect pitches' speed in case of consecutive failures.

As such, I first construct a set of dummy variables indicating whether there were more than 1,2 and up to 9+ consecutive failures. I then interact with the dummy, indicating one or more failures with frustration gained last period, two or more failures with the

[36]Admittedly, the frustration measure has a negligible impact on the adjusted R^2 of the remaining variance to explain. This is unsurprising and similar to the explanatory power in causal identification techniques after differencing away the trends.

[37]This is a sensible strategy as the frustration stock equals 0 in 80% of the sample)

Table 2: Effect of Frustration on Velocity and Quality

Dependent Variables:	Velocity	
Model:	(1)	(2)
Variables		
$\mathbf{F_t}$	0.1221***	0.1221***
	(0.0048)	(0.0034)
# Failures	0.0189***	0.0189***
	(0.0014)	(0.0008)
# Successes	0.0345***	0.0345***
	(0.0016)	(0.0007)
Attempt	-0.0143***	-0.0143***
	(0.0012)	(0.0003)
Δ Exp.	0.0270***	0.0270***
	(0.0071)	(0.0062)
Δ Score	0.0007	0.0007
	(0.0021)	(0.0012)
Δ InScore	-0.0887***	-0.0887***
	(0.0075)	(0.0060)
Fixed-effects		
Pitch Type	Yes	Yes
Game State	Yes	Yes
Player × Game	Yes	Yes
Batter	Yes	Yes
Inning	Yes	Yes
Fit statistics		
Observations	7,199,443	7,199,443
R^2	0.91726	0.91726
Within R^2	0.00235	0.00235

*Signif. Codes: ***: 0.01, **: 0.05, *: 0.1*

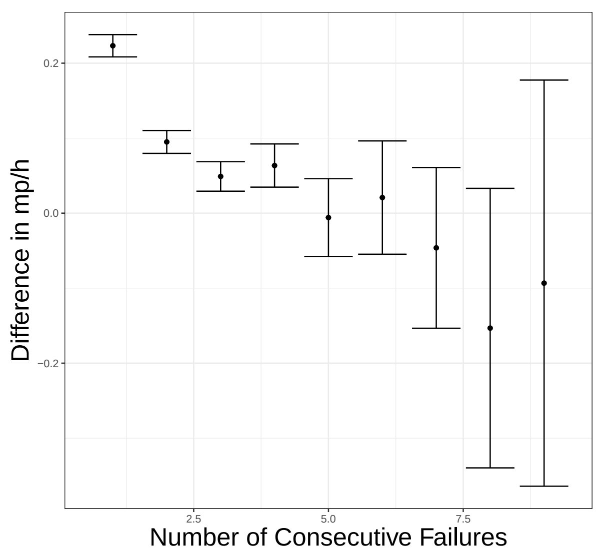
Column (1) is clustered at the pitcher level, and column (2) at the game level. $\mathbf{F_t}$ measures the change in the other team's expected score during the current and (possibly empty) sequence of consecutive failure. # Failures and # Successes count the number of successes and failures since the beginning of the inning. *Attempt* measures the number of pitches the pitcher has thrown since the start of the game. Δ Exp measure the change in the opposing team's expected score since the beginning of the inning. Δ Score measures the current difference in score, while Δ InScore measures the change in score since the beginning of the inning. *Pitch Type* is a categorical variable indicating the pitch type. Game State is a categorical variable indicating the number of batters out, the number of balls, strikes and whether a player is on base 1, 2 or 3. Player × Game indicates who is pitching during which game. *Batter* indicates who is currently facing the pitcher.

frustration gained two periods ago and so on. Generally, for an observation at time t, these interactions are:

$$\mathbb{1}[\#Consecutive\,Failures \geq n] \times \Delta f_{t-n}$$

These interactions allow the n^{th} frustration lag to have an effect only if there were at least n^{th} consecutive failures. The $n > 1$ coefficients should be significant if frustration accumulates over time. Moreover, if frustration decays with time, the magnitude of the interactions' coefficient should decrease with the number of lags. Indeed, if frustration decays, it seems natural that frustration gained five pitches ago has a lower impact on speed than the frustration stemming from the last failure.

Figure 4: Temporal Effect of Frustration



Each dot on the graph represent the interaction term $\mathbb{1}[\#Consecutive\,Failures \geq n] \times \Delta f_{t-n}$ for $1 \leq n \leq 8$. The 9^th dot represents the interaction between lags of Δf greater or equal to 9 and the dummy indicating more than nine consecutive failures. The confidence interval displayed on the graph are associated with 5% confidence level. Clusters are at the player level.

Figure 4 represents the value of the coefficients of the interactions and their confidence intervals (95%) for an OLS with these interactions, the dummy indicating the number of cumulative failures and the set of control presented in table 3. Frustration seems to have a temporal effect, with frustration from up to four failures ago still significantly impacting speed (at a 5% significance level). Second, frustration also seems to decay as the coefficient value gradually decreases with non-significant values if failures occurred more than three throws ago, indicating a substantial decay rate of frustration over time.

The mechanics of frustration: Emotional cost, Appraisal Tendencies or Controllability of the environment?

Three factors can influence pitch speed: future potential emotional cost, the effect of speed on the probability of success (γ) and appraisal tendencies μ_{xf}. According to the frustration measure, the future potential emotional cost, U^*_{ff}, should not influence behaviour. Indeed, the frustration level in case of failure is exogenous to the pitch's speed. Varying pitch speed should not affect his future emotional cost if he fails. As such, the anticipated emotional cost should not influence the pitcher's velocity decision.

We are left with two possible factors: appraisal tendencies of frustration and the effect of speed on the probability of success, i.e. the controllability of the environment. Let us start with the latter. Intuitively, faster pitches are more challenging to hit, so speed should positively affect the odds of success and the chance of alleviating frustration. The following regression tries to estimate the impact of speed on the probability of success, i.e. γ.

Table 3 presents the result of an OLS regressing the speed and the log of the speed of pitches on the probability of success. Table 1's footnotes describe all covariates. Speed has a statistically significant but economically small effect on the likelihood of success. Increasing the release speed by one per cent increases the chance of success by less than 0.0014%. To compare this to natural speed variations in the data, one can consider the average game and pitcher-specific standard deviation in speed at the inning level. Table 4 in Appendix 9.1 presents this standard deviation for different pitch types. The standard deviation is, in general, around 1. As such, according to columns (3) and (4), a pitcher would need to increase their pitch speed during an inning by a heroic five standard deviations to increase their probability of success by 1%. Of course, this estimate should be considered as a local effect, as pitchers cannot significantly increase their pitches' speed without losing precision or risking injury.

Given the low adjusted R^2, velocity does not seem to be a credible explanatory factor to predict success. Overall, Table 3 results are not surprising. MLB pitcher's market is competitive, making it hard to believe that a pitcher would not throw at the best of their capabilities given the strategic environment.

Moreover, Appendix 9.3 shows a regression result measuring how frustration affects the quality of the pitches.[38] In general, pitch quality tends to decrease with higher frustration

[38] The quality of the pitches is determined using a machine learning algorithm. I use the physical characteristics of the throw as an ex-ante predictor of success to measure their quality. See appendix 9.2 for more details on this.

Table 3: Effect of Speed on probability of success

Dependent Variable:	$P(Success)$			
Model:	(1)	(2)	(3)	(4)
Variables				
Speed	0.1463***	0.1463***	0.0020***	0.0020***
	(0.0283)	(0.0097)	(0.0003)	(0.0001)
# Failures	0.0193***	0.0193***	0.0193***	0.0193***
	(0.0006)	(0.0002)	(0.0006)	(0.0002)
# Successes	-0.0176***	-0.0176***	-0.0177***	-0.0177***
	(0.0007)	(0.0002)	(0.0007)	(0.0002)
Attempt	0.0058***	0.0058***	0.0058***	0.0058***
	(0.0004)	(8.77×10^{-5})	(0.0004)	(8.78×10^{-5})
Δ Exp.	-0.0209***	-0.0209***	-0.0210***	-0.0210***
	(0.0027)	(0.0015)	(0.0027)	(0.0015)
Δ Score	-0.0050***	-0.0050***	-0.0050***	-0.0050***
	(0.0005)	(0.0003)	(0.0005)	(0.0003)
Δ Inning Score	0.0081***	0.0081***	0.0081***	0.0081***
	(0.0027)	(0.0015)	(0.0027)	(0.0015)
Fixed-effects				
Pitch Type	Yes	Yes	Yes	Yes
Game State	Yes	Yes	Yes	Yes
Player × Game	Yes	Yes	Yes	Yes
Batter	Yes	Yes	Yes	Yes
Inning	Yes	Yes	Yes	Yes
Fit statistics				
Observations	7,199,443	7,199,443	7,199,443	7,199,443
R^2	0.05682	0.05682	0.05683	0.05683
Within R^2	0.00781	0.00781	0.00782	0.00782

*Signif. Codes: ***: 0.01, **: 0.05, *: 0.1*

Columns (1) and (3) are clustered at the pitcher level and columns (2) and (4) are clustered at the game level. Columns (1) and (3) Speed measure is the pitch's velocity when released by the pitcher in mph. Columns (3) and (4) Speed measure is the natural logarithm of the same variable. # Failures and # Successes count the number of successes and failures since the beginning of the inning. Attempt measures the number of pitches the pitcher has thrown since the beginning of the game. Δ Exp measure the change in the opposing team's expected score since the beginning of the inning. Δ Score measures the current difference in score, while Δ Inning Score measures the change in score since the beginning of the inning. *Pitch Type* is a categorical variable indicating the pitch type. Game State is a categorical variable indicating the number of batters out, the number of balls, strikes and whether a player is on base 1, 2 or 3. Player × Game indicates who is pitching during which game. *Batter* indicates who is currently facing the pitcher.

levels. As such, the controllability of the environment does not seem to be the main driver of the emotional reaction, as frustration would increase pitch quality in that case.

Empirical conclusion:

Overall, the analysis points to appraisal tendencies as the main culprit behind the emotional reaction. Frustrated pitchers throw faster simply because they have a higher marginal utility to do so. This also seems to translate into a loss in efficiency as the quality of their pitches tends to decrease. In Appendix 9.5, I carry out the same analysis at the individual level and take a closer look at frustration's effect on the pitches' quality. A higher effect of frustration on speed is associated with a more negative impact on quality. This rules out any rationale for saying that frustration motivates pitchers to reach a better optimum and strengthens the conclusion that appraisal tendencies are the source of the changes. In the same appendix, I show that frustration's effect on quality also seems to translate to lower career-level performance statistics. This indicates that frustration accumulation is more than an interesting behavioural phenomenon and could have labour market implications by affecting pitchers' productivity.

5 Conclusion and Discussion

This paper establishes a framework to study the inter-temporal effects of emotions on investment. I find that the impact of frustration depends mainly on the relative force of the negative emotional shock induced by the frustrating events, its appraisal tendencies and the decision-makers control over her environment. More importantly, negative emotions do not always trigger negative consequences in terms of investment, and positive appraisal tendencies stemming from them are not necessary nor sufficient to have a positive effect of frustration on motivation.

Overall, the empirical investigations seem to confirm that pitcher's behaviour is influenced by frustration and that frustration accumulates and decays. Frustration appears to increase pitches' speed at the average and individual levels and decrease their quality. A strong enough effect of frustration on quality can reduce a pitcher's career-level performance and productivity. This highlights the relevance of studying the dynamics of emotions in economics.

All models presented here assume a forward forward-looking and time-consistent decision-maker. This assumption is not innocuous but standard in the literature as most emotion models work exclusively regarding expected emotion (Loomes and Sugden, 1982; Bell, 1982, 1985; Loomes and Sugden, 1986; Caplin and Leahy, 2001). Yet, while individuals anticipate their emotions reasonably well, they are also prone to systematic mistakes (Loewenstein, 1999). Similarly, individuals only seem to be partially aware that the appraisal tendencies of their emotions are present. Moreover, their effect tends to diminish when they become aware of their existence. However, in the context of this paper, the deactivation of the appraisal tendencies seems to be more the exception than the rule, as they tend to persist when the emotional goal (a success) has not been fulfilled (Han et al., 2007).

Nonetheless, the mechanisms and trade-offs put forward here are surprisingly intuitive, albeit stylised. They also show to what extent "forward-lookingness" must be present to get very intuitive behaviour. For example, if an agent cannot predict that her frustration will disappear after a success, investing more effort to solve the problem would not make sense. Similarly, not anticipating the potential emotional cost of an action is a natural way of limiting the effect of appraisal tendencies effects on behaviour. In other words, emotions must be understood as dynamic processes. Going into models with several selves defined by their frustration could bring interesting results and could be pursued in further research to see how sophisticated time-inconsistent decision-makers could behave.

6 Models solutions

6.1 Solving the General model

To make the problem more tractable, let us introduce the following state variable: $\Phi_t = \phi(x_t, t) = \int_0^t \gamma x_s ds + rt$, where r is the time invariant discount rate, that might include a Poisson law fixed arrival rate π. In Section 3.1, we have $\gamma = 0$ and $r = \rho + \pi$. In section 3.2, $\gamma > 0$ and $r = \rho$. We can then rewrite the maximisation problem in the following way:

$$
\begin{aligned}
V &= \max_{x_t} \int_0^\infty e^{-\Phi_t} U(x_t, f_t) dt \\
\dot{f}_t &= x_t - \delta f_t \\
\dot{\Phi}_t &= \gamma \cdot x_t + r \\
& f_{t_0} \text{ given} \\
& \Phi_0 = 0
\end{aligned}
$$

The Hamiltonian of this problem is:

$$
\tilde{H}(x_t, f_t, \tilde{\lambda}_t, \tilde{\mu}_t) = e^{-\Phi_t} U(x_t, f_t) - \tilde{\lambda}_t (x_t - \delta f_t) - \tilde{\mu}_t (\gamma \cdot x_t + r)
$$

$$
\begin{aligned}
&\tilde{H}_x(\cdot) = e^{-\Phi_t} U_x(x_t, f_t) - \tilde{\lambda}_t - \tilde{\mu}_t \gamma = 0 \\
&\tilde{H}_f(\cdot) = e^{-\Phi_t} U_f(x_t, f_t) + \delta \tilde{\lambda}_t = \dot{\tilde{\lambda}}_t \\
&\tilde{H}_\Phi(\cdot) = -e^{-\Phi_t} U(x_t, f_t) = \dot{\tilde{\mu}}_t \\
&\lim_{t \to \infty} \tilde{H}(\cdot) = 0
\end{aligned}
$$

Where the last equation is a general transversality condition suited for endogenously discounted inter-temporal maximisation problem see (Michel, 1982; Barro and Sala-i Martin, 2004). Next, notice that:

$$
\frac{d\tilde{H}(\cdot)}{dt} = \dot{x}_t \tilde{H}_{x_t}(\cdot) + \dot{f}_t \tilde{H}_{f_t}(\cdot) + \dot{\tilde{\Phi}}_t \tilde{H}_{\Phi_t}(\cdot) - \dot{\tilde{\mu}}_t \dot{\Phi}_t - \dot{\lambda}_t \dot{f}_t = 0
$$

Together with the transversality condition, this means that on the optimal path, the maximised Hamiltonian $\tilde{H}^*(\cdot)$ is always equal to 0. As such, we can rewrite:

$$\tilde{\mu}_t = \frac{e^{-\Phi_t} U(x_t, f_t) - \tilde{\lambda}_t (x_t - \delta f_t)}{\gamma \cdot x_t + r}$$

Let me introduce the following variables : $\tilde{\mu}_t = e^{-\Phi_t}\mu_t$ and $\tilde{\lambda}_t = e^{-\Phi_t}\lambda_t$.

The first order conditions become:

$$U_x(x_t, f_t) - \lambda_t - \mu_t \gamma = 0 \tag{A.1}$$

$$U_f(x_t, f_t) + (\delta + r + \gamma \cdot x_t)\lambda_t = \dot{\lambda}_t \tag{A.2}$$

$$- U(x_t, f_t) + (\gamma \cdot x_t + r)\mu_t = \dot{\mu}_t \tag{A.3}$$

$$\lim_{t \to \infty} \tilde{H}(\cdot) = 0 \tag{A.4}$$

The transversality condition (A.4) then implies that:

$$\mu_t = \frac{U(x_t, f_t) - \lambda_t (x_t - \delta f_t)}{\gamma \cdot x_t + r} \tag{B.1}$$

To make the following computations more tractable let me drop the time index and the argument of the expected utility $U(\cdot)$ and of its derivatives. Inserting (B.1) in the first order condition (A.1) yields:

$$\begin{aligned} & U_x = \lambda + \gamma \frac{U - \lambda(x - \delta f)}{\gamma \cdot x + r} \\ \Longleftrightarrow\ & (\gamma \cdot x + r)U_x = (\gamma \cdot x + r)\lambda + \gamma \left(U - \lambda(x - \delta f)\right) \\ \Longleftrightarrow\ & (\gamma \cdot x + r)U_x - \gamma U = \left(\gamma \cdot x + r - \gamma(x - \delta f)\right)\lambda \\ \Longleftrightarrow\ & \lambda = \frac{(\gamma \cdot x + r)U_x - \gamma U}{r + \gamma \delta f} \end{aligned} \tag{B.2}$$

If I insert this result back in (B.1):

$$\begin{aligned}\mu &= \frac{U - \lambda(x - \delta f)}{\gamma \cdot x + r} \\ &= \frac{U}{\gamma \cdot x + r} - \frac{(x - \delta f)}{\gamma \cdot x + r}\lambda \\ &= \frac{U(\gamma \cdot x + r) - (x - \delta f)(\gamma \cdot x + r)U_x}{(r + \gamma\delta f)(\gamma \cdot x + r)} \\ &= \frac{U - (x - \delta f)U_x}{(r + \gamma\delta f)}\end{aligned}$$

As such, we can now get the re-express the co-state law of motion μ described in (A.3) as:

$$\begin{aligned}\dot{\mu} &= -U + (\gamma \cdot x + r)\frac{U - (x - \delta f)U_x}{r + \gamma\delta f} \\ &= \frac{-U(r + \gamma\delta f) + (\gamma \cdot x + r)(U - U_x\dot{f})}{r + \gamma\delta f} \\ &= -\frac{\gamma U(x - \delta f) - (\gamma \cdot x + r)\left(U_x\dot{f}\right)}{r + \gamma\delta f} \\ \dot{\mu} &= -\lambda\dot{f} \end{aligned} \tag{B.3}$$

Let us now differentiate the first order condition A.1 with regards to time to get the Euler equation :

$$\begin{aligned} U_{xx}\dot{x} - \dot{\lambda} - \gamma\dot{\mu} + \dot{f}U_{xf} = 0 \\ \iff \dot{x} = \frac{1}{U_{xx}}\left(\dot{\lambda} + \gamma\dot{\mu} - U_{xf}\dot{f}\right) \end{aligned} \tag{EE}$$

The Euler equation (EE), together with the frustration law of motion (2) form the canonical system of the inter-temporal maximisation problem. Next, let us linearise the system around the steady state to study its dynamics. These are given by

$$\begin{pmatrix} \dot{\hat{x}}_t \\ \dot{\hat{f}}_t \end{pmatrix} = J^* \begin{pmatrix} \hat{x}_t \\ \hat{f}_t \end{pmatrix}$$

Where the variables denoted by hats are the rescaled investment and frustration, such that steady state value are normalized to 0. J^* is the Jacobian of the canonical system evaluated at the steady-state (x^*, f^*):

$$J^* = \left.\begin{pmatrix} \frac{\partial \dot{x}(x,f)}{\partial x} & \frac{\partial \dot{x}(x,f)}{\partial f} \\ \frac{\partial \dot{f}(x,f)}{\partial x} & \frac{\partial \dot{f}(x,f)}{\partial f} \end{pmatrix}\right|_{(x,f)=(x^*,f^*)}$$

To get the expression of J^*, we need to compute the value of the derivative of λ in (B.2) with regard to f and x at the steady state:

$$\begin{aligned} \left.\frac{\partial \lambda}{\partial f}\right|_{(x^*,f^*)} &= \frac{\partial}{\partial f}\left.\left(\frac{(\gamma \cdot x + r)U_x - \gamma U}{r + \gamma\delta f}\right)\right|_{(x^*,f^*)} \\ &= U^*_{xf} - \frac{\gamma\left(U^*_f + \delta\lambda^*\right)}{(r + \gamma \cdot x^*)} && \text{(B.3)} \\ \left.\frac{\partial \lambda}{\partial x}\right|_{(x^*,f^*)} &= U^*_{xx} && \text{(B.4)} \end{aligned}$$

Next using (B.3) and (B.4), we can differentiate the co-state law of motion (A.2) with regards to x and f

$$\begin{aligned} \left.\frac{\partial \dot{\lambda}}{\partial f}\right|_{(x^*,f^*)} &= \frac{\partial}{\partial f}\left(U_f + (\delta + r + \gamma \cdot x^*)\lambda\right) \\ &= U^*_{ff} + (\delta + r + \gamma \cdot x^*)\frac{\partial \lambda}{\partial f} \\ &= U^*_{ff} + (r + \delta + \gamma \cdot x^*)\left(U^*_{xf} - \frac{\gamma\left(U^*_f + \delta\lambda^*\right)}{(r + \gamma \cdot x^*)}\right) \\ \left.\frac{\partial \dot{\lambda}}{\partial x}\right|_{(x^*,f^*)} &= \frac{\partial}{\partial x}\left(U_f + (\delta + r + \gamma \cdot x)\lambda\right) \\ &= U^*_{xf} + \gamma\lambda + (r + \delta + \gamma \cdot x^*)U^*_{xx} \end{aligned}$$

We can finally compute the elements of Jacobian matrix J^* using equations (B.2)-(B.4):

$$
\begin{aligned}
\left.\frac{\partial \dot{x}}{\partial f}\right|_{(x^*,f^*)} &= \frac{\left(U_{xx}\frac{\partial}{\partial f}\left(\dot{\lambda}+\gamma\dot{\mu}-U_{xf}\dot{f}\right)-U_{xxf}\cdot(0)\right)}{U_{xx}^2} \\
&= \frac{1}{U_{xx}^*}\left(\frac{\partial}{\partial f}\left(\dot{\lambda}+\gamma\dot{\mu}-U_{xf}\dot{f}\right)\right) \\
&= \frac{1}{U_{xx}^*}\left(U_{ff}^*+(\delta+r+\gamma\cdot x^*)\frac{\partial\lambda}{\partial f}+\gamma\delta\lambda^*+\delta U_{xf}^*\right) \\
&= \frac{1}{U_{xx}^*}\left(U_{ff}^*+(r+\delta+\gamma\cdot x^*)\left(U_{xf}^*-\frac{\gamma\left(U_f^*+\delta\lambda^*\right)}{(r+\gamma\cdot x^*)}\right)+\gamma\delta\lambda^*+\delta U_{xf}^*\right) \\
&= \frac{1}{U_{xx}^*}\left(U_{ff}^*+(r+\gamma\cdot x^*+2\delta)\left(U_{xf}^*-\frac{\gamma U_f^*}{r+\gamma\cdot x^*+\delta}\right)\right)
\end{aligned}
$$

Where I insert λ^*'s steady-state value determined by (A.2) in the two last lines. Similarly, I take the derivative of the Euler equation at the steady state with regard to investment.

$$
\begin{aligned}
\left.\frac{\partial \dot{x}}{\partial x}\right|_{(x^*,f^*)} &= \frac{\left(U_{xx}\frac{\partial}{\partial x}\left(\dot{\lambda}+\gamma\dot{\mu}+U_{xf}\dot{f}\right)-U_{xxx}\cdot(0)\right)}{(U_{xx})^2} \\
&= \frac{U_{xx}^*\left(U_{xf}^*+\gamma\lambda+(r+\delta+\gamma\cdot x^*)U_{xx}^*-\gamma\lambda-U_{xf}^*\right)}{{U_{xx}^*}^2} \\
&= r+\delta+\gamma\cdot x^*
\end{aligned}
$$

Let $\Omega_f^* = \left.\frac{\partial\dot{x}(x,f)}{\partial f}\right|_{(x^*,f^*)}$. Putting everything together the Jacobian of the system evaluated at the steady-state is:

$$
J^* = \begin{pmatrix} r+\delta+\gamma\cdot x^* & \Omega_f^* \\ 1 & -\delta \end{pmatrix}
$$

The eigenvalue of the Jacobian are given by:

$$
\begin{aligned}
\mu_{1,2} &= \frac{tr(J^*)\pm\sqrt{tr(J^*)^2-4\nabla(J^*)}}{2} \\
&= \frac{(\gamma\cdot x^*+r)\pm\sqrt{(\gamma\cdot x^*+r+2\delta)^2+4\Omega_f^*}}{2}
\end{aligned}
$$

Where $\nabla(J^*)$ is the determinant of the Jacobian matrix J evaluated at the steady-state.

7 Proofs

7.1 Proposition 1

Let us set $\gamma = 0$ and $r = \pi + \rho$ in Appendix 6.1. I get,

$$\mu_{1,2} = \frac{r \pm \sqrt{(r+2\delta)^2 + 4\Omega_f^*}}{2}$$

In this special case, we have $\Omega_f^* = \Omega_A^* = \frac{1}{U_{xx}}\left(U_{ff} + (r+2\delta)U_{xf}\right)$

For (1), consider the optimal path of the frustration associated with the smallest eigenvalue μ_1.

$$\hat{f}_t = ke^{\mu_1 t} \tag{10}$$

$$\dot{\hat{f}}_t = k\mu_1 e^{\mu_1 t} \tag{11}$$

Where $k < 0$, because $f_{t_0} < f^*$. Using the frustration law of motion to express x_t and substituting (10) and (11) yields

$$\hat{x}_t = (\delta + \mu_1)k \cdot \hat{f}_t$$

$k \cdot \hat{f}_t > 0$, as such, $\hat{x}_t$, will be increasing in $\hat{f}_t$ if

$$(\delta + \mu_1) > 0$$

$$\iff \frac{r + 2\delta - \sqrt{(r+2\delta)^2 + 4\Omega_A^*}}{2} > 0 \iff \Omega_A^* < 0$$

And similarly $\hat{x}_t$, will be decreasing in $\hat{f}_t$ if $\Omega_A^* > 0$. For (2), saddle path stability requires one positive and one negative eigenvalue. The highest eigenvalue will always be positive. On the other hand, the lowest will be negative if and only if:

$$r < \sqrt{(r+2\delta)^2 + 4\Omega_A^*}$$

$$\iff r^2 - r^2 - 4\delta(r+\delta) - 4\Omega_A^* < 0$$

$$\iff -\delta(r+\delta) < \Omega_A^*$$

7.2 Proposition 2

It is a special case of the general model. It follows the same logic as the proof of Proposition 1, with $\gamma > 0$, $r = \rho$.

8 Non-monotonic reactions to frustrating events

In real-life scenarios, successive failures are often accompanied by several inter-temporal dynamics, such as Bayesian updating or learning-by-doing. As such, if one wants to look at the effect of frustration accumulation on personal motivation, one might end up looking at a mix of different dynamics. This section studies the interaction one can get when several dynamics are present. In particular, I study the combined effect of learning-by-doing and frustration accumulation on investment and characterise when non-monotonicities of the optimal path can arise.

With that in mind, let us introduce a learning-by-doing mechanism in our system. Let w_t represent the stock of experience accumulated at time t. I assume that w_t follows a logistic growth: $\dot{w}_t = w_t(a - bw_t)$ where $a > b > 0$. The experience growth rate is slow when the decision-maker discovers the task, then increases when she tries to solve it and finally decreases as she perfects her solving mechanism. I suppose that the discount rate of frustration δ is not too large relative to the effect of a trial on experience $\delta < a$. Notice that experience only depends on time and not the agent's investment.[39] The expected instantaneous utility of the agent is:

$$U(x_t, f_t, w_t) = \pi u(x_t, f_t) - v(f_t) - c(w_t, x_t)$$

Learning by doing reduces the marginal cost of investment: $c_{xw}(x, w) < 0$ and has decreasing returns $c_{ww} > 0$, or alternatively $U_{ww}(x, f, w) < 0$. Finally, I focus on the case with negative appraisal effects $U_{xf} \leq 0$, which is the most interesting case.[40]

The inter-temporal optimisation problem is

[39] It is possible to develop the model with a learning-by-doing process that depends on investment. However, this involves solving a 3^{rd} degree polynomial to get the eigenvalues of the Jacobian, which greatly complicates any further analysis.

[40] Or at least, a case where non-monotonicities arise

$$V(t_0, f_{t_0}, w_{t_0}) = \max_{x_t} \int_0^{\infty} e^{-(\rho+\pi)t} U(x_t, f_t, w_t) dt$$
$$\dot{f}_t = x_t - \delta f_t$$
$$\dot{w}_t = w_t(a - b w_t)$$
$$f_{t_0}, w_{t_0} \text{ given}$$

I apply the same regularity conditions as before.[41] Let $X(t)$ be the optimal investment path functional solving this maximisation problem. The law of motion of experience is not affected by frustration and investment; it evolves independently.

The system optimisation follows the same lines as before. We can similarly characterise the canonical system and linearise it around the steady-state. This yields the following three-dimensional system. As before, one law of motion describes the evolution of the optimal investment, and the two other reiterates the law of motions of the two-state variables, frustration $\hat{f}_t$ and experience $\hat{w}_t$.

$$\dot{\hat{x}}_t = (\rho + \pi + \delta)\hat{x}_t + \Omega_A^* \hat{f} + \Omega_w^* \hat{w}_t \tag{12a}$$
$$\dot{\hat{f}}_t = \hat{x}_t - \delta \hat{f}_t \tag{12b}$$
$$\dot{\hat{w}}_t = -a \hat{w}_t \tag{12c}$$

Where Ω_A^* is define as in (6), $\Omega_w^* = \frac{(\rho+\pi+\delta+a)U_{xw}^*}{U_{xx}^*} < 0$ characterise the temporal complementarity between $\hat{x}_t$ and $\hat{w}_t$ in the same way Ω_A^* does for $\hat{x}_t$ and $\hat{f}_t$. The main difference with the system developed in section 3.1, besides the new learning-by-doing law of motion (12c), resides in this added term to the investment Euler equation (12a). Since $\Omega_w^* < 0$, any additional experience below the steady-state level $\hat{w}_t < 0$, increases the investment level. For example, a $\hat{w}_t$ of one unit below the steady-state, increases the investment level by Ω_w^*

In order to get a picture of how the dynamics play out, let us study the system using the phase diagrams in Figure 5. We are looking for conditions characterising non-monotonicities of the optimal investment path, that is, the cases where $X'(t)$ changes sign. The learning-by-doing effect is represented by a shift of the $\dot{\hat{x}}_t(w) = 0$ locus to the right, indicated by the thick dashed arrow on the graph, from $\dot{\hat{x}}_t(w_{t_0}) = 0$, to $\dot{\hat{x}}_t(w^*) = 0$, its

[41] That is, the Inada conditions and concavity of the Hamiltonian.

steady-state position.

Let us define the two zones, **I** (**I**ncreasing investment) and **D** (**D**ecreasing investment) in Figure 5. Zone **D** is the area above the $\dot{\hat{f}}_t = 0$ and below the $\dot{\hat{x}}_t(w_t) = 0$ loci, where investment decreases and frustration increases. Zone **I** is the area above the $\dot{\hat{f}}_t = 0$ locus and between $\dot{\hat{x}}_t(w_t) = 0$'s current position and its steady state position. Both investment and frustration increase in zone **I**. Notice that zone **I** is gradually replaced by zone **D** because the investment locus shifts to the right to its steady state position. So a point that is initially in zone **I** would end up in Zone **D** when the $\dot{\hat{x}}_t(w_{t_0}) = 0$ locus reaches its steady state position. This means that the sign of dynamics characterising the evolution of x_t might change when experience increases. Alternatively, their dynamic would change if the optimal investment at time t goes from one zone to another at $t' > t$.

Of course, such two-dimensional phase diagram analysis only partially shows how the system evolves because it cannot represent the dynamics' temporal speed. For example, suppose the initial values of the system are in zone **I**. In that case, will the optimal investment increase until it reaches the steady-state as in Casee 1 depicted in Figure5? Or will it first increase and then decrease, as depicted in Case 1 of figure 5. Understanding the behaviour of the optimal path $X(t)$ through time ultimately boils down to figuring out $\hat{f}_t$ and $\hat{w}_t$'s relative convergence speed and their relative effect on $\hat{x}_t$. Overall, all the different temporal behaviours of the system can be predicted by looking at whether the initial experience-frustration ratio $\frac{\hat{f}_0}{\hat{w}_0}$ is above or below a certain threshold Θ.[42] I focus on the "*hump-shaped*" investment path thereafter.

Let the frustration-experience ratio be low, with $\frac{\hat{f}_0}{\hat{w}_0} \geq \Theta$. This can be interpreted as experience being "far enough" from its steady-state value, relative to frustration, given both state variables' effect and convergence speed. For these starting values, $(\hat{x}_0, \hat{f}_0)$ will be in zone **I**, with $\hat{x}_0 \in X(0)$. Since the point is above the $\dot{\hat{x}}_t(w_{t_0}) = 0$ and $\dot{\hat{f}}_t = 0$ loci, frustration and investment increase. But, remember that zone **I** is gradually replaced by zone **D**. As such, there are two possibilities:

(1) If Ω_A^* is low[43], $\Omega_A^* < -\delta(\rho+\pi+\delta) - a(\rho+\pi+a)$, learning-by-doing's effect dominates all the way and frustration's negative effect is never revealed through the dynamics. In that case, the optimal investment $X(t)$ stays in zone **I** as in Case 4 in Figure 5.

(2) If Ω_A^* is large, $\Omega_A^* > -\delta(\rho+\pi+\delta) - a(\rho+\pi+a)$, frustration's effect dominates in the long run but not in the short run, and the optimal investment path is hump-shaped. Intuitively, the task is relatively easy to grapple with, and there is a lot to learn: the

[42] See appendix for the definition of Θ.

[43] Remember that $\Omega_A^* > 0$ with negative appraisal tendencies.

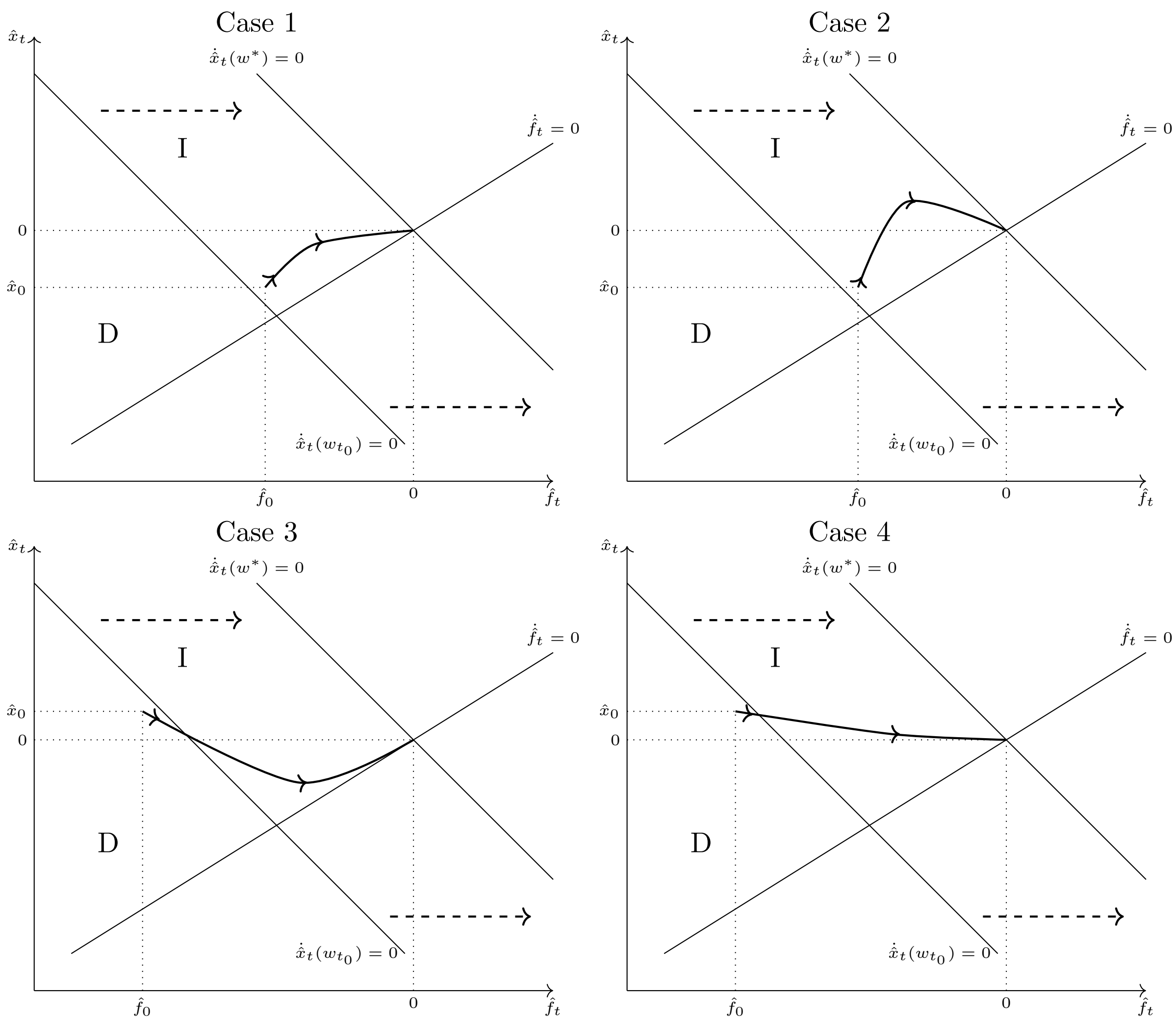


Figure 5: Examples of non-monotonic optimal investment paths a frustration-investment phase diagram.

learning process is rapid and decreases the marginal cost relatively quickly and significantly. However, once most of the learning occurred, the negative effect of frustration catches up, and investment decreases to its steady-state value. Graphically, the optimal investment level was in zone **I** before a specific $t' > 0$ and is in zone **D** afterwards, yielding a humped-shaped optimal investment path $X(t)$, **I** as in Case 2 in Figure 5.

A final and perhaps more surprising case happens when the effect of frustration on investment is moderate $\Omega_A^* < -\delta(\rho+\pi+\delta) - a(\rho+\pi+a)$, but the initial value of frustration is far from its steady-state value relative to $\hat{w}_0$. Investment first decreases quickly because of the emotional cost but is caught up by the learning mechanism. In this case, the optimal investment path is U-shaped. At first, the agent lowers her investment level because of the overwhelming frustration effect and the slow learning-by-doing mechanism. However, with time, the accumulation of experience becomes dominant relative to frustration, and investment provision increases to its steady-state level. The following proposition summarises the different cases.

Proposition 3. *Let f_{t_0} and w_{t_0} be below their steady-state values, then:*

- *if $\Omega_A^* > -\delta(\rho+\pi+\delta) - a(\rho+\pi+a)$, and $\frac{\hat{f}_0}{\hat{w}_0} \geq \Theta$ the optimal investment path $X(t)$ is monotone decreasing in time,*
- *if $\Omega_A^* > -\delta(\rho+\pi+\delta) - a(\rho+\pi+a)$, and $\frac{\hat{f}_0}{\hat{w}_0} < \Theta$ the optimal investment path $X(t)$ is first increasing and then decreasing in time,*
- *if $\Omega_A^* < -\delta(\rho+\pi+\delta) - a(\rho+\pi+a)$, and $\frac{\hat{f}_0}{\hat{w}_0} < \Theta$ the optimal investment path $X(t)$ is monotone increasing in time,*
- *if $\Omega_A^* < -\delta(\rho+\pi+\delta) - a(\rho+\pi+a)$, and $\frac{\hat{f}_0}{\hat{w}_0} \geq \Theta$ the optimal investment path $X(t)$ is first decreasing and then increasing in time.*

8.1 Proof of Proposition 3

Let $r = \rho + \pi$. The current value Hamiltonian of this problem is:

$$H(x_t, f_t, w_t, \lambda_t,) = U(x_t, f_t, w_t) - \lambda_t(x_t - \delta f_t)$$

The first-order conditions of the system are

$$\begin{cases} U_x(x_t, f_t, w_t) = \lambda_t \\ \dot{\lambda}_t = U_f(x_t, f_t, w_t) + (r+\delta)\lambda_t \end{cases} \tag{13}$$

Differentiating the the first equation of (13) with regards to time and inserting the law of motion of the costate yields the canonical system:

$$\begin{cases} \dot{x}_t &= \dfrac{1}{U_{xx}(x_t, f_t, w_t)}\Big((\dot{\lambda}_t - \dot{w}_t \cdot U_{xw}(x_t, f_t, w_t) - \dot{f}_t \cdot U_{xf}(x_t, f_t, w_t)\Big) \\ \dot{f}_t &= x_t^* - \delta f_t \\ \dot{w}_t &= w_t(a - bw_t) \end{cases}$$

The Jacobian of the system at the steady-state is:

$$\begin{aligned} J(x^*, f^*, w^*) &= \left.\begin{pmatrix} \frac{\partial \dot{x}(x,f,w)}{\partial x} & \frac{\partial \dot{x}(x,f,w)}{\partial f} & (\frac{\partial \dot{x}(x,f,w)}{\partial w} \\ \frac{\partial \dot{f}(x,f,w)}{\partial x} & \frac{\partial \dot{f}(x,f,w)}{\partial f} & \frac{\partial \dot{f}(x,f,w)}{\partial w} \\ \frac{\partial \dot{w}(x,f,w)}{\partial x} & \frac{\partial \dot{w}(x,f,w)}{\partial f} & \frac{\partial \dot{w}(x,f,w)}{\partial w} \end{pmatrix}\right|_{(x,f,w)=(x^*,f^*,w^*)} \\ &= \begin{pmatrix} r+\delta & \Omega_f^* & (r+\delta+a)\frac{U_{xw}^*}{U_{xx}^*} \\ 1 & -\delta & 0 \\ 0 & 0 & -a \end{pmatrix}. \end{aligned}$$

The eigenvalues of this system are μ_1 and μ_2, defined as in Section 7.1 and $\mu_3 = -a$. Let us define wlog that $\mu_1 < \mu_2$.

We can now compute the eigenvector $u_1 = (u_1^1, u_1^2, u_1^3)'$ and $u_3 = (u_3^1, u_3^2, u_3^3)'$ associated to the negative eigenvalues μ_1 and μ_3. These are found by solving the system $(J^* - \mu_i \times I) * u_i = 0$, where I is the 3 by 3 identity matrix, $i = 1, 3$:

$$\begin{cases} (r+\delta-\mu_i)u_i^1 + \Omega_f^* u_i^2 + (r+\delta+a)\frac{U_{xw}^*}{U_{xx}^*}u_i^3 = 0 \\ u_i^1 + (-\delta-\mu_i)u_i^2 = 0 \\ (-a-\mu_i)u_i^3 = 0 \end{cases} \tag{14}$$

For μ_1, given the third row of the system, $u_1^3 = 0$ since $\mu_1 \neq -a$. Using the second row, it is easy to find that $u_1^2 = \frac{\mu_1^1}{\delta+\mu_1}$. As such, $(u_1^1, u_1^2, u_1^3) = (1, \frac{1}{\delta+\mu_1}, 0)$ is one of the solutions of the system.

Next, for μ_3, normalise u_3^1 to 1 and use the second row of the system to get $u_3^2 = \frac{1}{\delta+\mu_3}$.

As for u_3^3, we can plug the previous result into the first row to get:

$$\begin{aligned}
-u_3^3(r+\delta-\mu_3)\frac{U_{xw}^*}{U_{xx}^*} &= \left((r+\delta-\mu_3)+\frac{\Omega_f^*}{\delta+\mu_3}\right)u_3^1 \\
&= \left(\frac{(r+\delta-\mu_3)(\delta+\mu_3)+\Omega_f^*}{\delta+\mu_3}\right) \\
&= \frac{1}{\delta+\mu_3}\left(\Omega_f^*+\delta(r+\delta)+\mu_3(r-\mu_3)\right) \\
&= \frac{1}{\delta+\mu_3}\left(\frac{1}{4}\left(r^2-r^2+4(\Omega_f^*+\delta(r+\delta))\right)+\mu_3(r-\mu_3)\right) \\
&= \frac{1}{\delta+\mu_3}\left(\frac{1}{4}\left(\left(\sqrt{r^2+4(\Omega_f^*+\delta(r+\delta))}\right)^2-r^2\right)+\mu_3(r-\mu_3)\right) \\
&= \frac{1}{\delta+\mu_3}\left(-\mu_1\left(r-r+\frac{r+\sqrt{r^2-4(\Omega_f^*+\delta(r+\delta))}}{2}\right)+\mu_3(r-\mu_3)\right) \\
&= \frac{1}{\delta+\mu_3}(\mu_3(r-\mu_3)-\mu_1(r-\mu_1)) \\
&= \frac{1}{\delta+\mu_3}(\mu_3-\mu_1)(r-\mu_3-\mu_1) \\
\iff u_3^3 &= \frac{U_{xx}^*}{U_{xw}^*}\frac{(\mu_1-\mu_3)(r-\mu_3-\mu_1)}{(r+\delta-\mu_3)(\delta+\mu_3)}
\end{aligned}$$

Given the eigenvalues and associated eigenvectors, we can express the solution of our system as:

$$\begin{cases} \hat{x}_t &= k_1 e^{\mu_1 t} + k_3 e^{\mu_3 t} \\ \hat{f}_t &= k_1 u_1^2 e^{\mu_1 t} + k_3 u_3^2 e^{\mu_3 t} \\ \hat{w}_t &= k_3 u_3^3 e^{\mu_3 t} \end{cases} \tag{15}$$

Where the initial values of the system determine k_1 and k_3. For what follows, it is important to note that we have $\delta+\mu_3<0$ by assumption and since $\Omega_A^*>0$ and thus $\delta+\mu_1<0$[44], as we are considering negative appraisal tendencies. Notice that:

$$\hat{f}_0 < 0 \iff k_1 > -k_3\frac{\mu_1+\delta}{\mu_3+\delta} \tag{16}$$

The following result gives conditions for system (15)'s investment path $\hat{x}_t$ to be non-monotonic in time:

[44] As shown Appendix 7.1

Lemma 1. *A system defined as in* (15) *with two negative eigenvalues* μ_1 *and* μ_3*, exhibits a non-monotonic optimal investment path* $\hat{x}_t$ *with regards to time if and only if* k_1 *and* k_3 *have opposite signs and either* $-\frac{k_1}{k_3}\frac{\mu_1}{\mu_3} < 1$ *and* $\mu_3 < \mu_1$ *or* $-\frac{k_1}{k_3}\frac{\mu_1}{\mu_3} > 1$ *and* $\mu_3 > \mu_1$*. Moreover,* $\dot{\hat{x}}_t$ *only changes sign once.*

Proof. A necessary condition for the optimal investment path to be non-monotonic is that at some t', $\dot{\hat{x}}_{t'} = k_1\mu_1 e^{\mu_1 t} + k_3\mu_3 e^{\mu_3 t} = 0$. This happens if and only if there is a positive t' solving:

$$\ln\left(-\frac{k_1}{k_3}\frac{\mu_1}{\mu_3}\right)\frac{1}{\mu_3-\mu_1} = t'$$

Since μ_1 and μ_3 are negative, this equation is well defined if k_1 and k_3 have opposite signs. Moreover, t' will be positive if either $-\frac{k_1}{k_3}\frac{\mu_1}{\mu_3} < 1$ and $\mu_3 < \mu_1$ or $-\frac{k_1}{k_3}\frac{\mu_1}{\mu_3} > 1$ and $\mu_3 > \mu_1$. Notice that there is only one t' satisfying this equation. As such, if t' characterises an extremum, it is unique.

For sufficiency, we need to verify that t' is an extremum, not an inflexion point. This can be easily checked by looking at the sign of the second derivative of $\hat{x}_t$

$$\ddot{\hat{x}}_t = \mu_1^2 k_1 e^{\mu_1 t} + \mu_3^2 k_3 e^{\mu_1 t}$$

$\hat{x}_t$ will change its curvature at the possible inflexion point t'' if $\ddot{\hat{x}}_{t''} = 0$. If t'' exists, it is defined by:

$$t'' = \ln\left(-2\frac{k_1}{k_3}\frac{\mu_1}{\mu_3}\right)\frac{1}{\mu_3-\mu_1} > t'$$

As such, t' is not an inflexion point.

□

Let us now prove the result of the proposition. First, as shown before, we have that:

$$u_3^3 = \frac{U_{xx}^*}{U_{xw}^*}\cdot\frac{-\Omega_f^* - \delta(r+\delta) - a(r+a)}{(\delta+\mu_3)(r+\delta-\mu_3)} \tag{17}$$

$$= \frac{U_{xx}^*}{U_{xw}^*}\cdot\frac{(\mu_1-\mu_3)(r-\mu_1-\mu_3)}{(\delta+\mu_3)(r+\delta-\mu_3)} \tag{18}$$

Equation 17-18 tell us that $-\Omega_f^* - \delta(r+\delta) - a(r+a)$ has the same sign as $(\mu_1 - \mu_3)$. Next, let us define the threshold of Θ of the proposition as:

$$\Theta = \frac{U^*_{xw}}{U^*_{xx}} \cdot \frac{(\delta + \mu_1 + \mu_3)(r + \delta - \mu_3)}{\mu_1(\delta + \mu_1)(r - \mu_1 - \mu_3)} > 0 \tag{19}$$

I can, therefore express u_3^3 in the following way:

$$u_3^3 = \frac{1}{\Theta} \frac{(\mu_1 - \mu_3)(\delta + \mu_1 + \mu_3)}{\mu_1(\delta + \mu_1)(\delta + \mu_3)}$$

When $t = 0$, it is possible to give an expression to k_1 and k_3:

$$\begin{aligned}
k_3 &= \frac{w_{t_0}}{u_3^3} = \frac{\mu_1(\delta + \mu_1)(\delta + \mu_3)}{(\mu_1 - \mu_3)(\delta + \mu_1 + \mu_3)} \Theta w_{t_0} \\
k_1 &= \frac{1}{u_1^2}\left(f_{t_0} - u_3^2 \cdot k_3\right) = (\delta + \mu_1)\left(f_{t_0} - \frac{\mu_1(\delta + \mu_1)}{(\mu_1 - \mu_3)(\delta + \mu_1 + \mu_3)} \Theta w_{t_0}\right)
\end{aligned}$$

Next, notice that if $f_{t_0} \geq \Theta w_{t_0}$, (and similarly for $f_{t_0} < \Theta w_{t_0}$):

$$\begin{aligned}
& f_{t_0} \geq \Theta w_{t_0} \\
\iff & f_{t_0} \geq \frac{(\mu_1 - \mu_3)(\delta + \mu_1 + \mu_3)}{(\mu_1 - \mu_3)(\delta + \mu_1 + \mu_3)} \Theta w_{t_0} \\
\iff & f_{t_0} - \frac{\mu_1(\delta + \mu_1)}{(\mu_1 - \mu_3)(\delta + \mu_1 + \mu_3)} \Theta w_{t_0} \geq -\frac{\mu_3(\delta + \mu_3)}{(\mu_1 - \mu_3)(\delta + \mu_1 + \mu_3)} \Theta w_{t_0} \\
\iff & k_1 \leq -k_3 \frac{\mu_3}{\mu_1}
\end{aligned}$$

The inequality switches on the first line because w_{t_0} is below its steady-state value. As such:

$$\frac{f_{t_0}}{w_{t_0}} \leq (>) \Theta \iff k_1 \leq (>) - k_3 \frac{\mu_3}{\mu_1} \tag{20}$$

I can now use the equivalence relation (20) and Lemma 1 to determine when non-monotonicities arise.

- If $\Omega_f^* > -\delta(r + \delta) - a(r + a)$, by expression (17)-(18), we have that $\mu_3 > \mu_1$ and $u_3^3 < 0$. Since $w_{t_0} < 0$, it must be that $k_3 > 0$ because $w_{t_0} = k_3 u_3^3$. Now given that $f_{t_0} < 0$, there are two cases:

1. $\frac{\hat{f}_0}{\hat{w}_0} < \Theta$ from expressions (20) and (16), k_1 is in $\left(-k_3\frac{\mu_1+\delta}{\mu_3+\delta}, -k_3\frac{\mu_3}{\mu_1}\right)$, and $k_1 < 0$. Then $-\frac{k_1}{k_3}\frac{\mu_1}{\mu_3} > 1$. By Lemma 1, the optimal path is non-monotonic. We have that:

$$\dot{\hat{x}}_0 = \mu_1 k_1 + \mu_3 k_3 > -k_3\frac{\mu_3}{\mu_1}\mu_1 + \mu_3 k_3 = 0$$

   Also, by Lemma 1, $\dot{\hat{x}}_t$ only changes sign once. As such, the optimal investment path is first increasing and then decreasing.

2. $\frac{\hat{f}_0}{\hat{w}_0} \geq \Theta$, from expression (20), $k_1 \geq -k_3\frac{\mu_3}{\mu_1}$,then $-\frac{k_1}{k_3}\frac{\mu_1}{\mu_3} \leq 1$ .By Lemma 2, the optimal path is monotone Since $\dot{x}_0 = k_1\mu_1 + k_3\mu_3 \leq 0$, the optimal path is monotone decreasing.

- If $\Omega_f^* < -\delta(r-\delta) - a(r+a)$ then, $\mu_1 > \mu_3$, $u_3^3 > 0$ and $k_3 < 0$. We then have two possibilities:

  1. $\frac{\hat{f}_0}{\hat{w}_0} \leq \Theta$, as such, $k_1 \in \left(-k_3\frac{\mu_1+\delta}{\mu_3+\delta}, -k_3\frac{\mu_3}{\mu_1}\right]$, then $-\frac{k_1}{k_3}\frac{\mu_1}{\mu_3} \leq 1$. By Lemma 1, the optimal path is monotonic. Since $\dot{x}_0 \geq 0$. It is first monotone increasing.

  2. $\frac{\hat{f}_0}{\hat{w}_0} > \Theta$, as such, $k_1 > -k_3\frac{\mu_3}{\mu_1}$ then $\dot{x}_0 < 0$ and $-\frac{k_1}{k_3}\frac{\mu_1}{\mu_3} < 1$ and $\mu_1 > \mu_3$, by Lemma 1, the optimal path is non-monotonic: it is first decreasing and then increasing.

9 Empirical Appendix

9.1 Standard Deviation of Speed per pitch type

Table 4: Standard deviation of Speed of pitches

Pitch Type	SD Speed	Number of Obs
Sinker	0.91	1549913
Slider	1.08	1103659
4-Seam Fastball	0.92	2505992
Split-Finger	0.96	110276
Cutter	0.94	412864
Changeup	0.95	742088
Curveball	1.09	587691
Knuckle Curve	1.06	151197

The first column indicates the pitch type. I only keep pitches that represent more than 1% of the observation of the dataset. The second column indicates the average standard deviation of the speed of a pitch type during a *game* $\times$ *inning* at the player level. The third column indicates the number of throws for each pitch type in the cleaned dataset.

Table 4 presents the standard speed variation for all pitches representing at least one per cent of the cleaned sample. The standard deviations are computed for each pitch type at the player-game-inning level. Table 4 presents these standard deviations' averages over the entire sample for each pitch type. The standard deviations are remarkably homogeneous and vary from 0.91 to 1.08 mph.

9.2 Machine learning algorithm to measure the quality of pitches

The pitch quality measure is the result of an estimation predicting pitches predictive success given by the physical characteristics of the motion of the throw. I use the eXtreme Gradient Boosting (XGBoost) algorithm (Chen and Guestrin, 2016) to do this. XGBoost is a gradient-boosting model based on decision tree ensembles. The tree ensemble is a set of classification trees where each leaf is assigned a prediction score. XGBoost then aggregates the prediction of all the trees to get the final prediction. The main difference with random forest models, which are also based on tree ensembles, is how each tree is optimised.

New trees aim to minimise the residual errors in the predictions from the existing sequence of trees. Each tree is sequentially learned by optimising a binary logistic objective function. I use the 2020-2021 seasons as a training dataset. I set the number of trees to 100. I tuned the algorithm's hyper-parameters using a random grid search and a 5-fold cross-validation. I focus on two hyperparameters. The first is the learning rate $\eta \in (0, 1]$, which reduces the weight of new features in the predictions to avoid over-fitting. The second controls the L2 regularisation term. Table 5 describes the features used in the machine learning algorithm. I train a separate algorithm for each pitch for which I have more than 10,000 observations in the training dataset.

Table 5: Features used in the estimation of the Success Probability

Features
Velocity of the pitch, in x, y and z-dimension, at y=50 feet.
The acceleration of the pitch in x, y and z-dimension, determined at y=50 feet.
Horizontal & vertical ball position when it crosses home plate (catcher's perspective).
Horizontal & vertical movement in feet from the catcher's perspective.
Out-of-hand pitch velocities.
Horizontal & vertical Release Position of the ball measured in feet(catcher's perspective).
Release position of pitch measured in feet (catcher's perspective).
Acceleration of the pitch, in feet per second per second, in x, y and z-dimension.
Top & bottom of the batter's strike zone when the ball is halfway to the plate.

Table 6 show the result of the machine learning algorithm when applied to the main dataset (2010-2019). I measure the accuracy as the proportion of observation in the dataset for which the algorithm predicts a probability of more than 0.5 for the event that actually happened (Success or Failure). The first column shows that the algorithm's accuracy is between 0.81 and 0.75, depending on the pitch type. The next two columns indicate the average probability predicted given the (ex-post) outcome. This shows that the algorithm does a good job differentiating the outcome.[45]

Table 6: Predictive Statistics of the Machine learning algorithm

Pitch name	Accuracy	Mean(Pred.prob(S)|S)	Mean(Pred.prob(S)|F)
Fastball	0.8096	0.7602	0.2722
Changeup	0.7637	0.7035	0.2938
Curveball	0.7741	0.7218	0.2885
Cutter	0.7794	0.7227	0.3008
Sinker	0.7914	0.7474	0.3065
Slider	0.7558	0.6959	0.2937
Split-Finger	0.7530	0.6790	0.3114
Knuckle Curve	0.7661	0.6862	0.2911

The first column indicates the pitch type for which the algorithm ran. The second indicates the proportion of observations for which the predicted probability tilted towards the right pitch outcome. That is the proportion of observation for which the pitches ended up in a failure (or success) and where the predicted probability of success was greater (or lower) than 0.5. The third and fourth columns indicate the average predicted probability of success given the outcome of the pitch ((S)uccess or (F)ailure).

9.3 Frustration effect on pitch quality

Let me comment on the effect of frustration on pitch quality. Overall, table 9.3 shows that one unit of frustration decreases the predictive probability of success by half a percentage point. Although this effect is substantial, the control strategy is less successful for these regressions. Indeed, the adjusted R^2 is much lower and around 8%. At the pitch level,

[45] Instead of glorified coin-toss machine-learning algorithm predicting successes or failures with probability 0.51.

Table 7: Effect of Frustration on Velocity and Quality

Dependent Variables:	Quality	
Model:	(1)	(2)
Variables		
$\mathbf{F_t}$	-0.0048***	-0.0048***
	(0.0007)	(0.0007)
# Failures	0.0138***	0.0138***
	(0.0004)	(0.0001)
# Successes	-0.0091***	-0.0091***
	(0.0003)	(0.0001)
Attempt	0.0030***	0.0030***
	(0.0002)	(5.19×10^{-5})
Δ Exp.	-0.0252***	-0.0252***
	(0.0012)	(0.0010)
Δ Score	0.0039***	0.0039***
	(0.0003)	(0.0002)
Δ Inning Score	0.0032***	0.0032***
	(0.0011)	(0.0010)
Fixed-effects		
Pitch Type	Yes	Yes
Game State	Yes	Yes
Player × Game	Yes	Yes
Batter	Yes	Yes
Inning	Yes	Yes
Fit statistics		
Observations	7,163,516	7,163,516
R^2	0.07660	0.07660
Within R^2	0.00438	0.00438

*Signif. Codes: ***: 0.01, **: 0.05, *: 0.1*

Column (1) is clustered at the pitcher level, and column (2) is clustered at the game level. $\mathbf{F_t}$ measures the change in the other team's expected score during the current and (possibly empty) sequence of consecutive failure. # Failures and # Successes count the number of successes and failures since the beginning of the inning. *Attempt* measures the number of pitches the pitcher has thrown since the beginning of the game. Δ Exp measure the change in the opposing team's expected score since the beginning of the inning. Δ Score measures the current difference in score, while Δ Inning Score measures the change in score since the beginning of the inning. *Pitch Type* is a categorical variable indicating the pitch type. Game State is a categorical variable indicating the number of batters out, the number of balls, strikes and whether a player is on base 1, 2 or 3. Player × Game indicates who is pitching during which game. *Batter* indicates who is currently facing the pitcher.

frustration significantly affects the most popular pitch type. However, it fails to do so for pitches with fewer observations[46].

9.4 Effect of frustration per pitch type

Table 8: Effect of frustration per pitch type

Dependent Variables:	log(Speed)		Quality		Observations
Model:	(1)	(2)	(3)	(4)	
Type of pitch					
Slider	0.0014***	0.0014***	-0.0032*	-0.0032*	1,103,659
	(0.0001)	(8.59×10^{-5})	(0.0017)	(0.0017)	
4-Seam Fastball	0.0017***	0.0017***	-0.0041***	-0.0041***	2,505,992
	(5.73×10^{-5})	(4×10^{-5})	(0.0012)	(0.0011)	
Split-Finger	0.0009**	0.0009***	-0.0103	-0.0103*	110,276
	(0.0004)	(0.0003)	(0.0063)	(0.0055)	
Cutter	0.0012***	0.0012***	-0.0032	-0.0032	412,864
	(0.0001)	(0.0001)	(0.0033)	(0.0027)	
Changeup	0.0008***	0.0008***	-0.0027	-0.0027	742,088
	(0.0001)	(9.11×10^{-5})	(0.0021)	(0.0021)	
Curveball	0.0012***	0.0012***	-0.0016	-0.0016	587,691
	(0.0002)	(0.0001)	(0.0024)	(0.0024)	
Knuckle Curve	0.0017***	0.0017***	0.0006	0.0006	151,197
	(0.0002)	(0.0002)	(0.0047)	(0.0046)	

*Signif. Codes: ***: 0.01, **: 0.05, *: 0.1*

The table's first column indicates which pitch type the regression was performed. The columns indicated by (1) and (2) display the OLS coefficient for frustration when the log of pitch speed, controlling for all fixed effects present in Table 2. The columns indicated by (3) and (4) perform the same regression but with Quality as its dependent variable. Columns (1) and (3) are clustered at the pitcher level, and columns (2) and (4) are clustered at the game level.

The effect of frustration on speed is robust whether you look at the effect at the pitch type level or at the aggregate level. For quality, though, one only observes a significant effect for the two most popular pitch types. This is quite natural as the quality of the machine learning algorithm decreases with sample size. Moreover, the quality OLS is still much noisier than the speed ones.

[46]Note that the quality of the machine learning algorithm measuring the quality of the pitches also decreases with sample size.

9.5 Individual level Analysis

Let me introduce an individual-level analysis of pitchers' reactions to frustration. I first select all pitchers for which I have more than 2000 registered pitches, for a total of 859 pitchers[47]. My first exercise is to perform the same regression as in column 1 of table 2 in the main text and 7 in Appendix 9.3.

Table 9 shows that more than one-third of the sample show a significant effect of frustration accumulation on their behaviours regarding pitch velocity. Note that frustration accumulation at a high level remains rare. As such, estimating a significant impact on a smaller sample size is much more challenging. The effects are also remarkably homogeneous. 98% of the significant frustration coefficients are positive. Regarding quality, the results are more modest, and only 8.6% of the sample exhibit a significant effect of frustration on the quality of the pitches. For the majority of pitchers, this effect is negative.

Table 9: Individual-level sensitivity to frustration

	Pos. Coeff.	Neg. Coef
Speed	0.379	0.006
Quality	0.020	0.066

The first row represents the proportion of pitchers displaying a significant effect of frustration on speed for positive and negative coefficients. The second row displays the same results for the Quality regressions.

Table 10 shows correlations for the individual-level coefficients. There is a significant and negative correlation between the effect of frustration on speed and its impact on pitch quality. A higher effect of frustration on speed is associated with a more negative impact on quality. This rules out any rationale for saying that frustration motivates pitchers to reach a better optimum. It seems that frustration increases the dis-alignment of emotional and performance incentives. For some players, this is limited to a slight change in speed when frustrated. For others, the effect on speed (among other things) decreases the quality of pitches. Overall, this goes towards the appraisal tendencies' interpretation of the results.

Next, I look at whether frustration sensitivity impacts performance at the career level.

[47]This creates some selection bias because I am effectively selecting for the most competitive pitchers in MLB. Table 9 shows the results.

I use two traditional measures of a pitcher's performance to do this. The first is ERA "Earned run average", which measures the average number of runs the pitcher allows during the last nine innings. Since the pitcher is a defensive player and wants to prevent the other team from scoring runs, a lower ERA means a better pitcher. Although it is arguably the most popular measure of a pitcher's performance, it also has some pitfalls. Foremost, it does not factor in the responsibility of the rest of its team in the number of runs allowed. As such, it is possible to have an excellent pitcher with a very high ERA if he is in a miserable team.

Table 10: Frustration coefficient: correlations

	Speed.	Quality	ERA	xFIP
Speed	1	-0.097***	-0.030	-0.058
Quality	-0.097***	1	0.105***	0.149***

The first row represents different correlations between the frustration coefficient for the Speed OLS. In contrast, the second row displays the same information for the Quality OLS. The first two columns provide the correlations between the coefficients of the different types of OLS. The two last columns display the coefficient correlations stemming from the two types of OLS and skills measures. ERA, "Earned run average", measures the average number of runs the pitcher allowed during the nine last innings. xFIP is a similar measure that focuses on the outcomes over which the pitchers have the most control.

To account for this, I complement the analysis using xFIP, which is short for Expected Fielding Independent Pitching. xFIP is similar to ERA but focuses on events a pitcher has the most control over, such as strikeouts, home runs, walks, or hit by pitches[48].

Table 10 shows the results. On its own, frustration's impact on speed cannot translate into a general performance decrease. However, frustration's influence on behaviour can be high enough to trigger negative consequences on pitch quality. In that case, frustration seems to be associated with a career-level decrease in the pitcher's performance.

[48]It also corrects for the Home-run-to-fly-ball rate season's league average.

References

Augenblick, N., 2016. The sunk-cost fallacy in penny auctions. The Review of Economic Studies 83 (1), 58–86.

Barro, R., Sala-i Martin, X., 2004. Economic growth second edition. The MIT Press, Cambridge MA.

Battigalli, P., Dufwenberg, M., 2007. Guilt in games. American Economic Review 97 (2), 170–176.

Battigalli, P., Dufwenberg, M., 2009. Dynamic psychological games. Journal of Economic Theory 144 (1), 1–35.

Battigalli, P., Dufwenberg, M., Smith, A., 2019. Frustration, aggression, and anger in leader-follower games. Games and Economic Behavior 117, 15–39.

Becker, G. S., Murphy, K. M., 1988. A Theory of Rational Addiction. Journal of Political Economy 96 (4), 675–700.

Bell, D. E., 1982. Regret in decision making under uncertainty. Operations Research 30 (5), 961–981.

Bell, D. E., 1985. Disappointment in decision making under uncertainty. Operations Research 33 (1), 1–27.

Breuer, J., Elson, M., 2017. Frustration-aggression theory. Wiley Blackwell.

Brooks, C., Sangiorgi, I., Saraeva, A., Hillenbrand, C., Money, K., 2023. The importance of staying positive: The impact of emotions on attitude to risk. International Journal of Finance & Economics 28 (3), 3232–3261.

Callen, M., Isaqzadeh, M., Long, J. D., Sprenger, C., 2014. Violence and Risk Preference: Experimental Evidence from Afghanistan. American Economic Review 104 (1), 123–148.

Campbell, J. Y., Cochrane, J. H., 1999. By Force of Habit: A Consumption-Based Explanation of Aggregate Stock Market Behavior. Journal of Political Economy 107 (2), 205–251.

Caplin, A., Leahy, J., 2001. Psychological expected utility theory and anticipatory feelings. The Quarterly Journal of Economics 116 (1), 55–79.

Chen, T., Guestrin, C., 2016. XGBoost: A scalable tree boosting system. In: Proceedings of the 22nd ACM SIGKDD International Conference on Knowledge Discovery and Data Mining. KDD '16. ACM, New York, NY, USA, pp. 785–794.

Cohn, A., Engelmann, J., Fehr, E., Maréchal, M. A., 2015. Evidence for Countercyclical Risk Aversion: An Experiment with Financial Professionals. American Economic Review 105 (2), 860–885.

Dalmia, P., Filiz-Ozbay, E., 2021. Your success is my motivation. Journal of Economic Behavior & Organization 181, 49–85.

Dillenberger, D., Rozen, K., 2015. History-dependent risk attitude. Journal of Economic Theory 157, 445–477.

Dollard, J., Miller, N. E., Doob, L. W., Mowrer, O. H., Sears, R. R., 1939. Frustration and aggression. Yale University Press.

Foellmi, R., Legge, S., Schmid, L., 2016. Do professionals get it right? Limited attention and risk-taking behaviour. The Economic Journal 126 (592), 724–755.

Geanakoplos, J., Pearce, D., Stacchetti, E., 1989. Psychological games and sequential rationality. Games and Economic Behavior, 60–79.

Goldberg, J. H., Lerner, J. S., Tetlock, P. E., 1999. Rage and reason: The psychology of the intuitive prosecutor. European Journal of Social Psychology 29 (5-6), 781–795.

Guiso, L., Sapienza, P., Zingales, L., 2018. Time varying risk aversion. Journal of Financial Economics 128 (3), 403–421.

Han, S., Lerner, J. S., Keltner, D., 2007. Feelings and consumer decision making: The appraisal-tendency framework. Journal of Consumer Psychology 17 (3), 158–168.

Heath, C., 1995. Escalation and de-escalation of commitment in response to sunk costs: The role of budgeting in mental accounting. Organizational Behavior and Human Decision Processes 62 (1), 38–54.

Imas, A., 2016. The Realization Effect: Risk-Taking after Realized versus Paper Losses. American Economic Review 106 (8), 2086–2109.

John Griffith, M. N., Shen, J., 2020. Emotions in the stock market. Journal of Behavioral Finance 21 (1), 42–56.

Keltner, D., Lerner, J. S., 2010. Emotion. In: Fiske, S., Gilbert, D., Lindzey, G. (Eds.), Handbook of Social Psychology. John Wiley & Sons, Inc.

Kreps, D. M., Porteus, E. L., 1978. Temporal Resolution of Uncertainty and Dynamic Choice Theory. Econometrica 46 (1), 185–200.

Kuppens, P., Verduyn, P., 2017. Emotion dynamics. Current Opinion in Psychology 17, 22–26.

Kőszegi, B., Rabin, M., 2006. A model of reference-dependent preferences. The Quarterly Journal of Economics 121 (4), 1133–1165.

Kőszegi, B., Rabin, M., 2009. Reference-Dependent Consumption Plans. American Economic Review 99 (3), 909–936.

Laibson, D., 2001. A cue-theory of consumption. The Quarterly Journal of Economics 116 (1), 81–119.

Lazarus, R. S., Folkman, S., 1984. Stress, appraisal, and coping. Springer.

Lerner, J. S., Keltner, D., 2000. Beyond valence: Toward a model of emotion-specific influences on judgement and choice. Cognition & emotion 14 (4), 473–493.

Lerner, J. S., Keltner, D., 2001. Fear, anger, and risk. Journal of Personality and Social Psychology 81 (1), 146.

Lerner, J. S., Li, Y., Valdesolo, P., Kassam, K. S., 2015. Emotion and decision making. Annual review of psychology 66, 799–823.

Lerner, J. S., Small, D. A., Loewenstein, G., 2004. Heart strings and purse strings: Carryover effects of emotions on economic decisions. Psychological science 15 (5), 337–341.

Loewenstein, G., 1999. Wouldn't It Be Nice?: Predicting Future Feelings. In: Kahneman, D., Diener, E., Norbert, S. (Eds.), Well-Being: Foundations of Hedonic Psychology. Russell Sage Foundation, pp. 85–106.

Loewenstein, G., 2000. Emotions in economic theory and economic behavior. American economic review 90 (2), 426–432.

Loomes, G., Sugden, R., 1982. Regret theory: An alternative theory of rational choice under uncertainty. The Economic Journal 92 (368), 805–824.

Loomes, G., Sugden, R., 1986. Disappointment and dynamic consistency in choice under uncertainty. The Review of Economic Studies 53 (2), 271–282.

Malmendier, U., Nagel, S., 2011. Depression Babies: Do Macroeconomic Experiences Affect Risk Taking? The Quarterly Journal of Economics 126 (1), 373–416.

Martens, N., Orzen, H., 2021. Escalating commitment to a failing course of action — A re-examination. European Economic Review 137, 103811.

Meier, A. N., 2022. Emotions and Risk Attitudes. American Economic Journal: Applied Economics 14 (3), 527–558.

Mesquita, B., Ellsworth, P. C., 2001. The role of culture in appraisal. Oxford: Oxford Univ. Press, Ch. The role of culture in appraisal.

Mesquita, B., Markus, H. R., 2004. Culture and emotion: Models of agency as sources of cultural variation in emotion. In: Feelings and emotions: The Amsterdam symposium. pp. 341–358.

Mesquita, B., Walker, R., 2003. Cultural differences in emotions: a context for interpreting emotional experiences. Behaviour Research and Therapy 41 (7), 777–793, cross-cultural assessment and abnormal psychology.

Michel, P., 1982. On the transversality condition in infinite horizon optimal problems. Econometrica: Journal of the Econometric Society, 975–985.

Negrini, M., Riedl, A., Wibral, M., 2022. Sunk cost in investment decisions. Journal of Economic Behavior & Organization 200, 1105–1135.

Passarelli, F., Tabellini, G., 2017. Emotions and Political Unrest. Journal of Political Economy 125 (3), 903–946.

Pe, M. L., Kuppens, P., 2012. The dynamic interplay between emotions in daily life: augmentation, blunting, and the role of appraisal overlap. Emotion 12 (6), 1320.

Post, T., van den Assem, M. J., Baltussen, G., Thaler, R. H., 2008. Deal or No Deal? Decision Making under Risk in a Large-Payoff Game Show. The American Economic Review 98 (1), 38–71.

Rozen, K., 2010. Foundations of Intrinsic Habit Formation. Econometrica 78 (4), 1341–1373.

Ryder Harl E., J., Heal, G. M., 1973. Optimal Growth with Intertemporally Dependent Preferences. The Review of Economic Studies 40 (1), 1–31.

Scherer, K. R., 2009. The dynamic architecture of emotion: Evidence for the component process model. Cognition and Emotion 23 (7), 1307–1351.

Segal, U., 1990. Two-Stage Lotteries without the Reduction Axiom. Econometrica 58 (2), 349–377.

Sharpe, S., Schwartz, C., 2020. MLB Pitch Classification.

Shirai, M., Soshi, T., Suzuki, N., 2021. Knowledge of sadness: Emotion-related behavioral words differently encode loss and failure sadness. Current Psychology 40, 895–909.

Smith, C. A., Ellsworth, P. C., 1985. Patterns of cognitive appraisal in emotion. Journal of Personality and Social Psychology 48 (4), 813.

Tversky, A., Kahneman, D., 1992. Advances in prospect theory: Cumulative representation of uncertainty. Journal of Risk and Uncertainty.

Wälde, K., 2018. Stress and Coping-An Economic Approach. CESifo Working Paper Series.

Wei, X., Palomaki, J., Yan, J., Robinson, P., 2016. The science and detection of tilting. In: Proceedings of the 2016 ACM on International Conference on Multimedia Retrieval. pp. 79–86.