Permanent exemption from social security contributions in Belgium: The role of hiring frictions

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Abstract

Since 2016, Belgium has permanently exempted new employers from paying social security contributions on the gross wage of one of their employees. This paper clarifies the extent to which the policy modified the behaviour of agents who would not have hired in the absence of the policy, and to what extent it affected the behaviour of agents who would have hired even without it. We find that only the latter group responds to the policy, but mostly by creating single-employee employers. This casts light on why the policy had a limited impact on the stock of firms with more than one employee. We also find that the increase in the cost of hiring caused by the policy is not substantial, and only marginally affects the firms' employment decisions.
Keywords: payroll taxes; size-dependent policies; hiring frictions; wage subsidies; competitive search theory.
JEL Codes: ...

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1 Introduction

In many advanced economies, the persistent issue of high unemployment rates and underwhelming growth performance has driven policymakers to prioritize job creation (Criscuolo et al., 2014). While most industrial policies offering subsidies target large existing firms (Acemoglu et al., 2018), there is a growing consensus that young firms play a substantial role in overall job creation (Bijnens and Konings, 2020; Decker et al., 2014; Criscuolo et al., 2014; Haltiwanger et al., 2013; Wong et al., 2005), suggesting it might be more effective to direct subsidies toward new employers who might have untapped potential to grow and innovate.

This paper assesses the potential of this type of policies by leveraging a permanent payroll tax exemption introduced in Belgium in 2016 for new employers. The purpose of this exemption was to rise the number of new employers, thereby promoting job creation. Under this policy, newly established employers - defined as firms with no employees in the four quarters before the moment they claim the exemption - were relieved from paying social security contributions on the gross salary of one employee.

Cockx and Desiere (2024) find a 31% increase in the number of new employers created immediately after the reform's implementation. Comparing a cohort of new employers created just before and after the reform implementation, Deng et al. (2024) confirm the increase in the number of new employers post-reform and find that employers created after the reform announcement are less productive on average than those created before. This is consistent with the reform pushing some agents, for whom it was not profitable to hire before the reform, to do so thanks to the exemption.[1] Having more employers with lower average performance does not preclude some from becoming very successful firms over time. In this regard, comparing the employment outcomes after three years for the cohort of new employers created before and after the reform, Deng et al. (2024) finds that the number of firms with at least five employees is 13% higher in the post-reform than in the pre-reform cohort.

To assess the effectiveness of the reform in promoting employment, it is key to understand how much the observed effects come from the relatively larger group of firms that would have hired anyway in the absence of the reform — we call them the *inframarginal* new employers — and those that would not have hired in its absence — the *marginal* new employers. On the one hand, we want to understand if subsidising firms that would not have hired in the absence of the reform only pushes them to hire one employee, or if they may hire additional employees thanks to the policy. On the other hand, we want to determine whether subsidising firms that would have hired anyway in the absence of the reform induces them to modify their employment decisions.

Therefore, this paper studies how *inframarginal* and *marginal* new employers contribute to the effects we observe and how hiring frictions shape firms' responses. In doing this, we will also assess to what extent the increase in the number of employers deter-

[1] Deng et al. (2024) find that the cohort of employers created after the policy implementation employ, on average, fewer employees, have lower turnover, and profits.

mines a rise in the wages firms need to offer to attract job seekers and whether these wage increases may counteract the possibly direct positive effects of the policy on employment. Finally, this paper evaluates if the short-term impact of the policy, where only some new firms are eligible, differs from the long-term outcomes where all firms were created after the policy implementation and, therefore, benefit from it.

To conduct our analysis, we develop a directed search model based on Kaas and Kircher (2015). Our model is estimated using panel data from firms, provided by the National Bank of Belgium (NBB). By employing a directed search model, we use a minimal set of variables to effectively describe agent behaviours and dynamics following the introduction of the exemption. However, the model still allows for a tractable representation of multi-worker firms and the inclusion of heterogeneity in their hiring and firing decisions, along with the introduction of labour market frictions. Therefore, the model allows us to separate the roles of the new infra-marginal and marginal employers and to identify the factors that shape their employment responses.

We show that the rise in the number of firms with one employee is driven by *marginal* employers. However, we find that to expand beyond one employee, these new employers need to experience positive idiosyncratic productivity shocks. We estimated a relatively low variance in the productivity shocks, which explains why only a small proportion of these new employers hire more than one employee. Few *inframarginal* employers respond to the reduction in the SSC. This is because we find that most of the firms with zero employees who would have hired in the absence of the policy were already filling their desired number of vacancies at the highest rate, and therefore, the policy had little room to incentivise them to post higher wages to attract more job-seekers. Similarly, for the *inframarginal* employers who increased their wages to attract more job-seekers due to the policy, the relatively low matching efficiency prevented them from achieving a significant increase in the filling rate and, therefore, a substantial increase in their number of employees.

We also find that the increase in the hiring cost caused by the policy is not substantial (it increases by about 1.36%) and only affects the firms' employment distribution in a minor way. Moreover, due to the small increase in hiring costs, incumbent employers who are not eligible for the exemption do not substantially modify their hiring decisions. This leads the economy to slowly adjust to the new steady-state value - it will take 13 (52) quarters for firms with 1 employee to achieve 1/2 (3/4) of their steady-state increase in number.

Our model contributes to the literature on size-dependent policies, which target firms above (or below) a certain threshold. There is an overall consensus that these policies result in the misallocation of labour, thereby reducing output and productivity. However, while some studies find significant negative effects (Guner et al., 2008; Braguinsky et al., 2011; Gourio and Roys, 2014; Garicano et al., 2016)[2], others find that the splillover effects

[2]Guner et al. (2008) develop a growth model with an endogenous size distribution of production units and find that policies that reduce the average size of establishments by 20% lead to reductions in output and output per establishment up to 8.1% and 25.6% respectively, as well as large increases in the number of establishments (23.5%). Braguinsky et al. (2011) shows that the high levels of employment

on firms not affected by a size-dependent reform may mostly compensate for the negative effects from misallocation Cahuc et al. (2023)[3] or that crowding-out effects reduce but do not completely cancel out the positive effects of targeting small firms (Rotemberg, 2019)[4]. Our paper contributes to this literature because, by design, our policy directly affects the extensive margin decision to become an employer, rather than the intensive margin decision regarding whether to hire more or fewer employees: The constraints preventing a firm from becoming an employer likely differ from those hindering the hiring of subsequent employees. Moreover, we not only distinguish the impact between eligible and ineligible employers but also differentiate, among eligible employers, between *infra-marginal* and *marginal* new employers. Understanding how the relatively large proportion of firms that would hire even in the absence of the reform react to a generous payroll tax change is essential to evaluate the policy's effectiveness.

This paper also contributes to the literature about the employment effects of payroll tax reductions while taking into account spillover effects and labour market frictions by equilibrium job search models (Shephard, 2017; Wang et al., 2023; Bíró et al., 2022; Cahuc et al., 2019, 2022).[5] These papers find that, although reallocation from non-eligible to eligible firms and spillover effects are often modest, they still need to be assessed to avoid biased estimates for the policy interventions. Our paper differs from these studies by evaluating the role of a permanent exemption rather than temporary reductions in SSC, potentially triggering stronger responses from agents. Moreover, the exemption is directly linked to the gross wages offered by firms and, consequently, directly affects their hiring and wage-setting behaviours.

This paper also complements the companion analysis by Desiere et al. (2024), who,

protection in Portugal operate as a tax on wages, and can produce a shift in the firm size distribution, relative to the distortion-free benchmark. Their results suggest that Portugal could achieve first-order productivity gains by moving to a less distorted labor market. In France, firms with 50 employees or more face substantially more regulation than firms that have less than 50. Gourio and Roys (2014) show that removing the regulation improves labor allocation across firms, leading in steady state to an increase in output per worker slightly less than 0.3%, holding the number of firms fixed. Looking at the same setup, Garicano et al. (2016) find that the welfare costs of the regulations are 3.4 (1.3) percent of GDP under partial real wage inflexibility (real wages perfectly flexible downward).

[3]The paper exploits a labor law reform implemented in Portugal in 2009 which restricted the use of fixed-term contracts for large firms. Accounting for spillovers on small firms yields an almost negligible employment impact of the reform.

[4]Studying the loosening of eligibility requirements for a variety of subsidies for small firms in India, the paper shows that these subsidies predicts large gains in firm output. However, crowding-out effects absorbed around two-thirds of the direct effects. Properly estimating crowding-out is therefore crucial for understanding the aggregate effects of firm level shocks.

[5]Shephard (2017) examines a series of reforms in the UK, implemented between 1997 and 2002, to increase in-work support for families with children, finding relatively modest equilibrium effects. Wang et al. (2023) examine a payroll-tax reduction for low-wage workers in France. They find reallocation and spillover effects to be moderate. Bíró et al. (2022) look at the general equilibrium effects of an employers' payroll tax cut for both younger and older workers in Hungary, finding an increase in wages at high-productivity firms. Cahuc et al. (2019) analyze a temporary hiring credit tied to monthly wages for firms with less than 10 employees in France: due to the short-term nature of the policy, they exclude the existence of general equilibrium effects. Cahuc et al. (2022) analyze a policy that limited the utilization of fixed-term contracts for companies employing over 750 individuals in Portugal, showing important spillover effects on smaller firms.

examining the same policy analysed in this paper, measures its general equilibrium effects on wages and aggregate output by assuming a simple frictionless general equilibrium model of occupational choices. We show that hiring frictions actually explain why *inframarginal* firms do not modify their hiring behaviour and, therefore, why the reaction of the **?** firms, which is the focus of Desiere et al. (2024), approximates the policy effects we observe in the data.

Finally, our model offers a theoretical contribution to the field of competitive search theory. Specifically, taking inspiration from Lucas (1978)'s setup, we incorporate the option for individuals to choose between entrepreneurship and salaried employment. This expansion allows us to assess reforms that may affect not only the choices made by established firms but also the trade-off between starting a new business and searching for a job.

This paper is organized as follows: Section 2 discusses the policy under consideration. Section 3 presents the model used in our analysis. Section 4 describes the data we use. In Section 5, we detail the estimation procedure. Moving forward, Section 6 is dedicated to simulating and discussing the policy's effects. Section 7 concludes.

2 The policy and some descriptive evidence

In October 2015, Belgium implemented an unforeseen and permanent exemption from payroll taxes for private sector firms that recruit their first employee after January 1st, 2016. Starting from that date onward, any firm (including new firms) that begins employing workers and had no employees in the preceding four quarters is permanently exempted from paying the employers' social security contributions (SSC) linked to the gross salary of one employee. The exemption is not granted to existing employers or to new firms created by existing employers[6]. The National Social Security Office (NSSO) is responsible for enforcing the correct application of the policy (Court of Audit, 2021).

This exemption stands out for several significant reasons. Firms keep the exemption even if they continue hiring, and firms with multiple employees can decide to whom to assign the exemption at the beginning of each quarter. Moreover, if the "subsidized" worker leaves, the firm can reassign the exemption to another worker. This means that once the firm claims the exemption, it is permanent as long as the firm keeps at least one employee. Lastly, until 2022, the exemption remained uncapped, enabling employers to maximize payroll tax savings by assigning it to the employee with the highest gross remuneration.

The tax savings granted by the exemption can be measured by the difference in the expected rate of SSC a new employer was expected to pay on the gross wages of one employee before and after the reform. Post-reform, new employers consistently paid a 2.9% SSC rate on the wage of one employee. In the pre-reform period, calculating the expected rate of SSC on the first employee is more complex. Indeed, the 2016 reform

[6]Moreover, in Belgium, an individual running a business activity is classified as self-employed and is, therefore, subject to a different SSC regime from that of employees and cannot claim the exemption.

replaced less generous temporary hiring subsidies for the first employee, which had been in effect, with modifications since 2004.[7]. Therefore, in the pre-reform period, the rate of SSC was lower at the beginning of the employment spell than later on. To compute the expected payroll tax rate when hiring their first employee, considering this temporary subsidy, we follow Cockx and Desiere (2024) and we compute the weighted average of SSC paid each quarter after hiring, with weights being the probabilities that a firm remains an employer and a yearly time discount rate of 5%. More details on how we computed the rate pre and post reform are in Appendix C. This resulted in a rate of 18.3% SSC pre-reform, meaning that the exemption reduced the rate of SSC by about 15pp. Given the magnitude of the SSC rate change, it is not surprising that approximately 72% of new employers in 2016 applied for this exemption within two quarters of hiring their first employee (López-Novella, 2021).

While for an empirical estimate of the impact of the reform on the employment distribution up to three years post-reform we refer to Deng et al. (2024), we can still grasp the first-order impact of the policy on firms' behaviours by plotting the evolution in the stock of firms. Figure 1 illustrates the changes in the number of firms employing 1, 2, and between 3 and 8 employees over time.[8] Firm size is defined as the headcount of employees in the firm on the last day of the quarter, regardless of whether they are part-time or full-time. Each quarter, we calculate the ratio of firms employing 1 employee (and likewise for those between 2 and 8 employees) to the number of firms with 1 employee during 2015Q3 - we use 2015Q3 as the pre-reform quarter because, as demonstrated by Cockx and Desiere (2024), when the reform was announced in 2015Q4, some firms decided to postpone hirings from 2015Q4 to 2016Q1.

Figure 1 highlights that before the reform, the stock of firms with 1 employee and the stock of bigger firms evolved similarly. However, the 2016 reform swiftly impacted the stock of firms with one employee, which increased by 8 to 10% compared to the 2015Q3 level. The stock of bigger firms did not react with the same magnitude but followed a similar, smaller upward trend.

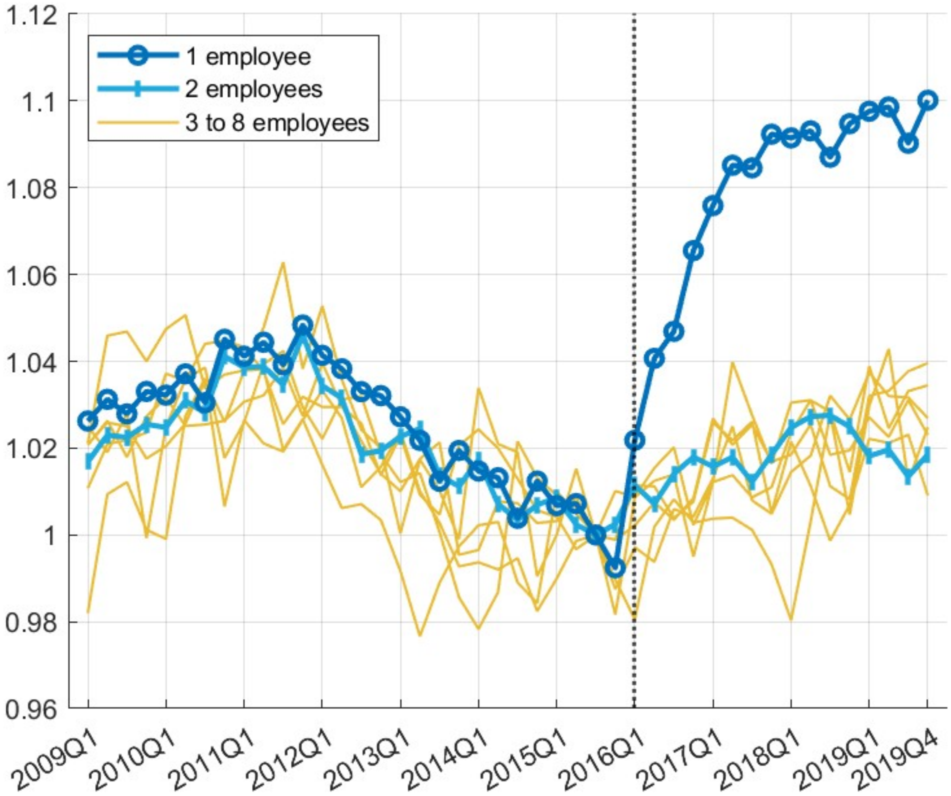
The exemption was part of broader reforms aimed at incentivizing increased hiring by reducing employers' SSC. Temporary subsidies for hiring the 2nd to the 5th employee have also been in place since 2004. In 2016, the government further enhanced these subsidies and introduced an additional temporary subsidy for hiring the 6th. However, these increases were minor compared to the introduction of the permanent exemption, which is, therefore, the only change in the SSC rate simulated in the paper.[9]

[7] In 2015, new employers hiring their first employee could claim a €1,550 quarterly payroll tax reduction for the first five quarters, €1,050 for the next four quarters, and €450 for the last four quarters.

[8] We plot only firms with 8 employees or fewer because firms with 3 to 8 employees follow the same trend pre-reform as firms with 1 or 2 employees, while this is not the case for firms with 9 to 15 employees. Similarly, the stock of firms with 0 employees followed a strong positive trend throughout our period of observation. See Figure A.1 for more details. This paper neither studies nor aims to explain this trend in the firms with 0 employees. In the model estimation, we assume that their number in the pre-reform period is equal to their average number in the two years pre-reform.

[9] For instance, firms hiring their second employee could claim a payroll tax reduction over thirteen quarters of at most €8,850 before and €13,750 after the 2016 reform. The subsidy for subsequent hires are less generous than for the second hire.

Figure 1: Evolution stock of firms



The figure illustrates the changes in the stock of firms with 1 to 8 employees. For each firms' group, we normalize the number of firms to 1 in 2015Q3. The vertical dashed line marks the beginning of the policy in 2016.

In addition, starting from 2016 the government gradually decreased the nominal rate of employers' SSC imposed on all private sector employers, from 32.4% to about 25% in 2018. Simultaneously, the government adjusted the reductions in employers' SSC granted when firms employ low-wage and high-wage employees.[10] From 2018 onward, these reforms affected the rate of SSC paid for employees beyond the first considerably. However, we leave these overall reductions in the SSC for further research.

3 The Model

The model is inspired by Kaas and Kircher (2015)'s directed search model, from which we adopt three distinct modelling assumptions. First, we introduce employers' payroll taxation, assuming that before the reform, employers pay a tax rate $\tau > 0$ on the wages of all their workers. After the policy implementation, new employers pay a tax rate of

[10]Specifically, on April 1, 2016, the government raised the maximum wage limit for firms to benefit from reduced SSC when occupying low-wage employees. In 2018, this threshold was further elevated, and the exemption for hiring high-wage employees was eliminated.

$0 \leq \tau_1 < \tau$ on the gross earnings of a single employee while still continuing to pay τ on the wages of the other employees. Secondly, we endogenize entrepreneurship into the directed search model, wherein firms originate not from a pool of potential entrepreneurs but from unemployed individuals who opt to start their own businesses instead of searching for a job. Thirdly, as this paper focuses on small and medium-sized firms, we assume that the law of large numbers doesn't apply. If we define the number of employees each firm can have with L, then $L \in \mathbb{N}_0$.

In our economy, there is a finite and constant mass of individuals. Individuals are infinitely-lived, risk-neutral, and discount future income with a factor $\beta < 1$. Time is discrete, and moreover, capital markets are perfect. At the beginning of each period, each agent can be employed, searching for a job, or running a firm. Firms produce a homogeneous good using only homogeneous labour, and the good market is perfectly competitive.

All employed agents provide one unit of labour per period and cannot search for a job in other firms while employed. Each period, each firm generates revenues $R(L, x)$, where $x \in X = [x_1, x_2, ..., x_n]$ is the firm's idiosyncratic productivity at the start of the period. The function R increases at a decreasing rate in L. Moreover, we assume that $R(L = 0, x) > 0$. The solo entrepreneurs are the business owners who operate their firms without having employees.

At the beginning of the period, a firm's owner must choose between exiting the market, terminating S employees, $0 \leq S \leq L$, or posting vacancies. These decisions will not impact the current period's workforce and production but will determine whether the firm is active and/or their number of employees at the start of the next period. If the business owner chooses to close their company, all employees lose their jobs at the end of the period. The business owner must then decide whether to start a new business or search for a job at the beginning of the next period. If the business owner chooses to fire employees, we assume that each of the L employees has an equal probability S/L of being fired. This assumption reduces the numerical complexity of our problem. The business owner does not pay firing costs for any of the employees fired. Finally, if the business owner decides to hire, they compete for job-seekers by posting vacancies. Following Kaas and Kircher (2015), each vacancy represents a long-term contract. Each long-term contract defines for each future period a state-contingent wage and a state-contingent retention probability. In the pre-reform period, firms pay SSC computed as τ times the wages offered. Consequently, τ increases the cost of offering a given wage to employees. We assume that neither firms, employees, nor job-seekers benefit from the SSC collected by τ. When advertising a job opening for the first time, business owners must pay a one time sunk cost associated with managing an employer-employee relationship for the first time, denoted as s_c.

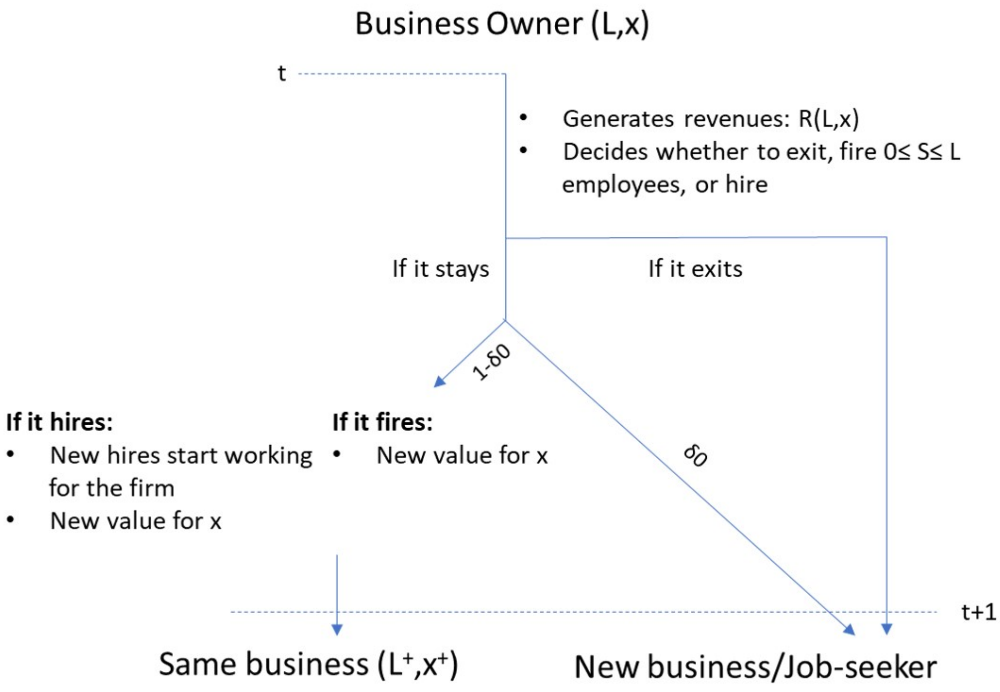
We define with λ, $\lambda \in \mathbb{N}_0$, the number of job-seekers each vacancy attracts. In turn, the number of job-seekers determines the probability $m(\lambda)$ that the vacancy will be filled by a standard matching function. The function $m(\lambda)$ increases at a decreasing rate as λ increases, with $m(\lambda = 0) = 0$ and $m(\lambda = +\infty) = 1$. The worker's probability of being hired is determined by $\frac{m(\lambda)}{\lambda}$, which strictly decreases as λ increases. In this framework, the number of job-seekers applying to a vacancy is directly tied to the expected payoff they get if hired. With the assumption that job-seekers see all available vacancies without any cost, a vacancy will attract more applicants as the expected payoff obtained if the individual is hired increases. Therefore, firms can increase $m(\lambda)$ by offering higher wages. By offering higher wages, they will also pay a higher amount of SSC. As a vacancy becomes more appealing and draws more applicants, the likelihood of any individual job-seeker getting hired for that vacancy decreases. This ensures that, in equilibrium, the expected payoffs from applying to various vacancies are equal.

We additionally assume that firms are perfectly committed to the contract they offer, meaning they cannot change the proposed state-dependent wages and retention probabilities after hiring. Moreover, we assume that the contracts offered by firms ensure that. once employed, the payoff received by workers in each period equals what they would receive if they were searching for a job or starting their own business. This last assumption is introduced to prevent firms from choosing contracts that frontload wages in order to maximize the amount of wages exempted from the SSC in the post reform period. Moreover, it allows for imperfect commitment from the employees, as we assume they choose to remain employed if it provides them with the same payoff as searching for another job or becoming business owners. This assumption also implies that a company will either lay off the employee(s) or exit the market if it is more profitable than offering an employee the present value they would receive while searching for a job or becoming business owners. We also assume that search for new hires is costly: following Kaas and Kircher (2015) the cost of posting vacancies, $C(V, L)$, is strictly convex in the number of vacancies V, $V \in \mathbb{N}_0$. This prevents firms from instantaneously growing large by posting many vacancies at a fixed cost. Moreover, we assume that the per-employee cost of vacancies decreases with L to acknowledge that posting a given number of vacancies is less expensive for firms with many employees than for smaller firms.

Even if the business owner decides not to shut down the firm but to fire/hire employees, the firm still faces an exogenous probability $\delta_0 > 0$ of shutting down at the end of each period. In this event, all their employees lose their jobs immediately. For brevity, let's define ϕ as the probability that an employee will still work for the firm at the beginning of the next period. Specifically, $\phi = 0$ if the firm decides to shut down; $\phi = (1-\delta_0)(1-S/L)$ if the firm chooses to fire $0 \leq S \leq L$ employees; and $\phi = (1-\delta_0)$ if the firm decides to hire.

At the beginning of each period, individuals not employed or running a business activity from a previous period must either seek employment from available job vacancies, in which case they will receive the value of home production denoted as b for the current period, and potentially work for a firm in the next period if successful. Alternatively, they can decide to start a firm. If an agent decides to start a firm, they will draw an initial idiosyncratic productivity level x form the discrete set X with probability σ_x. Following this decision, they will immediately need to decide whether to exit (and therefore have a firm only in the current period), stay without employees or hire employees. If a firm does not exit the market, its idiosyncratic productivity x follows a Markov process on the finite state space X, characterized by known transition probabilities $\pi(x^+|x)$, where superscript $+$ designates the next period's value. Figure 2 represents the timing of the

Figure 2: Timing of events for the business owners



event for the business owners.

3.1 Simplifying property

Proposition 1 *If the tax rate τ on the stream of wages is independent of the firm's size, individuals are risk neutral and the capital market is perfect, then business owners would make the same hiring/firing/exit decisions, and unemployed agents would make the same employment decisions whether i) firms post long-term contracts that specify a state-contingent wage and retention probability for each future period, or ii) contracts that only provide a "hiring bonus", $\bar{w}$, and a "continuation wage", $\underline{w}$, for each period the employee produces output. The continuation wage is set so that the value of staying in the current job equals the present value of being unemployed.*

Based on Proposition 1's result - see Appendix B for the proof - we assume firms offer contracts specifying a hiring bonus ($\bar{w}$) and a continuation wage ($\underline{w}$) instead of long-term state-contingent contracts. This makes our model numerically tractable because it ensures that the value an employee receives from the moment they provide one unit of output is the same across all firms. Therefore, when comparing different job vacancies, job-seekers solely consider the offered hiring bonus. Thus, λ depends solely on $\bar{w}$. Furthermore, this wage profile makes employees indifferent between moving to unemployment or remaining in employment. Instead, if a firm can't pay employees at least the amount they would receive in unemployment, it will either lay them off or exit the market.

We also assume that when hiring, each firm offers the same hiring bonus to all individuals hired at the same time. This assumption is mild. The increase in the hiring bonus needed to attract an additional job seeker (i.e., $\lambda + 1$ job seekers) rises with λ, making it progressively more costly for a firm to enhance the filling rate of a vacancy. As a result, firms have no incentive to increase the filling rate of one vacancy, leaving the filling rate of other vacancies unchanged.

3.2 Workers' problem in the pre-reform period

Let $\mathcal{I} = \{(L, x) | L \in \mathbb{N}_0, x \in X\}$. We define with $i \in \mathcal{I}$ all firms that share the same pair of values (L, x). Furthermore, let $\bar{\mathcal{I}}$ the subset of $\mathcal{I}$ where firms post at least one vacancy.

At the beginning of each period, agents in "unemployment" (i.e. not employed and not running a business from a previous period) can either apply for a vacancy or open their own business. Let's define with S_i the expected value an agent obtains by applying to a vacancy posted by firm i. The present value obtained by an agent deciding to open a business and receiving the initial idiosyncratic productivity draw x is denoted $J(L = 0, x)$.

Agents can observe all vacancies at no cost. When applying, they select the vacancy offering the highest expected value, denoted as $\max_{i \in \bar{\mathcal{I}}}[S_i]$. As a result, in equilibrium, all vacancies will offer the same expected value. Similarly, agents will only apply for a vacancy if the expected value of doing so is higher than the expected value of starting a business net of the sunk cost s_b to start a business (and vice versa). This cost serves as a proxy for the investment in time and/or resources necessary to open a business, such as completing administrative procedures, learning how to file taxes, etc. In equilibrium, the expected value of searching for a job and the net expected value of opening a business take the same value, denoted as U. Therefore, the present value in unemployment is given by:

$$U = \max[\sum_{x \in X} \sigma_x J(L = 0, x) - s_b, \max_{i \in \bar{\mathcal{I}}}[S_i]]. \tag{1}$$

The value that an agent receives when employed by firm i, denoted as W_i, is the sum of three components. Firstly, it consists of the continuation wage $\underline{w_i}$ the agent receives in the current period. Secondly, it consists of the expected value the agent receives if they remain employed in the next period, denoted as $E_{x^+}[W_i^+]$ - in the current period, the agent only knows the transition matrices $\phi(x^+|x)$. $E_{x^+}[W_i^+]$ is weighted by the probability that the agent might retain their job, ϕ_i. If the agent loses their job, they will become unemployed in the next period, resulting in them receiving the value in unemployment U^+. Therefore, if employed in firm i, the agent gets:

$$W_i = \underline{w_i} + \beta \phi_i E_{x^+}[W_i^+] + \beta(1 - \phi_i)U^+. \tag{2}$$

S_i is also the weighted sum of three components: the value of home production b the individual obtains during the job search, the expected value if the individual is still unemployed at the end of the period, U^+, and the expected value if the individual gets hired. This latter is determined by two factors: the hiring bonus, $\bar{w}_i$, and the expected value while employed, $E_{x^+}[W_i^+]$. Therefore, the value of applying to a vacancy posted by firm i is given by:

$$S_i = b + \beta(1-\delta_0)\frac{m(\lambda)}{\lambda(\bar{w}_i)}[E_{x^+}[W_i^+] + \bar{w}_i] + \beta(1-(1-\delta_0)\frac{m(\lambda)}{\lambda(\bar{w}_i)})U^+. \tag{3}$$

Given that, in equilibrium, $S_i = U$, $\forall i \in \bar{\mathcal{I}}$, and that we have assumed that $\underline{w_i}$ is such that $W_i = U$, we can rewrite equation (3) and equation (2) as follows:

$$U = b + \beta(1-\delta_0)\frac{m(\lambda)}{\lambda(\bar{w}_i)}[U^+ + \bar{w}_i] + \beta(1-(1-\delta_0)\frac{m(\lambda)}{\lambda(\bar{w}_i)})U^+ \tag{4}$$

and

$$U = \underline{w_i} + \beta\phi_i U^+ + \beta(1-\phi_i)U^+. \tag{5}$$

We can rearrange these two equations to determine the continuation wage,

$$\underline{w_i} = U - \beta U^+, \tag{6}$$

and the hiring bonus posted by firms,

$$\bar{w}_i = \frac{U - \beta U^+ - b}{\beta(1-\delta_0)\frac{m(\lambda)}{\lambda(\bar{w}_i)}}. \tag{7}$$

To find the steady state value for $\underline{w_i}$ and $\bar{w}_i$ we set $U = U^+$.

3.3 Firms' problem in the pre-reform period

Let's consider a firm that, at the beginning of the period, has the pair (L, x), where $L \in \mathbb{N}_0$ and $x \in X$ - for brevity, we omit the subscript i. Then, this firm chooses among three exclusive alternatives: exiting the market (e), firing workers (f), or hiring workers (h). The firm's payoff at the beginning of the period is:

$$J(L, x) = \max[e(L, x), f(L, x), h(L, x)]. \tag{8}$$

The firm's payoff from exiting the market, $e(L, x)$, is determined by two components. First, the current net revenues, which are the gross revenues minus wages and operating costs - we assume that the operating costs, $c(L)$, weakly increase at a decreasing rate in the number of employees. Second, the next period's payoff which is the value of entering unemployment if an agent decides to exit.

$$e(L,x) = R(L,x) - (1+\tau)\cdot \underline{w}\cdot L - c(L) + \beta \cdot U^+, \tag{9}$$

where $\underline{w} = U - \beta U^+$ comes from the workers' problem.

The first component of the firm's payoff when firing is still the current net revenues specified for the exit decision. The next period's payoff from firing is equivalent to the payoff from operating a firm with $L-S$ employees. If the business owner decides not to exit, they will still be forced to exit with exogenous probability δ_0. Therefore, the payoff the firm obtains if it decides to fire is:

$$\begin{aligned} f(L,x) = \max_{S\in\{0,1,..,L\}} \Big[& R(L,x) - (1+\tau)\cdot \underline{w}\cdot L - c(L) \\ & + \beta\cdot(1-\delta_0)\cdot E_{x^+} J(L-S, x^+) + \beta\cdot\delta_0\cdot U^+\Big]. \end{aligned} \tag{10}$$

The first component of the firm's payoff when hiring is still the current net revenues described for the exit decision. However, since the firm is hiring, it incurs the current extra cost $C(V,L)$. The next period's payoff when hiring is the value of running a firm with L^+ employees as a result of posting V vacancies with the promised hiring bonus $\bar{w}$, minus the total expected hiring bonuses the firm needs to pay for the new hires. Formally, we define with $Pr(L^+, L, m(\bar{w}), V)$ the probability of having L^+ employees at the start of next period, given that each of the V posted vacancies has a probability $m(\bar{w})$ to be filled - we use the notation $m(\bar{w})$ because $m(\lambda)$ and $\lambda(\bar{w})$. Similarly, let's define with $Pr(H, m(\bar{w}), V)$ the probability that the firm will actually hire H employees, given that each of the V posted vacancies has a probability $m(\bar{w})$ to be filled. Then, the payoff the firm obtains if it decides to hire is:

$$\begin{aligned} h(L,x) = \max_{V\in\mathbb{N}_0, \bar{w}\in\mathbb{R}} \Big[& R(L,x) - (1+\tau)\cdot\underline{w}\cdot L - c(L) - C(V,L) \\ & + \beta\cdot(1-\delta_0)\cdot [E_{x^+} \sum_{L^+\in\{L,\dots,L+V\}} Pr(L^+, L, m(\bar{w}), V)\cdot J(L^+, x^+) \\ & - \sum_{H\in\{1,\dots,V\}} Pr(H, m(\bar{w}), V)\cdot H\cdot(1+\tau)\cdot\bar{w}] + \beta\cdot\delta_0\cdot U^+\Big], \end{aligned} \tag{11}$$

where $\bar{w}_i = \frac{U - \beta U^+ - b}{\beta(1-\delta_0)\frac{m(\lambda)}{\lambda(\bar{w}_i)}}$ comes from the workers' problem.

3.4 Resource constraint

We assume that the population - i.e. the sum of the number of job-seekers, employees, and business owners remains constant over time. In detail, for the before steady state, we standardize the total number of business owners to 1. Then, we proceed to find the optimal number of job seekers and employed individuals by solving the problems faced by both firms and workers. The resulting total number of agents (which is the sum of job-seekers, employees, and business owners) is denoted by K, where $K > 1$.

Later, when we introduce the exemption, we assume that the total number of agents remains equal to K. In other words, we assume that offering an exemption for the first hire does not serve as an incentive for individuals in inactivity to start seeking jobs or initiate their own businesses.

3.5 Introducing the exemption for the first hire

In the before scenario, we assumed that all firms pay a rate $\tau > 0$ on the wages offered to their workers as employers' SSC. Now, let's modify the model such that eligible firms after the reform pay $0 \leq \tau_1 < \tau$ on the wage of one employee.

While all firms are eligible for the exemption in the after-steady state, during the transition, eligible firms are those that had zero employees at the time of the reform or were established only in the post-reform period. Firms with one or more employees at the time of the reform implementation continue to solve the optimisation problem in Subsection 3.3. Similarly, the worker's problem also remains unchanged, as the parameter τ does not directly affect their choices.

Introducing the exemption for the first hire in the directed search framework comes with certain challenges. In the model, if we don't impose any restrictions on how firms set wages for each hire across periods, and when and how the exemption can be transferred between employees, firms may strategically set wages and retention probabilities to maximize the exemption they can claim. For instance, in the model, a firm can optimize tax savings by assigning the exemption to each employee for a limited number of quarters. This involves setting a high wage when an employee is exempt and a low (or zero) wage for all the non-exempt quarters. This strategy is only viable under our assumption of risk-neutrality and perfect capital markets - i.e. job-seekers cares about the present value of the wage commitments, not about how wages are distributed over time. In reality, employees value the distribution of wages over time, making the described firm's strategy to maximize tax savings not feasible.

Therefore, to avoid fluctuations in the wage paid to each employee over time, we impose three additional assumptions for the after scenario for firms eligible for the exemption. Firstly, we assume that firms must offer the same contract, specifying the future wages and retention probability, to individuals hired at the same time. Secondly, we assume that a firm can assign the exemption to an employee only if it has zero employees when it posts vacancies. Finally, the firms with one employee or more do not anticipate that they may assign the exemption to one of the newly hired employees in the future at the moment they post their vacancies.

Under these assumptions, we can continue to assume, without loss of generality, that firms eligible for the exemption set the aforementioned hiring bonus and continuation wage. It is important to note that these assumptions create a gap between our model and reality. In reality, firms may not be able to change the wage of an employee from one quarter to another, but they can assign the exemption to the worker with the highest wage, regardless of when this worker was hired. Our assumption excludes this possibility, thus providing a lower bound for the tax savings firms can achieve.

3.5.1 Eligible firms' problem in the after-reform period

For eligible firms, the exemption first modifies the rate of employers' SSC applied to the continuation wages. Indeed, all eligible firms will pay $(1+\tau)\cdot \underline{w}\cdot \max[0, L-1] + \tau_1 \cdot \underline{w} \cdot \min[1, L]$ instead of $(1+\tau)\cdot \underline{w}\cdot L$. Therefore, the primary direct effect of the exemption is to reduce the total wage cost of one employee, enabling less productive firms, which previously could not afford to hire, to do so.

Second, if the firm has zero employees and hires H employees, the hiring bonus for one of these employees will be exempt from SSC. Since we assumed that the firm must offer the same wage to all employees hired at the same time, it will assign the exemption randomly to one of them. Therefore, the exemption reduces the cost of offering a higher hiring bonus because the firm will not pay SSC on this bonus. This represents the second direct effect of the policy: it lowers the cost of increasing the vacancy filling rate by raising wages. For firms with $L \geq 1$, this second effect does not occur, as the firm will pay the rate τ on the wages of all new hires. Formally, the payoff of a firm with $L = 0$ that decides to hire is:

$$\begin{aligned}
h(L=0, x) = &\max_{V\in[1,2,3\ldots],\bar{w}\in\mathbb{R}} \Bigg[R(L=0,x) - c(L=0) - C(V, L=0) \\
&+ \beta\cdot(1-\delta_0)\cdot\Big[E_{x^+}\sum_{L^+\in\{0,\ldots,V\}} Pr\big(L^+, L=0, m(\bar{w}), V\big)\cdot J(L^+, x^+) \\
&- \sum_{H\in\{1,\ldots,V\}} Pr\big(H, m(\bar{w}), V\big)\cdot H\cdot\big(\frac{1}{H}\tau_1 + \frac{(H-1)}{H}\cdot\tau\big)\cdot\bar{w}\Big] + \beta\cdot\delta_0\cdot U^+\Bigg];
\end{aligned} \tag{12}$$

While the payoff of a firm with $L > 0$ that decides to hire is:

$$\begin{aligned}
h(L, x) = &\max_{V\in[1,2,3\ldots],\bar{w}\in\mathbb{R}} \Bigg[R(L,x) - c(L) - C(V, L) \\
&- (1+\tau)\cdot\underline{w}\cdot\max[0, L-1] - \tau_1\cdot\underline{w}\cdot\min[1, L] \\
&+ \beta\cdot(1-\delta_0)\cdot\Big[E_{x^+}\sum_{L^+\in\{L,\ldots,V+L\}} Pr\big(L^+, L, m(\bar{w}), V\big)\cdot J(L^+, x^+) \\
&- \sum_{H\in\{1,\ldots,V\}} Pr\big(H, m(\bar{w}), V\big)\cdot H\cdot(1+\tau)\cdot\bar{w}\Big] + \beta\cdot\delta_0\cdot U^+\Bigg].
\end{aligned} \tag{13}$$

As in the before steady state, the term $Pr\big(H, m(\bar{w}), V\big)$ is the probability that posting V vacancies and choosing $m(\bar{w})$, a firm obtains H hires.

4 Data

Our objective is to estimate the model parameters such that the model i) closely mirrors the employment patterns of small firms observed in the data, ii) and simultaneously matches the total number of employees within the Belgian economy, without precisely representing the distribution of employment across large firms.[11]

Therefore, to calibrate the model, we primarily rely on firm-level employment statistics from a dataset provided by the National Bank of Belgium (NBB), supplemented by publicly available information from the Belgian Statistical Office (Statbel) and the Statistical Office of the European Union (EUROSTAT).

The dataset available from NBB is a quarterly panel dataset containing information at the firm level. It covers the period from the first quarter of 2009 to the fourth quarter of 2019. To be included in this dataset, firms must have had 15 or fewer employees, including zero, at the end of at least one quarter between 2009Q1 and 2019Q4.

The main variable used in our analysis is the number of paid employees each firm has at the end of each quarter. This data is recorded by the National Social Security Office (NSSO), which is responsible for administering SSC on gross wages paid by employees and employers. The NSSO provides detailed information on the number of part-time and full-time employees for each firm. Using these data, we categorize firms based on their number of employees (counted by head), and then we target their distribution and employment dynamics.

The firm-level dataset also includes information on the firms' total revenues for the quarter, the total wages paid each quarter, the total amount of paid employers' SSC, and the total amount of SSC reductions an employer has benefited from.

5 Estimation

We start by making some functional form assumptions. Then, we describe the parameters that are externally set or standardized. Finally, we describe the procedure implemented to internally estimate the remaining parameters.

[11]This decision was made because the policy is more relevant for very small firms than for big firms, and to limit our model's computational time.

5.1 Functional form assumptions

Each model's period corresponds to a quarter - we chose this frequency because this is the frequency of the NBB data.

As in Bilal et al. (2022), the matching function is Cobb–Douglas. Defining with γ_m the elasticity of the vacancy filling rate with respect to the number of job-seekers, and with μ the matching efficiency, the probability of filling a vacancy is $q(\lambda) = \mu \cdot (\frac{1}{\lambda})^{-\gamma_m}$, and the probability of a job-seeker being hired is $p(\lambda) = \mu \cdot (\frac{1}{\lambda})^{1-\gamma_m}$.

The firm's productivity takes the value x from the set $X = [x_1, x_2, ..., x_n]$, where all values between x_1 and x_n are equally spaced. Let's call Δx the difference between two consecutive values of x. The probability an agent obtains a certain value of $x \in X$ at entry is given by $\sigma_x \in \Sigma = [\sigma_{x_1}, \sigma_{x_2}, ..., \sigma_{x_n}]$. These values of $\sigma_x \in \Sigma$ are derived from a log-normal distribution with scale parameter μ_{ln} and shape parameter σ_{ln}, which is shifted to have its origin at x_1 and truncated at x_n - i.e. $\sigma_{x_1} = Pr(x \leq x_2 - \Delta x)$, $\sigma_{x_2} = Pr(x \leq x_3 + \Delta x) - Pr(x \leq x_2 - \Delta x)$ and so on, with $\sum_{x=1}^{n} \sigma_x = 1$. We chose this distribution among those commonly used in the literature to determine firms' idiosyncratic productivity (Dewitte et al., 2022) because of its simplicity and flexibility. Bilal et al. (2022) uses a Pareto distribution to determine the firms' productivity at entry.

Once productivity at entry is assigned, it follows a random walk process where $x^+ = x + \epsilon$, with $\epsilon \sim \mathcal{N}(0, \sigma_{\mathcal{N}})$. Based on this process, we compute $\phi(x^+|x)$. Notice that we discretise the random walk process so that x^+ can only take values $x \in X$.

Firms produce output with labour only. We assume that there is perfect competition in the good market and we normalize the good's price to one, such that the firms revenue are $R(L, x) = x \cdot L^{\alpha}$, $L \geq 0$. The per-period operating cost of employers is set to $c(L) = \mu_o \cdot L^{\gamma_o}$, $\mu_o > 0$ and $\gamma_o > 0$, for $L > 0$. We assume that the solo-entrepreneurs do no pay any per-period operating cost.

The cost of posting vacancies takes the form: $C(V, L) = \mu_V \frac{V^{\gamma_V}}{(L+1)^{\gamma_L}}$. The parameter μ_V is a scale parameter, while $\gamma_V \geq 1$ controls how fast $C(V, L)$ increases when the number of vacancies increases and $\gamma_L \geq 0$ how much the per employee cost of vacancies decreases when the number of employees increases. This specification is inspired by Kaas and Kircher (2015): it implies that the cost of posting vacancies is convex in the number of vacancies and that the average cost per employee of posting vacancies weakly decreases with the number of employees.

5.2 Parameters that are externally set or standardized

Following the literature, we set the quarterly discount rate $\beta = 1.3\%$ - which is equivalent to a yearly discount rate of about 5%. For the pre-reform period, we set the rate of employers SSC, τ, equal to 18.3% for all workers. For the post-reform period, we set the rate of employers' SSC to 2.9% for one employee for eligible firms - details on how we have computed these rates are in Appendix C. Setting $\tau = 18.3\%$ pre-reform, we impose the simplifying assumption that firms pay this rate for all employees beyond the first, if any. This is not actually the case. For example, firms with 6 to 15 employees pay an average rate of SSC of about 26.9% in the year pre-reform. Our model does not take into account this wedge, which will be then implicitly incorporated in the value of the estimated parameters. We set the number of levels for the idiosyncratic productivity n equal to 75. The choice of 75 has been done to have a fast code but enough "nodes" for the idiosyncratic productivity to match the firms' dynamics. We standardize $x_{low,1}$ to 1, and b to 0. We set the quarterly exogenous exit rate δ_0 to 1.03% for all firms. This is the quarterly exit rate computed from the NBB data for firms with 10 employees or more. Since firms with fewer than 10 employees have a higher exit rate, we still allow for firms to endogenously exit the market in response to a negative idiosyncratic productivity shock - Additional details regarding the computation of this exit rate can be found in Appendix D. The summary of the externally set or standardized parameters is reported in table 2. We set the elasticity matching function with respect to the number of job-seekers, γ_m, to 0.5, which is the value commonly estimated by the literature (Petrongolo and Pissarides, 2001) - our data did not allow us to estimate this parameter directly.

5.3 Parameters that are internally estimated

The 12 parameters left to be estimated are the scale factor of the matching function; the highest value of the idiosyncratic productivity $x_{low,75}$; the mean, μ_{ln}, and standard deviation, σ_{ln} of the log-normal distribution; the standard deviation of the shocks of the random walk process $\sigma_{\mathcal{N}}$; the production function elasticity α; the scale parameters μ_V and the elasticity parameter γ_V and γ_L of the cost of posting vacancy function; the scale parameter of the per-period operating cost of employers μ_o and the elasticity of the per-period operating cost wrt the number of employees γ_o; the exogenous quarterly exit rate equal for all firms, δ_0; the one-time sunk cost to open a firm s_b.

5.4 Estimation Procedure and validation exercise

We jointly estimate the parameters of our model to minimize a weighted sum of the squared distance between some empirical and respective simulated moments, where the simulated moments are obtained from the before model in steady state. In detail, following a similar approach to Bilal et al. (2022), if we name the vector of empirical moments θ, and $\hat{\theta}$ the model simulated counterparts, we minimize:

$$(\hat{\theta} - \theta)' W^{-1} (\hat{\theta} - \theta) \quad \text{s. to} \quad L \leq 50 \tag{14}$$

The matrix W contains the square of $min(\theta, \hat{\theta})$ on the main diagonal, with zeros elsewhere. We chose this weighting matrix because, other than some statistics from our NBB data, we also target aggregate statistics from Statbel and EUROSTAT, for whom is not possible to compute the corresponding standard deviation, as is commonly done in the method of simulated moments estimator (MSM). Therefore, minimising the percentage change between the simulated and observed statistics allows us to account for the varying scales of the moments employed. Moreover, by using $min(\theta, \hat{\theta})$, we take into account that some moments are bounded below to zero, thus avoiding giving more weight to the overestimation than the underestimation of the target moments.

Moreover, to reduce the numerical time required to solve the model, we impose an additional restriction that firms cannot have more than 50 employees.

5.4.1 Empirical and simulated moments: Description

The first moment we target is the proportion of firms with 0, 1, 2, 3, 4, and 5 employees in the economy over the total number of firms. This moment ensures that the model captures the distribution of "micro" firms. The second moment is the ratio of employees working in firms with strictly less than 10 employees to the total number of employees in the economy. This statistic ensures that our models replicate, on aggregate, the correct distribution of employment between our firms with 9 or fewer employees and the rest of employers - see Appendix E.1 for more details on these two moments.

The third moment is the proportion of new firms in a given quarter that have 0, 1, and 2 employees relative to the total number of new firms. The new firms are firms which did not exist in the previous quarter. We target this moment because the size of a new firm determines the relative importance of the SSC exempted amount compared to the total SSC the firm needs to pay for new hires. See Appendix E.2 for more details.

The fourth moment represents the firms' exit rate. We take firms with $L = 0$ in one quarter and check the proportion that exited the market after 1 year. We repeat this for firms with 1 and 2 employees. The fifth moment takes firms with $L = 0$ in a given quarter and checks the proportion that ended up with 0, 1, or 2 employees after four quarters. These moments are used to proxy employment dynamics - Refer to Appendix E.3 for more details.

The sixth moment is the unemployment rate sourced from the Labour Force Survey (LFS) conducted by Statbel.[12] The seventh is the the job vacancy rate (JVR) provided by EUROSTAT. The JVR is defined as the ratio of vacant job positions to the sum of vacant jobs and occupied positions. See Appendix E.4 for more details on the JVR. These last two moments play a crucial role in enabling the model to replicate the observed job-finding rate, job-filling rate, and the proportion of job-seekers over the total number of agents.

The eighth and final moment is the ratio of the average turnover in firms with 0 employees to firms with 1 employee. These moments ensure that we have firms in the model that remain with zero employees not because they are waiting to become more productive - and therefore they are waiting to have positive revenues when they start hiring - but because they generate some positive revenue while they have zero employees. See Appendix E.5 for more details.

[12] The average unemployment rate in Belgium for 2013 and 2014 is 8.50% for individuals between 15 and 64 years old (Statbel, 2022c).

Table 1: Match between observed and simulated moments

		From the data	Simulated
Firms with 0, 1, 2, 3, 4, and 5 over the total number of firms	N_0/N	79.09%	72.91%
	N_1/N	6.50%	4,54%
	N_2/N	3.51%	2.36%
	N_3/N	2.21%	1.93%
	N_4/N	1.52%	1.65%
	N_5/N	1.08%	1.45%
Proportion of employment in firms with [1,9] employees		11.63%	7.19%
Number of new firms with 0, 1, and 2 employees (EES) at entry	$N_{new,0}$	95.95%	94.65%
	$N_{new,1}$	2.30%	4.51%
	$N_{new,2}$	0.93%	0.83%
Firms with 0 EES that have 0, 1, and 2 EES in one year	$0 \rightarrow 0$	88.88%	94.13%
	$0 \rightarrow 1$	1.62%	1.30%
	$0 \rightarrow 2$	0.31%	0.38%
Firms with 0 EES that exit in one year	$0 \rightarrow$ Exit	8.89%	4.04%
	$1 \rightarrow$ Exit	4.73%	4.04%
	$2 \rightarrow$ Exit	4.43%	4.04%
Unemployment rate		8.50%	5.74%
Vacancy rate		2.45 %	8.81%
Revenues firms with 0 vs 1 EE		30.96%	25.18%

5.5 Empirical and simulated moments: Match

Table 1 compares the actual data with the simulated moments. The match is satisfactory for all presented moments. However, we overestimate the vacancy rate. This is because, by imposing the restriction that the largest firm has a maximum of 50 employees, we do not allow the presence of large firms that have achieved their optimal size - i.e. firms that have achieved their optimal size do not need to grow anymore and therefore post fewer vacancies.

While we estimate all our parameters together, implying that each parameter can potentially affect all the moments utilized for model estimation, certain parameters have a stronger impact on specific moments compared to others. In Appendix H, as outlined in Bilal et al. (2022), we illustrate the individual influence of each parameter on its related target moment and the objective function specified in equation 14. This analysis offers additional insights into the rationale behind selecting specific moments for estimating our set of parameters.

5.6 The estimated parameters

Table 2 summarises both the externally set/standardised parameters and estimated parameters. To interpret the estimated values, we need either to compare the parameters among each other or with the values estimated in the existing literature.

In greater depth, our values for the scale parameter of the matching function - 0.32 - is similar to the values estimated by Bilal et al. (2022) who respectively reported values of 0.195. This value of matching efficiency, together with an elasticity of the matching function equal to 0.5, for a number of job-seekers $\lambda = [1, 2, 3, 4, 5, 6, 7, 8, 9, 10]$, brings the vacancy filling rate to $q(\lambda) = [0.31, 0.45, 0.55, 0.63, 0.71, 0.77, 0.84, 0.89, 0.95, 1]$ and the job finding rate to $p(\lambda) = [0.32, 0.22, 0.18, 0.16, 0.14, 0.13, 0.12, 0.11, 0.11, 0.10]$. The production elasticity $\alpha = 0.89$ is only slightly higher than the estimated value obtained by Bilal et al. (2022), 0.817, or used by Schaal (2017), 0.85.

Regrettably, no reference value exists in the literature for the sunk costs of opening a firm. However, it is worth noting that our estimated sunk costs are about half the expected value of opening a firm $\sum_{x \in X} \sigma_x J(L = 0, x)$, suggesting that such costs are likely to be substantial. Moreover, a mean value of the log-normal distribution of 780 and a standard deviation of 113 imply that $\sigma = [0.74, 0.05, 0.03, ..., 0.0004]$, meaning most firms in our model start a business with very low productivity.

5.7 Validation exercise

As this paper studies a policy that has been implemented, its main validation comes from its ability to predict the changes observed in the data for the post-reform period. In Figure 3, we plot the simulated and observed changes in the stock of firms with 1, 2, and [3,8] employees. While Section 6 describes how we obtain the simulated firm dynamics and the change in the stock of firms in steady state, offering a more in-depth explanation of the results, we can already see that the model replicates the observed changes in the stock of firms satisfactorily. Although it overestimates the increase in the stock of firms with 1 employee, this increase follows a pattern similar to that observed in the data. Additionally, as the data does not show a significant increase in the stock of firms with 2 or [3,8] employees, the model reflects this as well.

Next, we turn to a pre-reform statistic not used to calibrate the model, namely the employment dynamics in new firms. In other words, we take firms with 0 (or 1) employees who were not existing in a previous quarter, and check how many of them exited the market within one year or had 0, 1, or 2 employees after one year. In Table **??**, the first column reports the statistics computed from the data. We can see that new firms grow faster than the average firm in the economy—see Table 1 for reference. The model correctly predicts this behaviour, as it also simulates that new firms grow faster than the average firm in the economy.

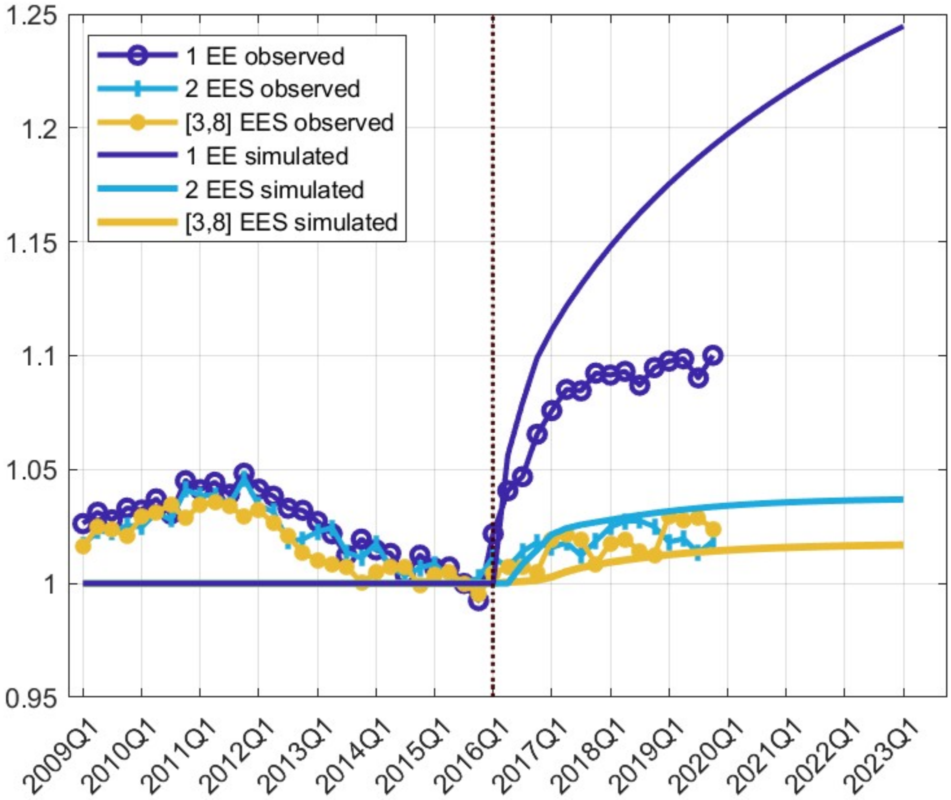
Table 2: externally/standardized/internally estimated parameters - unit of time: quarter

Parameter		Value
	Externally set/normalized parameters	
β	quarterly discount rate	0.987
τ	SSC rate pre-reform	0.183
τ_1	SSC rate post-reform	0.029
n	number of levels idiosyncratic productivity	75
x_1	lowest idiosyncratic productivity level	1
b	quarterly value of home production	0
δ_0	quarterly exogenous exit rate	0.010
γ_m	elasticity matching function wrt λ	0.5
	Internally set parameters	
x_n	highest value X	17.98
μ_{ln}	mean log-normal distribution	780
σ_{ln}	sd log-normal distribution	113
$\sigma_{\mathcal{N}}$	standard deviation shocks random walk	0.52
α	production function elasticity	0.89
μ_V	scale parameter cost of posting vacancies	12.33
γ_V	elasticity cost of posting vacancies wrt vacancy number	2.85
γ_L	elasticity cost of posting vacancies wrt number of employees	1.10
μ_m	scale parameter matching function	0.32
μ_o	scale parameter operating cost employers	1.32
γ_o	elasticity operating cost wrt L	1.26
s_b	sunk cost to open a firm	221

Table 3: Employment changes for new firms

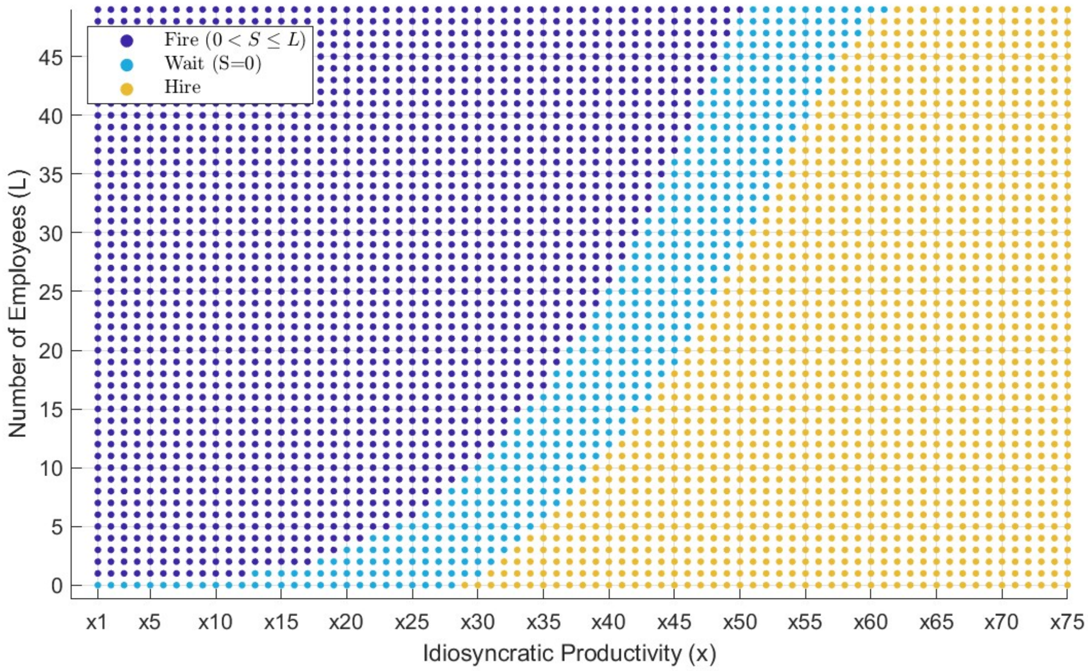
		From the data	Simulated
Firms with 0 (1) EES that exit in one year	$0 \rightarrow$ Exit	%	%
	$1 \rightarrow$ Exit	%	%
Firms with 0 EES that have 0, 1, or 2 EES in one year	$0 \rightarrow 0$	%	%
	$0 \rightarrow 1$	%	%
	$0 \rightarrow 2$	%	%
Firms with 1 EES that have 0, 1, or 2 EES in one year	$1 \rightarrow 0$	%	%
	$1 \rightarrow 1$	%	%
	$1 \rightarrow 2$	%	%

Figure 3: Evolution stock of firms



The figure illustrates changes in the stock of firms with 1, 2, and [3,8] employees. For each group of firms, we normalise the number of firms to 1 in 2015Q3. The vertical dotted line marks the beginning of the policy in 2016.

Figure 4: Firms' optimal policy



The dots represent the pairs of idiosyncratic productivity (x-axis) and the number of employees (y-axis) at which agents reduce the number of employees, wait, or hire more employees.

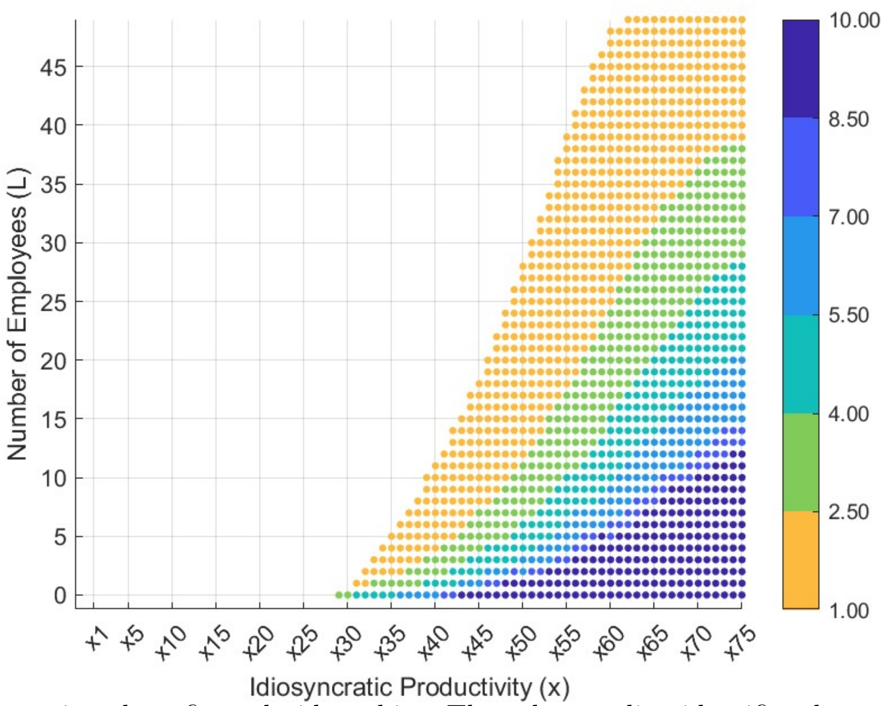
5.8 Some simulated statistics in the "before" steady state

Firms can either exit the market, reduce their workforce — i.e., set $0 < S \leq L$ — maintain the same number — i.e., set $S = 0$ — or hire employees. Figure 4 illustrates for which sets of (L, x) firms make one of these three choices. While no firms decide to exit the market[13], for any level of L, firing occurs at lower productivity than setting $S = 0$ and setting $S = 0$ happens at lower productivity than hiring. Additionally, due to decreasing returns to scale in the production function, the productivity required for hiring increases with L. While this graph determines the size distribution of firms along with the assumption governing firms' productivity at entry, it does not imply an equal distribution of firms at every grid point defined by the pairs (L, x).

Within the "hiring" and "firing" regions, firms with different pairs of (L, x) hire and fire at varying rates. Figure 5 shows the number of job-seekers per vacancy the firms want to attract. As expected, given L, the number of job-seekers increases with L, and given x, the number decreases with L. Indeed, it is the most productive firms, conditional on still being small, that benefit the most from filling their vacancies faster and, therefore, growing faster in size. Interestingly, we see that most of the firms with zero employees

[13]Since U is also what a business owner gets if they exit the market, the variable s_b also makes exiting less profitable for business owners. If we were to reduce s_b, we would get in the model some new business owners, as soon as they are created, would immediately decide to exit the market because what they get by operating in the market is lower than the payoff of exiting.

Figure 5: Number of job-seekers per vacancy

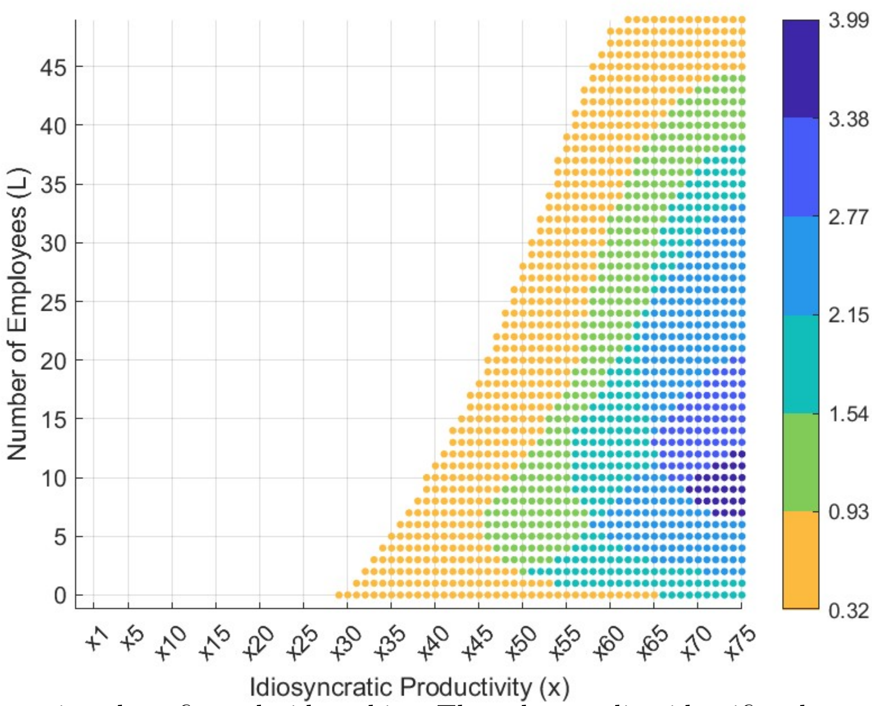


The dots represent the pairs where firms decide to hire. The colour coding identifies the number of job-seeker for each vacancy posted.

that want to hire already attract a high number of job-seekers (10 job-seekers in our simulation) and, therefore, fill their vacancies at a rate of about 100%. When we look at the number of vacancies posted by each firm in I.1, we find a similar story. Indeed, those selecting a higher number of vacancies are more productive given their employee count. However, since the average cost of posting vacancies decreases with L, the most productive firms post a number of vacancies that first increases and then decreases with L. A specular pattern emerges when we observe the number of individuals fired for each (L, x) pair in Figure I.2.

In graph 6, we plot the expected number of hires for firms that post vacancies, depending on the value of (L, x). The expected number of hires derives directly from the firm's decision on the number of vacancies and job-seekers. Hence, it is not surprising that, given L, more productive firms hire more employees, and that since the average cost of posting vacancies decreases with L, the number of hires first increases and then decreases with L for some highly productive firms. We will use this figure as a benchmark to demonstrate how the exemption altered hiring incentives for eligible firms.

Figure 6: Firms' expected number of hires



The dots represent the pairs where firms decide to hire. The colour coding identifies the expected number of hires for each pair.

6 Simulated impact of the reform

All else being equal, the exemption increases the payoff, $h(L, x, V, w)$, that (eligible) firms obtain if they hire. Consequently, the expected value of opening a business, $\sum_{x \in X} \sigma(x) J(L = 0, x)$, increases. However, if individuals get a higher expected value from opening a business, then firms need to offer a higher present value to the perfectly informed unemployed individuals to convince them to become job seekers: $\max_{\bar{i} \in \bar{\mathcal{I}}}[S_i]$ must increase to match $\sum_{x \in X} \sigma(x) J(L = 0, x)$. This results in an increase in the present value of unemployment U.

Only certain firms qualify for the exemption at the time of the policy implementation: those with no employees (regardless of having paid the sunk cost to become an employer) and those established after the policy enactment. In contrast, firms with more than 0 employees at the time of implementation cannot claim the exemption. However, these latter firms still need to pay the eventual increase in U. When hiring new employees, they must offer the same present value that subsidized firms offer. Additionally, they need to offer higher recurrent wages to their current workforce so that leaving the firm for unemployment is not more profitable than continuing to work. Firms will fire employees if it is more profitable than paying these higher recurrent wages. Over time, companies that were in the market when the policy was introduced gradually leave due to external shocks or voluntary exits. The economy eventually reaches a new steady state, where all firms have been established post-implementation and are eligible for the exemption.

Even if non-eligible firms exit gradually from the market, if the number of agents deciding to become business owners is strictly positive in each post-reform period, U jumps directly from its pre-reform steady-state value to its new post-reform steady-state value upon the announcement of the reform. Formally, when new firms enter the market in a given period, the equilibrium value of U for that period is determined by solving the optimal problem of these new firms, $J(L = 0, x)$. New firms face the same optimization problem each period following the reform, therefore, if their number is strictly positive in each post-reform period, U jumps to its new post-reform steady-state value.[14] This implies that after computing the steady-state value of U, to see how the market changes between the two steady states, we only need to follow these steps:

1. Using the before scenario model, we compute the steady-state distribution of firms and categorize them into two groups: firms with zero employees (referred to as the incumbent solo entrepreneurs) and firms with a strictly positive number of employees (referred to as the incumbent employers).

2. We assume that incumbent employers will never be eligible for the exemption. Therefore, their optimal policy is determined by solving the problem defined in sub-section (3.3) when U takes its post-reform steady-state value. Instead, the solo-entrepreneurs are eligible for the exemption. Therefore, their optimal policy is determined by solving the firms' problem specified in sub-section (3.5.1). To compute how the distribution of incumbent employers/solo entrepreneurs changes from quarter to quarter, we apply their optimal policy to their distribution at the beginning of each quarter and repeat this calculation from the moment the reform is announced.

3. We determined the number of new agents that become business owners each quarter after the reform as a residual such that the total number of agents in the economy remains constant over time.. These new firms face the firms' problem specified in sub-section (3.5.1).

6.1 Change in the firms' optimal policy - fixed U

In Figure 7, we show the change in the expected number of hires for eligible firms - i.e. firms with zero employees when the reform was announced and firms created after 2016 - assuming that U remains fixed at its pre-reform level[15]. Looking at the changes in firms'

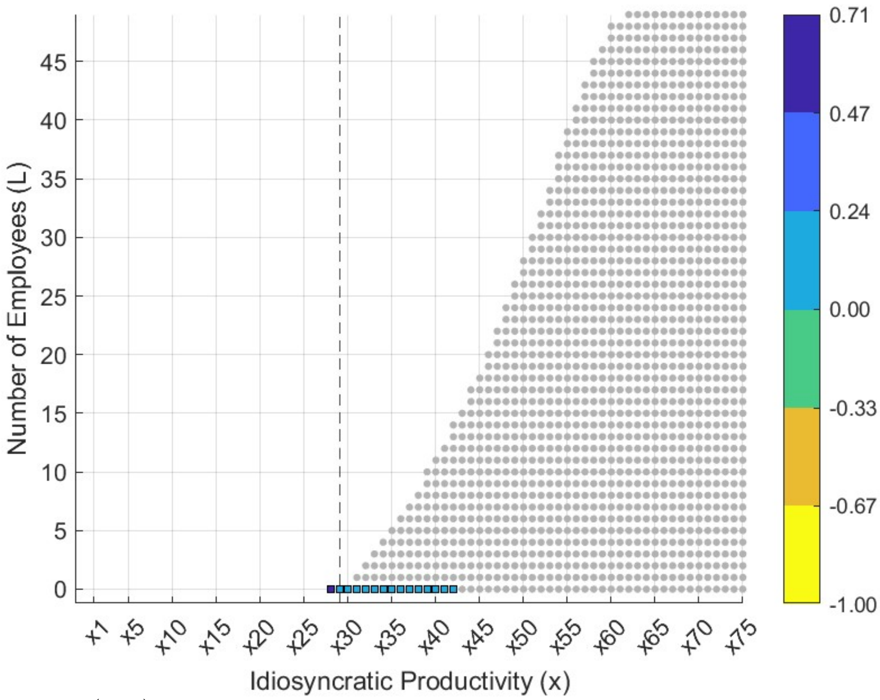
[14] We have numerically checked that the number of agents who decide to become business owners is strictly positive in each period after the reform.

[15] In our setup, U could remain unchanged only if we assume that there is a stock of agents outside the labour force - in our case, the labour force consists of employers, solo entrepreneurs, and the unemployed - such as those in inactivity, education, or retirement, who respond to the policy and become available for salaried employment, but to a lesser extent to create their own businesses. While we cannot rule out the possibility that some of these agents actually enter the labour force following the reform, our preferred specification is that the labour force is fixed.

optimal policies when U is fixed allows us to distinguish the first-order positive effect of making firms eligible for the exemption from the negative effects caused by a reduction in hiring due to the increase in the equilibrium value of U. The grey dots of Figure 7 represent the (L, x) pairs where firms hire the same expected number of individuals both before and after the reform. For pairs, represented by squares, where firms hire a different number of employees between the two scenarios, colour coding indicates the magnitude of the difference. The black dotted line represents the minimum productivity level at which firms with zero employees were hiring before the reform's implementation.

Some firms with zero employees that were not hiring before the reform have now started hiring - these are identified by the dark blue square on the left of the vertical dotted lines. Meanwhile, some firms with zero employees, which were already hiring before the reform, are now hiring at a slightly faster rate. The exemption makes it more convenient to offer a higher wage to attract their first employee, as they no longer have to pay SSC on this wage - these firms are identified by the blue squares on the right of the dotted line. Notably, not all firms with zero employees that were hiring before the reform increased their hiring rate in the post-reform period. This is because our model predicts - as shown in Figure 5 - that most firms with zero employees were already filling their vacancies at the highest rate before the reform and, therefore, had no incentive to attract more job-seekers.

Figure 7: Change in the after-reform number of hires - fixed U



The grey dot represents the (L,x) pairs where firms hire the same expected number of individuals both in the initial scenario and the simulated scenario. For pairs, represented by squares, showing a difference between the two scenarios, color coding indicates the magnitude of the difference. The black dotted line represents the minimum productivity level at which firms with 0 employees were hiring before the reform's implementation.

By definition, if we keep U fixed, the expected number of hires remains unchanged for firms that are not eligible for the exemption (i.e., incumbent employers). While they do not benefit from the exemption, they also incur no additional costs, so they behave as before the exemption implementation.

6.2 Change in the firms' optimal policy - increased U

If the labour force is fixed, the increase in the profit from opening a business due to the exemption must be matched by an increase in U to make job-seekers indifferent between opening a firm and starting a business. Allowing U to reach its new equilibrium value - 1.36% higher than the pre-reform value - impacts the optimal hiring decisions of firms.

Figure 8 shows how the expected optimal number of hires changes for firms with zero employees when the reform was announced and for firms created after the reform announcement. These two groups of firms share the same optimization problem, as they are both eligible for the exemption. The same colour coding defined previously applies. While some firms with zero employees still modify their hiring decisions, the increase in U makes hiring more expensive. Therefore, some firms with more than one employee reduce their hiring rates (the green, ocher and yellow squares). Although these points may appear randomly allocated, they correspond to the border of the areas where firms in the before steady state were either reducing the number of vacancies or job-seekers. These negative effects may counterbalance the direct positive effects of the exemption, making how the firms' distribution changes following the reform unclear.

Firms with at least one employee when the reform was announced are not eligible for the exemption but must pay the higher U for all their employees. Consequently, it's unsurprising that these firms either reduce the expected number of hires or keep it unaltered, but they never increase it - see Figure 9.

6.3 Comparing the before and after steady state

In the very long run, only firms eligible for the exemption remain in the market. Hence, we simulate where the economy is heading when U is at its new steady-state value, and all firms receive the exemption. Figure 10, black bars, shows the change in the number of firms by their number of employees when U increases to its new equilibrium value, while the dark grey bars represent the change when U is fixed. We clearly observe an increase in the stock of firms with 1 employee at the expense of solo entrepreneurs. However, there is little change in the stock of larger firms. When the equilibrium wage increases, there is a slight reduction in the stock of firms with more than 8 employees, leading to a moderate leftward shift in firm distribution. Overall, the effect of the change in the equilibrium wage is minimal.

The limited impact of the increase in U also signifies that the incumbent firms only limitedly change their hiring decisions; therefore, the new steady state will be reached slowly. For example, for firms with one employee, it will take 18 quarters (3.5 years) to

Figure 8: Change in the after-reform number of hires - increased U - Incumbent solo entrepreneurs and new firms

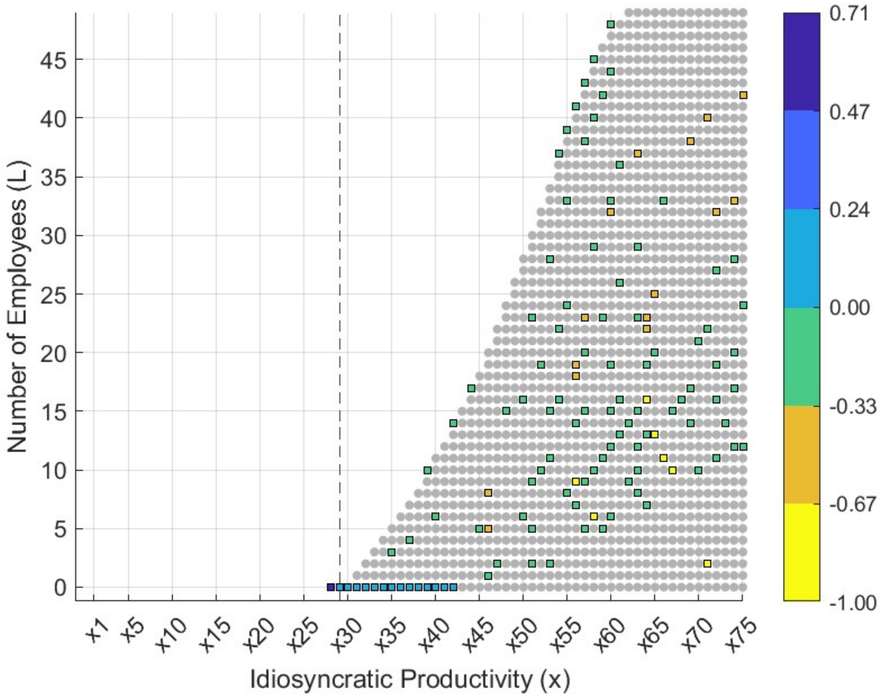
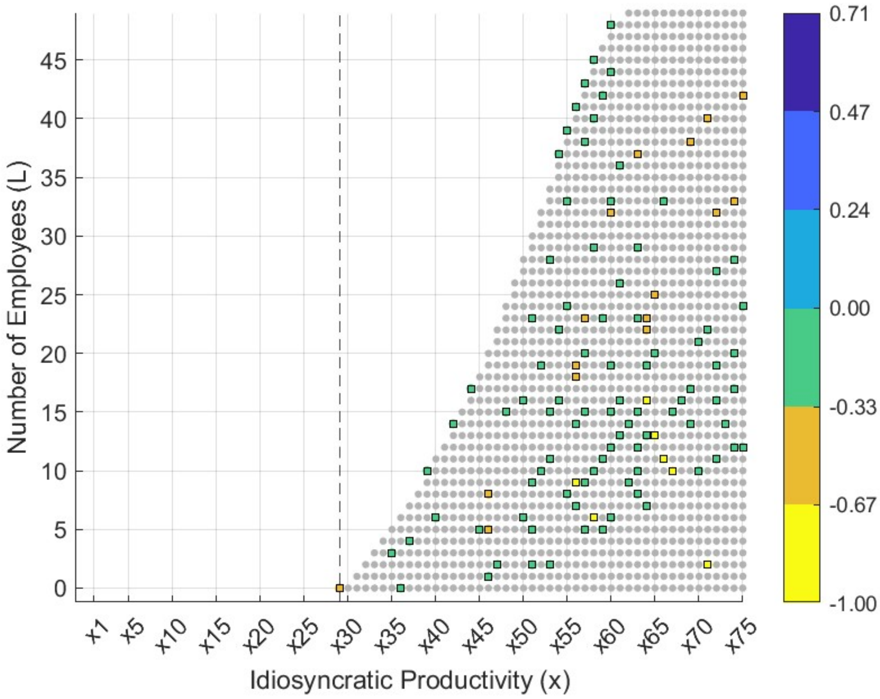


Figure 9: Change in the after-reform number of hires - increased U - incumbent employers



reach half of the increase we see in steady state and 52 quarters (about 13 years) to cover three-quarters of the adjustment.

In Figure 11, we report the overall change in the total number of agents, employees, job seekers, and firms between the before and after steady states when U reaches its new steady-state level (black bars) and when U is fixed (grey bars). We observe that when U increases, despite the rise in firms with one employee, the reduction in hiring by larger firms leads to an overall decrease in employment — by construction, the number of agents is fixed in this scenario. If U is fixed, and therefore larger firms do not reduce their hiring, the increase in the number of employers translates into a rise in both job seekers and employees.[16]

Finally, we see that the SSC collected by the government reduces by a sizable 5.40% when U is let free to increase and by a mostly identical 5.39% when U is fixed to the pre-reform level. If we look at the revenues generated by the firms, we see that total revenues decrease by -0.13% when U is free to increase and increases by 0.42% when U is kept fixed.

To understand why we see an increase in the number of firms with more than one employee but not in larger firms, we decompose the reform's effect between agents who would not have been productive enough to hire without the reform and those who would have hired regardless.

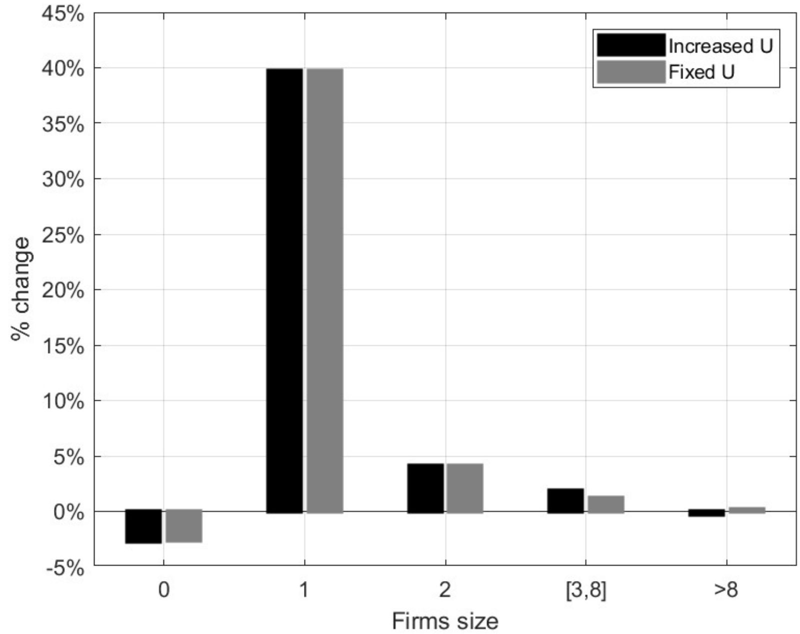
Agents who would not have been productive enough to hire. We assume that the policymaker can perfectly observe each agent's productivity x and grant the exemption only to firms whose productivity is below the level at which firms started hiring before the reform, denoted as $\bar{x}$ - we refer to these as the *marginal* new employers. Thus, we assume that only firms with $x < \bar{x}$ receive the exemption, while those with higher productivity do not.

Under this assumption, U increases by 0.23% in steady state, compared to the 1.36% increase when all firms are eligible. In Figure 12, light blue bars, we plot the change in the number of firms by their number of employees. Compared to the baseline (dark blue bars), we see that agents who were not productive to hire before the reform are responsible for most of the changes in the stock of firms observed in our baseline. Quite interestingly, the stock of firms with one employee increases even more than when all firms are eligible because, since U increases only by 0.23%, we have more marginal firms not hiring before the reform that start to hire than in the baseline situation.

However, only a few of the new employers created thanks to the exemption grow above one employee. To get some firms with productivity $x < \bar{x}$ that hired their first employee thanks to the exemption to employ additional employees, two conditions are necessary: Firstly, these agents need to experience a positive productivity shock after hiring their first employee, making it profitable for them to hire additional employees. Secondly, it should be less expensive for a firm with one employee to hire a given number of employees

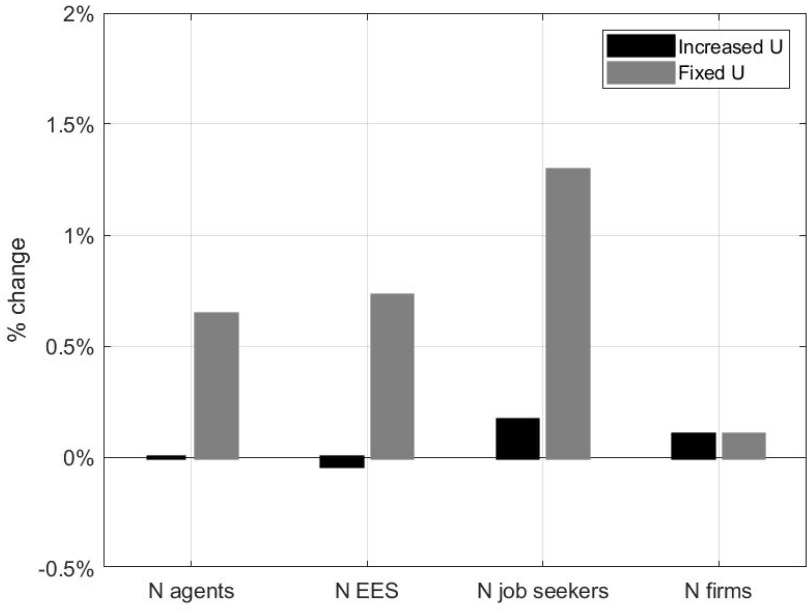
[16] In our model, if U does not increase, all individuals would choose to open a firm. To avoid the model reaching this degenerate solution while still obtaining meaningful statistics, we assume that the number of firms when U is fixed is equal to the number of firms in the scenario where U increases.

Figure 10: Change (in %) in the stock of firms



This figure shows the change in the stock of firms with 0, 1, 2, [3,8], and more than 9 employees between the before and after steady states.

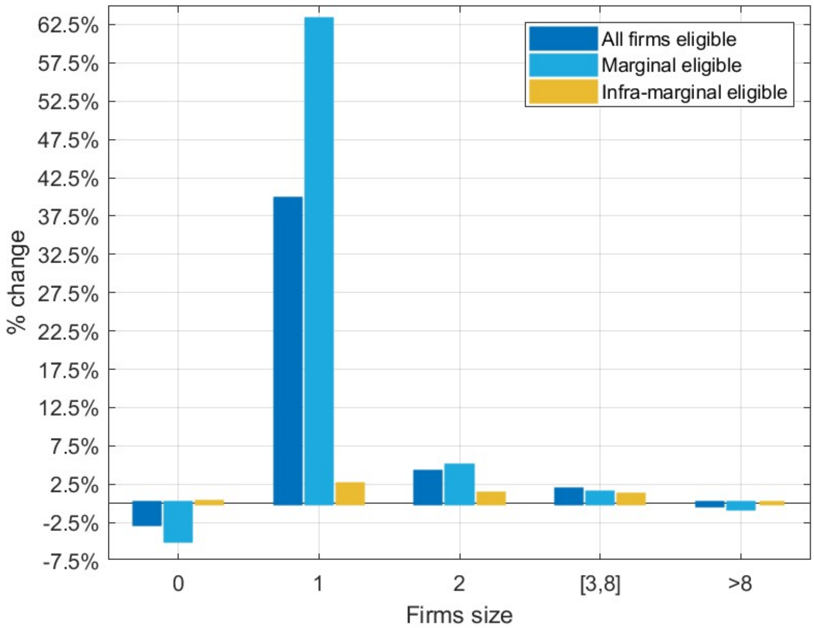
Figure 11: Change (in %) in the number of agents by type



This figure shows the change in the total number of agents in the labour force, employees, job seekers, and firms between the before and after steady states.

(e.g., one employee) than for a firm that starts with zero employees. If these conditions hold true, having more firms with one employee can lead some to grow further. In our model, we allow productivity to change—x follows a random walk—and the average cost of posting vacancies decreases as L increases. However, despite the estimated γ_L being relatively high, equal to 1.0958, the estimated variance of the idiosyncratic productivity shocks is quite small, equal to 0.52. Therefore, newly created employers have a limited chance of becoming more productive over time and thus hiring more employees.

Figure 12: Change (in %) in the stock of firms in steady state - decomposition



This figure shows the change in the stock of firms with 0, 1, 2, [3,8], and more than 9 employees between the before and after steady states. It describes three scenarios: the situation where all firms are eligible for the exemption; when only firms that were not productive enough to hire before the reform are eligible - *marginal* firms; and the situation where only firms that were productive enough to hire even before the reform are eligible - *infra-marginal* firms. In all scenarios, the labour force is fixed, and therefore, the U increases.

Agents who would have been productive enough to hire. As before, we assume that the government can perfectly observe the productivity of each agent and will assign the exemption only to firms whose productivity is above the threshold $\bar{z}$, where firms were starting to hire before the reform - we refer to these as the *inframarginal* new employers. In other words, we assume that only firms that would have hired even in the absence of the reform get the exemption. In this way, we can observe the effect on the firms' size distribution by incentivizing firms to hire at a faster rate than before.

In Figure 12, ochre bars, we plot the steady-state change in the number of firms in this scenario when U increases. The increase in the hiring rate by the *inframarginal* firms contributes only minimally to the change in the stock of firms. Two main reasons explain why the contribution of *inframarginal* is so limited. Firstly, given that the matching efficiency is relatively low, $\mu_m = 0.3157$, for firms that decide to attract one additional job seeker, the filling rate increases only by between 5% and 13% pp—the difference depends on how many job seekers the firm would have attracted in the absence of the reform. Secondly, the most productive firms with 0 employees were already filling their vacancies at a 100% rate, leaving no room for the policy to improve their job-filling rate.

7 Conclusions

We separately assess the behaviour of firms that would have hired even in the absence of the policy - the *inframarginal* new firms - and firms that start to hire thanks to the exemption - *marginal* firms.

We observe that while *marginal* employers contribute to the rise in the number of firms with one employee, only a small proportion of these firms grow beyond that stage. We attribute the limited growth of these new employers to the fact that they have a limited chance of becoming more productive over time, which is a necessary condition for them to hire more employees. Similarly, the exemption does not significantly alter the hiring behaviour of *inframarginal* employers. Indeed, our model predicts that most of these *inframarginal* firms with zero employees were already hiring at a high rate before the exemption's implementation, leaving no scope for them to increase hiring. For those who do increase hiring, low matching efficiency prevents them from obtaining a sizable increase in the filling rate of their vacancies by increasing their posted wages.

The fact that few new employers created by *marginal* firms hire more than one employee, and that the *inframarginal* employers - who still receive the exemption - only react to the policy in a limited way, casts doubt on the policy's effectiveness in fostering new successful employers.

We also observe that the increase in hiring costs caused by the policy is not substantial (U rises by 1.36%) and only minimally affects the firms' employment distribution. Therefore, the incumbent non-eligible employers will take time to exit the market, with the consequence that the economy only gradually adjusts to the new steady-state value; it will take 18 (52) quarters for firms with 1 employees to achieve 1/2 (3/4) of their steady-state increase in number.

Our model still overestimates the increase in the number of firms with one employee observed in the data. This is likely because the agents' behaviour is restricted by other frictions that we do not fully take into account, such as credit constraints that could reduce (or, in some cases, fortify) the agents' response. Alternatively, some agents may have different preferences for becoming employers, with some having a strong preference for remaining solo entrepreneurs despite changes in payroll taxation. We leave the analysis of these dimensions to future research.

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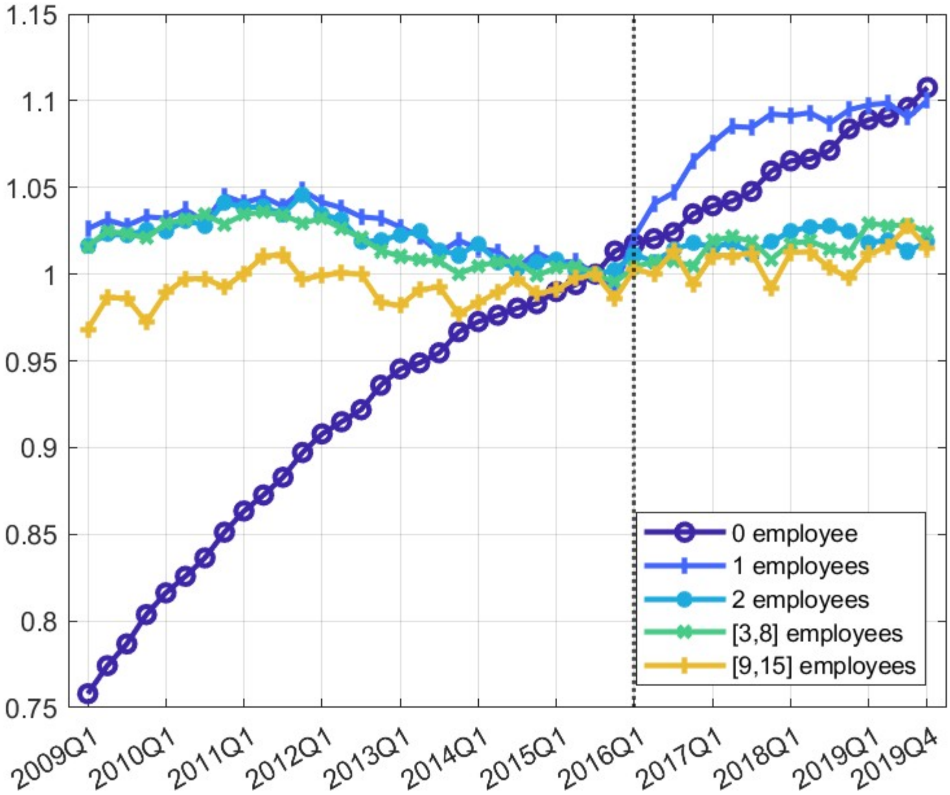
A Evolution stock of firms

Figure A.1: Evolution stock of firms



The figure illustrates the changes in the number of firms with 1 to 8, [9,15], and 0 employees. For each firms' group, we normalize the number of firms to 1 in 2015Q3. The vertical dashed line marks the beginning of the policy in 2016. Notice that in our dataset, we only have information for firms with 15 employees or less.

B Propositions' Proofs

Proposition 1 *If the tax rate τ on the stream of wages is independent of the firm's size, individuals are risk neutral and the capital market is perfect, then business owners make the same hiring/firing/exit decisions, and unemployed agents would make the same employment decisions whether i) firms post long-term contracts that specify a state-contingent wage and retention probability for each future period, or ii) contracts that only provide a hiring bonus, $\bar{w}$, and a "continuation wage", $\underline{w}$, for each period the employee produces output. The "continuation wage" is designed such that the value of staying in the current job equals the present value of being unemployed.*

Let's denote by $\{W_{i,t}\}_{i=1}^{L}$ the value in employment that a firm with L employees and productivity x has promised its employees for the current period t. Let's consider a contract for individual j that pays the wage $w_{j,t}$ at time t. Then, the current promised utility to agent j is equal to $W_{j,t} = w_{j,t} + \beta\phi_{j,t} Ex_{t+1}[W_{i,t+1}] + \beta(1-\phi_{j,t})U_{t+1}$. For simplicity, we assume that the firm's productivity is fixed over time, but the same reasoning applies when x changes according to a known Markov process. Let's also assume that U is at its steady-state value[17]

We show that the cost incurred when either keeping or firing worker j only depends on the current promised utility $W_{j,t}$ and the value in unemployment U, but not on the values that the wages $w_{j,\cdot}$ take over time. To show this, we look at two alternative situations, one where the firm fires individual j at time t and the other where the firm never fires individual j from t to $+\infty$ - remember that this worker can still lose their job due to the exogenous shock s_0 and exogenous firm's exit δ_0, so that the probability the worker retains their job from period to period, even if the firm does not fire them, is equal to $\phi_0 = (1-s_0)(1-\delta_0)$. Let's set $t = 0$.

If the firm fires individual j in the current period, then the firm needs to pay the current wage $w_{j,0} = W_{j,0} - \beta U$ to provide the promised utility $W_{j,0}$. Not surprisingly, the cost of firing only depends on the current promised utility and U. If the firm never fires the worker, it must provide $W_{j,0} = w_{j,0} + \beta\phi_0 W_{j,1} - \beta(1-\phi_0)U$ for the current period, $W_{j,1} = w_{j,1} + \beta\phi_0 W_{j,2} - \beta(1-\phi_0)U$ for the next period etc. If we recursively substitute the values of $W_{i,1}$, $W_{i,2}$, etc, we get that the total amount of wages the firm needs to pay for this individual j is equal to $\sum_{t=0}^{\infty} \beta^t \phi_0^t w_{j,t} = W_{j,0} - \frac{\beta(1-\phi_0)U}{1-\beta\phi_0}$. Again, the cost of retaining the worker does not depend on the value of future wages but only on the current value of $W_{i,0}$ and on U, thus making how wages are distributed not impacting the cost of retaining a worker or firing it. A similar reasoning applies to hiring.

[17] In search theory, even along the transitional dynamics, U immediately jumps to its new steady state following an unanticipated shock.

Figure C.1: The payroll tax rate for the first employee over the firm's lifetime: 2014 vs. 2016 cohort of new employers

C SSC rate pre and post-reform used in the model

To illustrate the generosity of the temporary pre-reform and permanent post-reform subsidy, we compute the median payroll tax rate for the first employee paid by quarterly cohorts of new employers who hired their first employee in 2014 (pre-refom cohorts) and 2016 (post-reform cohorts) in function of the number of elapsed quarters since their first hire (Figure C.1).[18] The black and red lines show the average annual payroll tax rate of the 2014 and 2016 cohorts.

The 2016 cohort benefits from the permanent payroll tax exemption and faces a constant payroll tax rate of 2.9% for the first employee. This rate is not exactly equal to zero because employers are only exempt from the base contribution, but they still have to pay specific contributions, such as contributions for short-time work or sector-specific training. The 2014 cohort benefits from temporary tax reductions for the first employee that are gradually phased out over thirteen quarters. As a result, the payroll tax rate for the 2014 cohort gradually increases with the firm's age and reaches a stable level fourteen quarters after having hired their first employee.

Following Cockx and Desiere (2024), we compute the *expected* payroll tax rate for the first employee at the time of hiring in the pre-reform and post-reform period. The expected payroll tax rate is defined as the weighted average of the payroll tax rate for each quarter after hiring (shown in Figure C.1), weighted by the firm's probability of still being an employer[19] and a quarterly discount rate set at 1.275% (e.g., Kaas and Kircher, 2015). These computations reveal that the expected payroll tax rate for the first employee of new employers is 18.3% in the pre-reform period and decreases to 2.9% in the post-reform period.

[18] We do not show the payroll tax rate of the 2015 cohort because this cohort was exempt from SSC for their first employee from 2016Q1 for at most thirteen quarters. When announcing the 2016 reform in October 2015, the government granted this temporary SSC exemption to the 2015 cohort of new employers to reduce the differential treatment of the 2015 and 2016 cohorts. Importantly, at the time of hiring, the 2015 cohort could not anticipate future government decisions and expected to be treated as the 2014 cohort.

[19] We compute the probability of still being an employer for the 2014 cohort of new employers for the first five years after hiring and assume that, after five years, 3% of the employers no longer employ workers in the following year, which corresponds to the Belgian average (Bijnens and Konings, 2020).

D Firms' exit rate

In this section, we describe how we compute the proportion of firms that exit the market within a year, grouped by the number of employees they have in a given quarter.

To compute these firms' exit rates, we follow a two-step procedure. Let's call N_i, $i \in [0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15]$ the number of firms with i employees that are in the market in a given quarter. We first count how many of these firms are no longer registered in the NBB dataset four quarters later. Table D.1, second column, reports the averages of these de-registration rates computed over four quarters, namely 2013q1/2014q1, 2013q2/2014q2, 2013q3/2014q3, and 2013q4/2014q4. We chose this pre-reform year because in 2015Q4, some firms modified their behaviour in anticipation of the policy Cockx and Desiere (2024).

To compute these firms' exit rates, we follow a two-step procedure. Let's call N_i, $i \in [0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15]$ the number of firms with i employees that are present in the market in a given quarter. We first compute how many of these firms are no longer registered in the NBB dataset four quarters later. Table D.1, second column, reports the averages of the exit rates computed over four quarters, namely 2013q1/2014q1, 2013q2/2014q2, 2013q3/2014q3, and 2013q4/2014q4. We chose this pre-reform year because in 2015Q4, we already had some anticipatory effects. We see that the de-registration rates are very low. This is because firms do not necessarily need to de-register from the NBB dataset if they are no longer active. Hence, these rates are not a good proxy for the firms' exit rate. To circumvent this caveat, in the third column of the same table, we compute the proportion of firms with i employees in a given quarter with 0 employees one year later. Interestingly, about 3.1-3.9% of firms with 10, 11, 12, 13, 14 and 15 see their workforce drop to 0 in one year. We decided to also consider these transitions as the exit rate. Therefore, we make the conservative assumption that, on top of the cancellation rate from the NBB datasets, an additional 3.6% of firms exit the market each period - where 3.6% is the average proportion of firms with [10,15] employees that in one year have 0 employees. These imputed yearly exit rates are reported in the last column of Table **??**. Since our model is quarterly, we then derive the quarterly rate, q, by applying the following formula to the yearly rates, y: $q = 1-(1-y)^(1/4)$. In our estimation procedure, we set $\delta_0 = 1.03\%$, which is the average imputed quarterly exit rate for firms with 10 or more employees, and then we target the imputed quarterly exit rate for firms with 0, 1, and 2 employees.

Table D.1: Yearly de-registration and exit rates

N EES	No longer in the dataset	With 0 EES	Imputed exit rate
0	5.33%	92.45%	8.89%
1	1.16%	21.48%	4.73%
2	0.87%	9.68%	4.43%
3	0.73%	6.54%	4.30%
4	0.62%	5.67%	4.18%
5	0.46%	4.93%	4.03%
6	0.52%	4.48%	4.08%
7	0.47%	4.05%	4.04%
8	0.39%	4.03%	3.95%
9	0.39%	4.16%	3.96%
10	0.42%	3.73%	3.99%
11	0.54%	3.96%	4.10%
12	0.52%	3.62%	4.08%
13	0.48%	3.68%	4.05%
14	0.49%	3.29%	4.05%
15	0.40%	3.11%	3.96%

E NBB, Statbel and EUROSTAT data

E.1 Firms' and employment distribution

From the NBB data, we compute the average number of firms with 1, 2, 3, 4, 5, 6, 7, 8, and 9 employees over 2013 and 2014. Our NBB data does not include information for larger firms; therefore, we complement these statistics with publicly available data from Statbel, the Belgian statistical office. Statbel (2022b) provides yearly statistics on the number of active firms and their corresponding number of paid employees. These statistics are restricted to for-profit firms in the industry and services sectors - sections B to N of NACE codes (rev 2) - excluding holding companies (NACE code K64.2). More information on the firms included in the Statbel statistics can be found at Statbel (2022a). In Table E.1, we highlight the firms' and employment distribution obtained by merging our granular NBB data for small firms and the Statbel data for larger firms.

E.2 Distribution of new firms

To measure the size distribution of new firms, we proceed as follows. Using the NBB data, we compute for each quarter between 2012Q3 and 2015Q3 the number of firms that did not exist in a previous quarter and report how many of these firms have 1, 2, 3, 4, 5, 6, 7, 8, and 9 employees. Notice that when a firm changes its identification number (CBE number), it will appear as a new firm in the NBB data. Therefore, we may mistake

Table E.1: Firms' and employment distribution

Firm size	N firms	N firms (%)	N EES	N EES (%)
0	655,083	79.09%		
1	53,829	6.50%	53,829	2.39%
2	29,055	3.51%	58,110	2.58%
3	18,296	2.21%	54,888	2.44%
4	12,567	1.52%	50,268	2.23%
5	8,962	1.08%	44,808	1.99%
6	6,834	0.83%	41,003	1.82%
7	5,311	0.64%	37,177	1.65%
8	4,267	0.52%	34,135	1.52%
9	3,480	0.42%	31,320	1.39%
[10,+∞]	30,557	3.69%	1,846,300	81.99%
total	828,240	100.00%	2,251,837	100.00%

These statistics are computed from our NBB data and publicly available Statbel statistics. They present the averages between 2013 and 2014.

firms that change CBE number as new firms. For this reason, as in Bijnens and Konings (2020), we assume that all firms which have 10 employees or more in the quarter they are created are not really new firms and, therefore, are not included in this statistic.[20] In Table E.2, we report the employment distribution of new firms.

E.3 Transition dynamics

To compute the proportion of firms with zero employees that have 0, 1, and 2 employees or exit in one year is relatively straightforward. Indeed, we first select all firms with 0 employees in 2013Q1 (we do the same for 2013Q2, 2013Q3, and 2013Q4), and then we compute how many are no longer in the market one year later, following the procedure explained in Appendix D. Similarly, we compute the number of firms that have 0, 1, and 2 employees in 2014Q1 - when computing the number of firms that have zero employees, we need to subtract the imputed number of firms with zero employees which are no longer active. Finally, we take the average of the results for all four quarters. We repeat the same for firms with 1 employee.

E.4 Vacancy rate

EUROSTAT provides the job-vacancy rate (JVR), which is the ratio of the number of vacant jobs over the sum of the number of vacant jobs and occupied posts. A 'job vacancy' is defined as a paid post that is newly created, unoccupied, or about to become vacant: (a)

[20] Geurts and Van Biesebroeck (2016) estimate that already 57% of new entrants between 10–19 employees are spurious, with this percentage increasing with the firm's size at entry

Table E.2: Employment distribution of new firms

size	N firms	N firms (%)
0	12,821	95.55%
1	308	2.30%
2	125	0.93%
3	59	0.44%
4	40	0.30%
5	24	0.18%
6	16	0.12%
7	11	0.08%
8	8	0.06%
9	6	0.04%
Total [0,9]	13,418	100.00%

These statistics are computed from our NBB data. They present the averages between 2012Q3 and 2015Q3.

for which the employer is taking active steps and is prepared to take further steps to find a suitable candidate from outside the enterprise concerned; and (b) which the employer intends to fill either immediately or within a specific period of time; An occupied post means a paid post within the organisation to which an employee has been assigned" (Eurostat, 2022). For 2013 and 2014, the JVR in the economy was, on average 2.46% (Statbel, 2022d). In our model, we count the number of vacancies as the vacant jobs.

The vacancy rate was about 2.8% in 2012, 2.6% in 2013, 2.2% in 2014, 2.4% in 2015, 2.8% in 2016, 3.4% in 2017, 3.5% in 2018, and 3.5% in 2019.

E.5 Turnover

Using the NBB data, we compute the average turnover in firms with 0 and 1 employee between 2013Q1 and 2014Q1, at 14,678 euros and 47,402 euros, respectively. Therefore, we obtain a ratio of 0.31. Notice that before computing the average turnover, we convert the quarterly nominal values into real terms using the Consumer Price Index (CPI) for 2013 for Belgium. Finally, note that the NBB knows the turnover only for firms liable to VAT. In Belgium, most firms can choose not to be liable to VAT if their annual turnover is less than 25,000 euros. This results in 27.85% (4.44%) of firms with 0 (1) employees for which the NBB does not know the turnover. Consequently, the ratio of the average turnover in firms with 0 employees over the average turnover in firms with 1 employee computed with the NBB data is likely to be an upper bound for ratio, computed in the model, that takes into account the whole universe of firms with 0 and 1 employees.

F Monopolistic competition

In Belgium, the good market rarely behaves as if it were perfectly competitive, but it shows finite price elasticity of demand (Aucremanne and Druant, 2005). Following Cahuc et al. (2019), we can assume that employers produce output with only labour, $y = x \cdot L^{\alpha}$, where α is the production function elasticity. However, each firm sells its output at a price $y^{\frac{1}{E}}$, where E is the price elasticity of demand. Therefore, the employers' revenues becomes $R(L, x) = x^{(\frac{E-1}{E})} \cdot L^{\alpha \frac{E-1}{E}}$, while solo entrepreneurs have revenues equal to $R(L = 1, x) = \mu_r \cdot x^{(\frac{E-1}{E})} \cdot 1^{\alpha \frac{E-1}{E}}$. Setting $E = 4.5$ (Aucremanne and Druant, 2005) – Cahuc et al. (2008) set $E = 4.3$ for France - we can re-estimate the model accordingly. In Table **??**, we report the new model parameters in the first column, while in the second column, we report the baseline estimated value parameters under perfect competition.

G The estimated parameters and their impact on the objective function and targeted moments

Let's denote by Y the set of our 14 estimated parameters and by y_j the estimated value of parameter j. In this section, for each parameter j, we compute $y_i/20$, and then we run the before steady-state model for 6 different values of the parameter, namely $[(1-3/20) \cdot y_j, (1-2/20) \cdot y_j, (1-1/20) \cdot y_j, y_j, (1+1/20) \cdot y_j, (1+2/20) \cdot y_j, (1+3/20) \cdot y_j]$. Then, we check the sensitivity of the objective function and the targeted moments based on these parameters' different values.

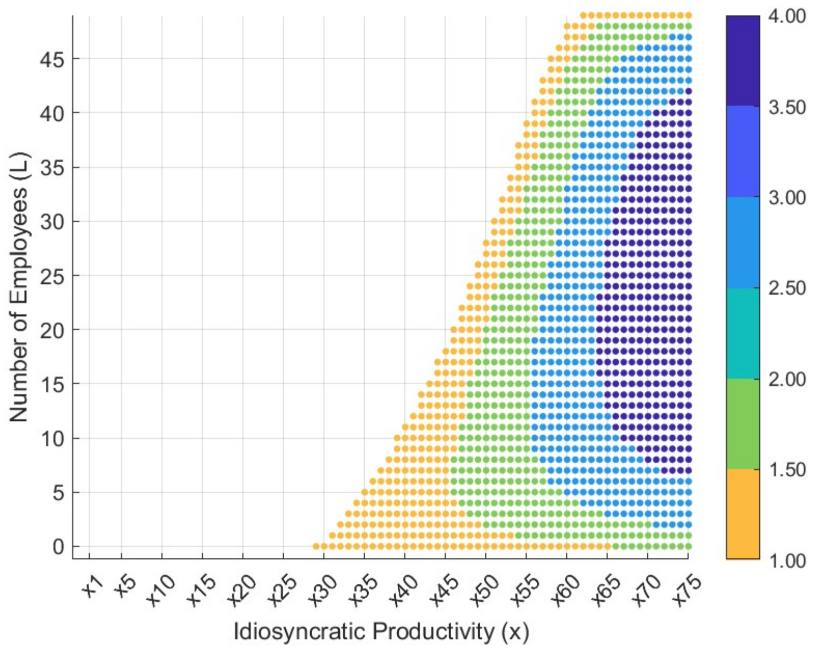
G.1 The objective function and the estimated parameters

In this section, we plot how the error computed with equation 14 evolves when we set, one by one, each of the estimated parameters to $[(1-3/20) \cdot y_j, (1-2/20) \cdot y_j, (1-1/20) \cdot y_j, y_j, (1+1/20) \cdot y_j, (1+2/20) \cdot y_j, (1+3/20) \cdot y_j]$. We normalize the value of the objective function to 1 for y_j. This analysis first provides evidence that we are (at least) at a local minimum for all the 14 parameters estimated and identifies which parameters affect the overall match of our model with the data the most.

H The targeted moments and the estimated parameters

In this section, we show what is the absolute and percentage change in the target moments when we set, one by one, each of the estimated parameters to $[(1-3/20)\cdot y_j, (1-2/20)\cdot y_j, (1-1/20)\cdot y_j, y_j, (1+1/20)\cdot y_j, (1+2/20)\cdot y_j, (1+3/20)\cdot y_j]$. This helps us to identify the parameters that each moment influences the most.

Figure I.1: Number of posted vacancies per employee



I Graphs steady state before

Figure I.2: Number of fired employees