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Università Bocconi MILANO

CORRELATION PREFERENCE

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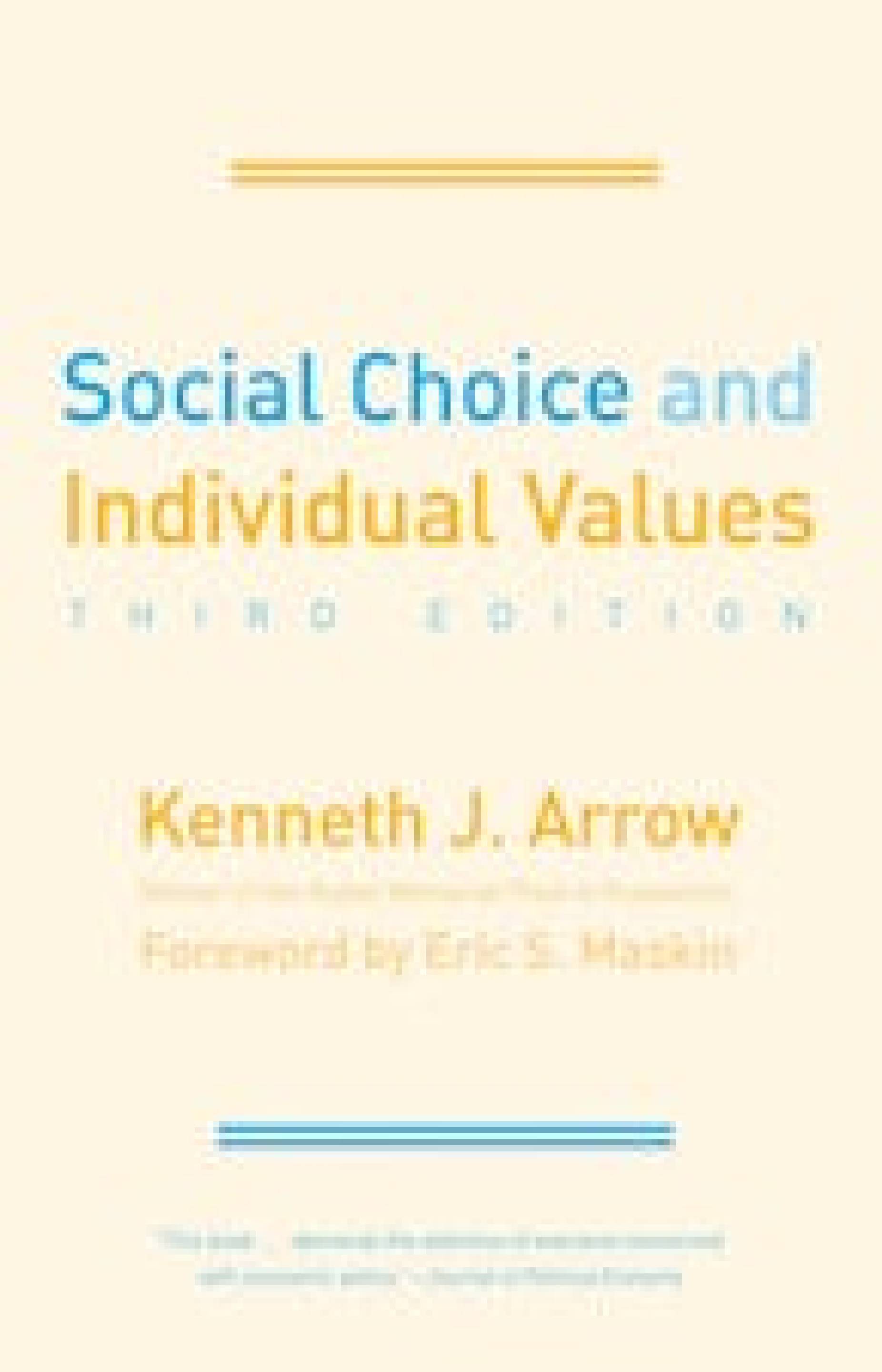
Aug 27@EEA-EMES

CORRELATION PREFERENCE

- **What's** missing?
- **What** is our model?
- **How** our model resolve it?



Arrow (1951, pp20) suggests the need for **a theory of choice** whose shape "cannot be foreseen"…

"It seems that the essential point is, and this is of general bearing, that, if conceptually we imagine a choice being made between two alternatives, we cannot exclude any (joint?) probability distribution over those two choices as a possible alternative. The precise shape of a formulation of rationality which takes the last point into account or the consequences of such a reformulation on the theory of choice in general or the theory of social choice in particular cannot be foreseen; ..."

Might Arrow have this in mind?

$$(x, p) \succcurlyeq^{\pi} (y, q)$$

π	y_1	$\cdots$	y_n	
x_1	$\pi(x_1, y_1)$	$\cdots$	$\pi(x_1, y_n)$	$p(x_1)$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
x_m	$\pi(x_m, y_1)$	$\cdots$	$\pi(x_m, y_n)$	$p(x_m)$
	$q(y_1)$	$\cdots$	$q(y_n)$	

Correlation Preference

- Traditional binary preference $\succcurlyeq$ typically modeled as **subset** of

$$\Delta X \times \Delta X$$

i.e., $p \succcurlyeq q$ (p preferred to q) *if* $(p, q) \in$ the subset $\succcurlyeq$.

Correlation Preference

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i.e., $p \succcurlyeq q$ (p preferred to q) *if* $(p, q) \in$ the subset $\succcurlyeq$.

- **Correlation Preference** $\succcurlyeq$ may be modeled as a subset Π of

$$\Delta(X \times X)$$

i.e., $p \succcurlyeq^{\pi} q$ for $(p, q) = (row, column)$ of π
(p preferred to q *at* $\pi \in \Pi$) *if* $\pi \in \Pi$.

Intransitivity - the Condorcet Paradox

This well-known paradox about the inherent intransitivity of majority voting in binary social choice is often credited with helping to usher in the tremendous literature on Social Choice originating with Arrow (1952).

"… we could as well build up our economic theory on other assumptions as to the structure of choice functions if the fact seemed to call for it" (Arrow, 1952)

Evidence of Correlation Preference

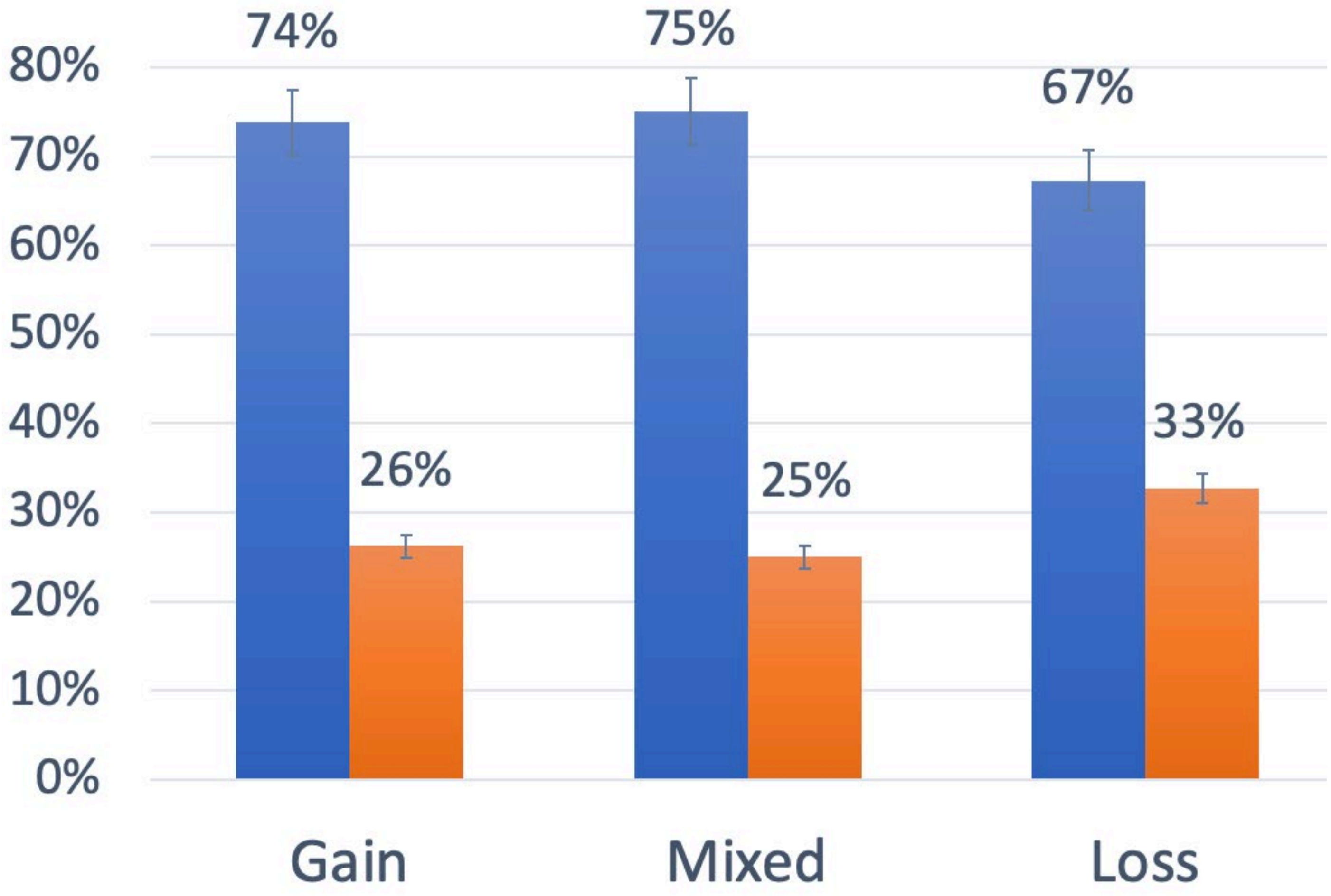
Non-Indifference bet. Same-Marginal Lotteries

(Loomes and Sugden, 1987; Loewenfeld and Zheng, 2024)

	1/3	1/3	1/3
p	$\mathcal{L}$	$\mathcal{M}$	$\mathcal{H}$
q	$\mathcal{H}$	$\mathcal{L}$	$\mathcal{M}$

$\pi(x, y)$	$x_1 = l$	$x_2 = m$	$x_3 = h$
$x_1 = l$	0	0	1/3
$x_2 = m$	1/3	0	0
$x_3 = h$	0	1/3	0

Evidence of Pure Correlation Preference: SML

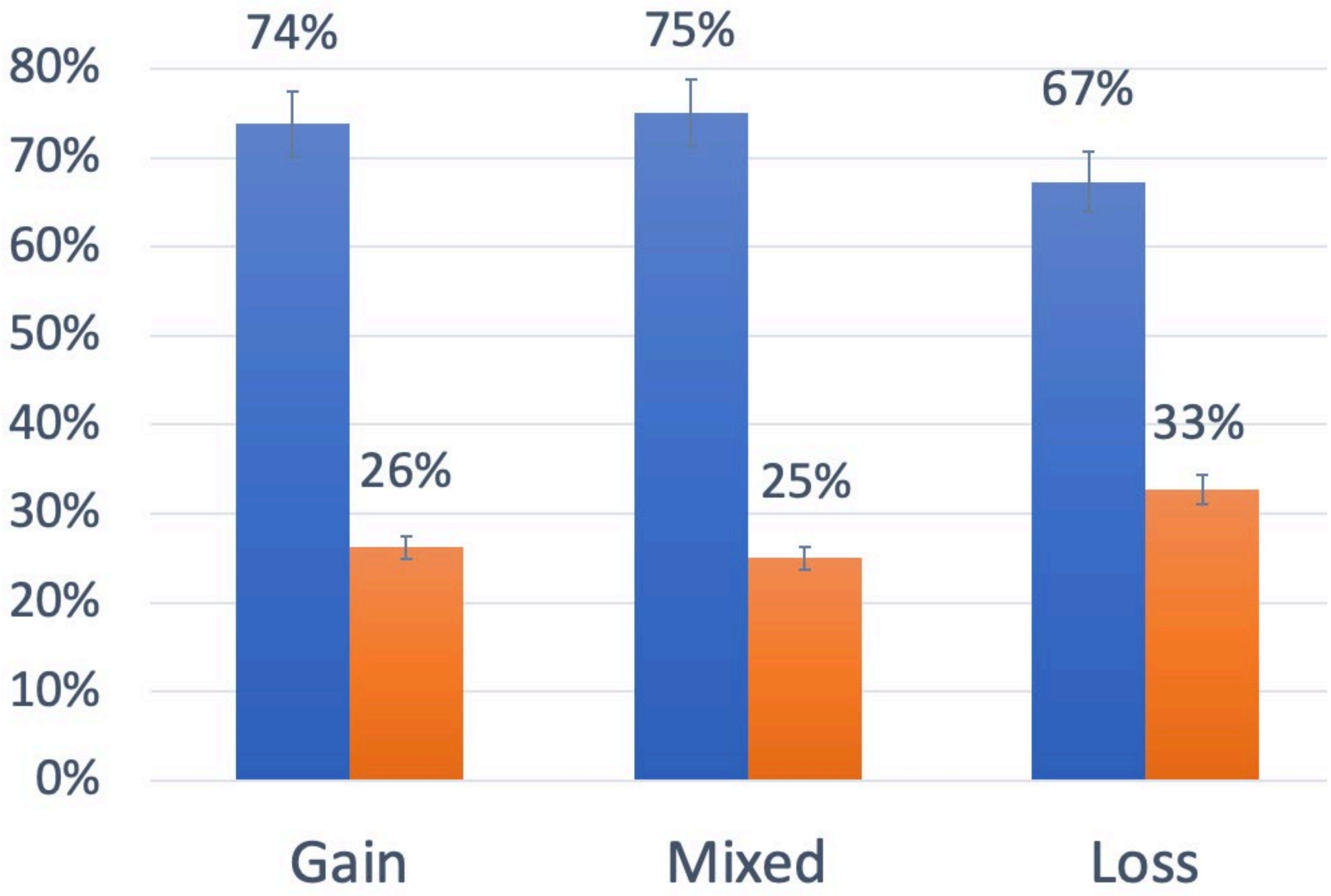


	1/3	1/3	1/3
p	$\mathcal{L}$	m	h
q	$\mathcal{H}$	ℓ	m

Which is more prevalent: ***p*** or ***q***?

Parameters (¥): Gain (100, 50, 0), Mixed (50, 0, -50), Loss (0, -50, -100)

Evidence of Pure Correlation Preference: SML



	1/3	1/3	1/3
p	$\mathcal{L}$	m	h
q	$\mathcal{H}$	ℓ	m

- *p* is more prevalent!
- Chinese proverb 田忌赛马
- Col. Blotto Game

Parameters (￥): Gain (100, 50, 0), Mixed (50, 0, -50), Loss (0, -50, -100)

Savage's Error and the Allais Paradox

Savage (1954, page 103) describes how he came to making his famous **choice error**:

Situation 1. Choose between

1. 500k for sure
2. (2,500k, 10%; 0, 1%; 500k, 89%)

Situation 2. Choose between

3. (500k, 11%; 0, 89%)
4. (2,500k, 10%; 0, 90%)

Savage revisits the problems with a state-act framework

Situation 1. Choose between

1. 500k for sure
2. (2,500k, 10%; 0, 1%; 500k, 89%)

Situation 2. Choose between

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	E		E^c
Situation 1	**1**	**2 – 11**	**12 – 100**
1	500k	500k	500k
2	0	2500k	500k
Situation 2	1	2 – 11	12 – 100
3	500k	500k	0
4	0	2500k	0

Applying STP*2 on the revisited problem

→ Either

#1 and #3

or

#2 and #4

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Savage revised his earlier choice. "I still feel an intuitive attraction to those preferences"

	E		E^c
Situation 1	**1**	**2 – 11**	**12 – 100**
1	500k	500k	500k
2	0	2500k	500k
Situation 2	1	2 – 11	12 – 100
3	500k	500k	0
4	0	2500k	0

Confession:

"…in reversing my preference between Gambles 3 and 4, I have corrected an 'error'…"

- Perhaps Correlation preference?

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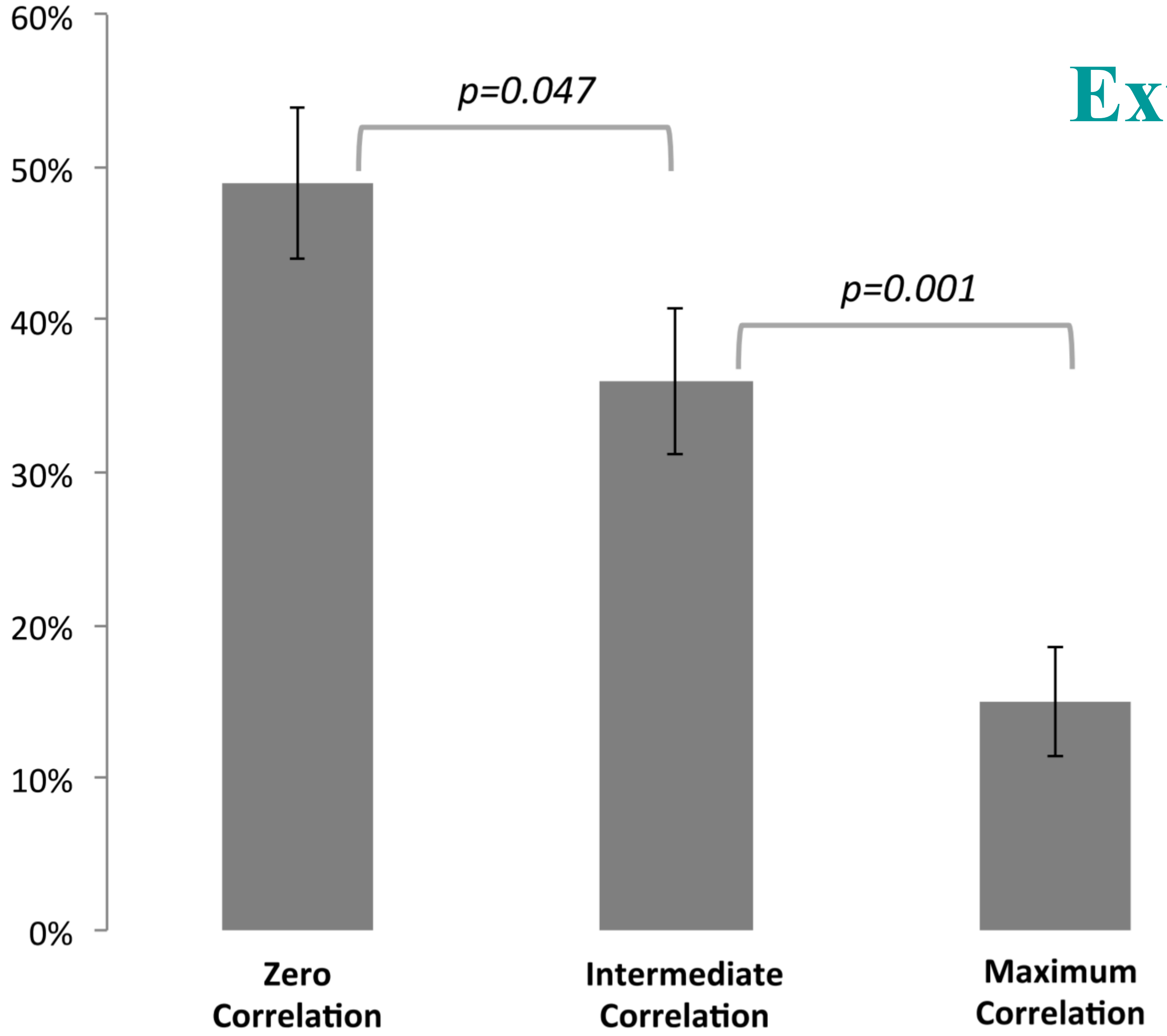
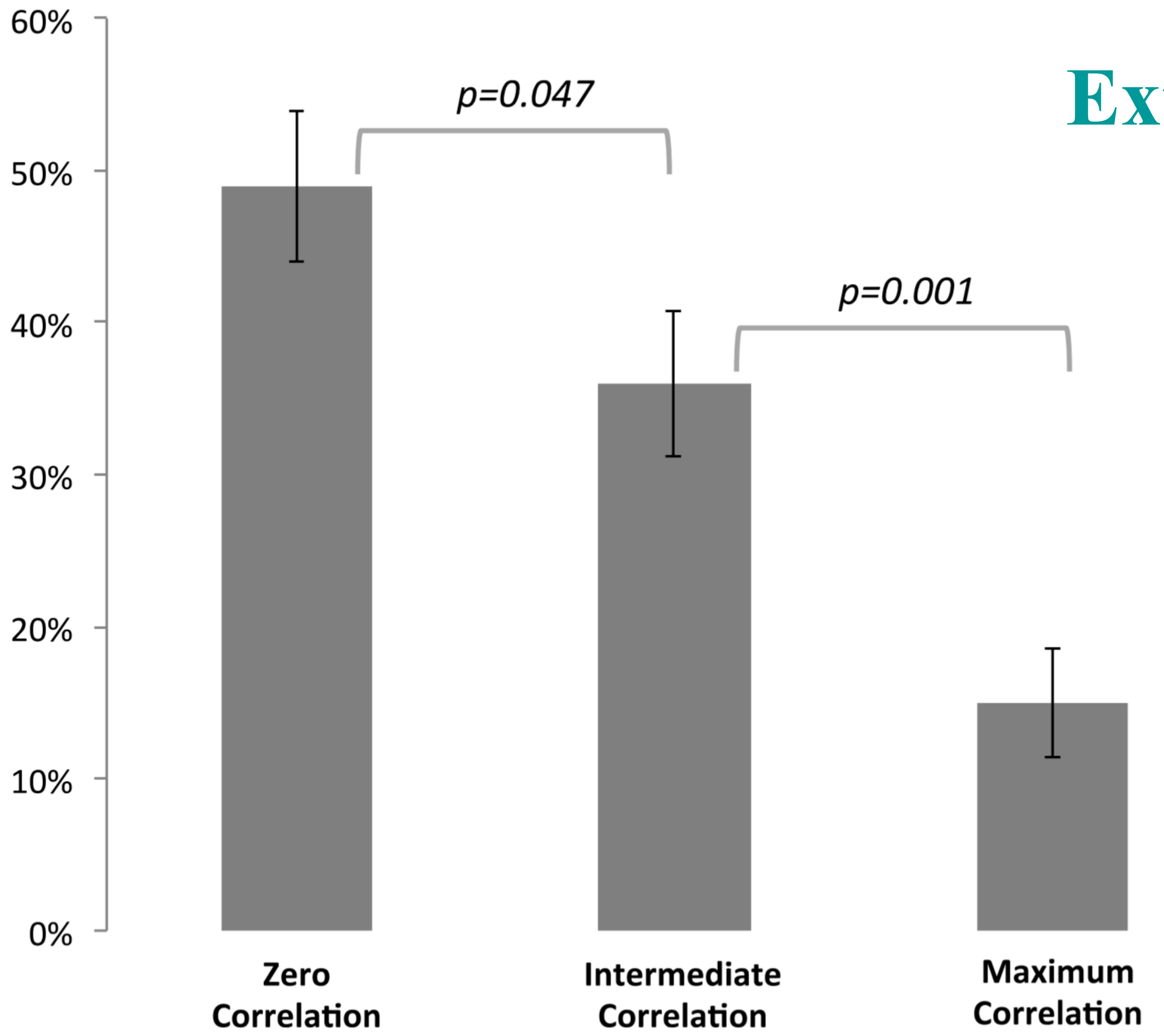
Extended Allais Paradox



Extended Allais Paradox

Different Rates of Allais between 'Independent' and 'Correlated'



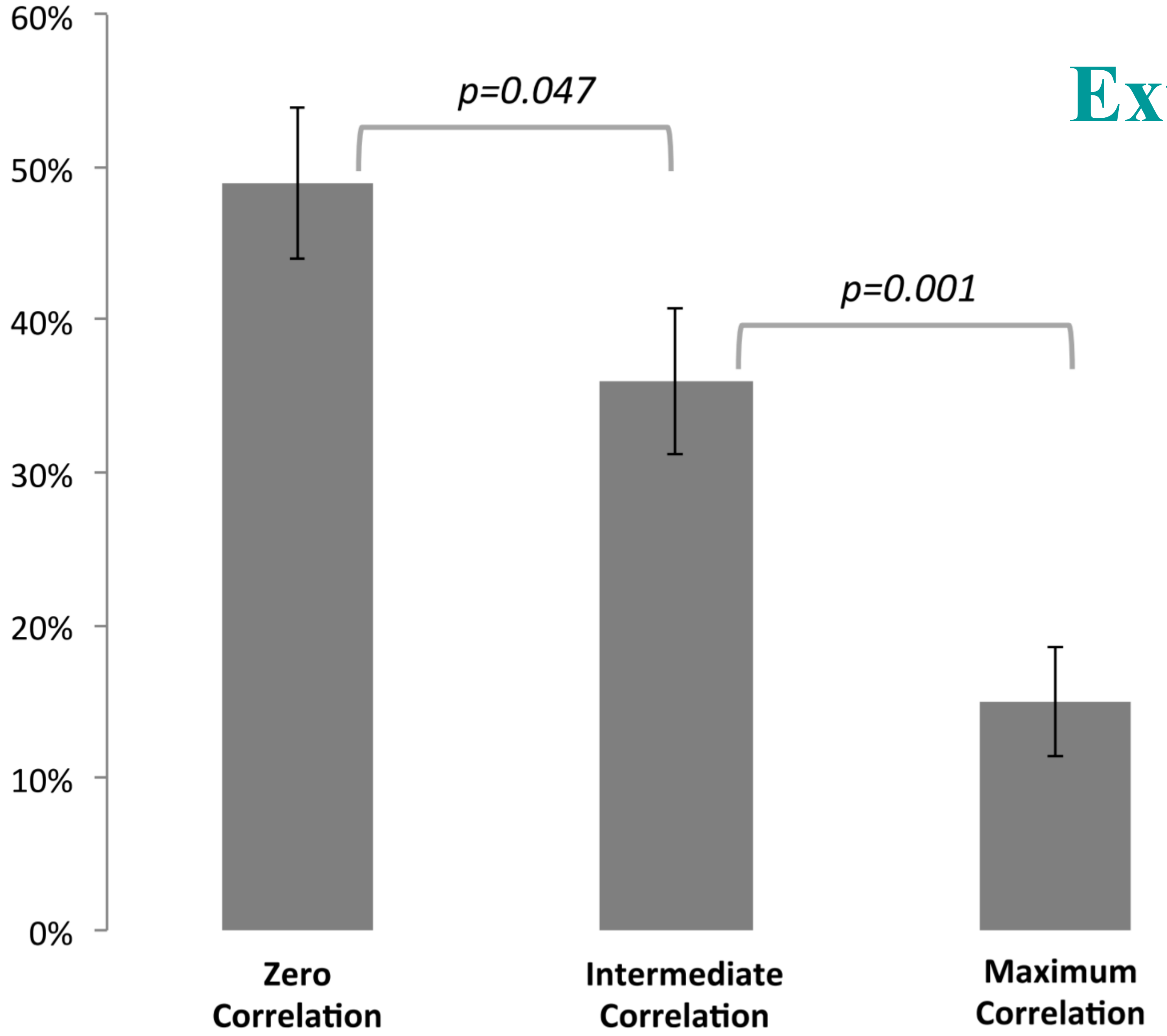
- Frydman and Mormann (2018): significantly different rates of Allais between 'independent' and 'maximally correlated'.

$\pi_\perp$	0	24
0	44.22%	22.78%
24		
25	21.78%	11.22%

π_{int}	0	24
0	65%	2%
24		
25	1%	32%

π_{max}	0	24
0	66%	1%
24		
25		33%

degree of correlation	independent	intermediate	maximal
FM18	48%	36%	15%
BMS22	48%	-	20%
LZ24	62%	-	18%



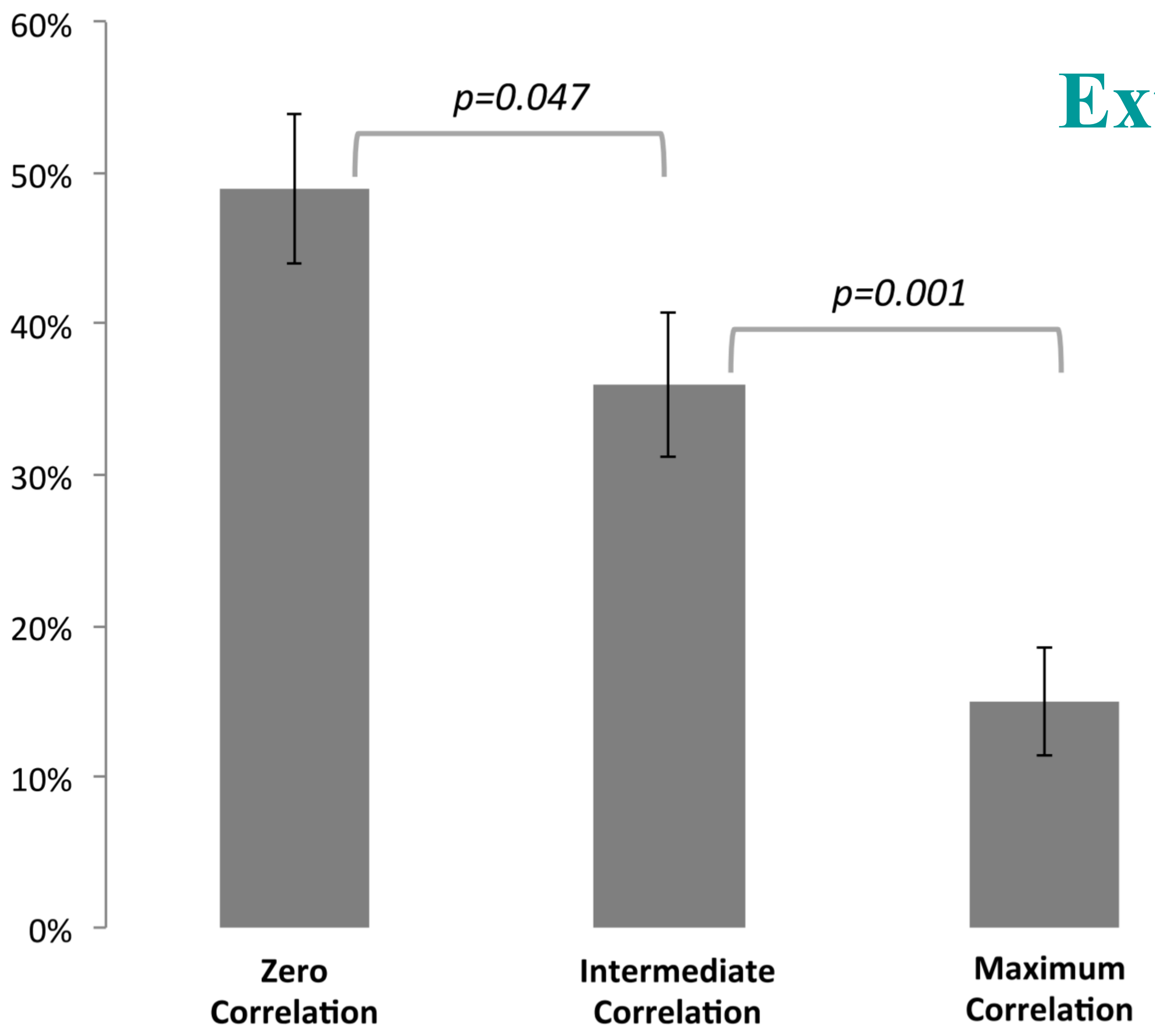
Extended Allais Paradox

Different Rates of Allais between 'Independent' and 'Correlated'

- Frydman and Mormann (2018): significantly different rates of Allais between 'independent' and 'maximally correlated'.
- Bruhin, Manai, and Santos-Pinto (2022); Lowenfeld and Zheng (2024): replicate significant difference in rates.

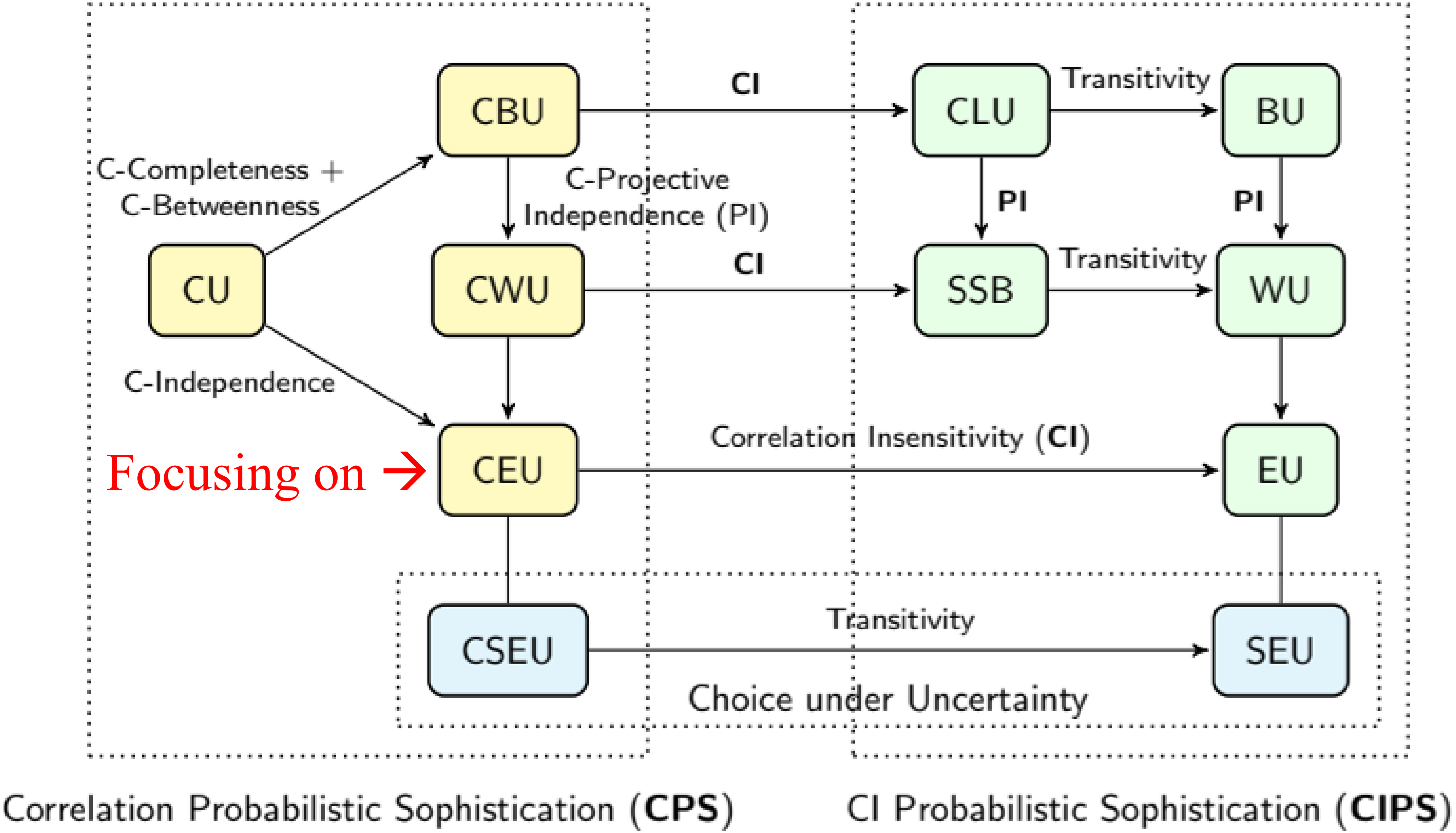
Extended Allais Paradox

Different Rates of Allais between 'Independent' and 'Correlated'

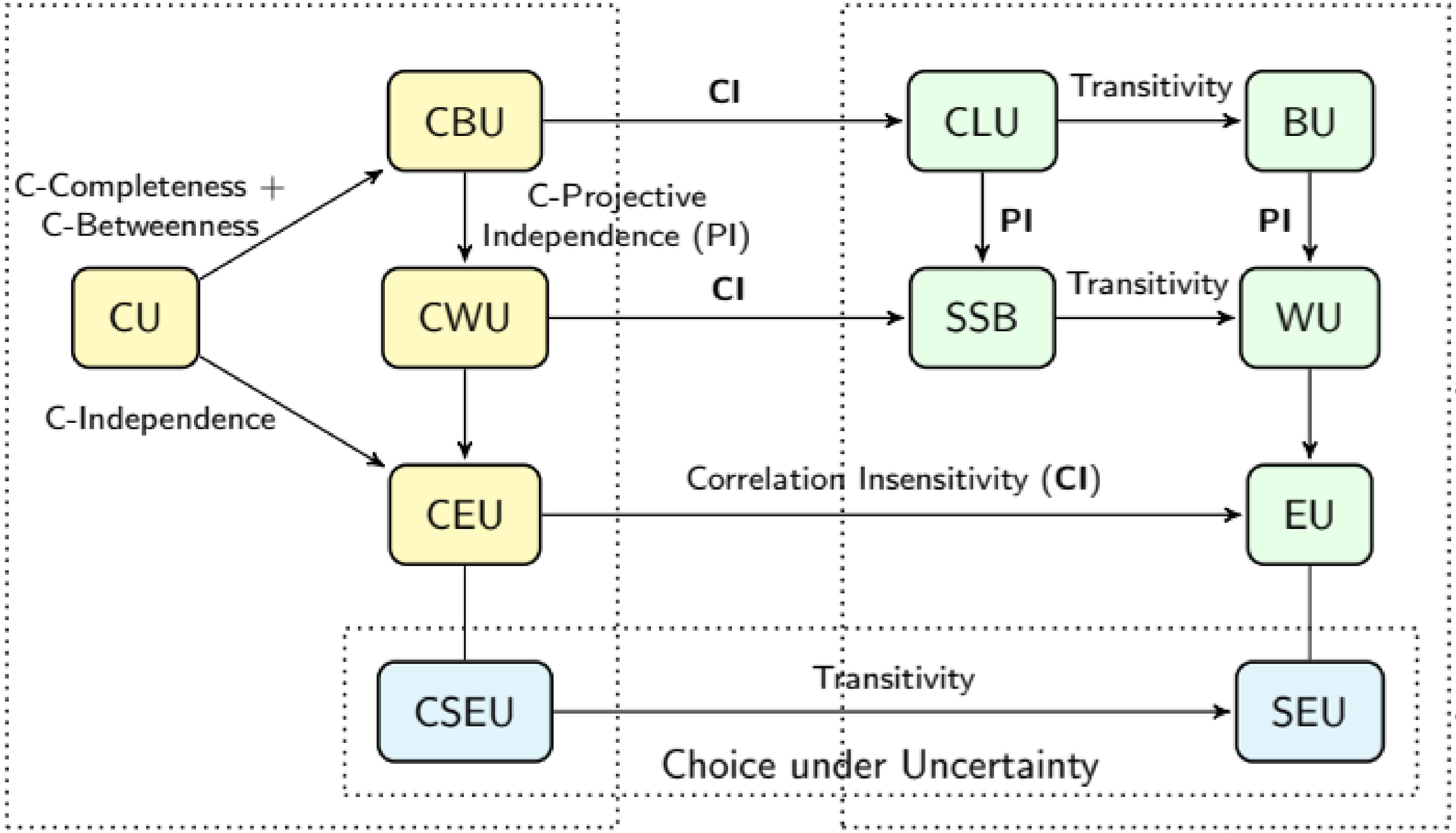


- Transitive NEU predicts the same rates of violations regardless of correlation!

Road Map


CBU
CI
CLU
Transitivity
BU
C-Completeness +
C-Betweenness
C-Projective
Independence (PI)
PI
PI
CU
CWU
CI
SSB
Transitivity
WU
C-Independence
Correlation Insensitivity (CI)
Focusing on →
CEU
EU
CSEU
Transitivity
SEU
Choice under Uncertainty
Correlation Probabilistic Sophistication (CPS)
CI Probabilistic Sophistication (CIPS)

Preview



- Classical Allais inspires literature on **transitive NEU** including WU, BU, PT/CPT, and RDU
  - Predicts correlation independent Allais behavior
- **CEU** → Accounts for violation of correlation reflexivity and independent Allais but not correlated Allais
- **CWU** → Accounts for violation of reflexivity as well as the correlation-sensitive Allais, i.e., **extended Allais paradox**.

‘Mother’ Representation (w/o C nor T)

Axiom 1 (Continuity). *Π is closed relative to $\Delta(X \times X)$ under the topology of pointwise convergence.*

Definition 1 (Correlation Utility Representation). *The correlation preference Π is represented by a correlation utility function $U : \Delta(X \times X) \to \mathbb{R}$ if U is continuous and for $\pi \in \Delta(X \times X)$, $\pi \in \Pi \iff U(\pi) \geqslant 0$.*

Proposition M *The correlation preference Π admits a correlation utility representation if and only if Π is continuous.*

Independence *Revisited*

- Samuelson (1952) and Fishburn (1975) offer a 4-lottery version of the classical **Independence Axiom**: $p \succsim q \wedge p' \succsim q' \Rightarrow$
$$\forall \alpha \in (0,1), \alpha p + (1-\alpha)p' \succsim \alpha q + (1-\alpha)q'$$

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- Expanding the domain to $\Delta(X \times X)$ yields: $p \succsim^{\pi} q \wedge p' \succsim^{\pi'} q' \Rightarrow$
$$\forall \alpha \in (0,1), \alpha p + (1-\alpha)p' \succsim^{\pi_\alpha} \alpha q + (1-\alpha)q'$$

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Without completeness, **Correlation Independence** will need to include the case of *p* (*p'*) being not preferred to *q* (*q'*).

- Restating the above yields

Axiom 3 (Correlation Independence). *For any* $\alpha \in (0,1)$,

(i) $\pi, \pi' \in \Pi \Rightarrow \alpha\pi + (1-\alpha)\pi' \in \Pi$;

(ii) $\pi, \pi' \in \check{\Pi} \Rightarrow \alpha\pi + (1-\alpha)\pi' \in \check{\Pi}$.

$\pi \in \ldots$	p vs. q
Π	$p \succeq q$
$\hat{\Pi}$	$p \succ q$
$\tilde{\Pi}$	$p \sim q$
$\check{\Pi}$	$p \not\succeq q$

Correlation EU w/o Completeness

Definition 1 (Correlation Expected Utility). *The correlation preference* Π *admits an correlation expected utility (CEU) representation if there exists a function* $\phi : X \times X \to \mathbb{R}$ *with* $\phi(x, x) = 0\ \forall x \in X$ *such that* $\forall \pi \in \Delta(X \times X)$, $\pi \in \Pi \iff \mathbb{E}_\pi \phi \geqslant 0$.

- Interpret $\phi(x, y)$ as utility of receiving x while foregoing y.

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- Interpret $\phi(x, y)$ as utility of receiving x while foregoing y.
- we say a CEU representation is *non-trivial* if there exists $(x, y), (x', y')$ such that $\phi(x, y) > 0$ and $\phi(x', y') < 0$.

Theorem E (Axiomatization of CEU). *A continuous correlation preference* Π *satisfies correlation independence, if and only if it admits a non-trivial CEU representation.*

- Expected Utility (EU) model: $\phi^{EU}(x, y) = u(x) - u(y)$.
- Regret Theory (RT) by Bell (1982) and Loomes and Sugden (1982): ϕ^{RT} is often assumed to satisfy *regret aversion* $\phi(x, y) > \phi(x, z) + \phi(y, z)$ for all $x > z > y$.
- Salience Theory (ST) by Bordalo et al. (2012): $\phi^{ST}(x, y) = \sigma(x, y)(x - y)$ with the *salience function* σ satisfying a set of *salience properties* (their Definition 1). Though with different behavioral/psychological fundamentals, ST representation also satisfies the regret aversion condition. An example would be $\sigma(x, y) = \frac{|x-y|}{|x|+|y|+\theta}$ as provided in the original paper.[13]
- The correlation sensitive representation by Lanzani (2022): This is the symmetric CEU model.

Examples of Symmetric CEU

- Expectations-based-Reference-Dependent (ERD) model from Kőszegi and Rabin (2007): $\phi^{ERD}(x, y) = (\lambda \cdot \mathbf{1}_{x>y} + \mathbf{1}_{x \leqslant y})(x - y)$, where $\lambda > 1$ captures decision maker's aversion to loss from x to y when $x > y$. Since $\lambda > 1$, ϕ^{ERD} is not skew-symmetric.

Completeness and Strong Independence

Axiom 2 (Correlation Completeness). *For all* $\pi \in \Delta(X \times X)$, $\pi \notin \Pi \Rightarrow \pi^T \in \Pi$.

Axiom 3* (Strong Independence). *For any* $\alpha \in (0,1)$,

(i) $\pi, \pi' \in \tilde{\Pi} \Rightarrow \alpha\pi + (1-\alpha)\pi' \in \tilde{\Pi}$;

(ii) $\pi \in \hat{\Pi}, \pi' \in \Pi \Rightarrow \alpha\pi + (1-\alpha)\pi' \in \hat{\Pi}$;

(iii) $\pi \in \check{\Pi} \bigcup \tilde{\Pi}, \pi' \in \check{\Pi} \Rightarrow \alpha\pi + (1-\alpha)\pi' \in \check{\Pi}$.

$\pi \in \ldots$	p vs. q
Π	$p \succeq q$
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Π	$p \succeq q$
$\hat{\Pi}$	$p \succ q$
$\tilde{\Pi}$	$p \sim q$
$\check{\Pi}$	$p \prec q$

- Thin indifference sets
- No inertia
- These axioms parallel closely the corresponding axioms in Lanzani (2022).

Axiomatizing Symmetric CEU

Corollary 1 (Axiomatization of Symmetric CEU). *A continuous correlation preference Π satisfies correlation completeness and correlation strong independence, if and only if it admits a symmetric CEU representation.*

- This is essentially Lanzani's (2022) Theorem 1.

Correlation Insensitivity and Transitivity in CEU

Theorem 2 (Correlation Insensitivity and EU). *For a correlation preference* Π, *the following are equivalent:*

(i) Π *admits a CEU representation and exhibits correlation insensitivity;*

(ii) Π *admits a symmetric CEU representation and exhibits SML correlation insensitivity;*

(iii) Π *admits a EU representation.*

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- Alternative axiomatization of EU
- Kantorovich duality

How well does CEU do?

- Accommodates SML preference ✓

How well does CEU do?

- Accommodates SML preference ✓
- But… not Extended Allais Paradox ✗
  - – correlation-sensitive Allais behavior

Axiomatizing Non-CEU Preference

- Development of NEU with transitivity:
  inspired by the Allais paradoxes without considering correlation sensitivity.

Axiomatizing Non-CEU Preference

- Development of NEU with transitivity:
  inspired by the Allais paradoxes without considering correlation sensitivity.
- ➡ value in going beyond CEU…
  to account for Extended Allais Paradox.

Betweenness ***Revisited***

- Transitive Betweenness, often state as $p \sim q \Rightarrow p \sim ap + (1-a)q$, for every a in $(0,1)$, characterizes a lottery-specific utility function.

Betweenness *Revisited*

- Transitive Betweenness, often state as $p \sim q \Rightarrow p \sim ap + (1\text{-}a)q$, for every a in $(0,1)$, characterizes a lottery-specific utility function.

- … Correlation Betweenness by restricting Strong Independence to joint densities with the same row marginal …

Axiom 5 (Correlation Betweenness). *For any* $\pi, \pi' \in \Delta(X \times X)$ *such that* $\pi_1 = \pi'_1$, *and* $\alpha \in (0,1)$,

(i) $\pi, \pi' \in \tilde{\Pi} \Rightarrow \alpha\pi + (1-\alpha)\pi' \in \tilde{\Pi}$;

(ii) $\pi \in \hat{\Pi}, \pi' \in \Pi \Rightarrow \alpha\pi + (1-\alpha)\pi' \in \hat{\Pi}$;

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$\pi \in \ldots$	p vs. q
Π	$p \geq q$
$\hat{\Pi}$	$p > q$
$\tilde{\Pi}$	$p \sim q$
$\check{\Pi}$	$p < q$

Correlation Betweenness Utility (CBU)

Definition 3 (Correlation Betweenness Utility). *The correlation preference* Π *admits a correlation betweenness utility (CBU) representation if for each* $p \in \Delta X$, *there exists* ϕ_p *such that* $\pi \in \Pi \iff \mathbb{E}_\pi \phi_{\pi_1} \geqslant 0$, *and* $\pi \in \tilde{\Pi} \iff \mathbb{E}_\pi \phi_{\pi_1} = 0$.

- lottery-specific ϕ parallels the lottery-specific utility in BU (Dekel, 1986)

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Theorem B (Axiomatization of CBU). *A continuous correlation preference* Π *satisfies correlation betweenness if and only if it admits a CBU representation.*

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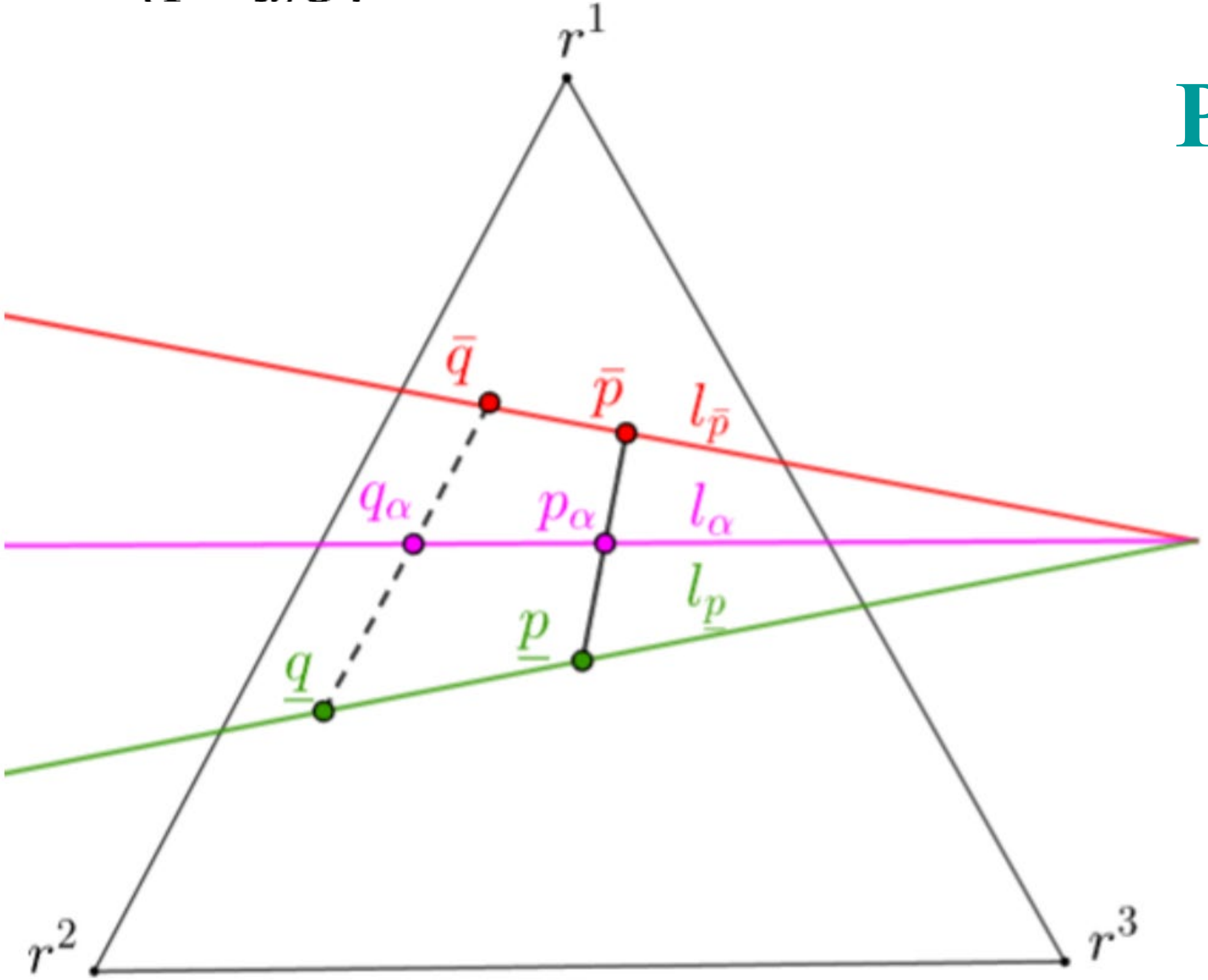
Theorem CI (Correlation Insensitivity and Betweenness Utility). *For* Π *admitting a CBU representation, it is correlation insensitive, complete and transitive if and only if it admits a betweenness utility representation.*

CBU + CI + ***T*** → BU

Projective independence: $\forall F, F', G, G' \in D(X)$, if $F \sim F'$ and $G \sim G'$, and if $\exists \alpha \in (0, 1)$ such that $\alpha F + (1-\alpha)G \sim \alpha F' + (1-\alpha)G'$, then $\forall \alpha \in (0, 1)$, $\alpha F + (1-\alpha)G \sim \alpha F' + (1-\alpha)G'$.



Projective Independence ***Revisited***

Chew, Epstein, Segal (1994)

underpinning **WU**

CWU = CEU + SSB

Definition 4. *The correlation preference* Π *admits a correlation weighted utility (CWU) representation on* $\Delta' \subset \Delta(X \times X)$ *if there exists a skew-symmetric kernel* ϕ *and a skew-symmetric bilinear kernel* $\Psi : \Delta X \times \Delta X \to \mathbb{R}$*, such that for* $\pi \in \Delta'$,

$$\pi \in \Pi \iff \mathbb{E}_\pi \phi + \Psi(\pi_1, \pi_2) \geqslant 0.$$

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C-Completeness + C-Betweenness + C-PI → **CWU**

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C-Completeness + C-Betweenness + C-PI → **CWU**

CWU + Correlation Insensitivity → SSB (Fishburn, 1982)

CWU + Correlation Insensitivity + ***T*** → WU

Allais Behavior, Correlation Sensitivity, and Linearity

		Correlation Sensitivity	
		Insensitive	Sensitive
Linearity of representation in (joint) probability	Linear	EU	CEU - SML preference
	Bilinear	SSB/WU - indep. Allais	CWU - corr. Allais

Resolving Extended Allais Paradox

THANK YOU

A Savagean Diamond

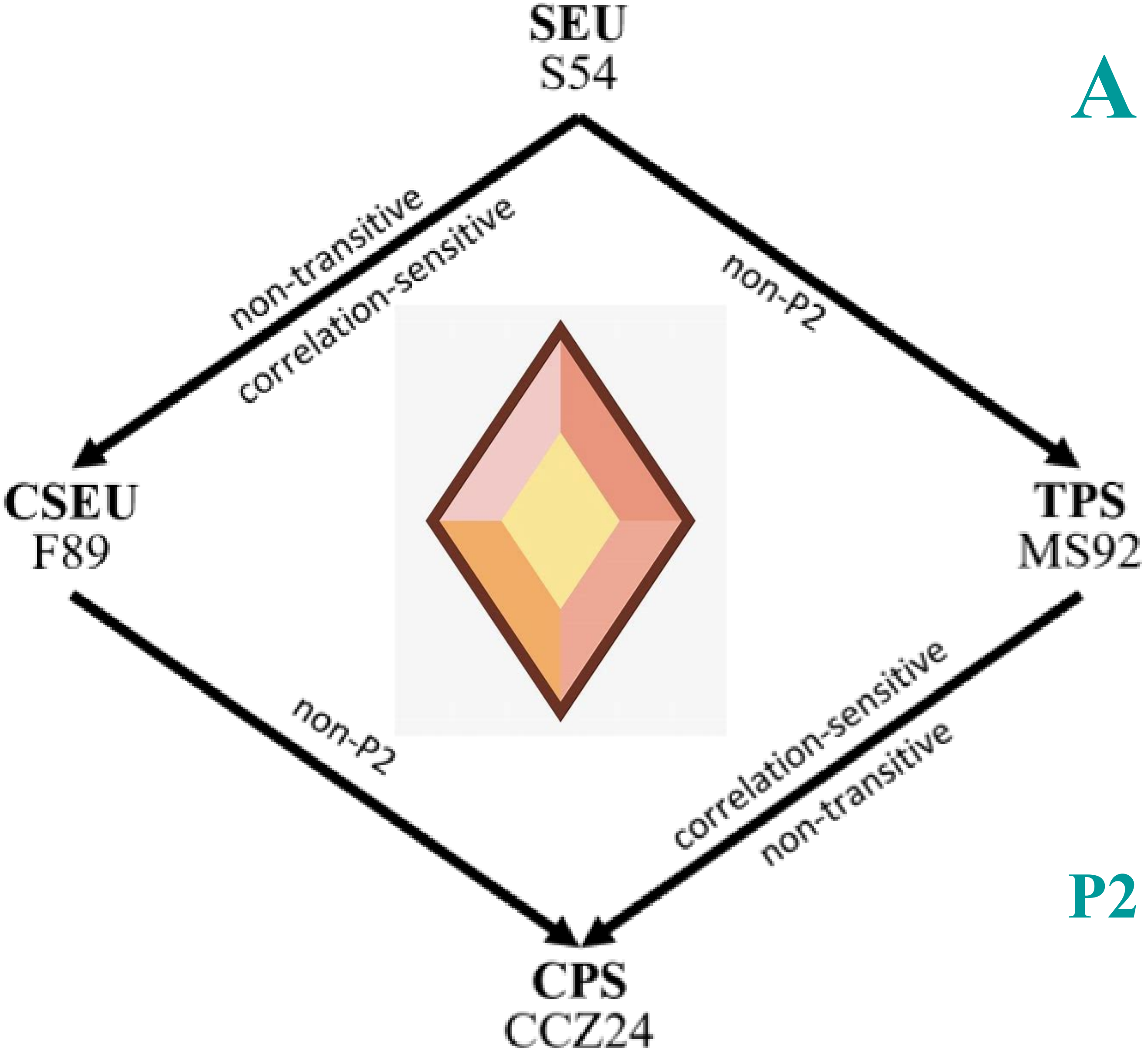


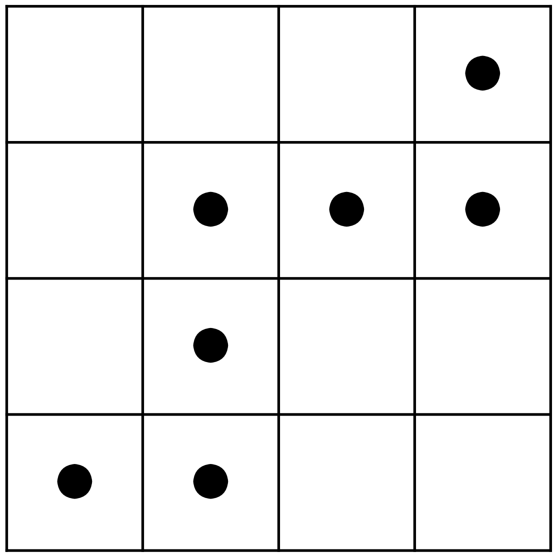
Figure 5: Parallelisms involving P2 and correlation sensitivity

Parallelisms involving P2 and Correlation Sensitivity

Small Worlds CPS

- **Transitive source preference** in a single grand world setting (Chew and Sagi, 2008) seems inherently **limiting** …
  - →**Small worlds CPS** from multiple sources of uncertainty.
  - →Evidence: familiarity bias, tends to relate to one's **identification** with the source of uncertainty.
  - →Relate to **limited attention**.
- Source preference through **multiple identities,** e.g., memberships such as family, nationality, jobs, and clubs relating to schools and hobbies.

Correlation Rank Dependence?



- Comonotonicity removes correlation-sensitivity
- Yaari's dual independence:
  - utility over (a pair of) outcome-induced rankings
  - correlation dual utility: **copula-sensitive** kernels
  - write $F(x,y) = C(F_1(x), F_2(y))$ $G_p(x) = Prob\ \{t > x\}$
  - $\pi \in \Pi \iff \int_{[0,1]^2} \psi^{C_\pi}(G_{\pi_1}(x), G_{\pi_2}(y))dxdy \geqslant 0$
- CI → Rank dependent SSB $\int_{[0,1]^2} \Psi(p,q)dG_{\pi_1}^{-1}(p)dG_{\pi_2}^{-1}(q) \geqslant 0$
- Adding T → Correlation extension of rank-dependent WU (Chew and Epstein, 1989).