Collaboration in Bipartite Networks*

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Abstract

This paper proposes a general framework for studying the impact of collaboration on team production. We build a micro-founded model for team production, where collaboration between agents is represented by a bipartite network. The Nash equilibrium of the game incorporates both the complementarity effect between collaborating agents and the substitutability effect between concurrent projects of the same agent. We propose a Bayesian MCMC procedure to estimate the structural parameters and illustrate the empirical and policy relevance of the model by analyzing the collaboration network of inventors in the semiconductor and pharmaceutical industries.

Keywords: Bipartite networks, R&D collaboration, spillovers, team production.

JEL: C31, C72, O31

1 Introduction

Collaboration between agents plays a prominent role in team production. Through a complex network of collaborations, agents generate spillovers not only to their coworkers but also to other agents indirectly connected to them. The aim of this paper is to develop a general structural model that helps us to understand how collaboration affects team production.

First, we build a micro-founded model for team production. The collaboration between agents is characterized by a bipartite network with two types of nodes: agents and projects. The effort that an agent spends on a project is represented by an edge in the bipartite network, and collaborating agents are connected through the projects they work on together. We characterize the equilibrium of the game where agents choose efforts in multiple and possibly overlapping projects to maximize utility. The equilibrium takes into account both the complementarity effect between collaborating agents and the substitutability effect between concurrent projects of the same agent.

Next, we propose an estimation procedure to recover the structural parameters of the model. There are three main challenges in estimating this model. First, the effort level of an agent in the production function is unobservable. To overcome this problem, we replace the unobserved effort level in the production function with the equilibrium effort level derived from the theoretical model. Second, the matching between agents and projects is likely to be endogenous. Estimating the production function without taking into account this potential endogeneity may incur a selection bias. To remediate the issue, we introduce a participation function to model the endogenous selection of agents into projects, allowing for both agent and project unobserved heterogeneity.[1] The resulting likelihood function involves high-dimensional integrals. This leads to the third challenge of the estimation, i.e., it is computationally cumbersome to apply a frequentist maximum likelihood method, even when resorting to a simulation approach. To bypass this difficulty, we adopt a Bayesian Markov Chain Monte Carlo (MCMC) approach to jointly estimate the production and participation

[1] As pointed out in Bonhomme (2020), a key feature of bipartite networks is two-sided heterogeneity.

functions.

Finally, we demonstrate the empirical relevance of our model. The proposed bipartite network model for team production has many potential applications including co-inventor networks in technology innovation (see, e.g., Singh 2005, Fleming et al. 2007, Singh & Fleming 2010), co-authorship networks in scientific research (see, e.g., Anderson & Richards-Shubik 2022), expert networks (The Economist 2011, Davenport et al. 1998), online labor markets (Horton 2010, Anderson 2017), networks of committees in the U.S. House of Representatives (Porter et al. 2005), networks of corporate board members (Conyon & Muldoon 2006), and networks of partners in crime (Billings et al. 2019).[2] In this paper, we provide an empirical illustration using the collaboration network of patent inventors in the semiconductor and pharmaceutical industries. We find that the estimated complementarity and substitutability effects are both statistically significant with the expected signs. The estimates are downward biased when the endogenous matching between researchers and projects is ignored. The direction of the bias is compatible with the intuition and consistent with the Monte Carlo simulation results. To show the importance of correctly estimating the structural model in policy analysis, we carry out a counterfactual study on the impact of innovation incentives on research output. We find that the effectiveness of innovation incentives tends to be understated when the complementarity effect is ignored and overstated when the substitutability effect is ignored. We also find the innovation incentive program is more effective in the pharmaceutical industry where the complementarity effect dominates the substitutability effect. Moreover, we derive the optimal incentive scheme and provide a ranking of the firms based on the effectiveness of the optimal innovation incentive program.

The theoretical model in our paper has a similar linear-quadratic payoff specification as Bimpikis et al. (2019), where firms compete in quantities à la Cournot across different markets. While the products sold by competing firms to the same market are substitutes in Bimpikis et al. (2019), the efforts spent by collaborating agents on the same project are

[2]See Section 2.4 for more detailed discussion.

strategic complements in our model. Chen et al. (2018) also consider a similar payoff function as ours to study agents' effort choices in multiple activities with complementarities between efforts of linked agents and substitutabilities between an agent's efforts across different activities. However, different from our model where different agents may participate in different activities (or projects), Chen et al. (2018) assume all agents in the network participate in the same set of activities.

Our empirical analysis encounters a similar challenge to the one addressed by Bonhomme (2021), i.e., individual contributions (or efforts) to the team production output cannot be directly observed in the data. Bonhomme (2021) assumes that the individual contribution is fixed across different projects and identifies individual contributions by tracking individuals who work on different projects over time. By contrast, we allow individual contributions (or efforts) to vary across projects and impute the unobserved individual contributions from the equilibrium of the model.

The rest of the paper is organized as follows. Section 2 introduces the theoretical model and characterizes the equilibrium. Section 3 presents the econometric methodology. The empirical implications of the model are discussed in Section 4, where Section 4.1 describes the data used in the empirical study, Section 4.2 gives the main estimation results, Section 4.3 reports the estimated marginal effects, and Section 4.4 conducts a counterfactual study on an innovation incentive program. Section 5 briefly concludes. The proofs, technical details, and robustness checks can be found in the online appendix.

2 Theoretical Model

2.1 Bipartite Network, Production Function, and Utility

Consider a *bipartite* network given by $\mathcal{G} = (\mathcal{N}, \mathcal{P}, \mathcal{E})$, where $\mathcal{N} = \{1, \dots, n\}$ denotes the set of agents, $\mathcal{P} = \{1, \dots, p\}$ denotes the set of projects, and $\mathcal{E}$ denotes the set of edges connecting agents and projects. In our model, an edge $e_{is} \in \mathcal{E}$ is the (non-negative) effort that agent i spends on project s. Let $\mathcal{N}_s$ denote the set of agents working on project s and $\mathcal{P}_i$ denote the set of projects agent i participates in. Let $|\cdot|$ denote the cardinality of a set.

The *production function* for project $s \in \mathcal{P}$ is given by

$$y_s(\mathcal{G}) = \sum_{i \in \mathcal{N}_s} \alpha_i e_{is} + \frac{\lambda}{2} \sum_{i \in \mathcal{N}_s} \sum_{j \in \mathcal{N}_s \setminus \{i\}} g_{ij} e_{is} e_{js} + \epsilon_s, \tag{1}$$

where $y_s(\mathcal{G})$ (or simply y_s) is the output of project s, α_i represents individual heterogeneity in productivity, $g_{ij} \in [0,1]$ measures the degree of compatibility between collaborating agents, and ϵ_s is a random shock. If λ is positive, then the marginal product of agent i's effort in a project increases with the efforts of other agents in that project. Hence, the coefficient λ captures the complementarity effect.[3]

We assume that the *utility* of agent i is given by

$$U_i(\mathcal{G}) = \underbrace{\sum_{s \in \mathcal{P}_i} \delta_s y_s}_{\text{payoff}} - \underbrace{\frac{1}{2} \left(\sum_{s \in \mathcal{P}_i} e_{is}^2 + \phi \sum_{s \in \mathcal{P}_i} \sum_{t \in \mathcal{P}_i \setminus \{s\}} e_{is} e_{it} \right)}_{\text{cost}}. \tag{2}$$

The utility function has a payoff/cost structure similar to Chen et al. (2018). The payoff is the weighted total output of the projects that agent i participates in, with the weights given by $\delta_s = |\mathcal{N}_s|^{-1}$, i.e., the individual payoff is discounted by the number of agents participating in project s (cf. Kandel & Lazear 1992, Jackson & Wolinsky 1996, Hollis 2001, Yuret 2017). Moreover, the cost in Equation (2) is quadratic in efforts, e_{is}, of agent i. The quadratic term, e_{is}^2, captures decreasing marginal returns from each project s, while the cross-product term, $\phi e_{is} e_{it}$, captures the interdependencies between efforts in different projects, $s \neq t$. The coefficient ϕ measures the degree of substitutability of an agent's efforts in different projects. If ϕ is positive, then the marginal cost (utility) of agent i's effort

[3] Our formulation for capturing complementarities of collaborating agents in Equation (1) follows the seminal model proposed in Ballester et al. (2006), and has been widely used in the theoretical and empirical literature studying games on networks (see Jackson & Zenou 2015, Bramoullé & Kranton 2016, for an overview and further discussion).

in a project increases (decreases) with the effort agent i spends on other projects.[4] This quadratic cost specification helps to capture the fact that the available time or resources of an agent are limited, and there exist substitutability effects between projects (albeit not perfect substitutes as a resource constraint would imply). It includes the convex separable cost specification as a special case if $\phi = 0$ (when there are no substitutability effects and projects are independent). The quadratic cost specification is very common in the literature (cf. Singh & Vives 1984, Bulow et al. 1985, Vives 2011, Chen et al. 2018, Bimpikis et al. 2019).[5,6] In Section 2.3 we will provide a simple example to understand better the intuition behind the complementarity and substitutability effects in our model.

2.2 Game and Equilibrium

The equilibrium analysis focuses on agents' strategic allocation of their efforts across the projects they are in, taking their assignments into projects as given. Let d_{is} be an indicator variable, such that $d_{is} = 1$ if agent i is in project s and $d_{is} = 0$ otherwise. Given $\{d_{is}\}$, the following proposition provides an equilibrium characterization of the agents' effort portfolio $e = (e_1', \cdots, e_p')'$, with $e_s = (e_{1s}, \cdots, e_{ns})'$ for $s = 1, \cdots, p$. Let

$$W = D(\mathrm{diag}_{s=1}^{p}\{\delta_s\} \otimes G)D, \qquad \text{and} \qquad M = D(J_p \otimes I_n)D, \tag{3}$$

where $\otimes$ denotes the Kronecker product, D is an np-dimensional diagonal matrix given by $D = \mathrm{diag}_{s=1}^{p}\{\mathrm{diag}_{i=1}^{n}\{d_{is}\}\}$, G is an $n \times n$ zero-diagonal matrix with the (i, j)th $(i \neq j)$

[4] Note that $\frac{\partial^2 U_i}{\partial e_{is} \partial e_{it}} = -\frac{1}{2}\phi$ for $s \neq t$ and thus efforts e_{is} and e_{it} are strategic substitutes when $\phi > 0$ (Bulow et al. 1985).

[5] For example, Bimpikis et al. (2019) analyze a multi-product Cournot competition model in which firms $i = 1, \ldots, n$ choose production quantities, $q_{ik} \geq 0$, across different markets, $k = 1, \ldots, m$, with a cost function of firm i given by $\left(\sum_{k=1}^{m} q_{ik}\right)^2 = \sum_{k=1}^{m} q_{ik}^2 + \sum_{k=1}^{m} \sum_{l \neq k}^{m} q_{ik} q_{il}$, capturing diseconomies of scope (cf. Bulow et al. 1985, Rawley & Simcoe 2010).

[6] A theoretical model with a similar cost specification as in Equation (2) but allowing for only two activities is studied in Belhaj & Deroïan (2014) and a generalization to multiple activities can be found in Chen et al. (2018). An empirical analysis is provided in Liu (2014) and Cohen-Cole et al. (2018). However, different from our model where different agents may participate in different projects, these papers assume that all the agents in the network participate in the same set of activities.

element being g_{ij}, and J_p is an $p \times p$ zero-diagonal matrix with off-diagonal elements equal to one. Let $\rho_{\max}(\cdot)$ denote the spectral radius of a square matrix.

Proposition 1. *Suppose the production function for each project $s \in \mathcal{P}$ is given by Equation (1) and the utility function for each agent $i \in \mathcal{N}$ is given by Equation (2). Let $L := L(\lambda, \phi) = \lambda W - \phi M$. Given $\{d_{is}\}$, if*

$$\rho_{\max}(L) < 1, \tag{4}$$

then the Nash equilibrium effort portfolio is given by

$$e^* = (I_{np} - L)^{-1} D(\delta \otimes \alpha), \tag{5}$$

where $\delta = (\delta_1, \cdots, \delta_p)'$ and $\alpha = (\alpha_1, \cdots, \alpha_n)'$.

The matrix $L = \lambda W - \phi M$ represents a weight matrix of the *line graph* $\mathcal{L}(\mathcal{G})$ for the bipartite network $\mathcal{G}$.[7] In the line graph $\mathcal{L}(\mathcal{G})$, each node represents the effort an agent invests into a project. The links between nodes with the same project are represented by the nonzero entries of W while the links between nodes with the same agent are represented by the nonzero entries of M. The matrix L is a weighted sum of the matrices W and M, with the weights being the complementarity effect (λ) and the substitutability effect (ϕ) respectively. The formulation of L highlights the importance of both effects (i.e., λ and ϕ) in the bipartite network. The condition in Equation (4) plays a similar role as the one in Theorem 1 of Ballester et al. (2006), which limits the rate at which spillovers decay across the bipartite network.

2.3 An Illustrating Example

We illustrate the equilibrium characterization of Proposition 1 with an example corresponding to the bipartite network $\mathcal{G}$ in Figure 1. In this bipartite network, there are 3 agents and

[7] Given a network $\mathcal{G}$, its line graph $\mathcal{L}(\mathcal{G})$ is a graph such that each node of $\mathcal{L}(\mathcal{G})$ represents an edge of $\mathcal{G}$, and two nodes of $\mathcal{L}(\mathcal{G})$ are connected if and only if their corresponding edges share a common endpoint in $\mathcal{G}$ (cf. e.g., West 2001).

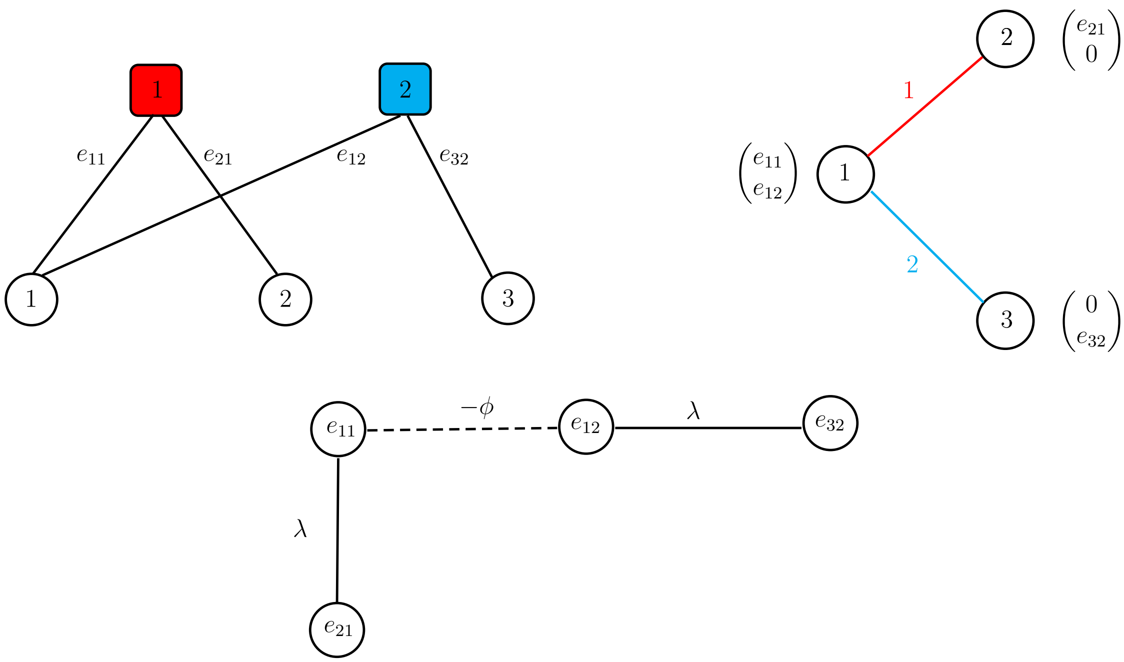


Figure 1: Top left panel: the bipartite network $\mathcal{G}$ of agents and projects analyzed in Section 2.3, where circles represent agents and squares represent projects. Top right panel: the projection of the bipartite network $\mathcal{G}$ on the set of agents. The effort levels of the agents for each project they are involved in are indicated next to the nodes. Bottom panel: the line graph $\mathcal{L}(\mathcal{G})$ associated with the bipartite network $\mathcal{G}$, in which each node represents the effort an agent invests into a project. Solid lines connect nodes with the same project while dashed lines connect nodes with the same agent.

2 projects, where agents 1 and 2 are collaborating in the first project and agents 1 and 3 are collaborating in the second project. For expositional purposes, let $g_{ij} = 1$ for all $i \neq j$.

Line Graph. The line graph $\mathcal{L}(\mathcal{G})$ of this bipartite network is depicted in the bottom panel of Figure 1. In the line graph, each node represents the effort an agent invests into a project. Solid lines connect nodes with the same project while dashed lines connect nodes with the same agent. Following Equation (3),

$$W = \begin{bmatrix} 0 & 1/2 & 0 & 0 & 0 & 0 \\ 1/2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/2 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/2 & 0 & 0 \end{bmatrix} \quad \text{and} \quad M = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

The nonzero entries of the matrices W and M correspond to, respectively, the solid lines and the dashed lines in the line graph. The matrix L is a weighted sum of the matrices W and M, given by

$$L = \lambda W - \phi M = \begin{bmatrix} 0 & \lambda/2 & 0 & -\phi & 0 & 0 \\ \lambda/2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -\phi & 0 & 0 & 0 & 0 & \lambda/2 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda/2 & 0 & 0 \end{bmatrix}.$$

The $(1,2)$th and $(2,1)$th elements of the matrix L represent the link between e_{11} and e_{21} with weight $\lambda/2$ in the line graph, the $(4,6)$th and $(6,4)$th elements represent the link between e_{12} and e_{32} with weight $\lambda/2$, and the $(1,4)$th and $(4,1)$th elements represent the link between e_{11} and e_{12} with weight $-\phi$. It is worth pointing out that, in the absence of the substitutability effect (i.e., $\phi = 0$), the line graph would be split into two independent sub-graphs with each one corresponding to the collaborators' efforts in a single project. Therefore, the substitutability effect provides a channel to capture the interdependence of efforts in different projects.

Equilibrium. In this example, the sufficient condition (4) for the existence of a unique equilibrium holds if $|4\phi| < 4 - \lambda^2$. Note that this condition reduces to $|\phi| < 1$ if $\lambda = 0$ and $|\lambda| < 2$ if $\phi = 0$. From Equation (5), the equilibrium effort portfolio is

$$e^* = \begin{bmatrix} e_{11}^* \\ e_{21}^* \\ e_{31}^* \\ e_{12}^* \\ e_{22}^* \\ e_{32}^* \end{bmatrix} = \frac{1}{(4-\lambda^2)^2 - 16\phi^2} \begin{bmatrix} 2(4-\lambda^2-4\phi)\alpha_1 + \lambda(4-\lambda^2)\alpha_2 - 4\lambda\phi\alpha_3 \\ \lambda(4-\lambda^2-4\phi)\alpha_1 + 2(4-\lambda^2-4\phi^2)\alpha_2 - 2\lambda^2\phi\alpha_3 \\ 0 \\ 2(4-\lambda^2-4\phi)\alpha_1 - 4\lambda\phi\alpha_2 + \lambda(4-\lambda^2)\alpha_3 \\ 0 \\ \lambda(4-\lambda^2-4\phi)\alpha_1 - 2\lambda^2\phi\alpha_2 + 2(4-\lambda^2-4\phi^2)\alpha_3 \end{bmatrix}.$$

In the following, we analyze how idiosyncratic heterogeneity, complementarity, and substitutability affect the equilibrium effort choices.

Idiosyncratic Heterogeneity: Marginal effects of α_i. As $|4\phi| < 4 - \lambda^2$,

$$\begin{aligned} \frac{\partial e_{11}^*}{\partial \alpha_1} &= \frac{\partial e_{12}^*}{\partial \alpha_1} = \frac{2}{4-\lambda^2+4\phi} > 0 \\ \frac{\partial e_{21}^*}{\partial \alpha_2} &= \frac{\partial e_{32}^*}{\partial \alpha_3} = \frac{2(4-\lambda^2-4\phi^2)}{(4-\lambda^2)^2-16\phi^2} > 0, \end{aligned}$$

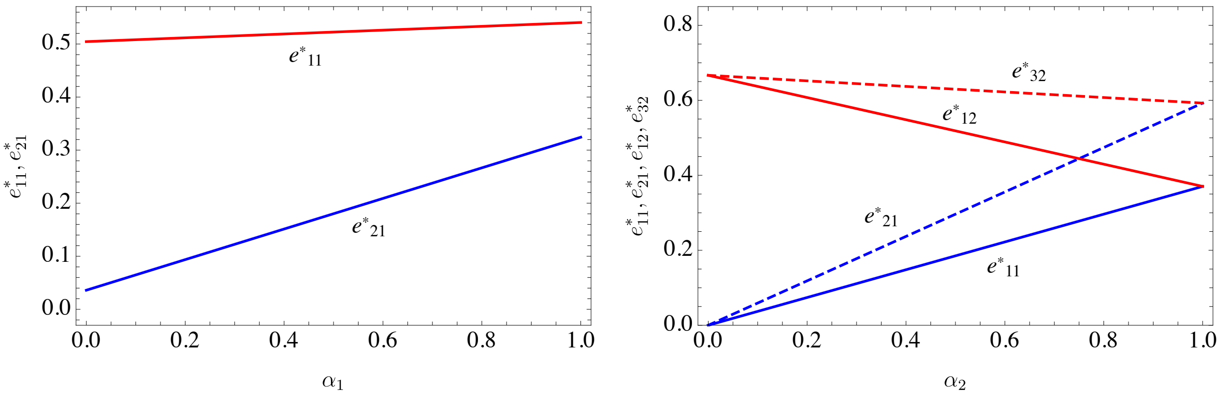


Figure 2: Left panel: equilibrium effort levels for agents 1 and 2 in project 1 for $\lambda = 0.25$, $\phi = 0.75$, $\alpha_2 = \alpha_3 = 1$, and varying values of α_1. Note that in this case, $e^*_{11} = e^*_{12}$ and $e^*_{21} = e^*_{32}$. Right panel: equilibrium effort levels for agents 1, 2 and 3 in projects 1 and 2 for $\lambda = 0.5$, $\phi = 0.75$, $\alpha_1 = \alpha_3 = 1$, and varying values of α_2.

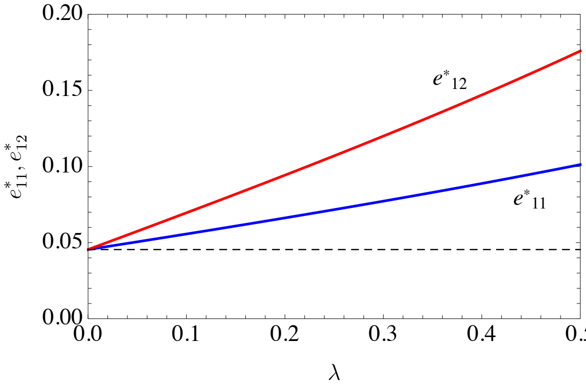
and, if the complementarity effect is positive (i.e., $\lambda > 0$),

$$\begin{aligned}\frac{\partial e^*_{21}}{\partial \alpha_1} &= \frac{\partial e^*_{32}}{\partial \alpha_1} = \frac{\lambda}{4-\lambda^2+4\phi} > 0\\ \frac{\partial e^*_{11}}{\partial \alpha_2} &= \frac{\partial e^*_{12}}{\partial \alpha_3} = \frac{\lambda(4-\lambda^2)}{(4-\lambda^2)^2-16\phi^2} > 0,\end{aligned}$$

which suggest that more productive agents raise not only their own effort levels but also the effort levels of their collaborators due to the complementarity effect. On the other hand, if the substitutability effect is also positive (i.e., $\phi > 0$),

$$\begin{aligned}\frac{\partial e^*_{12}}{\partial \alpha_2} &= \frac{\partial e^*_{11}}{\partial \alpha_3} = -\frac{4\lambda\phi}{(4-\lambda^2)^2-16\phi^2} < 0\\ \frac{\partial e^*_{32}}{\partial \alpha_2} &= \frac{\partial e^*_{21}}{\partial \alpha_3} = -\frac{2\lambda^2\phi}{(4-\lambda^2)^2-16\phi^2} < 0,\end{aligned}$$

which suggest that more productive agents induce lower effort levels spent by agents on other projects. It is worth noting that, without the substitutability effect (i.e., $\phi = 0$), agent i's productivity would have no effect on other agents' effort levels on a project that agent i is not involved in. This spotlights the important role of substitutability effect in the bipartite network. An illustration can be seen in Figure 2.



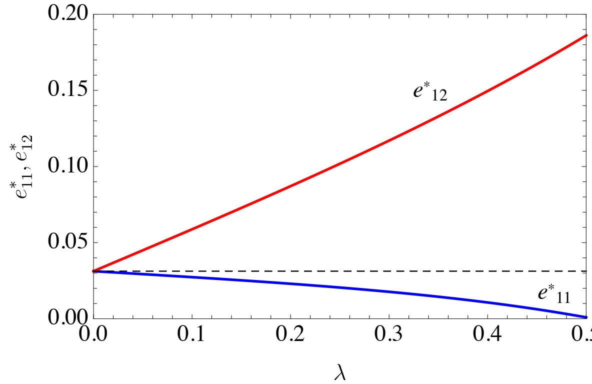


Figure 3: Equilibrium effort levels for agent 1 with $\alpha_1 = 0.1$, $\alpha_2 = 0.5$, $\alpha_3 = 1$, $\phi = 0.1$ (left panel), and $\phi = 0.6$ (right panel), for varying values of λ. The dashed lines indicate the effort levels for $\lambda = 0$.

Complementarity: Marginal effects of λ. The partial derivative of the equilibrium effort of agent 1 in project 1 with respect to the complementarity parameter λ is given by

$$\frac{\partial e^*_{11}}{\partial \lambda} = \frac{1}{\left[(4-\lambda^2)^2 - 16\phi^2\right]^2} \left\{ 4\lambda(4-\lambda^2-4\phi)^2\alpha_1 + \left[(16-\lambda^4-16\phi^2)(4-\lambda^2) + 32\lambda^2\phi^2\right]\alpha_2 \right.$$
$$\left. -4\phi\left[(4+3\lambda^2)(4-\lambda^2) - 16\phi^2\right]\alpha_3 \right\}.$$

Observe that the coefficient of α_3 is negative. Thus, when α_3 is large enough, $\partial e^*_{11}/\partial\lambda$ could be negative. The reason is that, with increasing λ, the complementarity effects between collaborating agents become stronger, and this effect is more pronounced for the collaboration of agent 1 with the more productive agent 3, than with the less productive agent 2. Moreover, when the substitutability effect ϕ is also large, agent 1 may spend even less effort on the project with agent 2, leading to a negative $\partial e^*_{11}/\partial\lambda$. An illustration can be seen in Figure 3.

Substitutability: Marginal effects of ϕ. The partial derivatives of the equilibrium efforts of agent 1 in projects 1 and 2 with respect to the substitutability coefficient ϕ are

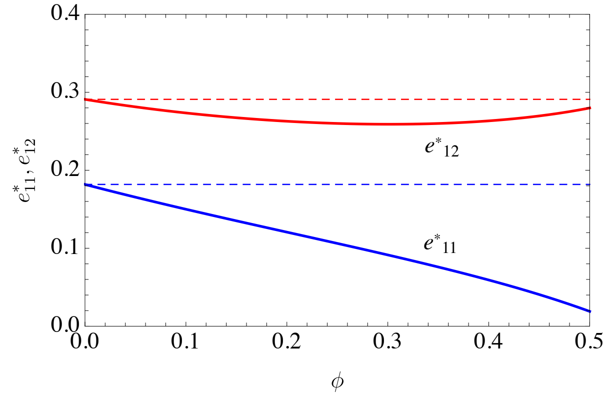


Figure 4: Equilibrium effort levels for agent 1 with $\alpha_1 = 0.2$, $\alpha_2 = 0.3$, $\alpha_3 = 0.8$, and $\lambda = 0.75$, for varying values of ϕ. The dashed lines indicate the effort levels for $\phi = 0$.

given by

$$\frac{\partial e^*_{11}}{\partial \phi} = -2\left[\frac{\lambda(\alpha_3 - \alpha_2)}{(\lambda^2 + 4\phi - 4)^2} + \frac{4\alpha_1 + \lambda(\alpha_2 + \alpha_3)}{(\lambda^2 - 4(\phi + 1))^2}\right],$$
$$\frac{\partial e^*_{12}}{\partial \phi} = 2\left[\frac{\lambda(\alpha_3 - \alpha_2)}{(\lambda^2 + 4\phi - 4)^2} - \frac{4\alpha_1 + \lambda(\alpha_2 + \alpha_3)}{(\lambda^2 - 4(\phi + 1))^2}\right].$$

Suppose $\alpha_3 > \alpha_2$. Then, $\partial e^*_{11}/\partial \phi$ is negative. That is, with increasing ϕ, agent 1 exerts lower effort in the project with a less productive collaborator. In contrast, $\partial e^*_{12}/\partial \phi$ can be positive or negative, depending on whether the first term is larger or smaller than the second term on the right-hand side of the second equation. With $\alpha_1 = 0.2$, $\alpha_2 = 0.3$, $\alpha_3 = 0.8$, and $\lambda = 0.75$, we can see in Figure 4 that, when the substitutability effect ϕ is small, both $\partial e^*_{11}/\partial \phi$ and $\partial e^*_{12}/\partial \phi$ are negative, and $\partial e^*_{11}/\partial \phi < \partial e^*_{12}/\partial \phi$. That is, increasing ϕ reduces efforts of agent 1 in both projects, and the effort reduction is more significant in the project with a less productive collaborator. When ϕ is larger, $\partial e^*_{12}/\partial \phi$ becomes positive while $\partial e^*_{11}/\partial \phi$ remains negative, indicating agent 1 reallocates effort to the project with a more productive collaborator as a result of the substitutability effect.

2.4 Applications of the Bipartite Network Model

In the following, we list some potential applications of our bipartite network model for team production. Empirical analysis in these applications shares some common challenges. For instance, the data usually only have information on the team production output but not on the individual contribution of each agent, and agents are likely to be sorted into different projects based on unobserved characteristics. These challenges motivate the general empirical model and estimation framework introduced in the next section.

Co-inventor Network. A large body of literature has emphasized the crucial role of networks of collaborating inventors on innovation outcomes (Singh & Fleming 2010, Singh 2005, Fleming et al. 2007, König 2016). We can treat patents and inventors as two distinct sets of nodes in a bipartite network connected by the effort that an inventor exerts in contributing to a patent. We then can use our framework to identify complementarity and substitutability effects in teams of inventors, and how these contribute to successful innovations.[8]

Co-authorship Network. In a co-authorship network, researchers and research projects can be treated as two distinct sets of nodes in a bipartite network connected by the research effort that a researcher spends on a project. Our model can be used to study the spillover effect of research effort taking into account the substitutability effect between concurrent projects of the same researcher. Hsieh et al. (2018) adopt this model to analyze the co-authorship network of economists registered in the Research Papers in Economics (RePEc) Author Service.

Expert Networks. An expert network consists of a group of professionals who are hired by companies in need of experts on some projects. They have become popular in the investment industry with professional firms (e.g. GLG or GlobalResearch) providing expert advice at a service fee (The Economist 2011). Moreover, Davenport et al. (1998) report several instances of firms building and managing expert networks within their organization. For example, Microsoft is mentioned as having developed an expert network that monitors the

[8] In Section 4, we will provide an empirical application of our model to the network of patent inventors in the semiconductor and pharmaceutical industries.

types of knowledge competencies necessary for software development projects and matches these projects with experts based on their expertise. The experts and projects can be represented as a bipartite network, and our framework can be used to analyze their performance and value to a firm.

Online Labor Markets. In recent years, numerous online labor markets have emerged that allow workers to sell their labor to a pool of buyers (Horton 2010). For example, Anderson (2017) analyzes a large online freelance labor market (Upwork), where workers with different skills are matched to employers searching for specific skills. Our bipartite network framework can be used to analyze how the interactions of workers with complementary skills in diverse teams create synergies and value for workers and employers.

Network of Committees. Porter et al. (2005) investigate the network of committees in the U.S. House of Representatives, with committees connected according to the common membership of the Representatives. Their model can be reframed as a bipartite network with the Representatives and the committees as two sets of nodes connected by each Representative's contribution to the committee's work. Since the bipartite network framework directly models the individual decision of each Representative instead of the aggregate outcome of a committee, it allows us to give a more intricate examination of the political correlation between Representatives.

Corporate Board Member Network Conyon & Muldoon (2006) study the network structure of boards of directors. They show the importance of the bipartite representation for understanding the "small world" characteristics (Jackson & Rogers 2005) of corporate board member networks. Our model, on the other hand, focuses on the interdependence of board members' input in the firm's operations and can be used to study the impact of board members' synergy on firm performance.

Partners in Crime Billings et al. (2019) examine social spillovers in crime using data on offenders who were arrested for the same crime. The offenders and the criminal incidents can be considered as two sets of nodes in a bipartite network. Using our micro-founded model, one can conduct counterfactual analysis to evaluate the effectiveness of different crime-reduction policies.

3 Estimation

Recall $d_{is} = \mathbb{1}(i \in \mathcal{N}_s)$, where $\mathbb{1}(\cdot)$ denotes an indicator function. Equation (1) can be rewritten as

$$y_s = \sum_{i \in \mathcal{N}} \alpha_i d_{is} e_{is} + \frac{\lambda}{2} \sum_{i \in \mathcal{N}} \sum_{j \in \mathcal{N} \setminus \{i\}} g_{ij} d_{is} d_{js} e_{is} e_{js} + \epsilon_s, \tag{6}$$

where ϵ_s is i.i.d.$(0, \sigma_\epsilon^2)$. In the empirical model, we assume agent i's productivity is given by

$$\alpha_i = \exp(x_i' \beta), \tag{7}$$

where x_i is a vector of observable individual attributes. Equation (7) is assumed to be an exponential function to guarantee that the productivity is positive.

There are three main challenges in estimating this model. First, the effort level e_{is} is usually unobservable to the econometrician. To overcome this problem, we replace e_{is} in Equation (6) with the equilibrium effort level e_{is}^* given by Equation (5) and estimate the equilibrium production function

$$y_s = \sum_{i \in \mathcal{N}} \alpha_i d_{is} e_{is}^* + \frac{\lambda}{2} \sum_{i \in \mathcal{N}} \sum_{j \in \mathcal{N} \setminus \{i\}} g_{ij} d_{is} d_{js} e_{is}^* e_{js}^* + \epsilon_s. \tag{8}$$

Equation (8) is highly nonlinear in the unknown parameters. Thus, it is difficult to derive easy-to-check sufficient conditions for identification as in Bramoullé et al. (2009). To get some intuition on what data variation identifies the complementarity parameter λ and the substitutability parameter ϕ respectively, we consider the following two cases. For simplicity, suppose $g_{ij} = 1$ for all $i \neq j$ and the productivities (α_i) are identical for all i. In the first case, suppose n agents collaborate on a single project. If $\lambda \neq 0$, then the (expected) equilibrium output of the project would change with n. Thus, the complementarity parameter λ can be identified from the output variation with n. In the second case, suppose a single agent works on p projects alone. If $\phi \neq 0$, then the (expected) equilibrium output of each project would change with p. Thus, the substitutability parameter ϕ can be identified from the output variation with p. Therefore, in the empirical application, when the structure of the bipartite network is sufficiently rich, we should be able to identify both complementarity and substitutability effects.

Second, d_{is} is likely to be endogenous. Intuitively, high-ability agents tend to work on more projects at the same time, and high-potential projects are usually harder to find and more challenging to work on. Furthermore, agents tend to be sorted into projects based on their abilities and other unobserved characteristics. Estimating Equation (8) without taking into account the potential endogeneity of d_{is} may incur a selection bias. To control for the endogenous selection, we introduce a participation function allowing for both agent and project unobserved heterogeneity in a similar way as Bonhomme (2020). More specifically, we assume

$$d_{is} = \mathbb{1}(z_{is}'\gamma + \xi\mu_i + \psi\eta_s + \kappa|\mu_i - \eta_s| + v_{is} > 0), \tag{9}$$

where z_{is} is a vector of observables measuring compatibility between agent i and other agents participating in project s, μ_i is an i.i.d.$(0, 1)$ agent-specific random component, η_s is an i.i.d.$(0, 1)$ project-specific random component, and v_{is} is an i.i.d.$(0, 1)$ error term independent of μ_i and η_s.[9] As the number of observations d_{is} is much larger than the number

[9]Our model follows a large literature in labor economics and related fields that analyze the matching of workers to firms (or jobs/projects) based on skills required to apply for a job or geographic restrictions (Petrongolo & Pissarides 2001, Anderson 2017, Bonhomme 2020). This literature has emphasized the role of controlling for unobserved individual and firm (or job/project) effects in the matching process as we do here (similar to e.g. Ahmadpoor & Jones 2019, Zacchia 2020, Bhaskarabhatla et al. 2021). Equation (9) can be seen as a variation of the so-called "beta model" of network formation (Chatterjee et al. 2011), including both node- and dyadic-specific shocks as well as covariates, as highlighted, for example, by Graham (2017) and

of random components μ_i and η_s, it is reasonable to assume that μ_i and η_s can be identified from Equation (9). To allow the agent and project unobserved heterogeneity to also affect production, we assume agent i's productivity depends on the agent-specific random component μ_i so that Equation (7) becomes

$$\alpha_i = \exp(x_i'\beta + \zeta\mu_i),$$

and the error term in Equation (8) can be written as

$$\epsilon_s = \varsigma\eta_s + u_s,$$

where u_s is an i.i.d.$(0, \sigma_u^2)$ error term independent of η_s. This specification has the following implications. First, if $\zeta > 0$ and $\xi > 0$, then an agent with higher ability (given by a higher μ_i) tends to participate in more projects. Second, if $\varsigma > 0$ and $\psi < 0$, then a project with higher potential (given by a higher η_s) has a higher threshold for agents to participate in. Finally, if $\kappa < 0$, then agents are more likely to join projects that match their abilities, i.e., agents are sorted into projects based on homophily of unobserved characteristics.

Third, with the unobserved heterogeneity, the joint likelihood function of production and participation involves high-dimensional integrals and is computationally cumbersome to evaluate. To bypass this difficulty, we follow the Bayesian approach of Zeger & Karim (1991). Let $\theta_d = (\gamma', \xi, \psi, \kappa)'$ and $\theta_y = (\lambda, \phi, \beta', \zeta, \varsigma, \sigma_u^2)'$. Let $f(d|\mu, \eta, \theta_d)$ denote the conditional probability of $d = [d_{is}]$ given $\mu = (\mu_1, \cdots, \mu_n)'$ and $\eta = (\eta_1, \cdots, \eta_p)'$, and $f(y|d, \mu, \eta, \theta_y)$ denote the conditional density of $y = (y_1, \cdots, y_p)'$ given d, μ, and η. Then, μ, η, and $\theta = (\theta_y', \theta_d')'$ can be sampled from the joint posterior density

$$p(\mu, \eta, \theta|y, d) \propto f(y|d, \mu, \eta, \theta_y) f(d|\mu, \eta, \theta_d) \pi(\mu)\pi(\eta)\pi(\theta_y)\pi(\theta_d),$$

Bonhomme (2021). In the empirical study in Section 4, the data indicates every agent only participates in projects within their employing company, suggesting that the agents are likely assigned to projects (informally or more formally by a manager) based on their compatibility (Aggarwal et al. 2020).

with the priors $\pi(\mu)$, $\pi(\eta)$, $\pi(\theta_y)$ and $\pi(\theta_d)$. The details of Bayesian estimation can be found in Appendix B.

4 The Bipartite Network of Patents and Inventors

Collaborations between inventors have been identified to be crucial for facilitating knowledge flows (Singh 2005), innovations and technological breakthroughs (Fleming et al. 2007, Singh & Fleming 2010, Bhaskarabhatla et al. 2021) as well as firm performance (Aggarwal et al. 2020, Almeida & Kogut 1999). To show the empirical relevance of our model, we provide the following illustration based on the collaboration network of inventors on U.S. patents in the semiconductor and pharmaceutical industries.

4.1 Data

In this empirical illustration, we use the dataset compiled by Bhaskarabhatla et al. (2021). The primary data used in their study consists of U.S. patents granted by the United States Patent and Trademark Office (USPTO), which includes information about the inventors' identities and the corresponding firm assignee for each patent.[10] Since USPTO does not provide unique identifiers for inventors and assignees, Bhaskarabhatla et al. (2021) use the procedures outlined in Li et al. (2014) and Hall et al. (2001) to disambiguate inventor names and standardize assignee names. Subsequently, they match the assignees in the USPTO patent data to Compustat data on publicly listed firms using the procedure in Bessen (2009). The Continuation patents are excluded due to their potential to inflate innovation productivity artificially. Each entry in the dataset represents a patent-inventor pairing, containing information on both the patent and the associated inventor(s). For each patent, we know the application number, application year, and granted year. We further merge the information of patent forward citation and patent value (Kogan et al. 2017) as measures of patent

[10] The sample contains all patents granted between 1973 and 2013.

output.[11] For inventors, the dataset includes the inventor's foreigner status, geographical location (at the state level for U.S.-based inventors), affiliated company, and industry classification (NAICS) code. Additionally, we gauge the ability of an inventor by the cumulative total of patent citations (value) garnered from all the granted patents, and the experience of an inventor by the number of years between the first patent application and the current application.

To prepare the data for the estimation, we undertake the following sample selection procedure. Firstly, we narrow our focus to patents filed in two specific industries: Semiconductors and related device manufacturing (with NAICS code 334413) and pharmaceutical preparation manufacturing (with NAICS code 325412). Notably, the semiconductor industry has the highest number of patent applications across all industries, while the pharmaceuticals industry leads in patent applications for non-computer-related sectors. Secondly, we focus on patents applied in 2003, in which the number of patent applications in the semiconductor industry is the highest compared to all other years.[12] Thirdly, we exclude patents with missing information on the application year or associated inventors, and inventors with missing information on their characteristics. Finally, we also drop inventors who only work on one solo-invented patent and the corresponding patents. The summary statistics of the data used for the estimation are reported in Tables 1 and 2. In the semiconductor industry our sample comprises 6,017 patents and 8,472 inventors. On average, each patent involves 2.58 inventors, and each inventor contributes to 1.83 patents. By contrast, the pharmaceuticals industry in our sample exhibits fewer patents (927) and inventors (2,888). Here, each patent is co-invented by 4.29 inventors on average, with each inventor contributing to 1.38 patents. These summary statistics imply that collaborations are more common in the pharmaceutical industry than in the semiconductor industry, and inventors in the pharmaceutical industry

[11]The patent citation and value data can be downloaded from the supplementary GitHub website of Kogan et al. (2017): https://github.com/KPSS2017/Technological-Innovation-Resource-Allocation-and-Growth-Extended-Data

[12]In Appendix E.1, we conduct a robustness check with patents applied in 2001, in which the number of patent applications in the pharmaceutical industry is the highest compared to all other years. The estimation results are qualitatively unchanged.

tend to concentrate their efforts on fewer innovation projects. In the empirical study, we will investigate whether these industries also differ in terms of the complementarity effects among inventors and substitutability effects across patents.

Figures 5 and 6 depict the networks of co-inventors in the semiconductor and pharmaceutical industries, respectively. Different colors indicate different company affiliations. We see that both networks are characterized by closely linked clusters of inventors with the same company affiliation. The extent to which these networks are clustered can be measured with the clustering coefficient, which is defined as the fraction of connected neighbors of a node, averaged across nodes in the network. The average clustering coefficient for the co-inventor network in the semiconductor industry is 0.64 while for the pharmaceutical industry it is 0.86, suggesting that the co-inventor network in the pharmaceutical industry is more clustered than in the semiconductor industry. In Section 4.2, we will analyze how these differences translate to the synergies between collaborating (linked) inventors in the network.

4.2 Estimation Results

In the benchmark empirical model, we assume that the compatibility between inventors is homogeneous, i.e., $g_{ij} = 1$ for $i \neq j$ in Equation (1).[13] Table 3 reports the estimation results of Equations (8) and (9) for semiconductors and pharmaceuticals. We use the logarithm of patent forward citations as the output of the production function.[14,15] Columns (A) and (C) report the estimates of the production function ignoring potential endogenous participation,

[13] In Appendix E.2, we conduct a robustness check with heterogeneous compatibilities between inventors, where g_{ij} is based on the Jaffe (1986) similarity measure between inventors i and j. The estimation results are qualitatively unchanged.

[14] There exists a well-documented positive correlation between the monetary value of a patent and the citations it receives (cf. e.g. Harhoff et al. 1999, Hall et al. 2005, Kogan et al. 2017, Higham et al. 2021). Based on our data sample we find that a one-unit increase in patent output (measured as the log of patent citations) is approximately associated with a \$1 million increase in economic value (in 1982 prices). See Appendix C for further details.

[15] In Appendix E.3, we conduct a robustness check with an alternative measure for the value of a patent introduced in Kogan et al. (2017) as the output of the production function. The estimation results are qualitatively unchanged.

Table 1: Sample statistics for semiconductors.

	Min	Max	Mean	S.D.	Obs.
Patents					
Forward Citations (log)	0.0000	5.7398	1.6047	1.1063	6017
Value (million dollars in 1982 prices)	0.0209	162.0552	7.4947	9.9904	6017
Number of inventors (in each patent)	1	13	2.5784	1.4803	6017
Inventors					
Foreigner	0	1	0.4214	0.4938	8472
Log life-time patent citations	0	9.3284	2.7749	1.7108	8472
Log life-time patent values (millions)	0	6.4146	1.9198	1.0437	8472
Seniority (decades since first granted patent)	0	2.5000	0.2275	0.4088	8472
Number of patents (for each inventor)	1	49	1.8312	1.9784	8472

Notes: Semiconductor and related device manufacturing (NAICS: 334413). This sample is based on patents applied in 2003. We drop inventors with a single solo-invented patent and the corresponding patents. We add one before taking the log of forward citations.

Table 2: Sample statistics for pharmaceuticals.

	Min	Max	Mean	S.D.	Obs.
Patents					
Forward Citations (log)	0.0000	5.1761	1.3639	1.2304	927
Value (million dollars in 1982 prices)	0.0100	563.4050	51.2777	60.5520	927
Number of inventors (in each patent)	1	24	4.2945	2.9925	927
Inventors					
Foreigner	0	1	0.2753	0.4467	2888
Log life-time patent citations	0	8.3156	2.4375	1.7482	2888
Log life-time patent values (millions)	0	6.8764	3.4198	1.2467	2888
Seniority (decades since first granted patent)	0	2.5000	0.2617	0.4495	2888
Number of patents (for each inventor)	1	18	1.3785	0.9459	2888

Notes: Pharmaceutical preparation manufacturing (NAICS: 325412). This sample is based on patents applied in 2003. We drop inventors with a single solo-invented patent and the corresponding patents. We add one before taking the log of forward citations.

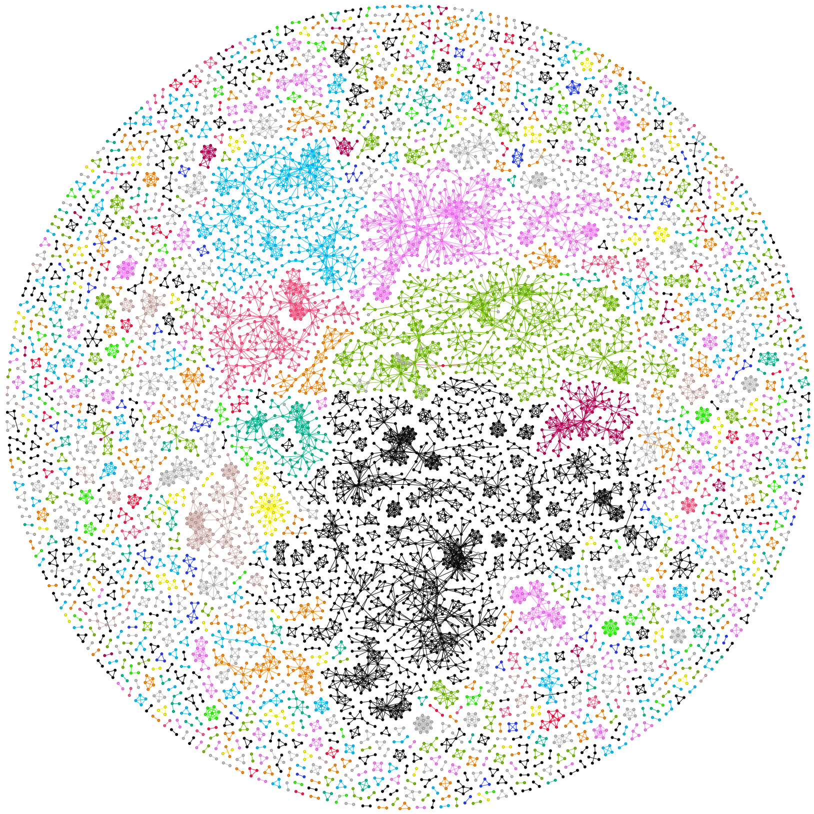


Figure 5: The network of co-inventors in the semiconductor industry (NAICS code 334413). Only inventors with at least one collaboration are shown. Different colors indicate different company affiliations. The network consists of 8,472 nodes and 14,617 links. The average degree is 3.52. The average clustering coefficient (i.e., the average fraction of connected neighbors of a node) is 0.64.

and Columns (B) and (D) report the joint estimates of the production and participation functions with both inventor- and patent-specific random components.[16]

For semiconductors the estimated complementarity effect (λ) and substitutability effect (ϕ) are both statistically significant when endogenous participation is controlled for. However, both effects are downwards biased when endogenous participation is ignored. A possible explanation for the downward bias is as follows. The estimated coefficients (ζ and ξ) of the inventor-specific random component suggest that high-ability inventors tend to participate in more projects. Therefore, ignoring endogenous project participation tends to

[16] The trace plot of the MCMC draws and the Geweke convergence test for the complementarity effect (λ) and the substitutability effect (ϕ) in Columns (B) and (D) of Table 3 are provided in Appendix Figure D.1. The test results suggest the Markov chains are converged.

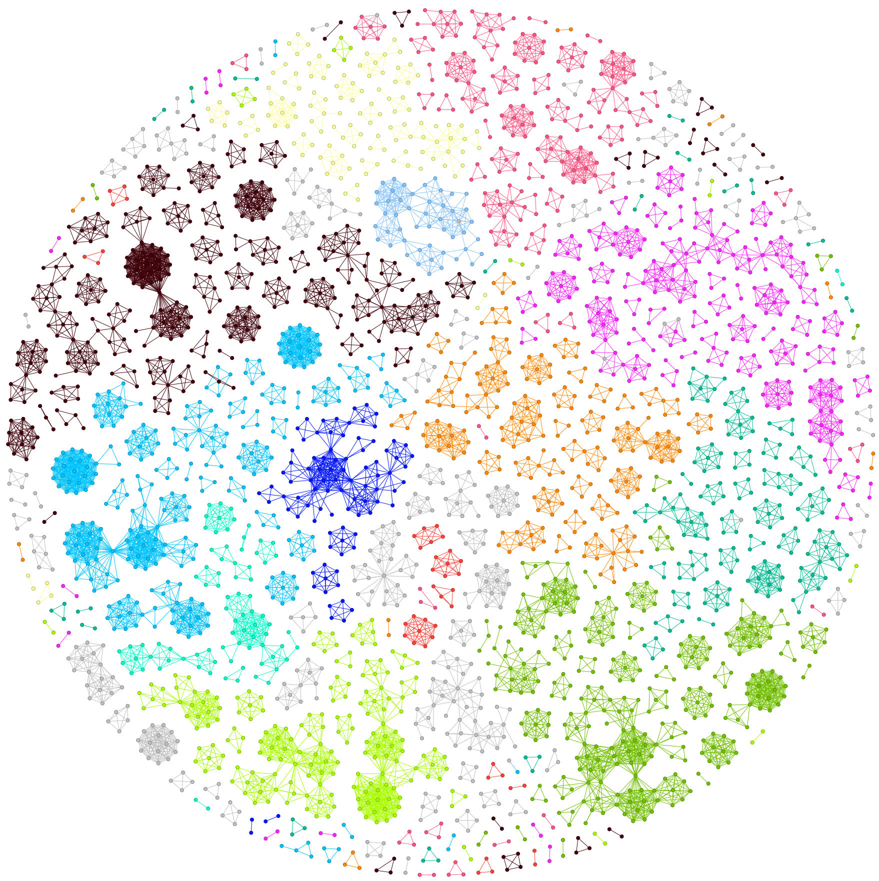


Figure 6: The network of co-inventors in the pharmaceutical industry (NAICS code 325412). Only inventors with at least one collaboration are shown. Different colors indicate different company affiliations. The network consists of 2,888 nodes and 9,164 links. The average degree is 6.36. The average clustering coefficient (i.e., the average fraction of connected neighbors of a node) is 0.86.

Table 3: Estimation results for semiconductors and pharmaceuticals using patent forward citations.

		Semiconductors		Pharmaceuticals	
		(A) Exogenous Participation	(B) Endogenous Participation	(C) Exogenous Participation	(D) Endogenous Participation
Production					
Complementarity	(λ)	0.1538***	0.1677***	0.2782***	0.3192***
		(0.0233)	(0.0245)	(0.0663)	(0.0751)
Substitutability	(ϕ)	0.0485***	0.0502***	0.1663***	0.1871***
		(0.0062)	(0.0062)	(0.0204)	(0.0348)
Constant	(β_0)	-0.2110***	-0.2167***	-0.3956***	-0.4208***
		(0.0224)	(0.0243)	(0.0610)	(0.0670)
Foreigner	(β_1)	-0.0494***	-0.0499***	-0.0726**	-0.0711*
		(0.0139)	(0.0138)	(0.0365)	(0.0362)
Log accu. citations	(β_2)	0.1559***	0.1569***	0.2212***	0.2237***
		(0.0060)	(0.0062)	(0.0099)	(0.0106)
Seniority	(β_3)	-0.3823***	-0.3890***	-0.7042***	-0.7191***
		(0.0339)	(0.0327)	(0.1102)	(0.1120)
Inventor effect	(ζ)	–	0.0106***	–	0.0289*
			(0.0027)		(0.0169)
Patent effect	(ς)	–	0.0129***	–	0.0283
			(0.0027)		(0.0170)
Error term variance	(σ_ϵ^2)	0.8867***	–	0.7174***	–
		(0.0162)		(0.0338)	
Error term variance	(σ_u^2)	–	0.8664***	–	0.7170***
			(0.0162)		(0.0337)
Participation					
Constant	(γ_0)	–	-5.1908***	–	-5.0033***
			(0.0455)		(0.0884)
Location	(γ_1)	–	1.5446***	–	5.2940***
			(0.0674)		(0.2421)
Past coauthors	(γ_2)	–	8.4770***	–	7.2150***
			(0.1589)		(0.3919)
Common co-authors	(γ_3)	–	15.1374***	–	12.8617***
			(0.1659)		(0.3785)
Inventor effect	(ξ)	–	1.4049***	–	1.1192***
			(0.0647)		(0.0767)
Patent effect	(ψ)	–	-2.8966***	–	-2.6903***
			(0.0432)		(0.0960)
Homophily effect	(κ)	–	-3.6539***	–	-1.6792***
			(0.0669)		(0.0827)
Sample size					
Patents		6,017		927	
Inventors		8,472		2,888	

Notes: Columns (A) and (B) show estimates for firms in the semiconductor and related device manufacturing industry (NAICS code: 334413). Columns (C) and (D) show estimates for firms in the pharmaceutical preparation manufacturing industry (NAICS code: 325412). The output of the production function is measured by the logarithm of patent forward citations. In Columns (A) and (C) we estimate the production function ignoring endogenous project participation. In Columns (B) and (D) we jointly estimate the production and participation functions with both inventor and patent random effects. We implement MCMC sampling for 25,000 iterations, leaving the first 5,000 draws for burn-in and using the rest of the draws for computing the posterior mean (as the point estimate) and the posterior standard deviation (in the parenthesis). The asterisks ***(**,*) indicate that the 99% (95%, 90%) highest posterior density range does not cover zero.

underestimate the substitutability effect because it fails to take into account that inventors simultaneously working on multiple projects are more likely to be high-ability ones. On the other hand, the estimated coefficients (ς and ψ) of the project-specific random component suggest that high-potential projects are harder to find and hold a higher threshold for inventors to participate in. The estimated coefficient κ suggests inventors are matched to projects based on homophily of unobserved characteristics. Since high-potential projects are scarce and inventors are sorted into projects according to their compatibility, most inventors in our data are collaborating on projects with relatively low potential. Therefore, the complementarity effect tends to be underestimated when endogenous participation is not taken into account.[17]

Regarding the effect of inventor characteristics on research output, we find that the cumulative total of citations garnered from all of the granted patents is a positive and significant predictor of the inventor's productivity (cf. e.g., Singh & Fleming 2010, Ductor 2015). On the other hand, seniority measured by the number of decades since the first patent application has a negative partial effect on research output. This finding mirrors Ductor (2015), who shows that career time has a negative impact on productivity, and it is consistent with the scientists' life-cycle effects documented in Levin & Stephan (1991).

Finally, from the estimation of the participation equation, we find that being in the same location, being collaborators in the past, and sharing common collaborators in the past all make collaboration more likely (cf. Freeman & Huang 2015).

For pharmaceuticals, the estimation results show a similar pattern. In particular, both the estimated complementarity and substitutability effects are statistically significant when endogenous participation is controlled for, and downwards biased when endogenous participation is ignored. In terms of magnitudes, we find that the estimated complementarity and substitutability effects are higher in the pharmaceutical industry relative to the semiconductor industry. This difference can be interpreted in the context of the existing empirical

[17] In Appendix F, we conduct Monte Carlo simulation experiments with different signs of ξ and ψ, and the pattern of estimation bias is consistent with the above explanation.

literature. For example, Pammolli et al. (2011) document a sharp decline in innovations generated per dollar of pharmaceutical research. Bloom et al. (2020) document that research productivity is declining across a variety of industries. However, the semiconductor industry is the industry with the least degree of diminishing returns in idea production. This indicates that successful innovation might be more difficult in the pharmaceutical industry than in the semiconductor industry. Jones (2009) argues that when innovation becomes more difficult, inventors tend to increase their degree of specialization and form more collaborations to compensate. Interpreting these findings in terms of our model, a higher difficulty of innovation could lead to higher complementarity (collaborations) and substitutability (specialization) effects, and this could be related to the higher magnitudes of the estimated effects for the pharmaceutical over the semiconductor industry.

4.3 Marginal Effects

From Equation (5), the marginal effect of the kth covariate of agent i on the equilibrium effort is given by

$$\frac{\partial e^*}{\partial x_{ik}} = (I_{np} - L)^{-1} D(\delta \otimes \frac{\partial \alpha}{\partial x_{ik}}),$$

where $\partial \alpha / \partial x_{ik}$ is an $n \times 1$ vector with the ith element being $\partial \alpha_i / \partial x_{ik} = \exp(x_i' \beta)\beta_k$ and other elements being 0.[18] As the agents are connected through the bipartite network, the change in an agent's covariate affects not only his/her own equilibrium effort but also the equilibrium efforts of other agents in the network. The former is known as the *direct* marginal effect, while the latter is known as the *indirect* marginal effect. In Tables 4 and 5, we report the average marginal effect (AME) of each covariate by first calculating the marginal effect for each individual and then taking an average across all individuals. For the kth covariate, the direct AME is given by

$$\frac{1}{n} \sum_{i \in \mathcal{N}} \sum_{s \in \mathcal{P}_i} \frac{\partial e^*_{is}}{\partial x_{ik}},$$

[18]The covariate x_{ik} is taken to be a continuous variable. If x_{ik} is a binary variable, then the marginal effect is given by $e^*(x_{ik} = 1) - e^*(x_{ik} = 0)$.

the indirect AME is given by

$$\frac{1}{n}\sum_{i\in\mathcal{N}}\sum_{j\neq i, j\in\mathcal{N},}\sum_{s\in\mathcal{P}_j}\frac{\partial e_{js}^*}{\partial x_{ik}},$$

and the total AME is given by

$$\frac{1}{n}\sum_{i\in\mathcal{N}}\sum_{j\in\mathcal{N},}\sum_{s\in\mathcal{P}_j}\frac{\partial e_{js}^*}{\partial x_{ik}}.$$

The benchmark marginal effects reported in the first columns of Tables 4 and 5 are calculated based on the estimates given in Columns (B) and (D) of Tables 3, respectively. To gain a deeper understanding of the magnitudes of the estimated complementarity and substitutability effects, we also calculate the marginal effects under the restrictions $\lambda = 0$, $\phi = 0$, and $\lambda = \phi = 0$ respectively. We find that ignoring the complementarity effect (i.e., λ is set to 0) leads to an attenuation bias (i.e., a bias towards zero) in the estimated marginal effects while ignoring the substitutability effect (i.e., ϕ is set to 0) leads to an exaggeration bias (i.e., a bias away from zero) in the estimated marginal effects. In the semiconductor industry, when both effects are ignored (i.e., both λ and ϕ are set to 0), the direction of the bias in the estimated total AME is the same as the case with ϕ set to 0, suggesting that the substitutability effect ϕ dominates the complementarity effect λ. By contrast, in the pharmaceutical industry, when both λ and ϕ are set to 0, the direction of the bias in the estimated total AME is the same as the case with λ set to 0, indicating that the complementarity effect dominates the substitutability effect. In Section 4.2, we found that the estimated complementarity effect is higher in the pharmaceutical industry. This pattern is corroborated in the current section where we further find that in the pharmaceutical industry the spillover effect dominates in the marginal effect analysis while in the semiconductor industry the substitution effect dominates. As discussed in Section 4.2, this is likely due to a higher difficulty of innovation in the pharmaceutical industry (Pammolli et al. 2011, Bloom et al. 2020), leading to higher synergy effects (complementarity) between team members in the pharmaceutical industry to compensate for the greater difficulty in discovering new ideas (Jones 2009).

Table 4: Marginal effects of inventor characteristics on efforts for semiconductors.

	Benchmark	$\lambda = 0$	$\phi = 0$	$\lambda = \phi = 0$
Direct AME				
Foreigner	-0.0398	-0.0396	-0.0455	-0.0453
Log accu. citations	0.1245	0.1240	0.1426	0.1419
Seniority	-0.3076	-0.3063	-0.3523	-0.3505
Indirect AME				
Foreigner	-0.0029	0.0000	-0.0038	0.0000
Log accu. citations	0.0092	0.0000	0.0119	0.0000
Seniority	-0.0226	0.0000	-0.0293	0.0000
Total AME				
Foreigner	-0.0427	-0.0396	-0.0493	-0.0453
Log accu. citations	0.1337	0.1240	0.1545	0.1419
Seniority	-0.3303	-0.3063	-0.3816	-0.3505

Notes: The marginal effects are calculated based on the estimates reported in Column (B) of Table 3.

Table 5: Marginal effects of inventor characteristics on efforts for pharmaceuticals

	Benchmark	$\lambda = 0$	$\phi = 0$	$\lambda = \phi = 0$
Direct AME				
Foreigner	-0.0206	-0.0202	-0.0248	-0.0243
Log accu. citations	0.0665	0.0652	0.0802	0.0785
Seniority	-0.2134	-0.2094	-0.2576	-0.2522
Indirect AME				
Foreigner	-0.0044	0.0000	-0.0060	0.0000
Log accu. citations	0.0141	0.0000	0.0196	0.0000
Seniority	-0.0449	0.0000	-0.0622	0.0000
Total AME				
Foreigner	-0.0249	-0.0202	-0.0308	-0.0243
Log accu. citations	0.0806	0.0652	0.0998	0.0785
Seniority	-0.2583	-0.2094	-0.3188	-0.2522

Notes: The marginal effects are calculated based on the estimates reported in Column (D) of Table 3.

4.4 Counterfactual Study

To illustrate the importance of accounting for the complementarity and substitutability effects in policy design and evaluation, we use our model to design an incentive program to promote innovations. Under this policy, we assume every inventor receives a reward, $r \in \mathbb{R}_+$, per unit of the output she generates.[19] Then, the utility function (2) of inventor i can be extended to

$$U_i(\mathcal{G}, r) = \sum_{s \in \mathcal{P}_i} (1+r)\delta_s y_s - \frac{1}{2}\left(\sum_{s \in \mathcal{P}_i} e_{is}^2 + \phi \sum_{s \in \mathcal{P}_i} \sum_{t \in \mathcal{P}_i \setminus \{s\}} e_{is} e_{it}\right). \tag{10}$$

Let $L(r) := L(r; \lambda, \phi) = \lambda(1+r)W - \phi M$. Following a similar argument as in the proof of Proposition 1, we can show that, if $\rho_{\max}[L(r)] < 1$, then the equilibrium effort portfolio is given by

$$e^*(r) = (1+r)[I_{np} - L(r)]^{-1} D(\delta \otimes \alpha). \tag{11}$$

It is worth pointing out that if the complementarity effect is ignored (i.e., $\lambda = 0$), then $L(r) = -\phi M$, which does not depend on r. In this case, the reward r only increases the equilibrium effort

$$e^*(r) = (1+r)(I_{np} + \phi M)^{-1} D(\delta \otimes \alpha)$$

by a factor of $(1+r)$. As the output is linear in $e^*(r)$ with $\lambda = 0$ in Equation (1), the impact of the inventive program on output is one-to-one. Intuitively, when $\lambda = 0$, the multiplier effect of the bipartite network is wiped out, and hence the impact of the incentive program is likely to be understated. On the other hand, if the substitutability effect is ignored (i.e., $\phi = 0$), then the cost of effort is understated and thus the impact of the incentive program on equilibrium effort is overstated. As a result, the impact of the incentive program on output

[19] Indeed, many firms, universities, and research funding institutions give awards or provide monetary incentives to promote high-quality research outputs. For example, Yuret (2017) documents subsidies for publications provided by the Scientific and Technological Research Council of Turkey. Researchers freely use these publication subsidies as pocket money, and, as in our model, the publication subsidy given to a researcher for an article is inversely proportional to the number of authors of the article.

tends to be overstated as well.

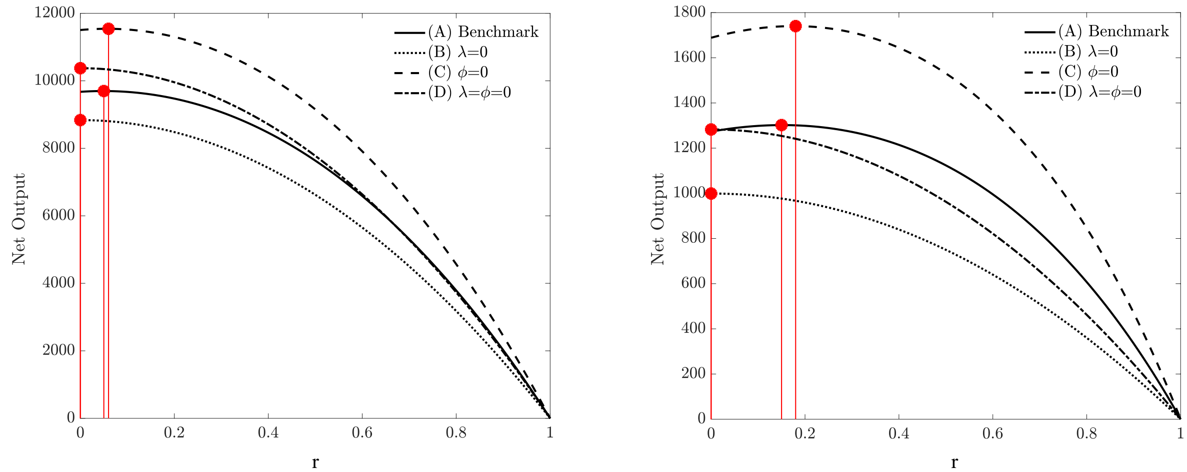
For a given reward rate r, the net total output can be computed as the total output, $\sum_{i \in \mathcal{N}} \sum_{s \in \mathcal{P}_i} \delta_s y_s(\mathcal{G}, r)$, minus the cost of the program, $\sum_{i \in \mathcal{N}} \sum_{s \in \mathcal{P}_i} r \delta_s y_s(\mathcal{G}, r)$. In Figure 7, net total output is shown as a function of the reward rate r varying from 0 to 1. Panel (a) depicts the case of the semiconductor industry, and Panel (b) depicts the case of the pharmaceutical industry. The solid curves (benchmark) are based on the estimates reported in Columns (B) and (D) of Table 3, respectively. The dashed curves correspond to the case that the complementarity effect is ignored (i.e., λ is set to 0). In this case, the net total output is understated (i.e., lower than the benchmark) at any rate between 0 and 1. The dotted curves correspond to the case that the substitutability effect is ignored (i.e., ϕ is set to 0). In this case, the net total output is overstated. When both effects are ignored (i.e., both λ and ϕ are set to 0), the net total output is depicted by the dash-dotted curve. In this case, net total output is overstated for the semiconductor industry but understated for the pharmaceutical industry, which is consistent with what we observe in the marginal effect analysis in Section 4.3. Hence, correctly estimating these two effects is crucial for policy design and evaluation.

Given the equilibrium effort portfolio $e^*(r)$ in Equation (11), we can derive the optimal reward rate r^* that maximizes the net total output. It is given by

$$r^* = \underset{r \in \mathbb{R}_+}{\operatorname{argmax}} \sum_{i \in \mathcal{N}} \sum_{s \in \mathcal{P}_i} (1 - r) \delta_s y_s(\mathcal{G}, r), \tag{12}$$

where $y_s(\mathcal{G}, r)$ is the output of project s with the equilibrium effort levels $e^*(r)$ given a rate of r.[20] In Figure 7, the maximum net total output is indicated by a vertical line which pinpoints the optimal rate r^*. The optimal rate is higher in the pharmaceutical industry (with an optimal rate of 0.1518) than in the semiconductor industry (with an optimal subsidy rate of 0.0490). The corresponding percentage increase of total output, which measures

[20]Given the parameter estimates and the data, Equation (12) can be solved numerically using standard optimization algorithms.



(a) Semiconductors (b) Pharmaceuticals

Figure 7: The net total output in the presence of a merit-based reward, $\sum_{i\in\mathcal{N}}\sum_{s\in\mathcal{P}_i}(1-r)\delta_s y_s(\mathcal{G},r)$, for the semiconductor industry and the pharmaceutical industry. The maximum at the optimal rate, r^*, is highlighted with vertical lines for different model specifications.

the log number of forward citations,[14] under the optimal rate is 4.97% for semiconductors and 14.22% for pharmaceuticals. This means that the increase in total output in the pharmaceutical industry is 2.8612 times higher than the increase in the semiconductor industry. The higher rate and its effect on total output reflect a stronger complementarity effect in the pharmaceutical industry that we have documented in Sections 4.2 and 4.3, respectively. The innovation incentive program, with the optimal reward rate r^*, internalizes the externality due to the complementarity effect in the effort decision of the inventors (cf. Equation (11)). The higher the complementarity effect, the more effective the program. On the other hand, when the complementarity effect is presumed to be absent, the innovation incentive program loses its purpose and the corresponding optimal reward rate is zero (cf. Figure 7).

The optimal reward rate, r^*, in Equation (12) can also be computed for each firm separately.[21] In this scenario, we assume that each firm implements its own innovation incentive program. Based on the optimal rates for every firm, we can then identify the firms in the

[21]Let $\mathcal{F}_i \subset \mathcal{N}$ denote the set of inventors affiliated with firm i. Then the optimal rate chosen by firm i is given by $r_i^* = \underset{r\in\mathbb{R}_+}{\operatorname{argmax}} \sum_{j\in\mathcal{F}_i}\sum_{s\in\mathcal{P}_j}(1-r)\delta_s y_s(\mathcal{G},r)$, where $y_s(\mathcal{G},r)$ is the output of project s with the equilibrium effort levels $e^*(r)$ given a rate of r from Equation (11).

semiconductor and pharmaceutical industries that stand to gain the most (in terms of research output) from this incentive program. Tables 6 and 7 list the top 10 firms in each industry, ranked by the difference in output before and after implementing the incentive program, displayed in the second column of the table.[22] In the semiconductor industry, Intel, a U.S.-based firm, occupies the top rank with an increase of 135.74 in patent output. The two following companies are Texas Instruments, another U.S.-based firm, with a gain of 58.50, and Infineon Technologies, a German semiconductor manufacturer, with a gain of 57.65. In the pharmaceutical industry, the top three firms benefiting the most from the program are all based in the U.S.: Bristol-Myers Squibb, Eli Lilly, and Wyeth, having gains of 31.25, 26.72, and 22.25 in the output, respectively. The optimal rates, as indicated in the sixth column, differ across firms. In Table 6, the optimal rate ranges from 0.029 to 0.057 for semiconductors, while in Table 7, it ranges between 0.118 and 0.191 for pharmaceuticals. In line with our previous findings, we consistently observe that the optimal rate is greater in the pharmaceutical industry compared to the semiconductor industry. The return on the innovation incentive program (RoIIP), as shown in the seventh column, is always higher than one for every firm. This means that the incentive program generates more benefits to the firm than costs. The highest-ranked firms are generally industry giants, having applied for the most patents, employing the most inventors, and having higher productivity. However, it must be noted that the ranking of firms would be different when using alternative measures (such as productivity) that do not take network externalities into account as we do here.

From a fixed-effect regression of the patent market value on the log number of forward citations, we find that a one-unit increase in the log number of forward citations (which is the measure of patent output in 3) in our sample is approximately associated with a \$1 million increase in economic value (in 1982 prices).[23] We can use this relationship to give a rough

[22]It's worth pointing out that the rankings in Tables 6 and 7 would vary if we were to consider the percentage of the total output (as shown in the third column) or the return on the program (as indicated in the seventh column) instead.

[23]See Footnote 14.

estimate of the economic gain from the innovation incentive program for the firm. Based on the reported increase in output per patent (at the fourth column) in Table 6, Intel, Texas Instruments, and Infineon Technologies in the semiconductor industry can gain \$92,000, \$85,000, and \$70,000, respectively, per patent from the incentive program. In contrast, from Table 7 we find that top pharmaceutical firms like Bristol-Myers Squibb, Eli Lilly, and Wyeth could gain even more, with potential gains of \$303,000, \$267,000, and \$297,000, respectively, per patent. This shows that the innovation incentive program that we analyze here can potentially yield considerable economic gains for a firm.

5 Conclusion

In this paper, we analyze the equilibrium efforts of agents who seek to maximize their utility when involved in multiple, possibly overlapping projects in a bipartite network. We show that both the complementarity effect between collaborating researchers and the substitutability effect between concurrent projects of the same researcher play an important role in determining the equilibrium effort level. To estimate the structural parameters of the model, we propose a Bayesian MCMC procedure that accounts for the endogenous selection of researchers into research projects. We then bring our model to the data by analyzing the collaboration network of patent inventors in the semiconductor and pharmaceutical industries and find empirical evidence for both complementarity and substitutability effects.

As our model has an explicit micro-foundation, it provides a formal framework for counterfactual analysis. To illustrate the importance of correctly estimating the structural model in policy evaluation, we conduct a counterfactual analysis of the impact of the innovation incentive program on research output. We find that the effectiveness of innovation incentives tends to be underestimated when the complementarity is ignored and overestimated when the substitutability is ignored. We also derive the optimal incentive scheme and show that economic gains of innovation incentives can be substantial for a firm.

The proposed bipartite network model provides a general framework to analyze complex interactions within diverse systems. By delineating the relationships between two distinct sets of nodes, the model offers a versatile structure that can be applied across various domains. Besides the direct applications of our model listed in Section 2.4, we believe that, with some modifications, this framework can also be used to analyze competition between multi-product firms (cf. Bimpikis et al. 2019, with firms and product markets depicted as two distinct sets of nodes), formation of syndicated loans (cf. Berlin et al. 2020, with financial institutes and borrowing firms depicted as two distinct sets of nodes), and spillovers from science to innovations (cf. Arora et al. 2021, with scientific publications and corporate patents depicted as two distinct sets of nodes).

Table 6: Top 10 firms in the semiconductor industry ranked by the impact of the incentive program on output.

Firm	Output Change[a] $\Delta Y_i(\mathcal{G}, r_i^*)$	Relative Output Change (%)[b] $\Delta Y_i(\mathcal{G}, r_i^*)/Y_i(\mathcal{G}, 0)$	Output Change per Patent[c] $\Delta Y_i(\mathcal{G}, r_i^*)/\lvert\mathcal{P}_i\rvert$	Output per Patent[d] $Y_i(\mathcal{G}, r_i^*)/\lvert\mathcal{P}_i\rvert$	Opt. Rate r_i^*	RoIIP_i^e $\Delta Y_i(\mathcal{G}, r_i^*)/$ $(r_i^* Y_i(\mathcal{G}, r_i^*))$	Patents	Inventors	Prod.[f]	R&D[g] Int.	Rank
Intel	135.742	5.400	0.092	1.790	0.049	1.046	1480	2325	378.181	0.145	1
Texas Instruments	58.499	5.730	0.085	1.577	0.052	1.049	685	1100	287.931	0.178	2
Infineon Technologies	57.650	5.440	0.070	1.362	0.049	1.047	820	1282	221.836	0.187	3
Taiwan Semiconductor	44.962	6.320	0.102	1.716	0.057	1.053	441	894	371.732	0.063	4
STMicroelectronics	34.877	5.740	0.080	1.466	0.052	1.049	438	764	158.381	0.171	5
Qualcomm	33.259	5.690	0.117	2.176	0.051	1.048	284	407	536.572	0.132	6
Micron Technology	25.906	3.040	0.057	1.915	0.029	1.028	458	463	186.223	0.212	7
Altera	18.175	6.010	0.114	2.004	0.054	1.051	160	262	414.640	0.216	8
Nvidia	15.936	5.990	0.120	2.120	0.054	1.051	133	190	998.874	0.150	9
National Semiconductor	13.991	5.050	0.075	1.563	0.046	1.044	186	266	204.443	0.178	10

[a] The difference in the output of a firm i with and without the incentive program, $\Delta Y_i(\mathcal{G}, r_i^*) = Y_i(\mathcal{G}, r_i^*) - Y_i(\mathcal{G}, 0)$, where $Y_i(\mathcal{G}, r) = \sum_{j \in \mathcal{F}_i} \sum_{s \in \mathcal{P}_j} \delta_s y_s(\mathcal{G}, r)$ is the output of firm i with the reward rate r, $\mathcal{F}_i \subset \mathcal{N}$ denotes the set of inventors affiliated with firm i, $y_s(\mathcal{G}, r)$ is the output of project s with the equilibrium effort levels $e^*(r)$ from Equation (11).

[b] The output difference, $\Delta Y_i(\mathcal{G}, r_i^*)$, divided by the total output before implementing the incentive program, $Y_i(\mathcal{G}, 0)$.

[c] The difference in firm output, $\Delta Y_i(\mathcal{G}, r_i^*)$, divided by the number of patents, $|\mathcal{P}_i|$, where $\mathcal{P}_i \subset \mathcal{P}$ denotes the set of patents of firm i, and $|\cdot|$ denotes its cardinality.

[d] The output of firm i, $Y_i(\mathcal{G}, r_i^*)$, divided by the number of patents, $|\mathcal{P}_i|$.

[e] Return on the innovation incentive program (RoIIP) is calculated as the output difference divided by the program cost: $\text{RoIIP}_i = \Delta Y_i(\mathcal{G}, r_i^*)/\left(r_i^* Y_i(\mathcal{G}, r_i^*)\right)$.

[f] Labor productivity is calculated as sales (in million dollars) divided by the number of employees (in thousands).

[g] R&D intensity is calculated as R&D expenditures (in million dollars) divided by total sales (in million dollars).

Table 7: Top 10 firms in the pharmaceutical industry ranked by the impact of the incentive program on output.

Firm	Output Change[a] $\Delta Y_i(\mathcal{G}, r_i^*)$	Relative Output Change (%)[b] $\Delta Y_i(\mathcal{G}, r_i^*)/Y_i(\mathcal{G}, 0)$	Output Change per Patent[c] $\Delta Y_i(\mathcal{G}, r_i^*)/\lvert\mathcal{P}_i\rvert$	Output per Patent[d] $Y_i(\mathcal{G}, r_i^*)/\lvert\mathcal{P}_i\rvert$	Opt. Rate r_i^*	RoIIP_i^e $\Delta Y_i(\mathcal{G}, r_i^*)/(r_i^* Y_i(\mathcal{G}, r_i^*))$	Patents	Inventors	Prod.[f]	R&D[g] Int.	Rank
Bristol-Myers Squibb	31.25	23.130	0.303	1.615	0.166	1.132	103	437	474.864	0.109	1
Eli Lilly	26.72	14.950	0.267	2.054	0.118	1.099	100	337	272.939	0.187	2
Wyeth	22.25	23.740	0.297	1.546	0.169	1.136	75	301	302.580	0.132	3
Novartis	20.71	20.820	0.280	1.624	0.153	1.126	74	279	316.574	0.151	4
Abbott Laboratories	20.71	25.010	0.351	1.754	0.176	1.139	59	309	272.656	0.093	5
Pfizer	20.30	20.990	0.214	1.232	0.154	1.125	95	313	370.393	0.270	6
Johnson & Johnson	18.70	14.860	0.302	2.331	0.118	1.100	62	138	378.499	0.134	7
Schering-Plough	16.52	28.200	0.330	1.502	0.191	1.150	50	220	273.246	0.176	8
Astrazeneca	15.11	15.350	0.153	1.147	0.121	1.102	99	253	307.738	0.181	9
Novo Nordisk	10.83	21.260	0.451	2.573	0.156	1.125	24	89	249.338	0.152	10

[a] The difference in the output of a firm i with and without the incentive program, $\Delta Y_i(\mathcal{G}, r_i^*) = Y_i(\mathcal{G}, r_i^*) - Y_i(\mathcal{G}, 0)$, where $Y_i(\mathcal{G}, r) = \sum_{j \in \mathcal{F}_i} \sum_{s \in \mathcal{P}_j} \delta_s y_s(\mathcal{G}, r)$ is the output of firm i with the reward rate r, $\mathcal{F}_i \subset \mathcal{N}$ denotes the set of inventors affiliated with firm i, $y_s(\mathcal{G}, r)$ is the output of project s with the equilibrium effort levels $e^*(r)$ from Equation (11).

[b] The output difference, $\Delta Y_i(\mathcal{G}, r_i^*)$, divided by the total output before implementing the incentive program, $Y_i(\mathcal{G}, 0)$.

[c] The difference in firm output, $\Delta Y_i(\mathcal{G}, r_i^*)$, divided by the number of patents, $|\mathcal{P}_i|$, where $\mathcal{P}_i \subset \mathcal{P}$ denotes the set of patents of firm i, and $|\cdot|$ denotes its cardinality.

[d] The output of firm i, $Y_i(\mathcal{G}, r_i^*)$, divided by the number of patents, $|\mathcal{P}_i|$.

[e] Return on the innovation incentive program (RoIIP) is calculated as the output difference divided by the program cost: $\text{RoIIP}_i = \Delta Y_i(\mathcal{G}, r_i^*)/\left(r_i^* Y_i(\mathcal{G}, r_i^*)\right)$.

[f] Labor productivity is calculated as sales (in million dollars) divided by the number of employees (in thousands).

[g] R&D intensity is calculated as R&D expenditures (in million dollars) divided by total sales (in million dollars).

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For Online Publication: Appendices for "Collaboration in Bipartite Networks"

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A Proof of Proposition 1

Proof of Proposition 1. Let d_{is} be an indicator variable such that $d_{is} = 1$ if agent i participates in project s and $d_{is} = 0$ otherwise. Substitution of Equation (1) into Equation (2) gives

$$\begin{aligned} U_i(\mathcal{G}) &= \sum_{s\in\mathcal{P}} d_{is}\delta_s \left(\sum_{j\in\mathcal{N}} \alpha_j d_{js} e_{js} + \frac{\lambda}{2} \sum_{j\in\mathcal{N}} \sum_{k\in\mathcal{N}\backslash\{j\}} g_{jk} d_{js} d_{ks} e_{js} e_{ks} + \epsilon_s \right) \\ &\quad - \frac{1}{2} \left(\sum_{s\in\mathcal{P}} d_{is} e_{is}^2 + \phi \sum_{s\in\mathcal{P}} \sum_{t\in\mathcal{P}\backslash\{s\}} d_{is} d_{it} e_{is} e_{it} \right). \end{aligned} \tag{13}$$

First, note that the marginal utility has to be non-positive at equilibrium, i.e.,

$$\frac{\partial U_i(\mathcal{G})}{\partial e_{is}}|_{e^*} = d_{is} \left(\delta_s \alpha_i + \lambda \delta_s \sum_{j\in\mathcal{N}\backslash\{i\}} g_{ij} d_{js} e_{js}^* - e_{is}^* - \phi \sum_{t\in\mathcal{P}\backslash\{s\}} d_{it} e_{it}^* \right) \leq 0,$$

where the inequality is strict only if $e_{is}^* = 0$ at equilibrium (corner solution). This set of inequalities can be written in matrix form as

$$-D(\delta \otimes \alpha) + (I_{np} - L)e^* \geq 0. \tag{14}$$

Second, if $e_{is}^* > 0$ at equilibrium, then $\frac{\partial U_i(\mathcal{G})}{\partial e_{is}}|_{e^*} = 0$, which implies

$$e^{*\prime}[-D(\delta \otimes \alpha) + (I_{np} - L)e^*] = 0. \tag{15}$$

Finally, the equilibrium effort has to be non-negative, i.e.,

$$e^* \geq 0. \tag{16}$$

Conditions (14), (15), and (16) constitute a linear complementarity problem (Samelson et al. 1958). If $\rho_{\max}(L) < 1$, the matrix $I_{np} - L$ is positive definite. It follows by Lemmas 2 and 3 in Bimpikis et al. (2019) that the unique equilibrium is given by the solution to the linear complementarity problem and the inactive links ($d_{is} = 0$) are strategically redundant and play no role in determining the equilibrium. Hence, it follows by a similar argument as in the proof of Theorem 1 in Bimpikis et al. (2019) that the game has a unique equilibrium with the equilibrium effort levels are given by Equation (5). ∎

B Bayesian Estimation

Since the likelihood function based on Equations (8) and (9) involves high-dimensional integrals, it is computationally cumbersome to apply a frequentist maximum likelihood method even when resorting to a simulation approach. As an alternative estimation method, the Bayesian Markov Chain Monte Carlo (MCMC) approach can be more efficient for estimating latent variable models (cf. Zeger & Karim 1991). We divide the parameter vector θ and unknown latent variables into blocks and assign the prior distributions as follows:

$$
\begin{aligned}
\lambda &\sim \mathcal{N}(0, \sigma_\lambda^2), \\
\phi &\sim \mathcal{N}(0, \sigma_\phi^2), \\
\beta &\sim \mathcal{N}(0, \Sigma_\beta), \\
\zeta &\sim \mathcal{N}(0, \sigma_\zeta^2), \\
\varsigma &\sim \mathcal{N}(0, \sigma_\varsigma^2), \\
\gamma &\sim \mathcal{N}(0, \Sigma_\gamma), \\
\xi &\sim \mathcal{N}(0, \sigma_\xi^2), \\
\psi &\sim \mathcal{N}(0, \sigma_\psi^2), \\
\kappa &\sim \mathcal{N}(0, \sigma_\kappa^2), \\
\sigma_u^2 &\sim \mathcal{IG}\left(\frac{\tau_0}{2}, \frac{\nu_0}{2}\right),
\end{aligned}
$$

and $\mu_i \sim \mathcal{N}(0,1)$ for $i \in \mathcal{N}$ and $\eta_s \sim \mathcal{N}(0,1)$ for $s \in \mathcal{P}$. We consider the normal and inverse gamma ($\mathcal{IG}$) conjugate priors, which are widely used in the Bayesian literature (Koop et al. 2007). The hyperparameters are chosen to make the prior distributions relatively flat and cover a wide range of the parameter space, i.e., we set $\sigma_\lambda^2 = \sigma_\phi^2 = 10$, $\Sigma_\beta = 10I$, $\sigma_\zeta^2 = \sigma_\varsigma^2 = 10$, $\Sigma_\gamma = 1000I$, $\sigma_\xi^2 = \sigma_\psi^2 = \sigma_\kappa^2 = 1000$, $\tau_0 = 2.2$, and $\nu_0 = 0.1$.

The MCMC sampling procedure combines the Gibbs sampling and the Metropolis-Hastings (M-H) algorithm. It consists of the following steps:

1. Draw the latent variable μ_i using the M-H algorithm based on $f(\mu_i|y, d, \theta, \mu_{-i}, \eta)$, for $i = 1, \ldots, n$.

2. Draw the latent variable η_s using the M-H algorithm based on $f(\eta_s|y, d, \theta, \mu, \eta_{-s})$, for $s = 1, \ldots, p$.

3. Draw γ using the M-H algorithm based on $f(\gamma|y, d, \theta\backslash\{\gamma\}, \mu, \eta)$.

4. Draw ξ using the M-H algorithm based on $f(\xi|y, d, \theta\backslash\{\xi\}, \mu, \eta)$.

5. Draw ψ using the M-H algorithm based on $f(\psi|y, d, \theta\backslash\{\psi\}, \mu, \eta)$.

6. Draw κ using the M-H algorithm based on $f(\kappa|y, d, \theta\backslash\{\kappa\}, \mu, \eta)$.

7. Draw λ using the M-H algorithm based on $f(\lambda|y, d, \theta\backslash\{\lambda\}, \mu, \eta)$.

8. Draw ϕ using the M-H algorithm based on $f(\phi|y, d, \theta\backslash\{\phi\}, \mu, \eta)$.

9. Draw β using the M-H algorithm based on $f(\beta|y, d, \theta\backslash\{\beta\}, \mu, \eta)$.

10. Draw ζ using the M-H algorithm based on $f(\zeta|y, d, \theta\backslash\{\zeta\}, \mu, \eta)$.

11. Draw ς using the M-H algorithm based on $f(\varsigma|y, d, \theta\backslash\{\varsigma\}, \mu, \eta)$.

12. Draw σ_u^2 using the conjugate inverse gamma conditional posterior distribution.

We collect the draws from iterating the above steps and compute the posterior mean and the posterior standard deviation as our estimation results.

C Citations and Patent Value

In this section, we relate the total number of forward citations that a patent receives to the estimated value of the patent. We use the data of patent forward citations and market values from Kogan et al. (2017). The regression results of the patent market value measured in million dollars in 1982 prices on the log number of forward citations (plus one), which is

Table C.1: Regression of patent market value on patent forward citations.

	(A)	(B)	(C)
log(1+citation)	2.5358***	2.5357***	0.9926***
	(0.6846)	(0.6820)	(0.3064)
Controls			
Employment	No	Yes	Yes
Fixed effects	C-T	C-T	C-T, F
N	788,853	760,885	760,885
$\bar{R}^2$	0.086	0.090	0.269

Notes: The dependent variable is the patent market value measured in million dollars in 1982 prices. In the controls, C-T denotes technology classification interacted with grant year fixed effects. F denotes firm fixed effects. Values in parentheses denote standard errors. Samples include granted patents between 1973 and 2013.

our measure of patent output used in Section 4.2, can be seen in Table C.1. Column (C) in Table C.1 suggests that a unit increase in patent output is associated with an additional $1 million dollars of patent value.

D MCMC Convergence Diagnostics

Figure D.1 shows MCMC trace plots and Geweke convergence diagnostics for the estimates of λ and ϕ in Columns (B) and (D) of Table 3. The Geweke convergence diagnostic tests for an equal mean of the first 10% versus the last 50% of the draws. The test results suggest that the Markov chains have converged. We also tried different proportions (e.g., 30% versus 70%), and the results are similar. The results can be obtained from the authors upon request.

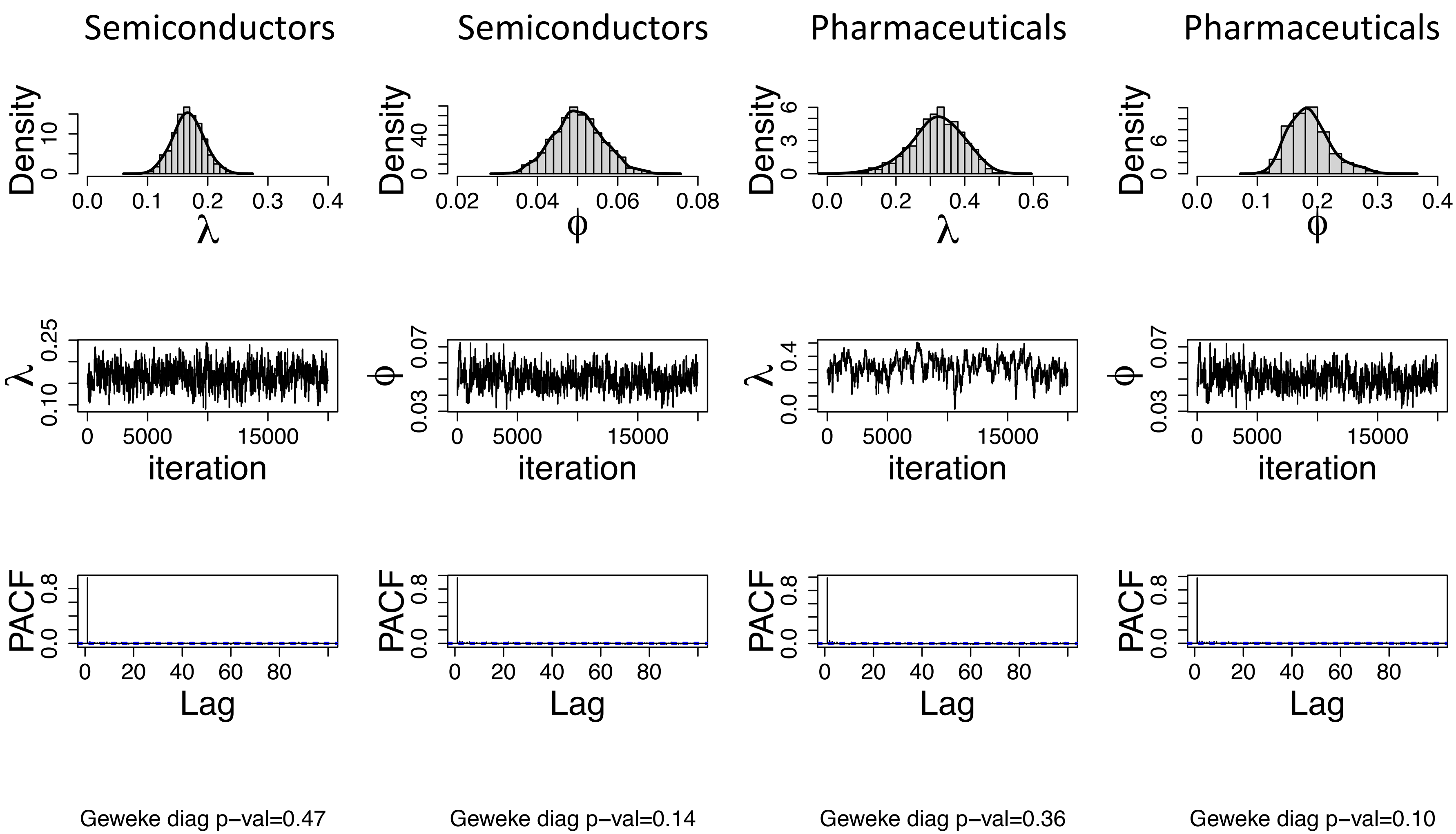

Figure D.1: MCMC trace plots and Geweke convergence diagnostics for estimates of λ and ϕ in Columns (B) and (D) of Table 3.

E Robustness Checks

In this section, we conduct three distinct robustness checks to assess the sensitivity of the estimation results.

E.1 Sample Period

For the first robustness check, we change the sample year from 2003 to 2001. As explained in Section 4.1, we select 2003 as the sample year for the main analysis due to the highest number of patent applications in the semiconductor industry during that year. Now we consider 2001 as an alternative sample year because it records the highest number of patent applications in the pharmaceutical industry compared to all other years. For the year 2001, our sample comprises 5,532 patents and 7,114 inventors in semiconductors, and 1,117 patents and 2,930 inventors in pharmaceuticals. The estimation results based on the sample in 2001 are presented in Table E.1. Although there are slight variations in the magnitude of estimates, the overall results align closely with those presented in Table 3.

Table E.1: Estimation results for semiconductors and pharmaceuticals: Year 2001 (for robustness check).

		Semiconductors		Pharmaceuticals	
		(A) Exogenous Participation	(B) Endogenous Participation	(C) Exogenous Participation	(D) Endogenous Participation
Production					
Complementarity	(λ)	0.1843***	0.2063***	0.1749***	0.2216***
		(0.0222)	(0.0215)	(0.0520)	(0.0473)
Substitutability	(ϕ)	0.0377***	0.0375***	0.0907***	0.0946***
		(0.0059)	(0.0062)	(0.0419)	(0.0138)
Constant	(β_0)	-0.0811***	-0.0961***	-0.2156***	-0.2483***
		(0.0202)	(0.0217)	(0.0444)	(0.0463)
Foreigner	(β_1)	-0.0274**	-0.0257**	0.0137	0.0162
		(0.0120)	(0.0125)	(0.0244)	(0.0254)
Log accu. citations	(β_2)	0.1242***	0.1224***	0.1740***	-0.1733***
		(0.0051	(0.0055)	(0.0067)	(0.0070)
Seniority	(β_3)	-0.1640***	-0.1869***	-0.3679***	-0.3487***
		(0.0215)	(0.0226)	(0.0547)	(0.0516)
Inventor effect	(ζ)	–	0.0165**	–	0.0415***
			(0.0070)		(0.0134)
Patent effect	(ς)	–	0.0166**	–	0.0198**
			(0.0060)		(0.0096)
Error term variance	(σ_ϵ^2)	1.0052***	–	0.8560***	–
		(0.0191)		(0.0367)	
Error term variance	(σ_u^2)	–	1.0016***	–	0.8367***
			(0.0193)		(0.0360)
Participation					
Constant	(γ_0)	–	-4.0168***	–	-1.9646***
			(0.0491)		(0.0568)
Location	(γ_1)	–	1.0446***	–	2.9024***
			(0.0624)		(0.1712)
Past coauthors	(γ_2)	–	7.5523***	–	8.4082***
			(0.1810)		(0.5784)
Common co-authors	(γ_3)	–	14.9260***	–	21.6027***
			(0.2411)		(0.6653)
Inventor effect	(ξ)	–	1.6989***	–	1.2851***
			(0.0701)		(0.1475)
Patent effect	(ψ)	–	-1.6971***	–	-1.8941***
			(0.0668)		(0.1275)
Homophily effect	(κ)	–	-5.0086***	–	-5.8276***
			(0.0772)		(0.1906)
Sample size					
Patents		5,532		1,117	
Inventors		7,114		2,930	

Notes: Columns (A) and (B) show estimates for firms in the semiconductor and related device manufacturing industry. Columns (C) and (D) show estimates for firms in the pharmaceutical preparation manufacturing industry. The output of the production function is measured by the logarithm of patent citations. In Columns (A) and (C) we estimate the production function ignoring endogenous project participation. In Columns (B) and (D) we jointly estimate the production and participation functions with both inventor and patent random effects. We implement MCMC sampling for 25,000 iterations, leaving the first 5,000 draws for burn-in and using the rest of the draws for computing the posterior mean (as the point estimate) and the posterior standard deviation (in the parenthesis). The asterisks ***(**,*) indicate that the 99% (95%, 90%) highest posterior density range does not cover zero.

E.2 Skill Complementarity

For the second robustness check, we replace $g_{ij} = 1$ with the similarity between inventors' research skills to measure their compatibility. Each patent has an International Patent Classification (IPC) number, which is used to classify the content of patents in a uniform manner. We compute the Jaffe (1986) technology similarity f_{ij} between inventors based on the IPC of their granted patents prior to 2003.[24] On average, the resulting Jaffe similarity in semiconductors is lower than that in pharmaceuticals. The estimation results based on heterogenous compatibility are presented in Table E.2. In general, the results exhibit qualitative similarities to those presented in Table 3. However, a notable difference is observed: the estimate of the complementarity effect in semiconductors is larger than in pharmaceuticals, as shown in Table E.2. This suggests that inventors in semiconductors require a stronger complementarity effect to compensate for the lower levels of compatibility between them.

[24] We computed the research skill proximity between inventors following Jaffe (1986) as

$$f_{ij} = \frac{\mathbf{P}_i^\top \mathbf{P}_j}{\sqrt{\mathbf{P}_i^\top \mathbf{P}_i}\sqrt{\mathbf{P}_j^\top \mathbf{P}_j}},$$

where $\mathbf{P}_i$ represents the IPC classes of inventor i and is a vector whose k-th component P_{ik} counts the number of patents inventor i has in IPC class k divided by the total number of patents of that inventor.

Table E.2: Estimation results for semiconductors and pharmaceuticals: Heterogeneous compatibility (for robustness check).

		Semiconductors		Pharmaceuticals	
		(A) Exogenous Participation	(B) Endogenous Participation	(C) Exogenous Participation	(D) Endogenous Participation
Production					
Complementarity	(λ)	0.2791***	0.2921***	0.2374***	0.2539***
		(0.0453)	(0.0437)	(0.0884)	(0.0869)
Substitutability	(ϕ)	0.0530***	0.0550***	0.2641***	0.2860***
		(0.0066)	(0.0072)	(0.0387)	(0.0423)
Constant	(β_0)	-0.1772***	-0.1778***	-0.2878***	-0.2875***
		(0.0211)	(0.0213)	(0.0488)	(0.0490)
Foreigner	(β_1)	-0.0507***	-0.0523***	-0.0664*	-0.0682*
		(0.0136)	(0.0141)	(0.0364)	(0.0375)
Log accu. citations	(β_2)	0.1540***	0.1544***	-0.7706***	-0.7989***
		(0.0059)	(0.0061)	(0.1165)	(0.1205)
Seniority	(β_3)	-0.3753***	-0.3772***	0.2269***	0.2281***
		(0.0327)	(0.0329)	(0.0110)	(0.0113)
Inventor effect	(ζ)	–	0.0097	–	0.0230*
			(0.0069)		(0.0145)
Patent effect	(ς)	–	0.0122***	–	0.0196**
			(0.0022)		(0.0092)
Error term variance	(σ_ϵ^2)	0.8883***	–	0.7231***	–
		(0.0162)		(0.0340)	
Error term variance	(σ_u^2)	–	0.8806***	–	0.7160***
			(0.0161)		(0.0344)
Participation					
Constant	(γ_0)	–	-5.3524***	–	-4.9922***
			(0.0533)		(0.0948)
Location	(γ_1)	–	1.5428***	–	5.0767***
			(0.0630)		(0.2669)
Past coauthors	(γ_2)	–	8.3868***	–	7.2488***
			(0.1730)		(0.3652)
Common co-authors	(γ_3)	–	14.7761***	–	12.7899***
			(0.1554)		(0.3774)
Inventor effect	(ξ)	–	1.6495***	–	1.0405***
			(0.0701)		(0.0991)
Patent effect	(ψ)	–	-2.9155***	–	-2.6083***
			(0.0469)		(0.1093)
Homophily effect	(κ)	–	-3.4693***	–	-1.5777***
			(0.0700)		(0.1054)
Sample size					
Patents		6,017		927	
Inventors		8,472		2,888	

Notes: Columns (A) and (B) show estimates for firms in the semiconductor and related device manufacturing industry. Columns (C) and (D) show estimates for firms in the pharmaceutical preparation manufacturing industry. The output of the production function is measured by the logarithm of patent citations. In Columns (A) and (C) we estimate the production function ignoring endogenous project participation. In Columns (B) and (D) we jointly estimate the production and participation functions with both inventor and patent random effects. We implement MCMC sampling for 25,000 iterations, leaving the first 5,000 draws for burn-in and using the rest of the draws for computing the posterior mean (as the point estimate) and the posterior standard deviation (in the parenthesis). The asterisks ***(**,*) indicate that the 99% (95%, 90%) highest posterior density range does not cover zero.

E.3 Patent Output

For the third robustness check, we substitute patent forward citations with patent values (Kogan et al. 2017) as the measure of patent output. The summary statistics of patent value are available in Tables 1 and 2. On average, the patent value is 7.50 (millions) in semiconductors and 51.28 (millions) in pharmaceuticals. The estimation results for patent values are in Table E.3. The results show that, after correcting the endogeneity bias, both the complementarity effect and the substitutability effect are positive and significant. These findings align with the results obtained using patent citations, as shown in Table 3. Notably, the estimated complementarity and substitutability effects remain higher in the pharmaceuticals compared to the semiconductors.

Table E.3: Estimation results for semiconductors and pharmaceuticals: Patent values (for robustness check).

		Semiconductors		Pharmaceuticals	
		(A) Exogenous Participation	(B) Endogenous Participation	(C) Exogenous Participation	(D) Endogenous Participation
Production					
Complementarity	(λ)	0.0235	0.2575***	0.1614**	0.3731***
		(0.0317)	(0.0334)	(0.0689)	(0.0392)
Substitutability	(ϕ)	-0.0053***	0.0052***	-0.0120	0.0418***
		(0.0016)	(0.0021)	(0.0121)	(0.0134)
Constant	(β_0)	-0.0395	-0.2776***	0.0213	-0.5337***
		(0.0309)	(0.0317)	(0.0967)	(0.0856)
Foreigner	(β_1)	-0.0432**	-0.0278	0.0563*	0.0204
		(0.0209)	(0.0194)	(0.0313)	(0.0291)
Log accu. citations	(β_2)	0.5323***	0.5509***	0.4801***	0.5461***
		(0.0086)	(0.0094)	(0.0167)	(0.0160)
Seniority	(β_3)	-1.1070***	-0.9047***	-0.2171***	-0.0855**
		(0.0430)	(0.0490)	(0.0681)	(0.0346)
Inventor effect	(ζ)	–	0.3405***	–	0.3126***
			(0.0185)		(0.0155)
Patent effect	(ς)	–	0.0431	–	0.6024*
			(0.0314)		(0.3598)
Error term variance	(σ_ϵ^2)	42.7614***	–	1054.4000***	–
		(0.7788)		(49.7417)	
Error term variance	(σ_u^2)	–	32.3299***	–	384.4710***
			(0.8089)		(22.2929)
Participation					
Constant	(γ_0)	–	-5.4096***	–	-5.2700***
			(0.0450)		(0.0946)
Location	(γ_1)	–	1.4750***	–	4.5117***
			(0.0623)		(0.2399)
Past coauthors	(γ_2)	–	8.2576***	–	7.1266***
			(0.1657)		(0.3479)
Common co-authors	(γ_3)	–	14.2455***	–	12.0131***
			(0.1472)		(0.3482)
Inventor effect	(ξ)	–	1.3245***	–	0.6691***
			(0.0980)		(0.0633)
Patent effect	(ψ)	–	-2.7347***	–	-2.3974***
			(0.0512)		(0.0789)
Homophily effect	(κ)	–	-3.2041***	–	-0.9441***
			(0.0503)		(0.0643)
Sample size					
Patents		6,017		927	
Inventors		8,472		2,888	

Notes: Columns (A) and (B) show estimates for firms in the semiconductor and related device manufacturing industry. Columns (C) and (D) show estimates for firms in the pharmaceutical preparation manufacturing industry. The output of the production function is measured by the logarithm of patent citations. In Columns (A) and (C) we estimate the production function ignoring endogenous project participation. In Columns (B) and (D) we jointly estimate the production and participation functions with both inventor and patent random effects. We implement MCMC sampling for 25,000 iterations, leaving the first 5,000 draws for burn-in and using the rest of the draws for computing the posterior mean (as the point estimate) and the posterior standard deviation (in the parenthesis). The asterisks ***(**,*) indicate that the 99% (95%, 90%) highest posterior density range does not cover zero.

F Monte Carlo Simulation

To show that the proposed Bayesian MCMC estimation approach in Appendix B can effectively recover the true parameters in Equations (8) and (9), we conduct a Monte Carlo simulation with 100 repetitions. In each repetition, we generate an artificial bipartite collaboration network of 300 agents ($n = 300$) and 400 projects ($p = 400$). The data generating process (DGP) runs as follows: we first simulate dyadic binary exogenous variables $z_{is} \in \{0, 1\}$ randomly with the probability $P(z_{is} = 1) = 0.64$; individual exogenous variable x_i from normal distribution $N(0, 4)$; and both agent and project latent variables μ_i and η_s from $N(0, 1)$. Then, we generate the artificial collaboration network and project output based on the participation function of Equation (9) and the production function of Equation (8).

In the Monte Carlo simulations, we consider three sets of parameters to see how the signs of the coefficients of agent and project latent variables affect the direction of the selection bias. In the first parameter specification, we set $\zeta > 0$ and $\xi > 0$ (i.e., an agent with higher ability μ_i tends to participate in more projects), $\varsigma > 0$ and $\psi < 0$ (i.e., a project with higher potential η_s has a higher threshold for researchers to participate in), and $\kappa < 0$ (i.e., agents are sorted into projects based on homophily of unobserved characteristics). The simulation results reported in Table F.1 confirm that all true model parameters can be effectively recovered by the employed Bayesian MCMC approach when endogenous project participation is controlled for through agent and project latent variables, and both the complementarity and substitutability effects are downward biased when endogenous project participation is ignored. The direction of the bias is the same as what we observe in the empirical study.

In the second parameter specification, we set $\zeta > 0$ and $\xi < 0$ (i.e., an agent with higher ability tends to participate in fewer projects), while holding the other parameters the same as the first specification. In this case, all true model parameters can still be effectively recovered by the employed Bayesian MCMC approach when endogenous project participation is controlled for. When endogenous project participation is ignored, the substitutability effect

Table F.1: Simulation results: Downward biases on λ and ϕ.

			Exogenous Participation		Endogenous Participation	
		DGP	Est.	S.D.	Est.	S.D.
Production						
Complementarity	(λ)	0.10	0.0315	0.0609	0.0997	0.0024
Substitutability	(ϕ)	0.10	-0.1051	0.1171	0.1007	0.0146
Constant	(β_0)	-1.00	-0.0584	0.6452	-0.9965	0.0620
x_i	(β_1)	0.50	0.3877	0.1754	0.4974	0.0131
Agent effect	(ζ)	1.00			1.0014	0.0262
Project effect	(ς)	0.50			0.5251	0.0459
Error variance	(σ_u^2)	0.50	236.3430	176.4898	0.4625	0.0438
Participation						
Constant	(γ_0)	-5.75			-5.7298	0.1008
z_{ij}	(γ_1)	0.50			0.4882	0.1041
Agent effect	(ξ)	1.00			1.2646	0.2290
Project effect	(ψ)	-1.00			-1.2657	0.2538
Homophily effect	(κ)	-0.50			-0.7721	0.2373

Notes: We perform Monte Carlo simulations with 100 repetitions. The reported values are the mean and the standard deviation of point estimates calculated across repetitions.

is overestimated because it is low-ability agents who are more likely to work on multiple concurrent projects.

In the third parameter specification, we set $\varsigma > 0$ and $\psi > 0$ (i.e., high-potential projects are easier to join than low-potential ones), while holding the other parameters the same as the first specification. In this case, all true model parameters can still be effectively recovered by the employed Bayesian MCMC approach when endogenous project participation is controlled for. When endogenous project participation is ignored, the complementarity effect is overestimated because agents are more likely to collaborate on high-potential projects.

From the simulation results, we can conclude (i) all true model parameters can be effectively recovered by the employed Bayesian MCMC approach when endogenous project participation is controlled for, and (ii) the pattern of bias is consistent with our intuition.

Table F.2: Simulation result: Upward bias on ϕ.

		DGP	Exogenous Participation Est.	Exogenous Participation S.D.	Endogenous Participation Est.	Endogenous Participation S.D.
Production						
Complementarity	(λ)	0.10	0.0539	0.0509	0.0934	0.0271
Substitutability	(ϕ)	0.10	0.8949	1.1015	0.0997	0.0342
Constant	(β_0)	-1.00	-0.6415	0.9804	-0.9829	0.0967
x_i	(β_1)	0.50	0.4004	0.3622	0.4913	0.0250
Agent effect	(ζ)	1.00			1.0042	0.0507
Project effect	(ς)	0.50			0.5259	0.0360
Error variance	(σ_u^2)	0.50	77.3531	76.722	0.4753	0.0354
Participation						
Constant	(γ_0)	-5.75			-5.7416	0.1063
z_{ij}	(γ_1)	0.50			0.4964	0.0971
Agent effect	(ξ)	-1.00			-1.2362	0.2490
Project effect	(ψ)	-1.00			-1.2485	0.2418
Homophily effect	(κ)	-0.50			-0.7283	0.2754

Notes: We perform Monte Carlo simulations with 100 repetitions. The reported values are the mean and the standard deviation of point estimates calculated across repetitions.

Table F.3: Simulation result: Upward bias on λ.

		DGP	Exogenous Participation Est.	Exogenous Participation S.D.	Endogenous Participation Est.	Endogenous Participation S.D.
Production						
Complementarity	(λ)	0.10	0.1277	0.0096	0.1005	0.0015
Substitutability	(ϕ)	0.10	-0.0197	0.0720	0.0968	0.0039
Constant	(β_0)	-1.00	-1.4053	1.2002	-1.0625	0.0338
x_i	(β_1)	0.50	0.5845	0.3356	0.5077	0.0124
Agent effect	(ζ)	1.00			0.9957	0.0221
Project effect	(ς)	0.50			0.4992	0.0460
Error variance	(σ_u^2)	0.50	904.6899	752.5478	0.5111	0.0412
Participation						
Constant	(γ_0)	-5.75			-5.7572	0.0894
z_{ij}	(γ_1)	0.50			0.4839	0.0938
Agent effect	(ξ)	1.00			0.9867	0.0488
Project effect	(ψ)	1.00			0.9734	0.0540
Homophily effect	(κ)	-0.50			-0.5036	0.0656

Notes: We perform Monte Carlo simulations with 100 repetitions. The reported values are the mean and the standard deviation of point estimates calculated across repetitions.