Reference Health and Investment Decisions

Chunli Cheng[1] Christian Hilpert[1] Alexander Szimayer[2] Peter Zweifel[3]

[1]Lingnan College, Sun Yat-sen University

[2]University of Hamburg

[3]University of Zürich

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Motivation

- Reference points influence economic decisions of individuals
- Form a fundamental building block of descriptive theories of choice under risk (Kahneman and Tversky, 1979; Köszegi and Rabin, 2006, 2007; Tversky and Kahneman, 1992)
- References in consumption and wealth influence individuals' consumption and portfolio choices (Barberis and Xiong, 2009; Berkelaar, Kouwenberg and Post, 2004; van Bilsen, Laeven and Nijman, 2020; Meng and Weng, 2018)
- Health, similarly, also has a benchmark value, influencing economic decisions (Harris and Kohn, 2018)

Our Analysis

- We build a dynamic model to show how health reference and its adaptation to decaying health influence individuals' consumption, medical spending, and financial investment choices
- Include projection bias, that is, individuals fail to anticipate their future health reference adaptation
- Calibrate our model to the Health and Retirement Study (HRS) data
- Pin down data-supported behavioural preferences of individuals
- Our model results replicate well both the magnitude and qualitative structure of the observed choices

Model: Health Shocks

- Agents face stochastically decaying health H_t, starting at $H_0 > 0$
- A point process N_t counts the health shocks, recorded at T_n, each of which
  - occurs with intensity λ
  - reduces health by $\theta_n \in [\underline{\theta}, \overline{\theta}]$, following a distribution F_θ
- Death occurs if H_t falls below a critical level H_D, with $0 \leq H_D < H_0$. Time of death T_D is the stopping time

$$T_D = \inf\{t \geq 0 : H_t \leq H_D\} = \inf\{T_n : H_{T_n} \leq H_D\}$$

Model: Medical Spending

- Agents seek medical treatment at each arrival of health shocks
- Investment in medication k
- Health improvement from medication $V(k, \theta, h, s) \geq 0$, with $V(0, \theta, h, s) = 0$, $V_k > 0$, $V_{k,k} < 0$, $\lim_{k \to 0} V_k(k, \theta, h, s) > 0$, $\lim_{k \to \infty} V_k(k, \theta, h, s) = 0$, $V_{k,\theta}(k, \theta, h, s) > 0$, $V_h > 0$
- The medication success ratio $s \in [0, 1]$ follows a Beta distribution
- Despite medication, shocks leave some lasting damage $\epsilon > 0$ such that $\theta - V(k, \theta, h, s) \geq \epsilon$

Model: Health Process

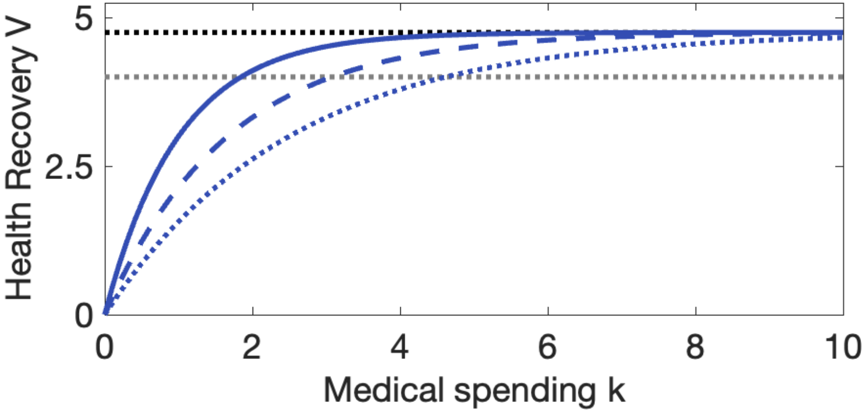
- Agents' health process evolves as

$$H_t = H_0 - \sum_{n=1}^{N_t} \left(\theta_n - V(k_n, \theta_n, H_{T_n -}, s_n)\right)$$

- Consider a concrete functional form

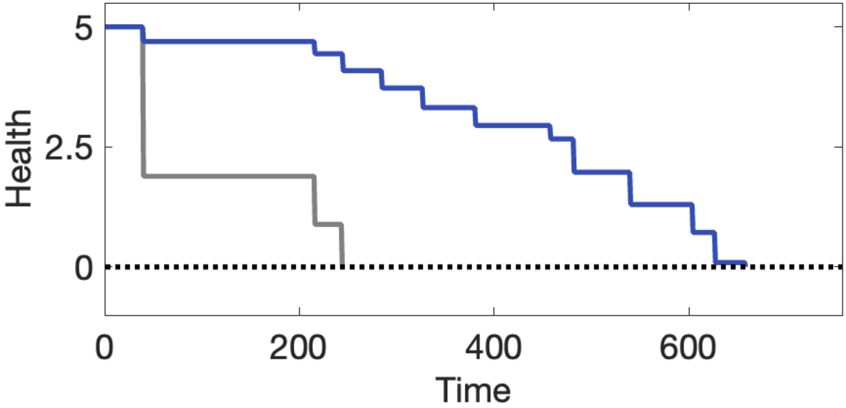
$$V(k, \theta, h, s) = (\theta - \varepsilon)\left(1 - e^{-\zeta \frac{kh}{\theta}}\right) s, \text{ with } \zeta > 0$$

Medical Treatment Effect, $\theta = 5$, $\epsilon = 0.25$, $\zeta = 1$



- Health $H = 5$ (solid), $H = 3$ (dashed), and $H = 2$ (dotted)

Health Decay, $H_0 = 5$, $H_D = 0$



- Health path without medication (grey) v.s. with medication (blue)

Model: Consumption and Investment

- Besides health management, agents consume at a rate c_t and invest a fraction a_t of their assets in a risky asset and the rest in a risk-free asset
- Between health shocks, their asset portfolio value X evolves as

$$dX_t = (a_t(\mu - r) + r)X_t dt - c_t dt + \beta H_t X_t dt + a_t \sigma X_t dW, \; X_0 = x_0 > 0$$

- Accounting for medical spending, their wealth equals (with $X_t > 0$)

$$X_t = x_0 + \int_0^t \left(X_s \left[a_s \left(\mu - r\right) + r + \beta H_s\right] - c_s\right) ds + \int_0^t X_s \, a_s \, \sigma \, dW_s - \sum_{n=1}^{N_t} k_n$$

Model: Reference Health

- Following Harris and Kohn (2018), the reference health value reflects past health realisations, giving rise to the following updating rule

$$B_{t\in[T_n,T_{n+1})} = (1-\omega)B_{T_{n-1}} + \omega H_{T_n}, \text{ with } T_0 = 0 \text{ and } B_{t\in[0,T_1)} = H_0,$$

where $\omega \in [0,1]$ measures the reference health adaptation speed

- Reference health summarises agents' health history

$$B_t = (1-\omega)^{N_t} H_0 + \sum_{n=1}^{N_t} (1-\omega)^{N_t-n} \omega H_{T_n}, \text{ for } t \geq T_1 \text{ and } B_{t\in[0,T_1)} = H_0$$

Model: Utility Function

- Utility embeds cross-partial derivatives $u_{c,h} < 0$, $u_{h,b} > 0$, and $u_{c,b} > 0$ empirically identified by Harris and Kohn (2018) as follows

$$u(c, h, b) = \frac{(ce^{h-b})^{1-\gamma}}{1-\gamma} + u_0, \;\; \gamma > 1$$

- Usual consumption and health derivative conditions $u_c > 0$, $u_{c,c} < 0$, $u_h > 0$, $u_{h,h} < 0$, and $u_b < 0$ hold
- Constant relative risk aversion w.r.t. consumption is γ; constant absolute risk aversion w.r.t. health is $\gamma - 1$

Optimal Control Problem

- Agents choose consumption, medical spending, and financial investment to maximise their lifetime utility

$$J(h, b, x; a, c, k) = \mathbb{E}_{(h,b,x)} \left[\int_0^{T_D} e^{-\rho t} u_t(c_t, H_t, B_t) \, dt \right]$$

- Here, $\rho > 0$ is the rate of time preference, (h, b, x) is the current status of the state space, and (a, c, k) are the controls

Optimal Decisions: Medical Spending

Proposition 1. Suppose that an optimal strategy $(a^\star, c^\star, k^\star)$ exists with value function

$$U(h, b, x) = \sup_{(a,c,k)} J(h-\theta+V(k,\theta,h,s), (1-\omega)b+\omega(h-\theta+V(k,\theta,h,s)), x-k; a, c, k),$$

then the (possibly not unique) optimal medical expenditure choice exists for sufficiently smooth U and V, which is characterised by

$$\begin{aligned} k^\star(h, b, x, \theta, \omega, \alpha_s, \beta_s) = \arg\max_{0\le k\le x} \int U(h - \theta + V(k, \theta, h, s), \\ (1-\omega)b + \omega(h - \theta + V(k, \theta, h, s)), x - k) dF_{\alpha_s,\beta_s}(s). \end{aligned}$$

Optimal Decisions: Consumption and Investment

Proposition 2. Under stated assumptions, the optimal consumption and investment choices are

$$c^\star(h,b,x) = \left(\frac{U_x(h,b,x)}{K_1(h,b,\gamma)}\right)^{-\frac{1}{\gamma}}, \qquad a^\star(h,b,x) = \frac{\mu - r}{\sigma^2}\,\frac{U_x(h,b,x)}{-x\,U_{xx}(h,b,x)}\,.$$

The value function is characterised by the ordinary differential equation

$$0 = \frac{1}{2}\,\frac{(\mu - r)^2}{\sigma^2}\,\frac{U_x(h,b,x)^2}{-U_{xx}(h,b,x)} + (r + \beta h)\,x\,U_x(h,b,x) + \frac{\gamma}{1-\gamma}\,U_x(h,b,x)^{\frac{\gamma-1}{\gamma}} K_1(h,b,\gamma)^{\frac{1}{\gamma}} + K_0 + \lambda\,\mathbb{U}(h,b,x) - (\rho + \lambda)\;U(h,b,x)\,,$$

where $K_1(h,b,\gamma) = e^{(h-b)(1-\gamma)}$ and $K_0 = u_0$.

Data

- Health and Retirement Study (HRS) data, 1996-2020
- Exclude individuals below 50 and above 95
- Consider individuals without initial health issues when they enter the survey
- Consider individuals who have positive financial wealth and at least once invest at least $5000 during their involvement in study
- Use females in our sample
- 1,874 individuals with 22,488 individual-biannual observations

Calibration: Financial and Health Parameters

- Financial parameters: 3-Month Treasury Bill Secondary Market Rate and the S&P 500 index are used to calibrate r, μ, and σ
- Use the multiple-correspondence health index by Kohn (2012) to construct a health index H
- Calibrate the intensity of health shocks λ, the distribution F_θ of health shock size θ, the medical success ratio s distribution, the medical cost level ζ, and the minimal health damage ϵ by using the index H and its changes

Calibration: Preference Parameters

- Preference parameters: risk aversion γ and reference adaptation speed ω
- Split our sample into four subgroups according to individuals' initial financial wealth in quartiles and use the median value in each subgroup as their representative initial financial wealth
- Calibrate γ and ω group-wise by minimising the sum of the squared deivations between the mean choices of model-implied medical spending and investment and the observed choices from the data across the health index quintiles

Parametrisation

Parameter	Value	Parameter	Value
Risk-free rate r	2.1%	Initial health H_0	5
Risky asset return μ	7.1%	Death threshold H_D	0
Volatility σ	19.5%	Health shock intensity λ	0.27
Minimum health shock $\underline{\theta}$	1	Maximum health shock $\bar{\theta}$	5
Minimum health damage ε	0.25	Earnings parameter β	0.31%
Discount parameter ρ	4%	Medical cost level ζ	0.6
Medical success beta α_s	2.89	Medical success beta β_s	4.95

Table: **Parametrization** This table shows the calibrated economic parameters for estimating model choices.

Results: Risky Investment Share (%)

	Model				Observed			
Health/Wealth	1	2	3	4	1	2	3	4
Poor	21.90	16.51	25.25	34.92	17.05	18.60	25.75	34.61
Fair	21.21	19.17	27.52	35.40	20.98	19.27	25.81	35.51
Good	20.29	21.04	29.72	35.95	21.49	23.89	31.95	35.72
Very good	21.24	24.32	31.73	35.87	23.10	23.32	31.15	36.79
Excellent	21.38	24.54	31.96	35.86	21.24	24.77	31.61	35.03
CRRA γ	2.70	2.52	3.00	3.22				
Adap. ω	0.53	0.72	0.68	0.72				
Error	28.96	14.26	8.74	2.48	Σ:	54.54		

Results - Medical Spending ($\$1,000$)

	Model				Observed			
Health/Wealth	1	2	3	4	1	2	3	4
Poor	3.94	4.02	3.92	4.00	3.47	3.23	3.78	3.41
Fair	2.85	2.88	2.78	2.83	2.40	2.67	2.64	3.11
Good	2.23	2.26	2.21	2.23	2.38	2.30	2.14	2.39
Very good	1.74	1.75	1.70	1.72	1.87	1.95	2.00	2.23
Excellent	1.70	1.72	1.67	1.69	1.73	1.73	1.74	1.95
CRRA γ	2.70	2.52	3.00	3.22				
Adap. ω	0.53	0.72	0.68	0.72				
Error	28.96	14.26	8.74	2.48	Σ:	54.54		

Model Extension: Projection Bias

- Reconsider the medical spending choice at the health shock time T_n: Agents choose k_n, resulting in a remaining lifetime utility

$$J(h-\theta_n+V(k_n,\theta_n,h,s),(1-\omega)b+\omega(h-\theta_n+V(k_n,\theta_n,h,s)),x-k_n;a,c,k).$$

- Solving for the optimal medical spending $k_n^\star$ assumes agents are able to anticipate their health reference updating
- Individuals present projection bias in forecasting their future life adaptation (Loewenstein, O'Donoghue and Rabin, 2003)

Model Extension: Projection Bias

- Agents likely fail to anticipate their health reference adaptation
- Extend our base model by including a reference adaptation anticipation parameter $\hat{\omega} \in \{0, 1\}$: At the health shock time T_n, the agent chooses k_n, resulting in a remaining lifetime utility

$$J(h - \theta_n + V(k_n, \theta_n, h), (1 - \hat{\omega})b + \hat{\omega}((1 - \omega)b + \omega B_{T_n}), x - k_n; a, c, k),$$

- $\hat{\omega} = 1$ is with anticipation (no projection bias) and $\hat{\omega} = 0$ is with no anticipation (projection bias)

Results: Risky Investment Share (%), No Anticipation

	Model				Data			
Health/Wealth	1	2	3	4	1	2	3	4
Poor	18.19	18.52	30.66	36.14	17.05	18.60	25.75	34.61
Fair	18.77	19.41	31.63	35.26	20.98	19.27	25.81	35.51
Good	19.93	23.70	31.91	35.59	21.49	23.89	31.95	35.72
Very good	22.74	24.52	28.33	34.92	23.10	23.32	31.15	36.79
Excellent	22.95	24.52	28.05	34.89	21.24	24.77	31.61	35.03
CRRA γ	3.22	3.60	3.60	3.34				
Adap. ω	0.69	0.35	1.00	0.55				
Error	12.18$\downarrow$	2.31$\downarrow$	78.77$\uparrow$	6.66$\uparrow$	Σ:	99.94		

Results: Medical Spending ($\$1,000$), No Anticipation

	Model				Observed			
Health/Wealth	1	2	3	4	1	2	3	4
Poor	3.96	4.05	3.87	4.05	3.47	3.23	3.78	3.41
Fair	2.86	2.86	2.81	2.88	2.40	2.67	2.64	3.11
Good	2.24	2.26	2.18	2.28	2.38	2.30	2.14	2.39
Very good	1.74	1.76	1.70	1.73	1.87	1.95	2.00	2.23
Excellent	1.71	1.73	1.67	1.72	1.73	1.73	1.74	1.95
CRRA γ	3.22	3.60	3.60	3.34				
Adap. ω	0.69	0.35	1.00	0.55				
Error	12.18↓	2.31↓	78.77↑	6.66↑	Σ:	99.94		

Conclusion

- This paper analyses the interplay of individuals' health and its reference value with their consumption, medical spending, and financial investment decisions in a dynamic optimal-control framework
- We find data-supporting evidence for adaptive reference points in health for all wealth subgroups and projection bias among poorer individuals
- Our model replicates well the magnitude and qualitative structure of the observed choices in data

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