# Constructing Fan Charts from the Ragged Edge of SPF Forecasts

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*The results presented here do not necessarily represent the views of Federal Reserve Bank of Cleveland, the Federal Reserve System, the Banco de España, the Deutsche Bundesbank, or the Eurosystem*

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# RESEARCH AGENDA

## Setup

**We observe point forecasts from the SPF**
(or a similar source)

**for fixed horizons and fixed events**
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## Problem

**How to construct fan charts**

i.e. term structures of expectations and uncertainty

**that are consistent with the SPF?**

. . . by modeling the distribution of SPF forecast errors
and filling in missing values

# TERM STRUCTURE OF SPF-CONSISTENT FORECASTS

$$\boldsymbol{Y_t \equiv F_t} \begin{bmatrix} \boldsymbol{y_{t-1}} \\ \boldsymbol{y_t} \\ \boldsymbol{y_{t+1}} \\ \vdots \\ \boldsymbol{y_{t+h}} \\ \vdots \\ \boldsymbol{y_{t+H}} \end{bmatrix}$$

**Details:**

- SPF-consistent means observed or imputed from the SPF
- $\boldsymbol{h}$ denotes a quarterly horizon
- Lagged outcome known to SPF: $\boldsymbol{y_{t-1} = F_t y_{t-1}}$
- Henceforth: $\boldsymbol{y_t}$ is a scalar outcome

# DATA: U.S. SPF

**Throughout, we look at point forecasts of the average respondent**

## Fixed-horizon forecasts

Predictions for quarterly outcomes, $\boldsymbol{y_t}$:

$\boldsymbol{F_t y_{t+h}}$ for $h = 0, 1, 2, 3, 4$

## Fixed-event forecasts

Predictions for calendar-year outcomes, $\boldsymbol{\bar{y}_t}$,

$\boldsymbol{F_t \bar{y}_{t+h}}$ for years 1 to 3 ahead

"fixed-event" forecast horizon shifts during the year:

if $t$ is in Q1: $h = 7, 11, 15$

if $t$ is in Q2: $h = 6, 10, 14$

etc.

**Time-varying SPF coverage**
**of different variables and horizons**

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3. forecast updates $\boldsymbol{\eta_t}$

# APPLICATIONS

## Supplement judgmental forecasts w/missing elements

- Example: FOMC's Summary of Economic Projections
- SEP fan charts combine policymaker point forecasts, with model-based measure of uncertainty

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## Measure reactions of SPF forecasts to shocks

How do economic shocks shape forecast updates $\boldsymbol{\eta_t}$?

(aka "SPF-consistent impulse responses")

# RELATED LITERATURE

**Survey uncertainty based on past forecast errors**

- Reifschneider & Tulip (2007/19), Clark, McCracken & Mertens (2020)
- Lahiri & Sheng (2010), Knüppel (2014), Jo & Sekkel (2019)
- Adams, Adrian, Boyarchenko, & Giannone (2021)

**Survey forecasts: term structures, densities and fixed events**

- Patton & Timmermann (2011), Kozicki & Tinsley (2012)
- Aruoba (2020), Crump, Eusepi, Moench, & Preston (2022)
- Dovern, Fritsche & Slacalek (2012), Ganics, Rossi & Sekhposyan (2019)
- Bassetti, Casarin, & Del Negro (2022), Cakmakli & Demircan (2022)

**Efficiency and calibration of survey forecasts**

- Croushore (2010,2023), Faust & Wright (2009), Clements (2018)
- Coibion & Gorodnichenko (2015), Mertens & Nason (2020)
- Andrade & LeBihan (2013), Fuhrer (2017), Bracha & Tang (2022)
- Hajdini and Kurmann (2022), Bianchi, Ludvigson & Ma (2022), Eva & Winkler (2023)
- Angeletos, Huo, and Sastry (2021), Broer & Kohlhas (2023), Farmer, Nakamura & Steinsson (2024)

See also Clark-Mertens handbook chapter (Edward-Elgar, forth.)

# AGENDA

1. State space model of SPF forecasts and outcomes
2. Term structures of expectations and uncertainty
3. Biased or unbiased SPF?
4. Conclusions

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1. **Map observed data** from SPF and realized series, $\boldsymbol{Z_t}$, **into latent state vector** of fixed-horizon forecasts, $\boldsymbol{Y_t}$:

$$\boldsymbol{Z_t} = \boldsymbol{C_t}\ \boldsymbol{Y_t}$$

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$$\boldsymbol{Y_t = \Psi\ Y_{t-1} + \eta_t}$$

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4. For finite $\boldsymbol{H}$, **assume flat term structure beyond** $\boldsymbol{H}$
5. **Specify DGP for** $\boldsymbol{\eta_t}$
   - **a)** In general: Bias in SPF and persistence in $\boldsymbol{\eta_t} \sim$ VAR
   - **b)** If SPF unbiased: $\boldsymbol{F_t = E_t}$ and $\boldsymbol{E_{t-1}\eta_t = 0}$

# MEASUREMENT EQUATIONS

## Measurement vector with two types of SPF data

- $\boldsymbol{Z_{q,t}}$: Quarterly fixed-horizon forecasts, including $\boldsymbol{y_{t-1}}$
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$$\boldsymbol{Z_t} = \begin{bmatrix} \boldsymbol{Z_{q,t}} \\ \boldsymbol{Z_{a,t}} \end{bmatrix} = \begin{bmatrix} \boldsymbol{C_{q,t}} \\ \boldsymbol{C_{a,t}} \end{bmatrix} \boldsymbol{Y_t}$$

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- Model can match any SPF data w/o need for measurement error

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- $\boldsymbol{C_{q,t}}$ and $\boldsymbol{C_{a,t}}$ are known from SPF's data definitions
- Model can match any SPF data w/o need for measurement error
- Horseshoe models for $\boldsymbol{n_t}$ to catch occasional discrepancies between $\boldsymbol{Z_{q,t}}$ and $\boldsymbol{Z_{a,t}}$

# ESTIMATION SETUP

1. Model applied **separately for each outcome variable** (RGDP, PGDP, CPI, UNRATE)
2. Estimated with **MCMC over growing samples** of **real-time data and SPF** that start in 1968Q3
   (FRB Phil.'s Real-Time Data Set for Macroeconomists)
3. Generate **out-of-sample predictive densities** from 1990Q1 onwards
4. Predictions **evaluated against 2nd release** outcomes for RGDP and PGDP and latest data for CPI, UNRATE

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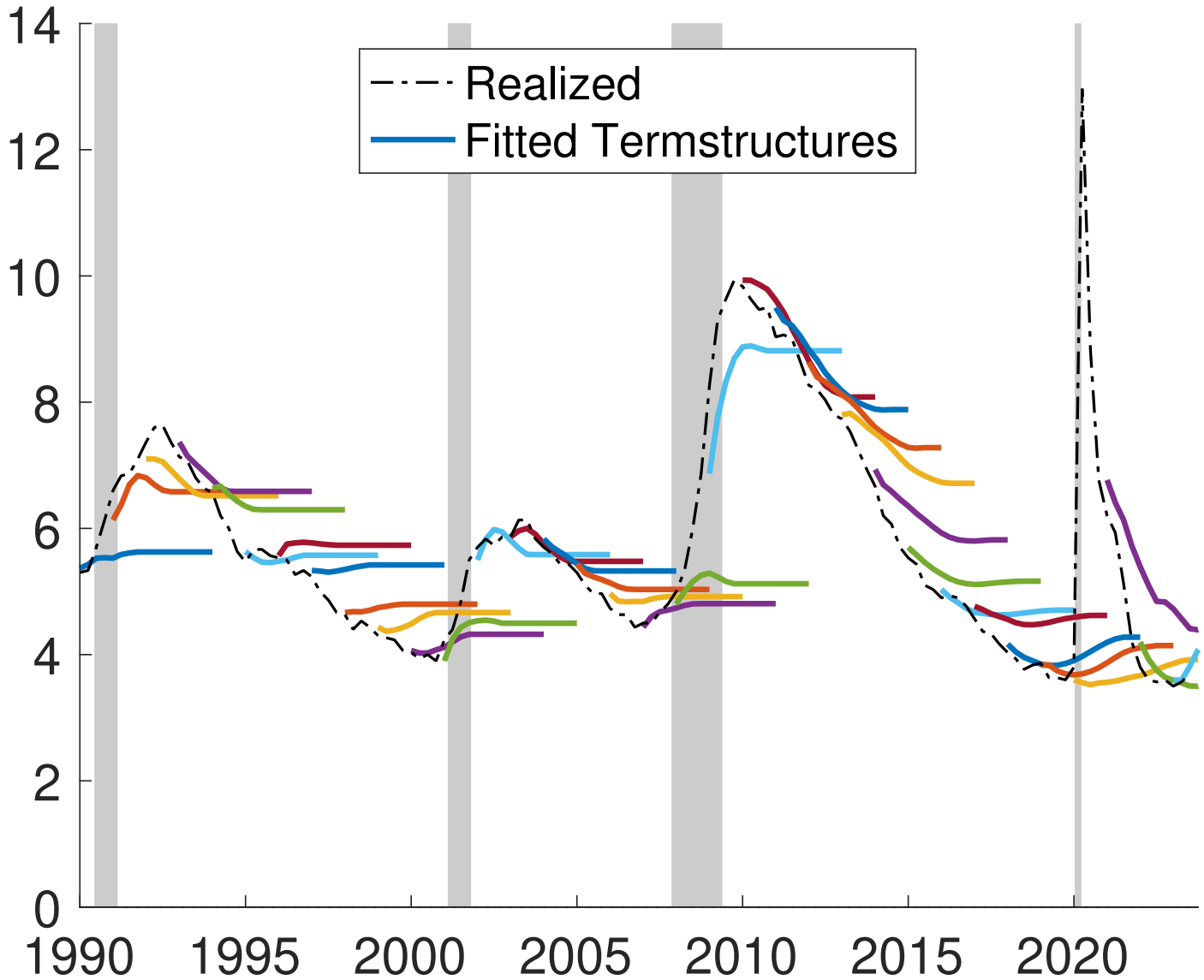
# TERM STRUCTURE OF FORECASTS

## Unemployment rate: Quarterly SPF-consistent forecasts, MDS model




Showing only forecast origins in Q1. Out-of-sample forecasts. NBER recessions shaded.

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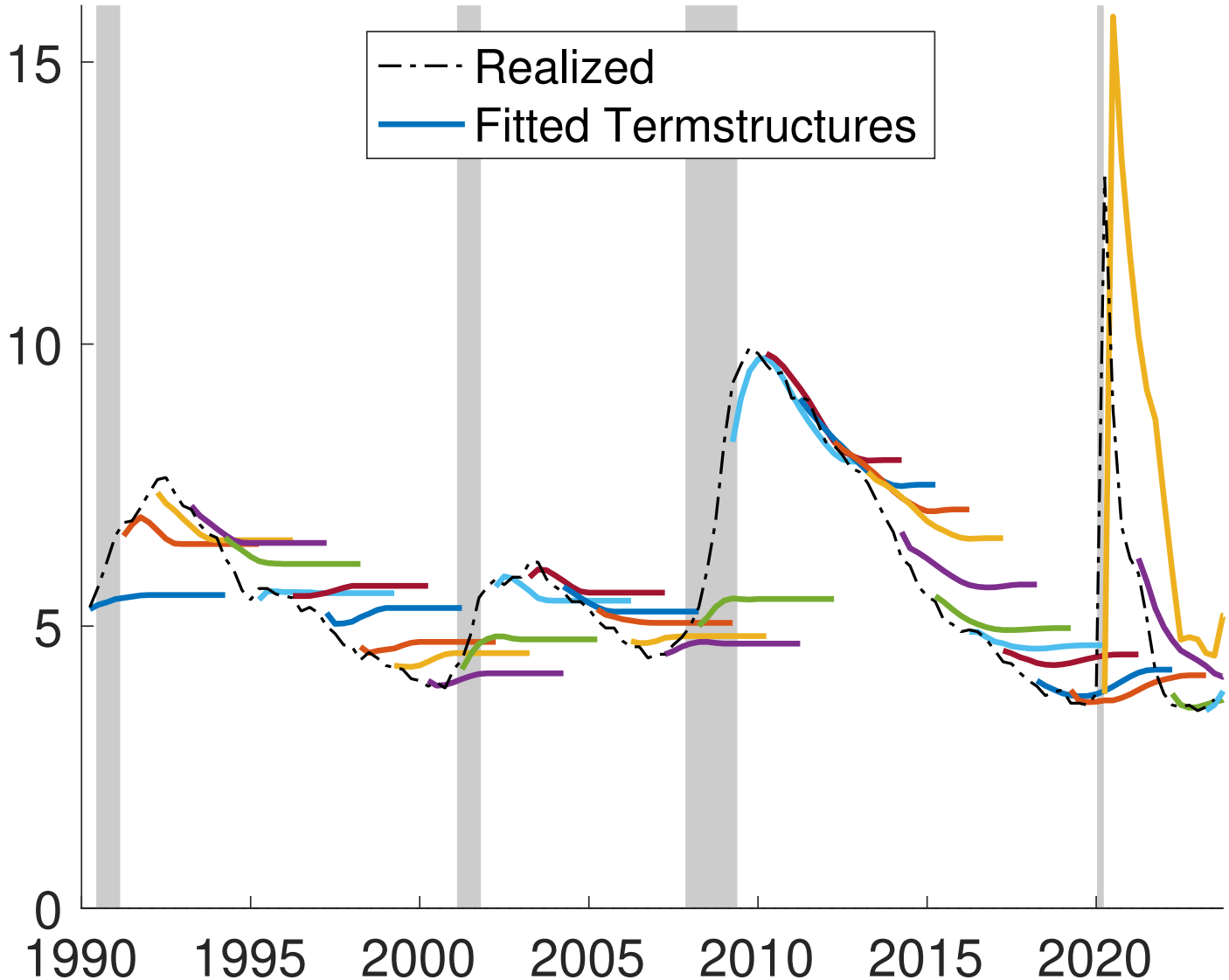
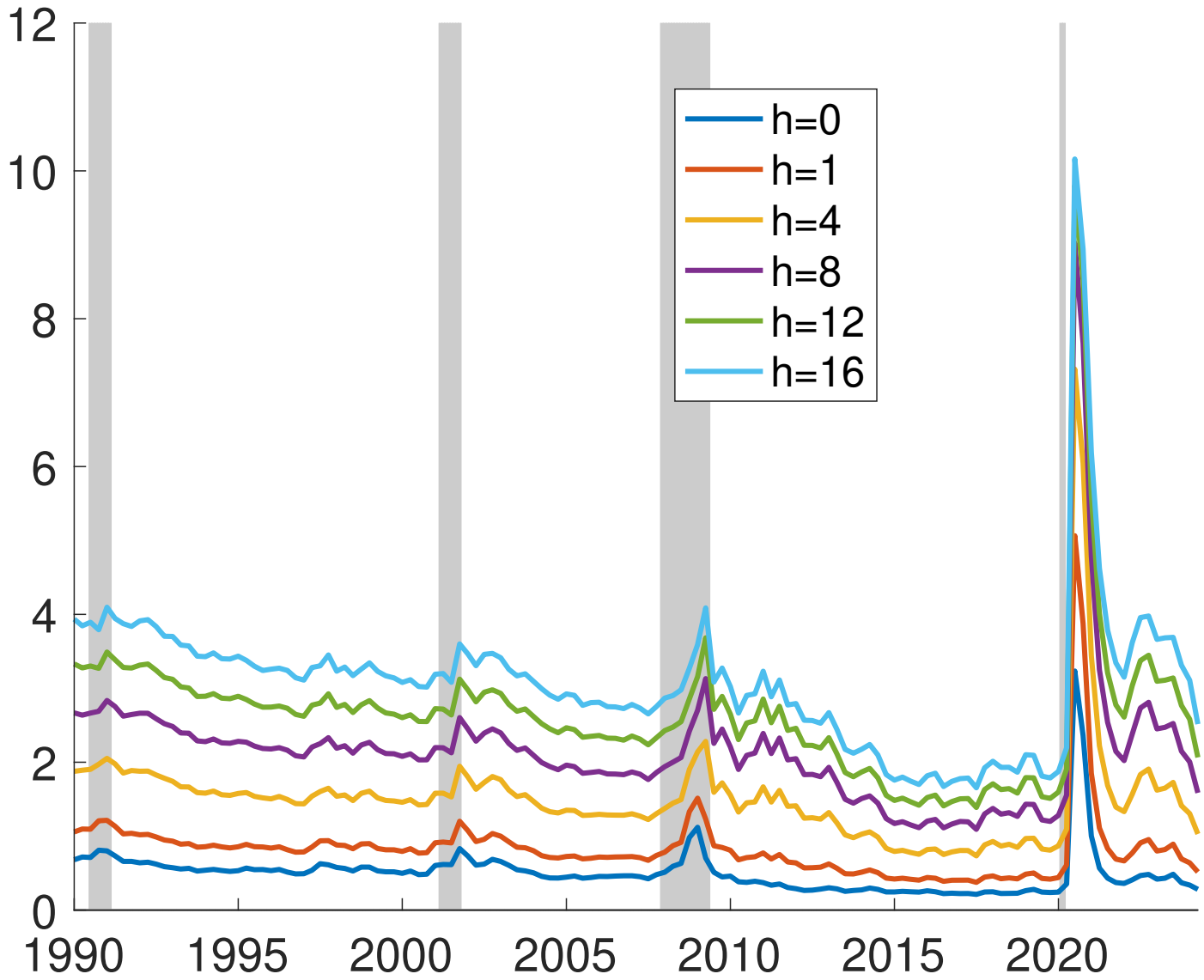



Showing only forecast origins in Q2. Out-of-sample forecasts. NBER recessions shaded.

# TERM STRUCTURES OF UNCERTAINTY

## Unemployment rate: Width of predictive 68% bands, MDS model

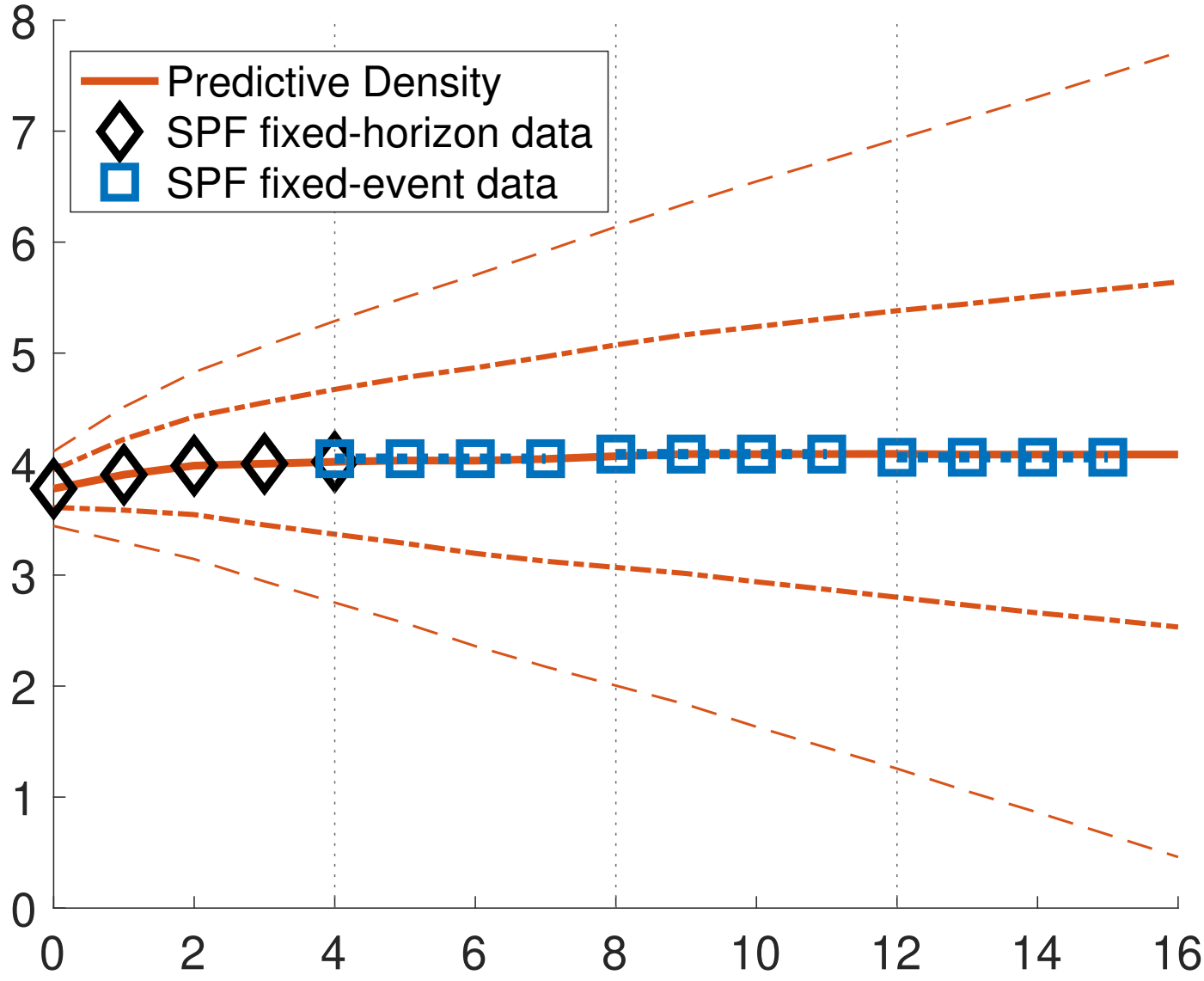



Out-of-sample forecasts. NBER recessions shaded.

# QUARTERLY SPF-CONSISTENT FAN CHARTS

2024Q1

## Unemployment rate, MDS model



**By construction, mid points of fan charts match observed SPF**

(up to measurement error in case of annual forecasts)

**Term structure of uncertainty** steadily rises with horizon (for UNRATE)

**and displays cyclical variations over time**

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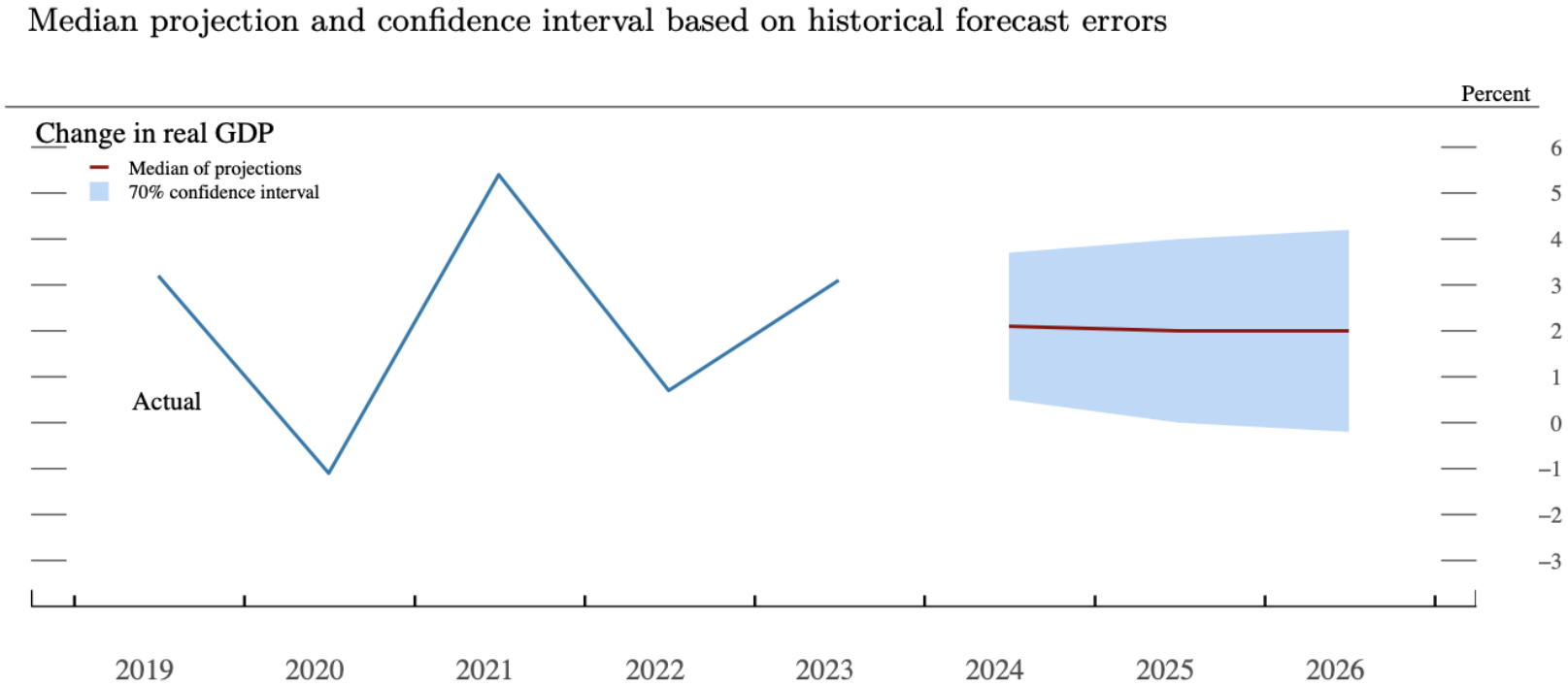
# FOMC "FAN CHARTS"

## From the "Summary of Economic Projections" (SEP), real GDP growth

## March 2024




- "Fan charts" published since March 2017
- Uncertainty ranges tabulated since first SEP in Oct 2007
- Bands: $+/-1$ historical RMSE of professional forecasters

Figure published with SEP, confidence intervals based on Reifschneider & Tulip (2007/17 FEDS, 2019 IJF)

# COMPARISON OF OUR MODEL BANDS AGAINST SEP

**We do the following:**

- From SEP: mid points and historical RMSE bands (Reifschneider & Tulip, 2007/19)
- From model: SPF-consistent forecasts and 68% bands
- Construct forecast errors from SEP and model
- Collect the above for every quarter since 2008 (no SEP for 2020Q1)

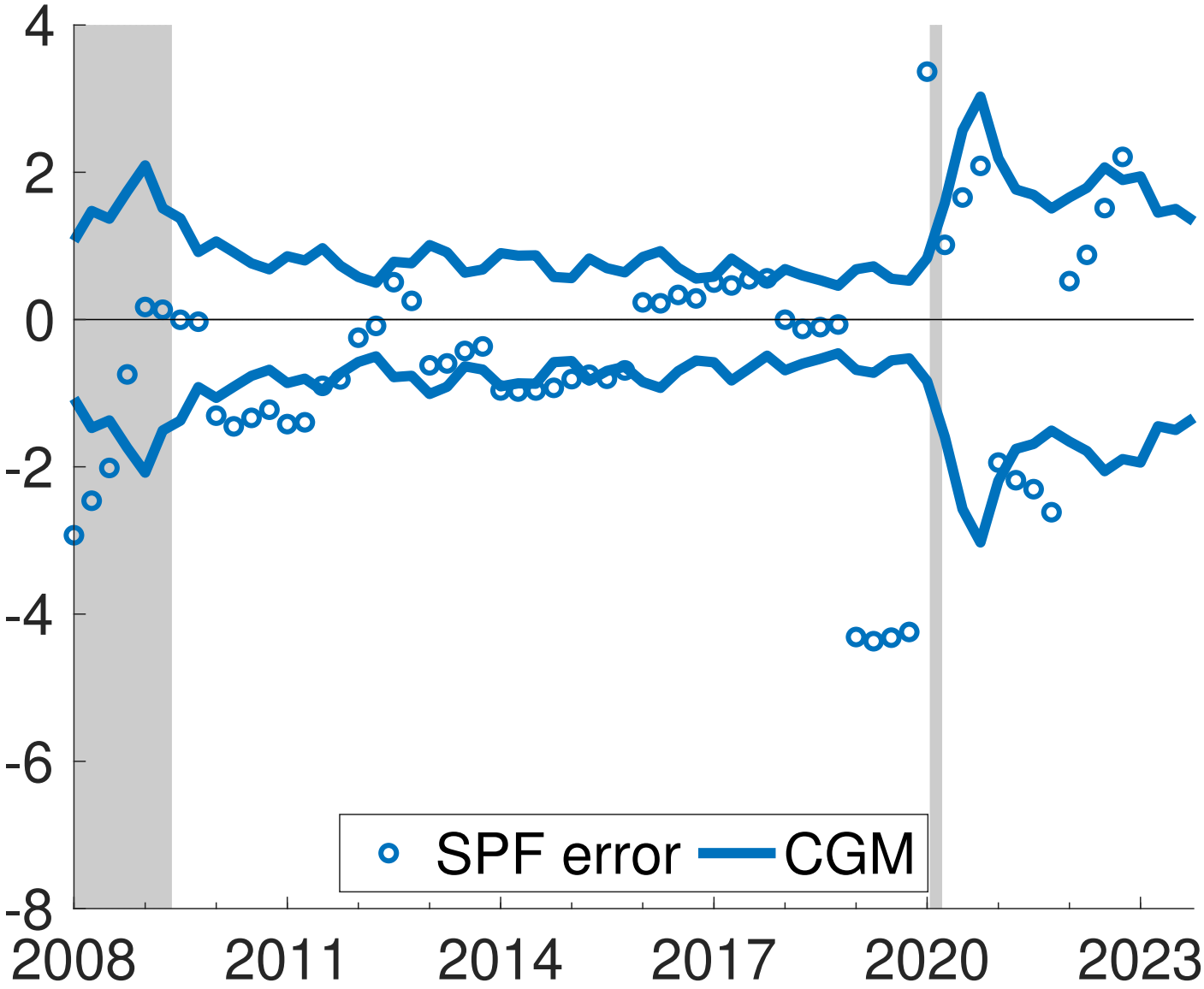
**Goal:**

**Which error bands have better coverage?**

(out of sample)

# SEP VS MODEL: ERROR BANDS AND REALIZED ERRORS

## Real GDP growth: next-year forecast




Note: 68% bands. Out-of-sample forecasts.

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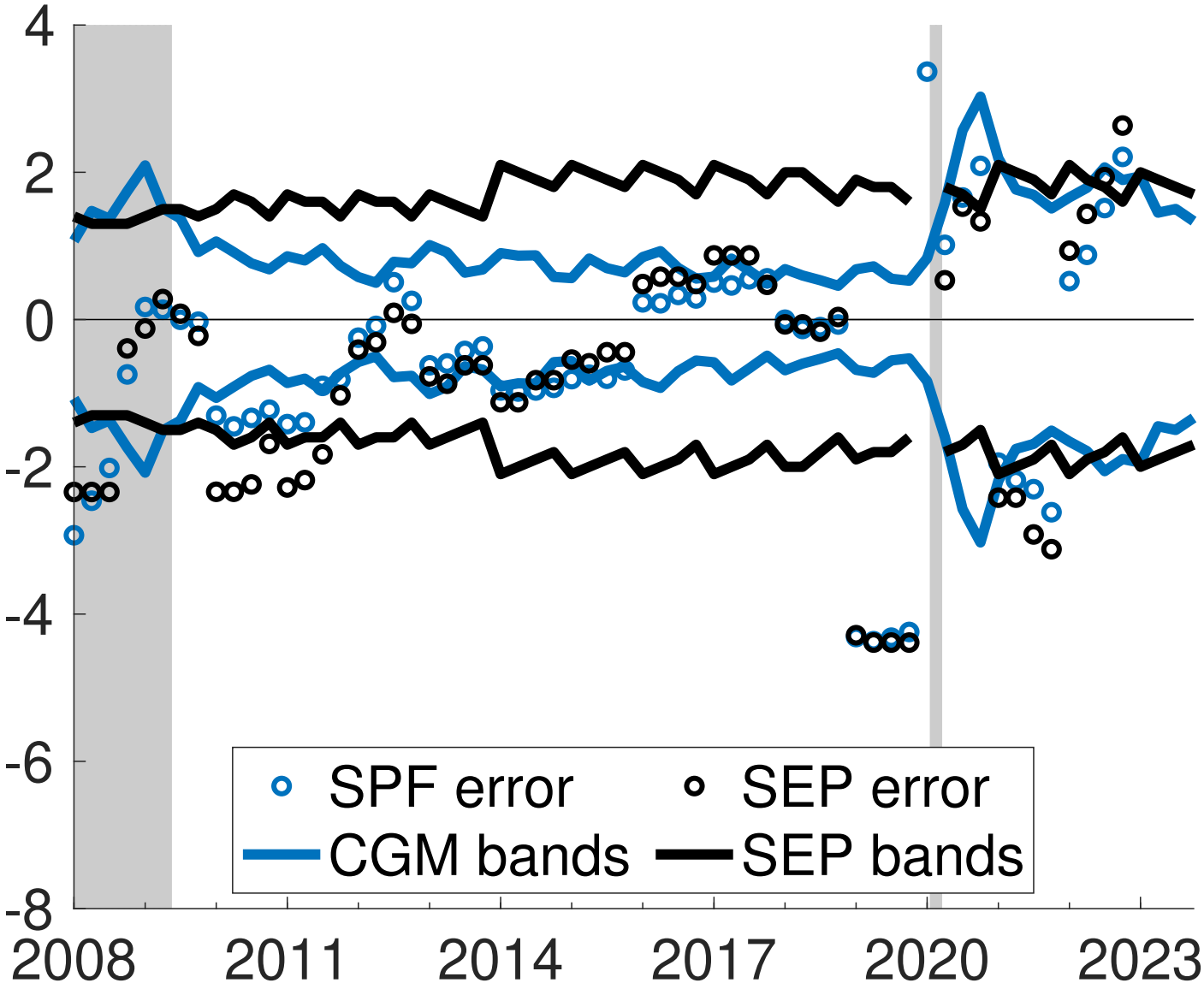


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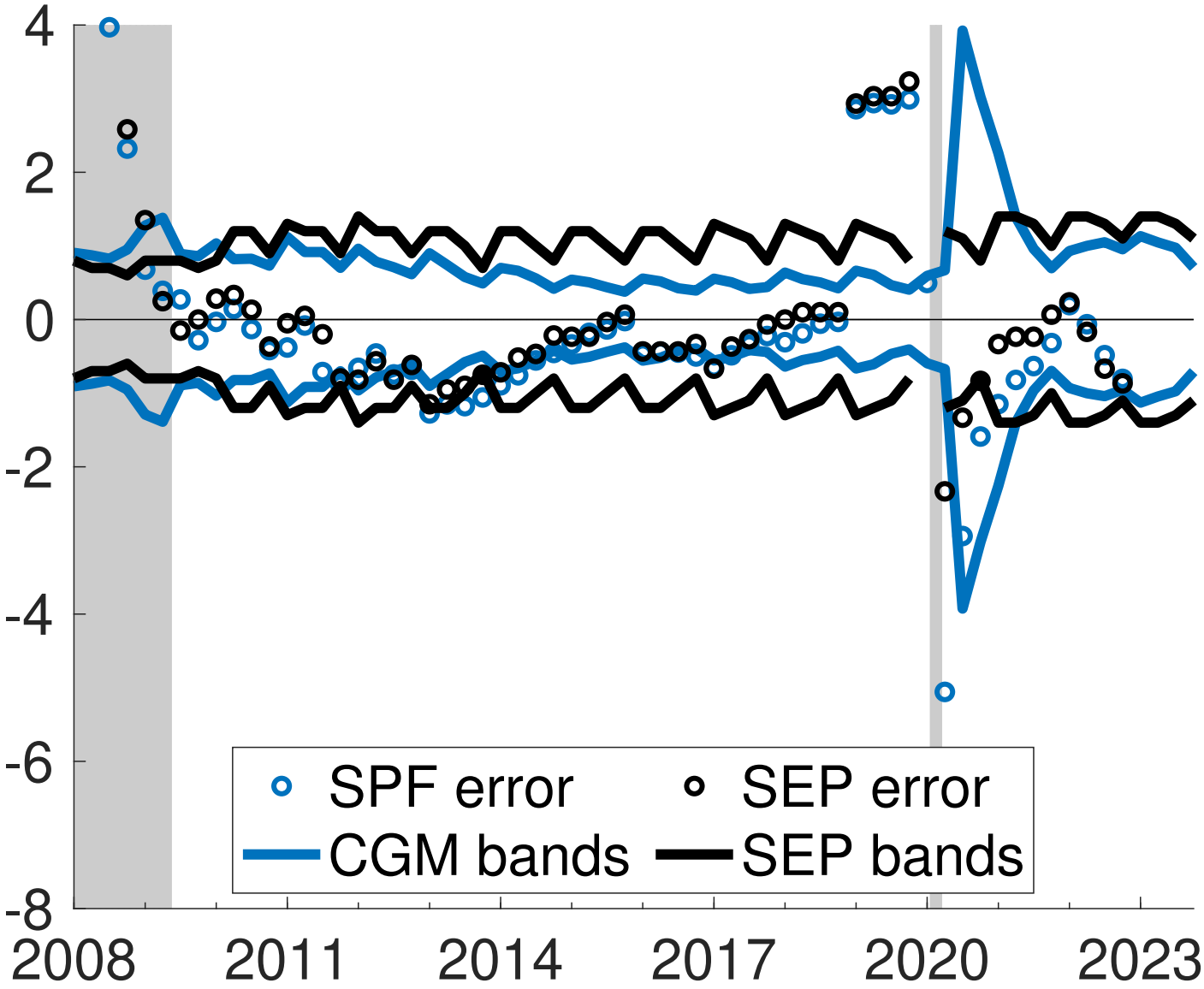


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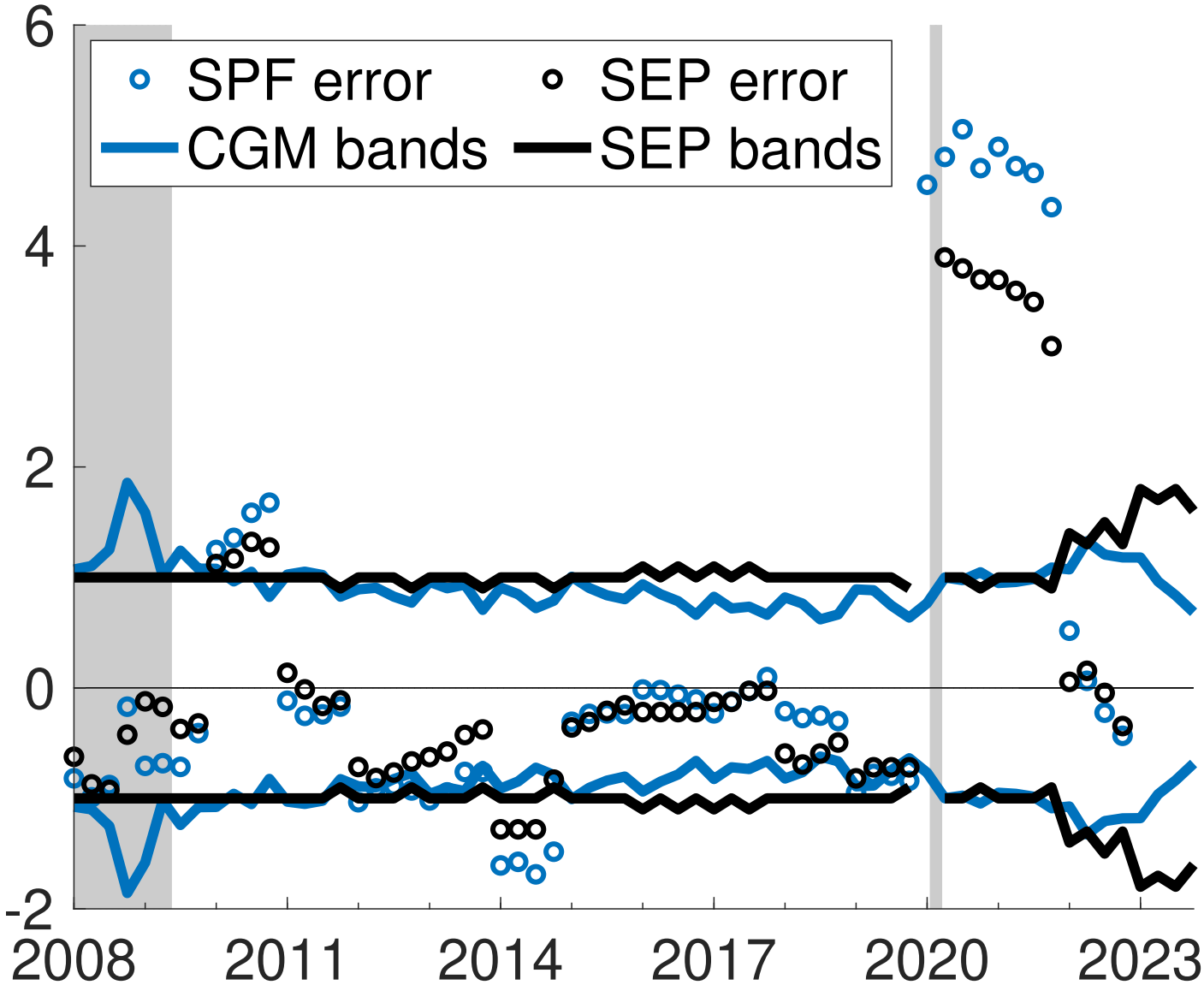
# SEP VS MODEL: ERROR BANDS AND REALIZED ERRORS

## Inflation: next-year forecast




Note: 68% bands. Out-of-sample forecasts. SEP for PCI, SPF for CPI inflation.

Similar forecast error patterns from SPF and SEP

Fixed events: clustering of errors and sawtooth bands

**SEP bands typically wider & w/too much coverage**

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We find

**significant predictability** of SPF forecast errors **in sample**,

which is, however, **hard to exploit out of sample**

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   - Out-of-sample predictions for SPF and outcomes
   - Predictable-error regressions implied by VAR model
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# RELATIVE FORECAST ACCURACY MDS VS VAR MODEL

## Values above one indicate that VAR does worse

| | RMSE | | | | CRPS | | | |
|---|---|---|---|---|---|---|---|---|
| $h$ | RGDP | UNRATE | PGDP | CPI | RGDP | UNRATE | PGDP | CPI |
| 0 | 1.01 | 1.12 | 0.99 | 0.91** | 1.02 | 0.95 | 1.00 | 0.93*** |
| 1 | 1.04 | 1.05 | 1.02 | 1.01 | 1.01 | 1.00 | 1.00 | 1.01 |
| 2 | 0.99 | 1.07 | 1.01 | 1.01 | 0.99 | 1.01 | 0.99 | 1.01 |
| 3 | 1.00 | 1.04 | 1.02 | 1.01 | 1.01 | 1.00 | 1.00 | 1.01 |
| 4 | 1.00 | 1.02 | 1.03 | 1.01 | 1.00 | 1.00 | 1.00 | 1.01 |
| 5 | 1.00 | 1.02 | 1.04 | 1.01 | 1.00 | 1.01 | 1.01 | 1.01 |
| 6 | 1.00 | 1.02 | 1.04 | 1.01 | 1.01 | 1.02 | 1.02 | 1.01 |
| 7 | 1.00 | 1.02 | 1.04 | 1.01 | 0.99 | 1.03 | 1.02 | 1.01 |
| 8 | 1.00 | 1.02* | 1.04 | 1.01 | 1.00 | 1.03 | 1.02 | 1.01 |
| 9 | 1.00 | 1.03* | 1.04 | 1.01 | 1.00 | 1.04 | 1.03 | 1.01 |
| 10 | 1.00 | 1.03 | 1.04 | 1.01 | 1.00 | 1.04 | 1.03 | 1.02 |
| 11 | 1.00 | 1.02 | 1.04 | 1.01 | 1.00 | 1.03 | 1.03 | 1.02 |
| 12 | 1.00 | 1.01 | 1.04 | 1.01 | 1.00 | 1.02 | 1.03 | 1.01 |
| 13 | 1.00 | 1.00 | 1.04 | 1.01 | 1.00 | 1.00 | 1.03 | 1.01 |
| 14 | 1.00 | 0.99 | 1.04 | 1.01 | 1.01 | 0.99 | 1.04 | 1.01 |
| 15 | 1.00 | 1.00 | 1.04 | 1.01 | 1.01 | 1.00 | 1.03 | 1.01 |
| 16 | 1.00 | 1.00 | 1.04 | 1.00 | 1.01 | 0.99 | 1.03 | 1.01 |

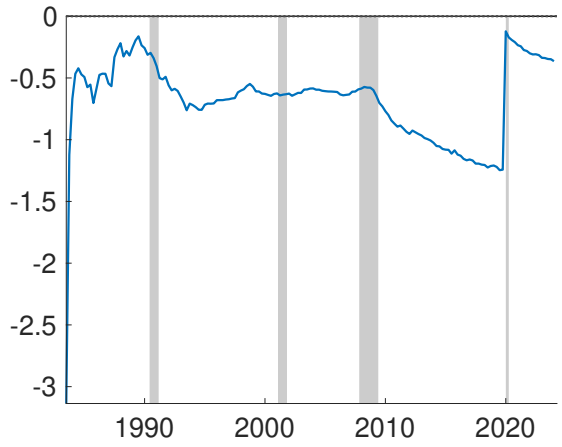
Note: Relative RMSE and CRPS of VAR model (with MDS in denominator). Quarterly forecast horizons, $h$. Evaluation window from 1990Q1 through 2023Q4 (and as far as realized values are available). Significance assessed by Diebold-Mariano tests using Newey-West standard errors with $h+1$ lags. ***, ** and * denote significance at the 1%, 5%, and 10% level, respectively.

# MARGINAL DATA DENSITIES

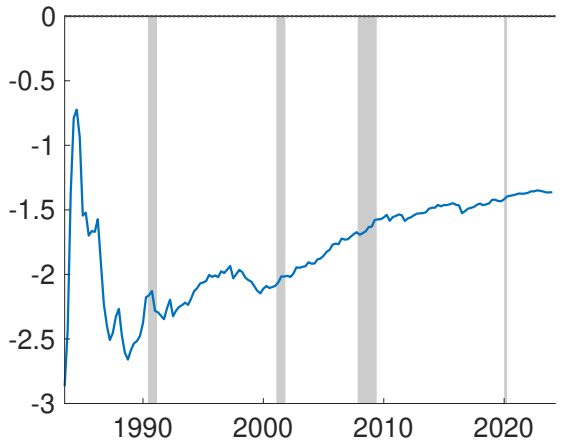
## Recursive mean differences: VAR less MDS

**MDS scores consistently better**

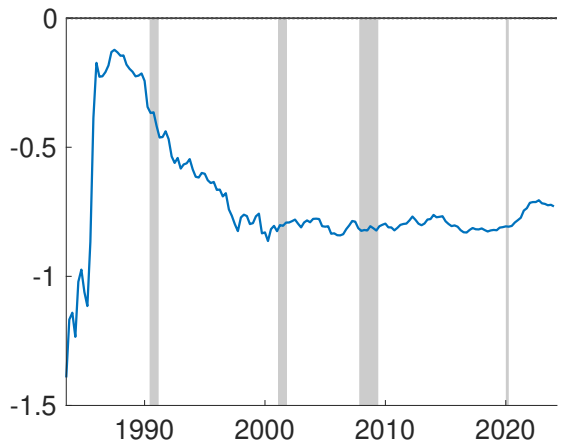
### RGDP



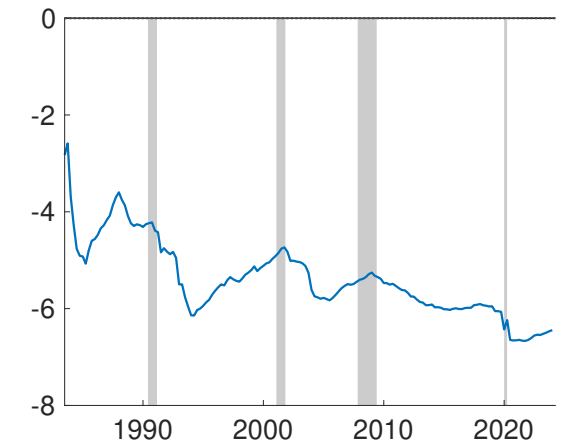

### CPI




### PGDP




### UNRATE



Considering predictions for future outcomes,

**MDS and VAR model are either on par,**
**or prefer the MDS model**

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# COIBION-GORODNICHENKO REGRESSIONS

## Coibion & Gorodnichenko (2015, AER)

CG regress forecast errors on last forecast update

$$y_{t+h} - F_t y_{t+h} = \alpha_h + \beta_h \left(F_t - F_{t-1}\right) y_{t+h} + \text{error}_{t+h}$$

and report significant slopes $\beta_h$

## CG slopes $\beta_h$ implied by VAR model

Conditional on parameter draws compute population regression

| Variable | 5% | 50% | 95% |
|---|---|---|---|
| RGDP | 0.02 | 0.12 | 0.23 |
| UNRATE | 0.06 | 0.15 | 0.27 |
| PGDP | 0.04 | 0.18 | 0.35 |
| CPI | 0.14 | 0.25 | 0.36 |

Similar in-sample fit as in empirical literature

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into a consistent term structure of expectations

- Matches observed SPF w/flexible outcome process
- Can be used to produce SEP-like fan charts
- Bayesian estimation with MCMC/Gibbs sampler

## Findings

- Error bands more nimble and more accurate than SEP
- Significant in-sample SPF bias
- But, potential bias hard to exploit out-of-sample

# ONGOING AND FUTURE WORK

## SPF-consistent responses to economic shocks

Our framework delivers estimates of SPF-consistent "shocks":

$$(\boldsymbol{F_t} - \boldsymbol{F_{t-1}}) \begin{bmatrix} \boldsymbol{y_t} \\ \boldsymbol{y_{t+1}} \\ \vdots \\ \boldsymbol{y_{t+H}} \end{bmatrix}$$

**To do:** Correlate with proxies for structural shocks
e.g., Gilchrist & Zakrajsek, 2012; Jarocinski & Karadi, 2020

## Incorporate SPF histograms

See ongoing companion work

## Application to other survey sources

ECB SPF, Consensus Economics and related sources ...

# APPENDICES

5. Data
6. Model details
7. Process for outcomes implied by our model
8. Term structures of expectations and uncertainty
9. Out-of-sample predictions of SPF point forecasts
10. Volatile imputations in noise-free models
11. Shifting endpoints of term structure of expectations
12. Incorporating histograms

# APPENDICES

# AVAILABILITY OF SPF POINT FORECASTS

| | Fixed-horizon | Fixed-event calendar years | | |
|---|---|---|---|---|
| Variable | $h = 0, \ldots, 4$ | next | 2-year | 3-year |
| Real GDP | 1968Q4 | 1981Q3 | 2009Q2 | 2009Q2 |
| Unemployment | 1968Q4 | 1981Q3 | 2009Q2 | 2009Q2 |
| GDP prices | 1968Q4 | 1981Q3 | NA | NA |
| CPI inflation | 1981Q3 | 1981Q3 | 2005Q3 | NA |

Note: Current-year SPF disregarded due to overlap w/quarterly fixed-horizon predictions.

# CALENDAR-YEAR FORECASTS FROM SPF



**Calendar-year data map into linear combinations of quarterly outcomes**

## For UNRATE and CPI: Average level

$$\bar{y}_t = \frac{1}{4} \times \sum_{j=0}^{3} y_{t-j}$$

Observe $F_t \bar{y}_{t+h}$ when $t+h$ is in Q4

## For RGDP and PGDP: Annual-average growth

$$\hat{y}_t = 100 \times \log\left(\frac{I_t + I_{t-1} + I_{t-2} + I_{t-3}}{I_{t-4} + I_{t-5} + I_{t-6} + I_{t-7}}\right)$$

$$\approx \sum_{j=0}^{6} w_j y_{t-j}$$

With "tent-shaped" weights $w_j$

# APPENDICES

# THREE ASSUMPTIONS ON THE TERM STRUCTURE

## 1) Flat term structure beyond horizon $H$

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## 3) Endpoint is common trend in outcomes and SPF

$$Y_t = \tilde{Y}_t + 1\, y_t^*\,, \qquad \lim_{j\to\infty} E_t\, \tilde{Y}_{t+j} = \bar{Y}$$

This means: deviations from rationality are mean-stationary

# STATE DYNAMICS

**Recall from previous slide:**

$$\boldsymbol{Y_t} = \tilde{\boldsymbol{Y}}_t + \mathbf{1}\, \boldsymbol{y}_t^* \,,$$
$$\boldsymbol{y}_t^* = \boldsymbol{y}_{t-1}^* + \boldsymbol{w}_t^*$$

**Accounting identities imply**

$$\tilde{\boldsymbol{Y}}_t = \left(\boldsymbol{I} - \tilde{\boldsymbol{\Psi}}\right) \bar{\boldsymbol{Y}} + \tilde{\boldsymbol{\Psi}} \tilde{\boldsymbol{Y}}_{t-1} + \tilde{\boldsymbol{\eta}}_t$$

- $\tilde{\boldsymbol{\Psi}}$ is a matrix of ones and zeros
- $\tilde{\boldsymbol{\eta}}_t$ are forecast updates adjusted for trend shocks and bias

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- $\tilde{\boldsymbol{\Psi}}$ is a matrix of ones and zeros
- $\tilde{\boldsymbol{\eta}}_t$ are forecast updates adjusted for trend shocks and bias

**Yet to be specified: Dynamics of $\tilde{\boldsymbol{\eta}}_t$**

# TWO MODEL VERSIONS: UNBIASED OR BIASED SPF

## 1) SPF has bias: predictable updates $\tilde{\eta}_t$

- Unconditional bias captured by $\bar{Y}$
- Conditional bias captured by VAR in $\tilde{\eta}$:

$$\tilde{\eta}_t = \tilde{\Pi}\,\tilde{\eta}_{t-1} + \tilde{\varepsilon}_t\,, \qquad \text{with} \quad \tilde{\varepsilon}_t \sim \mathcal{N}(0, \tilde{\Sigma}_t)$$

"VAR" model

## 2) SPF is unbiased: $\tilde{\eta}$ is unpredictable

$$\bar{Y} = 0\,; \tilde{\Pi} = 0$$

"MDS" model

since $\tilde{\eta}_t$ is a martingale difference sequence, $E_{t-1}\tilde{\eta}_t = 0$

# SHOCK DISTRIBUTIONS

## Overview

**Fat-tailed shocks with time-varying variances**
and conditionally Gaussian distributions

$$w_t^* \sim (0, \omega_t^*)\,, \quad \tilde{\varepsilon}_t \sim \mathcal{N}(0, \tilde{\Sigma}_t)$$

## Trend shock variances $\omega_t^*$

- horseshoe model allows for rare shifts in endpoints
- fat tailed prior w/substantial mass on zero

## Cyclical shock variance-covariance matrix $\tilde{\Sigma}_t$

- Two blocks: near- and far-term shocks
- Each block has a persistent SV factor and a short-lived inverse-gamma scale factor

# APPENDICES

# PROCESS FOR OUTCOMES IMPLIED BY MODEL

- The model describes joint dynamics of SPF and outcomes

- Innovations representation backs out process for $\boldsymbol{y_t}$:

$$\boldsymbol{y_t = \varepsilon_t + E(y_t|y^{t-1}) = \alpha(L)\varepsilon_t}$$

- In general: integrated moving average process (IMA)
- In case of MDS model: IMA$(1, H+2)$:

$$\boldsymbol{\Delta y_t = \Delta \varepsilon_t + \sum_{j=1}^{H+1} \tilde{\kappa}_{j+1} \cdot (\varepsilon_{t-j} - \varepsilon_{t-j-1})}$$

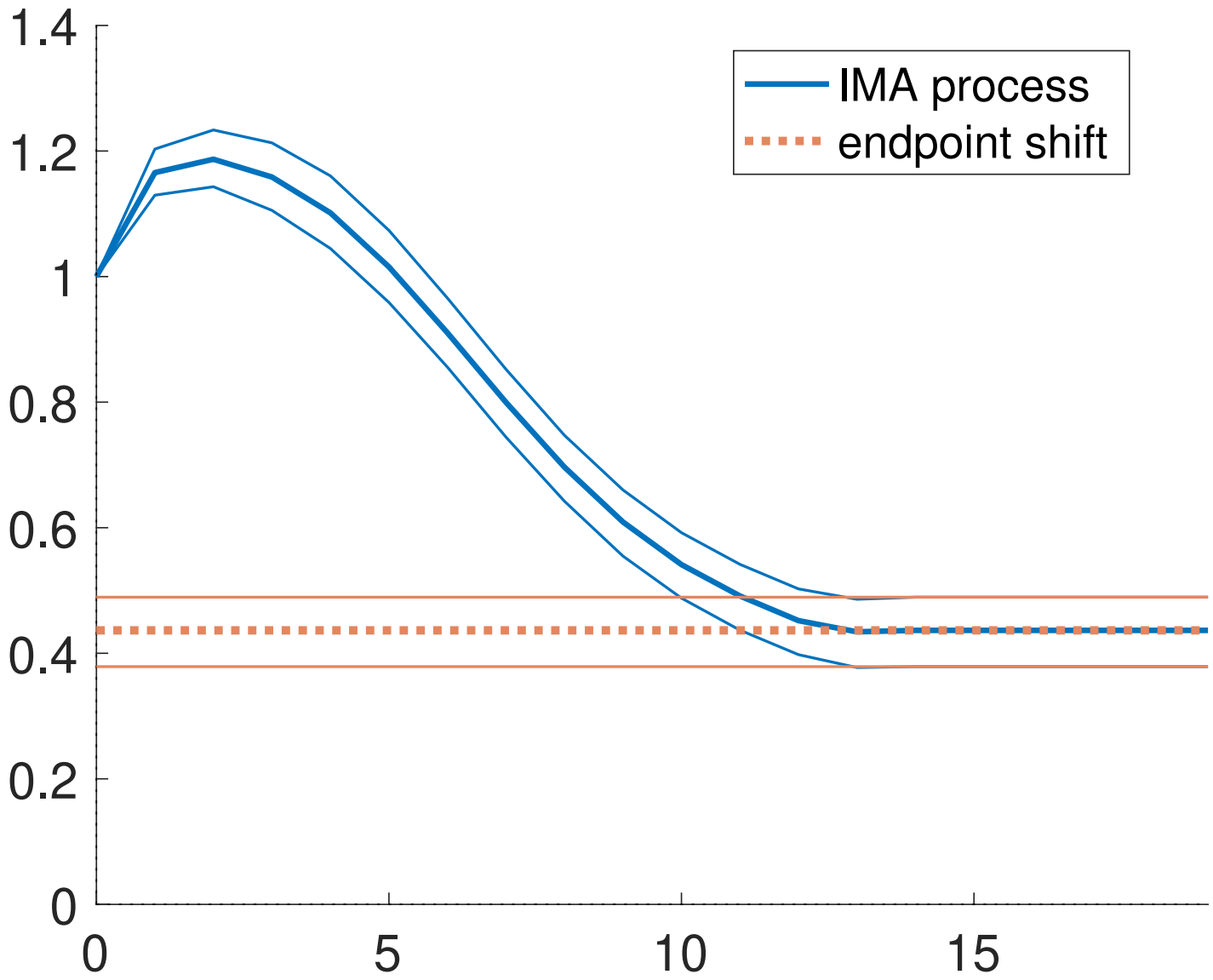
$$\boldsymbol{+ \kappa^* \cdot \varepsilon_{t-1}}$$

where $\tilde{\boldsymbol{\kappa}}_{j+1}$ and $\boldsymbol{\kappa}^*$ are Kalman gains on $\tilde{\boldsymbol{Y}}_{t+1}$ and $\boldsymbol{y}^*_{t+1}$

# IMPLIED PROCESS FOR UNRATE



## IMA coefficients



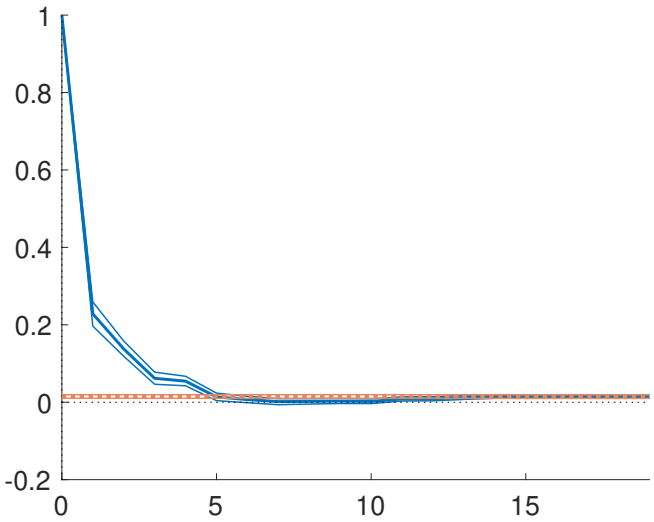

Note: Posterior median and 68% bands. Full-sample estimates.

# IMPLIED PROCESS FOR DIFFERENT VARIABLES

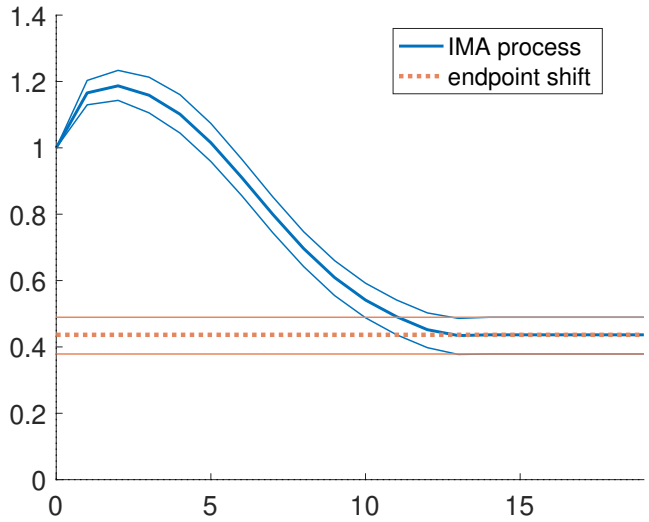
MDS

## IMA coefficients

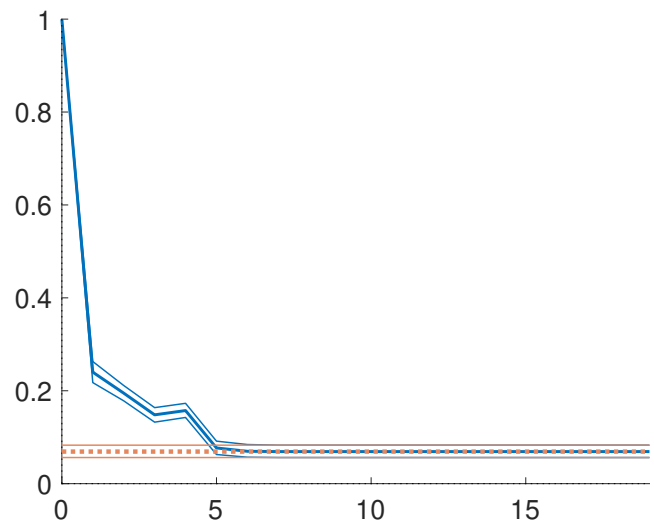
### RGDP



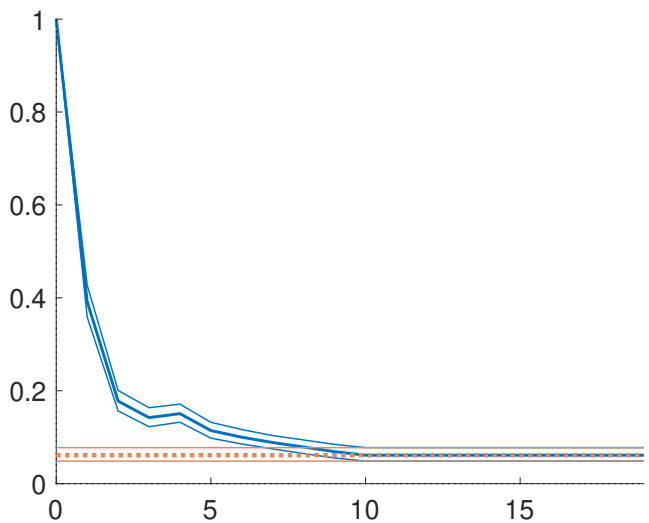

### UNRATE




### PGDP




### CPI




Note: Posterior median and 68% bands. Full-sample estimates.

**Unemployment rate:**

- Notable endpoint shift (40bp)
- Hump-shaped cyclical response, peaks about two quarters after impact

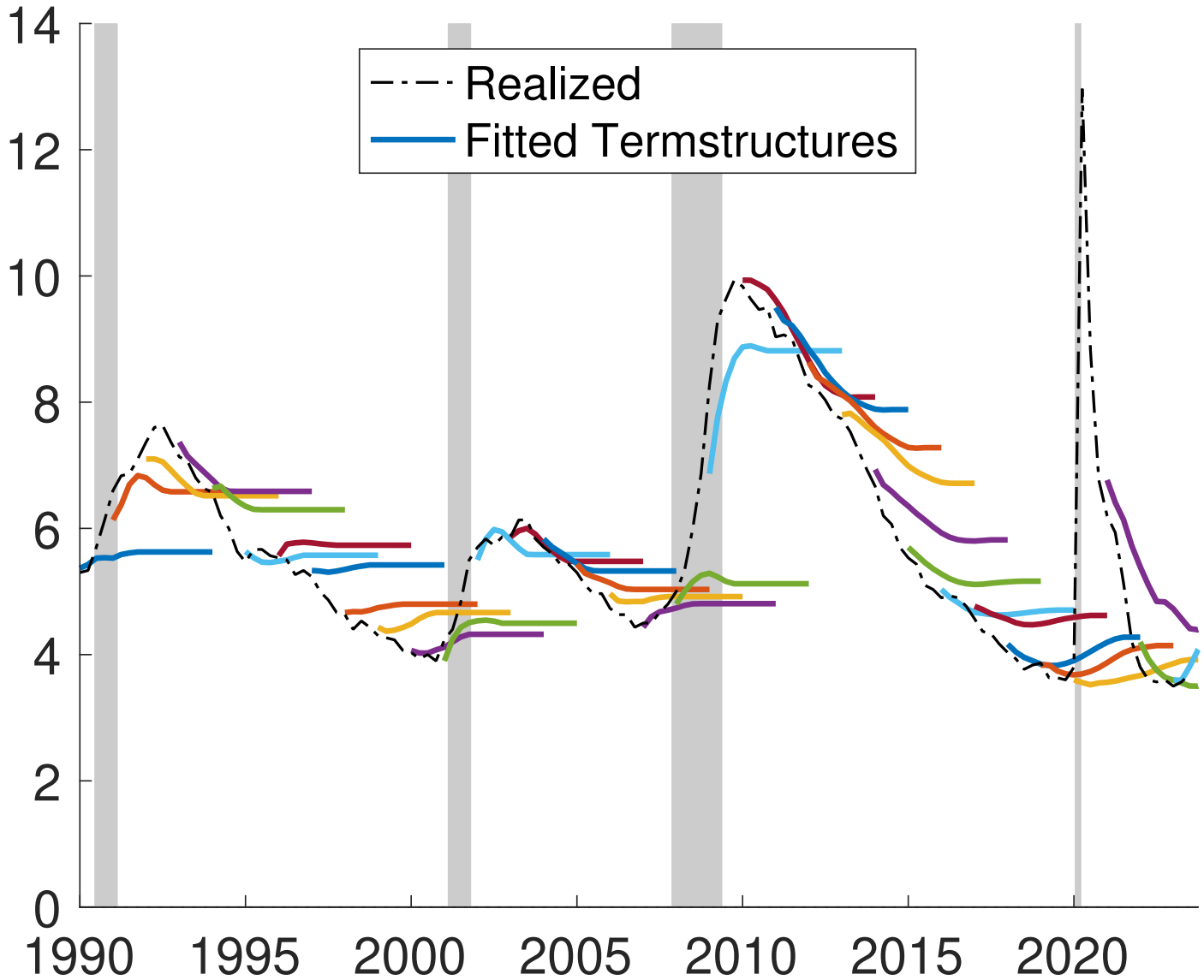
**Growth rates of real GDP, GDP prices and CPI:**

- Endpoint shift fairly small
- Largely monotonic decay after impact, peters out within five (RGDP, PGDP) or ten (CPI) quarters

# APPENDICES

# TERM STRUCTURE OF FORECASTS

## Unemployment rate: Quarterly SPF-consistent forecasts, MDS model




Showing only forecast origins in Q1. Out-of-sample forecasts. NBER recessions shaded.

# TERM STRUCTURE OF FORECASTS

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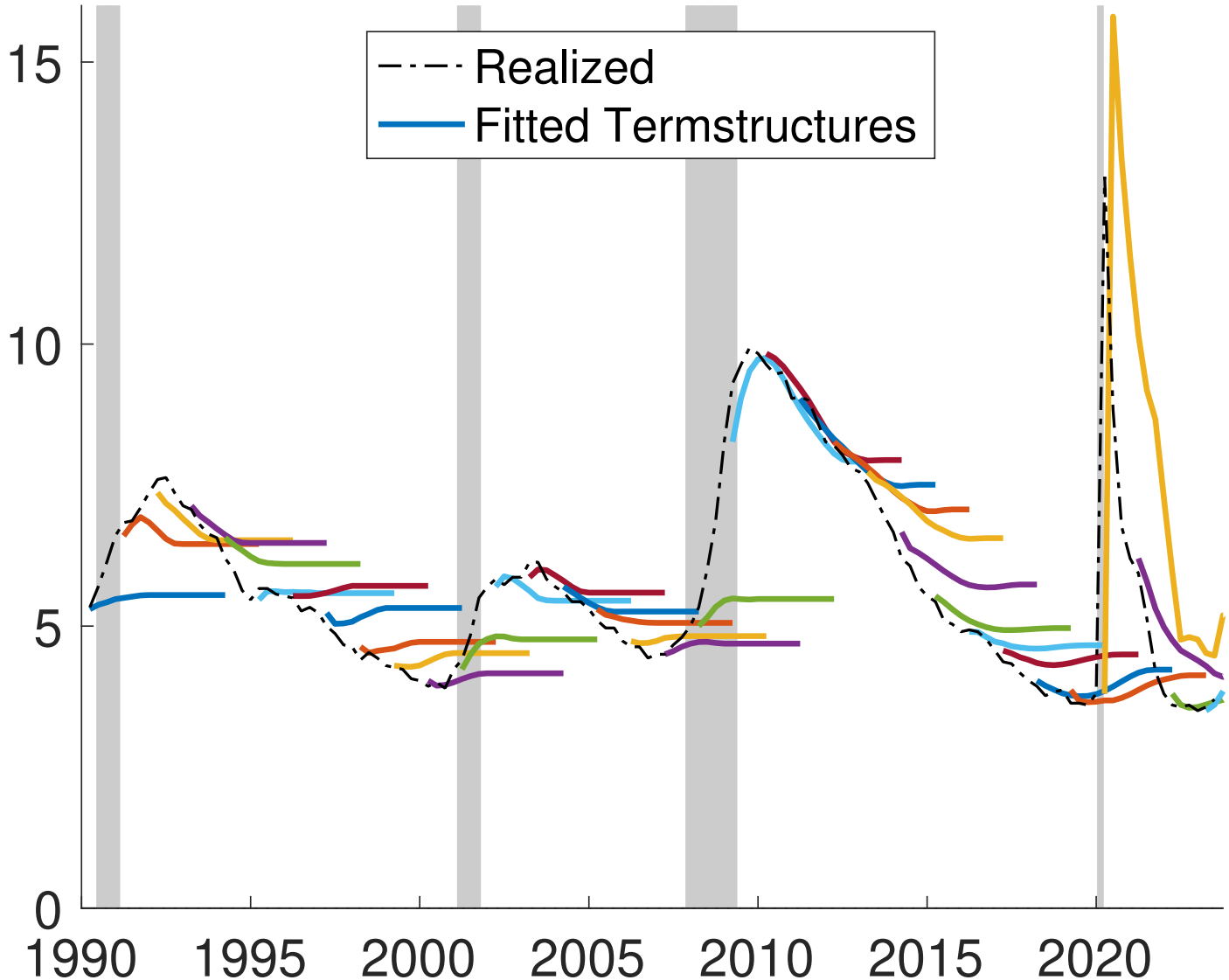
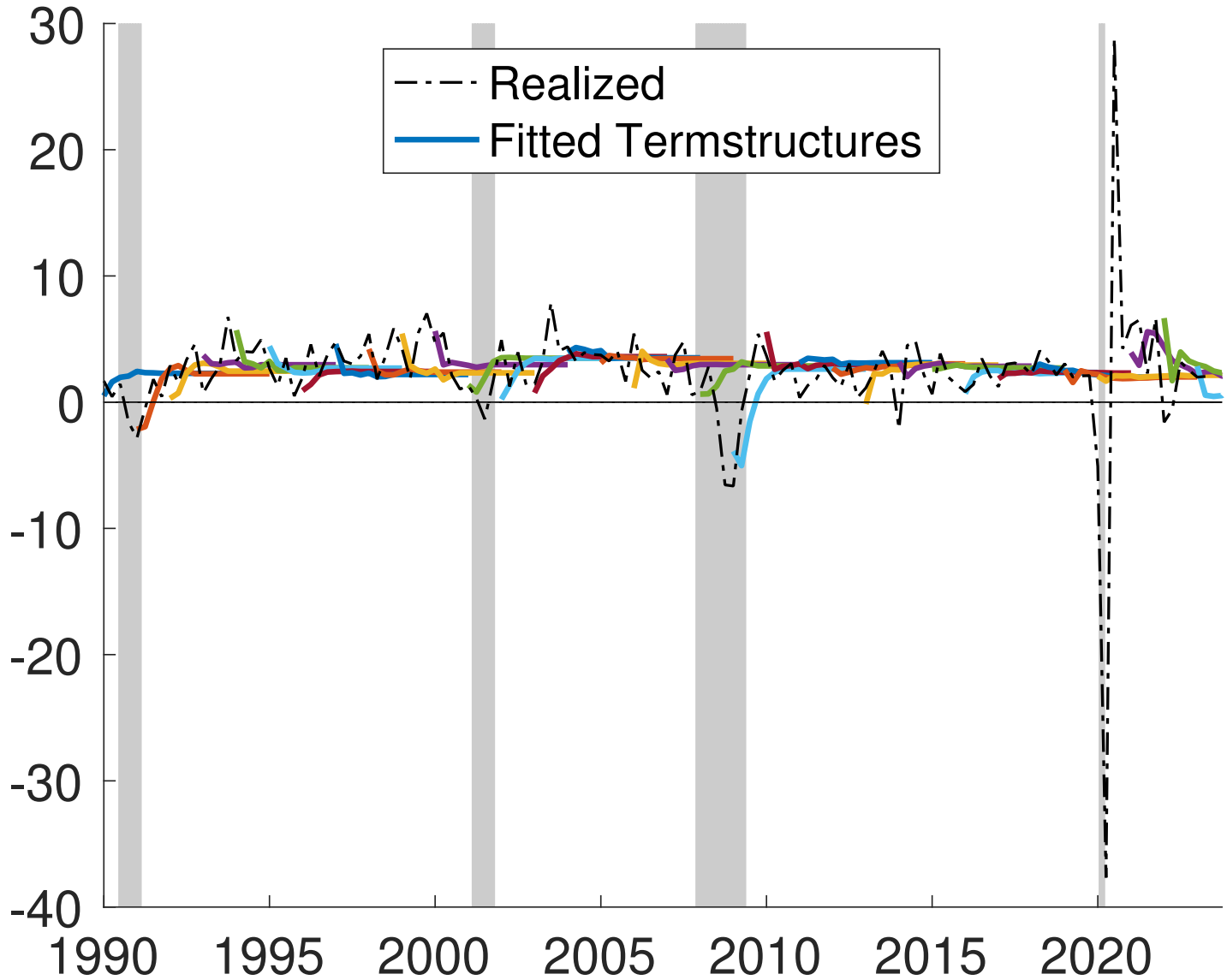



Showing only forecast origins in Q2. Out-of-sample forecasts. NBER recessions shaded.

# TERM STRUCTURE OF FORECASTS

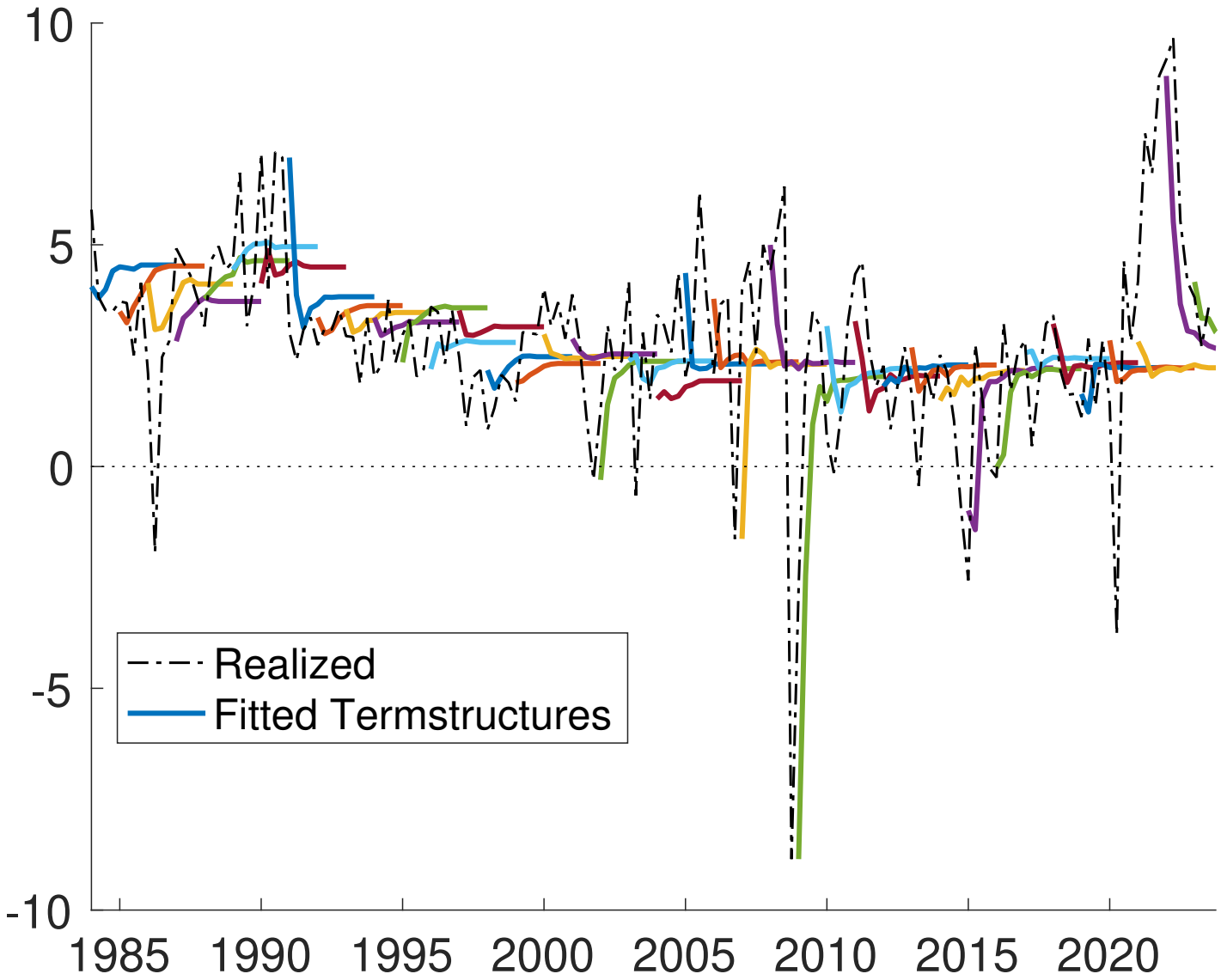
## GDP growth: Quarterly SPF-consistent forecasts, MDS model




Showing only forecast origins in Q1. Out-of-sample forecasts. NBER recessions shaded.

# TERM STRUCTURE OF FORECASTS

## CPI inflation: Quarterly SPF-consistent forecasts, MDS model

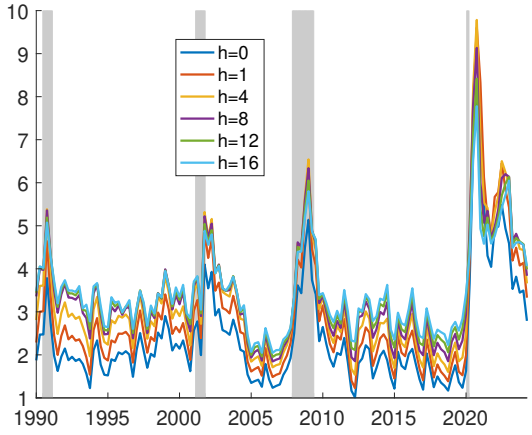



Showing only forecast origins in Q1. Out-of-sample forecasts. NBER recessions shaded.

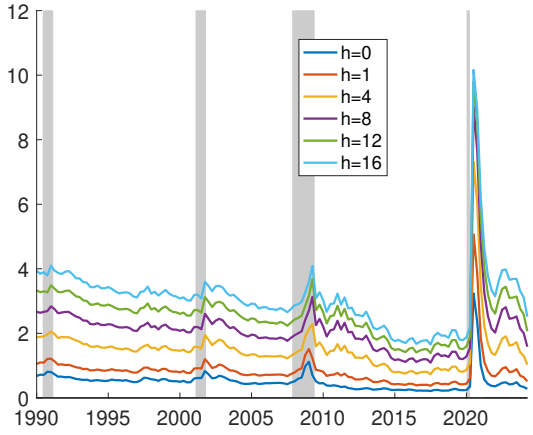
# TERM STRUCTURES OF UNCERTAINTY

## Width of predictive 68% bands, MDS model

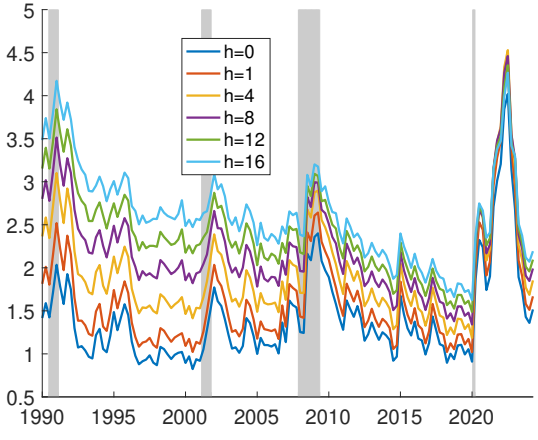
### RGDP



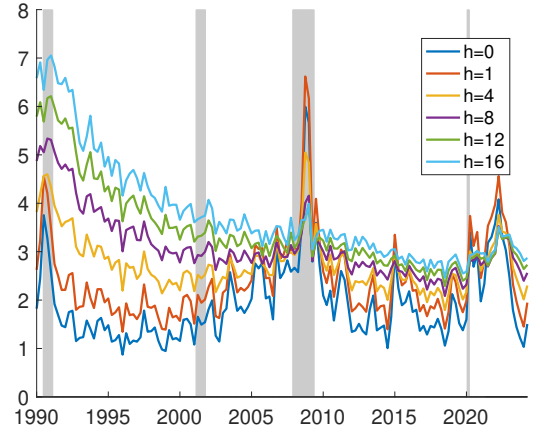

### UNRATE




### PGDP




### CPI




Out-of-sample forecasts. NBER recessions shaded.

Beyond $h = 8$, nearly flat term structure of uncertainty, except for UNRATE and pre-2000 CPI

Notable decline in inflation uncertainty in 1990s

Cyclical variations in uncertainty about real activity

# APPENDICES

# PREDICTING SPF POINT FORECASTS NEXT QUARTER

## Slopes of Mincer-Zarnowitz regressions

MDS and VAR slopes close to one, few rejections of $\beta = 1$

| | RGDP | | UNRATE | | PGDP | | CPI | |
|---|---|---|---|---|---|---|---|---|
| Forecast | MDS | VAR | MDS | VAR | MDS | VAR | MDS | VAR |
| h = 0 | 1.41 | 1.32 | 0.86 | 0.83 | 0.99 | 0.98 | 1.15 | 1.19 |
| | (0.24) | (0.25) | (0.08) | (0.10) | (0.05) | (0.06) | (0.16) | (0.15) |
| h = 1 | 1.02 | 1.02 | 0.91 | 0.86 | 1.02 | 0.99 | 1.01 | 1.00 |
| | (0.09) | (0.09) | (0.06) | (0.08) | (0.04) | (0.04) | (0.07) | (0.06) |
| h = 2 | 1.02 | 0.97 | 0.94 | 0.90 | **0.94** | **0.85** | 0.94 | 0.95 |
| | (0.08) | (0.08) | (0.06) | (0.07) | (0.03) | (0.03) | (0.04) | (0.05) |
| h = 3 | 0.94 | **0.80** | 0.96 | 0.91 | **0.92** | **0.86** | **0.92** | **0.92** |
| | (0.09) | (0.09) | (0.05) | (0.06) | (0.04) | (0.03) | (0.04) | (0.04) |
| h = 4 | **0.87** | **0.57** | 0.97 | 0.93 | **0.91** | **0.90** | 0.94 | **0.89** |
| | (0.06) | (0.09) | (0.05) | (0.06) | (0.03) | (0.04) | (0.04) | (0.03) |
| y = 1 | 0.94 | 0.91 | 0.96 | 0.93 | **0.93** | **0.91** | 0.98 | 0.96 |
| | (0.09) | (0.07) | (0.05) | (0.06) | (0.03) | (0.03) | (0.05) | (0.05) |
| y = 2 | 0.94 | 0.96 | 0.92 | 0.95 | — | — | 0.85 | **0.74** |
| | (0.09) | (0.10) | (0.07) | (0.04) | — | — | (0.11) | (0.09) |
| y = 3 | 0.95 | 0.59 | **0.76** | 0.95 | — | — | — | — |
| | (0.06) | (0.24) | (0.08) | (0.04) | — | — | — | — |

Note: Out-of-sample forecasts, evaluation window 1990Q1 – 2023Q4

# PREDICTING SPF POINT FORECASTS NEXT QUARTER

## Slopes of Mincer-Zarnowitz regressions

Noise-free models have slope further away from unity

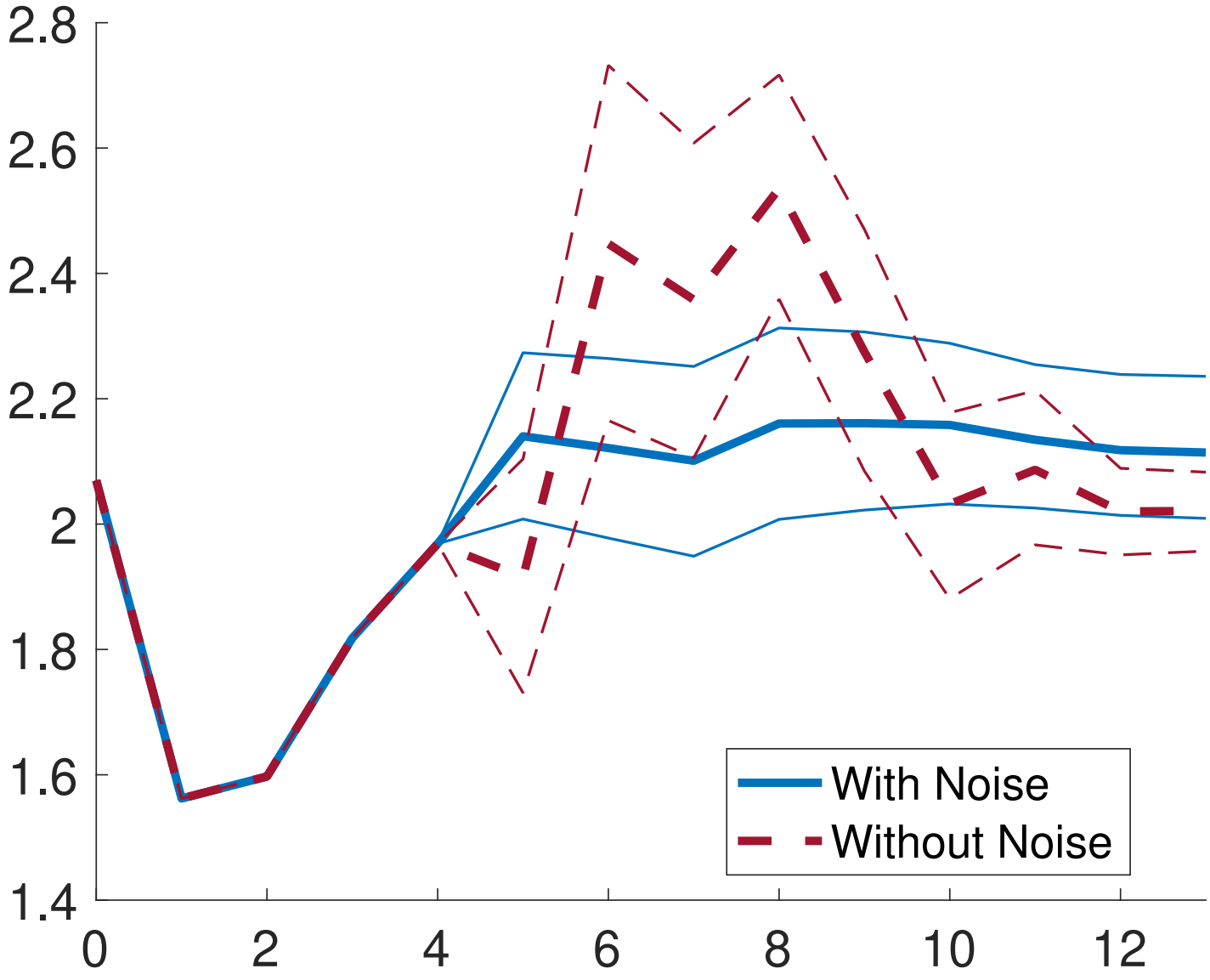
| | RGDP | | UNRATE | | PGDP | | CPI | |
|---|---|---|---|---|---|---|---|---|
| Forecast | MDS | VAR | MDS | VAR | MDS | VAR | MDS | VAR |
| h = 0 | 1.41 (0.24) | 1.35 (0.26) | 0.86 (0.08) | 0.85 (0.09) | 0.99 (0.05) | 0.96 (0.06) | 1.15 (0.16) | 1.21 (0.16) |
| h = 1 | 1.02 (0.09) | 1.02 (0.08) | 0.91 (0.06) | 0.88 (0.08) | 1.02 (0.04) | 0.98 (0.04) | 1.01 (0.07) | 1.03 (0.06) |
| h = 2 | 1.02 (0.08) | 0.97 (0.08) | 0.94 (0.06) | 0.90 (0.06) | **0.94** (0.03) | 0.99 (0.04) | 0.94 (0.04) | 0.93 (0.04) |
| h = 3 | 0.94 (0.09) | **0.37** (0.06) | 0.96 (0.05) | 0.96 (0.06) | **0.92** (0.04) | **0.37** (0.03) | **0.92** (0.04) | **0.75** (0.07) |
| h = 4 | **0.39** (0.07) | **0.40** (0.05) | 0.97 (0.05) | 0.93 (0.05) | **0.47** (0.06) | **0.89** (0.04) | **0.80** (0.06) | **0.82** (0.04) |
| y = 1 | 0.91 (0.05) | **0.83** (0.05) | 0.96 (0.06) | 0.94 (0.06) | 0.94 (0.04) | **0.80** (0.05) | 0.94 (0.04) | **0.86** (0.06) |
| y = 2 | 0.87 (0.07) | **0.73** (0.11) | 1.00 (0.04) | 0.94 (0.03) | — — | — — | **0.74** (0.12) | **0.66** (0.12) |
| y = 3 | **0.87** (0.06) | **0.58** (0.14) | 0.95 (0.04) | 0.94 (0.04) | — — | — — | — — | — — |

Note: Out-of-sample forecasts generated from models without measurement error, 1990Q1 – 2023Q4.

# APPENDICES

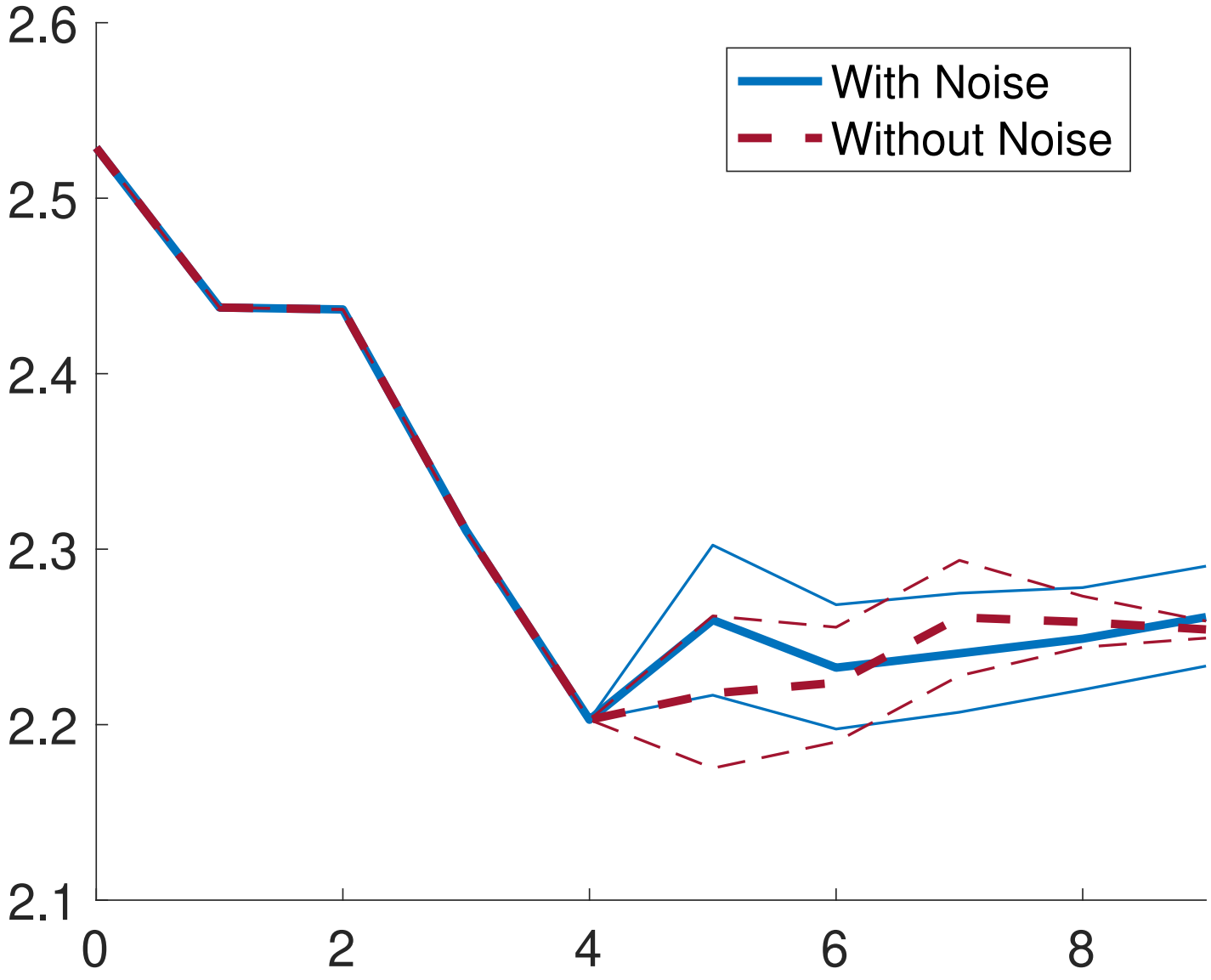
# VOLATILE IMPUTATIONS W/O NOISE

**RGDP per 2024Q1 with and without measurement noise in annual SPF**



# VOLATILE IMPUTATIONS W/O NOISE

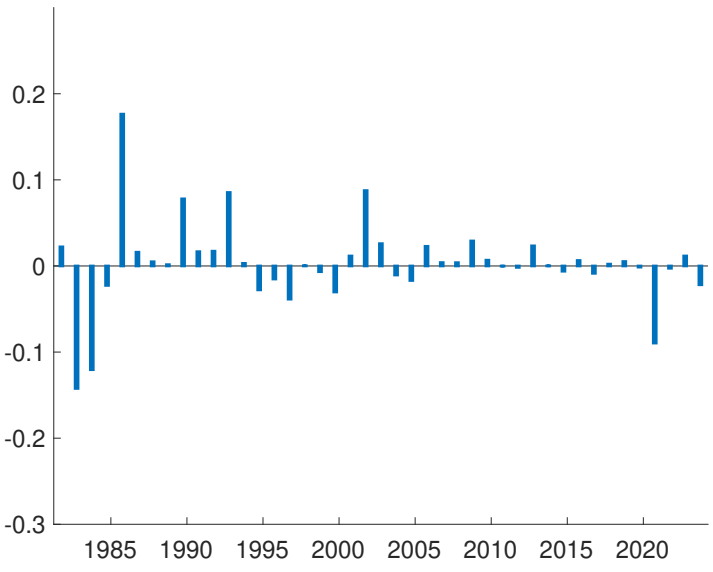
**CPI per 2024Q1 with and without measurement noise in annual SPF**



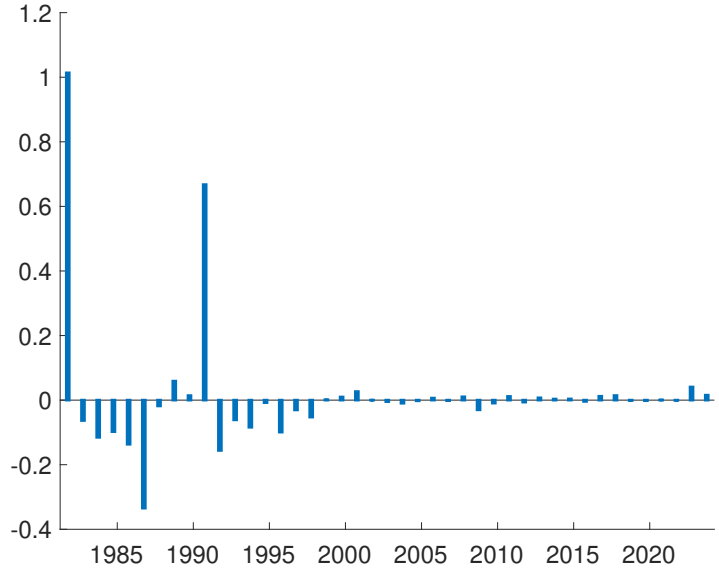
# OBSERVED DISCREPANCIES IN SPF DATA

## In Q4, perfect overlap between next-year SPF and $Z_{q,t}$

### RGDP




### CPI



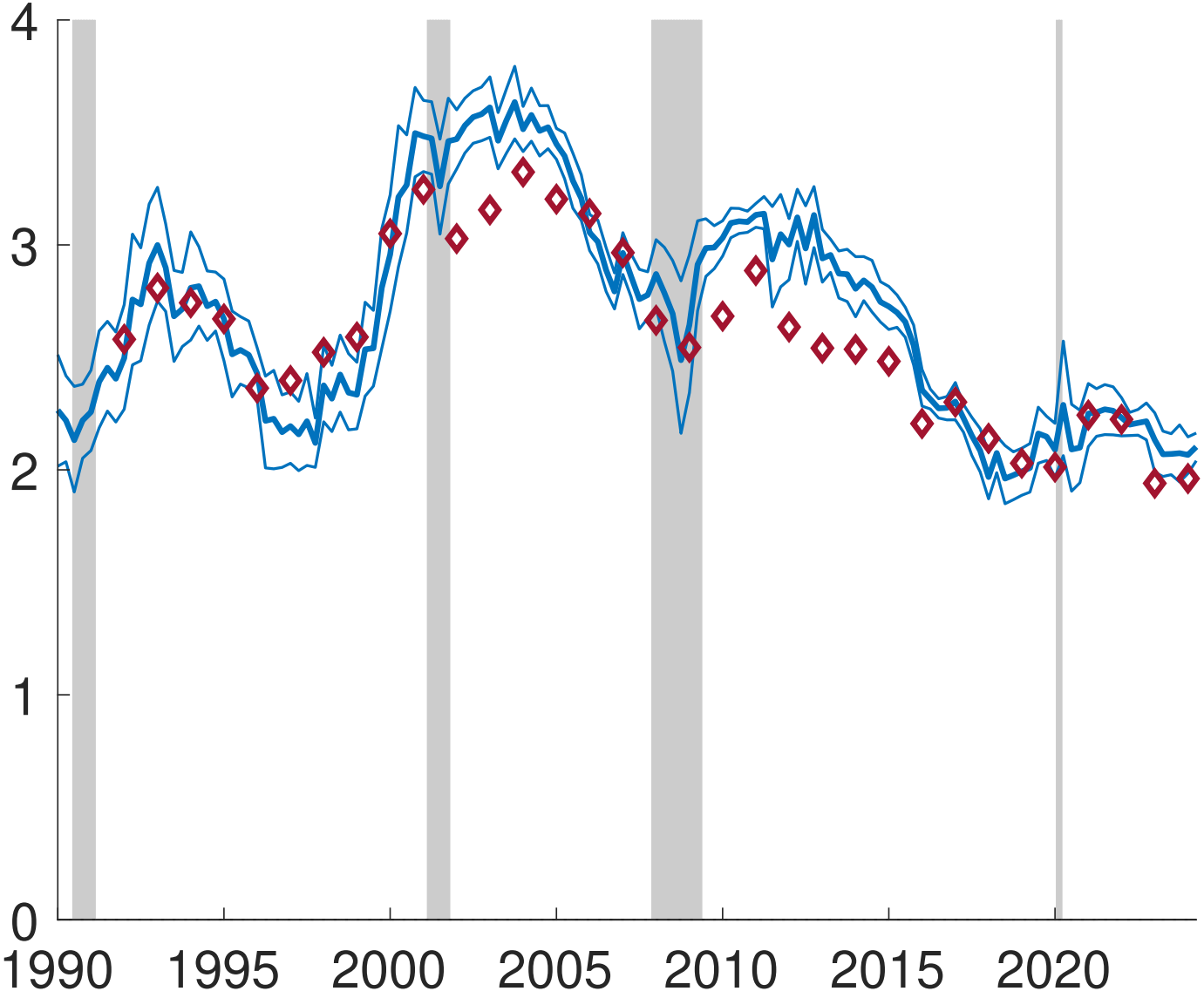

Irregular, but sizable discrepancies

Separate horseshoe models for data in different quarters (not only Q4)

# APPENDICES

# SHIFTING ENDPOINT ESTIMATES

## RGDP: Real-time estimates of $y_t^*$, Red diamonds are SPF long-run forecasts




Note: MDS model. Using available data since 1968Q4. NBER recessions shaded.

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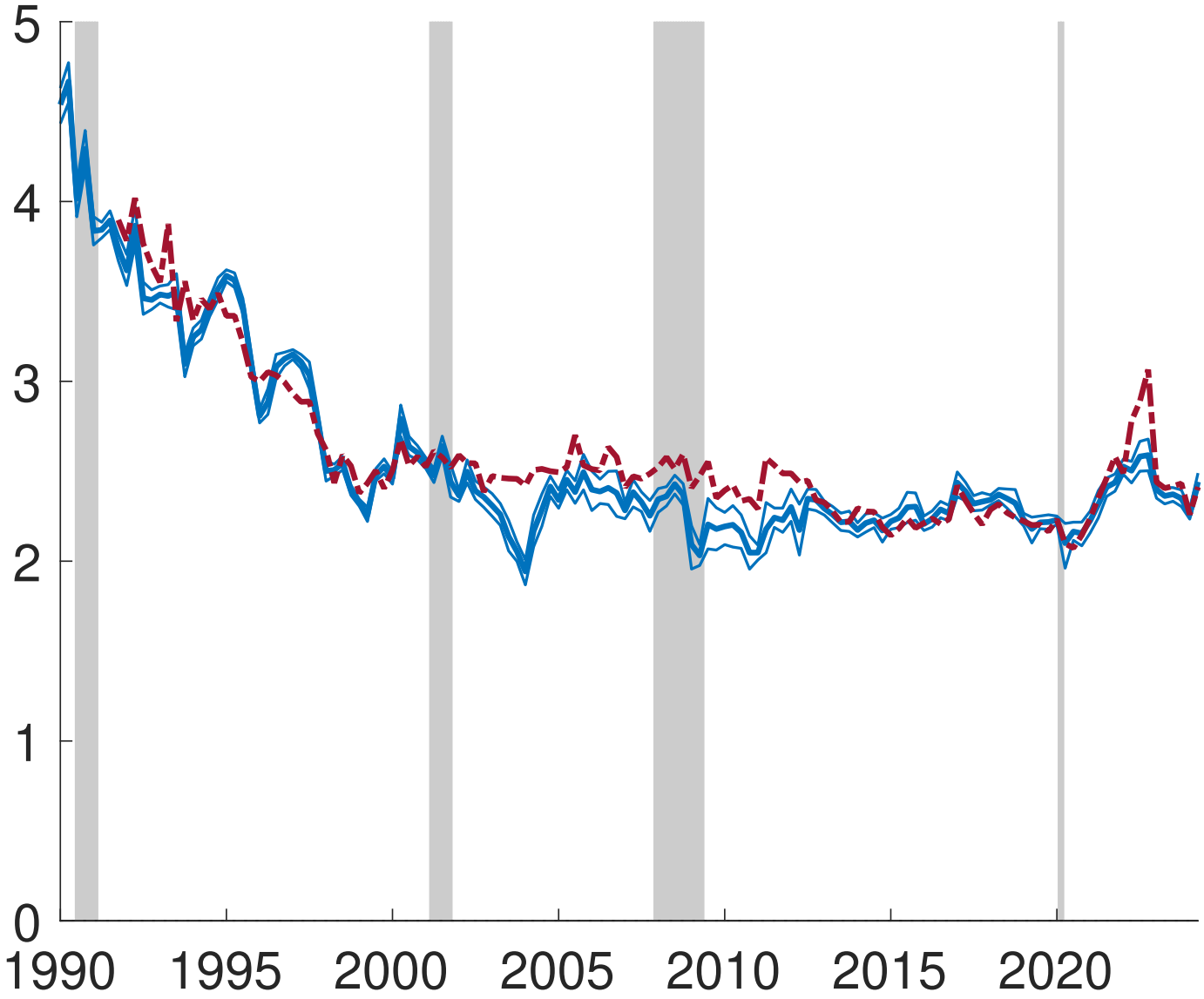
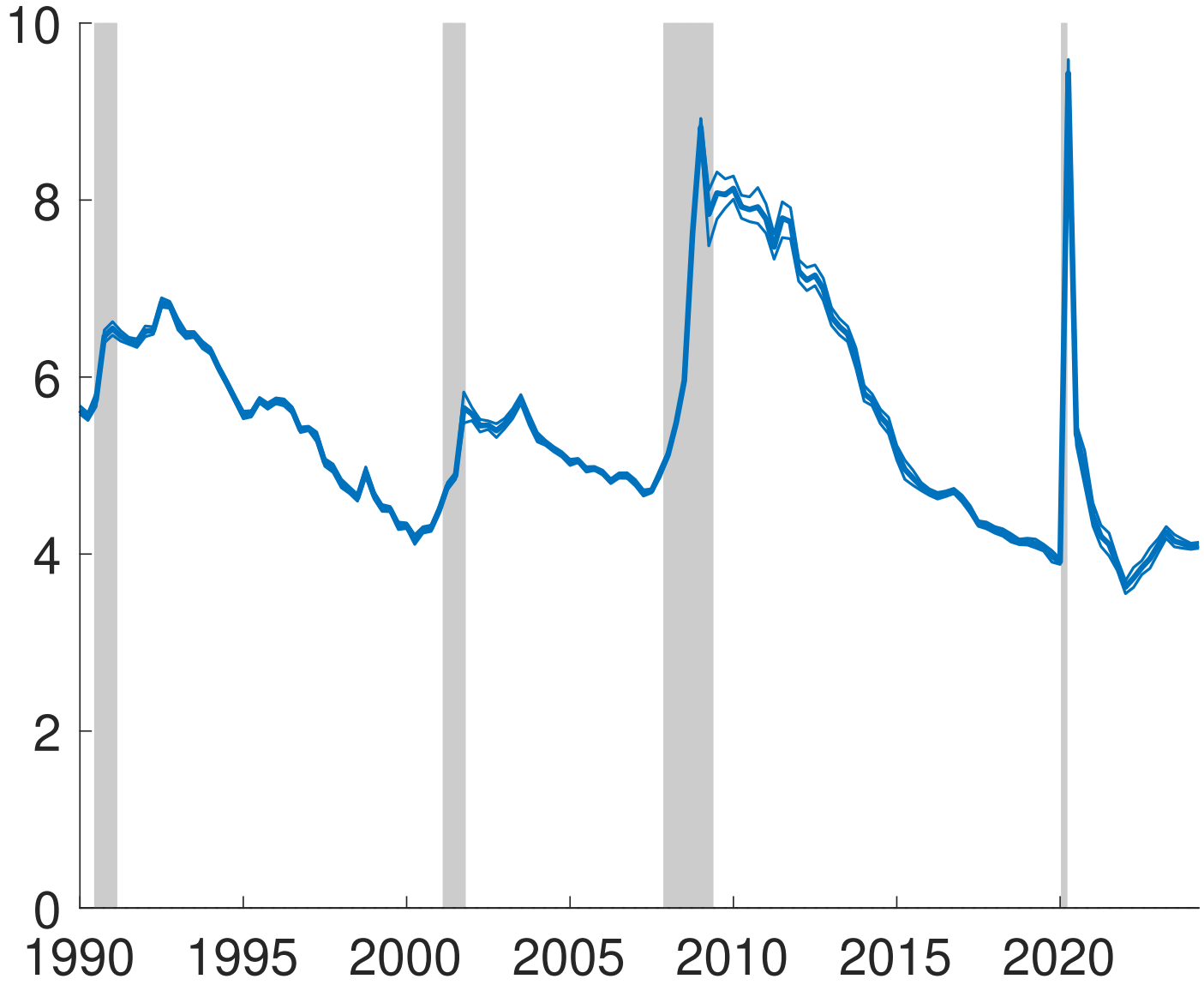



Note: MDS model. Using available data since 1968Q4. NBER recessions shaded.

# SHIFTING ENDPOINT ESTIMATES

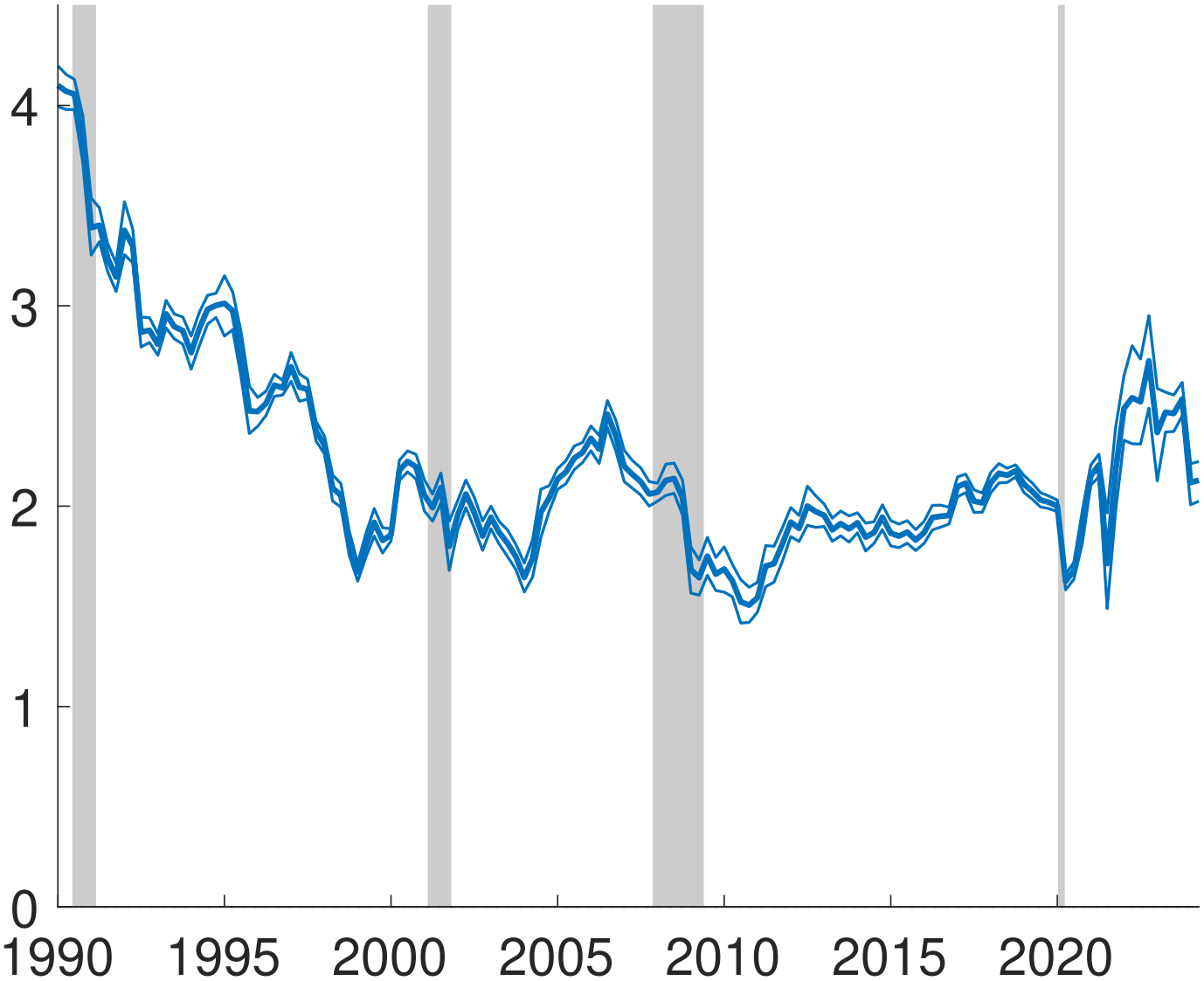
## UNRATE: Real-time estimates of $y_t^*$




Note: MDS model. Using available data since 1968Q4. NBER recessions shaded.

# SHIFTING ENDPOINT ESTIMATES

## PGDP: Real-time estimates of $y_t^*$




Note: MDS model. Using available data since 1968Q4. NBER recessions shaded.

# APPENDICES

# TILTING THE MODEL TO MATCH SPF HISTOGRAMS

**Clark, & Mertens (2023, work in progress)**

- SPF also collects subjective probability forecasts (in form of histograms)
- Potentially attractive data, though with varying views on its predictive value (Clark & Mertens, ORE, forth.)
- Typically: Fit parametric distribution to histogram

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**Clark, & Mertens (2023, work in progress)**

- SPF also collects subjective probability forecasts (in form of histograms)
- Potentially attractive data, though with varying views on its predictive value (Clark & Mertens, ORE, forth.)
- Typically: Fit parametric distribution to histogram
- **Our work:**
  - "Tilt" output of time series model (like CGM) to perfectly match entire histogram
  - . . . while otherwise preserving information embedded in model (aka "entropy")
  - New: Fast computation based on analytic solutions
  - So far: Center of histogram most informative

# SUMMARY

## Our contributions:

Model that transforms an arbitrary set
of fixed-event/-horizon SPF data
into a consistent term structure of expectations

- Matches observed SPF w/flexible outcome process
- Can be used to produce SEP-like fan charts
- Bayesian estimation with MCMC/Gibbs sampler

## Findings

- Error bands more nimble and more accurate than SEP
- Significant in-sample SPF bias
- But, potential bias hard to exploit out-of-sample