Providing Benefits to Uninformed Workers:
A Search Model with Learning about the Complex Attribute
(PRELIMINARY)

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Abstract

In this paper, I analyse a dynamic search model with learning about a "complex attribute" of a selected alternative. The model is applied to study the impact of workers' imperfect understanding of pecuniary benefits (e.g., employer-sponsored pensions or health insurance) at the time of job search on multidimensional compensation packages offered in equilibrium. First, an equilibrium with uniformly high benefits is more difficult to sustain than an equilibrium with uniformly low benefits, which ultimately harms firms' profits. Second, the presence of "uninformed workers" who do not update their beliefs about associated benefits when sampling a specific wage offer generates additional equilibria with spurious differentiation in benefits. I connect the model's predictions to the empirical literature on the provision of workplace pensions.

JEL: D83, D91, J31, J32, J33

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1 Introduction

Workplace benefits constitute an increasing share of total compensation. For example, in 2022 an average employee in the US received 31% of their total compensation in the form of various pecuniary benefits, most importantly health insurance, paid leave, and retirement benefits.[1]

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[1] Compiled from the data of US Bureau of Labor Statistics.

What is more, these benefits tend to be more unequally distributed than wages and thus wage-based measures of income inequality underestimate the extent of total compensation inequality (Kristal et al., 2020; Ouimet and Tate, 2023; Pierce, 2001).

At the same time, there is abundant evidence that an average individual is ill-equipped to evaluate complex financial contracts (Beshears et al., 2018; Lusardi and Mitchell, 2014). Focusing on workplace pensions, many individuals have poor understanding of the available benefits (Agnew et al., 2012; Gustman and Steinmeier, 2005), which may well explain some robust behavioural patterns, such as status quo bias in choosing contribution rates and investment allocations (Madrian and Shea, 2001; Samuelson and Zeckhauser, 1988; Thaler and Benartzi, 2004) and not taking full advantage of the available employer's match (Choi et al., 2011; Engelhardt and Kumar, 2007; Mitchell et al., 2007). However, there is also evidence indicating that employees' understanding of the available benefits improves with tenure, which could be driven by social interactions among colleagues (Duflo and Saez, 2002, 2003).[2]

In this paper, I study the impact of workers' imperfect understanding of workplace benefits at the time of job search, and subsequent learning, on multidimensional compensation packages offered in equilibrium. Methodologically, I build on and extend the literature on exploitative contracting (Heidhues and Kőszegi, 2018) by accounting for the dynamic effects of the agent learning about the complex attribute of a selected alternative. While the framework can be readily applied to a problem of a boundedly rational consumer learning about add-on price or quality of a product, here I adopt labels specific to the labour market application.

More precisely, I consider a setting in which each compensation package consists of a *simple attribute* ("wage") as well as a *complex attribute* ("benefits"). While wages are continuous, benefits can take one of two values (high or low). There is a continuum of homogeneous workers and firms. The firms' provision costs and the workers' preferences are such that offering high benefits to every worker is efficient.

A worker searching for a job observes the sampled wages perfectly, but learns the true value of the associated benefits only after accepting a particular offer. Thus, I effectively model the benefits component of a compensation package as an experience good. After learning the value of the benefits she receives, the worker can remain in current employment or search on the job. With benefits being unobservable prior to acceptance, an equilibrium with uniformly high benefits is more difficult to sustain than an equilibrium with uniformly low benefits, which ultimately harms firms' profits (Proposition 1). This result suggests a potential obstacle to delegating the provision of benefits to firms when workers are imperfectly informed.

[2]A recent survey found that 46% of respondents do not know what investment options are available in their workplace pension plan. This proportion decreases with age, ranging from 54% among individuals aged 18-34 to 37% among individuals above 65 (see Your Money Poll 2023, accessed 22.02.2024).

Next, I extend the model by introducing a fraction of "uninformed workers" who, in addition to learning the true value of benefits on the job, do not update their beliefs about associated benefits when sampling a specific wage offer.[3] The presence of uninformed workers generates additional equilibria with spurious differentiation in benefits. First, there may exist "perverse equilibria", in which high-wage jobs also provide high benefits, and vice versa (Proposition 2). In these equilibria, only uninformed workers accept dominated offers with low wages and low benefits, and these jobs necessarily induce higher turnover. Second, there may also exist "fictitious compensating differentials equilibria", in which high wages are paired with low benefits, and vice versa (Proposition 3). Although qualitatively similar to the prediction of the classical compensating differentials theory (Rosen, 1974, 1986), the logic behind these equilibria and their efficiency properties are starkly different.

Related literature. This paper contributes to several strands of the literature. First, modelling the benefits component of an offer as unobservable during job search, I build on and extend the literature in behavioural industrial organisation studying the design of products with hidden attributes, see Heidhues and Kőszegi (2018) for a review. In related papers, Heidhues et al. (2021) and Gamp and Krähmer (2022, 2023) introduced limited attention and misperceptions, respectively, into the model of consumer search.[4]

To the best of my knowledge, no previous work accounts for the possibility that an agent eventually learns the true value of the hidden attribute chosen by the provider and might search again for a better alternative. Thus, the methodological contribution of this paper is to analyse the equilibrium effects of learning about the hidden attribute and possible recontracting.[5]

Second, I contribute to the literature on behavioural labour economics (Dohmen, 2014)

[3] This corresponds to correlation neglect or cursedness (Eyster, 2019).

[4] Chen et al. (2022) study a two-period model where short-lived, rational consumers search for horizontally differentiated experience goods and firms have an incentive to build up reputation by providing high quality. They restrict attention to equilibria with uniform pricing in the first period, which precludes learning about the quality prior to purchase. Such equilibria can only be sustained when firms' costs are heterogeneous.

[5] In a recent paper, Schumacher (2024) departs from the standard assumption of myopia and analyses a model in which a boundedly rational consumer correctly anticipates the value of the hidden attribute on the equilibrium path, but potentially misperceives it for an alternative she never selects. In terms of other dynamic aspects of competition in markets with hidden attributes, Johnen (2019) and Murooka and Schwarz (2019) analyse the design of automatic-renewal contracts when consumers differ in their propensity to passively accept the automatic renewal after learning their (exogenous) utility from continued consumption. Johnen (2020) studies competition between firms who learn private information about their customers' naiveté. The consumers remain unaware that they are being exploited, however. Heidhues et al. (2023) analyse a model where firms offer an initial price and a switching price. Consumers observe all current and future prices perfectly, but may procrastinate on switching.

by modelling a multidimensional compensation package consisting of observable wage and unobservable benefits. The existing work, in contrast, analyses the effects of workers' present bias (Bubb and Warren, 2020; DellaVigna and Paserman, 2005; Englmaier et al., 2023a,b) and misperceptions of job finding probability (Spinnewijn, 2015) and value of the outside option (Jäger et al., 2024).

Third, the theoretical results of the present paper complement the empirical literature on the provision of workplace benefits (e.g., Cole and Taska, 2023; Ouimet and Tate, 2023) and compensating differentials (Lavetti, 2023). I discuss the model's implications for interpretation of the empirical patterns in section 4.

The remainder of this paper is structured as follows. In section 2, I characterise the boundedly rational worker's search behaviour for a given distribution of offers. In section 3, I endogenise the distribution of wages and benefits offered in equilibrium. In section 4, I discuss the empirical relevance of the model's predictions. Section 5 concludes.

2 Search model with complex attributes

2.1 Setup

Consider the following model of search and learning about the complex attribute of an offer. There are two periods, $t = 1, 2$. In period 1, an agent searches for a job for the first time, performing random, sequential search with perfect recall until she accepts an offer, as in McCall (1970). Each search imposes a fixed cost of $c_1 > 0$. Having accepted an offer, the agent derives utility from being in employment, thus learning the true value of the associated complex attribute. In period 2, she decides whether or not to search on the job at a cost $c_2 \geq c_1$.[6] If the agent is not searching on the job, she derives the same utility from being in employment as in period 1. Otherwise, she searches on the job until she accepts an alternative offer and subsequently derives the associated utility.

The total utility from being in employment is a function of an observable and a complex attribute. An agent who is sampling job offers can perfectly observe and compare their observable attributes. However, she only learns the true value of the complex attribute of a particular offer after accepting it. Denoting the observable attribute by w ('wage') and the complex attribute by b ('benefits'), the total utility of an agent i employed at firm k for a single period is:

[6]It is standard in the literature to assume that the search cost increases on the job, either due to classical economic reasons, such as a a higher opportunity cost of time (e.g., Burdett, 1978), or psychological factors, such as time-inconsistent preferences or loss aversion (e.g., Karle et al., 2023).

$$u_{ik} = w_k + b_k.$$

Suppose that the benefits chosen by the employer can be either 'high' or 'low', i.e. $b_k \in \{\underline{b}, \bar{b}\}$ for some $\bar{b} > \underline{b} > 0$.[7] The agent's objective is to maximise the expected utility from being employed in two periods, net of her search costs. For notational simplicity, I omit time discounting. The value of the outside option (staying out of the labour market) is normalised to 0.

2.2 Beliefs and search behaviour

In this section I analyse the search behaviour of an agent facing an exogenously given distribution of multidimensional job offers. While I focus on boundedly rational agents for whom the complex component of an offer effectively remains hidden while searching, there are many degrees of freedom in specifying how the agents form their beliefs about b_k. In this paper, I distinguish between "informed" and "uninformed workers", defined as follows. For an *informed worker*, the complex attribute of an offer remains hidden during search, effectively making it an experience good (Nelson, 1970). An informed worker nonetheless updates her beliefs about benefits after observing a particular wage offer, based on the joint distribution of wages and benefits. For an *uninformed worker*, the complex attribute of an offer is also an experience good, but she fails to take into account the correlation between the wage and benefits, even though she has correct beliefs about the marginal distributions of wages and benefits. In other words, I invoke a specific form of correlation neglect in order to model a worker who in a sense ignores the benefits while searching.[8] Note, however, that both informed and uninformed workers internalise the fact that they will learn the value of the complex attribute after acceptance. Thus, I depart from the approach prevalent in the behavioural industrial organization literature (Heidhues and Kőszegi, 2018), where it is standard to assume that a myopic agent simply ignores the hidden component of an offer. Towards the end of section

[7] The existing literature accounts for the possibility that a worker might learn the value of an idiosyncratic match quality, say ϵ_{ik}, on the job (Rogerson et al., 2005). As long as ϵ_{ik} is i.i.d. with mean zero and the worker's utility is additive, including this feature would not substantially affect the model's predictions. Nevertheless, its mechanics are radically different when the complex component of an offer is (at least partly) a choice variable of the employer.

[8] Such belief formation can be micro-founded with the costs of information acquisition. In that vein, Heidhues et al. (2021) analyse when a boundedly rational individual prefers to "browse" multiple offers superficially rather than "study" a single offer in detail. For evidence on correlation neglect and its relation to complexity see Enke and Zimmermann (2019). Formally, correlation neglect can be captured as manifestation of cursed beliefs (Eyster and Rabin, 2005) or analogy-based reasoning (Jehiel, 2005, 2022). See also Eyster (2019) for a taxonomy of errors that people make in strategic environments, summary of the lab and field evidence, and an overview of game-theoretic models incorporating these errors.

3, I discuss which predictions of the model depend on specific behavioural assumptions.

Solving the model backwards, I characterise a search strategy adopted by either type for a given distribution of offers.

Period 2. Worker i employed at firm k derives, and observes, the total utility of being employed $u_{ik} = w_k + b_k$. If the worker searches on the job, her value function of sampling a wage offer w is:

$$v_2^j(w) = \max\{w + \mathbb{E}^j[b \mid w]\ ;\ v_2^j - c_2\},$$

where $j \in \{I, U\}$ denotes the worker's type (informed or uninformed), $v_2^j = \int_w v_2^j(w)\phi(w)dw$, and $\phi(w)$ denotes the density function characterising some continuous distribution of wage offers.[9] The first expression inside the curly brackets represents the expected utility from accepting the offer. The second expression represents the expected utility from rejecting the offer and searching once more.

Notice that $\mathbb{E}^I[b \mid w] = \mathbb{E}[b \mid w]$, while $\mathbb{E}^U[b \mid w] = \mathbb{E}[b]$. Then, since $d\,\mathbb{E}^U[b \mid w]\,/\,d\,w = 0$, $v_2^U(w)$ takes a constant value of $v_2^U - c_2$ for low w and becomes strictly increasing for higher values of w. Consequently, the optimal search strategy of an uninformed worker is always characterised by a cutoff which prescribes to continue searching until the first offer with $w + \mathbb{E}[b] \geq R_2^U$ is sampled (McCall, 1970). In turn, an informed worker adopts a cutoff rule with R_2^I provided that $d\,\mathbb{E}[b \mid w]\,/\,d\,w > -1$. In this section, I will assume that this is indeed the relevant case.[10]

Then, for all $w + \mathbb{E}^j[b \mid w] < R_2^j$, $v_2^j(w) = v_2^j - c_2$, while for all $w + \mathbb{E}^j[b \mid w] \geq R_2^j$, $v_2^j(w) = w + \mathbb{E}^j[b \mid w]$. Combining this with the indifference at the cutoff, $R_2^j = v_2^j - c_2$, yields:

$$\begin{aligned}\int_w v_2^j(w)\phi(w)dw &= R_2^j \cdot \mathbb{P}[w + \mathbb{E}^j[b \mid w] < R_2^j]\ +\ \int_{w+\mathbb{E}^j[b \mid w] \geq R_2^j} (w + \mathbb{E}^j[b \mid w])\phi(w)dw \\ &= R_2^j\ +\ c_2,\end{aligned}$$

which solves for:

$$c_2 = \int_{w+\mathbb{E}^j[b \mid w] \geq R_2^j} (w + \mathbb{E}^j[b \mid w] - R_2^j)\phi(w)dw. \tag{1}$$

[9] Assuming that wage offers are continuously distributed simplifies notation and results in a clean illustration of the key intuition, but is not essential.

[10] If, instead, $d\,\mathbb{E}[b \mid w]\,/\,d\,w \leq -1$, then an informed worker might adopt a strategy of deliberately searching for low wages which in expectation are coupled with high benefits and thus provide higher total utility. This would not affect the main insight of Lemma 1, but would require a more cumbersome notation.

Intuitively, the optimal cutoff R_2^j equalises the marginal cost of search with the marginal benefit of search. Since the right-hand side of the above equality is strictly decreasing in R_2^j, the optimal cutoff is uniquely determined.

How does the agent's type affect her search strategy? Intuitively, whether an informed or an uninformed agent adopts a higher cutoff in period 2 should depend on whether the correlation between wages and benefits is positive or negative. For instance, if wages were negatively correlated with benefits, an uninformed worker would overestimate a benefit of sampling a higher wage offer, relative to an informed worker, and would adopt a higher cutoff as a result.

Formally, plugging either $\mathbb{E}^I[b \mid w] = \mathbb{E}[b \mid w]$ or $\mathbb{E}^U[b \mid w] = \mathbb{E}[b]$ into (1) yields:

Lemma 1: *If $d\,\mathbb{E}[b \mid w] \,/\, d\,w \lesseqgtr 0$ for all w, then $R_2^U \gtreqless R_2^I$. If $d\,\mathbb{E}[b \mid w] \,/\, d\,w = 0$, then $R_2^U = R_2^I$.*

Analogously to McCall (1970), a worker of type j searches on the job in period 2 when $u_{ik} < R_2^j$, but remains in current employment otherwise.

Period 1. The optimal search strategy in period 1, i.e. during the initial job search, depends on the anticipated behaviour once in employment. Even though both types internalise the fact that they will learn the true value of benefits on the job, the difference in beliefs about the joint distribution of wages and benefits can result in divergent search rules.

Accounting for this, the value function of sampling an offer with wage w in period 1 is:

$$v_1^j(w) = \max\,\{u_1^j(w), v_1^j - c_1\},$$

where:

$$\begin{aligned} u_1^j(w) = \big(w + \mathbb{E}^j[b \mid w]\big) \;&+\; \mathbb{P}^j[w + b \geq R_2^j \mid w] \cdot \big(w + \mathbb{E}^j[b \mid b \geq R_2^j - w, w]\big) \\ &+\; \mathbb{P}^j[w + b < R_2^j \mid w] \cdot \big(v_2^j - c_2\big) \end{aligned}$$

and $v_1^j = \int_w v_1^j(w)\phi(w)dw$. Conditional on accepting the offer, the agent expects to obtain the utility of $w + \mathbb{E}^j[b \mid w]$. Furthermore, if the realised utility from being employed turns out to exceed the type-dependent cutoff R_2^j, the agent expects to stay in the same employment and not engage in on-the-job search. Otherwise, she expects to search on the job, which yields an expected utility of $(v_2^j - c_2)$.

The optimal search strategy in period 1 is also given by a cutoff rule as long as $d\,u_1^j(w)\,/\,d\,w > 0$. Differentiating yields:

$$d\,u_1^j(w)/d\,w \;=\; \big(1 + d\,\mathbb{E}^j[b \mid w] \;/\; d\,w\big) \;+\; \mathbb{P}^j[w+b \geq R_2^j \mid w] \cdot \big(1 + d\,\mathbb{E}^j[b \mid b \geq R_2^j - w, w] \;/\; d\,w\big)$$
$$+\; d\,\mathbb{P}^j[w+b \geq R_2^j \mid w] \;/\; d\,w \cdot \big(w + \mathbb{E}^j[b \mid b \geq R_2^j - w, w] - \underbrace{(v_2^j - c_2)}_{=R_2^j}\big).$$

Clearly, $d\,u_1^U(w)/d\,w > 0$, while for an informed worker the sufficient condition for $u_1^I(w)$ to be strictly increasing in w is again $d\,\mathbb{E}[b \mid w] \;/\; d\,w > -1$. Then, proceeding as above yields the condition for the optimal cutoff in period 1:

$$c_1 = \int_{w \geq R_1^j} \big(u_1^j(w) - u_1(R_1^j)\big)\phi(w)dw. \tag{2}$$

The above specifies an optimal reservation wage R_1^j, because $u_1^j(w)$ embeds utility from all components of the compensation package.

Observe, however, that even imposing the sign of $d\,\mathbb{E}[b \mid w] \;/\; d\,w$ does not lead to a straightforward comparison between $d\,u_1^U(w)/d\,w$ and $d\,u_1^I(w)/d\,w$, and therefore between R_1^I and R_1^U. This illustrates that dynamic incentives have a dramatic impact on how correlation neglect affects search behaviour, relative to a static search problem that applies in period 2.

For example, if $d\,\mathbb{E}[b \mid w] \;/\; d\,w < 0$ for any w, then there are two opposing forces shaping how the informed workers search, relative to their uninformed counterparts. On the one hand, informed workers realise that higher wages are on average associated with lower benefits, which would make them less inclined to search intensely for a high-wage offer. On the other hand, $d\,\mathbb{E}[b \mid w] \;/\; d\,w < 0$ implies $R_2^I < R_2^U$ which means that the initial choice is more likely to be "sticky" in period 2, thus increasing the informed worker's incentive to search intensely for a good offer in period 1. Whichever of these two considerations dominates, depends ultimately on the specific distribution of offers. This will therefore be endogenised in the next section. For now, note the following:

Lemma 2: *If $d\,\mathbb{E}[b \mid w] \;/\; d\,w \neq 0$ for some w, then the comparison between R_1^U and R_1^I is ambiguous.*

3 Equilibrium analysis

In this section, I characterise equilibria of the following game. There is a unit mass of identical firms. The firms simultaneously choose the wage and benefits they offer to a unit mass of searching workers, committing to the same offer for both periods 1 and 2. The worker's problem is as outlined in the previous section. In particular, the workers do not observe the benefits while searching, but learn their true value on the job. Suppose that a fraction $(1-\lambda) \in (0,1)$ of workers are informed, while the remaining share λ are uninformed.

To focus attention, consider a variant of the model in which $\text{Var}[\epsilon_{ik}] \to 0$, so that there is no role for match-specific shocks. Offering a compensation package (w, b) costs the firm $w + (1 - \tau)b$ per period, if accepted by a worker. Here, $\tau \in (0, 1)$ represents tax advantages to providing workplace benefits.[11] Since the worker's utility aggregates received wages and benefits, it is socially desirable for the firms to offer high benefits to all workers.

Each firm produces $y > 0$ units of a numeraire good per hired worker using a constant-returns-to-scale technology. Thus, the profit from hiring a worker for a single period is:

$$\pi = y - w - (1 - \tau)b.$$

Let the equilibrium be defined as a tuple of strategies and beliefs, such that all players behave sequentially rationally given their beliefs. In contrast to a standard notion of Weak Perfect Bayesian Equilibrium (PBE), I allow a fraction of uninformed workers to display correlation neglect. Formally, the equilibrium is defined as follows:

Definition 1: *The workers' strategies summarised by* (R_1^j, R_2^j), $j \in \{I, U\}$, *the firms' strategies summarised by* (w_k, b_k), $k \in [0, 1]$, *and workers' beliefs about the distribution of offers* (w, b) *constitute a (pure-strategy)* PBE with correlation neglect, *if:*

1. *The informed workers' beliefs about the joint distribution of wages and benefits are derived from the firms' strategies.*

2. *The uninformed workers' beliefs about the marginal distributions of wages and benefits are derived from the firms' strategies, but they display correlation neglect in that they perceive these two components as independently distributed.*

3. *Given their beliefs, informed and uninformed workers adopt perceived-optimal cutoff rules satisfying (1) and (2).*

4. *Given the strategies adopted by other players, each firm chooses its offer* (w_k, b_k) *to maximise its expected profit.*

To clarify the above definition note the following. First, although a "fully cursed equilibrium" of Eyster and Rabin (2005) can also feature correlation neglect, this concept applies to static games with incomplete information (i.e., Bayesian games), while the above is a dynamic game with incomplete information. Second, correlation neglect can also arise in an "analogy-based expectation equilibrium" of Jehiel (2005), but this concept has been developed for dynamic games with complete information.

[11] For example, all OECD countries offer preferential tax treatment of private pension contributions, including those made by the employers into qualified workplace plans (OECD, 2020).

The workers' beliefs off-equilibrium-path, in particular beliefs about benefits b associated with a wage offer deviating from the equilibrium, determine the profitability of potential deviations. Consider the following two specifications inspired by McAfee and Schwartz (1994). First, workers have "passive beliefs" if, upon observing a deviation from w^*, they do not update their beliefs about the corresponding benefits b. Second, workers have "wary beliefs" if, upon observing a deviation, they suppose that the offer is associated with low benefits $b = \underline{b}$. To maintain logical consistency with the belief formation of informed and uninformed workers on the equilibrium path, I assume that informed workers have wary beliefs, while uninformed workers have passive beliefs. That is, following a deviation, the informed workers do not become less strategically sophisticated and the uninformed workers do not become more sophisticated.[12] Furthermore, since the probability of re-sampling the same offer is zero with continuum of firms, neither type updates their belief about the distribution of offers upon observing a deviation by a single firm.[13]

Assumption 1: *Let W^* denote the set of wages offered in equilibrium. Upon observing a deviation to $w_k \notin W^*$, informed workers hold wary beliefs about b_k and uninformed workers hold passive beliefs about b_k. Neither type updates their belief about the distribution of offers.*

3.1 Equilibria with a degenerate distribution of benefits

Consider first equilibria in pure strategies in which all firms offer an identical compensation package with some $w = w^*$ and $b = b^*$. Then, since correlation neglect plays no role when the distribution of benefits is degenerate, the workers of either type hold correct beliefs about the firms' offers. Furthermore, there is no role for learning and therefore each worker anticipates that she will stay in the same employment for two periods. Consequently, in period 1 a worker enters the market, searches exactly once, and accepts employment as long as $2(w^* + b^*) - c_1 \geq 0$.

It is easy to see that conditional on b^*, all firms would offer the same wage equal to the workers' reservation wage R_1. Offering a higher wage would not affect the firm's hiring probability or the worker's decision to stay, while offering a lower wage would result in the worker rejecting the offer. In equilibrium, this uniform wage has to make the workers exactly indifferent between accepting employment and not entering the labour market, which implies $w^* = c_1/2 - b^*$.[14] This is reminiscent of the main result in Diamond (1971).

[12] Classical refinements, such as the Intuitive Criterion (Cho and Kreps, 1987), arguably would be conceptually unappealing in a model with boundedly rational agents.

[13] This assumption is commonly made in the literature employing a sequential search model, see the discussion in Janssen and Shelegia (2020).

[14] Suppose instead that the uniform wage offered by all firms strictly exceeded w^*. Then, there would exist a

Under what conditions can the efficient outcome of high benefits being offered to all workers be supported in equilibrium? Suppose that $b^* = \bar{b}$ and $w^* = c_1/2 - \bar{b}$. Irrespective of off-path beliefs about a deviator's benefits, a firm that deviates to $b = \underline{b}$ cannot offer a wage lower than w^*, as this would result in all workers rejecting. Consider first a deviation whereby the firm does not adjust its wage offer and simply combines w^* with low benefits $\underline{b}$. A worker who accepted, either informed or uninformed, searches on the job and leaves if $c_2 < \bar{b} - \underline{b}$. However, the deviator is still better off as long as the profit from hiring a worker for a single period exceeds the profit from retaining the worker with $b = \bar{b}$:

$$y - w^* - (1-\tau)\underline{b} \ > \ 2(y - w^* - (1-\tau)\bar{b}) \iff$$

$$y - c_1/2 + (\bar{b} - \underline{b}) + \tau\underline{b} \ > \ 2\big(y - c_1/2 + \tau\bar{b}\big), \tag{3}$$

with the left-hand-side of the inequality taking into account that under such unilateral deviation there are no workers left in the market to re-hire in period 2.[15] On the other hand, if $c_2 \geq \bar{b} - \underline{b}$, the worker does not search on the job and the deviator is strictly better off.

Second, consider a firm that deviates to $\underline{b}$ and simultaneously raises its wage offer to some $\tilde{w}$ in order to prevent the worker from searching on the job. The highest $\tilde{w}$ that the deviator might consider lies along its iso-profit curve and is thus given by $\tilde{w} = w^* + (1-\tau)(\bar{b} - \underline{b}) = c_1/2 - \underline{b} - \tau(\bar{b} - \underline{b})$. Given their passive beliefs, such an offer is accepted by all uninformed workers in period 1 as $\tilde{w} > w^*$. If $c_2 > \tau(\bar{b} - \underline{b})$, there exists a deviation that makes the hiring of an uninformed worker strictly more profitable, because the deviator could offer some wage strictly below $\tilde{w}$, but above w^*, without losing the worker. On the other hand, an uninformed worker leaves following a deviation when $c_2 < \tau(\bar{b} - \underline{b})$. In this case, a deviation whereby the firm does not raise its wage would be preferable.

Despite their wary beliefs, informed workers accept the deviator's offer for any $\tilde{w}$ in period 1:

$$2(\tilde{w} + \underline{b}) \ \geq \ 2(w^* + \bar{b}) - c_1 \ = \ 0.$$

Intuitively, because the equilibrium compensation package (w^*, b^*) makes the worker indifferent between accepting the first offer and not entering the labour market at all, it also disincentivises further search upon observing a deviation to $\tilde{w}$. Moreover, because their wary beliefs actually materialise in this case, the informed workers' behaviour coincides with that of uninformed workers while on the job.

profitable deviation whereby a firm lowers its wage by some $0 < \epsilon < c_1$ and still hires all the workers it encounters, contradicting the equilibrium condition.

[15]The above assumes that aside from forgone productivity there are no other, 'direct' turnover costs. Introducing positive turnover costs would make it less profitable for the firms to offer compensation packages that induce a worker to leave.

In sum, an equilibrium in which all firms offer $b^* = \bar{b}$ exists if $c_2 \leq \tau(\bar{b} - \underline{b})$ and $2\big(y - c_1/2 + \tau\bar{b}\big) \geq y - c_1/2 + (\bar{b} - \underline{b}) + \tau\underline{b}$ are both satisfied. If either one of these conditions fails, there exists a strictly profitable deviation and thus all firms offering high benefits cannot be supported in equilibrium.

Next, an equilibrium with $b^* = \underline{b}$ always exists, provided that the hiring firms make non-negative profit. Notice that deviating to $\bar{b}$ would necessarily make a firm strictly worse off. This is due to the fact that with benefits being unobservable prior to acceptance, the deviator still has to offer a wage of at least $w^* = c_1/2 - \underline{b}$ in order to attract any workers, informed or uninformed. Thus, deviating to high benefits must raise the firm's labour cost without affecting the worker's decision to accept or to stay in employment. The condition for the firms to make non-negative profit is $y - w^* - (1-\tau)\underline{b} = y + \tau\underline{b} - c_1/2 \geq 0$, which is arguably very weak.

In sum, the following result obtains:

Proposition 1: *In any equilibrium with a degenerate distribution of benefits, the distribution of wage offers is also degenerate with $w^* = c_1/2 - b^*$.*

1. *An equilibrium in which all firms offer low benefits ($b^* = \underline{b}$) exists, provided that the firms make non-negative profits, i.e. $y + \tau\underline{b} \geq c_1/2$.*
2. *An equilibrium in which all firms offer high benefits ($b^* = \bar{b}$) exists if, in addition, $c_2 \leq \tau(\bar{b} - \underline{b})$ and $2\big(y - c_1/2 + \tau\bar{b}\big) \geq y - c_1/2 + (\bar{b} - \underline{b}) + \tau\underline{b}$ both hold.*

When the equilibrium distribution of benefits is degenerate, correlation neglect has no bite (on-path) and the distinction between informed and uninformed workers becomes inconsequential. As a result, the conditions above are independent of the distribution of types and one can interpret Proposition 1 as capturing the impact of modelling benefits as experience goods on the incentives of firms to offer either low or high benefits. Whenever the firms make non-negative profits when offering low benefits, and thus the equilibrium with $b^* = \underline{b}$ exists, the efficient equilibrium with high benefits $b^* = \bar{b}$ exists only if two additional conditions are met. First, the cost of searching on the job cannot be too high. Otherwise, a firm could profitably deviate to offering low benefits without losing a worker. Second, the firms must actually make a higher profit from employing a worker for both periods rather than losing her after one period of employment with slightly lower total compensation. In that sense, socially desirable equilibria with high benefits are more difficult to sustain.

Moreover, when the profits are non-negative in a low-benefit equilibrium and a high-benefit equilibrium exists, the firms are making strictly greater profits in the high-benefit equilibrium due to the associated tax advantages ($\tau > 0$). Thus, unobservability of benefits by the workers can be detrimental to the firms' profits.

3.2 Equilibria with spurious differentiation in benefits

Even though the firms are homogeneous, one might ask whether there exist equilibria with differentiated offers, since informed and uninformed workers may adopt different search strategies when the distribution of benefits is non-degenerate. Following the logic in Albrecht and Axell (1984) and the discussion above, when the two types adopt reservation wages $R_1^I \neq R_1^U$ in period 1, the firms that post the higher of the two wages $w_H \equiv \max\{R_1^I, R_1^U\}$ hire all workers who contact them, while the firms that post the lower of the two wages $w_L \equiv \min\{R_1^I, R_1^U\}$ hire one type only. Because of the trade-off between profits per worker and the number of hires, differential offers might co-exist in equilibrium as long as they generate equivalent total expected profits.

3.2.1 Perverse equilibria

Consider first the case of a "perverse equilibrium" in which $b(w_H) = \bar{b}$ and $b(w_L) = \underline{b}$. That is, jobs that pay higher wages also offer high benefits, and vice versa. Suppose that a fraction $p \in (0,1)$ of firms offer the high-wage, high-benefits jobs, while a fraction $(1-p)$ offer the low-wage, low-benefit jobs.

Notice that, for this distribution of offers, informed workers put probability 1 on specific benefits associated with a given wage offer, while uninformed workers mistakenly believe that any wage offer comes together with high benefits with probability p and with low benefits with probability $(1-p)$. Consequently, the two types have the following value of searching on the job in period 2:

$$v_2^I = \max\{p(w_H + \bar{b}) + (1-p)(w_L + \underline{b}) - c_2\,;\,(w_H + \bar{b}) - c_2/p\},$$
$$v_2^U = \max\{p(w_H + \bar{b}) + (1-p)(w_L + \underline{b}) - c_2\,;\,w_H + p\bar{b} + (1-p)\underline{b} - c_2/p\},$$

depending on whether they accept the first sampled offer or keep searching until an offer with w_H is encountered. Notice that $v_2^I \geq v_2^U$, which implies that informed workers are more inclined to search on the job and, if they do, to search for a high-wage offer in period 2. Intuitively, the fact that high wages are paired with high benefits increases the informed worker's valuation of sampling w_H, relative to her uninformed counterpart (see Lemma 1).

Further, since $c_2 \geq c_1$, informed workers never accept an offer in period 1 with the intention to search on the job, but uninformed workers might anticipate searching on the job for certain realisations of b and still accept. Clearly, neither type has an incentive to search on the job if the accepted offer turns out to provide $(w_H, \bar{b})$. Therefore:

$$u_1^I(w_H) = 2(w_H + \bar{b}), \qquad u_1^I(w_L) = 2(w_L + \underline{b}),$$
$$u_1^U(w_H) = (w_H + p\bar{b} + (1-p)\underline{b}) + p(w_H + \bar{b}) + (1-p)\max\{(w_H + \underline{b})\,;\,v_2^U\},$$

$$u_1^U(w_L) \;=\; (w_L + p\bar{b} + (1-p)\underline{b}) + p \max\{(w_L+\bar{b})\,;\, v_2^U\} + (1-p)\max\{(w_L+\underline{b})\,;\, v_2^U\},$$

which implies $(u_1^I(w_H) - u_1^I(w_L)) \;>\; (u_1^U(w_H) - u_1^U(w_L))$. Thus, if the two types search differently in period 1, it must be the case that informed workers are searching for $(w_H, \bar{b})$, while uninformed workers accept both w_L and w_H.

From the firms' perspective, the compensation package with a high wage and high benefits is strictly more costly to provide. Thus, for the equal profits condition to hold, the firms offering $(w_H, \bar{b})$ must either attract a larger mass of workers, retain a higher proportion of hired workers, or both. It can be shown that the first two cases result in a contradiction:

Lemma 3: *There do not exist perverse equilibria in which the differentiated offers induce the same turnover rate or attract the same mass of workers.*

The proof is provided in appendix A. Consequently, here I construct a perverse equilibrium in which the high-wage firms attract more workers and retain more of the workers that they hired. The first requires that only uninformed workers accept the low-wage jobs. The second requires that all uninformed workers who have accepted a low-wage job leave upon discovering low benefits.

In period 1, informed workers search for $(w_H, \bar{b})$ if:

$$u_1^I(w_H) - u_1^I(w_L) \;=\; 2((w_H - w_L) + (\bar{b} - \underline{b})) \;>\; c_1/p,$$

while uninformed workers accept also w_L if:

$$u_1^U(w_H) - u_1^U(w_L) \;=\; (w_H - w_L) + p\big((w_H+\bar{b}) - \max\{(w_L+\bar{b})\,;\, v_2^U\}\big) + (1 - p)\big(\max\{(w_H+\underline{b})\,;\, v_2^U\} - \max\{(w_L+\underline{b})\,;\, v_2^U\}\big) \;\leq\; c_1/p.$$

An uninformed worker leaves a job offering $(w_L, \underline{b})$ if $w_L + \underline{b} \;<\; v_2^U$. Since they accept the first encountered offer in period 1, $c_2 \geq c_1$ implies that the same must be true in period 2 and therefore $v_2^U = p(w_H + \bar{b}) + (1-p)(w_L + \underline{b}) - c_2$. Thus, an uninformed worker leaves $(w_L, \underline{b})$ if:

$$c_2 \;<\; p\big((w_H - w_L) + (\bar{b} - \underline{b})\big).$$

Firms are indifferent between making either one of the two offers if the following equal profits condition accounting for worker flows holds:

$$\begin{aligned}
\pi(w_H) \;&=\; (2\frac{1-\lambda}{p} + 2\lambda + \lambda(1-p)) \times (y - w_H - (1-\tau)\bar{b}) \;= \\
\pi(w_L) \;&=\; (\lambda + \lambda(1-p)) \times (y - w_L - (1-\tau)\underline{b}) \quad\Longleftrightarrow
\end{aligned}$$

$$(2\,\tfrac{1-\lambda}{p} + (3-p)\lambda)/((2-p)\lambda) \;=\; \tfrac{(y-w_L-(1-\tau)\underline{b})}{(y-w_H-(1-\tau)\bar{b})}.$$

To understand the above expression note the following. First, in period 1 each firm offering low wages is contacted by an uninformed worker with probability λ. Any such worker accepts the firm's offer, but leaves after one period of employment. In period 2, the mass of uninformed workers who are searching on the job is therefore $\lambda(1-p)$, which is again spread evenly across all firms. Thus, every low-wage firm expects to hire workers for a total number of $\lambda + \lambda(1-p)$ periods. Second, in period 1 the mass of $(1-\lambda)$ informed workers is spread evenly across the mass of p firms offering high wages. Once hired, informed workers stay in the same employment for two periods. In addition, a firm offering high wages is contacted by an uninformed worker with probability λ. Any such worker accepts the offer and remains in employment for two periods. Finally, in period 2 a high-wage firm expects to hire $\lambda(1-p)$ of uninformed workers who are searching on the job, thus yielding $2\,\frac{1-\lambda}{p} + 2\lambda + \lambda(1-p)$ of expected periods of employment.

While high-wage firms attract a strictly larger mass of workers, they also bear a strictly higher cost of employing each worker, which makes it possible for the two differentiated compensation packages to yield equivalent total profits.

Next, one needs to rule out profitable deviations by the firms. For $(w_H, \bar{b})$ to be offered in equilibrium, at least informed workers must leave employment upon discovering $\underline{b}$ paired with w_H, which should lower the deviating firm's profit. Since $c_2/p < (w_H + \bar{b}) - (w_L + \underline{b})$), $v_2^I = w_H + \bar{b} - c_2/p$ and thus the informed worker hired in period 1 searches on the job upon discovering low benefits if:

$$v_2^I \;>\; w_H + \underline{b} \;\iff\; c_2/p \;<\; (\bar{b} - \underline{b}).$$

To focus attention, consider a case when an uninformed worker would also leave $(w_H, \underline{b})$:[16]

$$c_2 \;<\; -(1-p)(w_H - w_L) + p(\bar{b} - \underline{b}).$$

Then, the deviator is not better off as long as:

$$\begin{aligned}
(2\,\tfrac{1-\lambda}{p} + 2\lambda + \lambda(1-p)) \times (y - w_H - (1-\tau)\bar{b}) \;\geq\; \\
(\tfrac{1-\lambda}{p} + \lambda + \lambda(1-p)) \times (y - w_H - (1-\tau)\underline{b}) \;\iff\; \\
(2\,\tfrac{1-\lambda}{p} + \lambda(3-p))/(\tfrac{1-\lambda}{p} + \lambda(2-p)) \;\geq\; \tfrac{(y-w_H-(1-\tau)\underline{b})}{(y-w_H-(1-\tau)\bar{b})}.
\end{aligned}$$

[16] I do not attempt to prove uniqueness here. Alternative off-path beliefs of uninformed workers would affect the firms' incentives to deviate as well as the worker's PC, see Appendix B for remaining derivations. These do not affect the formulation of Proposition 2, however.

For $(w_L, \underline{b})$ to be offered in equilibrium, the deviation to high benefits should be unprofitable. Such a deviation allows the firm to retain the uninformed workers that it hires in period 1 when:

$$c_2 \;\geq\; p(w_H - w_L) - (1-p)(\bar{b} - \underline{b}),$$

but is nonetheless unprofitable if:

$$(\lambda + \lambda(1-p)) \times (y - w_L - (1-\tau)\underline{b}) \;\geq\; (2\lambda + \lambda(1-p)) \times (y - w_L - (1-\tau)\bar{b}) \iff$$
$$(3-p)/(2-p) \;\leq\; \tfrac{(y - w_L - (1-\tau)\underline{b})}{(y - w_L - (1-\tau)\bar{b})}.$$

What about deviations to some wage offer different from w_L or w_H? Deviating to any $w < w_L$ or $w > w_H$ can never be profitable.[17] Otherwise, any wage offer $w \in (w_L, w_H)$ attracts only uninformed workers. When paired with low benefits, any such offer is dominated by $(w_L, \underline{b})$, because even $(w_H, \underline{b})$ does not allow to retain uninformed workers. Denote by $\tilde{w} < w_H$ wage such that:

$$\tilde{w} + \bar{b} \;=\; v_2^U \iff \tilde{w} \;=\; pw_H + (1-p)w_L - (1-p)(\bar{b} - \underline{b}) - c_2.$$

That is, $\tilde{w}$ is the lowest wage rate that allows to retain an uninformed worker when paired with high benefits. Notice that if $\tilde{w} \geq w_L$, then the deviation to $(\tilde{w}, \bar{b})$ is dominated by $(w_L, \bar{b})$, which is itself sub-optimal. If $\tilde{w} < w_L$, then such an offer is rejected by an uninformed workers.

Finally, I consider the worker's participation constraints (PC). Given their off-path beliefs, the condition for uninformed workers to accept w_L in period 1 becomes:

$$(1+p)(w_H - w_L) \;\leq\; c_1/p$$

and the uninformed worker's PC is binding as long as:

$$pu_1^U(w_H) + (1-p)u_1^U(w_L) - c_1 \;=$$
$$2pw_H + 2(1-p)w_L + p(3-p)\bar{b} + (1-p)(2-p)\underline{b} - c_1 - (1-p)c_2 \;=\; 0.$$

In turn, the PC of an informed worker who searches for w_H in period 1 is binding when:

$$u_1^U(w_H) - c_1/p \;=\; 2(w_H + \bar{b}) - c_1/p \;=\; 0.$$

[17] Offers with $w < w_L$ attract no workers. Deviating to some $w > w_H$ which is not accepted by informed workers due to their wary beliefs, combined with either $\underline{b}$ or $\bar{b}$, is dominated by $(w_L, \bar{b})$, which is itself sub-optimal. Deviating to $w > w_H$ high enough so that it also attracts (and retains) informed workers with wary beliefs is dominated by $(w_H, \bar{b})$.

Thus, in equilibrium, the informed worker's PC pins down w_H, while the uninformed worker's PC determines w_L. Then, the zero-profit condition pins down p.

To sum up, for such an equilibrium to exist, the following must hold simultaneously:[18]

(i) $c_1/p \;<\; 2((w_H - w_L) + (\bar{b} - \underline{b}))$,

(ii) $c_1/p \;\geq\; (1+p)(w_H - w_L)$,

(iii) $(2\frac{1-\lambda}{p} + (3-p)\lambda)/((2-p)\lambda) \;=\; \frac{(y-w_L-(1-\tau)\underline{b})}{(y-w_H-(1-\tau)\bar{b})}$,

(iv) $c_2 \;<\; p(\bar{b} - \underline{b}) - (1-p)(w_H - w_L)$,

(v) $(2\frac{1-\lambda}{p} + (3-p)\lambda)/(\frac{1-\lambda}{p} + (2-p)\lambda) \;\geq\; \frac{(y-w_H-(1-\tau)\underline{b})}{(y-w_H-(1-\tau)\bar{b})}$,

(vi) $c_2 \;\geq\; p(w_H - w_L) - (1-p)(\bar{b} - \underline{b})$,

(vii) $(3-p)/(2-p) \;\leq\; \frac{(y-w_L-(1-\tau)\underline{b})}{(y-w_L-(1-\tau)\bar{b})}$,

(viii) $2pw_H + 2(1-p)w_L + p(3-p)\bar{b} + (1-p)(2-p)\underline{b} - c_1 - (1-p)c_2 \;=\; 0$,

(ix) $2(w_H + \bar{b}) - c_1/p \;=\; 0$.

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The following results summarises the discussion above:

Proposition 2: *A perverse equilibrium, in which high wages are paired with high benefits, and vice versa, may exist. In any such equilibrium, uninformed workers accept all offers in period 1 and then leave low-wage, low-benefit jobs in period 2. Informed workers search for high-wage, high-benefit jobs in period 1 and remain employed in period 2.*

3.2.2 Fictitious compensating differentials equilibria

Next, consider the case of a "fictitious compensating differentials equilibrium" in which $b(w_H) = \underline{b}$ and $b(w_L) = \bar{b}$. That is, jobs that provide low benefits 'compensate' by paying higher wages, and vice versa. After characterising such equilibria, I explain why the resulting compensating differentials are indeed 'fictitious'.

[18]Note that $c_2 < -(1-p)(w_H - w_L) + p(\bar{b} - \underline{b})$ implies both $c_2 < p(\bar{b} - \underline{b})$ and $c_2 < p\big((w_H - w_L) + (\bar{b} - \underline{b})\big)$.

Suppose that a fraction $p \in (0,1)$ of firms offer the high-wage, low-benefits jobs, while a fraction $(1-p)$ offer the low-wage, high-benefits jobs. The two types have the following value of searching on the job in period 2:

$$v_2^I = \max\{p(w_H + \underline{b}) + (1-p)(w_L + \bar{b}) - c_2\,;\,(w_H + \underline{b}) - c_2/p\,;\,(w_L + \bar{b}) - c_2/(1-p)\},$$
$$v_2^U = \max\{p(w_H + \underline{b}) + (1-p)(w_L + \bar{b}) - c_2\,;\,w_H + p\underline{b} + (1-p)\bar{b} - c_2/p\},$$

depending on the adopted search strategy. Notice that in addition to accepting the first sampled offer, an informed worker could search specifically for either $(w_H, \underline{b})$ or $(w_L, \bar{b})$, depending on which compensation package provides higher total utility. An uniformed worker, in contrast, would only either accept the first offer or search specifically for w_H.

To demonstrate that such equilibria exist without relying on a strictly higher turnover of uninformed workers, I restrict attention to parametrisations under which $(w_H + \underline{b}) - c_2/p > (w_L + \bar{b}) - c_2/(1-p)$. This implies not only that an informed worker never targets $(w_L, \bar{b})$ in her search, but also that $v_2^U \geq v_2^I$. Intuitively, negative correlation between wages and benefits makes the uninformed worker more inclined to search on the job and to target high-wage jobs, in contrast to the case of perverse equilibrium with positive correlation.

In constructing these equilibria, it will be useful to distinguish between two cases, based on the cost ranking of the different job offers. Consider the following possibilities in turn.

Case 1: $w_H + (1-\tau)\underline{b} > w_L + (1-\tau)\bar{b}$. Since the high-wage, low-benefits job is more costly to offer, the equal profits condition requires that firms offering such a job attract a larger mass of workers, retain a higher proportion of hired workers, or both. Notice, however, that under $\tau > 0$, the ranking based on workers' utility does not need to coincide with the ranking based on firms' costs.

If it is the case that $w_H + \underline{b} \geq w_L + \bar{b}$, so that the high-wage, low-benefits jobs deliver greater utility to the workers, one can construct equilibria in which informed workers accept the first sampled offer and uninformed workers search (inefficiently hard) for w_H. Upon discovering that w_H is paired with $\underline{b}$, uninformed workers can either stay employed or search on the job. Neither case can be ruled out, demonstrating that higher turnover of uninformed workers is not necessary for the existence of such equilibria, see Appendix C.

To provide a stark illustration of a violation of the classical compensating differentials condition, here I construct a fictitious compensating differentials equilibrium in which $w_H + \underline{b} < w_L + \bar{b}$ and $w_H + (1-\tau)\underline{b} > w_L + (1-\tau)\bar{b}$, so that the jobs that are *more costly* to provide also deliver *lower utility*.

Equal profits condition requires that the more costly high-wage, low-benefit jobs attract more workers, since a compensation package with low benefits can never induce lower turnover than a compensation package with high benefits. In addition, because these jobs deliver lower total utility, the only way in which they may attract a greater mass of workers is for the informed workers to accept the first sampled offer in period 1 and for the uninformed workers to search specifically for w_H. Finally, focus attention on a case when no worker searches on the job.

Expecting to remain in employment, an informed worker accepts the first offer sampled in period 1 if:

$$u_1^I(w_L) - u_1^I(w_H) \; = \; 2((\bar{b} - \underline{b}) - (w_H - w_L)) \; \leq \; c_1/(1-p).$$

For $(w_L, \bar{b})$ to be offered in equilibrium, workers should search on the job upon discovering $(w_L, \underline{b})$. In this case, the relevant condition applies to the informed workers, who are the only ones accepting w_L initially. They would leave low-wage, low-benefit jobs if:

$$w_L + \underline{b} \; < \; p(w_H + \underline{b}) + (1-p)(w_L + \bar{b}) - c_2 \;\; \Longleftrightarrow \;\; c_2 \; < \; p(w_H - w_L) + (1-p)(\bar{b} - \underline{b}).$$

Turning to uninformed workers, these search for w_H in period 1 as long as:

$$u_1^U(w_H) - u_1^U(w_L) \; = \; (w_H - w_L) + p\big(\max\{(w_H + \underline{b})\,;\, v_2^U\} - v_2^U\big) + (1-p)\big((w_H + \bar{b}) - \max\{(w_L + \bar{b})\,;\, v_2^U\}\big) \; > \; c_1/p.$$

What is v_2^U in this case? Suppose that when searching on the job in period 2, uninformed workers also search for w_H, which requires:

$$(w_H - w_L) \; > \; c_2/p.$$

Then, $v_2^U = w_H + p\underline{b} + (1-p)\bar{b} - c_2/p$. Both types stay employed in high-wage, low-benefit jobs provided that:

$$w_H + \underline{b} \; \geq \; v_2^U \;\; \Longleftrightarrow \;\; c_2/p \; \geq \; (1-p)(\bar{b} - \underline{b}),$$

since $v_2^U > v_2^I$. Even though they never accept low wages in period 1, one needs to specify uninformed workers' expectations regarding staying in low-wage, high-benefit jobs. To focus attention, suppose that uninformed workers expect to leave such jobs:

$$w_L + \bar{b} \; < \; v_2^U \;\; \Longleftrightarrow \;\; c_2/p \; < \; (w_H - w_L) - p(\bar{b} - \underline{b}).$$

Then, the uninformed workers indeed search for w_H in period 1 as long as:

$$u_1^U(w_H) - u_1^U(w_L) \; = \; (w_H - w_L) + p\big((w_H + \underline{b}) - w_H - p\underline{b} - (1-p)\bar{b} + c_2/p\big) + (1-p)\big((w_H + \bar{b}) - w_H - p\underline{b} - (1-p)\bar{b} + c_2/p\big) \; > \; c_1/p \;\; \Longleftrightarrow$$

$$c_1/p \; < \; (w_H - w_L) + c_2/p,$$

which is trivially satisfied.

As for the firms, the equal profits condition boils down to:

$$\pi(w_H) = 2((1-\lambda)+\tfrac{\lambda}{p})(y-w_H-(1-\tau)\underline{b}) \;=\; 2(1-\lambda)(y-w_L-(1-\tau)\bar{b}) = \pi(w_L) \iff$$
$$(1-\lambda)\,/\,((1-\lambda)+\tfrac{\lambda}{p}) \;=\; \tfrac{(y-w_H-(1-\tau)\underline{b})}{(y-w_L-(1-\tau)\bar{b})}.$$

While a deviation from $(w_H,\underline{b})$ to $(w_H,\bar{b})$ is strictly unprofitable, a deviation from $(w_L,\bar{b})$ to $(w_L,\underline{b})$ is unprofitable as long as:

$$2(1-\lambda)(y-w_L-(1-\tau)\bar{b}) \;\geq\; (1-\lambda)(y-w_L-(1-\tau)\underline{b}) \iff$$
$$2 \;\geq\; \tfrac{(y-w_L-(1-\tau)\underline{b})}{(y-w_L-(1-\tau)\bar{b})}.$$

Deviation to some $w \in (w_L, w_H)$ would attract only informed workers and is therefore dominated by $(w_L, \bar{b})$.

To close the model, we take into account the workers' PC's, which must be binding in equilibrium (Albrecht and Axell, 1984). The PC of an uninformed worker is binding for:

$$u_1^U(w_H) - c_1/p \;=\; 2(w_H + p\underline{b} + (1-p)\bar{b}) - c_1/p \;=\; 0,$$

while that of an informed worker is binding for:

$$(1-p)u_1^I(w_L) + pu_1^I(w_H) - c_1 \;=\; 2((1-p)(w_L+\bar{b}) + p(w_H+\underline{b})) - c_1 \;=\; 0.$$

In sum, a fictitious compensating differentials equilibrium in which the high-wage, low-benefits job is strictly more costly to provide despite delivering lower utility to the workers may exist as long as the following conditions hold simultaneously:[19]

(i) $c_1/(1-p) \;\geq\; 2((\bar{b}-\underline{b}) - (w_H - w_L))$,

(ii) $c_2/p \;\geq\; (1-p)(\bar{b}-\underline{b})$,

(iii) $c_2/p \;<\; (w_H - w_L) - p(\bar{b}-\underline{b})$,

(iv) $(1-\lambda)\,/\,((1-\lambda)+\tfrac{\lambda}{p}) \;=\; \tfrac{(y-w_H-(1-\tau)\underline{b})}{(y-w_L-(1-\tau)\bar{b})}$,

(v) $2 \;\geq\; \tfrac{(y-w_L-(1-\tau)\underline{b})}{(y-w_L-(1-\tau)\bar{b})}$,

(vi) $2(w_H + p\underline{b} + (1-p)\bar{b}) - c_1/p \;=\; 0$,

(vii) $2((1-p)(w_L+\bar{b}) + p(w_H+\underline{b})) - c_1 \;=\; 0$.

[19]Note that $c_2/p < (w_H - w_L) - p(\bar{b}-\underline{b})$ implies both $c_2/p < (w_H - w_L)$ and $c_2 < p(w_H - w_L) + (1-p)(\bar{b}-\underline{b})$.

Thus, the three equalities representing the equal profits condition and the workers' PC's jointly determine w_L, w_H, and p. Then, if the remaining conditions in the form of inequalities are satisfied, then such a putative equilibrium is indeed incentive-compatible. It can be verified that the above are not contradictory.[20]

Such an equilibrium relies on the uninformed workers searching (inefficiently hard) for the high-wage offer, but not on a difference in turnover between the two types.

Case 2: $w_H + (1-\tau)\underline{b} \leq w_L + (1-\tau)\bar{b}$**.** When the high-wage, low-benefits job is less costly to provide, it must necessarily also deliver lower total utility. In this case, the equal profits condition requires that the low-wage, high-benefits jobs attract more workers, retain more workers that they attract, or both.

Here, I present in detail how to construct an equilibrium in which workers of both types remain in initially accepted employment, but low-wage, high-benefit jobs attract more workers. Given the corresponding utility ranking, for this to hold it is required that the uninformed workers accept both offers while the informed workers search specifically for $(w_L, \bar{b})$ in period 1.

Informed workers indeed search for $(w_L, \bar{b})$ if:

$$u_1^I(w_L) - u_1^I(w_H) \; = \; 2((w_L + \bar{b}) - (w_H + \underline{b})) \; > \; c_1/(1-p).$$

Suppose that when searching on the job in period 2 an informed worker would accept the first sampled offer, which requires:

$$(w_L + \bar{b}) - (w_H + \underline{b}) \; \leq \; c_2/(1-p).$$

Then, $v_2^I = p(w_H + \underline{b}) + (1-p)(w_L + \bar{b}) - c_2$. An informed worker therefore leaves $(w_L, \underline{b})$ if:

$$w_L + \underline{b} \; < \; p(w_H + \underline{b}) + (1-p)(w_L + \bar{b}) - c_2 \quad \Longleftrightarrow \quad c_2 \; < \; p(w_H - w_L) + (1-p)(\bar{b} - \underline{b}),$$

but stays in $(w_H, \underline{b})$ if:

$$w_H + \underline{b} \; \geq \; p(w_H + \underline{b}) + (1-p)(w_L + \bar{b}) - c_2 \quad \Longleftrightarrow \quad c_2 \; \geq \; (1-p)((w_L + \bar{b}) - (w_H + \underline{b})),$$

[20]For instance, as $p \to 1$, (i) and (ii) are trivially satisfied, while (vi) and (vii) coincide and solve for $w_H = c_1/2 - \underline{b}$. Then, (iv) implies $w_L = \frac{-\lambda}{1-\lambda}y + \frac{1}{1-\lambda}c_1/2 - (1-\tau)\bar{b} - \frac{\tau}{1-\lambda}\underline{b}$. The remaining conditions (iii) and (v) are satisfied for $\lambda = 0.50$, $y = 100$, $\underline{b} = 5.37$, $\bar{b} = 20.0$, $\tau = 0.11$, $c_1 = 4.30$, and $c_2 = 8.0$ (which imply $w_L = -114.67$ and $w_H = -3.20$), for example. Here, the wage rates are negative due to the normalisation of the workers' outside option to 0.

which coincides with the condition for accepting the first sampled offer. An informed worker obviously remains employed in $(w_L, \bar{b})$, which provides the highest total utility.

In turn, uninformed workers also accept the first sampled offer when searching on the job if:

$$c_2/p \ \geq \ (w_H - w_L),$$

which implies $v_2^U = v_2^I$. Then, upon learning the value of benefits the behaviour of uninformed workers coincides with that of informed workers and the former accept the first offer sampled in period 1 if:

$$u_1^U(w_H) - u_1^U(w_L) \ = \ (w_H - w_L) + p\big((w_H + \underline{b}) - p(w_H + \underline{b}) - (1-p)(w_L + \bar{b}) + c_2\big) + (1 - p)\big((w_H + \bar{b}) - (w_L + \bar{b})\big) \ = \ (2 - p^2)(w_H - w_L) - p(1-p)(\bar{b} - \underline{b}) + pc_2 \ \leq \ c_1/p.$$

As for the firms, the equal profits condition is:

$$\pi(w_H) \ = \ 2\lambda(y - w_H - (1-\tau)\underline{b}) \ = \ 2(\lambda + \tfrac{(1-\lambda)}{(1-p)})(y - w_L - (1-\tau)\bar{b}) \ = \ \pi(w_L) \ \iff \ \lambda/(\lambda + \tfrac{(1-\lambda)}{(1-p)}) \ = \ \tfrac{(y - w_L - (1-\tau)\bar{b})}{(y - w_H - (1-\tau)\underline{b})}.$$

Deviation from $(w_H, \underline{b})$ to $(w_H, \bar{b})$ is never profitable. Similarly, deviation to any $w \in (w_L, w_H)$ attracts only uninformed workers and is therefore dominated by $(w_L, \bar{b})$. Still, deviation from $(w_L, \bar{b})$ to $(w_L, \underline{b})$ is unprofitable as long as:

$$2(\lambda + \tfrac{(1-\lambda)}{(1-p)})(y - w_L - (1-\tau)\bar{b}) \ \geq \ (\lambda + \tfrac{(1-\lambda)}{(1-p)})(y - w_L - (1-\tau)\underline{b}).$$

Finally, the binding PC of informed workers implies:

$$u_1^I(w_L) - c_1/(1-p) \ = \ 2(w_L + \bar{b}) - c_1/(1-p) \ = \ 0,$$

while that of uninformed workers implies:

$$pu_1^U(w_H) + (1-p)u_1^U(w_L) - c_1 \ = \ p\big(2(w_H + p\underline{b} + (1-p)\bar{b})\big) + (1-p)\big(w_L + p\underline{b} + (1 - p)\bar{b} + (1-p)(w_L + \bar{b}) + p(p(w_H + \underline{b}) + (1-p)(w_L + \bar{b}) - c_2)\big) - c_1 \ = \ p(2 + p - p^2)w_H + (1-p)(2-p^2)w_L + (1-p)(2+p-p^2)\bar{b} + p(1+2p-p^2)\underline{b} - (1-p)pc_2 - c_1 \ = \ 0.$$

Overall, a fictitious compensating differentials equilibrium in which the low-wage, high-benefits job is strictly more costly to provide may exist as long as the following hold:

(i) $c_1/(1-p) \ < \ 2((w_L + \bar{b}) - (w_H + \underline{b}))$,

(ii) $c_2/(1-p) \;\geq\; (w_L+\bar{b})-(w_H+\underline{b})$,

(iii) $c_2 \;<\; p(w_H-w_L)+(1-p)(\bar{b}-\underline{b})$,

(iv) $c_2/p \;\geq\; (w_H-w_L)$,

(v) $c_1/p \;\geq\; (2-p^2)(w_H-w_L)-p(1-p)(\bar{b}-\underline{b})+pc_2$,

(vi) $\lambda/(\lambda+\frac{(1-\lambda)}{(1-p)}) \;=\; \frac{(y-w_L-(1-\tau)\bar{b})}{(y-w_H-(1-\tau)\underline{b})}$,

(vii) $2(y-w_L-(1-\tau)\bar{b}) \;\geq\; (y-w_L-(1-\tau)\underline{b})$,

(viii) $u_1^I(w_L)-c_1/(1-p) \;=\; 2(w_L+\bar{b})-c_1/(1-p) \;=\; 0$,

(ix) $p(2+p-p^2)w_H+(1-p)(2-p^2)w_L+(1-p)(2+p-p^2)\bar{b}+p(1+2p-p^2)\underline{b}-(1-p)pc_2-c_1 \;=\; 0$.

It can be verified that the above are indeed not contradictory.[21]

In contrast to the previous case, such an equilibrium relies on the uninformed workers searching inefficiently little. Again, the difference in turnover between the two types is not necessary to support it. To sum up, the following result obtains:

Proposition 3: *A fictitious compensating differentials equilibrium, in which high wages are paired with low benefits, and vice versa, may exist. If the high-wage, low-benefits package is strictly more costly to provide, any such equilibrium requires the uninformed workers to search (inefficiently hard) for the high-wage offers in period 1. Conversely, if the high-wage, low-benefits package is weakly less costly to provide, any such equilibrium requires the uninformed workers to search inefficiently little. Differences in turnover are not necessary, but can feature in such equilibria.*

The reason why in the above equilibria the compensating differential is fictitious is that it does not reflect the preferences of a marginal worker or the costs of a marginal firm. In the classical model of Rosen (1974, 1986), workers would have heterogeneous preferences for benefits and firms would have heterogeneous costs of providing benefits. Under perfect information, workers and firms sort into contracts with either high or low benefits, with the resulting difference in wage rates given by the indifference condition of a marginal worker and a marginal firm. With homogeneous preferences and costs, the classical framework yields

[21] For instance, as $p \to 0$, (iv) and (v) are trivially satisfied, while (viii) and (ix) coincide and solve for $w_L = c_1/2-\bar{b}$. Then, (vi) implies $w_L = \frac{\lambda-1}{\lambda}y+\frac{1}{\lambda}c_1/2-(1-\tau)\underline{b}-\frac{\tau}{\lambda}\bar{b}$. The remaining conditions (i), (ii), (iii), and (vii) are satisfied for $\lambda = 0.80$, $y = 50$, $\underline{b} = 7.20$, $\bar{b} = 26.50$, $\tau = 0.134$, $c_1 = 12.35$, and $c_2 = 14.85$ (which imply $w_L = -20.325$ and $w_H = -15.45$), for example.

an unambiguous prediction. Namely, all firms should offer high benefits, with wages set so as to satisfy the workers' participation constraint.

Accounting for information frictions on the workers' side therefore allows to generate spurious differentiation in benefits, despite the fact that all workers and all firms are identical. The fictitious compensating differentials underlying the derivation of Proposition 3 reflect the equivalence of *total profits* between firms attracting different numbers of workers, as well as the search strategies of boundedly rational workers who expect to learn the value of benefits only after accepting a particular offer.

3.3 Discussion

Framing of the model. Of course, the application to multidimensional compensation packages with salient 'wages' and hidden 'benefits' is just one way of framing predictions of the theoretical framework presented here. The same logic and results apply to settings where firms compete on observable 'price' and unobservable 'quality' of a subscription product.

Mixed strategies. Allowing the firms to adopt mixed strategies does not affect the conclusions drawn from the analysis above. First, if there does not exist a profitable, pure-strategy deviation from either equilibrium outlined in Proposition 1, there also does not exist a profitable deviation in mixed strategies. Second, the pure-strategy equilibria outlined in Propositions 2 and 3 are equivalent to a mixed-strategy equilibrium, in which each firm randomises over specific compensation packages. That is because the two offers generate identical expected profits.

Furthermore, in a deterministic setting, allowing for mixed strategies cannot be invoked to keep the benefits component 'hidden' from the workers. That is because for a given wage offer w, the firm would never have an incentive to independently randomise over the benefits component b. If the firm prefers the worker to stay for both periods, it would strictly prefer to offer the cheapest benefit level that prevents the worker from searching on the job. If the firm prefers the worker to leave, it would strictly prefer to offer the cheapest possible benefit.

Continuous benefits. How would the results be affected if instead of a binary choice of benefits $b \in \{\underline{b}, \bar{b}\}$, the firms could select any benefits level from the interval $b \in [\underline{b}, \bar{b}]$? Keeping in mind that for a given wage offer w, the optimal level of benefits to provide is uniquely determined, we can discuss the qualitative robustness of the main results above. First, for any given b, a putative equilibrium with a degenerate distribution of offers necessarily features $w(b)$, such that the worker's participation constraint binds. Then, to verify whether $(w(b), b)$ indeed constitutes an equilibrium, one only needs to consider downward deviations from b.

Suppose that the firm deviates by lowering the benefits by some $\epsilon > 0$. The deviating firm does not lose the worker, and the deviation is indeed profitable, as long as:

$$w(b) + b - c_2 \leq w(b) + b - \epsilon \iff \epsilon \leq c_2.$$

For $c_2 > 0$, one can always find a small enough increment $\epsilon \leq c_2$, such that the deviating firm is strictly better off. Thus, as long as $b > 0$, there must exist such profitable deviation and a result that is effectively an extreme version of Proposition 1 obtains. Namely, when workers cannot observe the benefits component while searching for a job, equilibria with benefits exceeding $\underline{b}$ do not exist due to unravelling. Similarly, equilibria with differentiated offers (Propositions 2 and 3) no longer exist if the firms offering high benefits can reduce those by an arbitrarily small amount.

Behavioural types. What is the role of specific assumptions about the workers' perceptions in generating the above results? On the equilibrium path, the informed workers associate the correct benefit level with each wage offer with probability 1. The assumption of unobservability of benefits prior to acceptance nonetheless determines the profitability of a firm deviating from high to low benefits. With observable benefits, such deviations would become less profitable, increasing the firms' incentive to offer high rather than low benefits.

The uninformed workers, in turn, may seem to act as if they were searching for jobs by ignoring the hidden benefits altogether. Nevertheless, the fact that they anticipate future learning feeds into the search strategy adopted by uninformed workers in the form of option value of searching on the job.[22]

To illustrate this point, recall that Gabaix and Laibson (2006) consider a model of add-on prices with two types of consumers. In equilibrium where the add-on prices are (endogenously) shrouded, "sophisticated" consumers apply Bayesian updating to form their beliefs. By contrast, "myopic" consumers completely ignore the add-on price. Consequently, the informed workers considered here are equivalent to sophisticated consumers who take into account the dynamic aspect of the game. Relative to myopic consumers, the uninformed workers expect to receive some positive benefit in addition to the wage, which relaxes their participation constraint. They also take into account future learning and the option value of searching on the job. Although the ranking of offers would be identical for myopic consumers and uninformed workers, what matters for the search intensity is the perceived marginal benefit from further search. It can be shown that in case of differentiated offers, myopic consumers can only search more intensely than uninformed workers:

[22] Recall that the distinction between informed and uninformed workers is material only in equilibria with a non-degenerate distribution of offers.

$$u_1^U(w_H) - u_1^U(w_L) = (w_H - w_L) + q\big(\underbrace{(w_H + \bar{b}) - \max\{(w_L + \bar{b})\,;\, v_2^U\}}_{\leq (w_H - w_L)}\big) + (1 - q)\big(\underbrace{\max\{(w_H + \underline{b})\,;\, v_2^U\} - \max\{(w_L + \underline{b})\,;\, v_2^U\}}_{\leq (w_H - w_L)}\big) \leq 2(w_H - w_L) = u_1^{Myop}(w_H) - u_1^{Myop}(w_L),$$

where $q \in \{p, 1-p\}$. Thus, modelling myopia, rather than correlation neglect, would strengthen the case for equilibria with fictitious compensating differentials in which Uninformed Workers search inefficiently hard (Proposition 3), but would weaken it for perverse equilibria in which they search inefficiently little (Proposition 2).

In contrast, the comparison with boundedly rational agents in Schumacher (2024) is less straightforward. Applying the concept of personal equilibrium (Spiegler, 2016), Schumacher (2024) considers a model where a boundedly rational consumer correctly anticipates the add-on price of a product she purchases on the equilibrium path, but potentially misperceives it for an alternative she never selects, believing that the product choice has no effect on the probability of being charged a positive add-on price. Since the consumer only purchases one product, she never has an opportunity to revise her beliefs. The notion of personal equilibrium applied to the current setting would imply that a worker has correct beliefs about benefits associated with wage offers that she accepts in equilibrium, and possibly expects the same benefits to materialise if she were to adopt a lower reservation wage. In general, this would give rise to decision rules that are not directly comparable to the behaviour of either informed or uninformed workers. For example, in case of a perverse equilibrium such a boundedly rational agent would require a strictly higher wage differential $(w_H - w_L)$ than an informed worker in order to search specifically for high-wage offers, while the opposite is true in a fictitious compensating differentials equilibrium. In turn, in any equilibrium with differentiated offers, an uninformed worker always learns new information on the equilibrium path.

Probabilistic learning. The above formulation of the model assumes that the worker learns the true value of workplace benefits in period 1. In reality, learning is more likely to be probabilistic, reflecting the impact of such factors as the information provided by one's union representatives (Gustman and Steinmeier, 2005) and peers (Duflo and Saez, 2002, 2003). Introducing probabilistic learning into the model should not affect its qualitative predictions, but it would further reduce the firm's incentive to provide high benefits.

4 Empirical relevance

In this section, I discuss the relevance of the model's theoretical predictions in light of the empirical work on the provision of workplace pension benefits.

Inefficiently low benefits. Modelling benefits as experience goods highlights a potential pitfall of delegating the provision of pension insurance to employers. Namely, if the workers find the benefits more difficult to understand and compare across offers than wages, the firms have weak incentives to offer high benefits (Proposition 1). Thus, such information frictions can result in an inefficiently low provision of benefits in equilibrium.

This prediction provides a complementary interpretation of the empirical results of Cole and Taska (2023). Combining data on job transitions with an online experiment, they find that a majority of workers have high willingness to pay (in terms of their total compensation) for access to a workplace pension plan and employer contributions. Consequently, a calibrated model of on-the-job search implies that 80% of firms could improve their hiring probability by shifting some of the total compensation towards higher benefits. While Cole and Taska (2023) explain this evidence of inefficiently low benefits by regulatory constraints on designing worker-specific compensation packages (most importantly, non-discriminatory rules in workplace pension provision), the fact that most workers place a high value on workplace benefits suggests that a reinforcing rationale could well be due to information frictions.[23]

Benefits and turnover. Dominated job offers with low wages and low benefits may co-exist in equilibrium with high-wage, high-benefit jobs, but they attract only the uninformed workers who leave upon learning the true value of benefits. The model therefore predicts that dominated jobs with low benefits must be characterised by higher turnover (Proposition 2).

This theoretical result resonates with robust evidence that lower benefits are associated with higher worker turnover, see for example Bennett et al. (1993), Lee et al. (2006), and Ouimet and Tate (2023). But why do workers accept these jobs in the first place? The model presented here proposes a specific mechanism based on information frictions on the worker side, specifically the difficulty of evaluating workplace benefits at the time of job search.

Estimating compensating differentials. Information frictions would make empirical estimates of the size of compensating differentials basically impossible to interpret (Lavetti, 2023), which could explain the mixed empirical success of the conceptually appealing theory (see, e.g., Schiller and Weiss, 1980; Montgomery and Shaw, 1997; Lamadon et al., 2022).

[23] By contrast, the school teachers' low willingness to pay for additional pension benefits would suggest an inefficiently high provision in the public sector (Fitzpatrick, 2015).

The above model formalises the idea that workers' imperfect understanding of the complex component of a compensation package may generate spurious differentiation in benefits and fictitious compensating differentials (Proposition 3).

5 Conclusion

To account for the workers' imperfect understanding of complex compensation packages, in this paper I set up and analyse a dynamic search model in which workplace benefits are not observable at the time of job search but become known after accepting a particular offer.

The model implies that under unobservabiliy of benefits prior to acceptance, an equilibrium with uniformly high benefits is more difficult to sustain than an equilibrium with uniformly low benefits, which ultimately harms firms' profits (Proposition 1).

In addition, the presence of "uninformed workers", who do not update their beliefs about associated benefits when sampling a specific wage offer, generates additional equilibria with spurious differentiation in benefits. First, there may exist "perverse equilibria", in which high-wage jobs also provide high benefits, and vice versa (Proposition 2). In these equilibria, only uninformed workers accept dominated offers with low wages and low benefits, and these jobs induce strictly higher turnover. Second, there may also exist "fictitious compensating differentials equilibria", in which high wages are paired with low benefits, and vice versa (Proposition 3). Depending on the cost ranking of these differentiated offers, such equilibria require the uninformed workers to search either inefficiently hard or inefficiently little, but do not rely on differences in turnover. The logic behind these equilibria and their efficiency properties are starkly different from the qualitatively similar prediction of the compensating differentials theory (Rosen, 1974, 1986).

There are several avenues for future research. In particular, extending the model to account for screening of workers of heterogeneous productivity, the effects of benefits on productivity, or monopsony powers resulting from benefit differentiation would improve our understanding of how information frictions may interact with the classical rationales for providing workplace benefits.

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A Proof of Lemma 3

First, consider a putative perverse equilibrium in which firms offering high-wage, high-benefits jobs attract the same mass of workers as firms offering low-wage, low-benefits jobs, but only high-wage jobs retain all workers that they attract in period 1.

Conditional on workers entering the labour market, the former is true only when both worker types accept the first offer that they encounter. As argued in the main text, when benefits are positively correlated with wages, informed workers have a higher perceived marginal benefit from further search upon sampling w_L. Thus, both types accept the first sampled offer as long as informed workers do so:

$$u_1^I(w_H) - u_1^I(w_L) \;=\; 2(w_H + \bar{b}) - (w_L + \underline{b}) - \max\{(w_L + \underline{b})\,;\, v_2^I\} \;\le\; c_1/p. \tag{A1}$$

Moreover, informed workers have a higher perceived valuation of on-the-job search. Thus, for any type to leave a low-wage, low-benefits job, the informed workers must do so. Since $c_2 \geq c_1$, the informed workers who search on the job accept the first encountered offer, as they do in period 1. These workers search on the job if:

$$\begin{aligned} w_L + \underline{b} \;<\; v_2^I \;&=\; p(w_H + \bar{b}) + (1-p)(w_L + \underline{b}) - c_2 \;\iff \\ c_2 \;&<\; p\big((w_H + \bar{b}) - (w_L + \underline{b})\big) \end{aligned} \tag{A2}$$

Then, combining (A1) with (A2) yields:

$$c_1 \;\geq\; p\big((w_H + \bar{b}) - (w_L + \underline{b})\big) \;+\; p\big(\underbrace{(w_H + \bar{b}) + v_2^I}_{>0}\big) \;>\; c_2,$$

which contradicts $c_2 \geq c_1$. Therefore, there does not exist a perverse equilibrium in which both job offers are accepted by both types in period 1, but at least one type leaves a low-wage, low-benefits job in period 2. Intuitively, that is because this type of equilibrium would rely on the informed workers accepting low-wage jobs in period 1, while realising that they will subsequently leave. This cannot be part of a worker's strategy in equilibrium if the search costs are (weakly) increasing on the job.

Second, consider a putative perverse equilibrium in which high-wage, high-benefits jobs attract more workers than low-wage, low-benefits jobs, but both job types retain the same proportion of workers. Given the positive correlation between benefits and wages, the former requires that informed workers search until they sample w_H in period 1, while uninformed workers accept the first sampled offer. The latter requires that no worker searches on the job in period 2.

Anticipating to stay, informed workers search for w_H in period 1 as long as:

$$u_1^I(w_H) - u_1^I(w_L) \;=\; 2((w_H - w_L) + (\bar{b} - \underline{b})) \;>\; c_1/p,$$

while uninformed workers accept also w_L if:

$$u_1^U(w_H) - u_1^U(w_L) \;=\; (w_H - w_L) + p\big((w_H + \bar{b}) - \max\{(w_L + \bar{b})\,;\, v_2^U\}\big) + (1 - p)\big(\max\{(w_H + \underline{b})\,;\, v_2^U\} - \max\{(w_L + \underline{b})\,;\, v_2^U\}\big) \;\leq\; c_1/p.$$

Uninformed workers remain employed in low-wage, low-benefit jobs if $(w_L + \underline{b}) \geq v_2^U$, under which the above simplifies to:

$$u_1^U(w_H) - u_1^U(w_L) \;=\; 2(w_H - w_L) \;\leq\; c_1/p.$$

Then, uninformed workers accept the first sampled offer also in period 2, which implies:

$$v_2^U \;=\; p(w_H + \bar{b}) + (1-p)(w_L + \underline{b}) - c_2 \;\leq\; w_L + \underline{b} \iff$$

$$c_2 \;\geq\; p\big((w_H + \bar{b}) - (w_L + \underline{b})\big). \tag{A3}$$

For high-wage firms to not deviate from offering high benefits, at least informed workers should leave employment upon discovering low benefits. When (A3) holds, informed workers searching on the job would accept the first encountered offer, which yields:

$$w_H + \underline{b} \;<\; v_2^I \;=\; p(w_H + \bar{b}) + (1-p)(w_L + \underline{b}) - c_2 \iff$$
$$c_2 \;<\; (1-p)(\underbrace{w_L - w_H}_{<0}) + p(\bar{b} - \underline{b}),$$

contradicting (A3). Intuitively, no perverse equilibrium can sustain the workers' strategies under which uninformed workers remain in employment upon discovering $(w_L, \underline{b})$, but informed workers leave $(w_H, \underline{b})$ and accept the first sampled offer.

Overall, in any perverse equilibrium, if it exists, the low-wage, low-benefits jobs must attract a lower mass of workers as well as induce strictly higher turnover.

□

B Remaining Derivations Underlying Proposition 2

Recall that uninformed workers searching on the job upon discovering $(w_L, \underline{b})$ is a prerequisite for a perverse equilibrium to exist (Lemma 3). Conversely, no worker has an incentive to search further upon discovering $(w_H, \bar{b})$.

Here, I consider alternative assumptions about off-path beliefs of uninformed workers, i.e. beliefs associated with $(w_L, \bar{b})$ and $(w_H, \underline{b})$, which can potentially be invoked to construct perverse equilibria.

Uninformed workers leave upon observing $(w_L, \bar{b})$. It can be shown that a decision rule under which uninformed workers leave low-paying jobs in period 2 irrespective of the realisation of benefits is necessarily inconsistent with them accepting low-wage offers in period 1. An uninformed worker leaves $(w_L, \bar{b})$ if:

$$w_L + \bar{b} \; < \; v_2^U = p(w_H + \bar{b}) + (1-p)(w_L + \underline{b}) - c_2 \quad \Longleftrightarrow \quad c_2 \; < \; p(w_H - w_L) - (1-p)(\bar{b} - \underline{b}),$$

or:

$$(w_H - w_L) > c_2/p + (1-p)/p\,(\bar{b} - \underline{b}).$$

But then, accepting w_L in period 1 requires:

$$u_1^U(w_H) - u_1^U(w_L) \; = \\ (w_H - w_L) + p\big((w_H + \bar{b}) - v_2^U\big) + (1-p)\big(\max\{(w_H + \underline{b})\,;\, v_2^U\} - v_2^U\big) \; \leq \; c_1/p.$$

To increase the odds that the above holds, suppose that uninformed workers would leave jobs offering $(w_H + \underline{b})$, i.e. $\max\{(w_H + \underline{b})\,;\, v_2^U\} = v_2^U$. Then, the above condition becomes:

$$c_1/p \; \geq \; (1 + p(1-p))(w_H - w_L) + p(1-p)(\bar{b} - \underline{b}) + pc_2 \; > \\ > \; (1+p)(c_2/p) + (1-p^2)((\bar{b} - \underline{b})/p) \; > \; c_2/p,$$

which contradicts $c_1 \leq c_2$. Thus, there cannot exist perverse equilibria in which uninformed workers believe that they will leave low-wage jobs in period 2 with certainty, but nonetheless accept those jobs during their initial search in period 1.

Uninformed workers stay upon observing both $(w_L, \bar{b})$ and $(w_H, \underline{b})$. If only informed workers were to leave a job offering $(w_H, \underline{b})$, instead of the expected package $(w_H, \bar{b})$, the high-paying firms would have a strictly higher incentive to deviate to low benefits, relative to the case considered in the main text. Then, the condition for the firms to not be able to deviate profitably becomes:

$$(2\,\tfrac{1-\lambda}{p} + (3-p)\lambda)/(\tfrac{1-\lambda}{p} + (3-p)\lambda) \; \geq \; \tfrac{(y - w_H - (1-\tau)\underline{b})}{(y - w_H - (1-\tau)\bar{b})},$$

while the remaining conditions for such an equilibrium to exist coincide with those outlined in the main text.

C Alternative Fictitious Compensating Differentials Equilibria

Here, I complement derivations provided in the main text by constructing fictitious compensating equilibria for alternative parametrisations.

Case 1^***:** $w_H + \underline{b} \geq w_L + \bar{b}$**.** Notice that:

$$w_H + \underline{b} \geq w_L + \bar{b} \implies w_H + (1-\tau)\underline{b} > w_L + (1-\tau)\bar{b}.$$

Thus, this case corresponds to an equilibrium in which the ranking of offers based on workers' preferences coincides with the ranking based on firms' costs. In what follows, I demonstrate that such an equilibrium may, but does not have to, feature a higher turnover of uninformed workers. In other words, their strictly higher turnover is not necessary to maintain such an equilibrium.

No turnover on the equilibrium path. First, construct an equilibrium in which no worker searches on the equilibrium path. Since the high-wage, low-benefits compensation package is strictly more costly to provide, the equal profits condition requires that it attracts a greater mass of workers. In turn, this requires that uninformed workers search explicitly for w_H, while informed workers accept the first sampled offer in period 1.

An informed worker accepts the first offer sampled in period 1 if:

$$u_1^I(w_H) - u_1^I(w_L) \; = \; 2((w_H - w_L) - (\bar{b} - \underline{b})) \; \leq \; c_1/p.$$

For $(w_L, \bar{b})$ to be offered in equilibrium, informed workers should search on the job upon discovering $(w_L, \underline{b})$, i.e.:

$$w_L + \underline{b} \; < \; p(w_H + \underline{b}) + (1-p)(w_L + \bar{b}) - c_2 \iff c_2 \; < \; p(w_H - w_L) + (1-p)(\bar{b} - \underline{b}).$$

Turning to uninformed workers, these search for w_H in period 1 as long as:

$$u_1^U(w_H) - u_1^U(w_L) \; = \; (w_H - w_L) + p\big(\max\{(w_H + \underline{b}) \, ; \, v_2^U\} - v_2^U\big) + (1-p)\big((w_H + \bar{b}) - \max\{(w_L + \bar{b}) \, ; \, v_2^U\}\big) \; > \; c_1/p.$$

To fix ideas, suppose that when searching on the job in period 2, uninformed workers also search for w_H, which requires:

$$(w_H - w_L) \; > \; c_2/p.$$

Then, $v_2^U = w_H + p\underline{b} + (1-p)\bar{b} - c_2/p$. Both types stay employed in high-wage, low-benefit jobs provided that:

$$w_H + \underline{b} \ \geq \ v_2^U \ \iff \ c_2/p \ \geq \ (1-p)(\bar{b} - \underline{b}),$$

since $v_2^U > v_2^I$. Even though they never accept low wages in period 1, one needs to specify uninformed workers' expectations regarding staying in low-wage, high-benefit jobs. To focus attention, suppose that uninformed workers expect to leave such jobs:

$$w_L + \bar{b} \ < \ v_2^U \ \iff \ c_2/p \ < \ (w_H - w_L) - p(\bar{b} - \underline{b}).$$

Then, the uninformed workers indeed search for w_H in period 1 as long as:

$$u_1^U(w_H) - u_1^U(w_L) \ = \ (w_H - w_L) + p\big((w_H + \underline{b}) - w_H - p\underline{b} - (1-p)\bar{b} + c_2/p\big) + (1 - p)\big((w_H + \bar{b}) - w_H - p\underline{b} - (1-p)\bar{b} + c_2/p\big) \ > \ c_1/p \ \iff$$
$$c_1/p \ < \ (w_H - w_L) + c_2/p,$$

which is trivially satisfied.

As for the firms, the equal profits condition boils down to:

$$\pi(w_H) = 2((1-\lambda) + \tfrac{\lambda}{p})(y - w_H - (1-\tau)\underline{b}) \ = \ 2(1-\lambda)(y - w_L - (1-\tau)\bar{b}) = \pi(w_L) \ \iff$$
$$(1-\lambda) \,/\, ((1-\lambda) + \tfrac{\lambda}{p}) \ = \ \tfrac{(y - w_H - (1-\tau)\underline{b})}{(y - w_L - (1-\tau)\bar{b})}.$$

While a deviation from $(w_H, \underline{b})$ to $(w_H, \bar{b})$ is strictly unprofitable, a deviation from $(w_L, \bar{b})$ to $(w_L, \underline{b})$ is unprofitable as long as:

$$2(1-\lambda)(y - w_L - (1-\tau)\bar{b}) \ \geq \ (1-\lambda)(y - w_L - (1-\tau)\underline{b}) \ \iff$$
$$2 \ \geq \ \tfrac{(y - w_L - (1-\tau)\underline{b})}{(y - w_L - (1-\tau)\bar{b})}.$$

Deviation to some $w \in (w_L, w_H)$ would attract only informed workers and is therefore dominated by $(w_L, \bar{b})$.

Finally, we take into account the workers' PC's. The PC of an uninformed worker is binding for:

$$u_1^U(w_H) - c_1/p \ = \ 2(w_H + p\underline{b} + (1-p)\bar{b}) - c_1/p \ = \ 0,$$

while that of an informed worker is binding for:

$$(1-p)u_1^I(w_L) + p u_1^I(w_H) - c_1 \ = \ 2((1-p)(w_L + \bar{b}) + p(w_H + \underline{b})) - c_1 \ = \ 0.$$

In sum, a fictitious compensating differentials equilibrium in which the high-wage, low-benefits jobs provide a strictly higher utility (and are strictly more costly to provide) may exist and feature no turnover on the equilibrium path as long as the following conditions hold simultaneously:[1]

(i) $c_1/p \;\geq\; 2((w_H - w_L) - (\bar{b} - \underline{b})),$

(ii) $c_2/p \;\geq\; (1-p)(\bar{b} - \underline{b}),$

(iii) $c_2/p \;<\; (w_H - w_L) - p(\bar{b} - \underline{b}),$

(iv) $(1-\lambda) \,/\, ((1-\lambda) + \frac{\lambda}{p}) \;=\; \frac{(y - w_H - (1-\tau)\underline{b})}{(y - w_L - (1-\tau)\bar{b})},$

(v) $2 \;\geq\; \frac{(y - w_L - (1-\tau)\underline{b})}{(y - w_L - (1-\tau)\bar{b})},$

(vi) $2(w_H + p\underline{b} + (1-p)\bar{b}) - c_1/p \;=\; 0,$

(vii) $2((1-p)(w_L + \bar{b}) + p(w_H + \underline{b})) - c_1 \;=\; 0.$

This coincides with the conditions underpinning the equilibrium constructed in the main text, except for (i) - the marginality condition for an informed worker searching in period 1. It can be verified that this set of conditions are also not contradictory.

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[2]

On-the-job search on the equilibrium path. Second, construct an equilibrium in which uninformed workers leave the high-wage, low-benefit jobs on the equilibrium path, as the informed workers never search on the job on the equilibrium path.

An informed worker accepts the first offer sampled in period 1 if:

$$u_1^I(w_H) - u_1^I(w_L) \;=\; 2((w_H - w_L) - (\bar{b} - \underline{b})) \;\leq\; c_1/p.$$

[1] Note that $c_2/p < (w_H - w_L) - p(\bar{b} - \underline{b})$ implies both $c_2/p < (w_H - w_L)$ and $c_2 < p(w_H - w_L) + (1-p)(\bar{b} - \underline{b})$.

[2] For instance, as $p \to 1$, (ii) is trivially satisfied, while (vi) and (vii) coincide and solve for $w_H = c_1/2 - \underline{b}$. Then, (iv) implies $w_L = \frac{-\lambda}{1-\lambda} y + \frac{1}{1-\lambda} c_1/2 - (1-\tau)\bar{b} - \frac{\tau}{1-\lambda}\underline{b}$. The remaining conditions (i), (iii), and (v) are satisfied for $\lambda = 0.50$, $y = 100$, $\underline{b} = 5.37$, $\bar{b} = 20.0$, $\tau = 0.11$, $c_1 = 4.30$, and $c_2 = 8.0$ (which imply $w_L = -114.67$ and $w_H = -3.20$), for example.

For $(w_L, \bar{b})$ to be offered in equilibrium, informed workers should search on the job upon discovering $(w_L, \underline{b})$, i.e.:

$$w_L + \underline{b} \ < \ p(w_H + \underline{b}) + (1-p)(w_L + \bar{b}) - c_2 \iff c_2 \ < \ p(w_H - w_L) + (1-p)(\bar{b} - \underline{b}).$$

They nonetheless remain in employment of high-wage, low-benefit firms when:

$$w_H + \underline{b} \ \geq \ p(w_H + \underline{b}) + (1-p)(w_L + \bar{b}) - c_2 \iff c_2 \ \geq \ -(1-p)(w_H - w_L) + (1-p)(\bar{b} - \underline{b}).$$

Turning to uninformed workers, these search for w_H in period 1 as long as:

$$u_1^U(w_H) - u_1^U(w_L) \ = \ (w_H - w_L) + p\big(\max\{(w_H + \underline{b})\,;\, v_2^U\} - v_2^U\big) + (1-p)\big((w_H + \bar{b}) - \max\{(w_L + \bar{b})\,;\, v_2^U\}\big) \ > \ c_1/p.$$

To fix ideas, suppose that when searching on the job in period 2, uninformed workers also search for w_H, which requires:

$$(w_H - w_L) \ > \ c_2/p.$$

Then, $v_2^U = w_H + p\underline{b} + (1-p)\bar{b} - c_2/p$ and an uninformed worker leaves upon discovering $(w_H, \underline{b})$ when:

$$w_H + \underline{b} \ < \ v_2^U \iff c_2/p \ < \ (1-p)(\bar{b} - \underline{b}).$$

An uninformed worker also expects to leave $(w_L, \bar{b})$ if:

$$w_L + \bar{b} \ < \ v_2^U \iff c_2/p \ < \ (w_H - w_L) - p(\bar{b} - \underline{b}).$$

Then, the uninformed workers indeed search for w_H in period 1 as long as:

$$u_1^U(w_H) - u_1^U(w_L) \ = \ (w_H - w_L) + p\big(v_2^U - v_2^U\big) + (1-p)\big((w_H + \bar{b}) - w_H - p\underline{b} - (1-p)\bar{b} + c_2/p\big) \ > \ c_1/p \iff c_1/p \ < \ (w_H - w_L) + (1-p)(\bar{b} - \underline{b}) + (1-p)c_2/p.$$

As for the firms, the equal profits condition is unaffected by churning of uninformed workers across high-wage firms:

$$\pi(w_H) = 2((1-\lambda) + \tfrac{\lambda}{p})(y - w_H - (1-\tau)\underline{b}) \ = \ 2(1-\lambda)(y - w_L - (1-\tau)\bar{b}) = \pi(w_L) \iff (1-\lambda)\,/\,((1-\lambda) + \tfrac{\lambda}{p}) \ = \ \tfrac{(y - w_H - (1-\tau)\underline{b})}{(y - w_L - (1-\tau)\bar{b})}.$$

While a deviation from $(w_H, \underline{b})$ to $(w_H, \bar{b})$ remains strictly unprofitable, a deviation from $(w_L, \bar{b})$ to $(w_L, \underline{b})$ is unprofitable as long as:

$$2(1-\lambda)(y - w_L - (1-\tau)\bar{b}) \; \geq \; (1-\lambda)(y - w_L - (1-\tau)\underline{b}) \iff$$
$$2 \; \geq \; \tfrac{(y-w_L-(1-\tau)\underline{b})}{(y-w_L-(1-\tau)\bar{b})}.$$

Deviation to some $w \in (w_L, w_H)$ would attract only informed workers and is therefore dominated by $(w_L, \bar{b})$.

Finally, we take into account the workers' PC's. For uninformed workers, this is:

$$u_1^U(w_H) - c_1/p \; =$$
$$w_H + p\underline{b} + (1-p)\bar{b} + (1-p)(w_H + \bar{b}) + p(w_H + p\underline{b} + (1-p)\bar{b} - c_2/p) - c_1/p \; = \; 0.$$

The PC of informed workers, in turn, is binding for:

$$(1-p)u_1^I(w_L) + pu_1^I(w_H) - c_1 \; = \; 2((1-p)(w_L + \bar{b}) + p(w_H + \underline{b})) - c_1 \; = \; 0.$$

In sum, a fictitious compensating differentials equilibrium in which the high-wage, low-benefits jobs provide a strictly higher utility (and are strictly more costly to provide) may exist and feature within-sector churning of uninformed workers on the equilibrium path as long as the following conditions hold simultaneously:[3]

(i) $c_1/p \; \geq \; 2((w_H - w_L) - (\bar{b} - \underline{b}))$,

(ii) $c_2 \; \geq \; -(1-p)(w_H - w_L) + (1-p)(\bar{b} - \underline{b})$,

(iii) $c_2/p \; < \; (1-p)(\bar{b} - \underline{b})$,

(iv) $c_2/p \; < \; (w_H - w_L) - p(\bar{b} - \underline{b})$,

(v) $(1-\lambda) \,/\, ((1-\lambda) + \frac{\lambda}{p}) \; = \; \frac{(y-w_H-(1-\tau)\underline{b})}{(y-w_L-(1-\tau)\bar{b})}$,

(vi) $2 \; \geq \; \frac{(y-w_L-(1-\tau)\underline{b})}{(y-w_L-(1-\tau)\bar{b})}$,

(vii) $w_H + p\underline{b} + (1-p)\bar{b} + (1-p)(w_H + \bar{b}) + p(w_H + p\underline{b} + (1-p)\bar{b} - c_2/p) - c_1/p \; = \; 0$,

(viii) $2((1-p)(w_L + \bar{b}) + p(w_H + \underline{b})) - c_1 \; = \; 0$.

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[3]Note that $c_2/p < (w_H - w_L) - p(\bar{b} - \underline{b})$ implies both $c_2/p < (w_H - w_L)$ and $c_2 < p(w_H - w_L) + (1-p)(\bar{b} - \underline{b})$.