

Crowding in Growth

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What is the effect of changes in government debt w/o Ricardian equivalence?

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Crowding in \geq Crowding out?

How big can "crowding in" be?

Environment: het. agents + inc. markets + two assets w/o nom. frictions

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 - for small increases, higher debt improves insurance, **crowding in** risky investment
 - for large increases, classical **crowding out** effects overweight
- **Quantitative results:** there exists potential for welfare and growth increases
 - higher debt can crowd in up to **0.5 percentage points** annual growth
 - welfare increases by **2.5 percentage points** of consumption equivalence

Interaction of heterogeneity and policy:

Woodford (1990), Heathcote (2005), Kitao (2008), Challe and Ragot (2010), Kaplan and Violante (2014), McKay and Reis (2016), **Bayer, Born, and Luetticke (2022)**

Optimal level of government debt with heterogeneous agents:

Aiyagari and McGrattan (1998), Flodén (2001), Krueger and Perri (2011), Gomes, Michaelides, and Polkovnichenko (2012), Röhrs and Winter (2015), Bhandari et al. (2016), Röhrs and Winter (2017), Dyrda and Pedroni (2023)

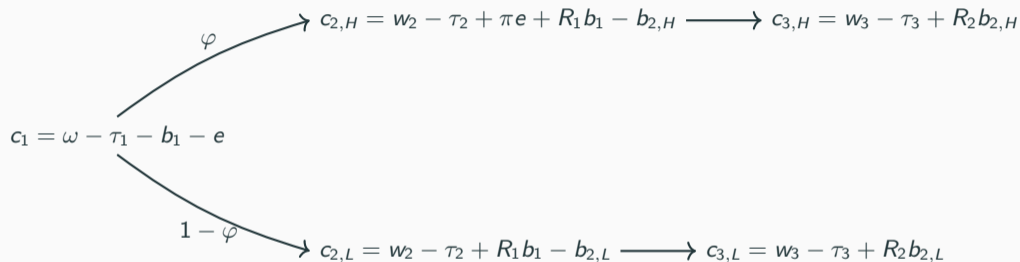
Endogenous growth and financial frictions literature:

Romer (1990), Buera and Shin (2013), Midrigan and Xu (2014), **Kung (2015)**, Bianchi, Kung, and Morales (2019), Anzoategui et al. (2019), Okada (2022)

Toy model

Three-period toy model

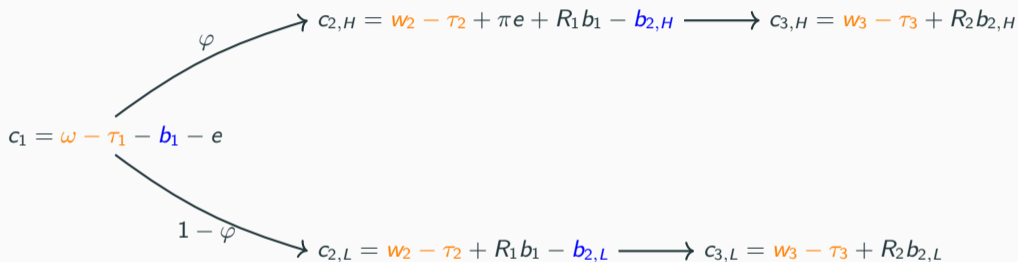
Figure 1: Setup of the three-period model



- ex-ante identical continuum of households that live for three periods
- maximize ex-ante life-time utility $V(c_1, c_{2,H}, c_{2,L}, c_{3,H}, c_{3,L})$

Three-period toy model

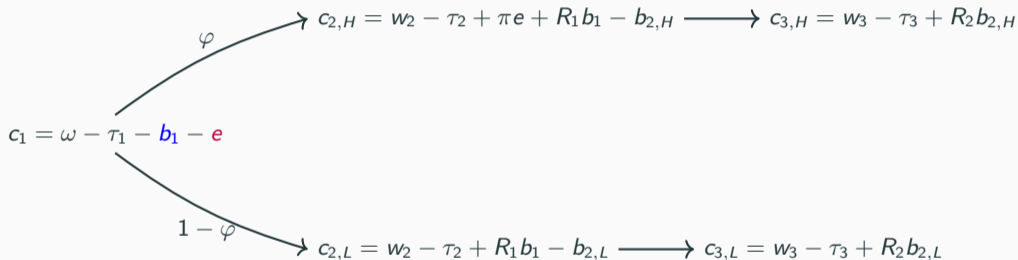
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- households obtain (after-tax) endowments $\omega - \tau_1$, $\omega_2 - \tau_2$, and $\omega_3 - \tau_3$
- can save in risk-free government debt $b_1 \geq 0$, $b_{2,i} \geq 0$ to smooth consumption

Three-period toy model

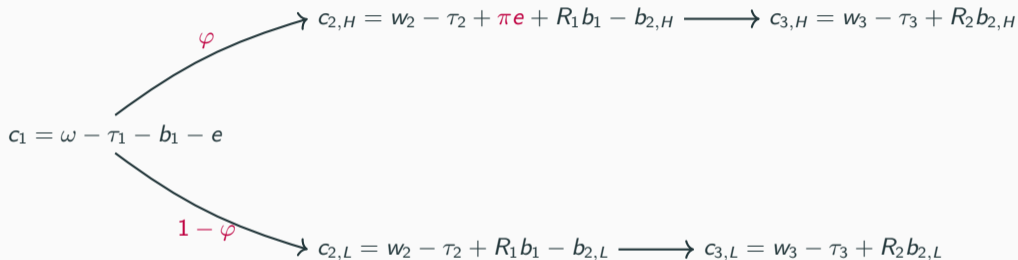
Figure 1: Setup of the three-period model



- portfolio choice between safe asset b_1 or risky asset e

Three-period toy model

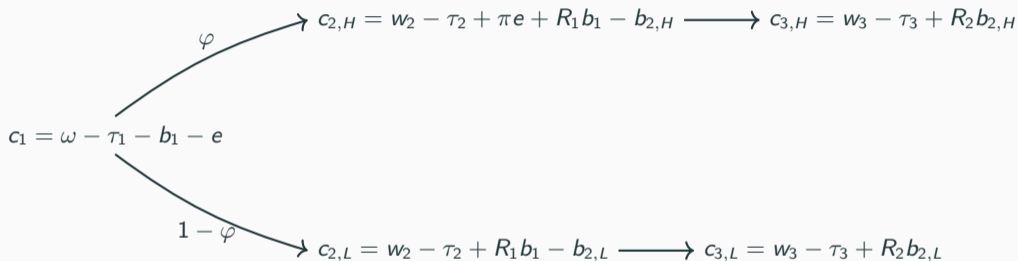
Figure 1: Setup of the three-period model



- with probability φ risky investment generates payoff πe , with $1 - \varphi$ investment lost
- successful households have declining income profile

Three-period toy model

Figure 1: Setup of the three-period model



- Government: $\tau_1 = -\mathcal{B}$, $\tau_2 = (R_1 - 1)\mathcal{B}$, and $\tau_3 = R_2\mathcal{B}$
- Firms: $w_t = w_t(\mathcal{E}_t)$ with $\mathcal{E}_t = \int_0^1 e_{it} di$ and $w_2 < w_3$

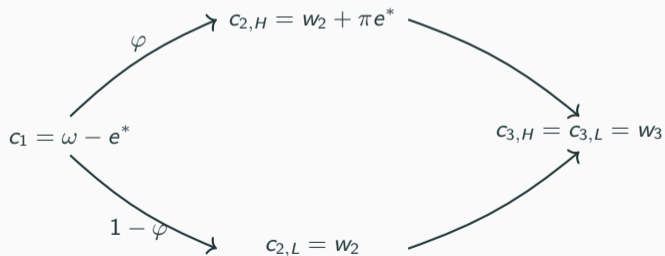
Risky investment and household utility increase with government debt

$$\frac{\partial V}{\partial \mathcal{B}} > 0 \quad \text{and} \quad \frac{\partial e^*}{\partial \mathcal{B}} > 0 \quad \text{if} \quad \mathcal{B} < \mathcal{B}^*.$$

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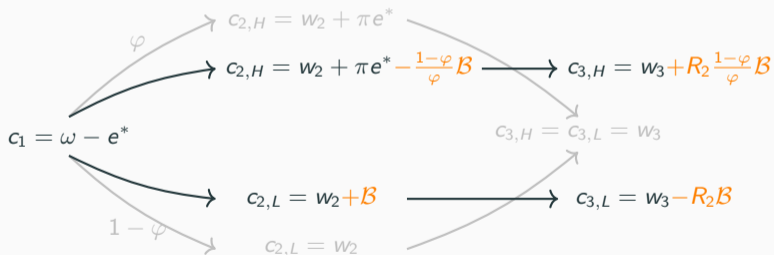
Figure 2: Allocations for $B = 0$



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Figure 2: Allocations for $0 < \mathcal{B} < \mathcal{B}^*$



- $\mathcal{B} \uparrow$ enables **smoother consumption** \Rightarrow **increases utility** from successful investment

Quantitative Model

Households

- obtain income from **idiosyncratic labor** $w_t h_{it} n_{it}$
- **risk-free** asset income $R_t a_{it}$ (government debt b_{it} and capital k_{it})
- **risky** equity income $\pi_t e_{it}$ from holdings of an intermediate goods variety

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Firms

Production

Innovators

- symmetric interm. firms produce **differentiated varieties** as monopolists determining π_t
- innovator **produces new variety/risky equity claims** at price q_t
- final goods bundler combines capital K_t , labor N_t , and **varieties** $\mathcal{E}_t = \int_0^1 e_{it} di$

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Government

Details

- supplies **bonds** B_t , **taxes** households τ_t^L , and has **wasteful expenditure** G_t

HHs maximize subject to labor income risk, return risk, and incomplete markets

$$V_t = \max_{\{c_{it}, n_{it}, a_{it}, e_{it}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u(c_{it}, n_{it})$$

$$\text{s.t.} \quad c_{it} + a_{it+1} + q_t e_{it+1} = (1 - \tau_t^L) w_t h_{it} n_{it} + R_t a_{it} + (q_t + \pi_t) e_{it}$$

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Key features: return risk + portfolio choice

Risk

Portfolio

VF

Interactions

Quantitative Exercise

Varying the debt-to-GDP ratio along a balanced growth path

Government budget constraint along BGP

$$(r_t - g_t)\tilde{B}_t = \tau_t^L \tilde{w}_t N_t - \tilde{G}_t$$

Varying the debt-to-GDP ratio along a balanced growth path

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Experiment: change debt \tilde{B}_t and adjust labor income tax τ_t^L or gov. expenditure \tilde{G}_t
alternative preferences, fixed growth rate, illiquid capital

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Questions:

- Does government debt **crowd in** risky investment and growth?
- Does crowding in **compensated classic crowding out** effects?

Table 1: Calibration Details (Quarterly Frequency)

Parameter	Value	Description	Source / Target
Households			
β	0.986	Discount factor	$K/Y = 9.0$ Auclert et al. (2021)
γ	2	Inverse Frisch	Chetty et al. (2011)
λ	0.3%	Portfolio adj. prob.	Income Gini = 0.5
ω	0.88	Scale labor disutility	$N_t = 1.0$ along BGP
ρ_h	0.98	Labor income persistence	Storesletten, Telmer, and Yaron (2004)
σ_h	0.16	Labor income std.	Storesletten, Telmer, and Yaron (2004)
Firms			
α	0.31	Capital share	62% labor income
ϵ	1.19	Substitution elasticity	profit share of 10%
δ	1.75%	Depreciation rate	Bayer, Born, and Luetticke (2022)
ρ	0.1	Growth to equity inv.	conservative value based on estimates
χ	0.1	New varieties scalar	Growth rate of 0.5% qtlly.
φ	92.5%	Prob. keeping equity	Güvönen, Kaplan, and Song (2014)
Government			
τ^L	37.8%	Tax rate level	$G/Y = 0.2$

Higher debt crowds out labor and capital...

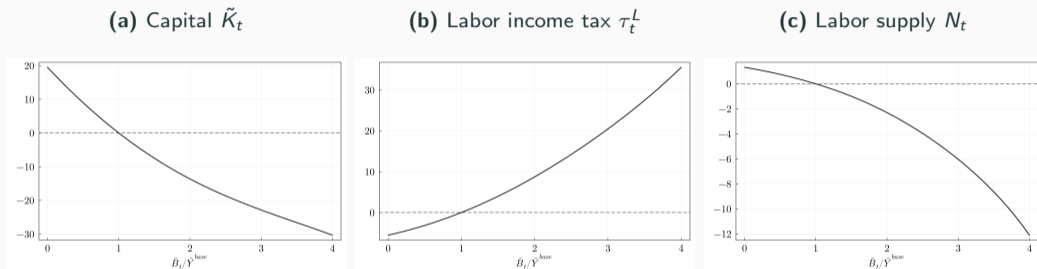


Figure 3: Varying government debt \tilde{B}_t and adjusting labor income tax τ_t^L residually

- higher debt $\tilde{B}_t \Rightarrow$ higher tax rate $\tau_t^L \Rightarrow$ **crowding out** of capital \tilde{K}_t and labor N_t

...but initially crowds in growth and welfare

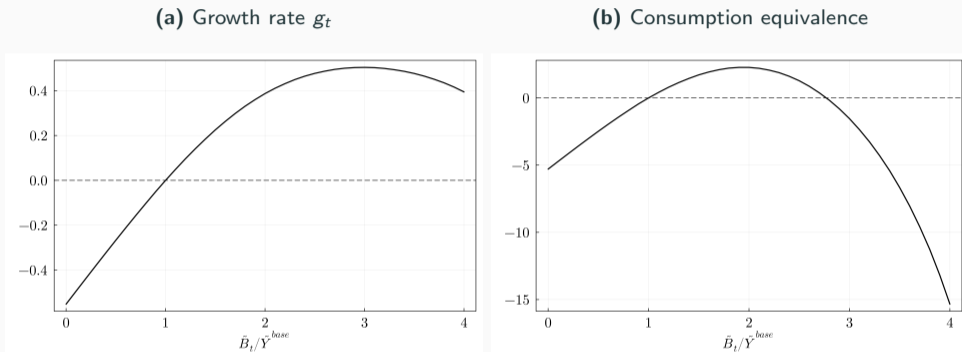
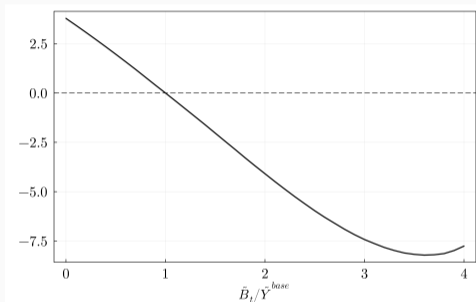


Figure 4: Varying government debt \tilde{B}_t and adjusting labor income tax τ_t^L residually

- debt **crowds in growth** up to 0.5 percentage points
- **consumption equivalence increases** up to 2.5 percentage points

Debt reduces wealth inequality and stimulates poor households' risky investment

(a) Wealth Gini



(b) Investment shares at 300% debt-to-GDP ratio

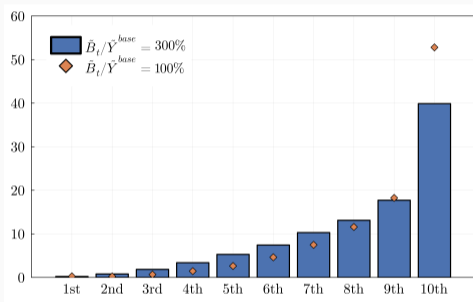
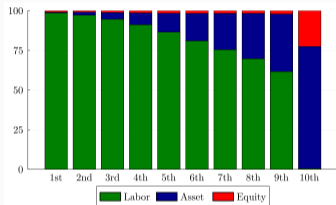


Figure 5: Wealth inequality and relative investment rates

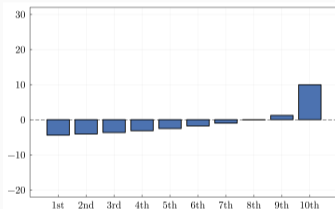
- higher debt **reduces wealth inequality** through asset accumulation
- households at lower end of the distribution **invest more** in risky equity

Highest wealth decile benefits in utility terms due to asset income

(a) Shares of income sources along wealth deciles



(b) CE along wealth deciles at 150% debt-to-GDP ratio



(c) CE along wealth deciles at 250% debt-to-GDP ratio

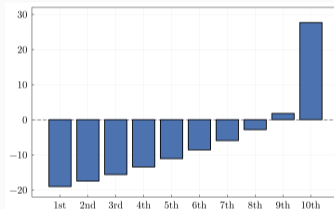
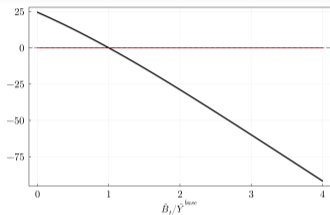


Figure 6: Consumption equivalence and income shares along wealth deciles

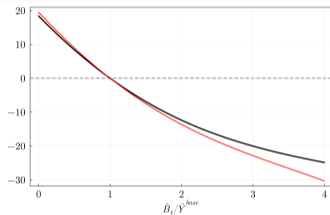
- equity holders benefit from $B \uparrow \Rightarrow$ crowding in equity investment
- households at the bottom of the distribution suffer

Changing the debt level and adjusting Government expenditure G_t

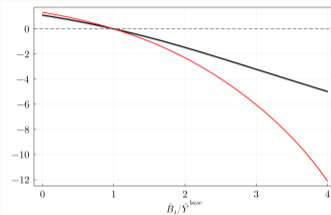
(a) Government expenditure \tilde{G}_t



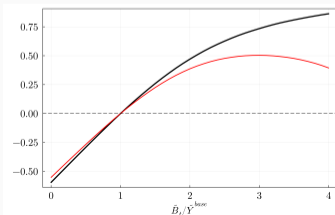
(b) Capital \tilde{K}_t



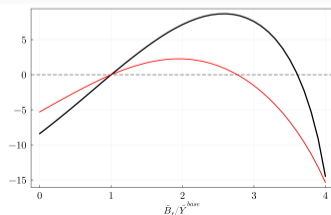
(c) Labor supply N_t



(d) Growth rate g_t



(e) Consumption equivalence








Conclusion







Model with risky investment features crowding in effect of public debt

- higher debt enhances household insurance and crowds in risky investment
- results in initially higher growth and welfare gains
- crowding out of capital and labor more important for higher levels



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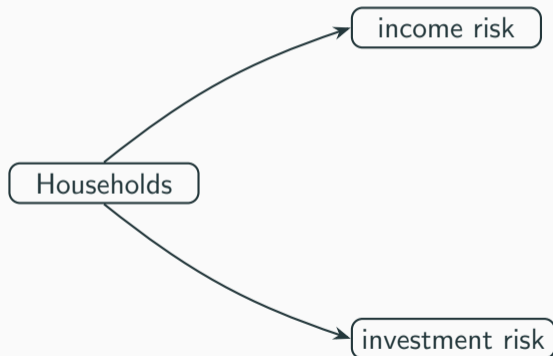
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Appendix

Households		Production	Government
Earn Income	Trade Assets	Development and Production of Goods	Fiscal Authority
<p>Wages w_t</p> <ul style="list-style-type: none"> Supply labor N_{it} Idiosyncratic risk h_{it} <p>Interest R_t</p> <ul style="list-style-type: none"> Liquid, riskless asset a_{it} <p>Profits π_{it}</p> <ul style="list-style-type: none"> From backyard technology i with e_{it} number of varieties j 	<p>Riskless, liquid a_{it}</p> <ul style="list-style-type: none"> Real, riskless, and liquid asset Borrowing constraint $a_{it} \geq \underline{A}$ <p>Risky, illiquid e_{it}</p> <ul style="list-style-type: none"> Only traded with prob. λ Irreversible $e_{it+1} \geq e_{it} \geq 0$ Fails with prob. $1 - \varphi$ Enhances growth g_t <p>⇒ Incomplete Markets</p>	<p>Intermediate Bundler</p> <ul style="list-style-type: none"> Buys differentiated goods Q_{ijt} Bundles goods into Q_t <p>Final Goods Producer</p> <ul style="list-style-type: none"> Use capital K_t, labor N_t, and bundle Q_t to produce Y_t <p>Innovator</p> <ul style="list-style-type: none"> Produces new varieties Δ_t Sells at price q_t to households <p>⇒ Endogenous TFP</p>	<p>Tax & Transfer</p> <ul style="list-style-type: none"> Taxes labor and profits τ^L Lump-sum transfers Tr_t <p>Liquidity provision</p> <ul style="list-style-type: none"> Supplies government bonds B_t <p>Govern. consumption</p> <ul style="list-style-type: none"> Wasteful government expenditure G_t

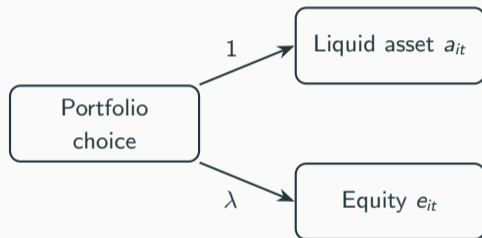
Figure 8: Idiosyncratic risk of the household



$$\log h_{it} = \rho_h \log h_{it-1} + \epsilon_t$$

lose all e_{it} with prob. $1 - \varphi$

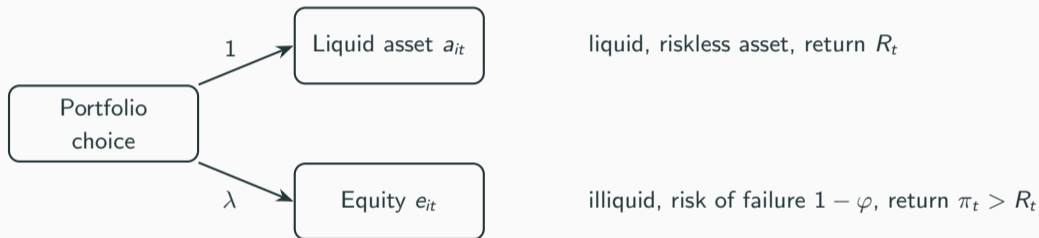
Figure 9: Households portfolio problem



liquid, riskless asset, return R_t

illiquid, risk of failure $1 - \varphi$, return $\pi_t > R_t$

Figure 9: Households portfolio problem



Tradeoff consumption vs. insurance against risk vs. investment opportunity in e_{it}

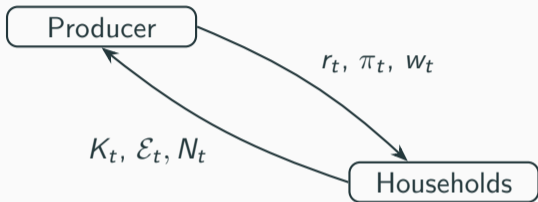
[Details](#)

Figure 10: Households interaction with other agents

Households

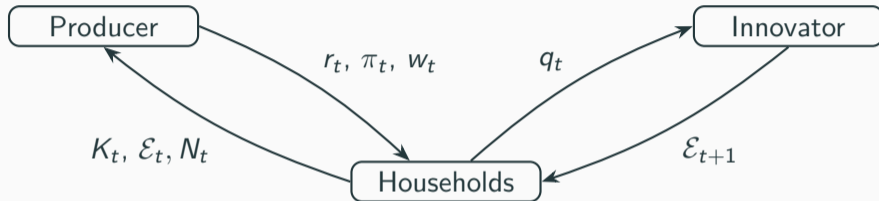
The diagram consists of a single rounded rectangular box with a thin black border, containing the word 'Households' in a simple, black, sans-serif font. The box is centered horizontally and vertically on the page.

Figure 10: Households interaction with other agents



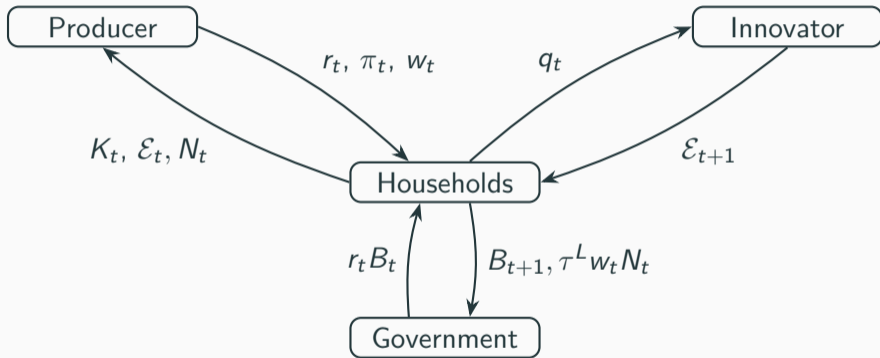
- supply safe asset K_t , risky asset \mathcal{E}_t , and labor N_t to firm
- receive interest income r_t , profits π_t , and wage w_t

Figure 10: Households interaction with other agents



- buy new varieties \mathcal{E}_{t+1} from innovator at price q_t

Figure 10: Households interaction with other agents



- raises debt B_{t+1} and collects taxes $\tau_t^L w_t N_t$
- repays interest cost of debt $r_t B_t$

- Households solve portfolio problem

$$\text{Cases: } \begin{cases} (1 - \lambda) : & V_t^n(a, e, h) = \max_{c, n, a'} u(c, n) + \beta W(a', e, h) \\ \lambda : & V_t^a(a, e, h) = \max_{c, n, e', a'} u(c, n) + \beta W_{t+1}(a', e', h) \end{cases}$$

$$\text{s.t. } c + a' = aR(a, R_t) + (1 - \tau^L)w_t h n,$$

and $a' \geq \underline{A}$

- Households solve portfolio problem

$$\text{Cases: } \begin{cases} (1 - \lambda) : & V_t^n(a, e, h) = \max_{c, n, a'} u(c, n) + \beta W(a', e, h) \\ \lambda : & V_t^a(a, e, h) = \max_{c, n, e', a'} u(c, n) + \beta W_{t+1}(a', e', h) \end{cases}$$

Consumption-saving decision

$$\begin{aligned} \text{s.t. } & c + a' = aR(a, R_t) + \pi e + (1 - \tau^L)w_t h n, \\ & \text{and } a' \geq \underline{A} \end{aligned}$$

with:

$$\begin{aligned} W(a, e, h) = & \varphi \left(\lambda \mathbb{E}_t[V_{t+1}^a(a, e, h')] + (1 - \lambda) \mathbb{E}_t[V_{t+1}^n(a, e, h')] \right) \\ & + (1 - \varphi) \left(\lambda \mathbb{E}_t[V_{t+1}^a(a, 0, h')] + (1 - \lambda) \mathbb{E}_t[V_{t+1}^n(a, 0, h')] \right) \end{aligned}$$

- Households solve portfolio problem

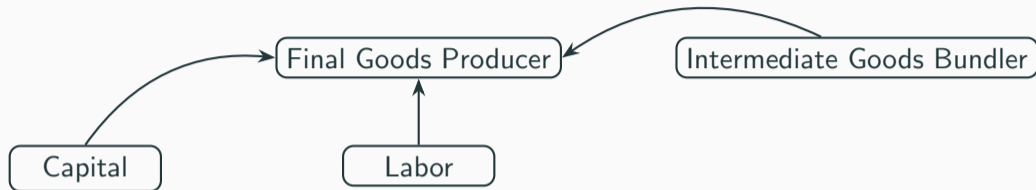
Cases:
$$\begin{cases} (1 - \lambda) : & V_t^n(a, e, h) = \max_{c, n, a'} u(c, n) + \beta W(a', e, h) \\ \lambda : & V_t^a(a, e, h) = \max_{c, n, e', a'} u(c, n) + \beta W_{t+1}(a', e', h) \end{cases}$$

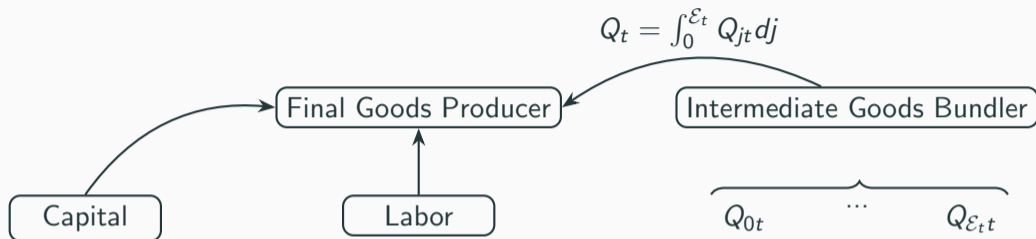
Safe-risky decision

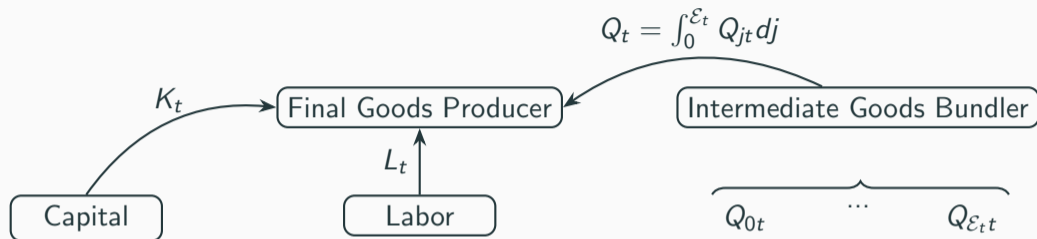
$$\begin{aligned} \text{s.t. } & c + a' + q_t e' = aR(a, R_t) + (q_t + \pi)e + (1 - \tau^L)w_t h n, \\ & \text{and } a' \geq \underline{A} \end{aligned}$$

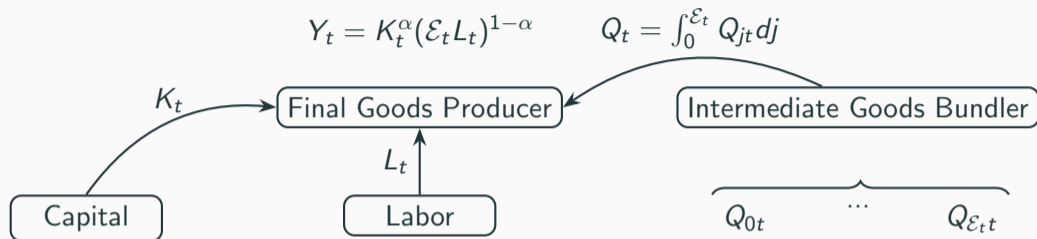
with:

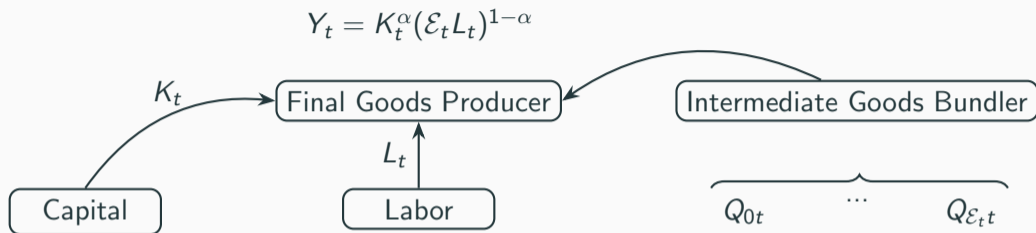
$$\begin{aligned} W(a, e, h) = & \varphi \left(\lambda \mathbb{E}_t[V_{t+1}^a(a, e, h')] + (1 - \lambda) \mathbb{E}_t[V_{t+1}^n(a, e, h')] \right) \\ & + (1 - \varphi) \left(\lambda \mathbb{E}_t[V_{t+1}^a(a, 0, h')] + (1 - \lambda) \mathbb{E}_t[V_{t+1}^n(a, 0, h')] \right) \end{aligned}$$







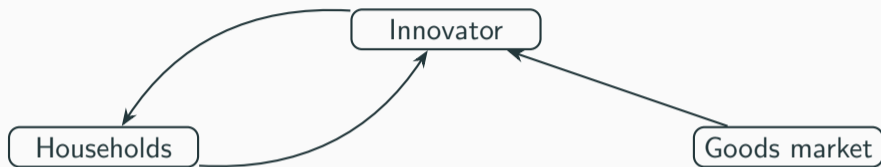


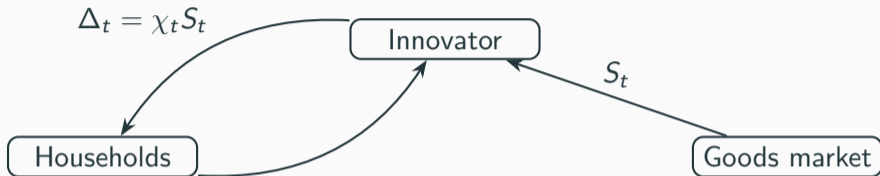


Prices are determined as:

$$r_t + \delta = \phi \alpha \frac{Y_t}{K_t}, \quad w_t = \phi(1 - \alpha) \frac{Y_t}{N_t}, \quad \text{and} \quad \pi_t = (1 - \phi) Y_t.$$

α denotes the capital share in income, $1 - \phi$ denotes the profit share. r_t , δ , w_t , and π_t represent the interest rate, depreciation, wage rate, and profits

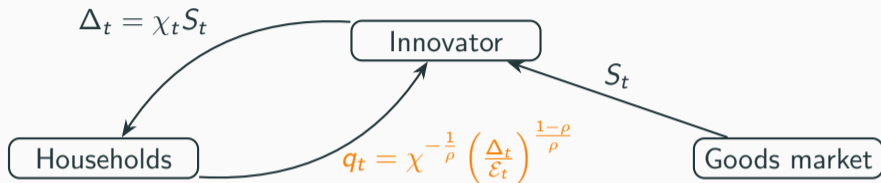




with **externality** as in Comin and Gertler (2006) and Kung (2015)

$$\chi_t = \chi \left(\frac{\mathcal{E}_t}{S_t} \right)^{1-\rho}$$

R&D expenditure S_t , existing varieties \mathcal{E}_t , ρ and χ as scalars



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Firms: produce according to $Y_t = K_t^\alpha (\mathcal{E}_t N_t)^{1-\alpha}$, with $\mathcal{E}_t = \int_0^1 e_{it} di$ as intermediate input

\mathcal{E}_t denotes the number of varieties in the economy

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- interest rate r_t and wage rate w_t from **marginal products** of final goods producer
- profits π_t from **monopolistic competition** of intermediate goods producer

[Details](#)

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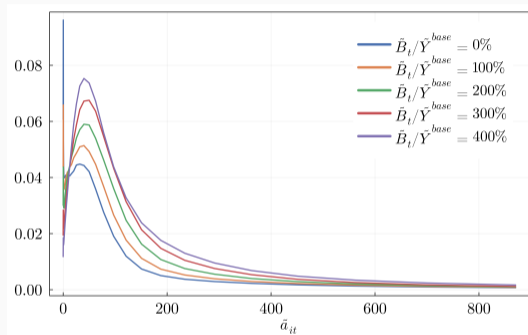
- interest rate r_t and wage rate w_t from marginal products of final goods producer
- profits π_t from monopolistic competition of intermediate goods producer

[Details](#)

Government: holds debt, consumes, and runs tax and transfer system

$$G_t + Tr_t + (1 + r_t)B_t = B_{t+1} + T_t$$

- **repays debt** $(1 + r_t)B_t$, **provides transfers** Tr_t and has **wasteful consumption** G_t
- collects **labor taxes** $T_t = \tau_t^L w_t \int_0^1 h_{it} n_{it} di$ τ_L and raises new debt B_{t+1}



- increasing amount of government debt triggers GE-effects
- distribution actually shifts to the left

compute welfare according to

$$W^* = \int W(a_{it}, e_{it}, h_{it}) d\Theta(a_{it}, e_{it}, h_{it}),$$

and use consumption equivalence for comparison

$$CE(B_t) = \exp((1 - \beta)(W^*(B_t) - W_0^*)) - 1,$$

where W_0^* is the welfare at the benchmark level