Credible Numbers: A Procedure for Reporting Statistical Precision in Parameter Estimates

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A simple example

Standard problem:

- Consider an econometrician who want to recover the "impact" of an explanatory variable X_1 on an outcome variable y, while observing other variables X_2 and X_3 .
- Assume the econometrician has retrieved a sample of \overline{T} observations of $(y_t, X_{1t}, X_{2t}, X_{3t})$ (where t indexes observations).

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Usually unknown information:

• The "true" relationship is:

$$
y=1+1.234X_1+X_2+X_3+\epsilon
$$

where ϵ is a non-modelled error term.

This sample has been generated by taking i.i.d. draws from the following distributions:

 $X_{1t} \sim \mathcal{N}(0, 1), X_{2t} \sim \mathcal{N}(0, 5), X_{3t} \sim \mathcal{N}(0, 10),$ and $\epsilon_t \sim \mathcal{N}(0, 5)$.

Current practice

• The econometrician specifies a handful of models:

Model 1: $y_t = \beta_0 + \beta_1 X_{1t} + \eta_{1t}$ Model 2: $y_t = \beta_0 + \beta_1 X_{1t} + \beta_2 X_{2t} + \eta_{2t}$ Model 3: $y_t = \beta_0 + \beta_1 X_{1t} + \beta_2 X_{2t} + \beta_3 X_{3t} + \eta_{3t}$

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• Results table for two different sample sizes:

 $T = 100$ $T = 1,000,000$

Actual precision - Monte Carlo simulation

Distributions of the digits of the coefficient $\hat{\beta}_1$ across 1,000 draws of datasets (Model 3).

A different approach?

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Outline

² [Proposed reporting rule](#page-27-0)

[Practical significance](#page-38-0)

- Let $\hat{\theta}$ be a scalar estimated from observed data by a researcher (e.g. a coefficient estimate).
- Being a finite real number, $\hat{\theta}$ can be decomposed as a series of digits $\{\hat{d}_i\}_{i=-\infty...D}$:

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- Current practice: pick a default value for i_0 , usually -3 .
- \bullet Our proposal: choose i_0 endogenously to somehow reflect the level of precision of the estimate.

- We propose to build on the literature on "equivalence testing" (Wellek, 2010).
- The objective of an equivalence test is to compare the following hypotheses:

H: $\theta \le a$ or $\theta \ge b$ versus $K: \theta \in]a, b[$ where θ is an unknown underlying parameter of a distribution from which i.i.d. observations are sampled.

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- To assess whether digit $i = -1$ is significant, we test: $H^{i=-1}$: $|\theta - 1.234| > 0.1$ versus $K^{i=-1}$: $\theta \in]1.134, 1.334[$

Last significant digit

Definition (Last significant digit)

We define the rank $i^*(\alpha)$ of the "last significant digit" as the smallest value of i such that digit \hat{d}_i is significant at level α :

$$
i^*(\alpha) \equiv \min_i \{ i \mid \hat{d}_i \text{ is a significant digit at level } \alpha \}
$$

[Number of sizable digits](#page-57-0)

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- We propose to adopt the widely used "two one-sided test" (TOST).

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- We propose to adopt the widely used "two one-sided test" (TOST).
- This test rejects the null hypothesis H_0^i at the level α if, and only if, the two following standard one-sided tests reject their null hypothesis at the level α :

$$
H_a^i: \theta = \hat{\theta} - 10^i \quad \text{versus} \quad K_a^i: \theta > \hat{\theta} - 10^i \quad (\psi_{ia})
$$
\n
$$
H_b^i: \theta = \hat{\theta} + 10^i \quad \text{versus} \quad K_b^i: \theta < \hat{\theta} + 10^i \quad (\psi_{ib})
$$

Application to standard OLS - Coefficient estimates

Consider the standard normal linear conditional mean model:

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y = X\beta + \epsilon \tag{1}
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where the conditional distribution of ϵ given the matrix X is a multivariate normal $T \times 1$ vector with mean zero and covariance matrix $\sigma^2 I_T$.

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where the conditional distribution of ϵ given the matrix X is a multivariate normal $T \times 1$ vector with mean zero and covariance matrix $\sigma^2 I_T$.

• The level α rank of the last significant digit for b_k is:

$$
i^*(\alpha) = \lceil \frac{\ln(SE(b_k)) + \ln(t_{1-\alpha}(T-K))}{\ln(10)} \rceil \tag{2}
$$

where $\lceil x \rceil$ denotes the ceiling of x.

More generally, a widely applicable asymptotic approximation for the size α rank of the last significant digit is:

$$
i_{\infty}^*(\alpha) = \lceil \frac{\ln(SE(b_k)) + \ln(Z_{1-\alpha})}{\ln(10)} \rceil \tag{3}
$$

where $Z_{1-\alpha}$, the $1-\alpha$ percentile of a $\mathcal{N}(0,1)$ random variable.

Let's go back to our simple example where the true relationship is:

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• We assume the researcher observes a sample of $T = 60$ observations of $(y_t, X_{1t}, X_{2t}, X_{3t})$ (where t indexes observations) that has been generated by taking i.i.d. draws from the following distributions: $X_{1t} \sim \mathcal{N}(0, 1), X_{2t} \sim \mathcal{N}(0, 5), X_{3t} \sim \mathcal{N}(0, 10),$ and $\epsilon_t \sim \mathcal{N}(0, 5)$.

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- We assume the researcher estimates and reports three models:

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Estimation results

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Removing non-significant digits:

- **•** generally simplifies the presentation of results tables.
- attracts more attention to the magnitude of the estimates.
- makes statistical precision a salient attribute of estimates.

[Scientific notations](#page-47-0)

Which digits are significant?

$$
\mathsf{T}=100
$$

 $T = 1,000,000$

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 \Rightarrow We propose that researchers should consider reporting digits only up to the first non-significant one.

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We retrieve all estimated coefficients and their associated standard error from articles published in the American Economic Review between 2000 and 2022.

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- We obtain a dataset of over 90,000 pairs (coefficient, standard error), as reported in the published articles.
- We focus on coefficient estimates, for which a lower bound of the last significant digit can be built from the sole knowledge of the estimated standard error (asymptotic bound).

Number of reported decimal digits

Figure: Number of decimal digits displayed for estimated coefficients.

Main result $(1/2)$

Figure: Fraction of the printed digits of estimated coefficients that are not significant digits at different levels of statistical significance.

Main result (2/2)

Figure: Fraction of the printed digits of estimated coefficients that are not significant digits at different levels of statistical significance.

[Number of sizable significant digits](#page-60-0) **[Robustness checks](#page-50-0)**

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[Econometrics](#page-13-0) **Econometrics [Conclusion](#page-45-0) Conclusion** [Proposed rule](#page-27-0) [Significance](#page-38-0) Significance Conclusion Conclusion

Thank you!

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N. Astier (PSE & ENPC) [Credible Numbersx](#page-0-0) - August 2024 24 / 24 / 24

[Appendix](#page-47-1)

Scientific notations

- The rank of the last significant digit may be a positive power of ten (e.g. $i^*(\alpha) = 2$).
- In such cases, the use of scientific notations would be warranted to avoid reporting non-significant zeros.
- Example: divide the units of X_1 , X_2 and X_3 by 1,000

Distribution of coefficients

Figure: Box plots of the magnitude of estimated coefficients (absolute value in log_{10} scale). Boxes locate the first, second and third quartiles of the distributions. Top whiskers (resp. bottom whiskers) are drawn at a distance of 1.5 times the interquartile range above the third quartile (resp. below the first quartile).

Distribution of standard errors

Figure: Box plots of the magnitude of estimated standard errors (absolute value in log_{10} scale). Boxes locate the first, second and third quartiles of the distributions. Top whiskers (resp. bottom whiskers) are drawn at a distance of 1.5 times the interquartile range above the third quartile (resp. below the first quartile).

Robustness checks - Main concerns

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2 Main coefficient estimates may be more precisely estimated that the coefficients on control variables.

Robustness checks - Approach

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- The authors retrieved the estimated main effect and corresponding standard error from over 300 meta-analysis studies.
- We restrict the sample to general interest and field journals in economics.
- \bullet The resulting dataset consists of 18,000+ pairs of coefficient and their associated standard error estimates

Robustness checks - Results

Figure: Histograms of the number of sizable significant digits for the different journals.

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Definition (Number of sizable significant digits)

Let $\hat{\theta}\equiv\sum_{i=-\infty}^{\infty}\hat{d}_{i}10^{i}$ be an estimate of the scalar population parameter $\theta.$ Let $D(\hat{\theta})$ equal the largest rank of its non-zero digits:

$$
D(\hat{\theta}) \equiv \max_{i} i \mathbf{1} \left[\hat{d}_i \neq 0 \right]
$$
 (4)

Let $i^*(\alpha)$ equal the rank of the last significant digit of $\hat{\theta}.$ We define the "number of sizable significant digits", $\nu(\alpha)$ as:

$$
\nu(\alpha) \equiv \max(D(\hat{\theta}) - i^*(\alpha) + 1, 0)
$$

Main result - Other

Figure: Distribution of the number of sizable significant digits.

Prevalence of empirical work

Figure: Evolution of the ratio of the number of reported empirical estimates in each year over the number of printed pages for that year.