# Fixed Costs, Product Heterogeneity, and The Force of Competition

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#### Abstract

Over the last few decades, fixed costs have increased significantly in the US and elsewhere. Yet, macroeconomists lack a framework to interpret this phenomenon. In this paper, we fill this gap by providing a macroeconomic model of fixed costs. The core of the framework is that goods are heterogeneous, as they differ in their fixed and variable costs. We show that, in a contestable equilibrium, the economy's fixed-cost share - i.e., the ratio of fixed-to-total costs - is determined endogenously both by the extensive margin, i.e., the set of goods being produced, and the intensive margin, i.e., the quantity produced of each good. These two margins also shape the effects of economic growth on the fixed-cost share: while intensive growth tends to reduce the fixed-cost share by raising the number of units produced, extensive growth tends to increase the fixed-cost share by activating goods with increasingly high fixed-costs. We use a numerical exploration of the model to rationalize the evolution of fixed costs in the US, and perform a normative analysis of the fixed-cost economy to characterize the distortions that affect the intensive and extensive margins of production in the contestable equilibrium.

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## 1 Introduction

A striking fact of the US economy during the last few decades is the rise of fixed costs. Figure 1 illustrates this by plotting the average ratio of fixed to total costs (henceforth, fixed-cost share) among Compustat firms between 1960 and 2020 (blue line). As the figure shows, the ratio roughly doubled during this period, from 16.9% in 1960 to 32.9% in 2020.

Naturally, this simple average may be misleading as an indicator of fixed costs in the overall economy, as it could be driven by a subset of small firms with very high fixed costs. To address this, Figure 1 also plots the weighted average of the ratio of fixed to total costs throughout the same period, where the weighting is done both by sales and by total costs (red and green lines, respectively). As the figure shows, even this weighted average has grown considerably during the period, increasing by about a third of its initial value. According to this measure, fixed costs represented 23.6% of total costs in 2020.



Figure 1: The evolution of the share of fixed costs in total costs in the US, 1960-2020

Note: The measures are constructed using Compustat. Fixed costs are represented by the "Selling, general and administrative expenses" (SG&A). Total costs are calculated as a sum of SG&A, Cost of Goods Sold (COGS) and capital expenditures. For the details on the data construction see Section 2 and Appendix B.

Besides having increased significantly in the aggregate, fixed-cost shares have also become more dispersed in the cross section of firms. Figure 2 plots the distributions of fixed-cost shares in 1980 and 2019. In 1980, the average firm in Compustat allocated 23% of its total costs to fixed costs, and very few firms had a fixed-cost share higher than 80%. In year 2019, however, the average firm's fixed costs accounted for 34% of its total costs, and about a tenth of all firms had a fixed-cost share higher than 80%. These broad trends are not exclusive to the US: fixed-cost shares have increased and become more dispersed in the world as a whole, and in individual economies, such as Europe, China, Japan and Canada (see Appendix D).



Figure 2: Share of fixed costs in total costs in the US, 1980 and 2019

Note: The measures are constructed using Compustat. Fixed costs are represented by the "Selling, general and administrative expenses" (SG&A). Total costs are calculated as a sum of SG&A, Cost of Goods Sold (COGS) and capital expenditures. For the details on the data construction see Section 2 and Appendix B.

These facts raise a number of key questions for macroeconomists. What can account for the significant rise in fixed costs observed over the last few decades? What are its macroeconomic effects? Do they reflect an underlying economic inefficiency, or are they simply a byproduct of larger markets?

This paper provides a framework to address these questions, with two central features: increasing returns and heterogeneous goods. In particular, each good requires some fixed number of workers to activate production (i.e., fixed costs), and an additional number of workers to produce each unit (i.e., marginal costs). Goods differ ex ante both in their fixed and marginal costs. A key feature of the framework is that the economy's ratio of fixed to total costs is determined endogenously, both by the extensive and intensive margins of production.

We study the determinants of fixed costs in a decentralized equilibrium under the assumption that markets are contestable. In equilibrium, producers cannot be atomistic, because the presence of increasing returns implies that there is space for a single participant in the market of each good. This participant does not have market power, however. If she tried to make profits by setting a price above average cost, another producer would step in and undercut her. Thus, the single producer in each market sets her price equal to average cost. This is the basic idea behind the concept of a contestable equilibrium, which is defined by three requirements: (i) the market clears; (ii) no producer makes losses; and (iii) no producer can enter the market and make profits. In a contestable equilibrium, fixed costs affect production both along the intensive and extensive margins. Since each producer sets her price equal to average costs, fixed costs determine the markup and thus the amount produced of each good. Since the revenues generated by each good must suffice to cover the costs of production, fixed costs also determine which goods are produced in equilibrium. Goods that have a low ratio of fixed-to-marginal costs are more likely to be produced than goods that have a high ratio of fixed-to-marginal costs.

The theory shows that economic growth—driven by an increase in market size or by technological innovation—has mixed effects on the economy's fixed-cost share. On the one hand, it expands the market of goods that were already produced. This expansion along the *intensive margin* naturally reduces the importance of fixed costs, which are spread over the larger number of units produced. On the other hand, economic growth also expands the variety of goods produced. This expansion along the *extensive margin* increases the importance of fixed costs, as goods with greater-than-average ratios of fixed-to-variable costs are activated and introduced into the production mix. Thus, the aggregate fixed-cost share can rise or fall with economic growth depending of the relative strengths of the intensive and extensive margins.

The contestable equilibrium is inefficient. For a benevolent planner, fixed costs are irrelevant to make production decisions along the intensive margin. The reason is that the planner equates the marginal benefit to the marginal cost of production: products with low marginal costs are thus produced in greater quantities than goods with high marginal costs. To determine which goods to produce, however, the planner does take fixed costs into consideration. Specifically, she only produces goods that generate a positive surplus, i.e., for which the total benefit in terms of utility exceeds the total cost of production, including fixed costs. A salient characteristic of the planner allocation is that it "spreads" fixed costs across all goods produced, so that there is cross-subsidization between goods with lower-than-average fixed costs and goods with higher-than-average fixed costs.

The contestable equilibrium is distorted along two dimensions relative to the planner allocation, both of which can be traced back to the absence of cost cross-subsidization across products. First, for a given measure of product variety, the market economy distorts production along the intensive margin. Since each producer has to generate enough revenues to cover her own fixed costs, goods with low ratios of fixed-to-variable costs are over-produced relative to goods with high ratios of fixed-to-variable costs. The planner instead takes only marginal costs into account when deciding on the intensive margin of production. Second, the market economy under-provides product variety. Once again, since goods with high ratios of fixed-to-variable costs are under-produced in the contestable equilibrium, market participants may not find it worthwhile to introduce them even though they generate a positive social surplus in the planner allocation. Too little product variety means that, relative to the planner allocation, the fixed-cost share is inefficiently low in the market economy.

Albeit simple, we show that the model can plausibly account for the increase in the salesweighted aggregate fixed-cost share in the US, from 0.17 in 1980 to 0.23 in 2019. We know from the theory that this increase can be the result of economic growth provided that the extensive margin is sufficiently strong. Using aggregate and Compustat data, we can gauge just how strong. Our baseline calculations imply that, to account for the rise in the fixed-cost share, the growth experienced by the US economy between 1980 and 2019 must have led to a high market share of "new" goods, i.e., goods that—given their costs structure—would not have been produced in 1980. In particular, these goods must have had a combined market share of 0.37 in 2019, and relatively high fixed-cost share of 0.43. In fact, if it were not for the extensive margin, our calculations suggest that the aggregate fixed-cost share in the US would have actually *fallen* by 0.06 between 1980 and 2019.

Finally, we use a numerical example to show that the model is able to reproduce both the evolution of the aggregate fixed-cost share in the US and its (Compustat-based) decomposition into intensive and extensive margins solely as the by-product of US economic growth. We also use this example to evaluate the welfare losses resulting from the presence of increasing returns: in our baseline parametrization, the contestable equilibrium entailed a welfare loss of 11% relative to the planner in 1980, which declined to 8.6% in 2019.

From a macroeconomic perspective, perhaps the works that are closest to ours are Dixit and Stiglitz (1977) and Krugman (1980). As in those seminal frameworks, our economy features different varieties of goods, not all of which are produced in equilibrium. Different from those frameworks, however, goods in our economy differ in their fixed and marginal costs of production. This feature is crucial to generate endogenous variations in the fixed-cost share in response to economic growth. The presence of heterogeneous goods also relates our work to Hopenhayn (1992) and Melitz (2003): one crucial difference is that in our framework goods are heterogeneous ex ante, not just ex post.

In our modelling of competition, we build on the literature on contestable markets (Baumol *et al.*, 1983). A key insight of this literature is that concentration is not akin to market power. In a world of increasing returns, markets are bound to be highly concentrated and it may be natural to have one market participant. This participant will not have market power, however, if the market is contestable: if she were to raise prices above average costs, a new participant would enter and undercut her price. We build on this literature by characterizing the contestable equilibrium in a macroeconomic model with heterogeneous goods, and analyzing its implications for the evolution of fixed costs.

Conceptually, our model relates to the vast literature on economies with increasing returns.

A result of this literature is that competitive equilibria are generically inefficient. JAUME COMPLETE

Our work also complements the recent literature that documents and tries to explain the rise of markups in the US and elsewhere in the world over the last few decades. De Loecker *et al.* (2020), for instance, document a significant increase in markups in the US. De Loecker *et al.* (2020) and Díez *et al.* (2021) show that this increase in markups has also taken place in other advanced economies. We relate to this work naturally because the significant rise in the fixed-cost shares that we document naturally implies increasing markups. Differently from existing work, however, we do not focus on markups and their relationship to higher profits or market power, but rather on providing a macroeconomic framework to characterize the equilibrium determinants of fixed costs.

On the measurement front, we relate to a series of recent papers that measure fixed costs and their evolution. Our approach is closest to De Loecker *et al.* (2020), who construct fixed costs directly from the data using the different cost categories provided by the Compustat database. The main advantage of this approach is that it is transparent: the categories of costs are well-defined, and it does not rely on a model-based estimations. In addition, these costs vary across firms (sectors) and time, allowing for cross-sectional analysis and decomposition. Like us, De Loecker *et al.* (2020) document that fixed costs have increased in recent decades.

De Ridder (2024) uses a model to derive a time-varying measure of fixed costs, and also documents a substantial increase both in the US and France in recent years. A different approach in corporate finance consists in measuring fixed costs (called the degree of operating leverage) as the elasticity of firm's operating income to sales (Mandelker and Rhee, 1984). García-Feijóo and Jorgensen (2010) provide a discussion of various challenges related to these estimation techniques; notwithstanding the challenges, Saibene (2016) finds a steady increase in the operating leverage estimates in the Compustat data. Using yet another approach, Abraham *et al.* (2024) rely on the properties of the primal and dual price-based and costbased Solow-residual to estimate price-cost margins and fixed costs. While their main findings highlight the importance of fixed costs in estimating price-cost margins and profits, they find that both the fixed costs and price-cost margins declined in Belgium between 1985 and 2014.<sup>1</sup>

The paper is organized as follows. Section 2 documents the rise of fixed costs over the last few decades. Section 3 develops the theoretical framework, characterizes the contestable equilibrium and discusses the inefficiencies associated with it. Section 4 revisits the data in light of the theory and shows that the increase in fixed costs can be rationalized as the by-product of US economic growth. Finally, Section 5 concludes. All proofs, data construction

<sup>&</sup>lt;sup>1</sup>The fall in price-cost margins (and, thus, markups) is also present in the Belgian data when using alternative measures (De Loecker and Warzynski, 2012; Cavalleri *et al.*, 2019), and therefore these trends might be specific to Belgium.

details and robustness exercises are relegated to the Appendix.

## 2 The Rise of Fixed Costs

In this section we discuss the main stylized facts about fixed costs, focusing both on timeseries and cross-sectional dimensions. We analyze the relationship between fixed costs and firm size, and look at the heterogeneity across different sectors of the economy. Before doing so, however, we discuss the measure of fixed costs that we use.

#### 2.1 The Measure of Fixed Costs

There is no universally accepted definition of fixed costs. Generally speaking, we would like to sort all types of costs that a firm faces into those that directly depend on the quantity of production (variable costs) and those that do not (fixed costs). Some costs, such as expenses on labor or raw materials can be defined as variable. This division, however, is not clear for all types of expenditures. A firm's quantity of production may affect relative spending on items that are seemingly independent from scale (such as, for example, expenses on research and development or insurance payments).

Data availability introduces another problem: even if we were able to suggest a reasonable division of every expense into variable and fixed, there is no detailed information on all types of expenses. For our main analysis we use Compustat database, which provides financial data on US publicly traded companies.<sup>2</sup>. Most of the expenses in the Compustat database are bundled together into broad categories.

Notwithstanding these challenges, we follow the literature (see De Loecker *et al.* (2020)) and obtain the measures of fixed and variable costs directly from the data. This approach is transparent and does not rely on modeling assumptions to estimate fixed costs. In addition, we can observe time-varying costs at the firm level, which allows us to establish important cross-sectional facts.<sup>3</sup>

As a proxy for fixed costs in Compustat we use the variable called "Selling, General, and Administrative Expense" or SG&A. Compustat dataset roughly follows the principle of the dependence on the production scale to categorize costs into this category. According to the Compustat User Guide definition, this item represents "all commercial expenses of operation (such as, *expenses not directly related to product production*) incurred in the regular course

<sup>&</sup>lt;sup>2</sup>See Appendix B for the details on data construction. We present results for other countries using the Worldscope database (see Appendix D).

<sup>&</sup>lt;sup>3</sup>Alternative measures used in De Ridder (2024) and Saibene (2016) confirm the rising fixed costs shares in the US economy.

of business pertaining to the securing of operating income". SG&A includes 29 items, among which, for example, are accounting expenses, advertising expenses, directors' fees and remuneration, engineering expenses, legal expenses, marketing expenses, research and development expenses, etc.<sup>4</sup>

To define the variable costs, we use the variable "Cost of Goods Sold" (COGS), which represents "all expenses that are *directly related to the cost of merchandise purchased or the cost of goods manufactured* that are withdrawn from finished goods inventory and sold to customers". COGS includes 37 items, among which are, for example, the following: amortization, direct labor, expenses associated with sales-related income, heat, light, and power, insurance and safety, licenses, lease, rent, salary expense, supplies, etc.

Finally, firms' capital expenditure is part of the total costs. To calculate capital expenditures we follow the the standard procedure in the literature (see, for example, De Loecker *et al.* (2020)): a measure of the user cost of capital is calculated using the federal funds rate, inflation, and depreciation rate, and multiplied by the gross capital variable in the Compustat database.<sup>5</sup> The average share of capital expenditure in total cost in our data is relatively stable, and fluctuates around 10% of the total cost throughout the period.<sup>6</sup>

To understand how these different types costs relate to each other and evolve over time, we define the total cost as the sum of SG&A, COGS and capital expenditures, and calculate the (weighted average) share of each component.

COGS is by far the largest expense category, with the average firm in the sample allocating around 70% of its expenditures in this category.<sup>7</sup> Roughly another 20% is allocated by firms as SG&A, which proxies for fixed costs. These expense categories evolved differently over time. Figure 3 plots the gross growth rates of these shares, relative to year 1960. As the figure demonstrates, SG&A share has grown the most (by 68% by year 2020). The share of COGS has declined by roughly 13%: from 75% in 1960 to 65.8% in 2020. The share of capital expenditure remained relatively stable over the entire time period.

<sup>&</sup>lt;sup>4</sup>Ideally, we would like to measure the expenses for all the individual components within SG&A and make a more precise division into variable and fixed costs. Unfortunately, only a very limited set of costs is reported by the Compustat firms, and data is missing for the majority of the variables. What we can establish is that the expenses on R&D contributed substantially to the growth of SG&A. Their (sales weighted) share within SG&A has increased from 4.9% in 1960 to 14.7% in 2020. Advertisement expenses, on the other hand, comprise a moderate share of SG&A, and grew from virtually zero to 4.7% in 2020.

<sup>&</sup>lt;sup>5</sup>The variable name is "Property, Plant, and Equipment – Total (Gross)" (PPEGT), and it represents "the cost of fixed property of a company used in the production of revenue before adjustments for accumulated depreciation, depletion, and amortization."

<sup>&</sup>lt;sup>6</sup>De Ridder (2024) demonstrates the robustness of the fixed cost share increase in the US and French data to different definitions of capital expenditure. Barkai (2020) carefully constructs measures of capital costs and finds a slight decrease in the alternative measures of capital costs, albeit small.

<sup>&</sup>lt;sup>7</sup>This number is calculated as the mean of the sales-weighted average proportions of costs across different years.



Figure 3: Gross growth rate of the (sales weighted) shares of different types of costs in total costs, 1960-2020

Note: The measures are constructed using Compustat. Total costs are calculated as a sum of SG&A, Cost of Goods Sold (COGS) and capital expenditures. For the details on the data construction see Appendix B.

In our baseline calculations (used to produce Figure 1 and Figure 2), we attribute capital expenditures entirely to variable costs. As a robustness exercise in Appendix C, we also check how our results would change if capital expenditures were instead included in fixed costs. The answer is that the overall increase in sales-weighted average fixed cost shares would be lower (37% as opposed to 68% in our baseline calculations), although from a larger base. Similarly, in Appendix C we show the levels and the growth rates of expenditure shares under an assumption of distributing the capital expenditures equally into fixed and variable costs (Figure 15 through Figure 19). Additionally, we show the counterparts of Figure 1 and Figure 2 under these alternative assumptions (Figure 11 through Figure 14). The main takeaway from this exercise is that the increase in fixed cost shares is large and persistent for all of these alternative definitions.

#### 2.2 Stylized Facts

The discussion so far has focused on the trends for average fixed cost shares, but there are interesting patterns and stylized facts behind these averages. How do fixed cost shares differ across industries? Is the rise of fixed cost sriven by changes within or across industries? What are the determinants of the fixed cost shares at the firm level? We start with a simple observation that fixed cost shares may differ across sectors of the economy. We define three broad sectors: agriculture and mining (NAICS codes 11 and 21), manufacturing (NAICS codes 31, 32, and 33), and services (all the remaining 2-digit NAICS codes). Figure 4 shows the evolution of the average shares of fixed costs *within* these broadly defined sectors. In the 1960s the fixed cost shares were roughly similar. In the manufacturing and services sectors they were steadily rising. In particular, in the the manufacturing sector the shares almost doubled, rising from 14% in 1960 to 26% in 2020. The average fixed cost shares in the agriculture and mining sector declined from 12.8% to 6.9%.



Figure 4: Sales-weighted share of fixed costs within different sectors, 1960-2020

Note: The measures are constructed using Compustat. Total costs are calculated as a sum of SG&A, Cost of Goods Sold (COGS) and capital expenditures. The sales weights are calculated within each sector. For the details on the data construction see Appendix B.

	Fixed Cost Share	$\Delta$ Fixed Cost Share	$\Delta$ Within	$\Delta Between$	$\Delta Cross$
1970	0.179	0.038	0.039	0.000	-0.001
1980	0.171	-0.008	-0.008	-0.000	-0.000
1990	0.210	0.039	0.040	0.000	-0.001
2000	0.223	0.013	0.012	0.000	0.001
2010	0.232	0.009	0.012	-0.002	-0.000
2020	0.236	0.004	0.003	0.001	0.000

Table I: Sectoral decomposition of the 10-year changes in fixed cost shares

At the same time, there were notable shifts in the shares of total sales across sectors. Services sector expanded substantially. In 1960 the total sales of services firms represented only 21.3% of the sample, in 2020 the their share rose to 48.1%, mostly at the expense of manufacturing, which shrank from 73.3% to 47.4%. To understand where the changes in the fixed cost shares come from, we adopt the following standard decomposition:

$$\Delta \phi_t = \sum_s m_{s,t-1} \Delta \phi_{s,t} + \sum_s \phi_{s,t-1} \Delta m_{s,t} + \sum_s \Delta \phi_{s,t} \Delta m_{s,t}$$

where  $\Delta \phi_t$  represents the change in sales-weighted fixed cost shares. This total change is decomposed into three components: within sector, between sectors, and a cross-term. Table I presents the results of the decomposition over 10-year intervals (starting in 1960). Most of the changes occur at the sector level, reflected in  $\Delta$ Within. In Table IV and Table V in Appendix C we perform the same decomposition at the 2 and 4-digit level of NAICS classification. Again, the within-sector component remains large. Thus, the importance of fixed cost shares is rising for all sectors of the economy, and is not driven purely by the expansion of firms in the high-fixed cost sectors.

	(1)	(2)	(3)	(4)	(5)	(6)
(log) Fixed cost share	$-1.141^{***} \\ (0.019)$	$-1.139^{***}$ (0.004)	$-1.144^{***}$ (0.018)	$-1.128^{***}$ (0.003)	$-1.386^{***}$ (0.025)	$-1.219^{***}$ (0.005)
$(\log) SG\&A$		$\begin{array}{c} 1.074^{***} \\ (0.001) \end{array}$		$\frac{1.081^{***}}{(0.001)}$		$\frac{1.078^{***}}{(0.002)}$
Constant	$\begin{array}{c} 10.135^{***} \\ (0.036) \end{array}$	$-1.086^{***}$ (0.019)	$\begin{array}{c} 10.131^{***} \\ (0.036) \end{array}$	$-1.141^{***}$ (0.020)	$9.764^{***} \\ (0.041)$	$-1.252^{***}$ (0.023)
Observations	269701	269701	269701	269701	252074	252074
R-squared	0.159	0.956	0.189	0.957	0.448	0.964
Time	No	No	Yes	Yes	No	No
Industry-Time	No	No	No	No	Yes	Yes

Table II: Dependent variable: (log) sales

Standard errors in parentheses

\* p < 0.10, \*\* p < 0.05, \*\*\* p < 0.01

Next we want to understand how fixed cost shares vary across firms. Figure 1 already pointed to two key features of the relationship between fixed costs and market size. First, they appear to be directly related along the time dimension: as the economy grows, so does

Note: The measures are constructed using Compustat data. Total costs are calculated as a sum of SG&A, Cost of Goods Sold (COGS) and capital expenditures. Fixed cost shares are calculated as the proportion of SG&A in the total costs. Industry is defined at the 6-digit NAICS level.

the importance of fixed costs. Second, they appear to be inversely related in the cross-section: since the weighted average of fixed costs is substantially lower than its simple average, it must be that – at least within the Compustat sample – smaller firms have higher fixed costs. Table II shows the the correlation between firms' sales and fixed cost shares. The even-numbered columns additionally control for the level of SG&A. Indeed, larger firms have higher levels and smaller shares of fixed costs. This relationship remains negative even within narrowly defined industries (6-digit NAICS codes), as can be seen from columns (5) and (6). Larger firms also have smaller fixed cost shares within sectors. Table VI through Table VIII in the Appendix run the regressions separately for each broadly defined sector of the economy, both for the whole sector and controlling for the 6-digit NAICS industries within a sector. Figure 20 through Figure 23 in Appendix C plot the series of cross-sectional coefficients of (log) sales on fixed cost shares for the whole economy and the broad sectors, demonstrating that the negative relationship between firm size and fixed cost shares also persists over time.

## 3 The fixed-cost economy

This section develops a theoretical framework that combines three ingredients: the production of goods requires fixed and variable costs; there is substantial product heterogeneity; and producers have no market power. We interpret this framework as a long-run equilibrium.

## 3.1 Utility maximization

We consider an economy populated by a mass L of individuals who are both consumers and workers. There is an infinite mass of consumption goods, indexed by  $z \in [0, \infty)$ . Preferences over consumption bundles are represented by this standard isoelastic utility function:

$$U = \int_0^\infty \frac{\sigma}{\sigma - 1} C\left(z\right)^{\frac{\sigma - 1}{\sigma}} dz,\tag{1}$$

where C(z) is the consumption of good z, and  $\sigma > 1$ . To finance their consumption bundle, individuals work and earn a wage W. Thus, their budget constraint is given by:

$$\int_0^\infty P(z) C(z) \, dz \le W,\tag{2}$$

where P(z) be the price of good z. Equation (2) simply says that the value of all purchases cannot exceed income.

Utility maximization implies the following consumption choices:

$$\lambda P(z) = C(z)^{-\frac{1}{\sigma}}, \qquad (3)$$

where  $\lambda$  is the Lagrange multiplier on the budget constraint, i.e., the marginal utility of income. Equation (3) says that individuals choose their consumptions by equating their marginal costs and benefits, both measured in terms of utils. This consumption policy delivers the following welfare or utility:

$$U = \frac{\sigma}{\sigma - 1} \lambda W. \tag{4}$$

That is, welfare is proportional to income W and its marginal utility  $\lambda$ . It is convenient to set  $\lambda = 1$  as our numeraire rule, so that the consumption of any good depends only on its own price (see Equation (3)), and welfare depends only on the wage (see Equation (4)).

#### **3.2** Producer competition

Labor is the only factor of production. The technology to produce good z is given by:

$$Q(z) = \max\left\{\frac{L(z) - \phi(z)}{\upsilon(z)}, 0\right\},\tag{5}$$

where Q(z) and L(z) denote the production of good z and the labor employed in it respectively. There is a fixed cost of  $\phi(z) \in [0, \infty)$  workers, and a marginal cost of  $v(z) \in (0, \infty)$ workers per unit produced. For reasons that will become apparent soon, we define a cost index  $I(z) \equiv \phi(z) v(z)^{\sigma-1}$  and order goods such that  $I(\cdot)$  is non-decreasing. To streamline the analysis, we assume  $v(\cdot)$  and  $\phi(\cdot)$  are continuous,  $I(\cdot)$  is strictly increasing, and  $\lim_{z\to\infty} I(z) = \infty$ .

Consistent with our interpretation of the model as a long-run equilibrium, we assume free entry and no sunk costs. Free entry means that, once the market clears, (i) no producer makes losses; and (ii) no producer could offer a lower price and make profits. The assumption of no sunk costs means that all costs must be considered when determining these losses and profits.<sup>8</sup>

Since technologies exhibit increasing returns, there is space only for a single active producer in each market. If no producer enters the the market for good z, we adopt the convention that  $P(z) = \infty$  which implies C(z) = 0. If a producer enters the market for good z, she sets the following price:

$$P(z) = \min P \text{ s.t. } P \ge \left( \upsilon(z) + \frac{\phi(z)}{Q} \right) W \text{ and } Q = P^{-\sigma}L.$$
 (6)

Equation (6) defines the lowest price that covers the average cost, given the demand for the

<sup>&</sup>lt;sup>8</sup>This notion of market equilibrium can be interpreted as the outcome of Bertrand competition. To see this, consider the following two-stage game. In the first stage, symmetric producers simultaneously decide whether to enter and post a price. In the second stage, they produce what consumers demand at the posted price. Producers choose not to enter if they anticipate losses. Otherwise, producers enter and post the lowest price that covers costs, that is, their average cost. Then, a single producer serves the entire market and the others do not produce.

product. This lowest price ensures that (i) the active producer does not make losses, and (ii) inactive producers cannot offer a lower price and make profits. This lowest price exists if and only if either the fixed or the marginal cost is low enough.

Let us define the marginal good  $\bar{z}$  as the good such that:<sup>9</sup>

$$I(\bar{z}) = \frac{1}{\sigma - 1} \left(\frac{\sigma}{\sigma - 1} W\right)^{-\sigma} L.$$
(7)

To gain some intuition about this good, recall that the monopoly price (which maximizes revenues net of variable costs) is given by:  $P_M(z) = \frac{\sigma}{\sigma-1}v(z)W$ . The marginal good has the largest cost index that, given labor costs W and the size of the market L, is compatible with the producer not making losses:  $\phi(\bar{z})W = (P_M(\bar{z}) - v(\bar{z})W)P_M(\bar{z})^{-\sigma}L$ . Defining this marginal good is useful because in the markets for goods with  $z < \bar{z}$  there is an active producer, while in the markets for goods with  $z > \bar{z}$  there is no active producer. Moreover,  $\bar{z}$ defines not only the identity of the marginal good, but also the measure of produced goods.



Figure 5: Market for good z.

Panel (a) in Figure 5 illustrates the market for a good such that  $z < \bar{z}$ . The supply schedule shows the price at which producers are willing to sell each quantity:  $P^S(z) = (v(z) + \phi(z)/Q)W$ . This schedule is downward-sloping because the average cost declines with production. The demand schedule shows the price at which consumers are willing to buy each quantity:  $P^D(z) = (Q/L)^{-\sigma}$ . This schedule is downward-sloping because of diminishing marginal utility of consumption. The supply and demand schedules cross twice. The upper crossing at  $(P_N, Q_N)$  cannot be an equilibrium. At this price-quantity combination, the active producer does not make losses but an inactive producer could offer a lower price between  $P_N$ 

<sup>&</sup>lt;sup>9</sup>That this good exists follows from the properties of the cost index I(z).

and  $P_E$  and make profits. The lower crossing at  $(P_E, Q_E)$  is the unique market equilibrium. At this price-quantity combination, the active producer does not make losses and there is no inactive producer that can enter and make profits.

Panel (b) in Figure 5 illustrates the market for a good such that  $z > \bar{z}$ . The supply and demand schedules do not cross. As a result, there is no active producer in the market. The equilibrium price is  $P(z) = \infty$  and there is neither production nor consumption. The difference between the markets in panels (a) and (b) is entirely driven by heterogeneity in costs. Since all goods enter the utility function symmetrically, the demand schedule is the same in both figures. But the supply schedule differs. In panel (a), the cost index is low and producers can offer the good at a price that is attractive to consumers and still cover costs. In panel (b), the cost index is high and producers cannot offer a price that is attractive to consumers and still cover costs.



Figure 6: Market for the marginal good.

Finally, Figure 6 illustrates the market for the marginal good  $\bar{z}$ . The supply and demand schedules cross only once. In this case, there might or might not be an active producer in the market. If there is an active producer, she must be charging the monopoly price so that no inactive producer can offer a lower price and make profits. If there is no active producer, no producer can offer a finite price and make profits. Since the marginal good has measure zero in the economy, whether this market is active or not has no general equilibrium consequences. For concreteness, and without loss of generality, we assume from now on that there is an active producer in the market for the marginal good.

## 3.3 General equilibrium

An equilibrium of the fixed-cost economy consists of a set of prices such that consumers and producers maximize and markets clear. The following lemma provides a characterization of the general equilibrium of the fixed-cost economy.

**Lemma 1.** An equilibrium of the fixed-cost economy consists of a measure of goods produced  $\bar{z}$ , a product price schedule P(z), and a wage W that solve the following set of equations:

$$P(z) = \begin{cases} \min P \quad s.t. \quad P^{1-\sigma}L = \left[\upsilon\left(z\right)P^{-\sigma}L + \phi\left(z\right)\right]W & \text{if } z \le \bar{z} \\ \infty & \text{if } z > \bar{z} \end{cases}, \tag{8}$$

$$I\left(\bar{z}\right) = \frac{1}{\sigma - 1} \left(\frac{\sigma}{\sigma - 1}W\right)^{-\sigma} L,\tag{9}$$

$$W = \int_{0}^{z} P(z)^{1-\sigma} dz,$$
 (10)

where  $I(z) = \phi(z)v(z)^{\sigma-1}$  and the production schedule is given by  $Q(z) = P(z)^{-\sigma} L$ .

Equations (8) and (9) summarize the discussion of product market equilibrium in the previous section. Equation (10) imposes the additional equilibrium condition that total income, i.e., WL, must equal total spending, i.e.,  $\int_0^{\bar{z}} P(z)^{1-\sigma} L dz$ . Together, these conditions implicitly define a set of equilibrium prices. The following proposition states that this set of prices exists and is well defined.

#### **Proposition 1.** The fixed-cost economy has a unique equilibrium.

The intuition behind Proposition 1 is straightforward. For a given wage W, Equations (8) and (9) uniquely determine the prize schedule P(z) and the marginal good  $\bar{z}$ . Moreover, these equations imply a continuous and decreasing relationship between aggregate spending, i.e.,  $\int_0^{\bar{z}} P(z)^{1-\sigma} L dz$ , and the wage W. Aggregate income WL is instead increasing in the wage. Thus, there is a unique value of W that satisfies Equation (10).

Having established that the fixed-cost economy has a unique equilibrium, we now turn to the analysis of its properties. We start by exploring the effects of market size, as measured by L.

#### **Proposition 2.** An increase in market size L leads to:

- (i) an increase in the production Q(z) of goods that were already produced,
- (ii) an increase in the measure of goods produced  $\bar{z}$ , and
- (iii) an increase in the wage W.

Proposition 2 is intuitive, as it simply describes the implications of increasing returns to scale. As the market size increases, fixed costs are shared by a larger measure of individuals. This effectively lowers production costs, which has two effects. The first one is an increase in the production of goods that were already produced before the increase in market size. The second one is an increase in product variety, as some goods that were not produced before are now produced. As a result, welfare increases, as measured by the wage.

This discussion suggests that a reduction in fixed costs should have effects that are similar to those of an increase in market size. The following proposition makes this idea precise.

**Proposition 3.** A uniform fall in fixed costs by a factor  $\gamma^{-1}$  (so that the fixed cost of good z is now given by  $\gamma^{-1}\phi(z)$  with  $\gamma > 1$ ) has the same equilibrium effects as an increase in market size by a factor of  $\gamma$  (so that the market size is now given by  $\gamma L$ ).

A uniform reduction in fixed costs is therefore equivalent to an increase in market size. The reason is that both changes reduce fixed costs per person. An increase in market size reduces this ratio because the same fixed costs are shared by a larger measure of individuals. A uniform reduction in fixed costs reduces this ratio because lower fixed costs are shared by the same measure of individuals. This proposition essentially says that what really matters for prices, production and welfare is fixed costs per person.

Proposition 3 requires the reduction of fixed costs to be uniform. Since all goods face the same market demand, an increase in market size has a uniform effect on all of them. Therefore, a reduction in fixed costs can only be equivalent to an increase in market size if it also affects all goods in the same way. Non-uniform reductions in fixed costs affect goods differently by definition. Their overall effects on prices and welfare are qualitatively similar to those of an increase in market size, but not identical from a quantitative viewpoint. Reductions in variable costs have quite different effects, though.

**Proposition 4.** A uniform fall in marginal costs by a factor of  $\gamma^{-1}$  (so that the marginal cost of good z is now given by  $\gamma^{-1}v(z)$  with  $\gamma > 1$ ) leads to:

- (i) an increase in the production Q(z) of goods that were already produced by a factor of  $\gamma$ ,
- (ii) no change in the measure of goods produced  $\bar{z}$ , and
- (iii) an increase in the wage W by a factor of  $\gamma^{\frac{\sigma-1}{\sigma}}$ .

Proposition 4 is surprisingly sharp, since it provides the exact quantitative effects of a reduction in marginal costs. If marginal costs decline by a factor  $\gamma^{-1}$ , the production of all goods that were already produced grows by a factor  $\gamma$ . This implies that there is no change in the labor allocation. The productivity of each worker (after the fixed cost has been paid)

has increased by a factor  $\gamma > 1$ , and so does the production of each good. Since the labor allocation does not change, neither does the measure of goods produced. Hence, the result that product variety is unchanged and the consumption of all goods increases by a factor  $\gamma$ . The wage, or welfare, grows by a smaller factor  $\gamma^{\frac{\sigma-1}{\sigma}}$  because there is diminishing marginal utility of consumption.

Interestingly, technological progress is similar to an increase in market size only if it affects fixed costs. In this case, both the production of all goods and product variety increase. But technological progress is quite different from an increase in market size if it affects marginal costs. In this case, product variety is unchanged. As we shall see later, this observation has important empirical implications.

This discussion has given us insights regarding the positive properties of the fixed-cost economy. We turn now to its normative properties.

#### **3.4** Market inefficiencies

A fundamental feature of the fixed-cost economy is that the market or laissez-faire equilibrium is inefficient. Since technologies are non-convex, the welfare theorems for competitive economies generically break down. To characterize these inefficiencies, we compare market outcomes to those that a benevolent social planner would achieve. This planner treats all individuals equally and chooses the labor and consumption allocation so as to maximize their aggregate utility, i.e.,  $U^{SP} = LU$ :

$$U^{SP} = L \int_0^\infty \frac{\sigma}{\sigma - 1} C\left(z\right)^{\frac{\sigma - 1}{\sigma}} dz \tag{11}$$

subject to the following resource constraints:

$$C(z) L \le \max\left\{\frac{L(z) - \phi(z)}{\upsilon(z)}, 0\right\} \quad \forall z,$$
(12)

$$\int_0^1 L(z) \, dz \le L. \tag{13}$$

The first constraint ensures that the planner has access to the same technologies as the market. The second constraint ensures that the planner has the same labor endowment as the market.

Let  $P^{SP}(z)$  and  $W^{SP}$  be the Lagrange multipliers associated to constraints (12) and (13) respectively. These are the shadow prices that the planner uses to guide its allocation of labor across goods. The following lemma provides a characterization of the planner's allocation.

**Lemma 2.** The social planner's allocation consists of a measure of goods produced  $\bar{z}^{SP}$ , a shadow product price schedule  $P^{SP}(z)$ , and a shadow wage  $W^{SP}$  that solve the following set

of equations:

$$P^{SP}(z) = \begin{cases} v(z) W^{SP} & \text{if } z \leq \bar{z}^{SP} \\ \infty & \text{if } z > \bar{z}^{SP} \end{cases},$$
(14)

$$I\left(\bar{z}^{SP}\right) = \frac{1}{\sigma - 1} \left(W^{SP}\right)^{-\sigma} L,\tag{15}$$

$$W^{SP}\left(1 - \frac{1}{L}\int_{0}^{\bar{z}^{SP}}\phi(z)\,dz\right) = \int_{0}^{\bar{z}^{SP}}P^{SP}(z)^{1-\sigma}\,dz,\tag{16}$$

where  $I(z) = \phi(z)\upsilon(z)^{\sigma-1}$  and the production schedule is given by  $Q^{SP}(z) = P^{SP}(z)^{-\sigma}L$ .

A comparison of Lemmas 1 and 2 reveals that the market and planner allocations differ. Thus, the fixed-cost economy is inefficient. Where do these inefficiencies come from? A more detailed look at the equations in these lemmas allows provides a sharp answer to this question.

The first source of inefficiency lies in the pricing decisions (see Equations (8) and (14)). The planner understands that, once the fixed cost has been paid, the cost of each additional unit of good z is equal to the cost of the additional units of labor needed to produce it, i.e.,  $v(z)W^{SP}$ . Thus, the shadow price  $P^{SP}(z)$  used by the planner is equal to the marginal cost of production. Instead, in the market equilibrium, prices equal average costs because producers need to recoup the fixed costs of production.

The second source of inefficiency lies in the entry decisions (see Equations (9) and (15)). Both the market and the planner compare the total benefits provided by a good to its total cost of production. They do not disagree on the costs, which are the sum of fixed and variable labor costs. But they do disagree on the total benefits. Whereas the planner takes into account the entire consumer surplus, the market only cares about total revenue.

If the planner decides how much to produce each good based only on marginal costs, how does she pay for the fixed costs? The answer comes from a comparison of Equations (10) and (16). In the market economy, total spending equals total income: WL. Instead, the planner effectively "taxes" a fraction of total income so as to pay for the fixed costs. Thus, in the planner's allocation, spending equals "after-tax income":  $W^{SP}L(1 - \int_0^{\overline{z}^{SP}} \phi(z) dz/L)$ .

The following corollary characterizes the implications of these distortions.

**Corollary 1.** The market equilibrium is inefficient: it features insufficient product variety, i.e.,  $\bar{z} < \bar{z}^{SP}$ , and excessive production of goods with low cost-index, i.e.,  $Q(z) > Q^{SP}(z)$  for z below some threshold.

Relative to the market allocation, therefore, the planner produces more of some goods and less of others. Despite these differences, the response of the planner allocation to changes in market size or in technology is qualitatively similar to that of the market allocation. Increases in market size and uniform declines in fixed costs increase both the quantity of each good produced and the measure of goods produced. A uniform decline in marginal costs increases the quantity of each good produced but do not affect the measure of goods produced.

**Proposition 5.** The effects on the planner's allocation of an increase in market size L, of a uniform fall in fixed costs  $\phi(z)$ , or a uniform fall in marginal costs v(z), are the same as in the market economy.

## 4 Understanding the rise of fixed costs

We began by highlighting the rise of fixed costs in the United States and elsewhere since 1980. We now analyze this increase through the lens of the theory.

We consider two points in time  $t \in \{t_0, t_1\}$ , and use subscript t to denote the value of a variable at time t. We allow for two sources of economic growth between  $t_0$  and  $t_1$ . The first is an increase in market size, as captured by the mass L of consumers in the economy: in the data, it could reflect either population growth or trade integration, both of which increase the number of consumers that can be reached by any given good. We capture this by assuming that  $L_{t_1} = \gamma_L L_{t_0}$  and  $\gamma_L > 1$ . The second is a technological innovation that leads to a reduction in the fixed-cost parameter  $\phi$  or in the marginal-cost parameter v.<sup>10</sup> We capture this as a uniform decline in fixed and marginal costs across goods, respectively:  $\phi_{t_1}(z) = \gamma_{\phi}\phi_{t_0}(z)$  and  $v_{t_1}(z) = \gamma_v v_{t_0}(z)$  for all z, for  $\gamma_{\phi} < 1$  and  $\gamma_v < 1$ .

Conditional on good z being produced at time t, i.e.,  $Q_t(z) > 0$ , its fixed-cost share is:

$$F_t(z) = \frac{\phi_t(z)}{\phi_t(z) + \upsilon_t(z)Q_t(z)},\tag{17}$$

i.e., the ratio of the good's fixed costs to its total costs of production. Naturally, a good's fixed-cost share varies inversely with the quantity produced. Moreover, note that a good's fixed-cost share can be high due to either high fixed costs  $\phi$  or low variable costs vQ. In fact, as the following Lemma shows, the relative ranking of goods according to fixed-cost shares, an endogenous object, is dictated by their relative ranking in terms of the cost index I(z), a purely technological characteristic.

**Lemma 3.** Consider the set of goods produced at time t, i.e.,  $z \leq \bar{z}_t$ . The fixed-cost share  $F_t(z)$  is continuous and increasing in the cost index  $I_t(z)$ , with  $F_t(\bar{z}_t) = \frac{1}{\sigma}$ .

<sup>&</sup>lt;sup>10</sup>Hsieh and Rossi-Hansberg (2023), for instance, argue that the technological revolution of recent decades has allowed firms – especially in the service sector – to exploit economies of scale in information gathering and processing, which can be interpreted as a reduction in fixed costs.

We are interested in understanding how economic growth affects the economy's aggregate fixed-cost share, which is defined as the ratio of fixed costs incurred in the entire economy to the economy's total cost of production:

$$F_{t} = \frac{\int_{0}^{\bar{z}_{t}} \phi_{t}(z) dz}{L_{t}}.$$
(18)

Since in equilibrium  $L_t = \int_0^{\bar{z}_t} (\phi_t(z) + v_t(z)Q_t(z)) dz$ ,  $F_t$  is simply the cost- or sales- weighted average of the fixed-cost shares across all goods produced.

As Propositions 2-4 show, economic growth weakly expands the set of goods that have at least one active producer in equilibrium, i.e.,  $\bar{z}_{t_1} \geq \bar{z}_{t_0}$ . Let the superscript O denote the set of "old" goods at  $t_1$ , i.e., goods  $z \leq \bar{z}_{t_0}$  that would be produced at  $t_0$ . Similarly, let the superscript N denote the set of "new" goods at  $t_1$ , i.e., goods  $z \in (\bar{z}_{t_0}, \bar{z}_{t_1}]$  that are produced at  $t_1$  but that would not be produced at  $t_0$ , and let  $\omega$  denote their equilibrium market share. It then follows that  $F_{t_1} = (1 - \omega)F_{t_1}^O + \omega F_{t_1}^N$  and therefore the change in the aggregate fixed-cost share between  $t_0$  and  $t_1$  can be expressed as:

$$F_{t_1} - F_{t_0} = \underbrace{(F_{t_1}^O - F_{t_0})}_{\text{intensive margin}} + \underbrace{\omega(F_{t_1}^N - F_{t_1}^O)}_{\text{extensive margin}},\tag{19}$$

where we follow the convention that  $F_{t_1}^N = 0$  if  $\bar{z}_{t_1} = \bar{z}_{t_0}$ .<sup>11</sup>

Equation (19) decomposes the change in the aggregate fixed-cost share into an *intensive* and an *extensive* margin. The intensive margin reflects the change in the fixed-cost share of old goods,  $F_{t_1}^O - F_{t_0}$ . The extensive margin reflects instead the difference between the fixed-cost share of new and old goods,  $F_{t_1}^N - F_{t_1}^O$ , weighed by the market share of new goods,  $\omega$ .

The intensive and extensive margins depend on the source of growth. Economic growth driven by an expansion in market size as captured by  $\gamma_L > 1$  or a decline in fixed costs as captured by  $\gamma_{\phi} < 1$  have isomorphic effects on the equilibrium allocation (see Proposition 3), and thus affect the intensive and extensive margins in the same way. In particular, the intensive margin associated to this type of growth is negative, since it leads to an increase in the equilibrium quantities of old goods  $z \leq \bar{z}_{t_0}$  and/or a decrease in their fixed costs (see Equation (17)). The extensive margin is instead positive because it follows from Lemma 3 that the fixed-cost share of new goods is higher than the fixed-cost share of old goods.

Economic growth driven by a decline in marginal costs as captured by  $\gamma_v < 1$  does not affect the intensive or extensive margins and thus has no effect on fixed-cost shares. This follows directly from Proposition 4, which shows that this type of growth does not affect the

<sup>11</sup>Formally, 
$$\omega = \frac{\int_{\tilde{z}_{t_0}}^{\tilde{z}_{t_1}} \left(\phi_{t_1}(z) + v_{t_1}(z)Q_{t_1}(z)\right) dz}{L_{t_1}}.$$

set of active goods or their variable costs vQ.

#### XX INSERT FIGURE XX

Figure ?? illustrates this discussion by depicting the fixed costs shares  $F_{t_0}(z)$  and  $F_{t_1}(z)$  across goods in response to economic growth driven by an expansion market size and/or a uniform decline in fixed costs. The negative intensive margin is captured by the downward shift in the fixed-cost share for all goods  $z \leq \bar{z}_{t_0}$ . The positive extensive margin is captured by the fact that  $F_{t_1}(z)$  is increasing in z: this implies, in particular, that  $F_{t_1}(z') > F_{t_1}(z)$  for all  $z \leq \bar{z}_{t_0}$  and  $z' \in (\bar{z}_{t_0}, \bar{z}_{t_1}]$ . The following proposition formalizes this result.

**Proposition 6.** Consider economic growth between  $t_0$  and  $t_1$  driven by:

- an increase in market size from  $L_{t_0}$  to  $L_{t_1} = \gamma_L L_{t_0}$  for  $\gamma_L > 1$ , or
- a uniform fall in fixed costs from  $\phi_{t_0}(z)$  to  $\gamma_{\phi}\phi_{t_0}(z)$  for  $\gamma_{\phi} < 1$ .

Then, the set of active goods expands,  $\bar{z}_{t_1} > \bar{z}_{t_0}$ , and:

- (intensive margin)  $F_{t_0}(z) > F_{t_1}(z)$  for all  $z \leq \overline{z}_{t_0}$ , and
- (extensive margin)  $F_{t_1}(z') > F_{t_1}(z)$  for all  $z \le \bar{z}_{t_0}$  and  $z' \in (\bar{z}_{t_0}, \bar{z}_{t_1}]$ .

If instead economic growth is driven by a uniform fall in marginal costs from  $v_{t_0}(z)$  to  $\gamma_v v_{t_0}(z)$ for  $\gamma_v < 1$ , the set of active goods is unchanged,  $\bar{z}_{t_1} = \bar{z}_{t_0}$ , and  $F_{t_0}(z) = F_{t_1}(z)$  for all  $z \leq \bar{z}_{t_0}$ .

Proposition 6 implies that the effect of economic growth on the economy's fixed-cost share depends on the relative strength of the intensive and extensive margins. Thus, according to the theory, the increase in the fixed-cost share observed in US data could be the result of economic growth provided that the extensive margin is sufficiently strong. To assess how strong, we next revisit the data through the lens of the theory.

#### 4.1 A theory-based look at the data

We perform a back-of-the-envelope calculation for the US between  $t_0 = 1980$  and  $t_1 = 2019$ . To do so, we first make assumptions about  $\gamma_L$ ,  $\gamma_{\phi}$ , and  $\gamma_{v}$ . As Proposition 6 established,  $\gamma_{v}$  is irrelevant for the evolution of fixed-cost shares and we thus set it equal to 1; regarding  $\gamma_L$  and  $\gamma_{\phi}$ , their effects on the extensive and intensive margins are isomorphic and thus only the ratio  $\gamma \equiv \frac{\gamma_L}{\gamma_{\phi}}$  matters for the evolution of fixed-cost shares. We set  $\gamma_L = 1.6$  to reflect the growth of the US labor force between 1980 and 2019. Choosing a value for  $\gamma_{\phi}$  is less straightforward. As a baseline, we assume that fixed costs (measured in terms of labor units) have declined uniformly by a third during the period, implying  $\gamma_{\phi} = 0.67$ . We also use  $\gamma_{\phi} = 1$  and  $\gamma_{\phi} = 0.5$  to gauge our results against alternative scenarios. This implies that we consider  $\gamma \in \{1.6, 2.4, 3.2\}.$ 

We use two observations that follow from the decomposition of Equation (19). The first is that, since the set of "old" goods in 2019, indexed by O, contains the same goods that would have been produced in 1980, it follows that:

$$F_{1980} = \gamma (1 - \omega) F_{2019}^O \tag{20}$$

Equation (20) says that there is a natural relationship between the fixed-cost share of goods produced in 1980 and the fixed-cost share of that same set of goods in 2019. All else equal, higher growth as captured by  $\gamma$  implies a stronger intensive margin, and thus a lower value of  $F_{2019}^O$  relative to  $F_{1980}$ . Combining this with the decomposition  $F_{2019} = (1 - \omega)F_{2019}^O + \omega F_{2019}^N$ , we obtain:

$$\omega F_{2019}^N = F_{2019} - \frac{F_{1980}}{\gamma}.$$
(21)

Given the fixed-cost shares in 1980 and 2019, and given a value of  $\gamma$ , Equation (21) backs out  $\omega F_{2019}^N$  as a residual. Since  $F_{2019} \approx 0.23$  and  $F_{1980} \approx 0.17$  in US data, and given our assumption that  $\gamma \in \{1.6, 2.4, 3.2\}$ , it implies that  $\omega F_{2019}^N \in \{0.126, 0.157, 0.173\}$ .

How plausible are these values? To assess this, we first rank the firms in Compustat in ascending order according to their fixed-cost shares in 2019, i.e.,  $F^1 < F^2 < ... < F^J$ , and use  $\omega^j$  to denote the ratio of firm j's costs to the costs of all firms. Let us assume, in line with the theory, that firms with higher fixed costs produce new goods, whereas firms with lower fixed costs produce old goods. For each *i*, define:

$$\widehat{F}^i = \sum_{j \ge i} \frac{\omega^j}{\widehat{\omega}^i} F^j \text{ for } i \text{ s.t. } \widehat{\omega}^i = \sum_{j \ge i} \omega^j.$$

If we think of *i* as denoting the "threshold" firm producing new goods,  $\hat{F}^i$  represents the aggregate fixed-cost share of new goods in Compustat, while  $\hat{\omega}^i$  represents the cost-weighted market share of these goods, i.e., their total costs as a share of the total costs of all goods.<sup>12</sup>

The red locus in Figure 7 depicts the relationship between  $\hat{F}^i$  and  $\hat{\omega}^i$  in Compustat in 2019, for i = 1, 2, ..., J. When i = 1,  $\hat{F}^i$  is equal to the aggregate fixed-cost share in Compustat in 2019. When i = J,  $\hat{F}^i \approx 0.99$ , which corresponds to the highest firm-level fixed-cost share observed in Compustat in 2019. The blue locus depicts instead all combinations of  $\omega$  and  $F_{2019}^N$  for which  $\omega F_{2019}^N = 0.157$ , i.e., combinations that satisfy Equation (21) when  $\gamma = 2.4$ . The two loci intersect at  $(F_{2019}^N, \omega) \approx (0.42, 0.37)$ . Through the lens of the theory, therefore,

<sup>&</sup>lt;sup>12</sup>That is, firms j > i produce new goods, while firms  $j \leq i$  produce old goods.



Figure 7: Market and fixed-cost shares of "new" goods in 2019

it is possible to explain the rise in the aggregate fixed-cost share as the result of US economic growth: this requires the market share of new goods to have been 37% in 2019, and the fixedcost share of new goods to have been 0.42. For the alternative values  $\gamma = 1.6$  and  $\gamma = 3.2$ , the same analysis respectively implies  $(F_{2019}^N, \omega) \approx (0.5, 0.25)$  and  $(F_{2019}^N, \omega) \approx (0.39, 0.44)$ .

Table III shows the implications of these calculations for the relative contributions of the intensive and extensive margins to the rise of the aggregate fixed-cost share. In the baseline scenario of  $\gamma = 2.4$ , our calculations imply that 63% of the market in 2019 was composed of goods that would have been produced in 1980 as well. As their markets grew, their average fixed-cost share declined from 0.17 to approximately 0.11. The remaining 37% of the market in 2019 was composed of goods that would not have been produced in 1980. Their average fixed-cost share was approximately 0.42, almost 4 times larger than that of the other goods. Taken jointly, these results imply that the extensive margin growth led to a 68% increase in the fixed-cost share of the US economy, but this was counteracted by a 34% decline due to the intensive margin growth. The net effect explains the 35% increase observed in the US aggregate fixed-cost share, from 0.17 to 0.23.

$\gamma$	Intensive margin growth	Extensive margin growth
1.6	-0.028	0.087
2.4	-0.058	0.117
3.2	-0.076	0.135

Table III: Decomposing the rise in the fixed-cost share between 1980 and 2019.

These findings suggest that it is possible to rationalize the rise of fixed costs in the US economy as the consequence of economic growth as captured by  $\gamma > 1$ . We now turn to a numerical example to illustrate how our theoretical model can generate this relationship between growth and fixed costs.

## 4.2 A numerical illustration

To fix ideas, we consider a class of economies in which goods differ in the fixed cost  $\phi$  but have the same marginal cost v = 1.<sup>13</sup> In particular, we consider three economies indexed by  $j \in \{l, m, h\}$ , which differ only in the shape of their fixed-cost schedule:

$$\phi_j(z) = \begin{cases} z^{\alpha_0} & \text{if } z \le z^* \\ z^{\alpha_j} + (z^{\star \alpha_0} - z^{\star \alpha_j}) & \text{if } z > z^* \end{cases}$$
(22)

where  $z^* > 0$  and  $\alpha_h > \alpha_m = \alpha_0 > \alpha_l > 0$ .



Figure 8: Schedule  $\phi_j(z)$  and threshold  $\bar{z}_j$  in economy j = l, m, h.

Each of these economies has the appealing property that its aggregate fixed-cost share converges to a stationary value as the economy grows. This is especially useful to understand the relationship between economic growth and the intensive and extensive margins identified in Proposition 6. Panel (a) of Figure 8 depicts the fixed-cost schedule in each of these economies. As the figure shows, the fixed costs in all of them coincide up to good  $z^*$  and diverge thereafter, rising faster in the economy with the higher value of  $\alpha$ .

 $<sup>^{13}\</sup>mathrm{Our}$  main insights are unchanged if product heterogeneity instead arises due to differences in marginal rather than fixed costs.



Figure 9: Aggregate fixed-cost share in economy j = l, m, h.

Panel (b) of Figure 8 shows the relationship between the threshold good that is active in economy  $j \in \{l, m, h\}$ ,  $\bar{z}_j$ , and economic growth as captured by  $\gamma$ . We know from Propositions 2 and 3 that the threshold increases with economic growth. The example is structured so that good  $z^*$  is activated exactly when  $\gamma = 1$ . The key observation is that, as  $\gamma$  increases above one, the rate at which new goods are introduced is faster for the economy with a lower value of  $\alpha$ . Intuitively, the introduction of new goods is more attractive in economy l than in economy h, since the production of these goods entails relatively lower fixed costs. As a result, the extensive margin is strongest in economy l, which has implications for the evolution of the aggregate fixed-cost share.

Figure 9 depicts the aggregate fixed-cost share in the three economies as a function of  $\gamma$ . The example is structured so that, before good  $z^*$  is activated, the aggregate fixed-cost share has already stabilized at its stationary value corresponding to the common fixed-cost schedule  $z_0^{\alpha}$ . The main insight is that, once the three economies diverge, the fixed-cost share rises the most in economy l, which – somewhat paradoxically – is precisely the one where the fixed cost  $\phi(z)$  is the lowest. The reason, as we already mentioned, is that in this economy new goods are introduced at the fastest rate. Eventually, the aggregate fixed-cost share stabilizes at the stationary value corresponding to the fixed-cost schedule  $z^{\alpha_j}$ , for  $j \in \{l, m, h\}$ .

Figure 9 shows that, within this class of economies, economic growth can lead to an increase in the aggregate fixed-cost share only if the rate at which  $\phi(z)$  rises with z declines within some range of goods. This is precisely what happens in economy l, which can numerically reproduce the documented rise in the US aggregate fixed-cost share, from 0.17 to 0.23, as a



Figure 10: Comparison of economy l with the planner.

result of our estimated increase in  $\gamma$  of 140%.<sup>14</sup> The model is also in line with the back-ofthe-envelope exercise performed in Section 4.1: 0.37 of the market in 2019 was composed of goods that would not have been produced in 1980, and the fixed-cost share of these goods in 2019 was 0.42.

Finally, we can use the model as a laboratory to evaluate the welfare losses due to the presence of increasing returns, as identified in Section 3.4. Figure 10 depicts the discrepancies between the market and the planner allocations in 1980 and 2019, for our parametrization of economy l. Panel (a) shows that the aggregate fixed-cost share is higher in the planner allocation for all values of  $\gamma$ . Panel (b), which plots the quantities Q(z) produced in the market and planner allocations in 1980 and 2019, can shed light on this result. Since the planner considers the marginal instead of the average cost of production, she allocates labor more efficiently and is thus able to produce a larger set of goods: this implies a higher aggregate fixed-cost share (see Equation (18)). Despite this difference, Panel (a) also shows that the qualitative behavior of the aggregate fixed-cost share is similar in the planner and the market allocations, as already hinted in Proposition 5: distortions due to increasing returns depress the fixed-cost share but they do not fundamentally affect its response to economic growth.

This numerical exercise also allows us to compute the welfare costs of the inefficiencies in the market allocation. Even though markets are contestable and there are no profits, our calculations suggest that costs of these inefficiencies may be sizable. Under our baseline parametrization, the market allocation entailed a 14.3% welfare loss relative to the planner in 1980, and a 12.7% welfare loss in 2019.

<sup>&</sup>lt;sup>14</sup>For this exercise, we set  $z^* = 1.5$ ,  $\alpha_0 = 1.38$ ,  $\alpha_l = \alpha_0 - \delta$  and  $\alpha_h = \alpha_0 + \delta$  with  $\delta = 0.62$ , and  $\sigma = 1.57$ .

The model developed here is stylized and not intended for a serious quantitative evaluation. Nonetheless, this section has shown – both through a theory-based look at the data and a simple numerical illustration – that it is possible to account for the rise in the aggregate fixedcost share in the US as the consequence of economic growth. Although growth expands the size of the market and thus reduces fixed-cost shares, it also facilitates the introduction of new products with relatively high fixed-cost shares: if the latter effect is sufficiently strong, growth leads to an increase in the aggregate fixed-cost share.

## 5 Conclusions

Over the last few decades, fixed costs have increased significantly in the US and elsewhere. Yet, macroeconomists lack a framework to interpret this phenomenon. We have attempted to fill this gap by developing a macroeconomic model of fixed costs.

The core of the framework is that goods are ex ante heterogeneous, as they differ in their fixed and marginal costs of production. The key finding is that, in equilibrium, the economy's fixed-cost share is determined endogenously both by the extensive margin, i.e., the set of goods being produced, and the intensive margin, i.e., the quantity produced of each good. These two margins also shape the effects of economic growth on the fixed-cost share: while intensive growth tends to reduce the fixed-cost share by raising the number of units produced, extensive growth tends to increase the fixed-cost share by activating goods with increasingly high fixed-costs. We show that the theory can rationalize the increase in the US fixed-cost share between 1980 and 2019 as a by-product of economic growth, which had a strong positive effect on fixed costs through the extensive margin.

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## A Appendix: Proofs

Proof of Lemma 1. See text.

Proof of Proposition 1. Given a wage W, Equations (23) and (24) uniquely pin down the price schedule P(z) and the marginal good  $\bar{z}$  (as well as the production schedule Q(z)). The equilibrium wage W is then determined by Equation (25). That there is a unique wage W satisfying Equation (25) follows from the fact that, as W goes from 0 to  $\infty$ , the right-hand side of Equation (25) decreases continuously (from  $\infty$  to 0), since P(z) increases for each  $z \leq \bar{z}$  and  $\bar{z}$  decreases in W.

Proof of Proposition 2. Consider an increase in market size from L to  $\gamma_L L$  for some  $\gamma_L > 1$ . Let  $\zeta \equiv W^{-\sigma} \gamma_L L$  and the fixed-cost share of good z be defined by  $F(z) \equiv \frac{\phi(z)}{\phi(z) + \upsilon(z)Q(z)}$ . Using Proposition 1, the equilibrium of our economy is characterized by:

$$F(z;\zeta) = \min F \text{ s.t. } F = \frac{\phi(z) v(z)^{\sigma-1}}{\phi(z) v(z)^{\sigma-1} + (1-F)^{\sigma} \zeta} \quad \forall z \le \bar{z},$$
(23)

$$F\left(\bar{z};\zeta\right) = \frac{1}{\sigma},\tag{24}$$

$$\int_0^{\bar{z}} \frac{\phi(z)}{F(z;\zeta)} dz = \gamma_L L,$$
(25)

where we make explicit the dependence of the fixed-cost share on  $\zeta$ , and where we have used the fact that the price schedule satisfies  $P(z) = (1 - F(z; \zeta))^{-1} v(z)W$  for  $z \leq \overline{z}$ .

The fixed-cost share  $F(z;\zeta)$  defined by Equation (23) is increasing in z on  $[0, \bar{z}]$  and decreasing in  $\zeta$  for all  $z \leq \bar{z}$ . By inspection of Equations (23)-(25), we see that an increase in market size must lead to an increase in  $\zeta$ . It follows that the quantity of each good produced increases, since  $Q(z) = (1 - F(z;\zeta))^{\sigma} v(z)^{-\sigma}\zeta$  and  $F(z;\zeta)$  decreases with  $\zeta$  for all  $z \leq \bar{z}$ ; the measure of active goods increases, since the fixed-cost share  $F(z;\zeta)$  increases in z and decreases in  $\zeta$  for all  $z \leq \bar{z}$ ; and the wage W increases, since otherwise there would be excess demand for labor: for a given wage W, the right-hand side of Equation (8) rises in  $\gamma_L$ .

Proof of Proposition 3. This follows from the observation that the schedule of fixed-cost shares  $F(z; \zeta)$  satisfying Equations (23)-(25) (which fully characterize the equilibrium allocations) are unchanged if instead of the rise in market size from  $L \mapsto \gamma_L L$  with  $\gamma_L > 1$ , we were to consider a uniform decrease in fixed costs  $\phi(z) \to \gamma_{\phi}\phi(z)$  with  $\gamma_{\phi} = \gamma_L^{-1} < 1$ .

Proof of Proposition 4. Consider a uniform fall in marginal costs from v(z) to  $\gamma_v v(z)$  for some  $\gamma_v < 1$ . Let  $\zeta \equiv W^{-\sigma} \gamma_v^{1-\sigma} L$  and the fixed-cost share of good z be defined by  $F(z) \equiv$   $\frac{\phi(z)}{\phi(z)+\gamma_{v}v(z)Q(z)}$ . Using Proposition 1, we can express the equilibrium of our economy as follows:

$$F(z;\zeta) = \min F \text{ s.t. } F = \frac{\upsilon(z)\,\upsilon(z)^{\sigma-1}}{\phi(z)\,\upsilon(z)^{\sigma-1} + (1-F)^{\sigma}\,\zeta} \quad \forall z \le \bar{z},$$
(26)

$$F\left(\bar{z};\zeta\right) = \frac{1}{\sigma},\tag{27}$$

$$\int_0^{\bar{z}} \frac{\phi(z)}{F(z;\zeta)} dz = L.$$
(28)

Technological progress as captured by  $\gamma_v$  affects Equations (26)-(28) only through its effect on  $\zeta$ . Since these equations uniquely pin down the equilibrium fixed-cost shares and  $\zeta$ , it follows that  $\gamma_v$  has no effect neither on the fixed-cost shares nor on  $\zeta$  nor on the measure of active goods  $\bar{z}$ . Since  $\zeta = W^{-\sigma} \gamma_v^{1-\sigma} L$ , it follows that the fall in the marginal costs is fully offset by a rise in the wage by a factor of  $\gamma_v^{\frac{\sigma-1}{\sigma}}$ . Finally, since the fixed-cost shares and  $\zeta$  remain unchanged, it follows that the quantity of each good produced  $Q(z) = (1 - F(z; \zeta))^{\sigma} \gamma_v^{-1} v(z)^{-\sigma} \zeta$ rises by a factor of  $\gamma_v^{-1}$ .

Proof of Proposition 2. First, that  $P^{SP}(z) = v(z)W^{SP}$  for each active good z follows by taking first order conditions to the planner problem with respect to L(z).

Second, that  $Q^{SP}(z) = P^{SP}(z)^{-\sigma}L$  for each active good z follows by taking first order conditions with respect to C(z) and combining it with the fact that constraint (12) must hold with equality, i.e.,  $Q^{SP}(z) = C(z)L$ .

Third, for a good to be activated by the planner, it must be that the total surplus generated by it,  $\frac{\sigma}{\sigma-1}C(z)^{\frac{\sigma-1}{\sigma}}L$ , exceeds its total cost of production,  $W^{SP}[v(z)Q^{SP}(z)+\phi(z)]$ . Given that, for each active good,  $P^{SP}(z) = v(z)W^{SP}$ ,  $Q^{SP}(z) = P^{SP}(z)^{-\sigma}L$ , and constraint (12) must hold with equality, we obtain that for the total surplus from activating good z to exceed its total cost of production it must be that  $I(z) \leq \frac{1}{\sigma-1} \left(W^{SP}\right)^{-\sigma}L$ . Since I(z) is continuous and strictly increasing in z, it follows that the marginal good  $\bar{z}^{SP}$  (and thus the measure of goods) activated by the planner satisfies  $I(\bar{z}^{SP}) = \frac{1}{\sigma-1} \left(W^{SP}\right)^{-\sigma}L$ .

Fourth, Equation (13) follows from the observations above combined with the fact that constraint (13) must clearly hold with equality at the planner allocation.

Finally, that the solution to the social planner's problem exists and is unique follows by similar arguments in the proof of Proposition 1. Given shadow wage  $W^{SP}$ , the price schedule  $P^{SP}(z)$  and the marginal good  $\bar{z}^{SP}$  (as well as the production schedule  $Q^{SP}(z)$ ) are uniquely pinned down by Equations (14) and (15). That there is a unique wage  $W^{SP}$  that solves the social planner's problem in turn follows from the observation that, as  $W^{SP}$  goes from 0 to  $\infty$ , the left-hand side of Equation (16) goes continuously from 0 to  $\infty$ , whereas the right-hand side goes continuously from  $\infty$  to 0, since  $P^{SP}(z)$  increases in  $W^{SP}$  for  $z \leq \bar{z}^{SP}$  and  $\bar{z}^{SP}$  decreases in  $W^{SP}$ .

Proof of Corollary 1. Suppose to the contrary that  $\bar{z}^{SP} \leq \bar{z}$ , which from Lemmas 1 and 2 holds if and only if:

$$W^{SP} \ge \frac{\sigma}{\sigma - 1} W. \tag{29}$$

Then, it must be that for all  $z \leq \bar{z}^{SP}$ , the quantity produced by the planner is lower than the quantity produced by the market:

$$Q^{SP}(z) = (\upsilon(z)W^{SP})^{-\sigma}L \le (1 - F(z))^{\sigma}(\upsilon(z)W)^{-\sigma}L = Q(z),$$
(30)

with strict equality for  $z < \bar{z}^{SP}$ , since the fixed-cost share F(z) in the market allocation is strictly increasing on  $[0, \bar{z}]$  and bounded above by  $\frac{1}{\sigma}$  (see Lemma 3). But if the planner produces less of each good and activates fewer goods, it must be that she leaves some labor unused, a contradiction. Thus,  $\bar{z}^{SP} > \bar{z}$ .

Next, we show that for z below some threshold,  $Q^{SP}(z) < Q(z)$ . To this end, it suffices to show that  $W^{SP}(1 - F(z)) > W$  where F(z) is the fixed-cost share of good z in the market allocation. Since welfare in the market equilibrium must be weakly lower than in the planner allocation, we have that:

$$\frac{\sigma}{\sigma-1}W = U \le U^{SP} = \frac{\sigma}{\sigma-1}W^{SP}\left(1 - \frac{\int_0^{\bar{z}^{SP}}\phi(z)dz}{L}\right),\tag{31}$$

where U is the per person welfare in the market equilibrium, and where  $U^{SP}$  is the per person welfare in the planner allocation. The last equality in Equation (31) follows from Lemma 2 and the expression for social welfare in Equation (11). Since  $\bar{z}^{SP} > \bar{z}$ , it follows that:

$$\frac{\int_{0}^{\bar{z}^{SP}}\phi(z)dz}{L} > \frac{\int_{0}^{\bar{z}}\phi(z)dz}{L} = \int_{0}^{\bar{z}}F(z)\frac{\phi(z)+\upsilon(z)Q(z)}{L}dz,$$
(32)

and since  $\int_0^{\bar{z}} \frac{\phi(z)+v(z)Q(z)}{L} dz = 1$ , it follows from Lemma 3 that the fixed-cost share F(z) of good z in the market allocation must be smaller than  $\int_0^{\bar{z}^{SP}} \phi(z) dz/L$  for z or, equivalently, I(z) below some threshold. Hence, for all such goods:

$$W^{SP}(1 - F(z)) > W^{SP}\left(1 - \frac{\int_0^{\overline{z}^{SP}} \phi(z)dz}{L}\right) \ge W,\tag{33}$$

which establishes the result.

*Proof of Proposition 5.* The proof is analogous to the proofs of Propositions 2-4.

Proof of Lemma 3. This result follows from the observation that we can express the equilibrium through Equations (23)-(25) in the proof of Proposition 2, where  $F(z; \zeta)$  is the fixed-cost share of good  $z \leq \overline{z}$ . It is then clear that the fixed-cost share is increasing in the cost-index I(z) or, equivalently, in z, with an upper bound  $\frac{1}{\sigma}$  at  $I(\overline{z})$  or, equivalently, at  $z = \overline{z}$ .  $\Box$ 

Proof of Proposition 6. See text.

# **B** Appendix: Data Construction

As the main data source we use Compustat Fundamentals, obtained from WRDS with access through Princeton University. Compustat provides standardized North American and global financial statement and market data for over 80,000 active and inactive publicly traded companies. The data contains all the variables for firms over the years 1959-2020.

We drop year 1959. For every firm, we keep only one observation per industry (cleaning out duplication), and we drop firms without industry information. We deflate the variables using the information on US GSP from World development indicators. We drop observations with negative values for sales, COGS or XSG&A. We construct the user cost of capital following De Loecker *et al.* (2020):

$$r_t = (I_t - \Pi_t) + \Delta$$

where  $I_t$ ,  $\Pi_t$ , and  $\Delta$  are the nominal interest rate, the inflation rate, and a depreciation rate. Compustat variable PPEGT is gross capital. The federal dunds rate and inflation rate are obtained from Federal Reserve; the depreciation is set at 12%.

As a baseline cleaning, we drop observations for firm-years that have missing values for sale or total costs. We also drop observations with zero values for COGS. We trim 1% of firms with the highest and the lowest sales values.

# C Appendix: Robustness



Figure 11: The evolution of the share of fixed costs in total costs, 1960-2020

Note: The measures are constructed using Compustat data. Fixed costs are represented by the sum of "Selling, general and administrative expenses" (SG&A) and capital expenditures. Total costs are calculated as a sum of SG&A, Cost of Goods Sold (COGS) and capital expenditures.



Figure 12: The evolution of the share of fixed costs in total costs, 1960-2020

Note: The measures are constructed using Compustat data. Fixed costs are represented by the sum of "Selling, general and administrative expenses" (SG&A) and half of the capital expenditures. Variable costs are represented by the sum of Cost of Goods Sold (COGS) and half of the capital expenditures.



Figure 13: Share of fixed costs in total costs, 1980 and 2018

Note: The measures are constructed using Compustat data. Fixed costs are represented by the sum of "Selling, general and administrative expenses" (SG&A) and capital expenditures. Total costs are calculated as a sum of SG&A, Cost of Goods Sold (COGS) and capital expenditures.



Figure 14: Share of fixed costs in total costs, 1980 and 2018

Note: The measures are constructed using Compustat data. Fixed costs are represented by the sum of "Selling, general and administrative expenses" (SG&A) and half of the capital expenditures. Variable costs are represented by the sum of Cost of Goods Sold (COGS) and half of the capital expenditures.



Figure 15: The evolution of the shares of the total cost components, 1960-2020

Note: The measures are constructed using Compustat data. Total costs are calculated as a sum of SG&A, Cost of Goods Sold (COGS) and capital expenditures.



Figure 16: The evolution of the shares of the total cost components, 1960-2020

Note: The measures are constructed using Compustat data. Fixed costs are represented by the sum of "Selling, general and administrative expenses" (SG&A) and capital expenditures. Variable costs are represented by Cost of Goods Sold (COGS). Total costs are calculated as a sum of SG&A, Cost of Goods Sold (COGS) and capital expenditures.



Figure 17: The evolution of the shares of the total cost components, 1960-2020

Note: The measures are constructed using Compustat data. Fixed costs are represented by the sum of "Selling, general and administrative expenses" (SG&A) and half of the capital expenditures. Variable costs are represented by the sum of Cost of Goods Sold (COGS) and half of capital expenditures. Total costs are calculated as a sum of SG&A, Cost of Goods Sold (COGS) and capital expenditures.



Figure 18: The gross growth rates of the shares of fixed and variable costs, 1960-2020

Note: The measures are constructed using Compustat data. Fixed costs are represented by the sum of "Selling, general and administrative expenses" (SG&A) and capital expenditures. Variable costs are represented by Cost of Goods Sold (COGS). Total costs are calculated as a sum of SG&A, Cost of Goods Sold (COGS) and capital expenditures.



Figure 19: The gross growth rates of the shares of fixed and variable costs, 1960-2020

Note: The measures are constructed using Compustat data. Fixed costs are represented by the sum of "Selling, general and administrative expenses" (SG&A) and half of the capital expenditures. Variable costs are represented by the sum of Cost of Goods Sold (COGS) and half of capital expenditures. Total costs are calculated as a sum of SG&A, Cost of Goods Sold (COGS) and capital expenditures.

	FC Share	$\Delta FCS$	$\Delta$ within	$\Delta between$	$\Delta cross$
1970	0.168	0.034	0.031	0.000	0.003
1980	0.150	-0.018	-0.004	-0.003	-0.011
1990	0.204	0.054	0.043	0.014	-0.003
2000	0.213	0.008	0.010	0.001	-0.002
2010	0.208	-0.005	-0.004	-0.001	0.000
2020	0.225	0.017	0.021	0.003	-0.007

Table IV: Sectoral decomposition of the 10-year changes in fixed cost shares, at the 2-digit NAICS level

 Table V: Sectoral decomposition of the 10-year changes in fixed cost shares, at the 4-digit

 NAICS level

	FC Share	$\Delta FCS$	$\Delta$ within	$\Delta between$	$\Delta cross$
1970	0.168	0.034	0.021	0.002	0.002
1980	0.150	-0.018	-0.009	-0.003	-0.007
1990	0.204	0.054	0.025	0.028	0.001
2000	0.213	0.008	0.001	0.009	-0.002
2010	0.208	-0.005	0.004	0.000	-0.005
2020	0.225	0.017	-0.007	0.025	0.002

Table VI: Dependent variable: (log) sales in agriculture and mining

	(1)	(2)	(3)	(4)	(5)	(6)
Fixed cost share	$-9.911^{***}$ (0.249)	$-6.324^{***}$ (0.091)	$-9.695^{***}$ (0.249)	$-6.349^{***}$ (0.092)	$-9.357^{***}$ (0.257)	$-6.302^{***}$ (0.100)
$(\log) SG\&A$		$\frac{1.095^{***}}{(0.005)}$		$\frac{1.100^{***}}{(0.006)}$		$\frac{1.101^{***}}{(0.007)}$
Constant	$12.663^{***} \\ (0.085)$	$\frac{1.941^{***}}{(0.058)}$	$12.625^{***} \\ (0.083)$	$\frac{1.906^{***}}{(0.062)}$	$12.531^{***} \\ (0.080)$	$\frac{1.879^{***}}{(0.068)}$
Observations	23093	23091	23093	23091	22106	22104
R-squared	0.337	0.930	0.374	0.934	0.478	0.939
Time	No	No	Yes	Yes	No	No
Industry-Time	No	No	No	No	Yes	Yes

Standard errors in parentheses

\* p < 0.10, \*\* p < 0.05, \*\*\* p < 0.01

Note: The measures are constructed using Compustat data. Total costs are calculated as a sum of SG&A, Cost of Goods Sold (COGS) and capital expenditures. Fixed cost shares are calculated as the proportion of SG&A in the total costs. Industry is defined at the 6-digit NAICS level.

	(1)	(2)	(3)	(4)	(5)	(6)
Fixed cost share	$-5.625^{***}$ (0.132)	$-4.322^{***}$ (0.027)	$-6.080^{***}$ (0.134)	$-4.324^{***}$ (0.027)	$-5.867^{***}$ (0.164)	$-4.275^{***}$ (0.033)
$(\log) SG\&A$		$\begin{array}{c} 1.037^{***} \\ (0.002) \end{array}$		$\frac{1.036^{***}}{(0.002)}$		$\frac{1.039^{***}}{(0.002)}$
Constant	$13.806^{***} \\ (0.049)$	$2.297^{***} \\ (0.023)$	$\begin{array}{c} 13.940^{***} \\ (0.049) \end{array}$	$2.313^{***} \\ (0.024)$	$13.880^{***} \\ (0.058)$	$2.260^{***} \\ (0.027)$
Observations R-squared Time Industry-Time	126835 0.212 No No	126826 0.975 No No	126835 0.252 Yes No	126826 0.976 Yes No	118814 0.453 No Yes	118806 0.980 No Yes

Table VII: Dependent variable: (log) sales in manufacturing

Standard errors in parentheses

\* p < 0.10, \*\* p < 0.05, \*\*\* p < 0.01

Note: The measures are constructed using Compustat data. Total costs are calculated as a sum of SG&A, Cost of Goods Sold (COGS) and capital expenditures. Fixed cost shares are calculated as the proportion of SG&A in the total costs. Industry is defined at the 6-digit NAICS level.

Table VIII: Dependent variable: (log) sales in services						
	(1)	(2)	(3)	(4)	(5)	(6)
Fixed cost share	$-4.451^{***}$ (0.102)	$-3.928^{***}$ (0.025)	$-4.769^{***}$ (0.100)	$-3.942^{***}$ (0.025)	$-5.014^{***}$ (0.127)	$\begin{array}{c} -4.014^{***} \\ (0.031) \end{array}$
$(\log) SG\&A$		$\frac{1.028^{***}}{(0.002)}$		$\frac{1.022^{***}}{(0.002)}$		$\frac{1.043^{***}}{(0.003)}$
Constant	$\begin{array}{c} 13.360^{***} \\ (0.043) \end{array}$	$\begin{array}{c} 2.315^{***} \\ (0.026) \end{array}$	$13.459^{***} \\ (0.042)$	$2.384^{***} \\ (0.027)$	$\begin{array}{c} 13.565^{***} \\ (0.046) \end{array}$	$2.185^{***} \\ (0.031)$
Observations R-squared	$115485 \\ 0.163$	$115442 \\ 0.962$	$115485 \\ 0.229$	$115442 \\ 0.963$	$106895 \\ 0.488$	$106857 \\ 0.972$
Time	No	No	Yes	Yes	No	No
Industry-Time	No	No	No	No	Yes	Yes

Table VIII: Dependent variable: (log) sales in services

Standard errors in parentheses

\* p < 0.10, \*\* p < 0.05, \*\*\* p < 0.01

Note: The measures are constructed using Compustat data. Total costs are calculated as a sum of SG&A, Cost of Goods Sold (COGS) and capital expenditures. Fixed cost shares are calculated as the proportion of SG&A in the total costs. Industry is defined at the 6-digit NAICS level.



Figure 20: Year-by-year coefficients for regressions of (log) sales on fixed cost shares, 1960-2020



Figure 21: Year-by-year coefficients for regressions of (log) sales on fixed cost shares in the agriculture and mining sector, 1960-2020



Figure 22: Year-by-year coefficients for regressions of (log) sales on fixed cost shares in the manufacturing sector, 1960-2020



Figure 23: Year-by-year coefficients for regressions of  $(\log)$  sales on fixed cost shares in the services sector, 1960-2020

# D Appendix: Other Countries

To understand the evolution and the distributions of the fixed cost shares in other countries we use Worldscope data. This dataset covers approximately 37,450 currently active companies in 75 countries, approximately 95% of global market capitalization. The data goes back to 1980 for some countries, while for others a good coverage may start at later years. The accounting definitions are the same as in Compustat for the variables representing for fixed and variable costs. In order to calculate capital costs, we use the same methodology as described in Appendix B.

Figure 24 through Figure 28 show the average fixed cost share in the world, Europe,<sup>15</sup> China, Japan, and Canada. The average fixed cost share has gone up across the board. Figure 29 through Figure 33 plot the distributions of fixed cost shares for the same regions early in the available sample and in 2019. Similarly to the US, the distributions for these countries have shifted to the right between the beginning and the end of the sample.



Figure 24: The evolution of the share of fixed costs in total costs in the world, 1980-2020

Note: The measures are constructed using Worldscope data. Fixed costs are represented by the sum of "Selling, general and administrative expenses" (SG&A) and capital expenditures. Total costs are calculated as a sum of SG&A, Cost of Goods Sold (COGS) and capital expenditures.

<sup>&</sup>lt;sup>15</sup>European region is defined as the following set of countries: Austria, Belgium, Bulgaria, Croatia, Cyprus, Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, Ireland, Italy, Latvia, Lithuania, Luxembourg, Malta, Netherlands, Norway, Poland, Portugal, Romania, Slovakia, Slovenia, Spain, Sweden, Switzerland, and the UK.



Figure 25: The evolution of the share of fixed costs in total costs in Europe, 1985-2020

Note: The measures are constructed using Worldscope data. Fixed costs are represented by the sum of "Selling, general and administrative expenses" (SG&A) and capital expenditures. Total costs are calculated as a sum of SG&A, Cost of Goods Sold (COGS) and capital expenditures.



Figure 26: The evolution of the share of fixed costs in total costs in China, 1989-2020

Note: The measures are constructed using Worldscope data. Fixed costs are represented by the sum of "Selling, general and administrative expenses" (SG&A) and capital expenditures. Total costs are calculated as a sum of SG&A, Cost of Goods Sold (COGS) and capital expenditures.



Figure 27: The evolution of the share of fixed costs in total costs in Japan, 1989-2020

Note: The measures are constructed using Worldscope data. Fixed costs are represented by the sum of "Selling, general and administrative expenses" (SG&A) and capital expenditures. Total costs are calculated as a sum of SG&A, Cost of Goods Sold (COGS) and capital expenditures.



Figure 28: The evolution of the share of fixed costs in total costs in Canada, 1986-2020

Note: The measures are constructed using Worldscope data. Fixed costs are represented by the sum of "Selling, general and administrative expenses" (SG&A) and capital expenditures. Total costs are calculated as a sum of SG&A, Cost of Goods Sold (COGS) and capital expenditures.



Figure 29: Share of fixed costs in total costs in the world, 1980 and 2019

Note: The measures are constructed using Worldscope data. Fixed costs are represented by the "Selling, general and administrative expenses" (SG&A). Total costs are calculated as a sum of SG&A, Cost of Goods Sold (COGS) and capital expenditures.



Figure 30: Share of fixed costs in total costs in Europe, 1990 and 2019

Note: The measures are constructed using Worldscope data. Fixed costs are represented by the "Selling, general and administrative expenses" (SG&A). Total costs are calculated as a sum of SG&A, Cost of Goods Sold (COGS) and capital expenditures.



Figure 31: Share of fixed costs in total costs in China, 1999 and 2019

Note: The measures are constructed using Worldscope data. Fixed costs are represented by the "Selling, general and administrative expenses" (SG&A). Total costs are calculated as a sum of SG&A, Cost of Goods Sold (COGS) and capital expenditures.



Figure 32: Share of fixed costs in total costs in Japan, 1986 and 2019

Note: The measures are constructed using Worldscope data. Fixed costs are represented by the "Selling, general and administrative expenses" (SG&A). Total costs are calculated as a sum of SG&A, Cost of Goods Sold (COGS) and capital expenditures.



Figure 33: Share of fixed costs in total costs in Canada, 1998 and 2019

Note: The measures are constructed using Worldscope data. Fixed costs are represented by the "Selling, general and administrative expenses" (SG&A). Total costs are calculated as a sum of SG&A, Cost of Goods Sold (COGS) and capital expenditures.