Multi-unit auctions with uncertain supply and single-unit demand

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Multi-unit auctions

- Trade commodities and financial instruments, e.g. electricity and treasury bonds.
- Traded volume of auctioneer, *Z,* could be uncertain.
- Non-competitive bids, e.g. wind/solar power or bids from small investors => uncertain volume for strategic bidders.
- Unknown value of traded good.
- Bidders have asymmetric (and potentially correlated) information about good's value and *Z*.

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Can auctioneer increase revenue?

- Should auctioneer disclose information relevant for value of good and its supply?
- For well-behaved equilibria, is uniform-price or pay-as-bid better?
- How to avoid inefficient, ill-behaved equilibria with prices at the collusive level in uniform-price auctions?

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Well-behaved symmetric equilibria

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Extension of Milgrom & Weber (1982)

- Auction sells uncertain number of units, *Z*.
- Single-unit demand, one bid per bidder.
- Each bidder *i* observes private signal X_i
- Good's value for buyer *i:* $u(\boldsymbol{S}, X_i, \boldsymbol{Y})$

  - ***Y*** is ordered vector of signals of competitors.

  - ***S*** is vector of non-observed signals
- Higher signals => Increases value of good.
- Signals and Z are correlated.
- ***S***, $X_i, \boldsymbol{Y}$ are affiliated conditional on *Z* and ***S***, $X_i, \boldsymbol{Y}$, -*Z* are affiliated => ***S***, $X_i, \boldsymbol{Y}, -Z, Y_z$ are affiliated.

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Signals and –Z are affiliated

=> Z and signals are required to be negatively correlated (or Z may be independent of the signals).

Motivation: auctioneer and alternative sellers (outside the model) tend to have shortage of the good at the same time, so that prices increase in alternative markets when Z is small, which increases the value of the good.

Results for symmetric equilibria

- Well-behaved, monotonic, efficient, symmetric equilibrium exists in uniform-price and pay-as-bid auction.
- One symmetric equilibrium in UPA, if auctioneer's supply is independent of bidders' signals.
- UPA better for auctioneer than PABA.
- Auctioneer gains by always and fully disclosing any signal S_k that it might observe, including Z.
- Revenue equivalence if signals are independent and independent of Z.

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Ill-behaved equilibria in uniform-price auctions with prices at collusive level

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Private value assumptions

- Good's value for buyer *i*: $u(X_i)$
- Bidders could be asymmetric, also ex-ante
- Range of signals/values is common knowledge
- $\boldsymbol{S}, X_i, \boldsymbol{Y}$ allowed to be affiliated or non-affiliated
- Uniform-price auction

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High-low equilibrium for certain Z

Z bidders bid high (above maximum value of low bidders) and win. Remaining, *n-Z* bidders, bid low (below lowest value of high bidders) and set the clearing price.

Two problems:

1) Low revenue for auctioneer.
2) Inefficient.

Special cases with partial high-low equilibria

- If Z=1, range of partial high-low equilibria exists at the bottom, for values below a threshold. All bidders bid their value above the threshold (Blume & Heidhues, 2004).

- If Z=n-1, range of partial high-low equilibria exists at the top, for values above a threshold.

=> Reduces efficiency and revenue of auctioneer.

Uncertain Z, such that $1<Z<n-1$, and symmetric value ranges

- Unique equilibrium, which is well-behaved (bid at value).

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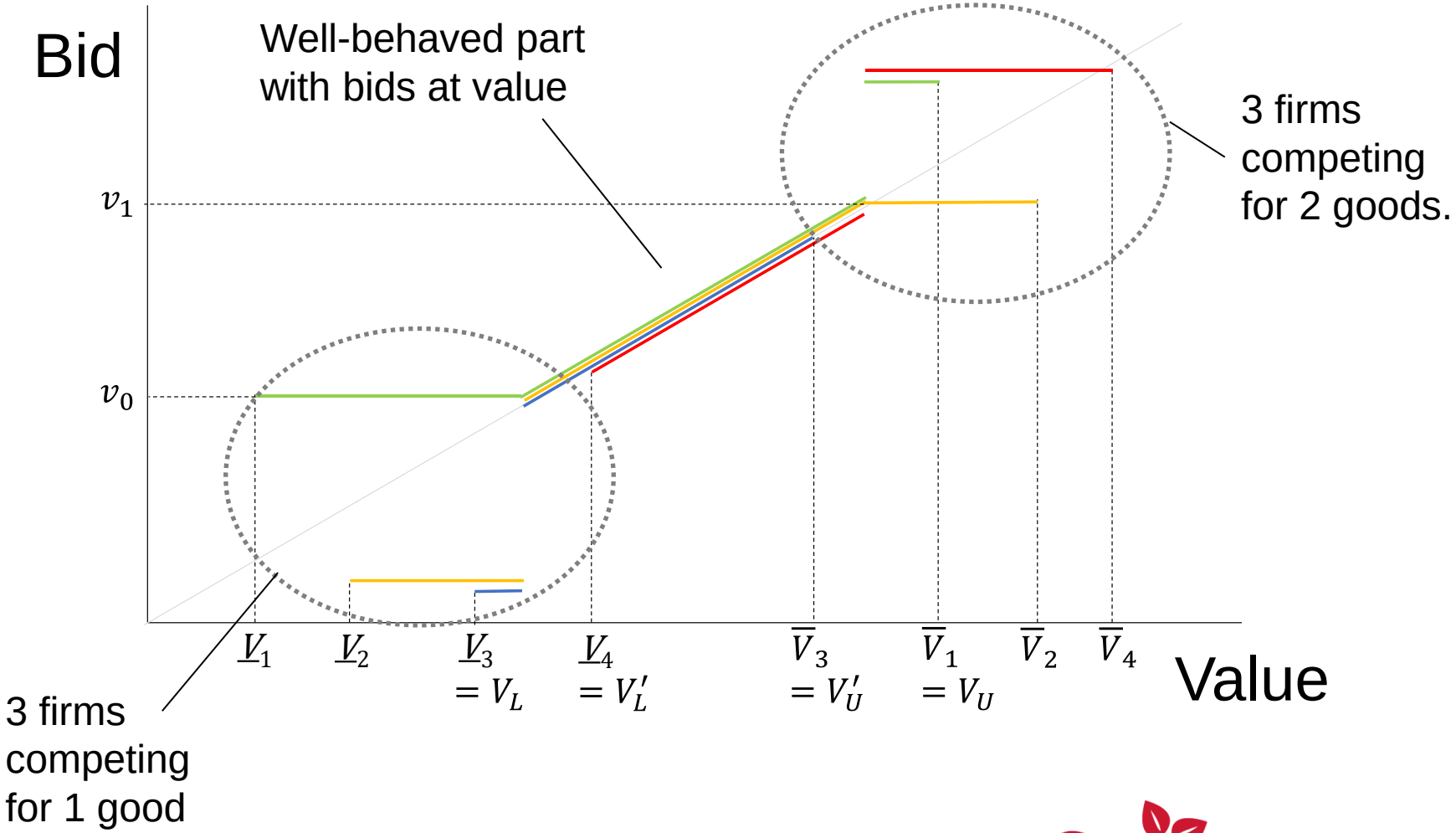
Equilibria for $1<Z<n-1$ and heterogeneous value ranges

Heterogeneous value ranges => continuum of partial high-low equilibria at the edges:

- Auction similar to $Z=1$ case for sufficiently low values (below lowest value of some firms) => range of partial high-low equilibria at the bottom.
- Auction similar to $Z=n-1$ case for sufficiently high values (above highest value of some firms) => range of partial high-low equilibria at the top.

Example: edge effect for *n*=4 and Z=2



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How to remove ill-behaved equilibria

- Effective price cap removes partial high-low equilibrium at the top.
- Effective price floor removes partial high-low equilibrium at the bottom (Blume & Heidhues, 2004).
- Effective price cap and floor give unique equilibrium.
- Price-sensitive supply can also give unique equilibria.

Invariance/symmetry result for reflected auctions

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Reflected auction

Equilibrium in uniform-price auction with *n* bidders and *Z* goods => equilibrium in a transformed auction with *n* bidders and *n-Z* goods, if sign of values and bids are reversed.

Also true if bidders are asymmetric ex-ante.

It does not matter how signals are correlated.

Consistent with:

- If *Z*=1, a partial high-low equilibrium exists at the bottom, for values below a threshold.
- If *Z*=*n*-1, a partial high-low equilibria exists at the top, for values above a threshold.

Summary

- Contribution 1: Milgrom & Weber (1982) is extended so that number of traded objects can be uncertain and correlated with bidders' signals.
- Contribution 2: Identifying ill-behaved equilibria in uniform-price auctions and showing how they can be removed (effective price cap and floor)
- Contribution 3: Identifying new symmetry/invariance property of multi-unit auctions.