# Across the borders, above the bounds: a non-linear framework for international yield curves

Laura Coroneo[1] Iryna Kaminska[2] Sergio Pastorello[3]

[1]University of York

[2]Bank of England

[3]University of Bologna

EEA 2024 - Rotterdam

28 August 2024

# Introduction

## Motivation

- Large literature on the analysis of the US monetary policy shocks transmission and its spillover effects
  (e.g. Craine and Martin 2008, Albagli, Ceballos, Claro and Romero 2019, Miranda-Agrippino and Rey 2020, Degasperi, Hong and Ricco 2020, Kearns, Schrimpf and Xia 2018 and Gourinchas, Ray and Vayanos 2022)

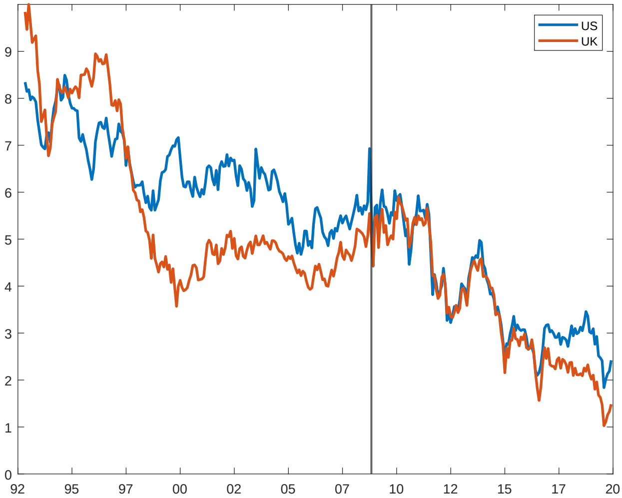
- These studies analyse the shock transmission via linear methods

- **Question**: how is the monetary policy shock transmission affected by the presence of the interest rate zero lower bound (ZLB)?

# Introduction

## Motivation

Figure: 10-year forward rates in US and UK



# Introduction

## This paper

- We extend the "shadow rate term structure model" framework to jointly modelling international yield curves
- We allow for non-linear shock transmission with an explicit ZLB constrain imposed:
  - Local projections $\rightarrow$ linear responses of state variables
  - Feed the state variable responses into the joint shadow rate model $\rightarrow$ (non-linear) responses of forward rates
- We assess the US monetary policy transmission mechanism and its spillover effects on the UK yield curve

# Introduction
## Results

1. US bond factors operate as global factors and account for a significant proportion of the variation of UK bond yields

2. Explicitly imposing ZLB restrictions on interest rates is key for the model performance and tractability

3. The post-2009 US monetary policy transmission mechanism and its spillover effects on the UK yield curve are non-linear and asymmetric

$\Rightarrow$ It is important to account for the ZLB and the non-linearities caused by it when evaluating the transmission and the spillover of shocks

# Introduction

## Related Literature

- Joint framework for international yield curves and importance of global factors (Diebold, Li, Yue (2008), Kumar and Okimoto (2011), Jotikasthira, Le, Lundblad (2015), Del Negro, Giannone, Giannoni, Tambalotti (2019), Coroneo, Garrett, Sanhueza (2018), Greenwood, Hanson, Stein, Sunderam (2020))

- Joint no-arbitrage term structure models (Egorov, Li, Ng (2011), Kaminska, Meldrum, Smith (2013), Sarno, Schneider, Wagner (2012), Chernov and Creal (2023), Gourinchas, Ray, Vayanos (2022))

- No-arbitrage term structure models with a lower bound for risk-free rates (Krippner (2015), Wu and Xia (2016), Bauer and Rudebusch (2016))

- Global transmission of U.S. monetary policy (Craine and Martin (2008), Albagli, Ceballos, Claro, Romero (2019), Miranda-Agrippino and Rey (2020), Degasperi, Hong, Ricco (2021), Kearns, Schrimpf, Xia (2022), Gourinchas, Ray, Vayanos (2022))

# Model

## State variables and physical dynamics

- State variables:
  - $n_0$ common factors $\boldsymbol{x}_t^0$ (global interest rate trends)
  - $n_i$ country-specific factors $\boldsymbol{x}_t^i$ (local trends)
- Under the physical measure $\mathbb{P}$, the vector of state variables follows:

$$\boldsymbol{x}_{t+1} = \boldsymbol{\mu} + \Phi \boldsymbol{x}_t + \Gamma \boldsymbol{\varepsilon}_{t+1}, \ \boldsymbol{\varepsilon}_{t+1} \overset{\mathbb{P}}{\sim} \mathcal{N}_{IID}(\mathbf{0}, \mathrm{I}_n) \qquad (1)$$

where $\boldsymbol{x}_t = \left(\boldsymbol{x}_t^{0'}, \boldsymbol{x}_t^{US'}, \boldsymbol{x}_t^{UK'}\right)'$, $\Gamma$ is lower triangular and $\Phi$ is lower triangular with a block diagonal lower right block (decomposability under $\mathbb{P}$, Egorov, Li and Ng, 2011).

$\Rightarrow$ The vectors $\boldsymbol{z}_t^i = \left(\boldsymbol{x}_t^{0'}, \boldsymbol{x}_t^{i'}\right)'$ for $i = US, UK$ also follow a Gaussian VAR(1) process under $\mathbb{P}$.

# Model

## Risk neutral dynamics

- Essentially affine stochastic discount factor (Duffee 2002)

$$M_{t+1}^{i} = \exp\left(-r_t^i - \frac{1}{2}\boldsymbol{\lambda}_t^{i\prime}\boldsymbol{\lambda}_t^{i} - \boldsymbol{\lambda}_t^{i\prime}\varepsilon_{t+1}^{i}\right), \quad i = US, UK,$$

where $\boldsymbol{\lambda}_t^i = \boldsymbol{\lambda}^i + \Lambda^i \boldsymbol{z}_t^i, \quad i = US, UK$

$\Rightarrow$ Under the risk neutral measures $\mathbb{Q}^i$, $i = US, UK$

$$\boldsymbol{z}_{t+1}^{i} = \boldsymbol{\mu}^{\mathbb{Q}^i} + \Phi^{\mathbb{Q}^i}\boldsymbol{z}_t^i + \Gamma^i \varepsilon_{t+1}^{\mathbb{Q}^i}, \quad i = US, UK, \tag{2}$$

# Model

## Interest rates

- Shadow short rates

$$s_t^i = \delta_0^i + \boldsymbol{\delta}_1^{i\prime} \boldsymbol{z}_t^i, \quad i = US, UK \tag{3}$$

- Observed short rates

$$r_t^i = \max(s_t^i, \underline{r}^i), \quad i = US, UK \tag{4}$$

# Model

## Interest rates

- Shadow short rates

$$s_t^i = \delta_0^i + \boldsymbol{\delta}_1^{i\prime} \boldsymbol{z}_t^i, \quad i = US, UK \tag{3}$$

- Observed short rates

$$r_t^i = \max(s_t^i, \underline{r}^i), \quad i = US, UK \tag{4}$$

- Under absence of arbitrage

$$p_{t,\tau}^i = E_t^{\mathbb{Q}^i}\left[\exp\left(-\sum_{j=0}^{\tau-1} r_{t+j}^i\right)\right], \quad i = US, UK$$

- Under (2), (3), and (4), the one period forward rate in country $i$ for a loan starting at $t+\tau$ is (Wu and Xia 2016) More

$$f_{t,\tau}^i \approx \underline{r}^i + \sigma_\tau^i g\left(\frac{a_\tau^i + \boldsymbol{b}_\tau^{i\prime} \boldsymbol{z}_t^i - \underline{r}^i}{\sigma_\tau^i}\right) \equiv h_\tau^i(\boldsymbol{x}_t), \quad i = US, UK \tag{5}$$

where $g(w) = wN(w) + n(w)$ with $N$ and $n$ the standard gaussian cdf and the pdf

# Model

## Depreciation rate

- $e_t$ log spot sterling-dollar exchange rate, i.e. the spot price of sterling in units of dollars.

- Under complete markets and absence of arbitrage (Backus, Foresi and Telmer 2001)

$$
\begin{aligned}
e_t - e_{t-1} &= \log M_t^{UK} - \log M_t^{US} \\
&= \left(r_{t-1}^{US} - r_{t-1}^{UK}\right) + \frac{1}{2}\left(\boldsymbol{\lambda}_{t-1}^{US'}\boldsymbol{\lambda}_{t-1}^{US} - \boldsymbol{\lambda}_{t-1}^{UK'}\boldsymbol{\lambda}_{t-1}^{UK}\right) + \\
&\quad + \boldsymbol{\lambda}_{t-1}^{US'}\varepsilon_t^{US} - \boldsymbol{\lambda}_{t-1}^{UK'}\varepsilon_t^{UK} \\
&= k(\boldsymbol{x}_t, \boldsymbol{x}_{t-1}) \qquad (6)
\end{aligned}
$$

# Estimation

## State space representation

- Measurement equation

$$\begin{pmatrix} \boldsymbol{f}_t \\ \Delta e_t \end{pmatrix} = \begin{pmatrix} \boldsymbol{h}(\boldsymbol{\xi}_t) \\ k(\boldsymbol{\xi}_t) \end{pmatrix} + \boldsymbol{u}_t, \quad \boldsymbol{\xi}_t = (\boldsymbol{x}'_t, \boldsymbol{x}'_{t-1})' \tag{7}$$

- Transition equation (companion form)

$$\boldsymbol{\xi}_t = \boldsymbol{\mu}_\xi + \Phi_\xi \boldsymbol{\xi}_{t-1} + \boldsymbol{v}_t, \;\; \boldsymbol{v}_t \sim \mathcal{N}_{IID}(\boldsymbol{0}, \Sigma_\xi) \tag{8}$$

- The measurement equation in (7) is nonlinear in the state variables: quasi maximum likelihood inference using the Extended Kalman filter
- Identification scheme similar to Dai and Singleton (2000) and Egorov, Li and Ng (2011): $\Gamma = \mathrm{I}_n$, $\boldsymbol{\delta}_1^{US} \geq 0$, $\boldsymbol{\delta}_1^{UK} \geq 0$ and $\boldsymbol{\mu} = 0_{n\times 1}$
- Assume that $\underline{r}^{US} = \underline{r}^{UK} = 0$

# Estimation

## US monetary policy transmission: linear responses of states and shadow rates

- Local projection on the exogenous shock $g_t$ (Jordà, 2005)

$$\boldsymbol{x}_{t+h} = \boldsymbol{\alpha}_h + \boldsymbol{\gamma}_h \boldsymbol{x}_{t-1} + \boldsymbol{\beta}_h g_t + \boldsymbol{\nu}_{t+h} \tag{9}$$

- Conditional expectations of the state variables

$$\begin{aligned} \boldsymbol{x}_{t+h|g_t=1} &\equiv E(\boldsymbol{x}_{t+h}|g_t = 1) = \boldsymbol{\alpha}_h + \boldsymbol{\gamma}_h \boldsymbol{x}_{t-1} + \boldsymbol{\beta}_h \\ \boldsymbol{x}_{t+h|g_t=0} &\equiv E(\boldsymbol{x}_{t+h}|g_t = 0) = \boldsymbol{\alpha}_h + \boldsymbol{\gamma}_h \boldsymbol{x}_{t-1} \end{aligned}$$

# Estimation

## US monetary policy transmission: linear responses of states and shadow rates

- Local projection on the exogenous shock $g_t$ (Jordà, 2005)

$$\boldsymbol{x}_{t+h} = \boldsymbol{\alpha}_h + \boldsymbol{\gamma}_h \boldsymbol{x}_{t-1} + \boldsymbol{\beta}_h g_t + \boldsymbol{\nu}_{t+h} \tag{9}$$

- Conditional expectations of the state variables

$$\begin{aligned}
\boldsymbol{x}_{t+h|g_t=1} &\equiv E(\boldsymbol{x}_{t+h}|g_t = 1) = \boldsymbol{\alpha}_h + \boldsymbol{\gamma}_h \boldsymbol{x}_{t-1} + \boldsymbol{\beta}_h \\
\boldsymbol{x}_{t+h|g_t=0} &\equiv E(\boldsymbol{x}_{t+h}|g_t = 0) = \boldsymbol{\alpha}_h + \boldsymbol{\gamma}_h \boldsymbol{x}_{t-1}
\end{aligned}$$

- State variables response

$$\Delta \boldsymbol{x}_{t+h|g_t} \equiv \boldsymbol{x}_{t+h|g_t=1} - \boldsymbol{x}_{t+h|g_t=0} = \boldsymbol{\beta}_h$$

- Shadow short rate response

$$\Delta s_{t+h|g_t} \equiv s_{t+h|g_t=1} - s_{t+h|g_t=0} = \boldsymbol{\delta}_1' \Delta \boldsymbol{x}_{t+h|g_t} = \boldsymbol{\delta}_1' \boldsymbol{\beta}_h$$

# Estimation

## US monetary policy transmission: non-linear responses of observed rates

- Short-rate response

$$\begin{aligned}\Delta r_{t+h|g_t} &\equiv r_{t+h|g_t=1} - r_{t+h|g_t=0}\\ &\approx \max\{\boldsymbol{\delta}_0' + \boldsymbol{\delta}_1' \boldsymbol{x}_{t+h|g_t=1}, \underline{r}\} - \max\{\boldsymbol{\delta}_0' + \boldsymbol{\delta}_1' \boldsymbol{x}_{t+h|g_t=0}, \underline{r}\}\end{aligned}$$

- Forward rates response

$$\Delta \boldsymbol{f}_{t+h,\tau|g_t} \equiv \boldsymbol{f}_{t+h,\tau|g_t=1} - \boldsymbol{f}_{t+h,\tau|g_t=0} \approx \boldsymbol{h}_\tau(\boldsymbol{x}_{t+h|g_t=1}) - \boldsymbol{h}_\tau(\boldsymbol{x}_{t+h|g_t=0})$$

- Forward premium response

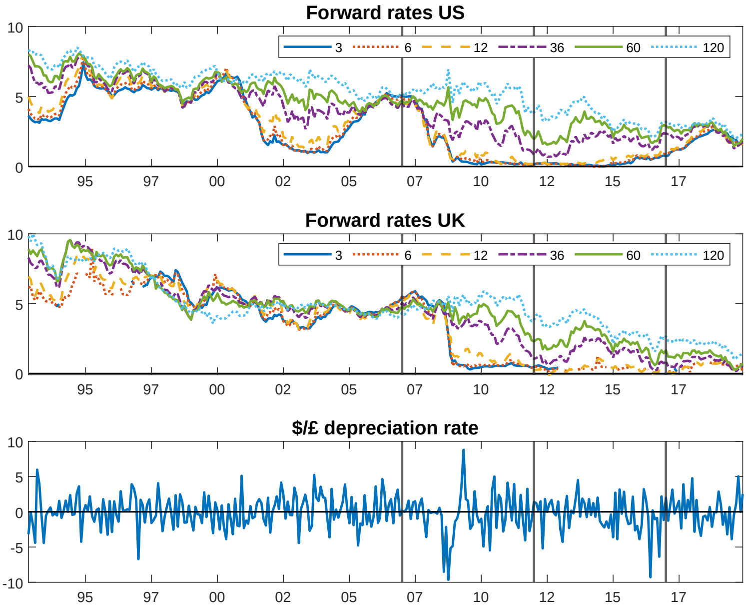
$$\begin{aligned}\Delta \boldsymbol{\pi}_{t+h,\tau|g_t} &\equiv \boldsymbol{\pi}_{t+h,\tau|g_t=1} - \boldsymbol{\pi}_{t+h,\tau|g_t=0}\\ &= \Delta \boldsymbol{f}_{t+h,\tau|g_t} - \Delta r_{t+h+\tau|g_t}\end{aligned}$$

- Confidence intervals by block-bootstrapping the residuals of (9), using an overlapping stationary circular scheme (Politis and Romano, 1994) with $\max b = \min\{h+1, 2\lfloor T^{1/4} \rfloor\}$.

# Data

- 1-month forward rates on US and UK government bonds for maturities of 3 and 6 months, 1, 3, 5 and 10 years constructed as in Wu and Xia (2016) obtained from the Federal Reserve Board and Bank of England
- Dollar-sterling exchange rate from the FRED data set
- End-of-month observations from October 1992 to December 2019
- Shocks from Kaminska, Mumtaz and Šustek (2021), constructed using high-frequency yield curve decomposition around FOMC announcements (end of month from Jan 1996 to Aug 2007)

# Data



Forward rates US
3
6
12
36
60
120
10
5
0
95
97
00
02
05
07
10
12
15
17
Forward rates UK
3
6
12
36
60
120
10
5
0
95
97
00
02
05
07
10
12
15
17
$/£ depreciation rate
10
5
0
-5
-10
95
97
00
02
05
07
10
12
15
17

Table: Cumulative proportion of variance explained by Principal Components

| | US rates | UK rates | All rates |
|---|---|---|---|
| PC1 | 0.871 | 0.893 | 0.857 |
| PC2 | 0.989 | 0.990 | 0.960 |
| PC3 | 0.998 | 0.998 | 0.979 |
| PC4 | 1.000 | 0.999 | 0.993 |
| PC5 | 1.000 | 1.000 | 0.997 |

Note: Average cumulative proportion of variance of US rates (first column), UK rates (second column), and all rates (third column) explained by the first five PCs extracted from US rates (first column), UK rates (second column), and jointly from US and UK rates (third column).

# Data

## Preliminary analysis

Table: Cumulative proportion of variance of UK rates explained by US PCs

| # US PCs | UK rates | UK PC1 | UK PC2 | UK PC3 |
|---|---|---|---|---|
| 1 | 0.795 | 0.886 | 0.030 | 0.013 |
| First 2 | 0.883 | 0.939 | 0.456 | 0.031 |
| First 3 | 0.891 | 0.946 | 0.456 | 0.258 |
| First 4 | 0.892 | 0.946 | 0.456 | 0.248 |
| First 5 | 0.903 | 0.956 | 0.475 | 0.320 |

Note: cumulative proportion of variance of UK rates and UK PCs explained by US PCs.

- 4 factors drive the joint dynamics of UK and US forward curves
  - US rates driven by 3 global factors
  - UK rates driven by 3 global factors and 1 UK-specific factor

# Results

## Fit Comparison

Table: Interest rates and depreciation rate fit

| | Interest rates | | Depreciation rate | |
|---|---|---|---|---|
| | Full | LB | Full | LB |
| **SRM with FX** | **0.167** | **0.156** | **1.462** | **1.169** |
| SRM w/o FX | 0.170 | 0.157 | - | - |
| GM with FX | 0.180 | 0.170 | 2.568 | 2.679 |
| GM w/o FX | 0.178 | 0.162 | - | - |

Note: this table reports RMSEs for interest rates (left panel) and the depreciation rate (right panel) from the joint shadow rate model with the depreciation rate (SRM with FX), the joint shadow rate model without the depreciation rate (SRM w/o FX), the joint Gaussian model with the depreciation rate (GM with FX), and the joint Gaussian model without the depreciation rate (GM w/o FX). Interest rates RMSEs refer to averages across maturities. Results are reported for the full sample (Oct 1992 - Dec 2019) and the lower bound sample (Dec 2008 - Dec 2019).

# Results

## Fit Comparison

Table: Correlation 10y rates

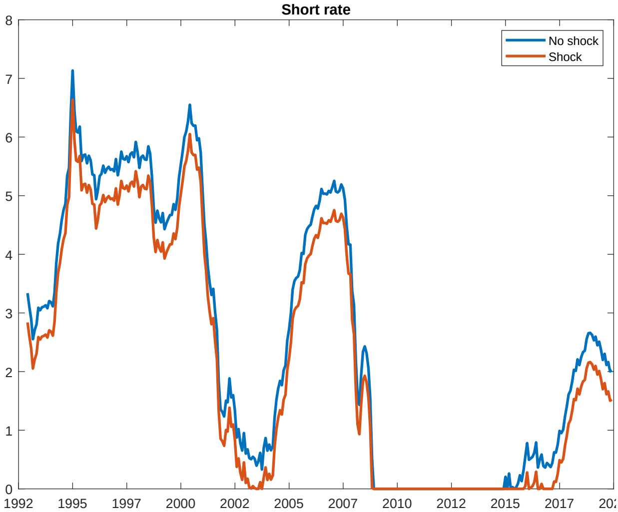
| | Full | Pre-LB | LB |
|---|---|---|---|
| Data | 0.873 | 0.792 | 0.966 |
| SRM with FX | 0.887 | 0.809 | 0.964 |
| SRM w/o FX | 0.885 | 0.811 | 0.973 |
| GM with FX | 0.888 | 0.840 | 0.934 |
| GM w/o FX | 0.899 | 0.874 | 0.936 |

Note: this table reports the correlation between the 10 year US rate and the 10 year UK rate. Results are reported for the full sample (Oct 1992 - Dec 2019), the pre-lower bound sample (Oct 1992 - Nov 2008) and the lower bound sample (Dec 2008 - Dec 2019).

# US monetary policy transmission

## Responses to a US target shock

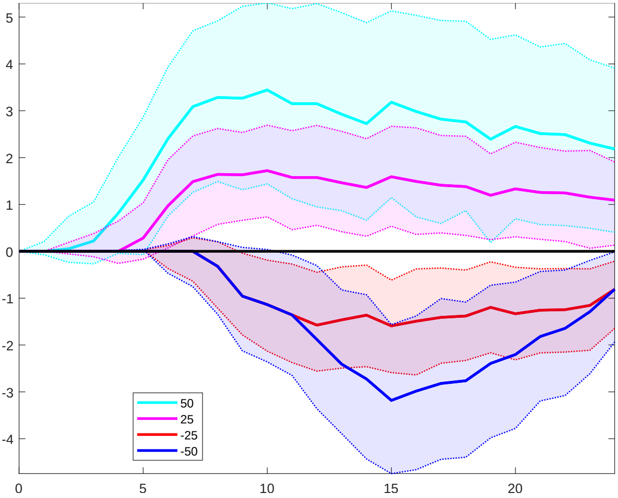
Figure: US short rate impact response to -50bp **US target/shadow rate shock**



# Results

## Responses to a US shadow rate shock

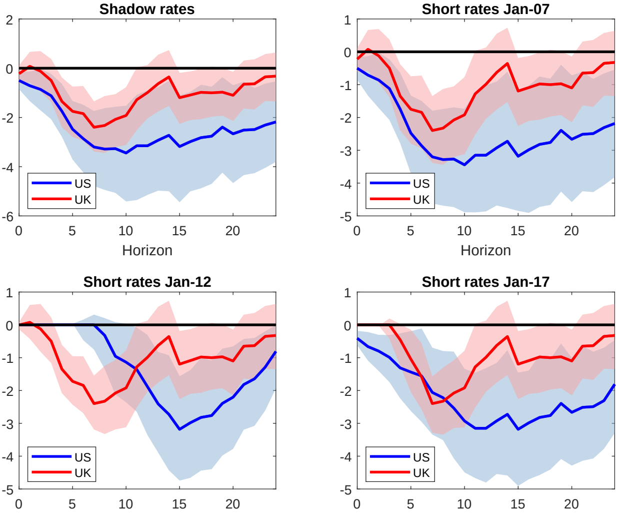
Figure: US short rate responses to a shadow rate shock on Jan 2012



# Results

## Responses to a US shadow rate shock

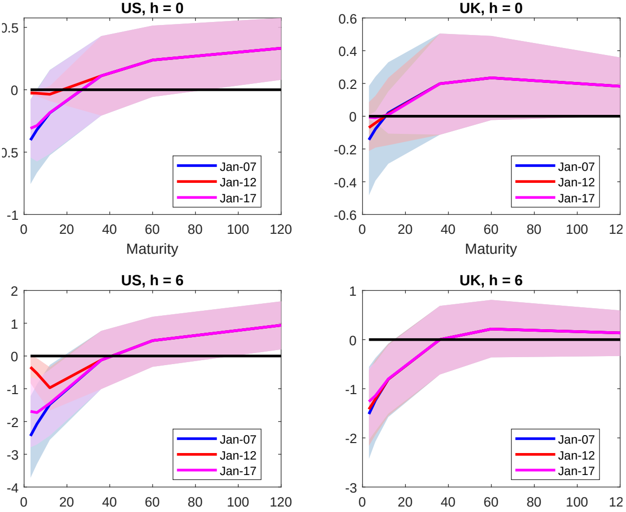
Figure: Shadow short rate and short rate responses to -50bp **US target shock**



# US monetary policy transmission

## Responses to a US shadow rate shock

Figure: Forward curve responses to -50bp **US shadow rate shock**



# US monetary policy transmission

## Responses to a US path shock

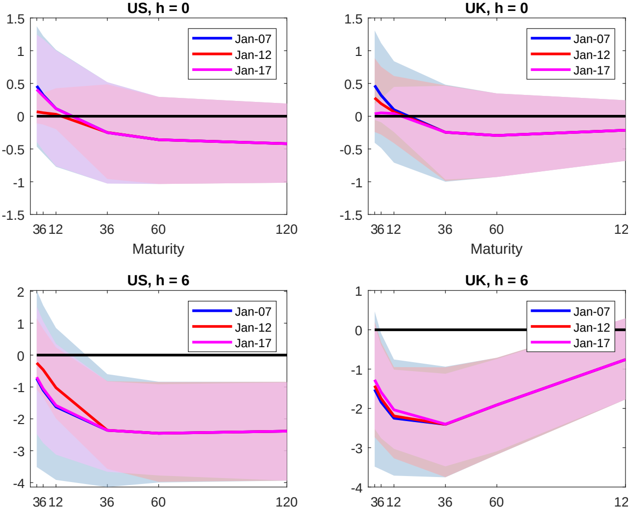



Figure: Forward curve response to -25bp **US path shock**

# Conclusion

- Propose a joint shadow rate term structure model for international yield curves
- Develop a new method to derive non-linear responses to shocks that is fully model consistent

$\Rightarrow$ The post-2009 US monetary policy transmission mechanism and its spillover effects on the UK yield curve are non-linear and asymmetric.

  - When close to the ZLB, a target US monetary policy shock is transmitted to short rates quicker in case of tightening than easing.
  - Increasing the shock magnitude in case of tightening delivers a proportionally increased response on interest rates, but the transmission impact is not proportional in the case of easing, with rates undershooting at shorter horizons.

# Model

## Interest rates

- Under (2), (3), and (4), the one period forward rate in country $i$ for a loan starting at $t+\tau$ is (Wu and Xia 2016) Back

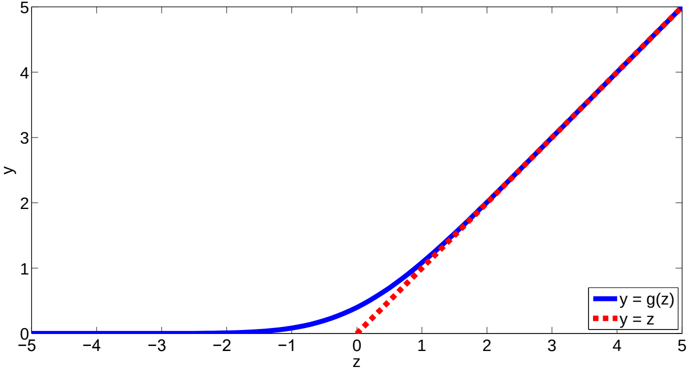
$$f^i_{t,\tau} \approx \underline{r}^i + \sigma^i_\tau g\left(\frac{a^i_\tau + \boldsymbol{b}^{i\prime}_\tau \boldsymbol{z}^i_t - \underline{r}^i}{\sigma^i_\tau}\right) \equiv h^i_\tau(\boldsymbol{x}_t), \quad i = US, UK, \quad (10)$$

where:

$$\begin{aligned}
g(w) &= wN(w) + n(w), \\
\boldsymbol{b}^i_\tau &= \left[\left(\Phi^{\mathbb{Q}^i}\right)^\tau\right]' \boldsymbol{\delta}^i_1, \\
a^i_\tau &= \delta^i_0 + \left(\sum_{j=0}^{\tau-1} \boldsymbol{b}^i_j\right)' \boldsymbol{\mu}^{\mathbb{Q}^i} - \frac{1}{2}\left(\sum_{j=0}^{\tau-1} \boldsymbol{b}^i_j\right)' \Gamma\Gamma' \left(\sum_{j=0}^{\tau-1} \boldsymbol{b}^i_j\right), \\
\left(\sigma^i_\tau\right)^2 &= \sum_{j=0}^{\tau-1} \boldsymbol{b}^{i\prime}_j \Gamma\Gamma' \boldsymbol{b}^i_j, \qquad (11)
\end{aligned}$$

# Model

## Interest rates: $g(\cdot)$




$$f^i_{t,\tau} \approx \underline{r}^i + \sigma^i_\tau g\left(\frac{a^i_\tau + \boldsymbol{b}^{i\prime}_\tau \boldsymbol{z}^i_t - \underline{r}^i}{\sigma^i_\tau}\right) = \begin{cases} \underline{r}^i_t, & \text{at the lower bound;} \\ a^i_\tau + \boldsymbol{b}^{i\prime}_\tau \boldsymbol{z}^i_t, & \text{when rates are high.} \end{cases}$$

Back

Backus, David K, Silverio Foresi, and Chris I Telmer (2001) 'Affine term structure models and the forward premium anomaly.' *The Journal of Finance* 56(1), 279–304

Dai, Q., and K. Singleton (2000) 'Specification analysis of affine term structure models.' *Journal of Finance* 55, 1943–1978

Duffee, G.R. (2002) 'Term premia and interest rate forecasts in affine models.' *Journal of Finance* 57, 405–443

Egorov, Alexei V, Haitao Li, and David Ng (2011) 'A tale of two yield curves: Modeling the joint term structure of dollar and euro interest rates.' *Journal of Econometrics* 162(1), 55–70

Kaminska, Iryna, Haroon Mumtaz, and Roman Šustek (2021) 'Monetary policy surprises and their transmission through term premia and expected interest rates.' *Journal of Monetary Economics* 124, 48–65

Wu, Jing Cynthia, and Fan Dora Xia (2016) 'Measuring the macroeconomic impact of monetary policy at the zero lower bound.' *Journal of Money, Credit and Banking* 48(2-3), 253–291