The Costs of Counterparty Risk in Long-Term Contracts*

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PRELIMINARY AND INCOMPLETE

Abstract

Promoting renewable energy investments relies on the availability of long-term contracts that ensure a predictable energy price. However, there are concerns regarding the liquidity of such contracts. This paper models the market for long-term power contracts and characterizes its main features. It shows that the buyers' counterparty risk enlarges the probability of default on the contracts, reducing contract liquidity and giving rise to under-investment in renewable energy. It also studies the effects of proposed market interventions, such as public support for long-term power contracts, public guarantees, regulatory-backed contracts, and the introduction of buyers' obligations to enter into these contracts.

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1 Introduction

In the summer of 2022, electricity spot market prices in Europe exceeded a record high of 700/MWh, well above the 50-60/MWh pre-crisis average (Fabra, 2023). In order to mitigate the incidence and occurrence of high electricity prices, the European Commission released a proposal for an electricity-market reform, emphasizing the need to promote long-term power contracts (European Commission, 2023). The objective was two-fold. First, to protect producers and consumers against revenue and cost shocks. Second, to promote the deployment of renewable energy, which involves long-lived capital-intensive investments and is, therefore, particularly vulnerable to the volatility of spot market prices.[1] As the European Commission put it, *"the ultimate objective is to provide secure, stable investment conditions for renewable and low-carbon energy developers by bringing down risk and capital costs while avoiding windfall profits in periods of high prices."* Long-term power contracts are not unique to European countries and they are also common in the electricity markets in the US, Canada, and Australia, to name just a few examples (Research and Markets, 2024).

There are two broad categories of long-term power contracts. Power Purchase Agreements (PPAs) are private long-term contracts between a consumer and a generator (typically renewable) who agree to buy and sell electricity at a fixed price over a certain number of years (typically, from 10 to 15). The use of PPAs has been increasing over time (Figure 1) but is still considered insufficient to boost renewable energy investments at the required speed and scale (Polo et al., 2023). Regulator-backed contracts, usually denoted as Contracts-for-Differences (CfDs), differ from PPAs in that the counterparty is not a private buyer but the regulator acting on behalf of several buyers (typically, all the buyers connected to the network).[2] Several regulators around the world have also relied on CfDs as a way to promote renewable investments.[3] This paper provides a

[1]Indeed, existing evidence indicates that decreasing exposure to price variations lowers the cost of capital, promoting renewable investments at lower costs (Gohdes et al., 2022; Dukan and Kitzing, 2023).

[2]PPAs and CfDs can differ in numerous dimensions beyond the identity of the counterparty. For instance, PPAs are commonly negotiated bilaterally, whereas CfDs are allocated through centralized auctions. However, nothing stops private sellers from using auctions to allocate PPAs. Similarly, PPAs can be tailored to the needs of the buyer and seller, whereas CfDs tend to be more standardized contracts. However, nothing stops regulators from auctioning CfDs with certain characteristics, e.g., baseload contracts instead of pay-as-produced contracts.

[3]CfDs were first introduced in the UK (see Kröger et al. (2022)). Since then, several countries have followed, including Denmark, Greece, Hungary, Ireland, Spain and Poland. Beyond Europe, CfDs have been used in Australia and Canada (Hastings-Simon et al., 2022).

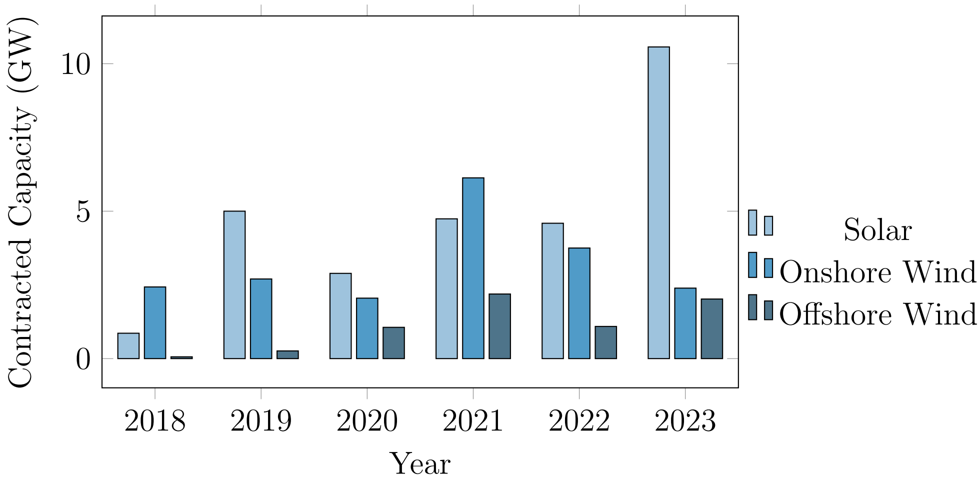


Figure 1: Volume of New PPA Contracts by Technology in Europe. Source: (Pexapark, 2024))

novel model of the PPA market and investigates the interaction between both types of long-term power contracts.

Despite the consensus on the need to promote long-term contracting, there is a lack of diagnosis as to why PPA markets have failed to provide enough liquidity for long-term power contracts. The report by the European Commission acknowledges that *"a barrier to the growth of this market is the credit risk that a consumer will not always be able to buy electricity over the whole period."* However, beyond expressing this concern, the implications of counterparty risk on the performance of PPA markets have not been explored in detail. Our paper puts buyers counterparty risk at the core of the analysis and uncovers how it affects PPA prices, the degree by which PPAs truly protect producers from spot price volatility considering the probability of contract default, and the resulting incentives to invest in renewable energy. We also use the model to analyze the properties of several public policies that have been proposed to overcome the market failures of the PPA market.

Our model considers sellers and buyers who trade one unit of electricity in a spot market. Risk-averse sellers suffer a utility loss when exposed to volatile prices. Buyers and sellers enter into fixed-price contracts that hedge them against the volatility of spot prices. However, buyers can decide to default on the contract if the spot price falls below the agreed price, exposing sellers to spot price volatility and the associated utility loss.[4]

[4]In our baseline model we normalize the costs of contract default to zero. However, in Section 3.2 we show that the model's main results remain unchanged if we add a collateral (as long as it is not too valuable) that the buyer forgoes in case of default.

Counterparty risk creates a trade-off for the seller, as increasing the contracts fixed price comes at the cost of raising the probability of contract default.

The equilibrium price for the contract is set through market clearing between the demand and supply for long-term contracts. Buyers are always willing to participate in fixed-price contracts as they can default if the spot price turns out to be below the contract price. However, the sellers willingness to invest and sign a fixed-price contract depends on their outside options. Sellers with low investment costs find entry at spot prices profitable even if accounting for their risk aversion. Hence, they invest regardless of whether contracts are available and sign contracts as long as they give them a utility at least as high as relying on the spot market. In contrast, since a contract is required to make the entry of high-cost sellers profitable, its price must allow the marginal seller to break even. It follows that the supply of contracts is weakly increasing in the contract price up to the point where further price increases reduce the sellers profits due to the higher risk of default. Investment beyond this point is never profitable, even if there exists excess demand for contracts at that price.

In this context, our model delivers two important results. First, it highlights the advantages that policymakers have attributed to long-term power contracts. Indeed, the market equilibrium with contracts is welfare superior to the no-contracts case. The reason is that contracts reduce the sellers disutility from spot price volatility, and trigger investment that would not have been undertaken in the absence of contracts. However, this does not imply that promoting the demand for long-term contracts is always welfare-enhancing. Indeed, there is a trade-off, as strengthening contract demand also pushes contract prices up in order to make additional investments profitable, at the cost of raising the probability of contract default for all contracts, including the inframarginal ones. This result uncovers a potential drawback of policies pursued by many countries aimed at boosting demand through a compulsory minimum in the long-term contracts that buyers must take. Although this requirement can promote investment, it might also create excessive counterparty risk.

Second, our model uncovers the cost of counterparty risk as a major market failure of PPA markets, leading to contract prices that are too high, excessive contract default, poor contract liquidity, and a weak ability to leverage investments in renewable energy. In our model, all sellers receive the market-clearing contract price, which is above the one that inframarginal sellers would require to sign the contract. Hence, contracts entail excessive counterparty risk. Therefore, there is scope to increase welfare through measures that allow for lowering contract prices and through that, a reduction in the probability of contract default. In turn, this intervention enhances the social profitability of entry, leading to increased investment in renewable energy.

In our model, collateral mitigates the counterparty-risk problem. Buyers must pledge some funds that will transferred to the seller in case of default. We show that it is often the case that a large collateral that eliminates the probability of default is inefficient. Collateral is costly and, therefore, its use should be optimally adjusted based on the default cost of the seller.

We explore various welfare-enhancing market interventions, some of which have been proposed in the regulatory debate, including *"requiring Member States to ensure that instruments to reduce the financial risks associated with the buyer defaulting on its long-term payment obligationsThese can be guarantee schemes at market prices, as well as public support for non-fossil fuels PPAs"* (European Commission, 2023). We show that these policy interventions can be welfare-enhancing as long as the funds that are required to provide public support or public guarantees are not too costly.

We first consider a regulator who can use subsidies contingent on the seller signing a fixed-price contract. We show that these subsidies have two welfare-enhancing effects. First, they can boost investment in cases where counterparty risk limits the seller's incentives to enter. Second, by boosting supply subsidies yield a lower equilibrium price in the market for long-term contract which mitigates the counterparty risk problem. When deciding on the optimal subsidy, the regulator must strike an optimal balance between the cost of the social funds associated to the subsidy and the previous two gains.

We then consider the role of public guarantees, which transfer the cost of counterparty risk from the seller to the regulator. Hence, as above, there is a trade-off between the benefits of protecting sellers against contract default and the public cost of the guarantees. However, a new trade-off arises in this case. Namely, since sellers do not internalize the full cost of their investments, excessive entry can occur, particularly if the cost of public funds is high.

Lastly, our model highlights that the most effective intervention is to promote regulator-backed long-term contracts. These contracts not only provide a counterparty risk-free option, but they may also improve the performance of the private PPA contracts market. The reason is simple: since contract demand is satisfied through another channel, the demand for PPAs goes down and the market clearing PPA price is lowered, mitigating counterparty risk for all PPA contracts. Furthermore, we show that the most effective CfDs are those aimed at promoting additional demand that would otherwise be absent from long-term markets.[5]

The remainder of the paper is organized as follows. In Section 2, we describe the model. In Section 3 we characterize the contract market equilibrium and assess its welfare properties. In Section 4, we analyze several market interventions, including public subsidies and public guarantees. Section 5 concludes.

2 Model Description

Consider a market for a homogeneous good. On the demand side, there is a unit mass of identical buyers with a maximum willingness to pay for one unit of the good equal to $v \geq 1$. On the supply side, there is a unit mass of (entrant) sellers, each capable of building one unit of capacity at a fixed cost c. Each unit of capacity allows to produce one unit of the good at a marginal cost normalized to zero. Entrants differ in their investment costs, which are independently drawn from a distribution function $G(c)$ with a positive density $g(c)$ in the interval $c \in [0, 1]$.

There is also a large amount of existing capacity capable of meeting total demand. Its marginal costs are denoted by p, and they are distributed according to $\Phi(p)$, with a positive and differentiable density $\phi(p)$ over the interval $p \in [0, 1]$. We further assume that the hazard rate,

$$\frac{\phi(p)}{1 - \Phi(p)},$$

is strictly increasing in p. The expected marginal costs of the existing capacity is denoted by $E[p]$. As a result, entry yields savings equal to the expected marginal costs of the existing capacity $E[p]$ minus the entrants' investment costs.[6]

The timing of the game proceeds as follows. First, at the investment stage, sellers

[5]We take the constraint on the maximum number of available CfDs as exogenous. The reasons behind the limit in the regulators' ability to auction off a higher number of CfDs are outside the scope of this model.

[6]New investments could generate positive or negative externalities not reflected in market prices. A parameter γ can be easily introduced to capture them. In this case, the savings from the investments would be given by $E[p] - c + \gamma$.

decide whether to enter or not after observing their investment cost c but before knowing the realization of the marginal costs of the existing capacity, p. Second, at the production stage, once p is observed, buyers and sellers trade the good in a perfectly competitive spot market, where the market price is given by the marginal cost of the marginal producer required to cover demand. Hence, unless demand is entirely served by entrants,[7] the equilibrium price in the spot market becomes p. Since entrants have zero marginal costs, they produce at full capacity, earning expected spot market revenues $E[p]$.

The volatility of spot market prices creates uncertainty over cost recovery, giving raise to a risk premium for the entrants denoted by $r \in (0, E[p])$.[8] Buyers do not incur in any investment and are thus assumed to be risk-neutral. Accordingly, at the investment stage, the expected profits of buyers (B) and sellers (S) can be formulated as

$$\Pi_B^0 = v - E[p],$$
$$\Pi_S^0 = E[p] - r - c.$$

Profitable entry requires spot market revenues to cover at least the investment cost c plus the risk premium r, i.e., only entrants with costs $c \leq c^0 \equiv E[p] - r$ decide to invest in equilibrium.

Notice that if sellers were not exposed to uncertain spot prices, they would invest until the marginal cost savings equaled the investment cost, i.e., for all $c \leq c^{FB} \equiv E[p]$. Hence, due to sellers' risk aversion ($r > 0$), the market solution is characterized by under-investment relative to the First Best. The resulting welfare loss can be decomposed as

$$W^{FB} - W^0 = rG\left(E(p) - r\right) + \int_{E(p)-r}^{E(p)} \left(E(p) - c\right) g\left(c\right) dc > 0, \tag{1}$$

where the first term captures the social cost originated by entrants' risk aversion and the second term measures the marginal cost savings that would have accrued under the efficient investment decision.

This result suggests that social welfare could be enhanced if sellers were sheltered from spot price fluctuations through fixed-price contracts. The next section explores the hedging opportunities these contracts provide.

[7]In a subgame perfect equilibrium, there can never be enough entry to fully cover demand. This is because the market price would drop to the entrants' (zero) marginal costs, preventing them from recovering their investment costs.

[8]The risk premium r could also be interpreted as the extra costs entrants have to pay to the lenders financing their investments.

3 Fixed-price Contracts

Suppose that buyers and sellers are allowed to sign a fixed-price contract prior to entry, enabling them to hedge their spot market transactions. In particular, the contract requires the seller to compensate the buyer for the difference between the spot price p and the contract price f if $p > f$, and vice versa if $f > p$. When both parties always honor the contract, the fixed-price contract insulates sellers from price uncertainty, allowing them to save the risk premium r.[9]

We assume that engaging in a fixed-price contract is costless for sellers but entails a heterogeneous transaction cost for buyers. In particular, a proportion $\theta \leq 1$ of buyers incur no cost, while the remaining proportion $1 - \theta$ face a transaction cost sufficiently high that they are never willing to participate.[10] To keep the discussion interesting, we focus on situations where demand for contracts exceeds the mass of sellers that would be willing to invest without contracts, i.e., $\theta > G(c_0) = G(E(p) - r)$.

At the contracting stage, the profits for buyers (B) and sellers (S) from hedging through a contract with a fixed price f become:

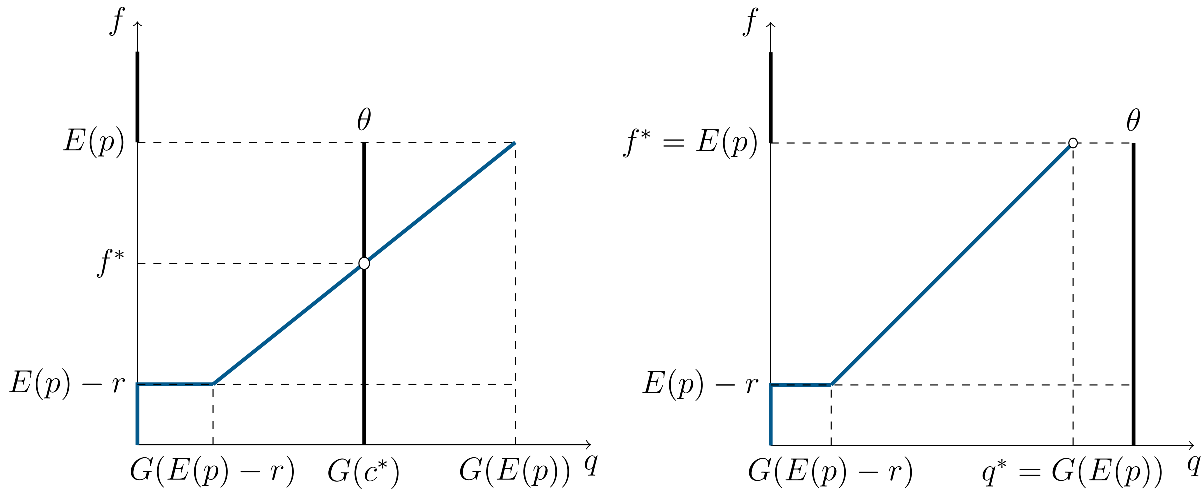
$$\Pi_B(f) = v - f,$$
$$\Pi_S(f; c) = f - c.$$

Buyers and sellers are willing to sign a fixed-price contract as long as $\Pi_B(f) \geq \Pi_B^0$ and $\Pi_S(f; c) \geq \Pi_S^0$, respectively. It follows that the demand for contracts is θ for prices $f \leq E[p]$, and zero otherwise, while the supply of contracts is given by the mass of entrants that can break even at each contract price, $G(f)$, for $f \in [E[p] - r, 1]$, and zero otherwise. It follows that, as $r > 0$, there is scope for contracting between buyers and sellers.

The equilibrium in the contract market is determined by the interplay between demand and supply. Two cases must be considered. First, if demand for contracts is low, i.e. $\theta \leq G(E[p])$, all demand is satisfied, $q^* = \theta$, at the market-clearing price $f^* = G^{-1}(\theta)$ (Figure 2a). Contracts make both buyers and sellers better off, allowing for investments

[9] For instance, in the context of power purchase agreements (PPA), the World Bank explicitly claims that "the structure and risk allocation regime under the PPA is central to the private sector participants ability to raise finance for the project, recover its capital costs and earn a return on equity."

[10] In practice, this partition might reflect differences related to the size of buyers, leading to lower average participation costs for larger buyers. In section 3.2, we show that differences in the costs of pledging collateral might have similar consequences.



(a) The no contract rationing case **(b)** The efficient contract rationing case

Figure 2: Market Clearing with a fixed-priced contract. In subfigure (a), the equilibrium price f^* is given by the cost of the marginal investor c^*. In subfigure (b), the equilibrium price is given by the highest price buyers are willing to pay, $E[p]$. There is contract rationing, but this is efficient given as contribution to welfare of the marginal investor $c^* = E[p]$ is zero.

that would not have occurred otherwise. Relative to the First Best, the only inefficiency stems from contract demand being inefficiently low, preventing cost-saving investments from taking place:

$$W^{FB} - W^* = \int_{G^{-1}(\theta)}^{E[p]} [E(p) - c]\, g(c)\, dc > 0. \tag{2}$$

Hence, an increase in contract demand θ up to $G(E[p])$ would allow to increase social welfare.

Second, when demand for contracts is high, $\theta > G(E[p])$, there is contract rationing, with only a fraction $q^* = G(E[p])$ of contract demand being served at the highest possible equilibrium contract price, $f^* = E[p]$ (Figure 2b). Importantly, contract rationing is efficient as further investment would involve investment costs c exceeding the marginal cost savings $E[p]$. Since the contract solution achieves the First Best, $W^* = W^{FB}$, the contribution of contracts to welfare is also given by (1).

These results are summarized in our following Proposition:

Proposition 1. *Assume no counterparty risk:*

(i) Without fixed-price contracts, there is equilibrium under-investment due to sellers' risk exposure: the marginal entrant has costs $c^* = E[p] - r < E[p]$.

(ii) *With fixed-price contracts, sellers' risk premia are eliminated, leading to efficient investment if contract demand is sufficiently high: for* $\theta \geq G(E[p])$, *the marginal entrant has costs* $c^* = E[p]$.

3.1 Buyer's Counterparty Risk

So far, we have assumed that buyers always honor the contract. However, if the spot market price p falls below the fixed price f, the buyer may be tempted to default on the contract,[11] introducing counterparty risk. As a result, fixed-price contracts only protect sellers from price uncertainty when the spot market price p is above the fixed price f. Sellers therefore face a risk premium $r\Phi(f)$, which increases with the contract price, as a higher price makes default more likely.

Expected profits for the buyers and sellers at the contracting stage can now be computed as

$$\Pi_B(f) = v - \int_0^f p\phi(p)dp - f(1 - \Phi(f)),$$
$$\Pi_S(f; c) = \int_0^f p\phi(p)dp + f(1 - \Phi(f)) - r\Phi(f) - c.$$

Buyers' expected profits are always decreasing in f. In contrast, under the hazard rate assumption, sellers' expected profits are quasiconcave in f (Figure 3). The quasiconcavity arises from the trade-off between higher revenue for the seller when f increases and the contract is honored and a higher probability of default due to counterparty risk. A price $\bar{f} < 1$ maximizes sellers' profits by striking a balance between these two forces. It follows that sellers never find it optimal to sign contracts at prices above $\bar{f}$.

Lemma 1. *The fixed price that maximizes sellers' profits,* $\bar{f}$, *is lower than* 1 *and it is decreasing in the sellers' risk premium* r.

The fixed-price contract must also yield higher utility than the spot market for both buyers and sellers to be willing to sign it. For buyers, the result is straightforward as they always have the option to default on the contract to avoid paying a higher price. Thus, $\Pi_B(f) \geq \Pi_B^0$ for all f.

[11] We abstract from the possibility that the seller may act opportunistically. In practice, financiers typically require sellers to have a fixed-price contract to obtain funding, which discourages them from later breaching the contract.

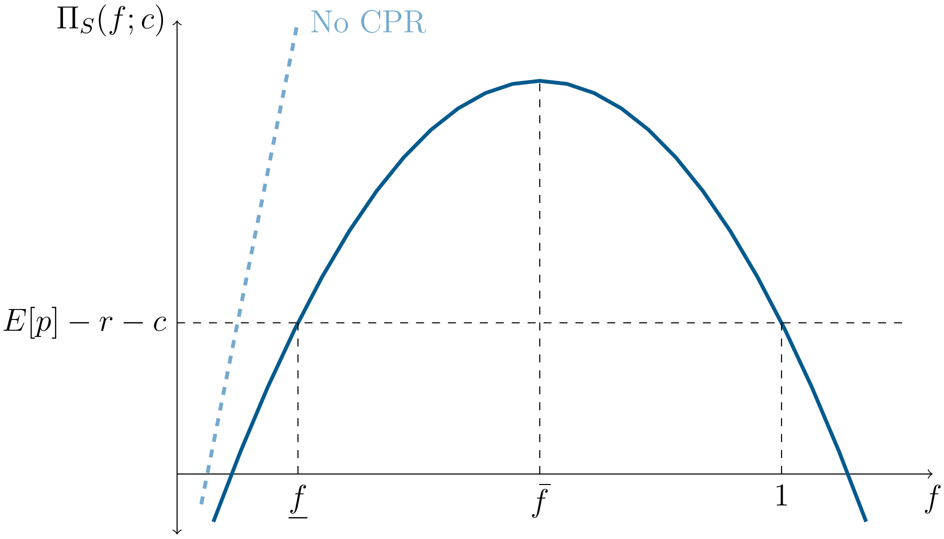


Figure 3: Seller's profits under a fixed-price contract as a function of f.

For sellers, the contract price must be above a certain threshold $\underline{f}$ to make it profitable relative to the spot market. Therefore, $\Pi_S(\underline{f}, c) = \Pi_S^0 = E[p] - r - c$. In turn, since the buyer can always guarantee to pay less than $E(p)$, sellers' profits can never exceed $E(p) - c$. As it will be shown later, this gap is important for the welfare analysis, as it implies that sellers can never fully capture the social value of their investments.

Lemma 2. *With counterparty risk and $r > 0$:*

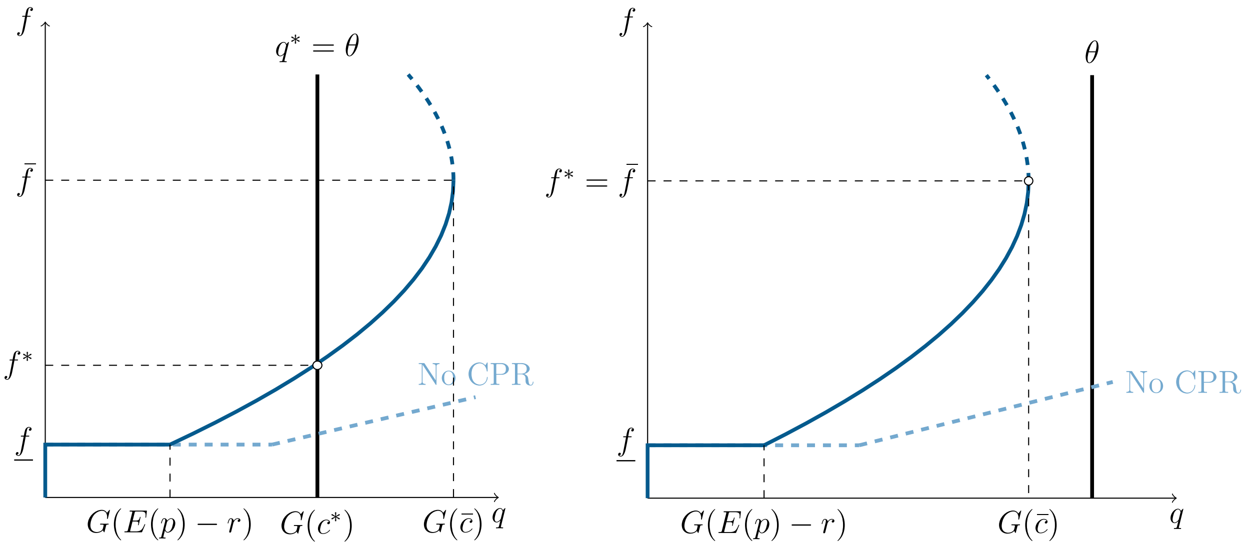
(i) Equilibrium contract prices f^* must be in the interval $[\underline{f}, \bar{f}]$, with

$$E[p] - r < \underline{f} < \bar{f} < 1.$$

(ii) Equilibrium profits $\Pi_B(f^*)$ must be in the interval $[E[p] - r - c, E[p] - c)$.

The intersection between the demand and supply for contracts determines the equilibrium price in the contract market. On the demand side, since buyers are always willing to sign contracts regardless of their price, the demand for contracts is θ.

On the supply side, sellers must be willing to sign the contract and carry out the corresponding investment. We can construct the supply for contracts by considering three cases as a function of the investment cost. First, if $\Pi_S^0 \geq 0$, i.e., if $c \leq E[p] - r$, the seller always invests regardless of whether a contract is signed or not. In this case, the seller accepts the contract as long as it is at least as profitable as the spot market, i.e., if $f \geq \underline{f}$.



(a) The no contract rationing case **(b)** The inefficient contract rationing case

Figure 4: Market clearing in the fixed-price contract market. In subfigure (a), the equilibrium price is given by the the break-even price of the marginal investor, with investment cost c^*. In subfigure (b), demand θ is above the mass of sellers $G(\bar{c})$ that can break even at that price. Contract rationing implies inefficient investment given that $\bar{c} < E[p]$. Without counterparty risk, the supply curve would be the dashed line.

Otherwise, the seller invests only if the contract price allows to recover the investment cost, which requires signing a contract at or above the investors' break-even price $\tilde{f}(c)$, implicitly defined by

$$\Pi_S(\tilde{f}; c) = 0. \tag{3}$$

Finally, if entry is not profitable at the profit-maximizing price, $\Pi_S(\bar{f}, c) < 0$, the entrant never invests as its investment cost is too high to break even. We use $\bar{c}$ to denote the highest investment cost for which break-even is possible, i.e., $\Pi_S(\bar{f}, \bar{c}) = 0$.

Figures 4a and 4b illustrate the supply function for contracts, which is weakly increasing in the contract price f. For prices above $\underline{f}$, the supply of contracts is constructed by mirroring the sellers' profit function in Figure 3. This curve bends backwards for $f > \bar{f}$ since, as stated in Lemma 1, sellers' profits decrease as the contract price increases beyond that point. Given that the contract price never exceeds the seller's profit-maximizing price $\bar{f}$, that region of the supply curve can be ignored.

These two figures capture the two cases that can emerge depending on whether there is contract rationing in equilibrium or not. Figure 4a corresponds to a situation where the demand for contracts is low, $\theta \leq G(\bar{c})$, and it is fully satisfied in equilibrium, $q^* = G(c^*) = \theta$. The market clearing price is determined by the break-even price of the marginal entrant, $\tilde{f}(c^*)$, as defined in (12). For higher demand levels, there is contract rationing in equilibrium, $q^* < \theta$, giving rise to the highest possible contract price, $f^* = \bar{f}$. Since the marginal entrant breaks even at that price, the equilibrium contract quantity is $q^* = G(\bar{c}) < \theta$. This case is illustrated in Figure 4b.

As expected, the equilibrium contract price increases with contract demand, particularly when there is no contract rationing. The equilibrium price is also strictly monotonic in the risk premium, as an increase in r reduces sellers' profits, thereby lowering their supply.

In the no contract-rationing case, equilibrium prices are higher than in the absence of counterparty risk. In the contract-rationing case, counterparty risk gives rise to both a higher contract price and a lower contract quantity. Importantly, in the presence of counterparty risk, contract rationing becomes inefficient as it leads to under-investment with respect to the First Best, adding to the inefficiency caused by price risk exposure:

$$W^{FB} - W^* = r\Phi\left(f^*\right) G(c^*) + \int_{c^*}^{E(p)} \left(E(p) - c\right) g\left(c\right) dc.$$

Regarding the contribution of contracts to total welfare, fixed-price contracts increase welfare relative to the no-contracts case. Social welfare improvements stem from the reduction in sellers' risk exposure and from new investments allowing for positive cost savings. Formally, the contribution of contracts to social welfare is

$$W^* - W^0 = (1 - \Phi(f^*))rG(E[p] - r) + \int_{E[p]-r}^{c^*} \left[E(p) - \Phi(f^*)r - c\right] g(c)\, dc. \qquad (4)$$

Counterparty risk reduces the contribution of fixed-price contracts to welfare, as sellers incur risk premia in case of default, also resulting in under-investment. Importantly, the resulting inefficiencies affect both the marginal entrant and the inframarginal ones, with costs $c < c^*$. At the margin, since entrants cannot fully capture the value of their investments, they under-invest. Additionally, the need to raise the equilibrium contract price above the investment cost of the marginal entrant increases the default probability of inframarginal contracts. This issue does not arise in the absence of counterparty risk, as contract prices would only affect the division of surplus between buyers and sellers, without impacting efficiency.

These results are summarized in our second Proposition:

Proposition 2. *With counterparty risk and* $r > 0$, *(i) Fixed-price contracts increase welfare relative to the no-contracts case, reducing sellers' risk exposure and under-investment.*

(ii) With fixed-price contracts, sellers' risk premia and underinvestment are not fully eliminated, implying lower welfare than under the First Best.

This result highlights buyers' counterparty risk as a source of inefficiency. It stands to reason that measures aimed at reducing counterparty risk should increase contract liquidity and reduce under-investment. We analyze this claim next.

3.2 Adding Collateral

Our previous analysis assumed that contract default was costless for the buyer. While this is a useful simplification, contracts usually include provisions that penalize the buyer in case of default. The buyer pledges a collateral $k > 0$, which is forfeited and transferred to the seller in case of default.

It is straightforward to see that a large collateral (e.g., $k = 1$) completely eliminates counterparty risk. However, such a large collateral is uncommon in practice due to the financial burden it imposes on the buyer. To account for this, let us now assume that collateral is costly, with a per-unit cost, ρ, which is heterogeneous among buyers according to a uniform distribution in $[0, 1]$. Different values of ρ might reflect heterogeneity in the cost of financing the collateral among buyers, which in turn could capture differences in the buyers' trustworthiness.[12]

The value of the collateral affects the profits of buyers and sellers', thus changing their optimal decisions. If the collateral exceeds the contract price, $k \geq f$, the buyer never finds it optimal to default. Hence, the utilities of buyers and sellers simplify to:

$$\Pi_B(f, k; \rho) = v - f - \rho k,$$
$$\Pi_S(f, k; c) = f - c.$$

Lower collateral, $k < f$, while reducing the probability of default, does not fully eliminate it as the buyer finds it optimal to default when $p < f - k$. In that case,

[12]For instance, in electricity markets, the main determinants of this heterogeneity are firm size and firm leverage. The cost of pledging collateral is much smaller for large technological companies and large utilities compared to small buyers.

expected profits become

$$\Pi_B(f,k;\rho) = v - f\,(1-\Phi(f-k)) - \int_0^{f-k} (p+k)\phi(p)\,dp - \rho k, \tag{5}$$

$$\Pi_S(f,k;c) = \int_0^{f-k} (p+k)\phi(p)\,dp + f\,(1-\Phi(f-k)) - r\Phi(f-k) - c. \tag{6}$$

Collateral affects the range of prices at which buyers and sellers are willing to trade. Consider sellers first. Since their profits increase with k, participation in the contract becomes more profitable the more collateral is pledged. As a result, the new minimum price the seller is willing to accept, now denoted $\underline{f}_S(k)$, decreases in k. Additionally, since default is less likely, the seller can profitably demand a higher price. Hence, the seller's profit-maximizing price, $\bar{f}_S(k)$, increases with k. Therefore, more collateral enlarges the range of prices at which sellers are willing to sign a fixed-price contract as well as the mass of entrants that can profitably enter the market.

This stands in contrast with the effect of collateral on buyers. Their participation constraint when $k \geq f$ is now given by

$$\Pi_B(f,k;\rho) - \Pi_B^0 = E(p) - f - \rho k \geq 0, \tag{7}$$

whereas when $k < f$, buyers' participation is driven by

$$\Pi_B(f,k;\rho) - \Pi_B^0 = \int_{f-k}^{1} (p-f)\phi(p)\,dp - k\Phi(f-k) - \rho k \geq 0. \tag{8}$$

This means that buyers no longer accept contracts regardless of their price, and the maximum price they are willing to pay, denoted $\bar{f}_B(k;\rho)$, is decreasing in the collateral requirement k.

These results are summarized below:

Lemma 3. *Assume there is counterparty risk, $r > 0$, and collateral k:*

(i) The lowest contract price sellers are willing to accept, $\underline{f}_S(k)$, decreases in k, while the sellers' profit-maximizing price, $\bar{f}_S(k)$, increases in k.

(ii) The highest contract price a buyer with collateral cost ρ is willing to accept, $\bar{f}_B(k;\rho)$ decreases in ρ and k, ranging from $E[p] - \rho$ for $k = 1$ to 1 for $k = 0$.

The heterogeneity of ρ between 0 and 1 implies that there is always some scope for trade. A buyer with zero collateral cost is willing to accept any contract with a fixed-price above $E[p]$, which is below the minimum price that a seller would be willing to accept, $\underline{f}_S(k) \leq E[p] - r$. However, buyers with high collateral costs could be excluded from the market as the maximum price they are willing to pay might be below the minimum acceptable price for a seller, i.e. $\bar{f}_B(k;\rho) < \underline{f}_S(k)$ for high ρ.

The demand curve for contracts with collateral k and a fixed-price f is given by the mass of buyers with $\rho \leq \hat{\rho}(f,k)$, a threshold that is implicitly defined by $\Pi_B(f,k;\hat{\rho}) = \Pi_B^0$. Since collateral costs are uniformly distributed and a mass θ of potential buyers are willing to take a fixed-price contract, $\theta\hat{\rho}(f,k)$ represents the demand curve. As expected, demand for contracts is now decreasing in f and k.

As in the baseline case, the supply function corresponds to the mass of sellers that can break even at each price. However, since sellers' profits increase with k, more collateral shifts the supply curve outward.

For high θ, the demand for contracts exceeds the maximum amount of contracts that sellers can profitably sell, i.e., if $\theta\hat{\rho}(\bar{f},k) > G(\bar{c}(k))$. Therefore, the equilibrium necessarily entails contract rationing. In this case, an increase in collateral k increases the supply of contracts by allowing sellers to profitably raise their price (Lemma 3).

For lower θ, the solution is interior, with market clearing determined by the intersection between demand and supply,

$$\theta\hat{\rho}(f^*,k) = G(c^*), \tag{9}$$

where, as before, c^* is related to f^* through the zero-profit condition $\Pi_S(f^*,k;c^*) = 0$. This solution is depicted in Figure 5.

Since higher k shifts the supply curve out and the demand curve in, the equilibrium contract price is decreasing in k. Hence, starting from an equilibrium with high f^* and low k such that $f^* > k$, an increase in k reduces f^* until $f^* = \hat{k}$, at which point the probability of default becomes zero. Further increases in k lead to $f^* < k$, ensuring that the probability of default remains zero.

Lemma 4. *There exists a unique $\hat{k}$ such that $f^*(k) \leq k$, implying $\Phi(f^* - k) = 0$, if and only if $k \geq \hat{k}$. If $r \leq E(p) - \hat{k}$, eliminating counterparty risk is not feasible.*

Hence, setting $k = \hat{k}$ is sufficient to fully eliminate the probability of default. However, this is not feasible for low r. Intuitively, asking for a high collateral reduces the demand for contracts, pushing contract prices down. Since $\hat{k}$ does not depend on r, for r low

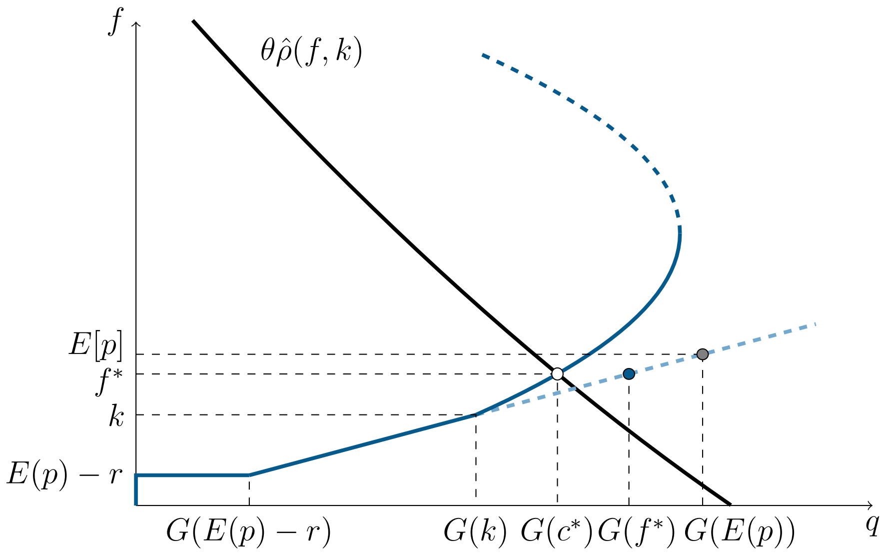


Figure 5: Market clearing when buyers pledge collateral. Contract demand is downward sloping because of the cost of collateral. Demand and supply intersect at $f^* > k$, so there is a positive probability of default. Without counterparty risk, the equilibrium would be at $(f^*, G(f^*))$ (blue dot). With costless collateral, the equilibrium would be at $(E[p], G(E[p])$ (grey dot).

enough, the equilibrium contract price would fall below the minimum price that makes sellers indifferent between hedging through contracts or not. Likewise, as shown in the Proposition below, even if setting $k = \hat{k}$ is feasible, sellers may be better off with lower collateral requirements that avoid the price reduction effect.

From a social welfare perspective, setting $k = \hat{k}$ need not be optimal either. The net welfare effect of increasing collateral depends on the costs associated with counterparty risk and the collateral itself. When r is low, the costs of counterparty risk are relatively small compared to the costs of collateral. Therefore, if the risk premium is sufficiently low, it is welfare optimal not to raise the collateral to a point where counterparty risk is entirely eliminated.

Proposition 3. *Assume there is counterparty risk, with $r > 0$ and collateral $k > 0$:*

(i) There exists a unique r_S such that sellers' profits are higher at some $k < \hat{k}$ than at $k = \hat{k}$ if and only if $r < r_S$.

(ii) There exists $r_W < r_S$ such that social welfare is higher at some $k < \hat{k}$ than at $k = \hat{k}$ if and only if $r < r_W$.

In general, relative to the First Best, the welfare loss when setting $k < \hat{k}$ is given by

$$W^{FB} - W^{*} = G\left(c^{*}\right)\Phi(f^{*} - k)r + \int_{c^{*}}^{E(p)} \left(E(p) - c\right)\phi(p)\,dp + \theta\frac{\hat{\rho}(f^{*})^{2}}{2}k. \qquad (10)$$

The costs of counterparty risk are captured by the first and second terms, representing the costs for sellers and the costs in terms of underinvestment, respectively.[13] The last term captures the costs of the collateral for the buyers.

Hence, while setting $k = \hat{k}$ eliminates the costs of counterparty risk, it does not achieve the First Best due to the cost of the collateral. Specifically, relative to the First Best, the welfare loss of setting $k = \hat{k}$ is $\theta\hat{k}^{3}/2$. This loss can be significant if $\hat{k}$ is large, such as when $E[p]$ is high.[14]

Corollary 1. *At the value of the collateral k that maximizes social welfare, total welfare is higher than without collateral but lower than under the First-Best.*

In sum, adding costly collateral does not eliminate the market failures associated with counterparty risk. Even when the optimal collateral eliminates the probability of default, the cost of collateral remains, leading to reduced demand and underinvestment. Furthermore, when the risk premium is sufficiently low, the optimal collateral also involves a positive probability of contract default, exposing sellers to costly risk. The resulting inefficiencies open the door to welfare-improving market interventions, as we discuss next.

4 Market Interventions

In this section, we consider several market interventions aimed at addressing the market failures uncovered in the previous section. For simplicity, we draw our analysis on the case where no collateral is possible, $k = 0$, in the understanding that the results would remain qualitatively unchanged under the optimal collateral as long as r is sufficiently low so that some meaningful counterparty risk would still exist.

[13] The costs of underinvestment can be further decomposed: without counterparty risk, the marginal investor would have had an investment cost $f^{*} > c^{*}$, and with costless collateral, the cost of the marginal investor would have shifted from f^{*} to $E[p]$. This decomposition can be observed in Figure 5, where the solid dots give the allocation without counterparty risk and with costless collateral.

[14] Note that *hatk* only depends on $E[p]$ and θ. In particular, it is the same for all mean-preserving price distributions, and it does not depend on the risk premium r.

4.1 Promoting Contract Demand

Our previous analysis revealed that weak demand for fixed-price contracts is one of the sources of under-investment. Specifically, cases with $\theta < G(E[p])$ result in inefficient investment, even in the absence of counterparty risk. Contract demand could grow if participation costs went down (e.g., though contract standardization)[15] or if contracting became compulsory for a larger fraction of buyers.[16] Disregarding potential changes in transaction costs, would increasing θ alleviate the market failures?

First, note that an increase in contract demand is ineffective when $\theta \geq G(\bar{c})$, as contract rationing occurs regardless. For lower values of θ, increasing contract demand boosts investment but also raises the equilibrium contract price, thereby increasing the default probability of all inframarginal contracts. As the following result shows, when θ is sufficiently high, welfare decreases with the demand for contracts.

Proposition 4. *There exists $\hat{\theta} < G(\bar{c})$ such that increasing contract demand reduces welfare for all $\theta \in [\hat{\theta}, G(\bar{c})]$.*

To interpret this result, it is useful to consider the case where θ is slightly below $G(\bar{c})$. In that situation, since the supply of contracts is perfectly inelastic, a small increase in demand significantly increases the equilibrium contract price. The resulting increase in the default probability for all inframarginal contracts outweighs the benefits of minimally increasing investment. This result illustrates a trade-off when considering demand mandates, as they increase investments at the cost of also increasing counterparty risk.

4.2 Regulator-Backed Contracts

An alternative way to address the market failures caused by buyers' counterparty risk is for the regulator to become the buyer itself by demanding fixed-price contracts, which are then passed on to the final buyers. Since the regulator has the authority to enforce payment of the contract price even if spot prices fall below it, counterparty

[15] In the context of electricity markets, contract standardization is often advocated as a way to promote long-term contracting. In line with this, the European energy regulator ACER is currently exploring whether "standardised PPAs will foster the transparency, efficiency and integration of the European internal energy market".

[16] An example of the contracting requirement is providing by the US, where utilities are required to have a certain share of total electricity generation from renewable energy - the so-called Renewable Portfolio Standards (RPS). This policy has become an important incentive for utilities to procure renewable energy through long-.term contracts.

risk is eliminated. For reasons outside of this model, we assume that the regulator cannot undersign more than θ^R contracts. In the demand side, the regulator allocates a proportion δ of the contracts to buyers that already participate in the fixed-price market. The rest goes to firms that in the benchmark model would not participate due to large transaction costs. The intervention of the regulator is assumed to eliminate this cost. We interpret δ as the extent of which regulator-backed contracts crowd out demand in the fixed-price market.[17]

Let f^R denote the fixed price that the regulator sets for these contracts. A seller signing a contract with the regulator obtain profits $f^R - c$ and it will accept it if $f^R - c \geq \Pi_S(f, c)$; otherwise, hedging in the fixed-price contract market is preferred. Because the regulator-backed contract creates efficiency gains $\Phi(f)r$ by eliminating counterparty risk, the price f^R can also satisfy the participation constraint for buyers, $v - f^R \geq \Pi_B(f)$. Note that luring buyers into a regulator-backed contract requires that it's priced strictly below the equilibrium market price, $f^R < f^*$. Intuitively, as buyers do not benefit from default when p is low, they need to be offered a lower fixed price to participate. Sellers benefit from a lower risk premium even if they receive a lower price.

The price f^R could be, in principle, parameterized based on how the regulator is willing to apportion the rents from eliminating counterparty risk between buyers and sellers. However, due to the inelastic demand, the socially optimal f^R is the highest price acceptable to the buyer,

$$f^R(f) = v - \Pi_B(f),$$

increasing in f. The reason is that a higher regulated price increases rents for sellers which expands supply. To simplify the discussion in the rest of this section we focus on this case.

Under a regulator-backed contract with a price f^R that is acceptable to both sellers and buyers, the former would have to be rationed. Let us assume that rationing is proportional. Therefore, the expected profits of a seller become

$$\Pi_S^R(f, \beta; c) = \beta(f^R - c) + (1-\beta)\Pi_S(f; c) = \int_0^f p\phi(p)dp + f(1-\Phi(f)) - (1-\beta)r\Phi(f) - c, \tag{11}$$

where β is the equilibrium probability that a seller has access to a regulator-backed contract. This means that the seller only incurs in the risk premium when trading in the

[17] The value $\delta = \theta$ corresponds to the case in which the regulator assigns contracts randomly.

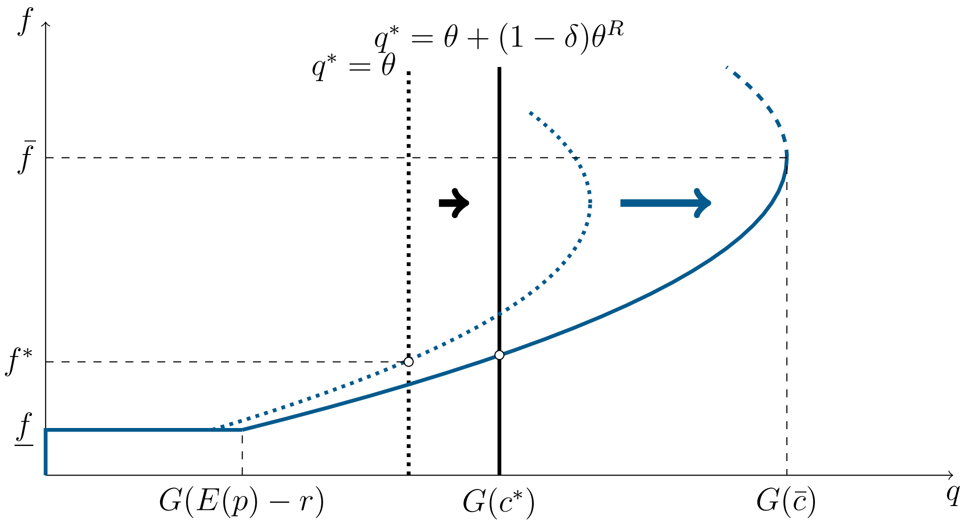


Figure 6: The effect of regulator-based contracts. The dotted lines depict supply and demand in the benchmark case. The solid lines indicate that as a result of these contracts, demand expands and supply, by increasing seller profits, also grows.

fixed-price contract market. Consider the case where, in equilibrium, entry equates the total demand for contracts, $q^* = \theta + (1-\delta)\theta^R$. Under proportional rationing, this implies that the probability becomes $\beta = \frac{\delta\theta^R}{\theta}$, as $\delta\theta^R$ contracts are allocated among the buyers that would participate even without the regulator-backed contract, θ.

In this case, equilibrium entry is determined by $G(c^*) = \theta + (1-\delta)\theta^R$ where $\Pi_S^R(f, \beta; c^*) = 0$. Notice that, for a given f, seller profits are increasing in δ as more access to regulator-backed contracts mitigates the resulting counterparty risk. However, because total demand is decreasing in δ it also means that the price f decreases, making the effects of a higher crowding out in principle ambiguous. The next result shows that the second effect dominates.

Lemma 5. *A higher δ reduces investment.*

This result has important implications. It is sometimes assumed that the role of regulator-backed contracts is of a substitute to market-based fixed-term contracts. Instead, our results indicate that they are most useful as complements. To the extent that they help in removing the obstacles that affect the demand for long-term contracts, the market expansion that regulator-based contracts engender is more effective in fostering investment. At the same time, by reducing the price in the market they still help in reducing counterparty risk compared to the situation where demand exogenously expanded.

4.3 Public Subsidies

Counterparty risk creates risk premia and prevents sellers from capturing the social value of their investments, causing underinvestment. Here we study the effects of subsidized entry as a way to mitigate this inefficiency.

Importantly, unconditional subsidies, i.e., those paid to all investors regardless of whether they sign a fixed-price contract, are ineffective in boosting fixed-price contracts as they do not affect sellers' participation constraints.[18] Therefore, we focus on uniform subsidies $T \geq 0$ paid to sellers by signing a fixed-price contract. In the tradition of the market-regulation literature, we assume that this subsidy implies a per-unit social cost of funds $\lambda \geq 0$.

Subsidies affect the supply of contracts through two channels. First, the sellers' participation constraint becomes

$$\Pi_S(f; c) + T \geq \Pi_S^0 = E[p] - r - c.$$

Hence, the minimum contract price that sellers are willing to accept, $\underline{f}$, is decreasing in T. Furthermore, since supply is expanded as more sellers can break even at every contract price f, the market-clearing price (weakly) goes down. This price is now implicitly defined by the solution to the new break-even constraint for the marginal investor,

$$\Pi_S(\tilde{f}(c^*); c^*) + T = 0. \tag{12}$$

if $c^* < \bar{c}$, or to the profit-maximizing price $\bar{f}$ otherwise. Note that while $\bar{f}$ remains unchanged with or without subsidies, the mass of sellers that can now profitably enter the market at that price shifts outward. Therefore, total investment increases with T.

In sum, subsidies (weakly) increase investment and (weakly) reduce contract prices, mitigating the counterparty risk of all inframarginal contracts. Nevertheless, these positive welfare impacts must be weighed against the social costs of the subsidies.

Formally, the regulator chooses T to maximize the contribution of contracts to social welfare, considering how subsidies affect the equilibrium price $f^*(T)$, minus the social costs of the subsidies. The regulator's problem is

$$\max_T \left[(1 - \Phi(f^*))rG(E(p) - r) + \int_{E(p)-r}^{c^*} [E(p) - \Phi(f^*)r - c]g(c)\, dc - \lambda G(c^*)T \right].$$

[18]Unconditional subsidies are pervasive. For instance, in the US, renewable producers receive a Production Tax Credit (PTC) per unit of renewable output produced or investment subsidies, regardless of whether output is backed by long-term contracts (Aldy et al., 2023; Chen, 2024). While these subsidies promote investments, they do not address counterparty risk.

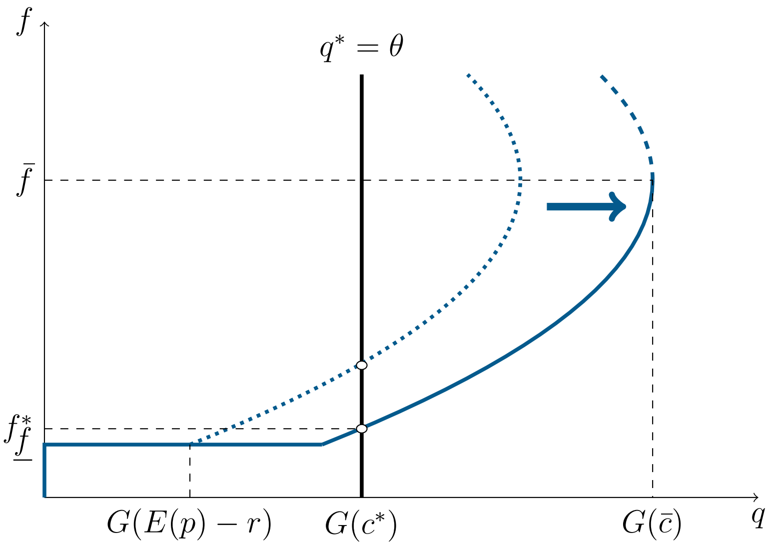


Figure 7: The Effect of Public Subsidies.

The optimal solution reflects a *rent-investment-counterparty risk* trade-off. Raising T reduces the equilibrium contract price, thereby reducing counterparty risk for all contracts and enabling new cost-saving investments. However, raising T also engenders a social cost due to the usage of public funds, implying that the subsidy should be decreasing in λ.

The optimal subsidy has corner solutions for extreme values of λ. If λ is sufficiently close to 0, social welfare always increases in T up to the point where $f^* = 0$, fully eliminating counterparty risk and achieving efficient investments. Therefore, it is optimal to fully subsidize sellers through a transfer $T^* = E[p]$ that makes the marginal seller with investment cost $E[p]$ willing to invest and accept the contract.

At the other extreme, when λ is sufficiently high, transfers are very costly and using them is always suboptimal, i.e., $T^* = 0$. As a result, the market solution prevails.

The previous results are summarized in the following proposition.

Proposition 5. *The optimal transfer T is decreasing in λ.*

4.4 Public Guarantees

Suppose now that, instead of offering a conditional subsidy, the regulator can provide public guarantees. These guarantees are designed to secure revenue f for the seller even if the buyer defaults on the contract. In other words, public guarantees act as a payment to the seller that compensates for the revenue shortfall $f - p$ in the event of default. As in the previous case, the disbursement of public funds is subject to a social cost $\lambda \geq 0$.

Because the seller no longer faces counterparty risk, the seller's utility under the fixed-price contract becomes $\Pi_S(f) = f - c$, regardless of the value of $f \in [0, 1]$. The buyer's utility remains unchanged.

Suppose first that demand is low so that, without guarantees, there is no contract rationing, i.e. $\theta \leq G(\bar{c})$. The following condition determines whether public guarantees increase or reduce social welfare,

$$W^G - W^* = \theta \left[\Phi(f^*)r - \lambda \int_0^{f^G} (f^G - p)\phi(p)dp \right] \lesseqgtr 0,$$

where f^G is the equilibrium contract price that arises when public guarantees are in place. Since the seller's profits are now higher, the break-even competitive price is lower than under the market solution without guarantees for any given demand θ, $f^G < f^*$. Therefore, public guarantees involve a trade-off between eliminating counterparty risk for the seller, as captured by the first term in brackets, and incurring the social cost of the public funds, as captured by the second term in brackets. Clearly, for public guarantees to be optimal, λ must be sufficiently low.

Alternatively, for higher contract demand $\theta > G(\bar{c})$, public guarantees induce new investment because the seller can now profitably charge a price above $\bar{f}$ without facing counterparty risk. Interestingly, this case implies a new trade-off, as shown in the following expression:

$$W^G - W^* = G(\bar{c})\Phi(\bar{f})r - \theta\lambda \int_0^{f^G} (f^G - p)\phi(p)dp + \int_{\bar{c}}^{f^G} (E(p) - c)g(c)dc \lesseqgtr 0.$$

The first two terms correspond to the benefits of reducing counterparty risk at the expense of increasing the costs of public funds, as identified in the previous scenario. The main difference here is that, in this case, counterparty risk is eliminated for sellers who would have participated even without public guarantees, i.e., $G(\bar{c})$. However, the cost of public funds applies to the guarantees provided to all sellers θ, including those who would not have entered the market without the guarantees.

The last term is noteworthy as it captures the social value of new investment absent counterparty risk. However, if contract demand is sufficiently large, i.e., if $\theta > G(E[p])$, then $E(p) < c$ for some sellers, particularly the marginal entrant, $c = f^G$. This scenario implies that inefficient entry might occur, as the marginal cost savings induced by some of the new capacity fall below the investment cost. This outcome reflects a form of moral hazard, where sellers do not fully internalize the social costs of increasing f. In particular, the resulting high equilibrium contract price encourages excessive contract default, leading to socially inefficient investments.

[ADD A STATEMENT OF A PROPOSITION]

5 Concluding Remarks

This paper is one of the first attempts to model the markets for long-term power contracts. One of its main contributions is to study the consequences of counterparty risk by the buyer in combination with the presence of a spot market where buyers and sellers can trade and investment is endogenous. Our paper shows that inefficiencies arise both in investment and in excessive counterparty risk. Interventions aimed at fostering investment might exacerbate this cost.

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A Proofs

Proof of Lemma 1: We first show that the profit function of the seller is quasiconcave. Notice that

$$\frac{\partial \Pi_S}{\partial f}(f) = 1 - \Phi(f) - \phi(f) r.$$

The previous derivative is 0 when

$$\frac{\phi(f)}{1-\Phi(f)} = \frac{1}{r}.$$

This solution, which we denote as $\bar{f}$, is unique due to the hazard rate assumption. Moreover, it corresponds to an interior maximum since

$$\frac{\partial^2 \Pi_S}{\partial^2 f}(\bar{f}) = -\phi(\bar{f}) - \phi'(\bar{f}) r < 0,$$

where the inequality comes from the fact that, due to the increasing hazard rate of $\Phi(p)$ and the definition of $\bar{f}$,

$$\phi'(\bar{f}) > -\frac{\phi(\bar{f})^2}{1-\Phi(\bar{f})} = -\phi(\bar{f})\frac{1}{r}.$$

Notice that $\frac{\partial \Pi_S}{\partial f}(1) = -\phi(1) r < 0$ and $\frac{\partial \Pi_S}{\partial f}(0) = 1 - \phi(0) r > 0$ if $r < \frac{1}{\phi(0)}$, so that the profit function cannot be monotonically increasing or decreasing.

Finally, since $\frac{\partial \Pi_S}{\partial f \partial r} < 0$, using the Implicit Function Theorem we can establish that $\bar{f}$ and $\Pi_S(\bar{f})$ are decreasing in r. □

Proof of Lemma 2: First notice that when $r = 0$, $\Pi_S(f; c)$ is always increasing in f and equal to Π_S^0 if and only if $f = 1$. Hence, $\bar{f} = \underline{f} = 1$.

Suppose now that $r > 0$. From the FOC condition $\frac{\partial \Pi_S}{\partial f}(\bar{f}, c) = 0$ we can obtain, using the Implicit Function Theorem,

$$\frac{\partial \bar{f}}{\partial r} = -\frac{1}{1 + \frac{\phi'(\bar{f})}{\phi(\bar{f})} r} > -\frac{1}{1 - \frac{\phi(\bar{f})}{1-\Phi(\bar{f})} r}.$$

Using the definition $\Pi_S(\underline{f}, c) = \Pi_S^0$ we obtain, again using the Implicit Function Theorem, that

$$\frac{\partial \underline{f}}{\partial r} = -\frac{1}{1 - \frac{\phi(\underline{f})}{1-\Phi(\underline{f})} r}.$$

To show that $\bar{f} > \underline{f}$ first notice that, using the previous expressions, $\left.\frac{\partial \bar{f}}{\partial r}\right|_{r=0} > \left.\frac{\partial \underline{f}}{\partial r}\right|_{r=0}$, meaning that, by continuity, for values of r close to 0, $\bar{f} > \underline{f}$. Suppose now towards a contradiction that there exists a value of r' for which $\bar{f} < \underline{f}$. In that case, again by continuity, there exists a value $\tilde{r} < r'$ for which $\bar{f} = \underline{f}$ and for all $r \in (\tilde{r}, r']$ $\bar{f} < \underline{f}$. At $\tilde{r}$, however, $\left.\frac{\partial \bar{f}}{\partial r}\right|_{r=\tilde{r}} > \left.\frac{\partial \underline{f}}{\partial r}\right|_{r=\tilde{r}}$, which is a contradiction.

Finally, notice that

$$E[p] - r - c = \Pi_S^0 = \Pi_S(\underline{f}, c) < \underline{f} - c.$$

Likewise, notice that $\Pi_B(f) + \Pi_S(f; c) = v - r\Phi(f) - c$. Since the buyer can always guarantee to pay at most $E[p]$, then $\Pi_B(f) \geq v - E[p] - c$. Hence, it follows that $\Pi_S(f; c) < E[p] - c$. □

Proof of Proposition 2: Regarding part (i), the contribution of contracts to welfare relative to the no-contracts case is given in equation (4) as

$$W^* - W^0 = (1 - \Phi(f^*))rG(E[p] - r) + \int_{E[p]-r}^{c^*} [E(p) - \Phi(f^*)r - c]\, g(c)\, dc > 0.$$

The first term is positive. Regarding the second term, note that for the marginal seller obtains profits,

$$\begin{aligned}
\Pi_S(f^*; c^*) &= \int_0^{f^*} p\phi(p)\, dp + (1 - \Phi(f^*))\, f^* - r\Phi(f^*) - c^* \\
&= E(p) - \int_{f^*}^1 (p - f^*)\, \phi(p)\, dp - r\Phi(f^*) - c^* = 0.
\end{aligned}$$

Hence,

$$c^* = E(p) - \int_{f^*}^1 (p - f^*)\, \phi(p)\, dp - r\Phi(f^*) < E(p) - r\Phi(f^*).$$

As the term in the integral is decreasing in c, the second term is also positive.

With respect to (ii), the market inefficiency relative to the First Best solution is:

$$W^{FB} - W^* = r\Phi(f^*)\, G(c^*) + \int_{c^*}^{E(p)} (E(p) - c)\, g(c)\, dc > 0.$$

In equilibrium, there is underinvestment since $c^* \leq \bar{c} < E(p)$ and counterparty risk is not fully mitigated since $f^* \leq \bar{f} < 1$. □

Proof of Lemma 3: We will denote the highest and lowest prices sellers are willing to accept for a fixed-price contract with collateral k as $\bar{f}_S(k)$ and $\underline{f}_S(k)$, respectively. In turn, the highest price a buyer with cost of collateral ρ is willing to pay for a contract with collateral k is denoted $\bar{f}_B(k, \rho)$.

Regarding the seller, the lowest acceptable price, $\underline{f}_S(k)$, satisfies (7) with equality. Since

$$\frac{\partial \Pi_S(f, k; c)}{\partial k} = r\phi(f - k) + \Phi(f - k) > 0,$$

it follows that $\underline{f}_S(k)$ must be decreasing in k

The highest price the seller is willing to accept, $\bar{f}_S(k)$, is its profit maximizing price. Since

$$\frac{\partial \Pi_S(f, k; c)}{\partial f} = (1 - \Phi(f - k)) - \phi(f - k)\, r,$$

the first-order condition implies that

$$\frac{\phi\left(\bar{f}_S(k) - k\right)}{1 - \Phi\left(\bar{f}_S(k) - k\right)} = \frac{1}{r_S}.$$

The cross-derivative with respect to f and s is

$$\frac{\partial \Pi_S(f, k; c)}{\partial k \partial f} = r_S \phi'(f - k) + \phi(f - k),$$

which, evaluated at $\bar{f}_S$ and using the FOC can be re-written as

$$\frac{\partial \Pi_S(f, k; c)}{\partial k \partial f} = \frac{1 - \Phi\left(\bar{f}_S(k) - k\right)}{\phi\left(\bar{f}_S(k) - k\right)} \phi'\left(\bar{f}_S(k) - k\right) + \phi\left(\bar{f}_S(k) - k\right) > 0.$$

The sign makes use of the hazard rate condition, which implies that

$$\left(1 - \Phi\left(\bar{f}_S(k) - k\right)\right) \phi'\left(\bar{f}_S(k) - k\right) + \phi^2\left(\bar{f}_S(k) - k\right) > 0.$$

Hence, it follows that $\bar{f}_S(k)$ is increasing in k.

The highest price a buyer with cost of collateral ρ is willing to accept, $\bar{f}_B(k, \rho)$, satisfies

$$\Pi_B(\bar{f}_B(k, \rho), k; \rho) = v - E(p).$$

Since profits are decreasing in k and f, and the right-hand side is a constant, it follows that $\bar{f}_B(k, \rho)$ must be decreasing in k and ρ. For $k = 0$, we revert to the baseline model, with buyers accepting the contract regardless of the price, $\bar{f}_B(0, \rho) = 1$ for all ρ. For $k = 1$, which fully eliminates counterparty risk, $\Pi_B(\bar{f}(1, \rho), k; \rho) = v - f - \rho$. Hence, $\bar{f}_B(1, \rho) = E[p] - \rho$. □

Proof of Lemma 4: In a interior solution, defined as an outcome with positive counterparty risk, f^* is obtain from equation (9). Since $\hat{\rho}(f, k)$ is decreasing in f and k and c^* is increasing in k, this implies that $f^*(k)$ is strictly decreasing in k. As this function is continuous and $f^*(0) > 0 > f^*(1) - 1$, we have that there is a unique value of k, denoted as $\hat{k}$, such that $f(\hat{k}) = \hat{k}$.

For this contract to eliminate counterparty risk it must lead to $f^*(\hat{k}) = \hat{k} > \underline{f} = E(p) - r$. When this is not the case, eliminating counterparty risk is incompatible with sellers participating in the fixed-price contract.

□

Proof of Proposition 3: From Lemma 4, we only need to consider thresholds that exceed $E(p)-\hat{k}$. When the following thresholds do not meet this constraint, the relevant one is the maximum of both.

With respect to part (i), the derivative of the seller's profits in (6) with respect to k is

$$\frac{d\Pi_S\left(f^*,k;c\right)}{dk}=\left[\Phi\left(f^*-k\right)+r\phi\left(f^*-k\right)\right]+\left[1-\Phi\left(f^*-k\right)-r\phi\left(f^*-k\right)\right]\frac{df^*}{dk}.$$

which evaluated at $\hat{k}$, where $f^*=\widehat{k}$, simplifies to

$$\left.\frac{d\Pi_S\left(f^*,k;c\right)}{dk}\right|_{k=\widehat{k}}=r\phi\left(0\right)+\left(1-r\phi\left(0\right)\right)\frac{df^*}{dk}. \tag{13}$$

The first term is how much a higher collateral reduces the cost of default. The second one captures how much it reduces profits in the absence of default by lowering the equilibrium price.

To compute $\frac{df^*}{dk}$ we use the Implicit Function Theorem on the market clearing condition which can be written as

$$\Psi(f,k)\equiv kG(c^*)-\theta\left(\int_{f-k}^{1}(p-f)\phi(p)dp-k\Phi(f-k)\right)=0.$$

where $c^*=\int_0^{f-k}(p+k)\phi(p)dp+f(1-\Phi(f-k))-r\Phi(f-k)$. We can compute

$$\begin{aligned}\frac{d\Psi}{dk}&=G(c^*)+kg(c^*)\left[\Phi(f-k)+r\phi(f-k)\right]+\theta\Phi(f-k),\\ \frac{d\Psi}{df}&=kg(c^*)\left[1-\Phi(f-k)-r\phi(f-k)\right]+\theta\left(1-\Phi(f-k)\right).\end{aligned}$$

Evaluated at $k=\hat{k}=f^*\left(\hat{k}\right)$ we can compute

$$\left.\frac{df}{dk}\right|_{k=\hat{k}}=-\frac{\left.\frac{d\Psi}{dk}\right|_{k=\hat{k}}}{\left.\frac{d\Psi}{df}\right|_{k=\hat{k}}}=-\frac{G(\hat{k})+\hat{k}g(\hat{k})r\phi(0)}{\hat{k}g(\hat{k})(1-r\phi(0))+\theta}.$$

Replacing in (13), we obtain that eliminating counterparty risk decreases seller profits if and only if

$$r<r_S\equiv\frac{1}{\phi\left(0\right)}\frac{G(\hat{k})}{\theta+G(\hat{k})}.$$

Regarding part (ii), total welfare can be written as

$$W(k)=\int_0^{\Pi_S(f^*,k;c^*)}\Pi_S\left(f^*,k;c\right)g(c)dc+\theta\int_0^{\hat{\rho}}\left(\Pi_B(f^*,k;\rho)-\Pi_B^0\right)d\rho.$$

The derivative with respect to k evaluated at $f^*(\hat{k}) = \hat{k}$ becomes

$$W'(k) = G(c^*)\frac{d\Pi_S\left(f^*, k; c\right)}{dk} + \theta\hat{\rho}\left[-(1-\Phi(f-k))\frac{df}{dk} - \Phi(f-k) - \frac{\hat{\rho}}{2}\right]$$

where we are using the fact that $\Pi_S\left(f^*, k; c^*\right) = 0$ and $\Pi_B(f^*, k; \hat{\rho}) - \Pi_B^0 = 0$.

When we evaluate this derivative at $k = \hat{k}$ it becomes

$$W'(\hat{k}) = G(\hat{k})\left[\left.\frac{d\Pi_S\left(f^*, k; c\right)}{dk}\right|_{k=\hat{k}} - \left.\frac{df}{dk}\right|_{k=\hat{k}} - \frac{G(\hat{k})}{2\theta}\right].$$

Replacing from part (i) we obtain that the derivative is increasing in k if and only if

$$r < r_W = \frac{1}{\phi(0)}\frac{G(\hat{k})(\theta + g(\hat{k})\hat{k})}{2\theta^2 + 2g(\hat{k})\hat{k}\theta + 2G(\hat{k})\theta + G(\hat{k})g(\hat{k})\hat{k}}.$$

Furthermore,

$$r_S - r_W = \frac{1}{\phi(0)}\frac{G(\hat{k})\theta\left(\theta + g(\hat{k})\hat{k} + G(\hat{k})\right)}{(\theta + G(\hat{k}))(2\theta^2 + 2g(\hat{k})\hat{k}\theta + 2G(\hat{k})\theta + G(\hat{k})g(\hat{k})\hat{k})} > 0.$$

□

Proof of Proposition 4: Consider a demand $\theta \in (G(E(p) - r), G(\bar{c})]$. In that case, welfare could be written as

$$W = (1 - \Phi(f^*(\theta)))rG(E(p) - r) + \int_{E(p)-r}^{c^*} \left[E(p) - \Phi(f^*(\theta))r - c\right] g(c)dc,$$

where we explicitly indicate that the equilibrium price, $f^*(\theta)$, increases in θ.

The derivative concerning the demand can be obtained as

$$\frac{\partial W}{\partial \theta} = -r\phi(f^*(\theta))\theta\frac{\partial f^*}{\partial \theta} + \left[E(p) - c^* - \Phi(f^*(\theta))r\right] g(c^*).$$

When evaluated at $\theta = G(\bar{c})$, $\frac{\partial f^*}{\partial \theta} \to \infty$ since $f^* = \bar{f}$ at which the first-order condition of the seller's profit-maximizing problem is satisfied. As a result, $\left.\frac{\partial W}{\partial \theta}\right|_{\theta=G(\bar{c})} < 0$. By continuity, there exists $\theta^* < G(\bar{c})$ for which social welfare is also decreasing in θ. □

Proof of Lemma 5: Since $c^* = G^{-1}(\theta + (1-\delta)\theta^R)$ we have that for a given c,

$$\Pi_S^R(f, \beta; c) = c^* - c = G^{-1}(\theta + (1-\delta)\theta^R) - c$$

which is decreasing in δ. □

Proof of Proposition 5: TBW □.