

Redistribution & Unemployment Insurance

Antoine Ferey (Sciences Po)

August 2024

Motivation

Governments operate large redistribution and social insurance policies

- ▶ How to design income taxes and transfers?
- ▶ How to design unemployment insurance? disability insurance? pensions?

Policymakers and academics tend to separate these two questions ...

- ▶ Often separate laws and reforms, managed by separate entities
- ▶ Optimal tax (Mirrlees, 1971) vs. social insurance (Chetty, Finkelstein, 2013)

... and, at the same time, recognize they are somewhat intertwined

- ▶ E.g., social insurance policies are often redistributive
- ▶ *“One should ideally analyze tax and social insurance policies in a unified framework rather than optimizing each program [...] separately.”* (Chetty, Finkelstein, 2013)

This paper jointly analyzes redistribution and unemployment insurance

- ▶ How do tax-transfer and UI policies interact? What are the implications?

This paper



1. Derive Pareto-efficiency condition linking tax-transfer and UI benefits

- ▶ Extends Baily-Chetty formula for optimal UI in presence of redistribution
- ▶ Replacement rates decrease with earnings: 100% at origin, $\leq 0\%$ at top
- ▶ Replacement rates depend on redistribution: UI is a redistributive policy!

2. Characterize optimal tax-transfer schedule with UI benefits

- ▶ Extends canonical tax formulas in extensive-intensive margin models
- ▶ Additional effects really matter only when unemployment rates are high
- ▶ At low incomes, implications depend on search vs participation elasticities

3. Empirically assess policy implications in the US and in France

- ▶ US features decreasing replacement rates, consistent with theory 
- ▶ FR features stable replacement rates, room for Pareto-improvement 

Related literature

Optimal redistribution: Mirrlees (1971); Diamond (1980, 1998); Saez (2001, 2002); Choné, Laroque (2010, 2011); Jacquet, Lehmann, VdLinden (2013); Golosov, Tsyvinki, Werquin (2014); Sachs, Tsyvinski, Werquin (2019); Bierbrauer, Boyer, Hansen (2022)

Optimal UI: Baily (1978); Chetty (2006, 2008); Schmieder, Von Wachter (2016); Lawson (2017); Landais, Michailat, Saez (2018); Spinnewijn (2015, 2019); Landais, Spinnewijn (2021)

→ bridge between optimal redistribution and optimal UI literatures

Optimal redistribution with unemployment: Boone, Bovenberg (2004, 2006); [Hungerbühler], Lehmann, Parmentier, VdLinden ([2006], 2011); Boadway, Cuff (2018); Kroft, Kucko, Lehmann, Schmieder (2019); Hummel (2019); Da Costa, Maestri, Santos (2022)

Optimal UI with redistribution: Uren (2018); Setty, Yedid-Levi (2020); Haan, Prowse (2021)

→ joint design of nonlinear redistribution and nonlinear UI policies

Redistribution & social insurance, pooling: Rochet (1991); Cremer, Pestieau (1996); Boadway, Leite-Montero, Marchand, Pestieau (2006); Netzer, Scheuer (2007)

—, **dynamic:** Golosov, Tsyvinski (2006); Farhi, Werning (2013); Golosov, [Shourideh], Troshkin, Tsyvinski ([2013], 2016); Findeisen, Sachs (2016); Ndiaye (2020); Stantcheva (2020)

→ no pooling logic ; no inverse Euler equation nor absorbing state

Outline

1. Heuristic derivation of Pareto-efficiency
2. Extensions of Pareto-efficiency
3. Labor supply and optimal tax-transfer schedule
4. Empirical application

Heuristic derivation of Pareto-efficiency

Setting

Population of individuals with heterogeneous earnings when employed, z

- ▶ Spend fraction of time $e(z)$ employed and $1 - e(z)$ unemployed
- ▶ $u_e(c_e(z))$ and $u_u(c_u(z))$: consumption utilities
- ▶ $k(z)$ and $\psi(e(z), z)$: disutility from work and from job search

$$V(z) := e(z) \left[u_e(c_e(z)) - k(z) \right] + (1 - e(z)) \left[u_u(c_u(z)) - \psi(e(z), z) \right]$$

Simplifying assumptions

- ▶ No labor supply, earnings z are exogenous
- ▶ Job search decisions are utility-maximizing and interior, $0 < e(z) < 1$
- ▶ No heterogeneity within earnings levels and no risk, unique $e(z)$
- ▶ No savings, $c_e(z) = z - T_e(z)$, never expiring UI, $c_u(z) = B_u(z)$

$$\mathcal{R}(z) := e(z)T_e(z) - (1 - e(z))B_u(z)$$

Impact of tax-benefit reforms

Impact of tax-benefit reforms on individuals' utility

- ▶ Tax increase $dT_e(z)$ decreases utility
- ▶ Benefit increase $dB_u(z)$ increases utility
- ▶ Changes in job search $de(z)$ do not affect utility (envelope argument)

$$dV(z) = -e(z)u'_e(c_e(z)) dT_e(z) + (1 - e(z)) u'_u(c_u(z)) dB_u(z)$$

Impact of tax-benefit reforms on government's revenue

- ▶ Tax increase $dT_e(z)$ increases revenue
- ▶ Benefit increase $dB_u(z)$ decreases revenue
- ▶ Changes in job search $de(z)$ affect revenue

$$d\mathcal{R}(z) = e(z)dT_e(z) - (1 - e(z)) dB_u(z) + de(z) (T_e(z) + B_u(z))$$

Sufficient statistics

Job search semi-elasticities

- ▶ Measure changes in the time spent unemployed, $1 - e(z)$

$$\mu_e(z) := \frac{1}{1 - e(z)} \frac{\partial(1 - e(z))}{\partial T_e(z)} \quad \mu_u(z) := \frac{1}{1 - e(z)} \frac{\partial(1 - e(z))}{\partial B_u(z)}$$

- ▶ Given the structure of the baseline model,

$$\frac{\mu_e(z)}{u'(c_e(z))} = \frac{\mu_u(z)}{u'(c_u(z))}$$

- ▶ Empirical literature measures $\mu_u^{elast} = 0.5$ (Schmieder, von Wachter, 2016)

$$\mu_u^{elast}(z) := B_u(z)\mu_u(z)$$

Pareto-improving tax-benefit reforms

Can we increase the government's revenue without affecting welfare?

- ▶ A tax-benefit reform that leaves individuals' utility constant satisfies

$$dV(z) = 0 \iff dT_e(z) = \frac{(1 - e(z)) u'_u(c_u(z))}{e(z) u'_e(c_e(z))} \underbrace{dB_u(z)}_{=1}$$

- ▶ This tax-benefit reform leads to a change in time spent employed

$$de(z) = -(1 - e(z)) [\mu_e(z) dT_e(z) + \mu_u(z) dB_u(z)] = -\frac{1 - e(z)}{e(z)} \mu_u(z)$$

- ▶ This tax-benefit reform leads to a change government's revenue

$$d\mathcal{R}(z) = (1 - e(z)) \left[\frac{u'_u(c_u(z))}{u'_e(c_e(z))} - 1 - \frac{\mu_u(z)}{e(z)} (T_e(z) + B_u(z)) \right]$$

There exists a Pareto-improving reform if and only if $d\mathcal{R}(z) \neq 0$

Pareto-efficiency

Proposition 1. A Pareto-efficient tax-benefit system satisfies at each $z > 0$

$$\overbrace{\frac{u'_u(c_u(z))}{u'_e(c_e(z))} - 1}^{\text{insurance value}} = \overbrace{\frac{\mu_u^{\text{elast}}(z)}{e(z)^2}}^{\text{job search responses}} \left[1 + \overbrace{\frac{e(z)T_e(z) - (1 - e(z))B_u(z)}{B_u(z)}}^{\text{net contribution to tax-benefit system}} \right]$$

Redistribution & UI: heterogeneous z , net contribution broadly increasing z

- ▶ Fiscal externality of UI depends on redistribution → interaction!
- ▶ Leads to replacement rates, $\frac{c_u(z)}{c_e(z)}$, that decrease with earnings

Baily-Chetty formula for UI: representative z , net contribution is 0

- ▶ Fiscal externality of UI is independent of redistribution
- ▶ Leads to a unique replacement rate, $\frac{c_u}{c_e}$, constant across earnings

$$\overbrace{\frac{u'_u(c_u)}{u'_e(c_e)} - 1}^{\text{insurance value}} = \overbrace{\frac{\mu_u^{\text{elast}}}{e^2}}^{\text{job search responses}}$$

Policy implications

1. Optimal replacement rate at origin of the income distribution is 100%

Corollary. If (i) $u'_e(\cdot) = u'_u(\cdot)$, (ii) $T_e(z)$, $B_u(z)$ and $e(z)$ are continuous, and (iii) job search elasticity $\mu_u^{elast}(z)$ are locally constant at very low incomes, then

$$\lim_{z \rightarrow 0} B_u(z) = - \lim_{z \rightarrow 0} T_e(z).$$

2. Simple policy rule with log utility highlights the interaction!

Corollary. With logarithmic consumption utilities, $u_e(c) = u_u(c) = \log(c)$,

$$B_u(z) = \frac{e(z)}{e(z) + \mu_u^{elast}(z)} z - T_e(z).$$

Illustration with log utility and linear taxes

Parametric example with log utility and linear taxes

- ▶ Linear tax rate τ
- ▶ Lump-sum transfer R_0

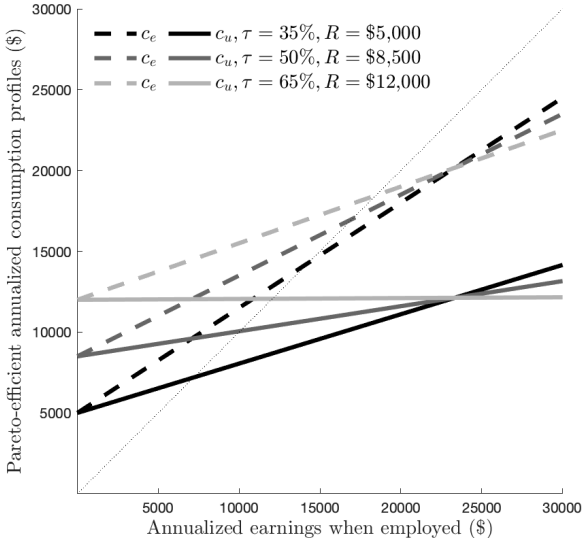
$$c_e(z) = z - T_e(z) = (1 - \tau)z + R_0$$

$$c_u(z) = B_u(z) = \left(\frac{e(z)}{e(z) + \mu_u^{elast}(z)} - \tau \right) z + R_0$$

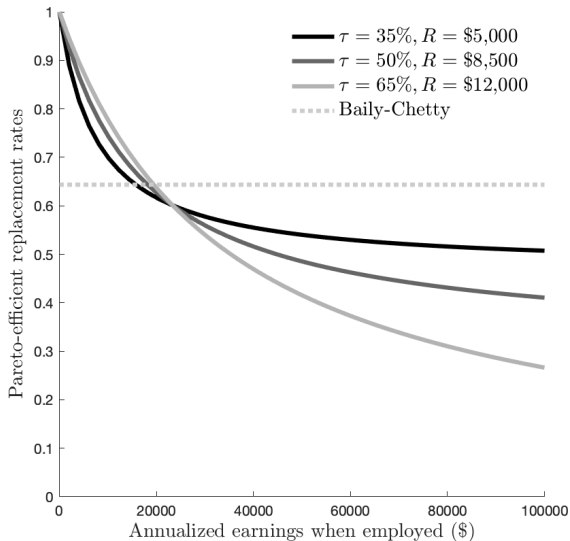
Tax-transfer schedule shapes UI benefits: UI is a redistributive policy!

- ▶ Higher transfers, $R_0 \nearrow \implies$ higher benefits when unemployed, $B_u \nearrow$
- ▶ Steeper tax schedule, $\tau \nearrow \implies$ flatter profile of benefits when unemployed
- ▶ Replacement rates decrease with earnings, starting from 100% at origin

P-efficient consumption profiles ($e = 95\%$, $\mu_u^{elast} = 0.5$)



P-efficient replacement rates ($e = 95\%$, $\mu_u^{elast} = 0.5$)



Extensions of Pareto-efficiency

Extensions of Pareto-efficiency

Savings: stronger decrease in replacement rates, 100% at origin, $\leq 0\%$ at top

- ▶ Introduce liquid savings $s(z)$ and illiquid assets $a(z)$
- ▶ General sufficient statistics formulas spanning many microfoundations

Internalities & externalities: additional corrective terms going either way

- ▶ Wedge between privately chosen and optimal job search
- ▶ Covers individual's biases but also general equilibrium effect

Multidimensional heterogeneity: ambiguous effects going either way

- ▶ Heterogeneity within earnings levels, distribution of e at each z
- ▶ Optimality condition that involves horizontal redistribution

[Skip](#)

Savings: Setting

Individuals: earnings z , liquid savings $s(z)$, illiquid assets $a(z)$

- ▶ Consumption when employed, $c_e(z) = z - T_e(z) - s(z) - a(z)$
- ▶ Consumption when unemployed, $c_u(z) = B_u(z) + \frac{e(z)}{1-e(z)}s(z)$
- ▶ Additional utility from illiquid assets, $V(z) + U(e(z)a(z))$

Benchmark 1: privately-optimal savings and assets (often used in macro)

$$u'_e(c_e(z)) = u'_u(c_u(z))$$

$$u'_e(c_e(z)) = U'(e(z)a(z))$$

Benchmark 2: exogenous savings and assets (often used in public econ)


$$\frac{\partial s(z)}{\partial T_e(z)} = \frac{\partial s(z)}{\partial B_u(z)} = 0$$

$$\frac{\partial a(z)}{\partial T_e(z)} = \frac{\partial a(z)}{\partial B_u(z)} = 0$$

Savings: Pareto-efficiency

Proposition 2. A Pareto-efficient tax-benefit system satisfies at each $z > 0$

$$K_r(z) \frac{u'_u(c_u(z))}{u'_e(c_e(z))} - 1 = \left[1 + K_r(z) K_\mu(z) \frac{1-e(z)}{e(z)} \right] \frac{\mu_u^{elast}(z)}{e(z)} \left[1 + \frac{e(z) T_e(z) - (1-e(z)) B_u(z)}{B_u(z)} \right]$$

- ▶ $K_r(z)$ such that $dT_e(z) = K_r(z) \frac{1-e(z)}{e(z)} \frac{u'_u(c_u(z))}{u'_e(c_e(z))} dB_u(z)$ leaves utility constant
- ▶ $K_\mu(z)$ such that job search semi-elasticities verify $\frac{\mu_e(z)}{u'_e(c_e(z))} = K_\mu(z) \frac{\mu_u(z)}{u'_u(c_u(z))}$
- ▶ $K_r(z) = 1$ in both benchmarks, and $K_\mu(z) = 1$ only in exogenous case 

Presence of savings reduces insurance value of UI

- ▶ Savings unambiguously push for lower benefits
- ▶ Since savings grow with earnings, stronger decrease in replacement rates

Adjustment factors have ambiguous effects

- ▶ In general, $K_r \leq 1$, which pushes for lower benefits (UI crowds out savings)
- ▶ When $K_\mu \leq 1$ it pushes for higher benefits (dampening of μ_u^{elast})

Savings: Policy implications

1. Optimal replacement rates at top incomes are negative

Corollary. If individuals above earnings \bar{z} have enough savings to be perfectly self-insured against unemployment, then at $z \geq \bar{z}$,

$$B_u(z) = -T_e(z).$$

2. Simple policy rule with log utility now includes savings rates

Corollary. If consumption utilities are logarithmic and if $K_r(z) = 1$, denoting $\rho_s(z) := \frac{s(z)}{z}$ and $\rho_a(z) := \frac{a(z)}{z}$ the savings rates and $\mathcal{K}_s(z) \sim K_\mu(z)$, we get

$$B_u(z) = \frac{e(z)}{e(z) + \mu_u^{elast}(z)\mathcal{K}_s(z)} \left(1 - \frac{\rho_s(z)}{1 - e(z)} - \rho_a(z) \right) z - T_e(z),$$

Skip

Labor supply and the optimal tax-transfer schedule

Labor supply and the optimal tax-transfer schedule

Anatomy of labor supply decisions

- ▶ Extensive margin: whether to participate in labor market, given fixed cost χ
- ▶ Intensive margin: how much to income to earn, given earnings ability ω

Labor supply decisions have (almost) no impact on replacement rates

- ▶ Extensive margin responses do not affect Pareto-efficiency condition
- ▶ Intensive margin responses adds income effects term, quantitatively small

Labor supply decisions at the heart of optimal tax theory

- ▶ Extend classical optimal tax formulas in extensive-intensive margin models

Labor supply: Individuals

Individual choose their earnings and search efforts

- ▶ Heterogeneous earnings ability ω , work cost $k(z; \omega)$, search cost $\psi(e, z; \omega)$

$$V(\omega) := \max_z \left[\max_e e \underbrace{\left[u_e(z - T_e(z)) - k(z; \omega) \right]}_{\text{employed}} + (1-e) \underbrace{\left[u_u(B_u(z)) - \psi(e, z; \omega) \right]}_{\text{unemployed}} \right]$$

Individuals choose to participate in the labor market if and only if

- ▶ Heterogeneous participation costs χ , social assistance R_0

$$\underbrace{V(\omega) - \chi}_{\text{participating}} \geq \underbrace{u_0(R_0)}_{\text{not participating}} \iff \chi \leq \tilde{\chi}(\omega) \equiv V(\omega) - u_0(R_0)$$

New sufficient statistics

- ▶ Compensated earnings semi-elasticity ζ_e , income effect parameter η_e
- ▶ Cross-partial effect of earnings on job search ξ_z^{1-e}
- ▶ Participation semi-elasticities, π_e

Labor supply: Pareto-efficiency is (almost) unaffected

Proposition 3. A Pareto-efficient tax-benefit system satisfies at earnings z

$$\begin{aligned} \frac{u'(c_u(z))}{u'(c_e(z))} - 1 &= \frac{\mu_u^{elast}(z)}{e(z)^2} \left[1 + \frac{e(z)T_e(z) - (1 - e(z))B_u(z)}{B_u(z)} \right] \\ &+ \frac{\eta_e(z)}{e(z)} \left[\frac{B'_u(z)}{1 - T'_e(z)} \frac{u''(c_u(z))}{u''(c_e(z))} - \frac{u'(c_u(z))}{u'(c_e(z))} \right] \\ &\times \left[\underbrace{(e(z)T'_e(z) - (1 - e(z))B'_u(z))}_{\text{marginal rate of net contribution}} - \underbrace{(T_e(z) + B_u(z))}_{\text{employment tax}} \xi_z^{1-e}(z) \right] \end{aligned}$$

New corrective term related to earnings income effects

- ▶ Empirically negligible at low earnings
- ▶ More relevant at higher earnings, but low replacement rates anyway

All important aspects do not appear in formula!

- ▶ Participation responses: irrelevant as joint reforms keep utility V constant
- ▶ Earnings decisions: need eligibility thresholds \underline{e} against double deviations

Labor supply: Government

Objective: weighted sum of utility with social welfare function $G(\cdot)$

$$\int_z \left[\underbrace{\int_{\chi \leq \tilde{\chi}(z)} G(V(z) - \chi)}_{\text{participating}} + \underbrace{\int_{\chi \geq \tilde{\chi}(z)} G(u(R_0))}_{\text{not participating}} \right] dF_{\chi, z}(\chi, z)$$

Resource constraint: exogenous expenditure requirement Exp

$$\int_z \left[\underbrace{\int_{\chi \leq \tilde{\chi}(z)} (e(z)T_e(z) - (1-e(z))B_u(z))}_{\text{participating}} - \underbrace{\int_{\chi \geq \tilde{\chi}(z)} R_0}_{\text{not participating}} \right] dF_{\chi, z}(\chi, z) \geq Exp$$

New sufficient statistic: social marginal welfare weight

$$g_e(z) := \frac{\overline{G'(V(z) - \chi)}}{\lambda} u'_e(c_e(z))$$

Extensive margin only: Optimal tax formula

Proposition 4. An optimal tax-transfer schedule $T_e(z)$ satisfies at each $z > 0$,

$$\underbrace{(T_e(z) + R_0)}_{\text{participation tax}} \pi_e(z) - (1 - e(z)) \underbrace{(T_e(z) + B_u(z))}_{\text{employment tax}} (\pi_e(z) - \mu_u(z)) = e(z)(1 - g_e(z)).$$

Generalizes optimal tax formula in extensive margin models

- ▶ Usually assume $e(z) = 1$ as in Diamond (1998), Saez (2002), ...
- ▶ EITC policy is desirable, $R_0 < -T_e(z)$ if and only if $1 < g_e(z)$

$$(T_e(z) + R_0) \pi_e(z) = (1 - g_e(z))$$

Net impact on tax-transfer schedule is ambiguous

- ▶ Unemployment dampens mechanical effect
- ▶ Unemployment dampens participation effect, optimal $T_e \nearrow$
- ▶ Job search effect calls for job search incentives, optimal $T_e \searrow$

Extensive-intensive margins: Optimal tax formula

Proposition 5. The optimal marginal tax rate $T'_e(\cdot)$ satisfies at earnings z

$$\begin{aligned} & \left[\underbrace{(e(z)T'_e(z) - (1 - e(z))B'_u(z))}_{\text{marginal rate of net contribution}} - \underbrace{(T_e(z) + B_u(z))}_{\text{employment tax}} \xi_z^{1-e}(z) \right] \zeta_e(z) z h_z(z) \\ &= \int_{x \geq z} \left\{ \underbrace{e(x)(1 - g_e(x))}_{\text{mech. eff.}} + \underbrace{(1 - e(x))(T_e(x) + B_u(x))}_{\text{employment tax}} (\pi_e(x) - \mu_e(x)) - \underbrace{(T_e(x) + R_0)}_{\text{particip. tax}} \pi_e(x) \right. \\ & \quad \left. + \underbrace{(e(x)T'_e(x) - (1 - e(x))B'_u(x))}_{\text{marginal rate of net contribution}} \eta_e(x) - \underbrace{(T_e(x) + B_u(x))}_{\text{employment tax}} \xi_x^{1-e}(x) \eta_e(x) \right\} h_z(x) dx \end{aligned}$$

Extends standard ABC formula (Diamond, 1998; Saez, 2001)

- ▶ Optimality condition for the schedule of marginal tax rates $T'_e(z)$
- ▶ Top marginal tax rates are (almost) unaffected
- ▶ Large **search** responses may imply negative marginal tax rates

Empirical application

Testing Pareto-efficiency in the US

Pareto-efficiency condition: baseline + log utility (CRRA=1; Chetty, 2006)

$$\frac{\overbrace{B_u(z)}^{\text{replacement rate}}}{z - T_e(z)} = \left(1 + \frac{\mu_u^{\text{elast}}(z)}{e(z)^2} \left[1 + \frac{\overbrace{e(z)T_e(z) - (1 - e(z))B_u(z)}^{\text{net contribution to tax-benefit system}}}{B_u(z)} \right] \right)^{-1}$$

Compute actual replacement rates from tax-benefit schedules

- ▶ OECD tax-benefit calculator TaxBEN, focus on childless singles
 - T_e = income tax + contributions - EITC - social assistance
 - B_u = unemployment benefits - income tax + social assistance

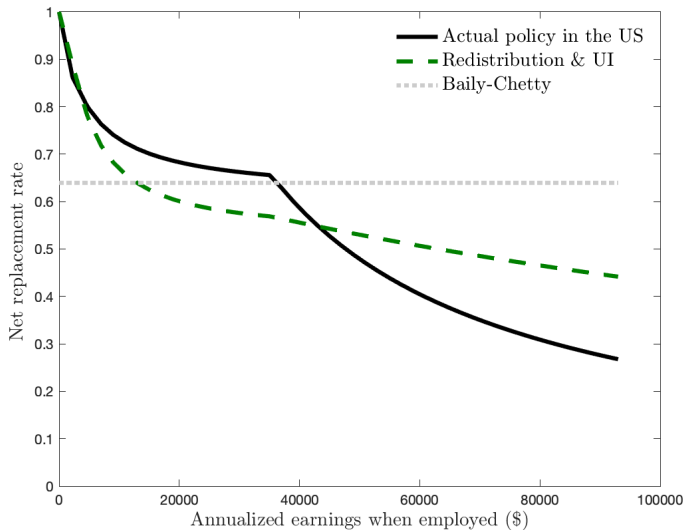
Compute Pareto-efficient replacement rates (Redistribution & UI)

- ▶ Search elasticity $\mu_u^{\text{elast}} = 0.5$ (Schmieder & von Wachter, 2016)
- ▶ Link unemployment rates and earnings through education (BLS, CPS)

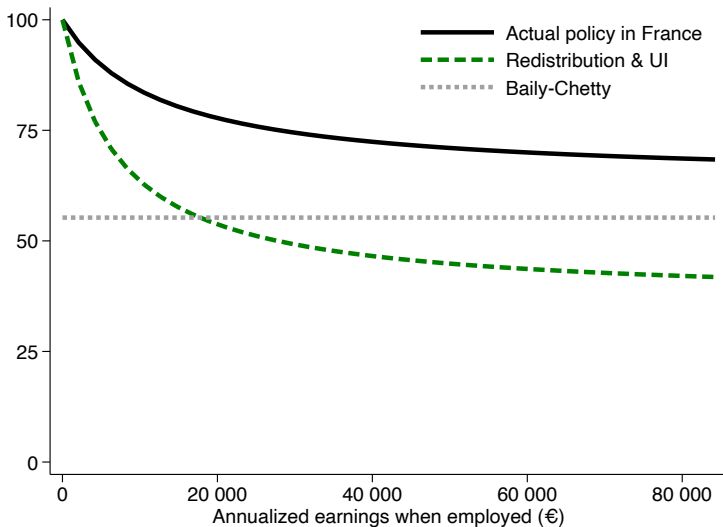
Compute Baily-Chetty optimal replacement rate (UI only)

- ▶ Net contribution to tax-benefit system = 0, average unempl't rate = 5.81%

Testing Pareto-efficiency in the US [Back](#)



Testing Pareto-efficiency in France [Back](#)



Discussion

US: actual replacement rates decrease with earnings, broadly efficient

- ▶ Low-income: improve schedule of linearly increasing benefits with a cap?
- ▶ High-income: adding savings would lower efficient replacement rates
 - Simulating optimal policies requires a calibrated structural model

FR: actual replacement rates stable with earnings, broadly inefficient

- ▶ Low-income: high transfers & high UI is broadly efficient
- ▶ High-income: high taxes & high UI creates room for Pareto-improvement!
 - Interactions between redistribution & UI call for policy coordination

Conclusion

Conclusion

Analyze the interactions between redistribution and UI policies

- ▶ Unifying framework bridging canonical models of optimal tax & optimal UI
- ▶ Sufficient statistics characterization of optimal policies

Pareto-efficiency implies a tight link between optimal tax and optimal UI

- ▶ Efficient replacement rates decrease with earnings
- ▶ Efficient replacement rates are (in part) shaped by redistribution

Interactions between redistribution & UI call for policy coordination

- ▶ US: actual replacement rates decrease with earnings, broadly efficient
- ▶ FR: actual replacement rates stable with earnings, broadly inefficient

Redistribution & Unemployment Insurance

Antoine Ferey (Sciences Po)

August 2024

Steady-state representation of dynamic model Back

Population of individuals with **heterogeneous earnings** when employed, z

Employed at t : exogenous probability $q(z)$ to become unemployed at $t+1$

- ▶ Utility, $u_e(\cdot)$, from consumption, $c_e(z)$
- ▶ Costs of working, $k(z)$

Unemployed at t : endogenous probability $p(z)$ to become employed at $t+1$

- ▶ Utility, $u_u(\cdot)$, from consumption, $c_u(z)$
- ▶ Costs of **searching**, $\tilde{\psi}(p(z), z)$

Lemma 1. This (stationary) dynamic model converges to a steady state, where the utility of an individual with earnings z when employed is

$$\underbrace{\frac{p(z)}{q(z)+p(z)}}_{e(z)} \underbrace{\left[u_e(c_e(z)) - k(z) \right]}_{\text{employed}} + \underbrace{\frac{q(z)}{q(z)+p(z)}}_{1-e(z)} \underbrace{\left[u_u(c_u(z)) - \tilde{\psi}(p(z), z) \right]}_{\text{unemployed}}$$

Appendix: Other sufficient statistics Back

Participation semi-elasticities: changes in the number of participants, $h_z(z)$

$$\pi_e(z) \equiv -\frac{1}{h_z(z)} \frac{\partial h_z(z)}{\partial T_e(z)} \quad \pi_u(z) \equiv \frac{1}{h_z(z)} \frac{\partial h_z(z)}{\partial B_u(z)}$$

$$\frac{\pi_e(z)}{e(z) u'(c_e(z))} = \frac{\pi_u(z)}{(1-e(z)) u'(c_u(z))}$$

Social marginal welfare weights: changes in utility (value of public funds λ)

$$g_e(z) \equiv \frac{\overline{G'(V(z) - \chi)}}{\lambda} u'_e(c_e(z)) \quad g_u(z) \equiv \frac{\overline{G'(V(z) - \chi)}}{\lambda} u'_u(c_u(z))$$

$$\frac{g_e(z)}{u'(c_e(z))} = \frac{g_u(z)}{u'(c_u(z))}$$

Appendix: Optimal tax schedule $T_e(z)$ Back

Proposition 1b. The optimal tax schedule satisfies at earnings z

$$\underbrace{(T_e(z) + R_0)}_{\text{participation tax}} \pi_e(z) - (1 - e(z)) \underbrace{(T_e(z) + B_u(z))}_{\text{employment tax}} (\pi_e(z) - \mu_e(z)) = e(z) \underbrace{(1 - g_e(z))}_{\text{mechanical effect}}$$

Redistribution benchmark (e.g., Saez, 2002): no unemployment, $e(z) = 1$

- ▶ Trade-off: redistribution vs. participation

$$\underbrace{(T_e(z) + R_0)}_{\text{participation tax}} \pi_e(z) = \underbrace{(1 - g_e(z))}_{\text{mechanical effect}}$$

Redistribution & UI: unemployment + job search

- ▶ Unemployment dampens the mechanical effect → reduces $T_e(z)$
- ▶ New term weighting participation vs search responses → ambiguous

Appendix: Proof of optimal tax formula Back

Reform: increase taxes by $dT_e(z)$

1. Impact on social welfare (envelope argument)

$$\underbrace{-e(z) u'_e(c_e(z)) dT_e(z)}_{dV(z)} \frac{\overline{G'(V(z) - \chi)}}{\lambda} h_z(z)$$

2. Mechanical effect on government budget

$$e(z) h_z(z) dT_e(z)$$

3. Fiscal externality from participation responses

$$\underbrace{\left(e(z) T_e(z) - (1 - e(z)) B_u(z) + R_0 \right)}_{\text{"total" participation tax}} \pi_e(z) dT_e(z) h_z(z)$$

4. Fiscal externality from search responses

$$\underbrace{\left(T_e(z) + B_u(z) \right)}_{\text{employment tax}} (1 - e(z)) \mu_e(z) dT_e(z) h_z(z)$$

Appendix: Optimal benefit schedule $B_u(z)$

Proposition 1c. The optimal benefit schedule satisfies at earnings z

$$(1-e(z)) \underbrace{(T_e(z) + B_u(z))}_{\text{employment tax}} (\pi_u(z) + \mu_u(z)) - \underbrace{(T_e(z) + R_0)}_{\text{participation tax}} \pi_u(z) = (1-e(z)) \underbrace{(g_u(z) - 1)}_{\text{mechanical effect}}$$

UI only: exogenous labor $\pi_u(z) = 0$ + no redistributive concerns in $g_u(z)$

- ▶ Trade-off: insurance vs. **search**

$$\underbrace{(T_e(z) + B_u(z))}_{\text{employment tax}} \mu_u(z) = \underbrace{(g_u(z) - 1)}_{\text{mechanical effect}}$$

Redistribution & UI: redistributive concerns + **participation responses**

- ▶ Redistributive concerns in $g_u(z)$ call for progressivity in optimal $B_u(z)$
- ▶ **Participation responses** induce additional fiscal externalities → ambiguous

Appendix: Proof of Pareto-efficiency condition Back

Reform: increase benefits by $dB_u(z) = 1$ and taxes by $dT_e(z) = \frac{(1-e(z))u'_u(c_u(z))}{e(z)u'_e(c_e(z))}$

1. No impact on expected utility thus on participation (envelope argument)

$$dV(z) = -e(z) u'_e(c_e(z)) dT_e(z) + (1 - e(z)) u'_u(c_u(z)) dB_u(z) = 0$$

2. Mechanical effect on government budget

$$[e(z)dT_e(z) - (1 - e(z))dB_u(z)] h_z(z) = (1 - e(z)) \left[\frac{u'_u(c_u(z))}{u'_e(c_e(z))} - 1 \right] h_z(z)$$

3. Fiscal externality from search responses

$$T_e(z) + B_u(z) = \frac{B_u(z)}{e(z)} \left[1 + \frac{e(z)T_e(z) - (1 - e(z)) B_u(z)}{B_u(z)} \right]$$

with the magnitude of search responses (using $\frac{\mu_e(z)}{u'_e(c_e(z))} = \frac{\mu_u(z)}{u'_u(c_u(z))}$)

$$(1 - e(z)) [\mu_e(z) dT_e(z) + \mu_u(z) dB_u(z)] h_z(z) = (1 - e(z)) \frac{\mu_u(z)}{e(z)} h_z(z)$$

Appendix: Application to log utility

Pareto-efficient unemployment benefits

$$B_u(z) = \frac{e(z)}{e(z) + \mu_u^{elast}(z)} z - T_e(z)$$

Globally optimal tax schedule (assuming utilitarian SWF, $g_e(z) = \frac{1}{\lambda} \frac{1}{z - T_e(z)}$)

$$T_e(z) = \frac{e(z)}{e(z) + \pi_e^{elast}(z)} \left[z - \frac{1}{\lambda} + (1 - e(z)) \frac{\pi_e^{elast}(z) - \mu_u^{elast}(z)}{e(z) + \mu_u^{elast}(z)} z \right] - \frac{\pi_e^{elast}(z)}{e(z) + \pi_e^{elast}(z)} R_0$$

Back

Appendix: Pareto-efficiency condition with savings

Back

$$K_r(z) = \frac{1 + \frac{e(z)}{1-e(z)} \frac{u'_e(c_e(z))}{u'_u(c_u(z))} \left[\left(\frac{u'_u(c_u(z))}{u'_e(c_e(z))} - 1 \right) \frac{\partial s(z)}{\partial B_u(z)} + \left(\frac{U'(e(z)a(z))}{u'_e(c_e(z))} - 1 \right) \frac{\partial a(z)}{\partial B_u(z)} \right]}{1 - \left(\frac{u'_u(c_u(z))}{u'_e(c_e(z))} - 1 \right) \frac{\partial s(z)}{\partial T_e(z)} - \left(\frac{U'(e(z)a(z))}{u'_e(c_e(z))} - 1 \right) \frac{\partial a(z)}{\partial T_e(z)}}$$

$$K_\mu(z) = \frac{\mathcal{K}_T(z) + \left[\mathcal{K}_s(z) \frac{\partial s(z)}{\partial T_e(z)} + \mathcal{K}_a(z) \frac{\partial a(z)}{\partial T_e(z)} \right]}{\mathcal{K}_B(z) + \frac{u'_e(c_e(z))}{u'_u(c_u(z))} \left[\mathcal{K}_s(z) \frac{\partial s(z)}{\partial B_u(z)} + \mathcal{K}_a(z) \frac{\partial a(z)}{\partial B_u(z)} \right]}$$

where, omitting arguments of functions to economize on space,

$$\mathcal{K}_T = 1 - e \frac{u''_e}{u'_e} \left[\frac{\partial s}{\partial e} + \frac{\partial a}{\partial e} \right] - e \left[\left(\frac{u'_u}{u'_e} - 1 \right) \frac{\partial^2 s}{\partial T_e \partial e} + \left(\frac{U'}{u'_e} - 1 \right) \frac{\partial^2 a}{\partial T_e \partial e} \right]$$

$$\mathcal{K}_B = 1 - \frac{u''_u}{u'_u} \left[\frac{s}{1-e} + e \frac{\partial s}{\partial e} \right] - e \frac{u'_e}{u'_u} \left[\left(\frac{u'_u}{u'_e} - 1 \right) \frac{\partial^2 s}{\partial B_u \partial e} + \left(\frac{U'}{u'_e} - 1 \right) \frac{\partial^2 a}{\partial B_u \partial e} \right]$$

$$\mathcal{K}_s = \frac{u''_u}{u'_e} \frac{e}{1-e} \left[\frac{s}{1-e} + e \frac{\partial s}{\partial e} \right] + \frac{u''_e}{u'_e} e \left[\frac{\partial s}{\partial e} + \frac{\partial a}{\partial e} \right] + \left(\frac{u'_u}{u'_e} - 1 \right) \left[1 + e \frac{\partial^2 s}{\partial s \partial e} \right] + e \left(\frac{U'}{u'_e} - 1 \right) \frac{\partial^2 a}{\partial s \partial e}$$

$$\mathcal{K}_a = \frac{U''}{u'_e} e \left[a + e \frac{\partial a}{\partial e} \right] + \frac{u''_e}{u'_e} e \left[\frac{\partial s}{\partial e} + \frac{\partial a}{\partial e} \right] + \left(\frac{u'_u}{u'_e} - 1 \right) e \frac{\partial^2 s}{\partial a \partial e} + \left(\frac{U'}{u'_e} - 1 \right) \left[1 + e \frac{\partial^2 a}{\partial a \partial e} \right]$$

Appendix: New sufficient statistics Back

Earnings responses to tax-benefit reforms

- ▶ Compensated earnings semi-elasticities

$$\zeta_e(z) \equiv -\frac{1}{z} \frac{\partial z}{\partial T'_e(z)} \quad \zeta_u(z) \equiv \frac{1}{z} \frac{\partial z}{\partial B'_u(z)}$$

$$\frac{\zeta_e(z)}{e(z) u'(z - T_e(z))} = \frac{\zeta_u(z)}{(1 - e(z)) u'(B_u(z))}$$

- ▶ Income effects parameters

$$\eta_e(z) \equiv \frac{\partial z}{\partial T_e(z)} \quad \eta_u(z) \equiv -\frac{\partial z}{\partial B_u(z)}$$

$$\frac{\eta_e(z)}{(1 - T'_e(z)) e(z) u''(z - T_e(z))} = \frac{\eta_u(z)}{B'_u(z) (1 - e(z)) u''(B_u(z))}$$

Cross-partial effect of earnings on job search

$$\xi_z^{1-e}(z) \equiv \frac{\partial(1 - e(z))}{\partial z}$$

Appendix: Optimal allocations in mechanism design Back

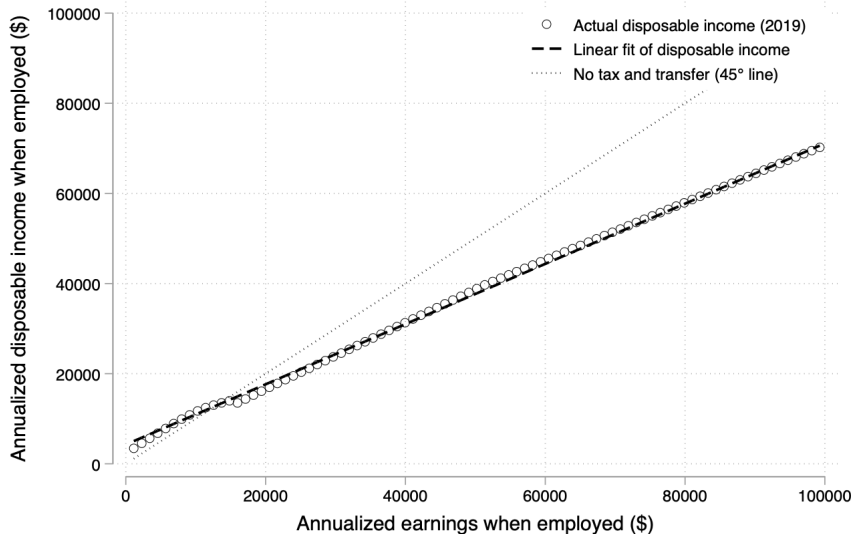
Second-best allocations: $\{z(\omega), c_e(\omega), c_u(\omega), e(\omega)\}_\omega$ and c_0

- ▶ Planner dictates $e(\omega)$ and provides full insurance $c_e(\omega) = c_u(\omega)$
 - Involves unrealistic discontinuities in allocations across values of $e(\omega)$

Third-best allocations: $\{z(\omega), c_e(\omega), c_u(\omega)\}_\omega$ and c_0 given \underline{e}

- ▶ Threshold mechanism: allocation is independent of e as long as $e \geq \underline{e}$
 - Threshold mimicks eligibility requirements of actual UI systems
- ▶ If $\underline{e} = 0$, incentive compatibility restricts insurance to be lump-sum
 - Upward deviations: work one day, enjoy unemployment insurance forever
- ▶ If $\underline{e} > 0$, eligibility requirements restore the possibility to provide insurance
 - Eliminate upward deviations (+ smoothes and concavifies the problem)

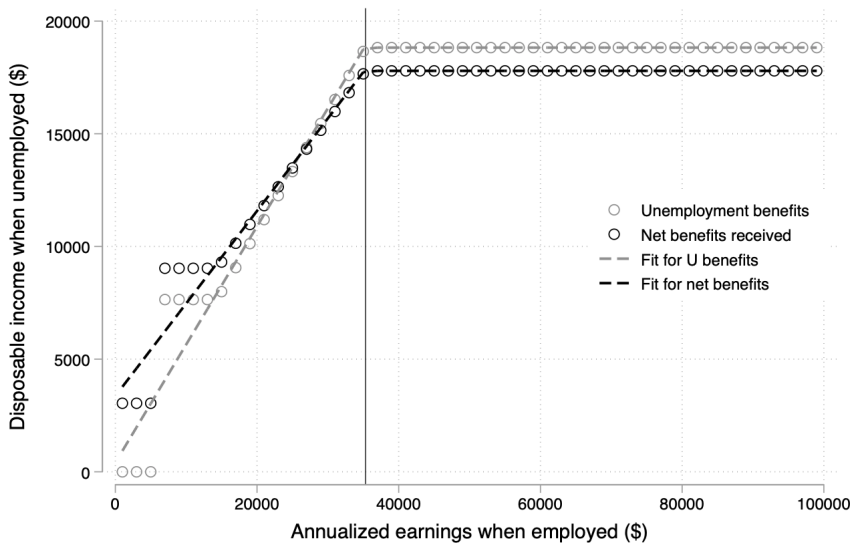
Appendix: Tax schedule in the US

[MTR](#)[Back](#)

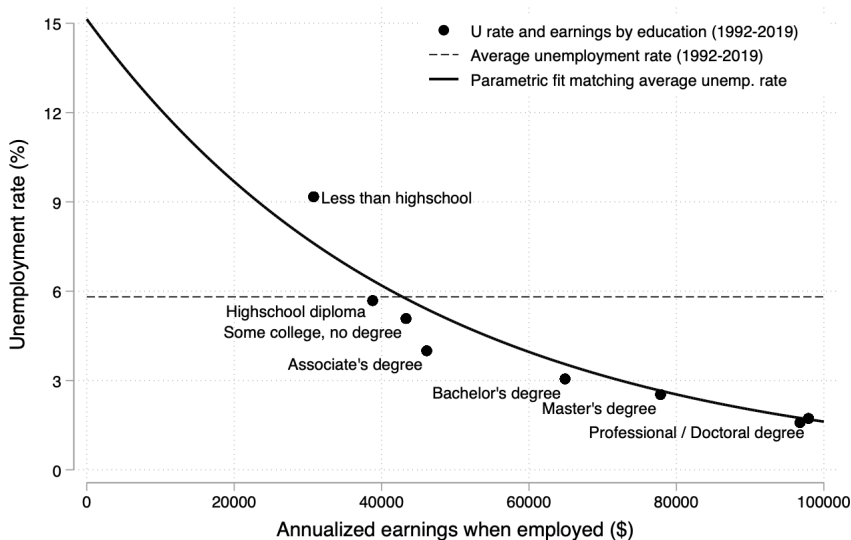
Appendix: Benefit schedule in the US

[Back to Intro](#)

[Back to Application](#)



Appendix: Unemployment rates by earnings in US Back



Appendix: Simulating counterfactual policies Back

Simulating counterfactual policies

- ▶ Specify linear tax schedule, $T_e(z) = \tau z - R_0$
- ▶ Analyze feasible Pareto-efficient policies for different tax rates τ

Calibrate search costs $\psi(e, z)$

- ▶ Unemployment rates across earnings
- ▶ Search elasticity $\mu_u^{elast} = 0.5$

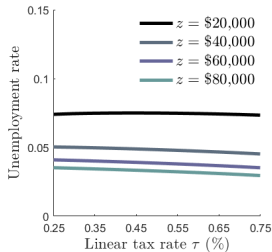
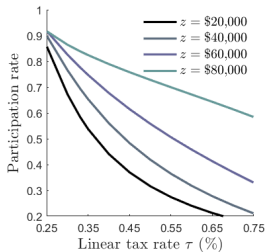
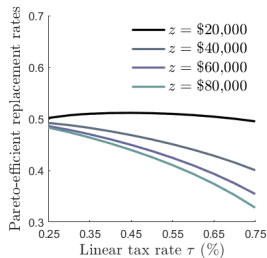
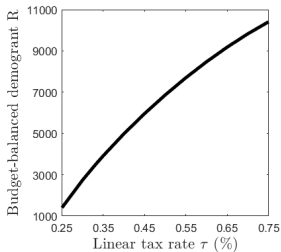
Calibrate fixed participation costs χ

- ▶ Participation rates across earnings
- ▶ Participation elasticity decreasing from $\pi_e^{elast} = 0.5$ to 0 above \$100,000

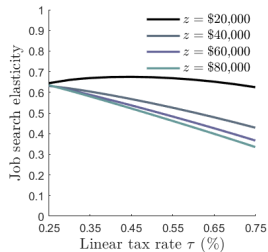
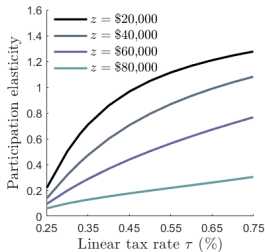
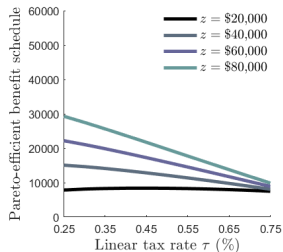
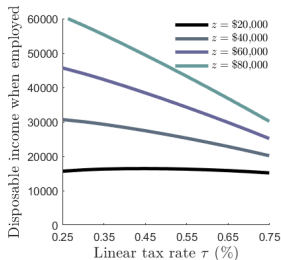
Calibrate earnings distribution $h_z(z)$

- ▶ CPS microdata, variable *usual weekly earnings*, append Pareto-tail

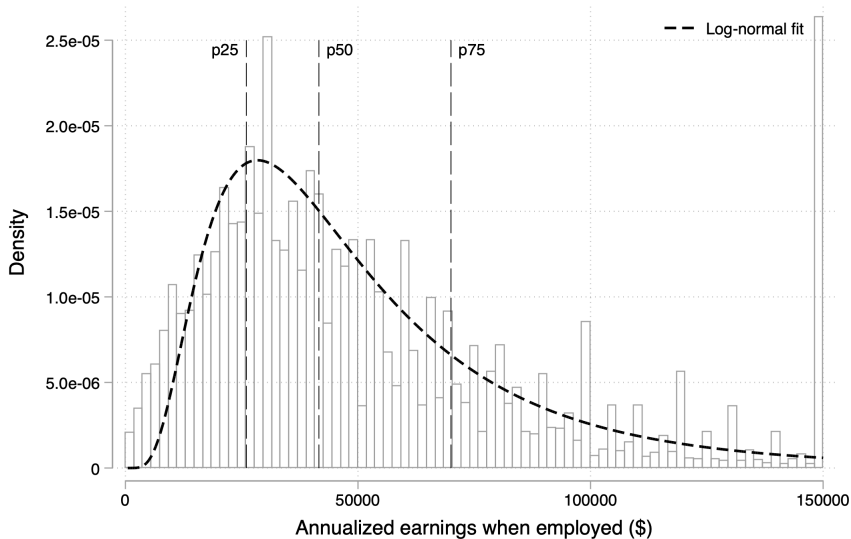
Appendix: Simulating counterfactual policies Back



Appendix: Simulating counterfactual policies Back



Appendix: Earnings distribution in US Back



Appendix: Participation rates across earnings in US



Appendix: Marginal tax rates in the US Back

