Reach for Yield by U.S. Public Pension Funds[1]

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Abstract

We study whether U.S. public pension funds increase the riskiness of their asset portfolios and net worth in a low interest-rate environment or when they are underfunded. We present a theoretical model that relates funds' asset risk to risk-free rates, underfunding, and state sponsors' fiscal condition. To study risk-taking empirically, we create a new methodology for inferring funds' risk from limited information on their returns and portfolio weights. We find that funds on average took more asset risk when risk-free rates and funding ratios were lower, and when they were affiliated with fiscally-weaker state sponsors. In contrast, we find that the risk of changes in funds' net worth is not explained by underfunding and is only weakly explained by low interest rates. Our net worth results are consistent with funds sometimes increasing their asset risk to hedge their liabilities.

Keywords: U.S. public pension funds, reach for yield, Value at Risk, underfunding, duration-matched discount rates, state public debt.

JEL Classification: E43, G11, G23, G32, H74.

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1. Introduction

How do low interest rates affect the investment behavior of institutional investors? How is this behavior influenced by investors' financial condition, and what are its potential consequences for financial stability? This paper studies these questions in the context of state and municipal U.S. public pension funds (henceforth PPFs, funds, or plans). We investigate two aspects of funds' risk-taking. First, we examine the determinants of funds' asset risk, which is the risk of fluctuations in the value of funds' asset portfolios. Second, we study funds' net worth risk, which is the risk of fluctuations in the present value of funds' assets relative to their liabilities.

For our analysis of asset risk, we investigate whether PPFs reach for yield (RFY) by holding riskier asset investment portfolios to increase their expected returns when interest rates on relatively safe assets are low. In addition, we study how the extent of funds' underfunding and the fund sponsors' fiscal condition affect the funds' asset risk-taking behavior, and how such behavior in turn may affect the funds and their sponsors. To study these relationships, we first present a simple theoretical model relating funds' asset risk-taking to the level of risk-free rates, to their underfunding, and to the fiscal condition of their state sponsors. The theory identifies two distinct channels through which interest rates and other factors may affect asset risk-taking: funding ratios and risk premia. The theory also shows that the effect of state finances on funds' risk-taking depends on states' risk-shifting incentives. We use the theory to interpret our empirical findings. To study the determinants of funds' risk-taking behavior empirically, we create a new methodology for measuring funds' asset portfolio risk, which is a key contribution of our paper. Specifically, we use the total returns and portfolio weights that funds report each year to identify the relevant mix of market indexes that best describes funds' returns in each asset category. We use the indices returns measured at a daily frequency together with funds' portfolio weights to impute funds' annual Value at Risk (VaR). To analyze the determinants of funds' risk-taking, we perform a panel regression analysis to assess how the VaR of funds' asset portfolios is related to the level of risk-free rates, to funds' underfunding as measured in Rauh (2017), and to the states' fiscal condition. We also study the implications of our asset risk-taking results for state finances.

In addition to our analysis on risk-taking in the asset portfolio, which is the usual focus of research on excess risk-taking by pension funds, we analyze whether underfunding or lower risk-free rates are associated with more net worth risk. Net worth risk is an important notion of PPFs' risk because it measures the risk of the shortfall that taxpayers may have to pick-up if the fund falls short of its obligations, and the risk to pension beneficiaries if the fund defaults on its obligations. Studying net worth risk may also provide an alternative interpretation of our findings–and those in the literature–on asset risk-taking. In particular, PPFs may sometimes find it optimal to increase asset risk to hedge liabilities and thus reduce net worth risk. Hence, our approach is important because we study not only whether PPFs take more asset risk in response to low risk-free rates and underfunding, but also whether higher asset risk is consistent with pension funds' hedging to reduce net worth risk.

A PPF is underfunded if the present value of its assets is less than the net present value of liability payments to its pension holders. In such a case, state sponsors are limited in their ability to close the funding gap by reducing promised pension benefits, because in most states public employee retirement benefits are either guaranteed by state constitutions or constitute a contractual obligation between the sponsor and plan members.[2] Sponsors do, however, have many other choices: their funds can invest in assets with higher expected returns—but also risk—hoping this will close the gap; they can require greater contributions from future pension beneficiaries; or sponsors can provide higher contributions to the plan, which they would fund by current or future taxation, by borrowing, or by cutting expenditures on other governmental programs and prerogatives. The choice that is ultimately made is a political decision.[3,4]

Our theoretical analysis models the political decision in a stylized setting. Specifically, we model the asset portfolio choice of a public pension plan that is acting on behalf of its sponsors and can invest in risky and risk-free assets. The model captures the tradeoff that plan sponsors

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[2] Munnell and Quinby (2012) provide an analysis of the restrictions on the reduction of pension benefits to public sector employees and retirees.

[3] Political considerations can also affect the PPFs' investment behavior because governmental sponsors of PPFs have discretion regarding the level of contributions to the fund and the setting of funds' target asset return. Government accounting standards require that plan sponsors develop a plan to fully fund public pension plans over a period no greater than thirty years. The plan requirements are not binding. Many plan sponsors do not adhere to the funding schedules specified in the plan. Moreover, standards governing public sector pension plans provide sponsors with considerable discretion in the choice of accounting assumptions. Naughton, Petacchi and Weber (2015) provide evidence that plan sponsors use this discretion to reduce reported levels of underfunding and contributions. Kelley (2014) finds that political factors have a significant influence on plan funding levels.

[4] A large share of the public pension plan board members consists of political appointees and elected officials. Return objectives of state public sector pension plans are often set by state legislatures in the budgeting process. Similar processes are used by local government pension plan sponsors. For details see Andonov et al (2017).

face when choosing between their constituents paying higher future taxes to support pension beneficiaries, or by the plan taking more risk in the hopes that the risky assets perform well and reduce the extent of underfunding. In the model, funds' incentives to take risk operate through two main channels. The first channel operates through funding ratios, defined as the ratio of the present value of funds' assets relative to their liabilities. When funding ratios are lower for any reason—including but not limited to lower assets, higher future liabilities, or lower interest rates—funds may choose to take more risk in the hopes of "catching up". This is the reach-for-yield channel in our model. The second is a risk-premium channel that operates if lower risk-free interest rates increase risk premia and boost plans' incentives to take risk. The risk-premium channel is conceptually separate from reach-for-yield. The model also captures the possibility that some sponsors may choose to default on their non-pension debt in order to more easily make required payments to pension fund beneficiaries. Theoretically and empirically, we examine how the possibility of default, the level of interest rates, the funding ratio, the amount of non-pension debt relative to state income, and their interactions jointly determine funds' asset risk. In addition, we empirically analyze how underfunding and low interest rates are related to funds' net worth risk.

Our empirical findings show that lower funding ratios and lower interest rates on safe assets caused PPFs to increase asset risk. Interpreted through our model, we find evidence for both a reach-for-yield channel acting through funding ratios as well as a channel capturing how interest rates affect risk premia. Second, our findings show an interaction between underfunding and low interest rates, i.e. the effect of a lower funding ratio on asset risk was more pronounced when interest rates were relatively low, such as during the last five years of our sample (2012-2016). Third, PPFs affiliated with state or municipal sponsors with weaker public finances—as reflected by higher levels of public debt or worse credit ratings—also took more asset risk. In line with our model implications, we find a notable interaction between public finances and interest rates, as PPFs from states in worse fiscal condition took more asset risk especially during periods of low interest rates.

Our modeling of state finances suggests that if states can default on their non-pension debt, states with high debt-to-income ratios may choose to take higher asset risk in their pension funds because they can shift the risk of poor fund performance away from taxpayers and toward state debt holders. On the other hand, our model also implies that states may choose to take less asset risk if state debt is high but they cannot default on it, or if the penalties for defaulting are large. Viewed through the model, our empirical analysis of state finances is mostly consistent with higher state debt-to-income ratios leading to higher asset risk in pension plans' portfolios, and thus with risk-shifting. Therefore, our analysis implies that, because PPFs in states with weaker financial conditions take more risk, they run the risk of further weakening state finances. We quantify that the potential loss to the states if a 1-in-20 years episode of adverse returns had occurred in 2016 would have been on average about 20% of states' debt, as opposed to only 13% in the period before the Global Financial Crisis.

A related theory of why U.S. PPFs' asset risk increased during the recent low-yield environment is that sponsors attempt to mask their PPFs' extent of underfunding, and may do so by holding riskier assets with higher returns to reduce the reported value of their liabilities by increasing the rate at which liabilities are discounted under regulatory rules. Specifically, GASB accounting rules allow U.S. PPFs to discount their liabilities based on the expected return on their assets. Andonov et al. (2017)'s cross-country study provides evidence consistent with the theory that reach for yield among U.S. PPFs is driven by the GASB accounting treatment.[5] However, Boubaker et al. (2018) find that given their asset holdings, PPFs tend to significantly exaggerate the expected returns on their assets; which means they do not necessarily have to hold riskier assets in order to mask a part of their underfunding. This phenomenon is partly illustrated in Figure 1 (panel a), which shows that PPFs' expected return targets have declined little during 2009-2016 when Treasury yields were relatively low.

To perform our empirical analysis, we use Rauh's (2017), method to discount the value of funds' liabilities with risk-free rates motivated by Brown and Pennacchi's (2016) argument that if the purpose of the discount rate measure is to assess by how much a fund is underfunded, then the

[5] Greenwood and Vissing-Jorgenson (2018) show that discount rate regulation affect funds' investments because if discount rates are related to yields on particular instruments that are used for discounting liabilities, then funds have an incentive to invest in those instruments to hedge against changes in discount rates.

appropriate discount factor for the liabilities is the risk-free rate. This is the discount rate (adapted to liability duration) that we use in our study.[6]

Our findings on net-worth risk differ somewhat from our findings on asset risk. In particular, we don't find evidence for higher net-worth risk based on pension underfunding and we only find weak evidence that low interest rates are consistent with more net-worth risk. This evidence is more consistent the higher asset risk-taking as a hedging strategy—i.e., PPFs adjust asset risk to hedge the riskiness of liabilities—and complements the findings of Ali and Wang (2019) for interest rate hedging by insurance companies.

Our study makes three key contributions to the literature on financial institutions' risk-taking behavior. First, we present a stylized theoretical model to help interpret our empirical results. Importantly, the model highlights the role of risk-shifting as a determinant of PPFs' risk-taking behavior. In addition, the theory highlights that risk-taking occurs through separate channels linked to funding ratios and risk premia, and illustrates the importance of distinguishing between these two channels.

Second, we use a new and more flexible approach for measuring funds' asset-risk based on the limited data that are publicly available at the annual frequency. To do so, we assume that funds' returns in each asset category (e.g., equities, fixed income, alternatives, etc.) consist of the return on an unknown category return index–which depends on a mix of market indices and is common across funds—plus a fund-specific component. We estimate the category return indices and their constituents, as well as the funds' residual risks econometrically. Then we use the estimated category return indices measured at the daily frequency, the funds' portfolio weights, and the fund-specific components to estimate funds' risk. Unlike our approach, some papers in the literature measure funds' asset-risk while assuming that a particular index (such as the S&P 500 for equities) is representative of funds' returns in each asset category. Other papers measure asset risk using less comprehensive measures, such as the share of equities or risky assets in the funds' investment portfolios, without accounting for all asset classes, or for the time-varying riskiness of each asset class. In contrast, our approach to risk measurement is more

[6] It has also been argued that strong legal protections for public plan beneficiaries justify treating these benefits as nearly risk-free. See Novy-Marx and Rauh (2011) for a wide range of considerations in assessing the riskiness of PPF benefits from the perspective of taxpayers.

comprehensive because it accounts for all asset classes for which data is reported, identifies market indices relevant for each asset class (rather than assuming them), and allows for time-varying correlations of returns across asset classes.

Third, we analyze the determinants of public funds' net-worth risk, and are one of the first papers to do so, while we illustrate the econometric challenges of doing so. Our findings for net-worth risk contrast with our results on risk-taking based on assets suggests more work is needed to better interpret the literature's findings that solely focus on financial intermediaries' risk-taking based on asset portfolios --- and to ascertain whether the increased asset risk can be explained by evidence of hedging against changes in net worth.

1.1 Literature

Our paper is related to three strands of the literature. The first is the literature on PPFs' risk taking, and how it is related to plan underfunding, interest rates, and other factors Boubaker et al. (2018) and Mohan and Zhang (2014) measure funds' risk using aggregate market beta coefficients, with different betas assumed for each asset category. Andonov et al. (2017) measure funds' risk as the share of risky assets in the portfolio. Pennacchi and Rastad (2011) measure funds' risk based on the tracking error volatility between the value of assets and liabilities. The volatility of assets is measured under the assumption that returns in each asset category are determined by specific return indices. Relative to these papers, our approach improves the measurement of risk in two ways. First, measures of risk based on covariation with the market, such as beta, or on the share of risky assets held do not measure the time series aspect of risk. For example, high covariation with the market or large holdings of risky assets in the portfolio increase a PPF's riskiness when the market is expected to be more volatile, but the afore-mentioned measures do not account for such time variation in riskiness. In contrast, our approach based on funds' Value-at-Risk accounts for it, as described in our risk measurement section. Second, we use an approach to measuring the variance of asset portfolios that is similar to Pennacchi and Rastad (2011), but instead of assuming that returns in each category are driven by a particular index, our approach is more flexible. We believe our method of measuring the riskiness of pension funds' portfolios has potential to improve on other methods used when the data are limited. An additional similarity with Pennacchi and Rastad (2011) is they focus on the tracking error volatility between the return of assets and liabilities. This is equivalent to studying the risk of net worth, as we do, when the value of assets and liabilities match. Our analysis considers the case of risk of net worth in the more general setting when assets and liabilities do not match. The more general setting raises difficult econometric issues which we make a first attempt to address in this paper.

The second strand of related literature concerns how the risk-taking behavior of financial intermediaries varies with macroeconomic conditions. As is the case with our study, these papers rely on cross-sectional differences between institutions in their response to changes in macroeconomic conditions to identify risk-taking behavior. Becker and Ivashina (2015) examine reach-for-yield behavior among life insurance companies. They find that life insurers tend to assume greater levels of investment credit risk during economic expansions and that this effect is more pronounced among more poorly capitalized firms. DiMaggio and Kacperczyk (2017) find evidence of greater risk-taking by money funds when interest rates are low. This effect is stronger for independent funds than for funds affiliated with insurance companies, commercial or investment banks. They argue that reputational considerations tend to moderate reach-for-yield behavior by affiliated funds. Studies of commercial banks also find evidence of increased risk-taking in low-rate environments, but the effect of financial condition on risk taking is mixed. Jiménez et al. (2014) examine lending activity by Spanish banks. They find that lower overnight rates induce banks to do more risky lending. This effect is stronger among more poorly capitalized institutions. Dell'Ariccia et al. (2017) examine commercial banks in the United States. They also find evidence of greater risk-taking in a low rate environment; however the increase in risk-taking is more prevalent among well-capitalized institutions. Unlike for banks, they posit that financial intermediaries with negative maturity mismatches, such as insurance companies and pension funds, should switch to risky assets in response to monetary easing, and that this behavior should be most pronounced for the least capitalized financial institutions, which we demonstrate in our paper. Closer to our study, Boubaker et al. (2018) study reach-for-yield by PPFs in a framework that models the evolution of PPFs' asset category risk-exposures and monetary policy innovations in a Bayesian VaR framework with Markov regime switching. Consistent with our results, they find that monetary easing is consistent with greater asset-risk. Because of differences in how riskiness is measured, and our very different modeling methodologies, we view our two approaches as complementary.

We also examine the possibility that higher asset-risk reflects hedging when we study pension funds' net-worth risk.   Therefore, our paper is related to Ozdagli and Wang (2019), who study how the risk of life insurers asset portfolios changes as interest rates decline. They find that life insurers buy higher yield bonds as rates decline, but do so not to take on credit risk, but rather to increase the duration of their assets to close the duration gap between assets and liabilities, i.e. they increase asset-risk for hedging reasons. Our paper is also related to Khetan, Li, Neamțu, and Ishita (2023), who show that U.K. pension funds and insurance companies increase their asset duration through interest rate swaps when interest rates decline to close the duration gap with liabilities.

The third strand of related literature concerns the effect of PPFs' obligations on state and local finances. Increased risk-taking and reach-for-yield behavior increase the exposure of plan sponsors to large declines in asset values, and hence increases the volatility of contributions necessary to fund pension promises. A growing literature considers the impact of pension costs, underfunding, and investment losses on state and local government borrowing costs (Novy-Marx and Rauh, 2012, and Boyer, 2018). Several academic and policy studies have examined the effect of a decline in asset prices on the required contributions of plan sponsors (Novy-Marx and Rauh, 2014, Boyd and Ying, 2017, and Mennis et. al., 2018). Measures of PPFs' risk-taking presented herein should be useful in future work concerning the vulnerability of PPFs and plan sponsors to adverse shocks in asset prices.

The remainder of the paper proceeds as follows. Section 2 presents our stylized theoretical model. Section 3 present our data and our methodology for measuring PPFs' asset portfolio risk and plan underfunding. Section 4 contains our empirical analysis of how PPFs' risk has changed over time and in the cross section. It examines the relationship between asset portfolio risk, the interest rate environment, plan underfunding, and the financial condition of fund sponsors. Section 5 quantifies the contributions of underfunding and low risk-free rates to the funds' asset risk-taking behavior, and implications for state finances. Section 6 contains our analysis on risk-taking based on net worth. Section 7 concludes.

2. Theoretical Model

To guide our thinking about the determinants of risk-taking by public pension plans, and to provide some intuition for how to interpret the findings from our econometric analysis, here we present a very simple and stylized "benchmark" two-period model of risky portfolio choice for a state or municipal pension plan that must meet future obligations (liabilities) at a future date t by choosing an asset portfolio at date 0. The risk in the model is that assets fall short of liabilities at date t. Because the value of liabilities are fixed at date t, the risk in the model is captured by focusing on asset-risk alone, as we do below. In section 6, the value of assets and liabilities are both stochastic and we consider the more general setting in which net-worth risk is the focus.

We refer to the plan sponsor as the state throughout. For our modeling, we assume there is no conflict of interest between the state and the manager of its pension investments, and that therefore the pension plan is managed in accordance with the wishes of plan sponsors. Therefore we model the state as controlling the amount of assets managed by the pension fund, and how those assets are invested. Multi-period treatments of the pension fund's portfolio choice problem are contained in Pennacchi and Rahstad (2011) and D'Arcy et al (1999); following their approach, the state is assumed to choose the pension assets to maximize a utility function that is based on the preferences of its citizens, denoted as the representative citizen RC, hereafter. We interpret the representative citizen as the median voter within the state or municipality associated with a pension plan, but we acknowledge the utility function could have richer interpretations. In particular, it may embed preferences on how the plans' and sponsors' actions affect conflicting special interests such as plan beneficiaries, state taxpayers, and the holders of state debt.[7] Because of these potential conflicts and other potential imperfections, we don't interpret maximization of the utility function as maximization of social welfare, but we regard it as a useful modeling device for our positive analysis. In our modeling below, we rely

[7] The preferences could reflect median voters, vocal interest groups or a blend of the median voter and the median voter as in Kelley (2014); or they could represent the preferences of median voters or fund managers as in Pennacchi and Rahstad (2011). Or they could represent the preferences of the median voter in a setting where there are information asymmetries among voters regarding pay structure as in Glaeser et al (2014).

primarily on a median voter interpretation, but we allow for the possibility that the interests of the median voter and state debt holders may be in conflict.[8]

There are two dates in the model. Date 0, which represents today, and date t, a date t years in the future. The RC is endowed with income Y_0 and Y_t. The income Y_t is assumed to be net of all tax payments other than those that may need to be made to support pension beneficiaries, or to pay off state debt. To simplify the analysis, Y_t and Y_0 are assumed to be known at date 0.[9] In addition, the state has zero coupon debt with face value D_t that must paid at date t and it has a pension liability of L_t that must be paid to its workers at date t. We model the debt D_t in two different cases. In the first, the debt is risk-free and the state will pay it out of state income Y_t. In the second, the state can choose to default on its debt and will do so if the taxes needed to support its pension plan and state debt are too high. We first focus on the risk-free debt case and later turn to the case with risky debt. At date 0, the state pension plan has assets A_0 to invest on behalf of its pension beneficiaries. At date t, the portfolio grows to value $A_0 R_{p,t}$, where $R_{p,t}$ is the gross return on the portfolio, then the portfolio is liquidated and the full proceeds from liquidation are turned over to workers.[10] The pension's liabilities consist of a single lump sum payment L_t that must be paid to beneficiaries at time t. If the liabilities toward workers exceed the proceeds from asset liquidation, the difference is paid as a transfer from taxpayers to the pension beneficiaries. The taxes to support the pension plan at date t are given by $T_t = Max(L_t - A_0 R_{p,t}, 0)$. The consumption of the RC at date t is then given by income at date t less debt payments and taxes, $C_t = Y_t - D_t - T_t$. Similarly, consumption at date 0 is $C_0 = Y_0 - A_0$.

[8]Our framework also implicitly allows for intergenerational conflicts in which some generations use pension debt to finance their current consumption while shifting the costs of servicing the pension debt to future generations. We don't fully model the intergenerational conflict because it is beyond the scope of our paper.
[9] If instead Y_t is stochastic, then the investment choices of the pension fund would be used to hedge against the risk of Y_t as pointed out by Lucas and Zeldes (2009).
[10] The workers are assumed to get all of the cash flows from their asset portfolio even if it exceeds the liabilities. This assumption follows the practice that only workers have access to the cash flows that have been set aside in their pension funds.

The state chooses A_0 and the proportion of its portfolio to invest in risky assets, ω, to maximize the discounted expected utility of consumption of the RC subject to the constraint that the pension liabilities and debt are paid off:

$$Max_{A_0,\omega}\, U_0(C_0) + E_0 \delta^t U_t(C_t) \qquad (1)$$

where δ is the instantaneous rate at which the RC discounts the future, and U_0 and U_t are strictly increasing concave functions of utility over consumption in each period. The optimization is also equivalent to minimizing the expected utility loss due to tax payments at period t.[11] In this view of the problem, the utility functions can be interpreted as incorporating the costs of distortionary taxes.[12]

In our theoretical analysis the pension fund can invest in only two assets.[13] There is a risk-free asset with instantaneous net return r_f which we treat as fixed between dates 0 and t. One dollar invested in the risk-free asset at date 0 grows to $e^{r_f t}$ at date t. In addition, the pension fund can invest in a risky asset, which we think of as an equity index whose log-return between dates 0 and t is normally distributed:

$$\ln(R_t) \sim N\left(\left[r_f + \lambda - .5\,\sigma^2\right]t, \sigma^2 t\right)$$

where λ is the market price of risk, which is the reward for exposure to stock-market risk, and σ is the instantaneous standard deviation of the return on the risky asset. This assumption implies the gross return R_t has the functional form:

$$R_t = Exp\left(\, (r_f + \lambda - .5\sigma^2)t + \sigma\sqrt{t}\,\epsilon\,\right), \quad (2)$$

where $\epsilon \sim N(0,1)$.

The pension fund invests ω percent of its wealth in the risky asset and $1 - \omega$ percent in the risk-free asset subject to the constraint that $0 \leq \omega \leq 1$. This constraint rules out the use of leverage

[11] Utility at time t is decreasing and convex in taxes T_t.
[12] Distortionary tax representations of the problem in two period and multi-period settings are contained in Lucas and Zeldes (2009) and Epple and Schipper (1981).
[13] We consider a larger number of investment types in our empirical analysis.

by the fund as well as the use of short sales. The resulting return on the asset portfolio is given by:

$$R_{p,t} = (1-\omega)e^{r_f t} + \omega e^{(r_f+\lambda-.5\sigma^2)t+\sigma\sqrt{t}\,\epsilon}$$

Substituting $R_{p,t}$ into the expression for utility, given a choice of A_0 at time 0, the maximization for the choice of ω, the share of risky assets in the portfolio, after simplification reduces to:

$$Max_{\omega}\, E_0\, U_t\left(Y_t - D_t - \; Max\left(L_t - A_0\left[(1-\omega)e^{r_f t} + \omega e^{(r_f+\lambda-.5\sigma^2)t+\sigma\sqrt{t}\,\epsilon}\right], 0\right)\; \right) \quad (3)$$

Our analysis studies how the riskiness of the portfolio depends on risk-free interest rates, plan funding, and state finances. We measure pension underfunding by its funding ratio, which is the present value of the fund's assets divided by the present value of its liabilities. When the funding ratio is 1 or over, a plan is fully funded; when it is less than one, then the plan is underfunded. Because, as discussed further below, payments to beneficiaries are very likely to be paid in full, we treat them as risk-free and discount their value using risk-free rates. Furthermore, we proxy for the debt burden of state finances to the representative citizen by the ratio D_t/Y_t, which is the state debt to income ratio, SDI_t; and L_t/Y_t, which is the pension debt to income ratio, PDI_t.[14] With these transformations, the portfolio choice problem in equation (3) can be rewritten as:

$$\max_{\omega} E_0\, U_t[Y_t \times \left(1 - \frac{D_t}{Y_t} - \frac{L_t}{Y_t} Max\left(1 - \frac{A_0}{\frac{L_t}{e^{r_f t}}}\left[(1-\omega) + \omega e^{(\lambda-.5\sigma^2)t+\sigma\sqrt{t}\epsilon}\right], 0\right)\right)]$$

$$= \max_{\omega} E_0\, U_t\left[Y_t \times \left(1 - SDI_t - PDI_t \times Max\left(1 - FR_0\left(r_f, A_0, L_t\right)\right)\left[(1-\omega) + \omega e^{(\lambda-.5\sigma^2)t+\sigma\sqrt{t}\epsilon}\right], 0\right)\right)\right] \quad (4)$$

where $FR_0\left(r_f, A_0, L_t\right) = A_0/(\frac{L_t}{e^{r_f t}})$ is the funding ratio, which is the present value of fund assets divided by the present value of fund liabilities.

Equation (4) illustrates the role of the funding ratio in determining fund risk. If the funding ratio is greater than or equal to 1, corresponding to a fully-funded pension plan, equation (4) shows that by investing only in the risk-free asset (by setting $\omega = 0$), the proceeds from the asset

[14] In our empirical analysis we treat pension underfunding and state indebtedness as potentially different determinants of risk-taking. This is consistent with the economics literature that argues that pension debt is different from other forms of debt because its reporting is more opaque (see Glaeser and Ponzetto (2014)).

portfolio are sufficient to pay off the pension liability. If instead the funding ratio is less than one because of low assets, high future liabilities, low risk-free rates, or for any other reason, then the equation shows the plan's obligations cannot be met by investing in risk-free assets alone. Instead, the plan could attempt to meet its obligations by taking on more risk, i.e. reaching for yield, as described in Rajan (2005) or Yellen (2011), and/or plan sponsors must pay more to pension beneficiaries through taxes at time t.[15]

Equation (4) allows risk-free interest rates to also affect risk-taking by altering the risk-premium. To denote this possibility, we model the risk-premium as a function of the risk-free rate: $\lambda = \lambda(r_f)$, and refer to this as the risk-premium channel. A series of papers tracing back to Campbell (1987) study whether equity risk premia are time varying and predictable from interest rates or other variables.[16] Our reading of the literature is that in OLS univariate return predictability regressions, the evidence for risk premia predictability is weak, and the regression coefficients are time varying and unstable (Welch and Goyal, 2008, and Paye and Timmermann, 2006). However, in more recent univariate forecasting regressions that use weighted least squares to improve estimator efficiency and / or impose other sensible restrictions from economic theory (Campbell and Thompson (2008) and Pettenuzzo et al (2014)), there is statistically significant evidence that risk premia are forecastable based on Treasury Bill and Treasury Bond yields (Johnson 2019). In addition, when univariate forecasts are combined, forecastability improves, especially near recessions (Rapach et al., 2010). For illustrative purposes, in our theoretical analysis we rely on an estimate of how Treasury-bill rates affect the equity premium in a univariate regression with economic restrictions based on Pettenuzzo et al (2014). They estimated the following relationship towards the end of their data sample:[17]

[15] Rajan (2009) discusses the need for private insurance companies to increase portfolio risk when rates fall: "Insurance companies may have entered into fixed rate commitments. When interest rates fall, they may have no alternative but to seek out riskier investments – if they stay with low return but safe investments, they are likely to default for sure on their commitments, while if they take riskier but higher return investments, they have some chance of survival." Similarly, Yellen (2011) states "[I]mportant classes of generally unlevered investors (for example, pension funds) are reportedly finding it difficult in the present low rate environment to meet nominal return targets and may be reaching for yield by assuming greater interest-rate and credit risk in their portfolios."

[16] For reviews of this literature see Rapach et al (2013), and Timmermann (2018).

[17] The estimates reported in the published version of Pettenuzzo et al (2014) end in the mid-1980s. We thank the authors for providing us with estimates from December 2010, the end of their sample, and the middle of our sample. We use their constrained estimates above. Their estimates for the unconstrained equation are $\lambda = .008 - .09 \times r_f$ where r_f is the yield on 3-month Treasury bills.

$$\lambda(r_f) = .004 - .007 \times r_f, \qquad (5)$$

where r_f is the yield on 3-month U.S. Treasury bills, which we will interpret as the risk-free rate. Inserting this expression for λ inside equation (4) shows that the short-term risk-free rate affects risk-taking through the funding ratio and the equity premium.

Because our model (see equation 4) shows funding ratios are determined by the value of funds' assets, the face value of funds liabilities, and risk-free rates, plans' funding ratios can vary independently from risk-free rates.[18] Therefore, in our theoretical analysis we treat funding ratios and risk-free rates as separate determinants of funds' risk-taking. Moreover, the reach-for-yield channel operates through the funding ratio; the risk-premium channel operates through interest rates, and the two interact in determining risk-taking.

As a first cut for providing intuition on how the funding ratio and interest rates affect risk-taking, we assume the RC has a utility function for time t that has a power utility form:

$$U_t(C_t) = -(C_t)^{-k} \text{ for } k \geq 1.$$

We focus on the case of k=10 in all of numerical analysis below, and for now focus on the case of the State Debt-to-Income Ratio = 3%, and we assume that state debt is risk-free and will not be defaulted upon. We then numerically solve for the optimal portfolio choices and risk-taking as a function of the funding ratio and the risk-free rate. The main results from the numerical analysis are presented in Figure 2. The figure shows that risk, here measured by the proportion of the portfolio invested in risky assets, increases in pension underfunding. This finding is consistent with reach for yield behavior because risk increases as the funding ratio decreases. In addition, holding underfunding fixed, lower risk-free rates are associated with more risk-taking through the risk-premium channel from equation (5). Figure 2 also shows there is an interaction effect: The marginal effect of underfunding on risk-taking is larger when interest rates are lower, and the marginal effect that interest rates have on risk-taking is more pronounced when underfunding is higher. This shows the reach-for-yield and risk premium channels interact, and one should account for the interaction that these different channels have on risk taking. These

[18] For example, during the 2007-09 crisis, many funds' risky assets performed poorly, reducing funding ratios by amounts that varied based on plans' exposures to risky assets.

findings provide justification for our empirical specification that studies how funding ratios, interest rates, and their interactions affect risk taking.

Equation (4) also shows state finances measured by debt to be paid at date t as fraction of state income SDI_t also affects risk-taking. To investigate the role of state finances on risk-taking, we consider two circumstances, the first being when the state will not default on its debt. We model this case with our assumption that its debt will be covered by taxes on income at date t. Under our assumptions, Figure 3 shows that a lower funding ratio increases risk for a range of levels of state debt. This is consistent with the reaching-for-yield implication in Figure 2. The figure also shows that greater state debt relative to income leads to reduced risk-taking by the state's pension fund. The intuition for this result is when the state is more indebted, then the taxes it faces if the pension fund performs poorly have a much greater effect on the utility of the representative citizen than it does if the state is less indebted. To avoid the more severe consequences, the pension fund takes less risk if the taxpayers have to make up the shortfall for large losses by the pension fund.

A more general model of state finances would account for the possibility that some states might actually default on and/or renegotiate their debt, and that all else equal larger pension liabilities increase the risk of default or renegotiation (see Boyer, 2018). To model this in a simple way, we assume that when a state defaults on its debt, it defaults on all of its debt, and when it does so it incurs a penalty measured in utility terms that is proportional to the amount of its debt. Specifically, the representative citizen receives the following utility when it does not and does default at date t:

$$U_t(.) = \begin{cases} U_t(Y_t - D_t - T_t) & No\ Default \\ U_t(Y_t - T_t) - \gamma \times\ D_t & Default \end{cases}$$

where $\gamma \times D_t$ ($\gamma > 0$) is the penalty for defaulting, and T_t represents the taxes if any that need to be paid to pension beneficiaries at date t. The state will choose to default on its debt if doing so raises its utility. Analysis below shows the state will choose to default on its debt when its pension fund assets perform poorly enough. The option for the state to default shifts some of the downside risk of the pension fund's performance from state taxpayers to the holders of the state's debt. The ability to shift some of the downside risk of the pension fund to debt holders affects PPFs' incentives to take risk. Risk shifting is only valuable for the state if it sometimes chooses to default on its debt. Therefore, to understand incentives to shift risk, it is necessary to understand when default is valuable. Algebra shows that to a second-order approximation the state will choose to optimally default when the taxes required to support the pension plan satisfy the condition:

$$\gamma \leq U_t'(Y_t - T_t) - 0.5 * U_t''(Y_t - T_t) * D_t.$$

Rearrangement shows default is optimal when:

$$\gamma \leq U_t'(Y_t - T_t) \times \left(1 + .5 \times CRRA \times \frac{SDI_t}{1 - \frac{T_t}{Y_t}} \right),$$

where $CRRA$ is the coefficient of relative risk aversion. Because utility is increasing and concave, the condition shows for a given risky asset portfolio, the default condition is more likely to be satisfied when γ and income Y_t are smaller, or when taxes T_t, relative risk aversion $CRRA_t$, or the state debt to income ratio SDI_t are greater. In addition, the determinants of T_t such as the funding ratio and risk-free rates also affect the default condition. Because these variables affect the decision to default, they also affect incentives to shift risk to state bondholders.[19] To further examine how the possibility of shifting risk through default affects the pension funds' investment decisions, we numerically solve for the pension funds' optimal investments when the state can default on its debt. The results are presented in Figures 4 and 5. Figure 4 shows that for low funding ratios, as the amount of debt to income increases from 0, risk-taking goes down, just as it did in Figure 3. But, unlike Figure 3, when the debt-to-income ratio increases enough, risk-taking suddenly becomes much higher. This non-monotonicity is due to the risk-shifting effect. Moreover, Figure 4 shows an interaction between state debt-to-

[19] A related but mechanism for risk-shifting to occur comes from recalling that pension claims are low risk to beneficiaries because they are very senior in the state's debt structure. As noted by Ivanov and Zimmerman (2018) when states are more indebted, they tend to issue more senior debt (bank debt) to increase their debt capacity, and the borrowers that do this are high risk. Because pension liabilities are very senior, paying a greater share of workers' salary through pension benefits represents another channel to shift risk from state taxpayers to state bond holders.

income and pension underfunding: risk-shifting occurs for higher levels of state debt when a pension plan is more underfunded. In Figure 5, an additional interaction effect exists between state debt and the risk-free rate. More specifically, the effect that risk-free rates have on risk-taking through the risk premium channel can depend on the state debt-to-income ratio. This interaction is present when the state cannot default on its debt (not shown) and also in some circumstances when the state can default on its debt. In particular, Figure 5 shows that when the state debt-to-income ratio is high enough, a decline in the risk-free rate leads to a non-monotonic increase in risk-taking.

In our empirical examination of the role of state finances as a driver of pension fund risk-taking, we study whether higher state debt to income is associated with higher pension fund risk-taking as would be consistent with the state pension funds shifting risk to debt holders. In addition, we examine if there is an interaction effect between the ratio of debt-to-income and interest rates.

In summary, the theory in this section provides motivation for our empirical analysis and its interpretation. Five main results emerge from our theoretical analysis:

1. The effect that the funding ratio has on risk-taking captures the effect of reach for yield. In our regression analysis we interpret the coefficient on the funding ratio as capturing reach-for-yield effects.
2. After controlling for funding ratios, interest rates may also affect risk-taking because they affect risk-premia. We interpret the coefficient on interest rates in our regression analysis as the risk-premium effect.
3. The reach-for-yield and risk-premium effects interact in theory. We allow for their interaction in our empirical specification.
4. How state finances affect risk-taking depends on whether states can shift risk to their debt holders. If they do shift risk, then state debt-to-income ratios that are large enough lead to higher risk-taking, especially for underfunded pension plans. If states don't shift risk, then greater state debt is predicted to lead to lower risk.
5. The effect that state finances have on risk-taking also interacts with the risk premium channel of risk-free rates. If states can default on their debt, then for state debt-to-income ratios that are high enough, lower risk-free rates lead to higher risk-taking.

The appendix provides further comparative statics results for the case when state debt is risk free.

3. Data and Measurement of Risk, Underfunding

This section describes our data and the methodology we use to measure pensions funds' asset-risk as well as their funding ratio. We measure net-worth risk using a similar approach that is discussed in section 6.

3.1 Data

Publicly available data on PPFs' investment performance, risk-taking, and the value of liabilities is limited and incomplete. In this section we describe our data on PPFs and the methods we use to measure the PPFs' asset-risk and underfunding despite the data limitations. Our main data set on state and local public pension plans is the Public Plans Database (PPD) from the Center for Retirement Research at Boston College.[20] Our PPD data contains plan-level annual data from 2001 through 2016 for 170 public pension plans: 114 administered by states and 56 administered locally. This sample covers 95 percent of public pension plan membership and fund assets nationwide.[21] The data set includes annual (by fiscal year) observations on the returns on each fund's assets, the percentage of the fund's portfolio invested in six main asset categories (equities, fixed income, real estate, cash, alternatives, and other), the market (fair) value of funds' assets, and the actuarial value of funds' liabilities.[22] Our sample ends in 2016 because the structure of the PPD data was changed after this date, and our data for rediscounting liabilities (see below) is hand-collected for 2014-2016. While non-trivial, it is feasible to update the methodology and data collection for more recent years.

Descriptive statistics on our PPD data are contained in Table 1. On average across time and funds, the largest asset holdings were equities and fixed income (54 and 27 percent of total assets, respectively), followed by alternatives and real estate (10 and 5 percent, respectively).

[20] The PPF data is available at: http://publicplansdata.org/public-plans-database/download-full-data-set/.
[21] The sample of plans is a carry-over from the Public Fund Survey (PFS), which was constructed with an eye toward the largest state-administered plans in each state, but also includes some large local plans such as New York City ERS and Chicago Teachers.
[22] The data on holdings in some asset categories are further subdivided into foreign and domestic subcategories. Because the subcategories are not well populated, we combine like subcategories into the six broader categories noted above.

The value of assets represented only 80 percent of the actuarial value of liabilities, pointing to substantial underfunding. Figure 1 also shows the evolution of these variables over time: As shown in panel (b), the ratio of actuarial assets to liabilities (henceforth funding ratio, or FR) declined from about 100 percent in 2001 to little more than 70 percent in 2016. In panel (c), the shares of equities and fixed income assets in funds' portfolios declined, while the share of alternative assets rose steeply from 5 percent in 2001 to almost 20 percent in 2016.

The data on funds' risk exposures is coarse. Therefore, to infer funds' risk, in the next subsection we bring in additional information by assuming that funds' returns in each asset category can be decomposed into the return on an asset-category index and fund-specific risk. Furthermore, we assume that each category index return is spanned by a linear combination of returns on tradable market indices and estimate the linear combination for each category. Although we could choose a wider set of tradeable indices for our analysis, for now we rely on the 17 tradable indices, which are detailed in Table 2.

To measure underfunding, we would like to compare the market value of funds' assets with the value of their obligations to pension beneficiaries. However, as noted in the introduction, the actuarial value of liabilities is measured using GASB standards that discount liability cash flows based on the properties of the funds' assets, not their liabilities. This approach to liability valuation is inconsistent with finance theory and has been widely criticized (see for example Brown and Wilcox, 2009). Following the recommendation of Brown and Pennacchi (2016) we instead discount their promised cashflows using risk-free rates since doing so measures the cost of providing the promised cashflows to beneficiaries with certainty. Comparing this risk-free valuation of a fund's liabilities against the present value of its assets is a sensible way of ascertaining the extent to which a pension plan is underfunded.[23] We do not observe the exact timing of funds' promised cashflows, but we can approximate risk-free discounting by applying Rauh's (2017) methodology: we infer the discount rates that each fund should have used from

[23] For the purposes of assessing the extent of plan underfunding, it does not make sense to value liabilities using their market value. For example, if a pension fund was very likely to default on its obligations because of poor pension funding, then the market value of the funds liabilities would be essentially 0, while the market value of its assets is positive. But, in this case the fact that the market value of assets is greater than the market value of the funds liabilities does not imply the fund is well-funded. In fact, the opposite is true. Conversely, if the market value of assets is greater than value of liabilities with risk-free discounting, then the assets value can, if invested in risk-free assets, be used to provide the promised liability cashflows with certainty.

data on the U.S. Treasury zero coupon yield curve.[24] In addition we infer funds interest-rate sensitivity. Using the two together we approximate what the value of funds liabilities would have been if they had been discounted using risk-free rates. We refer to the entire process as rediscounting funds' liabilities. The information on the sensitivity of funds' liabilities to interest rate changes comes from GASB Statement 67 or from funds' Comprehensive Annual Financial Report (CAFR). This sensitivity information is only available starting in 2014, when GASB 67 required funds to report interest rate sensitivities.[25] GASB 67 data was available for 108 of the 170 funds in the Boston College dataset.[26] As a robustness check, following Lucas (2017), we also revalue the liabilities with discount factors based on a high-quality corporate bond yield curve.[27]

We provide further information on our data in the discussion of results. The following two subsections discuss how we measure funds' risk and how we rediscount (i.e. revalue) their liabilities.

3.2 Risk Measurement

There are many possible measures of funds' asset-portfolio risk that could conceivably be used in our analysis. We have chosen to focus on funds' 5% annual value-at-risk (VaR), which measures the minimum potential loss that a fund could sustain over a one-year horizon with 5% probability. For example, if the probability a fund could lose 12% or more over the next year is 5%, then the funds' 5% value-at-risk is 12%. Put differently, an annual loss of 12% or more is expected to occur in one out of every 20 years for the fund. An advantage of using VaR to measure portfolio risk is that it is comprehensive: it depends on the joint distribution of the

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[24] The zero coupon Treasury yield curve is computed using the methodology in Gurkaynak et al (2006). This data is updated daily, and is provided by the Federal Reserve Board at: https://www.federalreserve.gov/pubs/feds/2006/200628/200628abs.html.

[25] GASB Statement 67 disclosures require plans to disclose their Net Pension Liability (NPL) under alternative assumptions of the discount rate being 1 percentage higher and 1 percentage lower.

[26] We gathered CAFRs from public pension fund websites. In many instances, GASB 67 information was not included in the materials posted on the website.

[27] We use the return on the Citigroup Treasury Model Curve, which is created through a multistep process, starting with the Citigroup Corporate Index and taking corporate bonds rate AA-, AA, and AA+ by S&P. The data includes yields ranging from half a year to 30 years, reported monthly.

returns on all of the assets in the fund's portfolio. In addition, VaR changes through time as the joint distribution of asset returns changes.

By contrast, some of the other risk measures that have been used in the literature on pension funds' risk-taking are less comprehensive or do not capture time variation in risk. For example, some papers in the literature have measured risk as the share of equities in a fund's portfolio. This measure is not comprehensive because it does not consider all portfolio assets. In addition, it does not capture time variation in the risk of portfolio assets. For example, two portfolios that have the same equity shares have different value at risk in 2006 and 2009 because market conditions in 2009 were much more volatile than in 2006. A risk measure that only focused on the share of equity holdings would be insensitive to this difference.

In fairness, an important part of the reason VaR has rarely been used in the academic literature on pension fund risk is because computing funds' VaR requires data on both funds' risk exposures and on the returns of the assets the funds are exposed to. The data available are much more limited; we know funds' total annual return as well as portfolio weights for six asset categories: equities, fixed income, real estate, cash, alternatives, and other. Another complicating factor is the data is not time-synchronous because returns and weights are both measured at the end of each fund's fiscal year.[28]

An important contribution of our paper is that we develop a methodology to measure funds' VaR despite the data limitations. To do so, we make the following assumptions:

Assumption 1: Each fund i's return for asset category c at time t, $r_{c,i,t}$, can be expressed as the projection of the return onto a risk-category index $r_{c,t}$ that is common across funds plus a fund-specific residual return $\epsilon_{c,i,t}$ that is not correlated with any of the category return indices:

$$r_{c,i,t} = \alpha_{c,i} + r_{c,t} + \epsilon_{c,i,t} \qquad (7)$$

[28] In addition, in the Public Plans Database, returns are sometimes reported gross of fees, sometimes net of fees, and sometimes it is unclear which reporting convention is being utilized. This inconsistency adds noise to our measures of risk, which should bias us against finding a relationship between our measures of risk and the hypothesized determinants of funds risk-taking. Ideally, since fees charged for managing funds are not necessarily related to funds risk', the returns data would be net of fees.

Assumption 2: The return of each risk-category index $r_{c,t}$ is spanned by the return of publicly available asset return indices indexed by j:

$$r_{c,t} = \alpha_c + \sum_j r_{j,t}\theta_{c,j} \qquad (8)$$

Assumption 1 implies that, within each risk category, fund returns for that category can be decomposed into a category index that is common across funds, plus a fund specific component that is uncorrelated with all of the category return indices.[29] Assumption 2 conveys that the returns for each category index can be spanned by the returns of publicly available indices. Assumption 2 should be satisfied if the category return indices are well diversified, so they only depend on pervasive risk-factors (i.e., risk factors that affect a significant part of the economy), if the publicly-available indices are well diversified, and if the same factors that drive the returns on the category indices also drive the returns of the publicly-available indices.

Using the arbitrage pricing theory from finance, the intercepts in equations (7) and (8) should be nonzero only if they represent compensation for non-diversifiable risks that are priced by the market but not included as regressors in the equation; i.e. it is compensation for the risks in the residuals of the equations. Because the residuals in equation (7) are by assumption fund-specific and not captured by the category return indices, they should receive a zero price, hence $\alpha_{c,i} = 0$ for all c and i. Relatedly, in equation (8) because there are no residuals by assumption, α_c= 0 for all c. The implications of this reasoning are summarized in the following assumption:

Assumption 3: $\alpha_c = 0$ for all c, and $\alpha_{c,i} = 0$ for all c and i.

To illustrate how these assumptions make it possible to compute fund risk, note that each fund i's asset return can be written as the sum of its portfolio weights times its asset return in each category:

$$r_{i,t} = \sum_c w_{i,c,t} r_{c,i,t}.$$

[29] Our analysis admits a slightly more general framework in which funds return within each category have the form:

$$r_{c,i,t} = \alpha_{c,i} + \beta_c r_{c,t} + \epsilon_{c,i,t},$$

with β_c common across funds. Without loss of generality, we have normalized β_c to 1 in our analysis.

Substituting in for each fund's category return from Assumption 1 and the decomposition of the category return from Assumption 2, this equation can be rewritten as a regression equation in which the right-hand side variables are an intercept term and funds' portfolio weights interacted with the returns on the publicly-available indices:

$$r_{i,t} = \sum_c w_{i,c,t}(\alpha_{c,i} + r_{c,t} + \epsilon_{c,i,t})$$

$$= \sum_c w_{i,c,t}\left[\alpha_{c,i} + \left(\alpha_c + \sum_j r_{j,t}\theta_{c,j}\right) + \epsilon_{c,i,t}\right]$$

$$= \sum_c w_{i,c,t}\left[\alpha_{c,i} + \alpha_c\right] + \sum_c \sum_j w_{i,c,t}\, r_{j,t}\theta_{c,j} + \sum_c w_{i,c,t}\, \epsilon_{c,i,t}$$

$$= \alpha_{i,t} + \sum_c \sum_j w_{i,c,t}\, r_{j,t}\theta_{hc,j} + \epsilon_{i,t}$$

$$= \alpha + \sum_{c=1}^{C} \sum_{j=1}^{J} w_{i,c,t}\, r_{j,t}\theta_{c,j} + u_{i,t}, \qquad (9)$$

where

$$u_{i,t} = \epsilon_{i,t} + \left(\alpha_{i,t} - \overline{\alpha_{i,t}}\right), \qquad (10)$$

and $\alpha = \overline{\alpha_{i,t}}$ is the average value of $\alpha_{i,t}$ when averaged over all pension funds and time periods.

Equation (9) is a regression equation with a fairly large number of regressors. In particular, in what follows we estimate the intercept in equation (9) and the slope coefficients $\theta_{c,j}$ corresponding to 17 publicly traded indices (J=17) interacted with the portfolio weights for 6 categories of assets (C=6), for a total number of J×C=102 slope coefficients.

The estimated $\theta_{c,j}$ coefficients identify the stochastic part of the category return indices denoted by $\widetilde{r_c}$, whose time t realization is given by $\widetilde{r_{c,t}} = \sum_j r_{j,t}\theta_{c,j}$. The regression residuals $u_{i,t}$ that are recovered from estimation of the regression consist of two pieces. The first piece is $\epsilon_{i,t}$, which is the stochastic part of each fund's returns that is not explained by the category return indices. Additionally, $\epsilon_{i,t}$ is a component of the risk of funds' investments. The second piece is $\left(\alpha_{i,t} - \overline{\alpha_{i,t}}\right)$, which is a function of funds' portfolio weights and of the regression intercepts in equations (7) and (8). Importantly, the second piece of the residual is not a source of investment risk for the funds, hence if the residual has this component, then the funds' risk will be slightly overstated because of it. It turns out the second piece of the residual will be uniformly equal to zero if the regression intercepts in equations (7) and (8) are zero, as assumed using finance theory in Assumption 3. This implies that under Assumptions 1 – 3, the regression residuals in equation (9) will only contain the stochastic part of funds' investment risks that are not explained by the category return indices. In summary, our methodology can estimate the stochastic part of the funds' category returns and residual risks. Given these estimates, we can compute funds' value at risk through time. In what follows we describe how we estimate funds' value at risk and how we estimate the regression parameters in equation (9).

To estimate funds' value at risk, we make the following assumptions for the return of the category return indices and the idiosyncratic risks:

Assumption 4: Let $R_{c,t}$ denote the C×1 vector of category return indices in year t and let $\epsilon_{i,t}$ denote the residual return on fund i's investment portfolio in year t. Then,

$$R_{c,t} \sim N(\mu_t, \Sigma_t)$$

$$\epsilon_{i,t} \sim N(0, \sigma_\epsilon^2)$$

$$Cov\left(R_{c,t}, \epsilon_{i,t}\right) = 0.$$

Note that the category return indices and the residual returns are modeled as Gaussian for simplicity. Additionally, for simplicity the residual returns are for now modeled as independently and identically distributed (i.i.d.) across funds and time, and by Assumption 1 their covariance with the category indices is 0. There is scope to relax the conditions in Assumption 4 if needed.

In this paper we have chosen to measure funds' value-at-risk as the 5th percentile of the unexpected component of a fund's return distribution, and then express this quantity as a loss.

This is best illustrated using an example. If fund $i's$ return has distribution $r_i \sim N(\mu_i, \sigma_i^2)$, then the unexpected component of the fund's return is the return less its expected value $r_i - \mu_i$. Furthermore the 5th percentile of the funds unexpected return distribution is $\Phi^{-1}(.05)\sigma_i = -1.65\sigma_i$. Expressed as a loss, $VaR(.05) = 1.65\ \sigma_i$.

Using analogous reasoning, a fund with portfolio weights $w_{i,t}$ at the beginning of time *t* has annual 5 percent value at risk given by

$$VaR_{i,t}(5\%) = 1.65\sqrt{w'_{i,t}\Sigma_t w_{i,t} + \sigma_\epsilon^2} \quad (11)$$

In order to compute VaR, we need measures of Σ_t and σ_ϵ^2. Because our data on pension fund returns is annual and has less than 20 annual time series observations, it would not be possible to estimate a time-varying Σ_t matrix using the short span of annual data on publicly-available indices that is used to estimate equation (9). To overcome this problem, we estimate Σ_t using daily data on the public indices returns. In particular, using the estimated coefficients for $\theta_{c,j}$ and daily data on public indices returns, we construct daily series of the returns on the category indices. Let $R_{c,d_1,t-1}, \ldots R_{c,d_N,t-1}$ denote the vector of estimated daily returns on the category indices for each trading day of the calendar year $t-1$. Our estimate of the annual variance-covariance matrix in calendar year t conditional on daily returns in year $t-1$ is:

$$\Sigma_t = 250 \times \frac{1}{N}\sum_{k=1}^{N} R_{c,d_k,t-1} * R'_{c,d_k,t-1}. \qquad (12)$$

Thus, Σ_t is equal to the conditional variance-covariance of daily category index returns scaled up by 250, the number of business days per year, to make it a variance-covariance matrix of annualized category index returns.[30]

Although we can estimate Σ_t using daily returns for the estimated category indices, we cannot estimate σ_ϵ^2 using daily data because we only observe pension fund returns annually, and hence can only observe annual residuals from equation (9). Therefore, to estimate σ_ϵ^2 we simply rely

[30] In estimating Σ_t we made an assumption that expected daily returns are equal to 0. This assumption approximates the reality that expected returns at a daily frequency are close to 0. Because high frequency estimation of expected returns is very noisy, when estimating the variance-covariance matrix of high frequency returns it is better to set expected returns to 0 rather than trying to estimate them.

on the estimated residuals from equation (9) and estimate σ_ϵ^2 as the sample variance of the residuals.

Our approach for estimating each fund's value at risk has many advantages. The first and most important is we estimate the category return indices that best explain funds' annual returns and portfolio weights. This improves on other approaches that do not measure time variation in risk or assume that the returns in different asset categories are the returns of a particular traded index**.**

The second advantage is by using daily data we are able to overcome some of the limitations in estimating funds' risk on the basis of annual data. In particular, our approach produces estimates of funds' risk that vary through time because of changes in the variance-covariance matrix, and because of changes in funds' asset composition.

The third advantage of our approach is that we can infer the riskiness of funds by taking into account their different definitions of fiscal years. Because the fiscal years of different pension funds can end on different dates, their reported total annual returns span different intervals of a calendar year. Therefore, in the regression described by equation (9), it is important to match the total annual returns with annual market index returns computed in a manner consistent with the fiscal year definition for each fund. We then compute the VaR measure for calendar years to allow for a consistent comparison of riskiness across funds. To construct our risk measures, it is necessary to estimate the $\theta_{c,j}$ coefficients from equation (9). This task involves the estimation of a large number of parameters with a relatively small sample of data. Therefore, we follow a method that avoids statistical problems associated with overfitting. Based on the fact that to some extent some of the 17 indices are correlated with each other, and given that there might be some common factors driving these indices, we assume that the dependent variable in equation (9) can be closely approximated by using a small subset of these indices for each asset category, which is the approximate sparsity assumption in our paper.[31] This assumption allows us to use a penalized estimation method to estimate the model parameters.

[31] As discussed by Hastie et al. (2015), there are two settings of the sparsity condition. One is the so-called hard sparsity, in which only a small number of the true coefficient parameters are nonzero. This assumption is overly restrictive, so they also consider the other one, which is the so-called weak sparsity where the true coefficient parameters can be closely approximated by vectors with few nonzero entries, in order words, coefficients can be

We use a two-step procedure to estimate the model in equation (9). First, we use a penalized regression method to select the most relevant subset of indices for each asset category. Second, we proceed with the estimation based on the selected indices only. In the first step, we use LASSO regression for equation (9) (i.e., Least Absolute Shrinkage and Selection Operator, see Frank and Friedman, 1993, Tibshirani, 1996; and James et al., 2013) to select the most relevant indices for each asset class and shrink the coefficients on the other variables to 0, essentially eliminating them from the regression. [32, 33] After using LASSO, the number of relevant explanatory variables shrinks considerably. Then in the second step, we apply OLS estimation with only the selected indices used as explanatory variables to obtain the estimates of $\theta_{c,j}$.

This two-step procedure estimator, the OLS post-LASSO estimator, is well known in the literature on high-dimensional sparse models. It has been shown that the OLS post-LASSO estimator performs at least as well as the LASSO estimator in terms of the rate of convergence, and has the advantage of a smaller bias (see Belloni and Chernozhukov, 2013). [34] This desirable characteristic holds even if LASSO may omit some components: as long as these components have relatively small coefficients, the OLS post-LASSO estimator still benefits from a high rate of convergence and smaller bias. Regarding the penalized selection method for the first stage, LASSO is a popular and powerful approach but not the only one. Researchers can choose different penalized methods from a variety of choices, for example, threshold LASSO, the Dantzig selector, etc.[35]

Finally, once we obtain the estimates of coefficients $\theta_{c,j}$ with the OLS post-LASSO estimator (reported in Table 3), Assumption 2 and the data on returns of publicly-traded indices allow us to

estimated based on a subset of the explanatory variables and letting the coefficients of the rest explanatory variables being zero. This weakly sparsity is more general and has been widely used in the literature of Lasso-type penalized methods. The approximate sparsity condition used in both this paper and Belloni and Chernozhukov (2013) is the weakly sparse condition.

[32] In James et al (2013), the main description of LASSO can be found on pages 219 through 227. Useful introductory notes can also be found at: https://onlinecourses.science.psu.edu/stat857/node/158/

[33] Lasso has been used and its properties have been researched in many papers, for instance, Bickel et al. (2009), Meinshausen and Yu (2009), Van de Geer (2008), Zhang and Huang (2008), and so on.

[34] Belloni and Chernozhukov (2013) investigate the properties of OLS post-LASSO in the mean regression problem; Belloni and Chernozhukov (2011) also studies the post-penalized procedures, but different problem of median regression.

[35] Belloni and Chernozhukov (2013) also consider using the threshold LASSO for the first stage. For more details about the Dantzig selector, readers can refer to Bickel et al. (2009), Candes and Tao (2007), and among many others.

compute the daily returns of category indices $R_{c,d_1,t-1}, \ldots R_{c,d_N,t-1}$ for each asset class c and calendar year $t-1$. Moreover, we identify each fund's residual risk, then use the estimated joint dynamics of category indices and the residual risk to compute funds' VaR as described above.

Figure 6 shows funds' VaR through time with a time-varying variance-covariance matrix for asset returns across different asset categories estimated via equation (12). This is the main measure of asset portfolio risk in our empirical analysis.[36] The VaR changes over time due to changes in both market conditions and portfolio weights, while the alternative measure in panel (b) only changes due to the funds' portfolio weights. Moreover, we find that the majority of the VaR variation comes from the asset allocation risk component (about 85% on average) rather than the fund-specific risk component in equation (11), implying that the changes in market behavior and portfolio weights explain most of funds' riskiness.

3.3 Measurement of Underfunding

In this subsection, following Brown and Pennacchi (2016) we revalue the PPFs' liabilities using a risk-free discount rate so that the plans' funding ratios reflect their funding shortfalls relative to the value of the cashflows that funds have promised to their beneficiaries. To do so, following the approach in Rauh (2017) and Novy-Marx and Rauh (2011) we rediscount PPFs' liabilities using a discount rate that is equal to the yield of a zero-coupon U.S. Treasury bond whose duration matches the duration of the funds' liabilities. This is the correct discount factor to use if all of the beneficiaries' cash flows occurred at the duration of the liabilities, and is otherwise a reasonable approximation of the correct discount factor. To adjust the value of the liabilities, it is necessary to approximate how they would be valued if discounted using the risk-free rate appropriate for the liabilities' duration instead of the discount rate used to value the liabilities for accounting purposes.

The extent of each pension plan's underfunding is measured by its funding ratio, which is an estimate of the present value of its assets over its liabilities:

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[36] Alternatively, we compute an VaR measure that is based on the annualized variance-covariance matrix of the monthly returns computed for the entire sample period and hence does not change over time, shown in the Appendix Figure A.1. This measure changes over time only due to the funds' portfolio weights, hence it displays less variation than the main VaR measure.

$$Funding\ Ratio\ (FR) = \frac{Actuarial\ Assets\ (AA)}{Total\ Pension\ Liabilities\ (TPL)}. \qquad (13)$$

In this expression, a lower ratio reflects greater underfunding. We use two measures of total pension liabilities (TPL) to compute funding ratios. One is the amount of TPL_r reported by the PPFs themselves, which are discounted based on their reported expected rates of return r on their asset portfolios. The other measure is the one we obtain following the approach in Rauh (2017) denoted as $TPL_{r'}$ in equation (14), which is an approximation of what *TPL* should be if discounted at the correct duration-matched, risk-free, zero-coupon rate r'. Rauh computes $TPL_{r\prime}$ using a second-order Taylor series of how TPL_r should change if discounting takes place at rate r' instead of r. Expressed in terms of duration and convexity, the second-order Taylor series has form:

$$TPL_{r'} = TPL_r - TPL_r * Duration * \Delta r + 0.5 * TPL_r * Convexity * (\Delta r)^2 \qquad (14)$$

where $\Delta r = r' - r$, with r' denoting the duration-matched zero-coupon Treasury yield and r the funds' original discount rate.[37] To compute the second-order approximation, we use estimates of duration and convexity available only for the most recent years (2014-2016).[38] GASB 67 provides information on the value of funds' TPL when valued at its reported discount rate r as well as when discount rates increase or decrease by one percentage point. With this information, we approximate the duration and convexity as:

$$Duration = -\frac{TPL_{\{r+0.01\}} - TPL_{\{r-0.01\}}}{0.02 * TPL_r}, \qquad (15)$$

$$Convexity = \frac{TPL_{\{r+0.01\}} + TPL_{\{r-0.01\}} - 2 * TPL_r}{(0.01)^2 * TPL_r}. \qquad (16)$$

We use data on PPFs' TPL and funding status for the year 2015 to illustrate the importance of rediscounting the PPFs' liabilities so that funding ratios better reflect funding shortfalls. In Figure 7 (left panel), the chart plots funds' reported TPL_r (on the horizontal axis) against their rediscounted $TPL_{r'}$ obtained like in equation (14) (on the vertical axis). If rediscounting made

[37] The original discount rate r is available in the GASB statement 67 reports as "Current Discount Rate." Alternatively, r' is the rate on the Treasury yield curve that matches the duration of each PPFs' liabilities.
[38] PPFs started reporting the sensitivity of the value of their liabilities to interest rate changes under GASB 67 in 2014.

no difference, all observations would line up along the 45 degrees line. However, most observations are situated above the 45 degree line, showing that rediscounting boosts the present value of liabilities to almost double the reported amounts. Also in Figure 7 (right panel), the chart shows a similar relationship between the funding ratios measured using reported and rediscounted liabilities for 2015. In this case, the observations fall below the 45 degrees line, showing that rediscounting boosts liabilities and, as a result, reduces the funding ratios by almost half. Notably, the plot observations do not suggest a linear relationship between the original and adjusted funding ratios. Put differently, re-computing funding ratios with rediscounted liabilities is not equivalent to a linear rescaling of the original funding ratios. Therefore, in linear regressions to explain risk-taking, the rescaled funding ratios will change the ability of the funding ratio variable to explain risk-taking, so our linear regressions can in theory reveal which measure of underfunding better explains funds' risk-taking. Relatedly, if funds' risk-taking behavior is in practice related to the measure of underfunding that uses risk-free rates for discounting liabilities, our approach corrects for the measurement error that occurs if liabilities are discounted at expected rates of return that differ across funds (because of differences in funds' asset portfolios) and are not risk-free.

4. Empirical Analysis of Risk-Taking Behavior

In this section we empirically study how the PPFs' asset risk is related to plan underfunding, to risk-free interest rates, and to state finances. We use three separate frameworks for our empirical analysis. First, we study the cross-sectional relation between PPFs' riskiness and lagged funding ratios across funds at one point in time (the year 2016), for which the best data is available to measure duration and convexity, rediscount liabilities, and measure plan underfunding.[39] Second, we estimate the cross-sectional relation between risk-taking and funding ratios for each year in the sample. Third, in a panel data context, we study the importance of funding ratios, risk-free rates, and the fiscal condition of the funds' state sponsors as determinants of risk-taking, while allowing for interactions among them. We use our theoretical analysis from Section 2 to interpret our empirical findings.

[39]We use a recent year for analysis because we need to rely on GASB 67 data, a relatively new reporting, in order to rediscount reported liabilities.

4.1 Cross-Sectional Results for 2016: Asset Risk vs. Underfunding

Figure 8 illustrates the cross-sectional relation between the VaR-based measure of risk for 2016 on the vertical axis and the one-year lagged funding ratios on the horizontal axis, using either the original TPL_r reported by funds (left panel) or the $TPL_{r'}$ obtained with rediscounted liabilities (right panel) to compute funding ratios.

Several conclusions emerge from the comparison of the two panels. First, the link between funds' riskiness and the one year-lagged funding ratios is negative and statistically significant in both cases, i.e., funds with ex-ante lower funding ratios had asset portfolios with higher risk. This is consistent with funds' reaching for yield, as described in Section 2. Second, rediscounting liabilities shifts the entire distribution of funding ratios to the left, as discussed earlier. Third, the approach with rediscounted liabilities results in a steeper slope coefficient and higher statistical significance for the link between riskiness and funding ratios, as well as a higher regression R-squared (right panel). The increased statistical significance and higher R-squared are consistent with an interpretation that funding ratios, when computed using liabilities discounted with risk-free rates, are related to funds' risk-taking. Furthermore, it supports the idea that funding ratios based on liabilities discounted with expected returns on assets are measured with error, since classical measurement error reduces both goodness of fit and statistical significance.

4.2 Cross-Sectional Results over Time: Asset Risk vs. Underfunding

To examine the relation between PPFs' riskiness and funding ratios over time, the analysis is constrained by the fact that the duration and convexity data needed to rediscount liabilities, as well as total pension liabilities were only collected starting in 2014 (based on a new GASB accounting requirement). To overcome this data limitation, we assume that the duration and convexity of TPLs reported for the most recent years are relevant for PPFs' liabilities over time. Thus, we extrapolate the average duration and convexity computed for 2014-2016 across the full sample period, while adjusting them for fund- and state-level demographics over time.[40] Then

[40] To adjust duration and convexity for trends in fund characteristics and state-level demographics, we first take the mean of duration and convexity over the period 2014-2016. Then we estimate cross-sectional regressions for

we use the adjusted duration and convexity along with the duration-matched, zero-coupon Treasury yields to rediscount Actuarial Liabilities (instead of TPLs), and thus to recompute time-varying funding ratios as Actuarial Assets divided by rediscounted Actuarial Liabilities going back to 2001 (previously shown in Figure 1, panel b).

Using the time-varying funding ratios obtained under this approach, Figure 9 shows the magnitude and statistical significance of slope coefficients from simple univariate regressions between PPFs' riskiness and lagged funding ratios for each year during 2002-2016, with the results reported in the Appendix Table A.1 (panel a). The results show a negative link between PPFs' riskiness and funding ratios, with the link gaining statistical significance for the interval 2012-2016, which largely coincides with the post-crisis period of low risk-free rates.

As a robustness test, we use the funding ratio computed with rediscounted TPLs available for 2015 as a time-invariant proxy for the funds' underfunding status held fixed over the entire sample period, motivated by the fact that the funds' relative underfunding has been persistent over the sample period.[41] The results shown in the Appendix Table A1 (panel b) are largely similar to the baseline results in Figure 9 and Table A1 (panel a).

4.3 Panel Data Results: Asset Risk vs. Underfunding, Interest Rates, and State Finances

For our panel analysis, we use the following specification to examine the determinants of funds' asset riskiness:

$$VaR_{i,t} = \alpha + \beta * FR_{i,t-1} + \gamma * TrYield_t + \delta * FR_{i,t-1} * TrYield_t + \mu_i + \epsilon_{i,t} \qquad (17)$$

with the following explanatory variables: (i) $FR_{i,t-1}$ is the time-variant funding ratio computed as Actuarial Assets divided by Actuarial Liabilities rediscounted like in Section 3.3; and (ii) the 5-year Treasury yield as a proxy for the level of risk-free rates, and alternatively an indicator

duration and convexity on the funds' beneficiaries-to-members ratio and the states' life expectancy at birth in 2015. For both duration and convexity, we find a negative and statistically significant relation with the beneficiaries-to-members ratio, and a positive and significant relation with life expectancy (i.e., fewer beneficiaries relative to members at the fund level and relatively higher life expectancy at the state level were associated with higher duration). Finally, we use the cross-sectional estimates from 2015 and historical fund- and state-level data to obtain time series for duration and convexity over 2001-2015.

[41] Data shows that funds that were relatively more underfunded prior to the 2008 global financial crisis tended to remain more underfunded in the post-crisis period.

variable for the post-GFC period of low interest rates. These variables mimic the determinants of risk-taking behavior considered in the model in Section 2. We use the 5-year Treasury yield for our main results because it seems best aligned with the maturity in PPFs' fixed income portfolio assets, given that the average maturity of the U.S. investment-grade fixed income securities is almost 6 years (for instance, see the Bloomberg Barclays Aggregate Bond Index); we also show robustness results with the 1-year and the 10-year Treasury yields. Due to multicollinearity between funding ratios and interest rates, it would not be feasible to identify their effects on risk-taking separately without further alterations. Therefore, we demean the funding ratios by the cross-sectional mean of each year to remove the time-series variation associated with interest rates.[42] We cluster the standard errors at both the fund and year levels. In some specifications, we also include the fund fixed effect μ_i.

In Table 4, the coefficient estimates on lagged funding ratios are negative and statistically significant at the 1 percent level in the regressions without fund fixed effects (columns 1-4). Although their statistical significance decreases in the presence of fund fixed effects (columns 5-8), the funding ratio coefficients remain negative. Given the persistence of underfunding status across funds over time, i.e., funds that started by being underfunded remained underfunded, it is not surprising that the fund fixed effects obscure the explanatory power of funding ratios, which is why we report regression results with and without fund fixed effects. Regarding the role of risk-free rates, the coefficients on Treasury yields are negative and statistically significant (columns 1, 2, 5, and 6), while those on the post-crisis dummy are positive (columns 3, 4, 7, and 8), indicating that PPFs took more risk when the risk-free rates were lower. The interactions between Treasury yields and funding ratios are positive and statistically significant (columns 2 and 6), while those between the post-GFC dummy and funding ratios are negative and statistically significant (columns 4 and 8), showing that funds with relatively lower funding ratios engaged in more risk-taking especially during periods with low risk-free rates. Interpreted through our model in Section 2, these results provide empirical support for the role of

[42] The Variance Inflation Factor (VIF) for the original funding ratio is 10.9, which exceeds the threshold level of 10 and thus suggests a high degree of multicollinearity. After time-demeaning, the VIF for the funding ratio drops to 1.

underfunding and risk-free rates as drivers of risk-taking through the reach-for-yield and risk-premium channels, both individually and interacted with each other, as illustrated in Figure 2.[43]

To examine the role of sponsor states' public finances as an additional determinant of funds' asset risk, we introduce relevant measures of state finances in equations (17) and (18):

$$VaR_{i,t} = \alpha + \beta * TrYield_t + \gamma * State_{i,t-1} + \delta * TrYield_t * State_{i,t-1} + \epsilon_{i,t} \qquad (18)$$

$$VaR_{i,t} = \alpha + \beta * FR_{i,t-1} + \gamma * State_{i,t-1} + \delta * FR_{i,t-1} * State_{i,t-1} + \eta * TrYield_t + \epsilon_{i,t} \quad (19)$$

Specifically, $State_{i,t-1}$ measures either the states' debt-to-income ratios or state bond ratings, with higher values implying worse ratings.[44] The variables are demeaned relative to the sample average for each year. They enter the regressions both in levels and interacted with the risk-free rates or funding ratios.[45]

The results for equation (18) are in Table 5, columns 1-4. The coefficients for state debt-to-income ratios and state bond ratings are positive and statistically significant. They show that funds sponsored by states with higher debt-to-income ratios or worse bond ratings took more asse risk on their portfolios. Like before, the coefficient on the Treasury yield is negative and statistically significant. Importantly, the coefficients on the interacted terms between state finances and Treasury yields are negative and statistically significant, showing that during periods of low risk-free interest rates, especially the funds with state sponsors with higher debt-to-income ratios and worse bond ratings take more risk. These results support the model implications with risky state debt and risk-shifting discussed in Section 2 and illustrated in Figure 5. Specifically, for a level of state debt that is large enough, lower risk-free rates provide an incentive for funds to take more risk, as their sponsor states shift the risk to debtholders.

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[43] For the economic significance of these results, see Appendix B.

[44] In the main results, we use the full sample of state and local government funds, and assume that the states' fiscal situation applies to the local governments as well. Alternatively, we re-estimate the regressions with the restricted sample of state funds that are directly matched with their sponsoring states' fiscal situation, while removing local government funds for which we do not have reliable measures of debt-to-income or bond ratings. While the sample size decreases without the local government funds, the results are similar.

[45] We do not control for funding ratios in equation (18), because funding ratios are correlated with the states' debt-to-income ratios. Also, not including funding ratios allows us to increase sample size by adding the PPFs for which we do not have rediscounted liabilities.

The results for state finances interacted with funding ratios as in equation (19) are presented in Table 5. Columns 5-6.[46] Consistent with the model implications discussed in Section 2 and illustrated in Figures 3 and 4, the coefficients on funding ratios are negative and statistically significant, while the coefficients on state bond ratings as a measure of states' fiscal strength are positive and statistically significant. Funds with lower funding ratios or residing in states with worse bond ratings took more risk. The interaction between funding ratios and state bond ratings has a positive coefficient, which departs from the predictions associated with risk-shifting. Overall, the majority of empirical results support the role of state public finances as a determinant of risk-taking behavior through risk-shifting.

4.4 Robustness Tests

We perform a set of robustness tests that involve alternative measures of asset risk as the dependent variable, including the alternative VaR-based measure of asset risk computed with the long-term average variance-covariance of returns, the time-varying VaR measure of asset risk computed with portfolio shares that abstract from valuation changes,[47] and a more traditional measure of risk such as the share of alternative assets in PPF portfolios. We also include alternative explanatory variables, such as funding ratios obtained by rediscounting liabilities with duration-matched, high-quality corporate bond yields instead of Treasury yields, and the 1-year and 10-year Treasury yields used instead of the 5-year yield as proxies for risk-free rates. Our robustness tests support the main results from Section 4.3.

For the cross-sectional relation between the PPFs' riskiness and lagged funding ratios in 2016, the negative link from Figure 8 is robust when using the alternative VaR measure computed with the long-term average variance-covariance matrix of monthly returns that is fixed over time (see

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[46] We do not interact the funds' funding ratios with the states' debt-to-income ratios because the two variables are correlated in the cross-section, which makes it not feasible to identify their effect on risk-taking separately: more underfunded funds tent to reside in states with more debt relative to income.

[47] We decompose the change in portfolio weights $\Delta w_{i,a,t}$ (fund *i*' portfolio share of asset type *a* held at time *t*) into two parts: (1) a more passive change component driven by valuation changes, and (2) a residual change component that more closely resembles active portfolio reallocations that involve trading. The valuation-driven weight change is defined as: $ValuationChange_{i,a,t} = w_{\{i,a,t-1\}} * \frac{1+R_{a,t}}{\sum_{j=1}^{6} w_{\{i,j,t-1\}} * (1+R_{j,t})} - w_{\{i,a,t-1\}}$, where $R_{i,t}$ is the net return of asset class a. The reallocation change is defined as: $Reallocation_{i,a,t} = w_{i,a,t} - ValuationChange_{i,a,t}$. Subsequently, we cumulate the reallocation changes to obtain the weights that abstract from valuation changes. See Ahmed et al. (2016) for similar decompositions applied to international portfolio flows.

the Appendix Figure A.2, panel b); when using the time-varying VaR measure of asset risk computed with portfolio shares that abstract from valuation changes (Figure A.2, panel b); and also when using funding ratios rediscounted with high-quality corporate bond yields as the explanatory variable (Figure A.2, panel c).

For the panel data analysis, the negative coefficients on funding ratios and Treasury yields, as well as positive coefficients on the interactions between the two are preserved when the VaR measure of risk is computed with portfolio shares that abstract from valuation changes (Table 6, column 1), when funding ratios are based on liabilities rediscounted with corporate bond yields (Table 6, column 2), and also when the 1-year and 10-year Treasury yields are used (Table 6, columns 3 and 4). We also consider using a more traditional measure of risk as the dependent variable, such as the share of risky assets in the composition of PPF portfolios. In Table 6 (columns 5-8), we find some evidence of asset risk-taking behavior with the share of alternatives as the measure of risk, albeit considerably weaker than with the VaR measure of risk. Only the negative coefficients on 1-year, 5-year, and 10-year Treasury yields, as well as the positive coefficients on the post-GFC dummy variable preserve statistical significance, which is consistent with the rising share of alternatives in funds' portfolios over time. However, the coefficients on funding ratios are not statistically significant, and neither are the interacted terms between funding ratios and risk-free rates.

5. Pension Funds' Net-Worth Risk

In this section, we extend our analysis to cover the riskiness of a pension fund i's net worth, NW, given by

$$NW_{i,t} = A_{i,t} - L_{i,t}, \quad (20)$$

where $A_{i,t}$ and $L_{i,t}$ are the present values of a fund's assets and liabilities at the beginning of year t. There are two motivations for focusing on net worth as an alternative to our measure of asset risk used in the baseline empirical analysis. First, it may be the most relevant measure of risk to taxpayers (and perhaps current workers) because it represents the amount taxpayers are expected to contribute to the pension fund in present value terms if the fund's net worth becomes negative.[48] Second, focusing on asset risk alone can give a distorted picture of net-worth risk, as higher asset-risk may represent either reach for yield or liability hedging. For example, the change in net worth of fund p during year t is given by

$$\Delta NW_{i,t} = A_{i,t}R_{a,i,t} - L_{i,t}R_{l,i,t}, \quad (21)$$

where $R_{a,i,t}$ and $R_{l,i,t}$ represent the net annual returns on fund i's assets and liabilities respectively during year t. If the returns are distributed Normally, then the value at risk of its net worth at the 5% confidence level is given by:

$$VaR(NW)_{i,t} = 1.65\sqrt{A_{i,t}^2\sigma_{R_{a,i,t}}^2 + L_{i,t}^2\sigma_{R_{l,i,t}}^2 - 2A_{i,t}L_{i,t}\sigma_{R_{a,i,t}}\sigma_{R_{l,i,t}}\rho_{R_{a,i,t},R_{l,i,t}}}, \quad (22)$$

where, $\sigma_{R_{a,i,t}}^2$, and $\sigma_{R_{l,i,t}}^2$, are the variances of the returns on fund i's assets and liabilities, and $\rho_{R_{a,i,t},R_{l,i,t}}$ is the returns' correlation. When the correlation is positive and if $A_{i,t}\sigma_{R_{a,i,t}} < L_{i,t}\sigma_{R_{l,i,t}}\rho_{R_{a,i,t},R_{l,i,t}}$, then straightforward differentiation shows that an increase in the standard deviation of asset returns (which we associated with an increase in the asset risk of the pension fund in Section 4) while holding return correlations constant reduces the value-at-risk of net worth, i.e. higher asset variance can sometimes reduce net-worth variance.[49] Put differently, the increased VaR of asset returns may not represent more risk taking, but instead hedging of liabilities.

To investigate if the increased variance of assets we found above could represent hedging rather than reach for yield, below we shift our focus to the Value-at-Risk of net worth. We run regression analysis to identify how funds' net-worth risk is related to their underfunding, as measured by their funding ratios $\frac{A_t}{L_t}$, and to the level of risk-free interest rates. In doing so, we must overcome two challenges to identifying these effects. The first is heteroskedasticity driven by differences in funds' sizes. Specifically, inspection of the expression for $VaR(NW)$ shows that it is homogeneous of degree 1 in pension fund size. That is, if we scale up the size of a

[48]For example, if $NW_t = -\$1$ billion, then the expected present value of taxpayer's contribution to the fund is \$1 billion.

[49] The intuition is if the assets and liabilities are positively correlated, they partially hedge each other, and thus if the variance of liabilities increases, the variance of assets may need to be increased to reduce the fund's net-worth risk.

fund's assets and liabilities by multiplying them by a scalar s, then $VaR(NW)$ is also scaled up by s.[50] This suggests if the funds being analyzed have different funding ratios but also different sizes, the size differences, if not accounted for, will create significant cross-sectional variance in $VaR(NW)$, which would make it difficult to detect the role of underfunding and other variables in risk-taking. Therefore, it is necessary to account for size differences by adding size controls or equivalently by scaling the measure of net-worth risk to account for differences in pension funds' sizes. Following the latter approach, instead of focusing on the $VaR(NW)$, we focus on $VaR(NW/s)$, i.e. the VaR of changes in net worth normalized by a measure of fund scale or size, such as assets or liabilities. The normalization appears to alleviate the severe heteroskedasticity present in unscaled net-worth regressions, but it creates a second challenge.

The second challenge is that scaling NW by a measure of fund size can create a mechanical relationship between our risk measure and funding ratios, i.e. the relationship is by construction and not a proof that funding ratios are related to more risk-taking. To illustrate the issue, if we divide $\Delta NW_{p,t}$ in equation (21) above by liabilities, then the normalized $VaR\left(\frac{NW}{L}\right)$ becomes:

$$VaR\left(\frac{NW}{L}\right)_{p,t} = 1.65\sqrt{\left(\frac{A_{p,t}}{L_{p,t}}\right)^2 \sigma^2_{R_{a,p,t}} + \sigma^2_{R_{l,p,t}} - \left(\frac{A_{p,t}}{L_{p,t}}\right) \sigma_{R_{a,p,t}}\sigma_{R_{l,p,t}}\rho_{R_{a,p,t},R_{l,p,t}}}. \quad (23)$$

Observation shows $VaR\left(\frac{NW}{L}\right)$ appears to be a mechanical function of the funding ratio $\frac{A}{L}$, whether or not actual fund risk-taking is a function of the funding ratio. If we normalize ΔNW by a different measure of fund size such as assets, or some average of assets and liabilities, a similar problem emerges.

There is no straightforward approach that we are aware of to address the mechanical issue. The way we do so here is we adopt as our null hypothesis that funds' portfolio choices and their funding ratios are independent. In an environment that imposes this null, we simulate the distribution of the mechanical relationship between net-worth risk and funding ratios, as explained in Section 5.1. Then using our actual data, we test if the relationship between

[50] For example, if a fund is scaled up by a factor $s = 2$, then its assets and liabilities both double, and its $VaR(NW)$ doubles.

$VaR\left(\frac{NW}{L}\right)$ or $VaR\left(\frac{NW}{A}\right)$ and the funding ratio is stronger than we might expect mechanically, and whether the direction of deviation is consistent with more asset risk-taking or with hedging.

5.1 Simulation Approach

Our approach is to simulate pension fund data that are similar to our actual pension fund data, while imposing the null that the simulated funds' net-worth risk is not related to funding ratios. To do so, we create the largest balanced panel dataset using our pension fund sample.[51] Then:

1. For each fund i = 1,…N in the panel, we create a simulated fund by matching fund i's asset and liability returns data with the assets and liabilities of a randomly chosen (without replacement) fund $j \neq i$.
2. We repeat step 1 for 1,000 times to create 1,000 random simulated data sets.

In each simulated data set, fund i's net-worth risk taking strategy is unrelated to its assets and liabilities and any function thereof, such as the funding ratio, by construction, given the random matching in step 1. Hence, any relationship between funds' net-worth risk (normalized by the matched assets or liabilities) and funding ratios that we find in our regression analysis using the simulated data is either mechanical or purely coincidental.

To perform our statistical analysis, we first estimate the regression 1,000 times using normalized risk estimates and funding ratios constructed from our simulated data, which reflect the mechanical relation between net worth risk and funding ratios. Then we compare the regression estimates obtained using actual data with the distribution of estimates using the data simulated 1,000 times under the null. If the regression estimate from actual data is low (or high) relative to the distribution of simulation estimates, we conclude that the link between risk taking and funding ratios is weaker (or stronger) than implied by any mechanical relation between them. We acknowledge this approach is imperfect, but to the best of our knowledge this is the only way to analyze the risk of net worth while addressing the challenges of heteroskedasticity and the possibility of a mechanical relationship. Our findings are in the next subsection.

[51] The balanced panel is chosen to maximize the number of time series observations times the number of funds in the panel.

5.2 Regression Findings for the Value-at-Risk of Net Worth

In this section we analyze whether the Value-at-Risk of Net Worth is related to risk-free interest rates and plans' underfunding. To study these relationships, it is necessary to compute the Value-at-Risk of net worth. Our approach for doing so is an extension of how we compute the Value-at-Risk of asset portfolios.[52]

Tables 8 and 9 show the results from balanced panel regressions of funds' riskiness of net worth on the time-varying funding ratio and the long-term Treasury yields, with the riskiness of net worth computed using our earlier value-at-risk approach.[53] Net worth is normalized either by assets (Table 8) or by liabilities (Table 9). The tables report the coefficient estimates using the real data; for each coefficient estimate, the tables also report left-tail and right-tail p-values, in parentheses.[54] The p-values show how the real data estimate compares with the distribution of estimates arising from simulated data like in Section 5.1, which reflect the mechanical relation—rather than the economic one—between net-worth risk and regressors including funding ratios. A small left-tail (right-tail) p-value would indicate that the coefficient estimate with real data is probably lower (higher) than is consistent with the mechanical relation described by the distribution of coefficient estimates from the simulated data.

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[52] The Value-at-Risk of net-worth normalized by liabilities given in equation (24) is computed by plugging in estimates for the different terms in equation (24). $A_{i,t}$ and $L_{i,t}$ are the beginning of year t values for assets and liabilities. These are assumed equal to the values reported by funds for the end of year $t-1$. The liabilities are rediscounted using the approaches reported in section 4. $\sigma^2_{R_{a,i,t}}$ which equals $w'_{i,t}\Sigma_t w_{i,t} + \sigma^2_\epsilon$ from equation (11) is computed using the approach for equation (11). To compute $\sigma^2_{R_{l,i,t}}$, the return on the funds' liabilities, we assume this return is the same as the return on a zero-coupon bond that has a maturity equal to the duration of the funds' liabilities. Using daily zero-coupon U.S. yield-curve estimates that is updated following Gurkaynak, Sack, and Wright (2006), we infer the daily return of the liabilities for each day in year $t-1$. We then estimate the daily sample variance of the return on the liabilities and annualize it. This follows the same approach as in equation (12) but applied to the return on liabilities. The term $\sigma_{R_{a,i,t}}\sigma_{R_{l,i,t}}\rho_{R_{a,i,t},R_{l,i,t}}$ is $Cov_t(R_{a,i,t}, R_{l,i,t})$. To estimate it, we assume the idiosyncratic component of funds asset returns (the part not spanned by the return on the category indices) is uncorrelated with the return on liabilities. Under this assumption, for each year t, we estimate the sample covariance between funds daily returns due to the category indices and the daily returns on the liabilities using daily return data from year $t-1$, while applying asset portfolio weighs from the end of year $t-1$. To estimate Value-at-Risk of net worth normalized by assets, we follow an analogous approach.

[53] The results in Tables 8 and 9 correspond to the results for asset risk presented in Table 4, columns 1-2 and 5-6.

[54] The left- and right- p-values are the areas to the left- and right- of the estimated coefficient using real data under the parameters simulated distribution when the relationship between the risk measure and the regressors is purely mechanical. A small upper or lower p-value is evidence that the estimated relationship using real data is unlikely to just be mechanical.

In Table 7, under model (1) without fund fixed effects, the left- and right-tail p-values are 0.927 and 0.073 around the negative coefficient estimate of -0.881 on funding ratios with real data; these values suggest that the real estimate, despite its negative value, is not any lower than the negative mechanical relationship expected between the riskiness of net worth and the funding ratio. Similar results emerge when adding interactions between the funding ratio and Treasury yields (column 2), when using fund fixed effects (columns 3 and 4), and when running univariate regressions (columns 5 and 6). Figure 10, panel (a) illustrates this relation: the real-data estimate of the funding ratio coefficient rests toward the right tail of the distribution of coefficient estimates with simulated data, suggesting that the real relation between net-worth risk and funding ratios is not any more negative than implied by the mechanical relation. The result provides no evidence that more underfunded funds take more risk in their net worth.[55]

Table 8 shows similar results when funds' net worth is normalized by liabilities rather than assets. In models (1)-(4), the coefficient estimates on funding ratios with real data are not lower that the positive mechanical relationship implied by the distribution with simulated data, since the left-tail and right-tail p-values are roughly equal.[56] Overall, the results do not provide evidence that more underfunded funds take more risk in their net worth.

We find mixed evidence on whether pension funds take more risk in their net worth in response to lower Treasury yields: In Table 7, when net worth is normalized by assets, the coefficient on Treasury yields is negative indicating that lower Treasury yields increase risk-taking. In addition, the left-tail p-value for Treasury yields is lower than 10 percent suggesting the relationship is not mechanical. Figure 10, panel (b) illustrates this result, showing that the real-data coefficient estimate on the Treasury yield lies in the left tail of the distribution of coefficient estimates resulting from simulated data, potentially indicating a negative relation

[55] The result is largely consistent with those in the Appendix Table A.3, which shows real-data coefficient estimates from cross-sectional regressions of net-worth risk (normalized by assets) on time-varying funding ratios for each year (like we did for asset risk in Table 4, panel d), along with the corresponding p-values. Except for years 2013 and 2015, the left-tail p-value is greater than 10 percent, providing no evidence that more underfunded funds take more net-worth risk than implied by the mechanical relation between the two variables.

[56] The result is largely consistent with those in the Appendix Table A.4, which shows real-data coefficient estimates from cross-sectional regressions of net-worth risk (normalized by liabilities) on time-varying funding ratios for each year and the corresponding p-values.

between net-worth risk and Treasury yields. However, the result is not robust when net worth is normalized by liabilities in Table 8.

7. Conclusion

This paper examines the determinants of risk-taking behavior by U.S. public pension funds. To motivate and interpret our empirical analysis, we developed a simple theoretical model that relates funds' risk-taking to their underfunding (the reach-for-yield channel), to interest rates (the risk-premium channel), and to the condition of state public finances. To measure risk, we developed a new methodology for inferring funds' asset exposures and compute their Value-at-Risk on the basis of limited public data. In addition, to create meaningful measures of underfunding, we rediscount their liabilities with discount rates that better match the riskiness of liabilities. We analyze risk based on asset returns alone, or alternatively, based on the risk of changes in the net worth of pension funds. When measuring pension funds risk based on asset returns alone, we find evidence consistent with both the reach-for-yield and risk-premia channels of risk-taking behavior, as funds take more asset risk in response to underfunding and low interest rates on safe assets. The effects of low interest rates on asset risk-taking are especially pronounced for funds that are more underfunded or are affiliated with states with weaker public finances.

Our measure of asset risk also allows us to compute the losses that would be suffered by public pension funds under a severely adverse economic scenario, which would place an additional burden on the public finances of sponsoring states. Based on our results, we infer that the potential loss transferable to the states if a 1-in-20 years adverse return event were to occur at the end of our sample period (after several years of low Treasury yields) would have been equivalent on average to about 19% of state and local government' debt, in contrast to just 13 percent before the 2008 Global Financial Crisis (GFC).   We attribute up to one-fourth of funds' recent asset risk-taking behavior to the lower yields on risk-free assets and lower funding ratios since the GFC.

Our findings on net-worth risk are novel and differ somewhat from our findings on asset-risk. In particular, we don't find evidence of higher net-worth risk for funds with lower funding ratios, and we find only weak evidence that low interest rates are consistent with more net-worth risk-taking. These findings suggest some of the evidence on higher asset risk-taking may not represent reaching for yield, but instead may be consistent with some pension funds hedging their liabilities and net worth. The extent to which changes in pension funds' asset risk represent hedging of liabilities rather than reach for yield should be thoroughly explored in future research.

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Table 1: Summary Statistics for Public Pension Fund Variables

Variable	Mean	Median	St Dev
Returns (Annual)	.062	.085	.109
% Equities	.536	.551	.111
% Fixed Income	.273	.262	.090
% Real Estate	.054	.053	.045
% Cash	.024	.013	.037
% Alternatives	.098	.073	.104
% Other	.012	0	.030
Actuarial assets ($ mil)	15.949	6.682	28.045
Actuarial liabilities ($ mil)	20.044	9.094	33.876
Funding ratio, original	.800	.808	.197
Funding ratio, rediscounted	.507	.492	.1537
State debt-to-income ratio	.083	.079	.039
State bond rating	2.852	3	1.558

Notes: Table 1 is mostly based on the Public Plans Data dataset from the Center for Retirement Research (CRR) at Boston College, and the Center for State and Local Government Excellence (SLGE). The data are publicly available at www.publicplansdata.org. *Returns (Annual)* is the InvestmentReturns_1yr variable in the dataset, which reports each fund's returns in a given fiscal year. *%Asset Class* shows the percentage allocation to funds' portfolios a particular asset class. The dataset provides a breakdown for six asset classes: equities, fixed income, real estate, cash, alternatives, and other. Their allocation shares are found as equities_tot, FixedIncome_tot, RealEstate, CashAndShortTerm, alternatives, and other (note capitalization) respectively in the original dataset. *Actuarial assets* and *Actuarial liabilities* are found under ActAssets_GASB and ActLiabilities_GASB in thousands of dollars, which we convert to millions. *Funding ratio, original* is given by ActFundedRatio_GASB, which is ActAssets_GASB divided by ActLiabilities_GASB in the dataset. *Funding ratio, rediscounted* is the time-varying funding ratio computed as ActAssets_GASB divided by rediscounted ActLiabilities_GASB, where the rediscounting uses the duration and convexity computed as follows. We compute each fund's duration and convexity during 2014-2016 as in equations (15) and (16), based on GASB data available only for these years; we take averages for duration and convexity over 2014-2016, then extrapolate them for the interval 2001-2016 adjusted for demographics as discussed in Section 4.2. We build *State debt-to-income ratio* with state income from the Bureau of Economic Analysis and state debt from the United States Census Bureau. Finally, *State bond rating* is the Standard and Poor's rating by state-year, coded numerically as AAA = 1 through BBB = 8, i.e., with higher values indicating worse ratings.

Table 2: List of Publicly-Traded Market Indices

Index name	Symbol
HFRX Global	HFRXGlobal
Bloomberg Commodities	BCOM
Bloomberg Commodities total returns	BCOMTR
Thomson Reuters/CoreCommodity Index	CRY
Thomson Reuters/CoreCommodity Index total returns	CRYTR
Credit Suisse Hedge Index	HEDGNAV
Barclays Hedge Fund	BGHSHEDG
ICE USD LIBOR 3 Mon	ICELIBOR3Mon
SIFMA Minu Swap Index	MUNIPSA
S&P 500 Index	SPX
Russel 3000	Russel3000
FTSE All World Excluding US	FTAW02
Dow Jones Global Index	W1DOW
ICE BoAML US Broad Market Ind	US00
ICE BofAML Global Broad Market	GBXD
Citi World Government Bond Ind	SBWGU
FTSE NAREIT All Equity REITS I	FNER

Notes: The table provides information on the set of publicly traded indices that are used to estimate funds' category-index risk exposures. See Section 3.2 of the text for further details.

Table 3. Coefficient Estimates for $\theta_{c,j}$ from OLS post-LASSO Estimation

Asset Category	Index	OLS Coefficient	St. Err.
Equities	Russel 3000	1.02***	(0.13)
Equities	ICE LIBOR 3 Mon	-0.042	(0.035)
Equities	W1DOW	-1.11***	(0.29)
Equities	FTAW02	0.86***	(0.17)
Equities	SBWGU	0.019	(0.033)
Equities	US00	-0.47***	(0.067)
Fixed Income	Russel 3000	0.30***	(0.10)
Fixed Income	FTSE	0.12***	(0.039)
Fixed Income	SBWGU	-0.27***	(0.088)
Fixed Income	BCOMTR	0.27***	(0.10)
Fixed Income	CRYTR	-0.11	(0.082)
Real Estate	Russel 3000	-3.29***	(0.98)
Real Estate	FTSE	0.45***	(0.12)
Real Estate	W1DOW	7.13***	(2.29)
Real Estate	SBWGU	-3.30**	(1.29)
Real Estate	US00	-0.70***	(0.14)
Real Estate	BCOMTR	0.19	(0.11)
Cash	Russel 3000	0.23	(0.18)
Cash	FTSE	0.063	(0.17)
Alternatives	Russel 3000	0.11	(0.56)
Alternatives	FTSE	0.25***	(0.057)
Alternatives	W1DOW	-0.32	(1.38)
Alternatives	SBWGU	0.60	(0.80)
Alternatives	US00	-0.38***	(0.070)
Other	FTSE	0.33**	(0.13)
Other	BCOMTR	-1.62*	(0.85)
Other	CRY	-2.70	(2.46)
Other	CRYTR	4.14	(2.56)
		-0.26***	(0.040)
Observations		2,473	
R-squared		0.897	

Robust standard errors in parentheses, with *** p<0.01, ** p<0.05, * p<0.1

Notes: The table provides the coefficient estimates for $\theta_{c,j}$ from the OLS post-LASSO estimation of equation (9), for each asset category *c* and selected market index *j*. See Section 3.2 of the text for further details.

Table 4. Determinants of PPF's asset-risk: underfunding and risk-free rates, using time-varying funding ratios

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
VARIABLES				VaR assets				
FR	-0.020***	-0.054***	-0.021***	-0.0052***	-0.040	-0.090	-0.047	-0.037
	(0.00031)	(0.0019)	(0.0012)	(0.00034)	(0.036)	(0.059)	(0.035)	(0.030)
TRY 5yr	-0.021**	-0.021**			-0.021**	-0.021**		
	(0.0075)	(0.0075)			(0.0075)	(0.0075)		
FR * TRY 5yr		0.013***				0.015**		
		(0.0012)				(0.0060)		
Dummy post-GFC			0.054*	0.053*			0.054*	0.053*
			(0.029)	(0.029)			(0.029)	(0.029)
FR * Dummy post-GFC				-0.034***				-0.045**
				(0.0011)				(0.018)
Constant	0.21***	0.21***	0.13***	0.13***	0.21***	0.21***	0.13***	0.13***
	(0.033)	(0.033)	(0.011)	(0.011)	(0.033)	(0.033)	(0.011)	(0.011)
FR	Varying	Varying	Varying	Varying	Varying	Varying	Varying	Varying
FR rediscounted	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Fixed effects	No	No	No	No	Yes	Yes	Yes	Yes
Observations	1,289	1,289	1,289	1,289	1,289	1,289	1,289	1,289
Funds	111	111	111	111	111	111	111	111
R-squared	0.209	0.210	0.184	0.184	0.225	0.226	0.200	0.201

Robust standard errors in parentheses, *** p<0.01, ** p<0.05, * p<0.1

Notes: The regressions examine the link between PPFs' asset-risk, underfunding, and risk-free rates, using the following panel specification: $VaR_{i,t} = \alpha + \beta * FR_{i,t-1} + \gamma * TrYield_t + \delta * FR_{i,t-1} * TrYield_t + \mu_i + \epsilon_{i,t}$. Columns 1-4 show results from regressions without fixed effects, while columns 5-8 show results with fund fixed effects. The data are annual. The dependent variable is the VaR measure of asset-portfolio risk depicted in Figure 6. The time-varying funding ratios are based on actuarial liabilities rediscounted as in Sections 3.3 and 4.4. The funding ratios are demeaned by the cross-sectional mean of each year. The measure of risk-free rates is given by the 5-year Treasury yield. In columns 3, 4, 7, and 8, the Treasury yield is replaced by a post-GFC dummy variable that takes the value of 1 for years 2010-2016 and zero otherwise. The sample excludes years 2002, 2003, and 2009. Standard errors are double-clustered at the fund and year level.

Table 5. Determinants of PPF's asset-risk: state public finances, underfunding, and risk-free rates

	(1)	(2)	(3)	(4)	(5)	(6)
VARIABLES			VaR assets			
TRY 5yr	-0.021**	-0.021**	-0.021**	-0.021**	-0.021**	-0.021**
	(0.0076)	(0.0076)	(0.0075)	(0.0075)	(0.0075)	(0.0076)
State DTI	0.033***	0.039***				
	(0.0025)	(0.0022)				
TRY 5yr * State DTI		-0.0023**				
		(0.00080)				
State bond rating			0.00085***	0.0017***	0.00072***	0.00073**
			(0.000017)	(0.000048)	(0.000051)	(0.00026)
TRY 5yr * State rating				-0.00036***		
				(0.000013)		
FR					-0.017***	-0.021***
					(0.00043)	(0.00077)
FR * State bond rating						0.021***
						(0.0026)
Constant	0.21***	0.21***	0.21***	0.21***	0.21***	0.21***
	(0.033)	(0.034)	(0.033)	(0.033)	(0.033)	(0.033)
FR					Varying	Varying
FR rediscounted					Yes	Yes
Fixed effects	No	No	No	No	No	No
Funds	138	138	138	138	110	110
Observations	1,656	1,656	1,653	1,653	1,277	1,277
R-squared	0.209	0.209	0.209	0.209	0.210	0.212

Robust standard errors in parentheses, *** p<0.01, ** p<0.05, * p<0.1

Notes: The regressions examine the link between PPFs' asset-risk, state finances, underfunding, and risk-free rates. The dependent variable is the VaR measure of asset risk. State finances are measured as either the state debt to income ratios or state bond ratings; they are demeaned by the cross-sectional mean of each year. For state bond ratings, higher values reflect worse ratings. The regressions allow for interactions between state finances and risk-free rates (columns 2 and 4), or between state finances and funding ratios (column 6). The sample excludes years 2002, 2003, and 2009. Standard errors are double-clustered at the fund and year level.

Table 6. Determinants of PPF's asset risk: robustness with VaR measure and traditional measures of risk

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
VARIABLES	VaR assets				Share of alternative assets in PPF portfolios			
FR	-0.051***	-0.054***	-0.035***	-0.081***	0.021	-0.012	0.017	0.016
	(0.0013)	(0.0038)	(0.00078)	(0.0054)	(0.11)	(0.038)	(0.078)	(0.16)
TRY 5yr	-0.024**	-0.021**			-0.037***			
	(0.0082)	(0.0075)			(0.0057)			
FR * TRY 5yr	0.012***	0.012***			-0.0082			
	(0.00082)	(0.0013)			(0.027)			
Dummy post-GFC						0.10***		
						(0.015)		
FR * Dummy post-GFC						0.018		
						(0.077)		
TRY 1yr			-0.017**				-0.026***	
			(0.0067)				(0.0043)	
FR * TRY 1yr			0.0085***				-0.0081	
			(0.0015)				(0.018)	
TRY 10yr				-0.018				-0.049***
				(0.011)				(0.0066)
FR * TRY 10yr				0.018***				-0.0060
				(0.0016)				(0.038)
Constant	0.22***	0.21***	0.19***	0.22***	0.21***	0.058***	0.16***	0.28***
	(0.036)	(0.033)	(0.026)	(0.045)	(0.019)	(0.011)	(0.014)	(0.025)
Robustness	VaR active shares	FR corporate yields	TRY 1yr	TRY 10yr	Traditional measure of risk		TRY 1yr	TRY 10yr
FR	Varying	Varying	Varying	Varying	Varying	Varying	Varying	Varying
FR rediscounted	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Fixed effects	No	No	No	No	No	No	No	No
Observations	1,289	1,289	1,289	1,289	1,277	1,277	1,277	1,277
Funds	111	111	111	111	111	111	111	111
R-squared	0.227	0.210	0.238	0.097	0.199	0.206	0.167	0.219

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Notes: The table shows robustness checks for the results on asset risk. In columns (1), (3) and (4), the dependent variable is the standard VaR measure of asset-risk depicted in Figure 6. As explanatory variables, we use funding ratios with liabilities rediscounted with duration-matched, high-quality corporate bond yields instead of Treasury yields as the explanatory variable (column 1); and the 1-year and 10-year Treasury yields instead of the 5-year Treasury yield (columns 3 and 4). In column (2), the VaR is computed with portfolio shares that abstract from valuation changes as the dependent variable (marked as "VaR active shares"), while the explanatory variables are the same as in Table 4, column (2). In columns (5) to (8), the dependent variable is the share of alternatives instead of the VaR as the measure of risk, while the explanatory variables are the same as in Table 4, column (2). The sample excludes years 2002, 2003, and 2009. Standard errors are double-clustered at the fund and year level.

Table 7. Determinants of PPF's net-worth risk (normalized by assets): underfunding and risk-free rates

VARIABLES	(1) VaR	(2) VaR	(3) VaR	(4) VaR	(5) VaR	(6) VaR
FR	-0.881 ***	-1.483 ***	-0.955 ***	-1.758 ***	-0.893 ***	
	(0.927,0.073)	(0.574,0.426)	(0.970,0.030)	(0.947,0.053)	(0.928,0.072)	
TRY 5yr	-0.087 ***	-0.085 ***	-0.086 ***	-0.084 ***		-0.087 ***
	(0.075,0.925)	(0.049,0.951)	(0.080,0.920)	(0.027,0.973)		(0.075,0.925)
FR * TRY 5yr		0.215 ***		0.235 ***		
		(0.845,0.155)		(0.360,0.640)		
Constant	0.679 ***	0.674 ***			0.468 ***	0.689 ***
	(0.628,0.372)	(0.433,0.567)			(0.072,0.928)	(0.390,0.610)
FR	Varying	Varying	Varying	Varying	Varying	N/A
FR rediscounted	Yes	Yes	Yes	Yes	Yes	Yes
Fixed effects	No	No	Yes	Yes	No	No
Observations	1092	1092	1092	1092	1092	1092
Funds	91	91	91	91	91	91
R-squared	0.563	0.583	0.606	0.627	0.188	0.381

Robust standard errors in parentheses
*** $p < 0.01$; ** $p < 0.05$; * $p < 0.1$

Notes: The regressions examine the link between PPFs' net-worth risk, underfunding, and risk-free rates, using the following panel specification: $VaR_{i,t} = \alpha + \beta * FR_{i,t-1} + \gamma * TrYield_t + \delta * FR_{i,t-1} * TrYield_t + \mu_i + \epsilon_{i,t}$. Columns 1-2 show results from regressions without fixed effects, columns 3-4 show results with fund fixed effects, and columns 5-6 again show results without fixed effects and the funding ratio and Treasury yield included one at a time. The tables report the coefficient estimates using real data; for each coefficient estimate, the table also reports in parentheses left-tail and right-tail p-values (the areas under the simulated distribution described in Section 5.1 that is to the left- and right-of the estimated coefficient using real data). A small upper or lower p-value is evidence that the estimated coefficient is inconsistent with what would be expected from a mechanical relationship between the regressors and risk measures. The data are annual. The dependent variable is the VaR measure of net-worth risk normalized by assets. The time-varying funding ratios are based on actuarial liabilities rediscounted as in Section 3.3. The funding ratios are demeaned by the cross-sectional mean of each year. The measure of risk-free rates is given by the 5-year Treasury yield. The sample excludes years 2002, 2003, and 2009. Standard errors are double-clustered at the fund and year level.

Table 8. Determinants of PPF's net-worth risk (normalized by liabilities): underfunding and risk-free rates

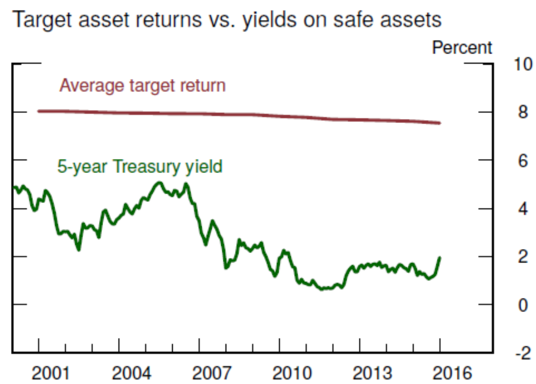
	(1)	(2)	(3)	(4)	(5)	(6)
VARIABLES	VaR	VaR	VaR	VaR	VaR	VaR
FR	0.089 ***	0.151 ***	-0.016	0.052	0.087 ***	
	(0.814,0.186)	(0.642,0.358)	(0.463,0.537)	(0.892,0.108)	(0.814,0.186)	
TRY 5yr	-0.019 *	-0.019 *	-0.019 *	-0.019 *		-0.019 ***
	(0.528,0.472)	(0.579,0.421)	(0.640,0.360)	(0.278,0.722)		(0.638,0.362)
FR * TRY 5yr		-0.022 ***		-0.020 *		
		(0.584,0.416)		(0.078,0.922)		
Constant	0.256 ***	0.257 ***			0.210 ***	0.255 ***
	(0.505,0.495)	(0.462,0.538)			(0.518,0.482)	(0.111,0.889)
FR	Varying	Varying	Varying	Varying	Varying	N/A
FR rediscounted	Yes	Yes	Yes	Yes	Yes	Yes
Fixed effects	No	No	Yes	Yes	No	No
Observations	1092	1092	1092	1092	1092	1092
Funds	91	91	91	91	91	91
R-squared	0.157	0.159	0.201	0.202	0.014	0.142

Robust standard errors in parentheses
*** $p < 0.01$; ** $p < 0.05$; * $p < 0.1$

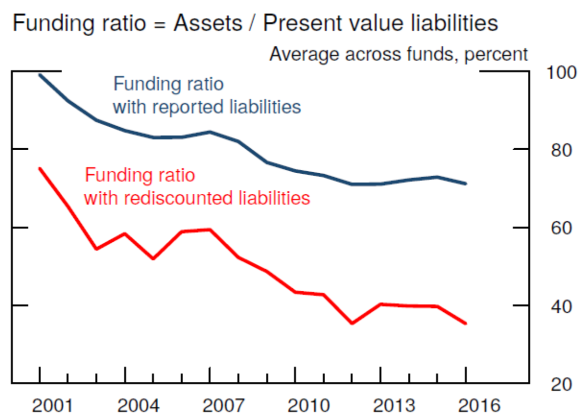
Notes: The regressions examine the link between PPFs' risk-taking of net worth, underfunding, and risk-free rates, using the following panel specification: $VaR_{i,t} = \alpha + \beta * FR_{i,t-1} + \gamma * TrYield_t + \delta * FR_{i,t-1} * TrYield_t + \mu_i + \epsilon_{i,t}$. Columns 1-2 show results from regressions without fixed effects, columns 3-4 show results with fund fixed effects, and columns 5-6 again show results without fixed effects and the funding ratio and Treasury yield included one at a time. The tables report the coefficient estimates using real data; for each coefficient estimate, the table also reports in parentheses left-tail and right-tail p-values (the areas under the simulated distribution described in Section 5.1 that is to the left- and right-of the estimated coefficient using real data). A small upper or lower p-value is evidence that the estimated coefficient is inconsistent with what would be expected from a mechanical relationship between the regressors and risk measures. The data are annual. The dependent variable is the VaR measure of net worth risk normalized by liabilities. The time-varying funding ratios are based on actuarial liabilities rediscounted as in Section 3.3. The funding ratios are demeaned by the cross-sectional mean of each year. The measure of risk-free rates is given by the 5-year Treasury yield. The sample excludes years 2002, 2003, and 2009. Standard errors are double-clustered at the fund and year level.

Figure 1. PPFs' portfolio allocation and expected return targets

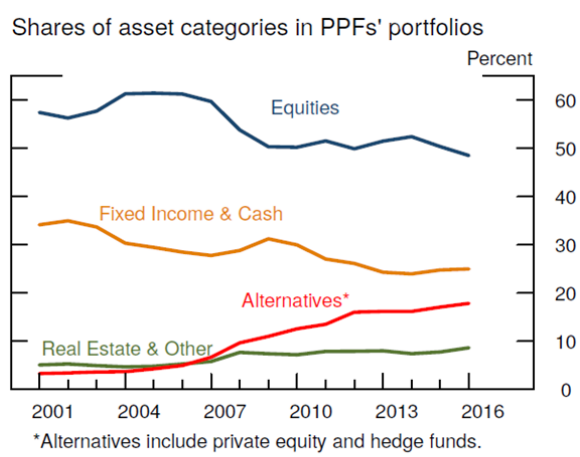
Panel (a)



Panel (b)

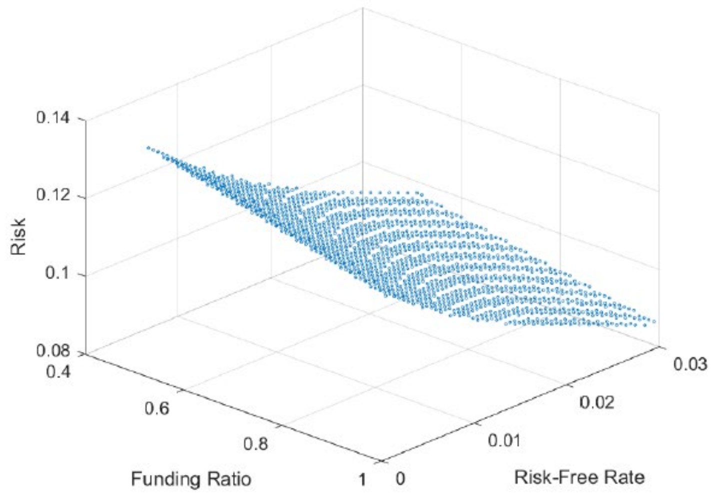


Panel (c)



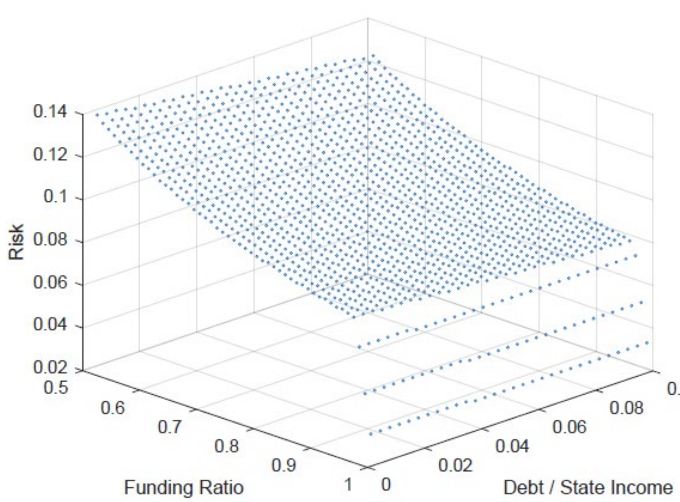
Notes: The figure presents summary information on PPFs' expected return targets (panel a), the ratio of assets to actuarial liabilities (panel b), and the change aggregate asset allocations over time (panel c).

Figure 2: Risk vs Risk Free Rate and Pension Funding Ratio when State Debt is Risk-Free



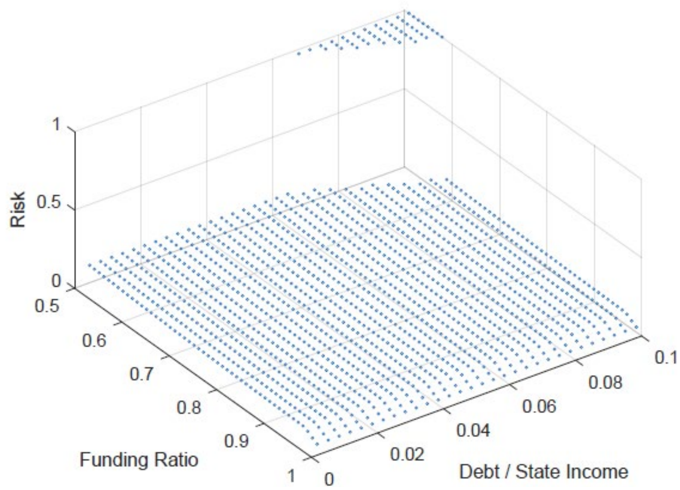
Notes: For the theoretical model in Section 2, when states are unable to default on their debt, the Figure presents the relationship between pension fund asset risk, the plans funding ratio, and the risk-free interest rate. Risk in the figure is measured as the proportion of risky asset in the fund's asset portfolio. The funding ratio is the present value of fund liabilities discounted by the risk-free rate. For further details see Section 2 of the text.

Figure 3: Risk as a Function of Debt to State Income and Pension Funding Ratio when State Debt is Risk-Free



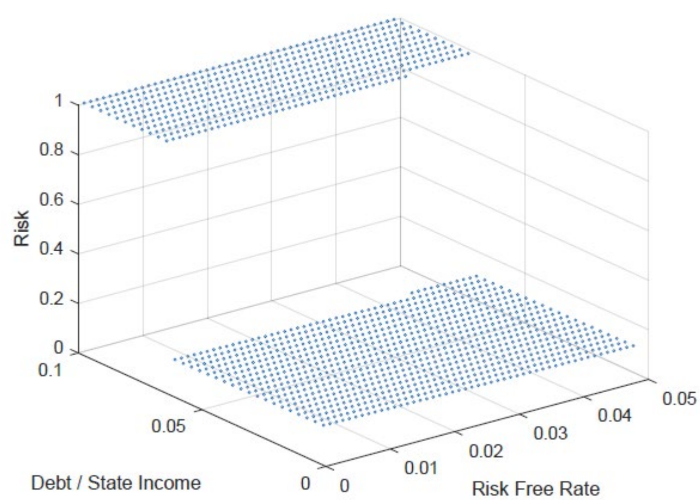
Notes: For the theoretical model in Section 2, when states are unable to default on their debt, the Figure presents the relationship between pension fund asset risk, the plans funding ratio, and the ratio of state debt to state income. Risk in the figure is measured as the proportion of risky asset in the fund's asset portfolio. The funding ratio is the present value of fund liabilities discounted by the risk-free rate. Debt to state income is measured as debt at date *t* divided by state income at date *t*. For further details see Section 2 of the text.

Figure 4: Pension Fund Risk vs Pension Funding Ratio and Debt to State Income when State Debt is Risky



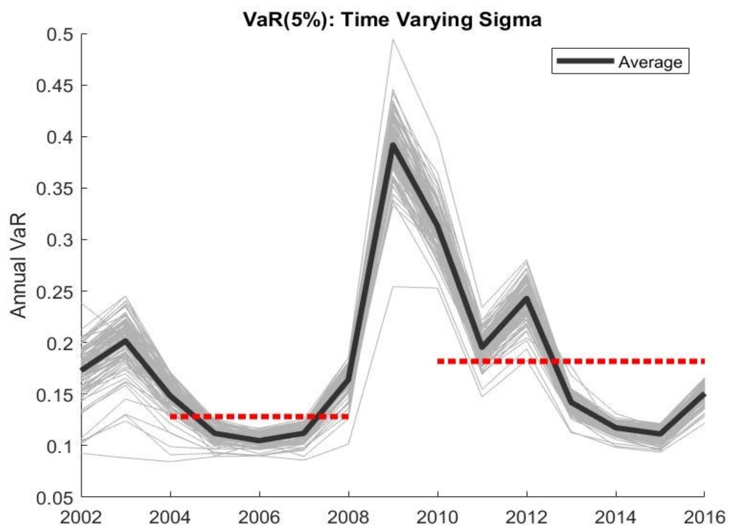
Notes: For the theoretical model in Section 2, when states can choose to default on their debt, the Figure presents the relationship between pension fund asset risk, the plans funding ratio, and the ratio of state debt to state income. Risk in the figure is measured as the proportion of risky asset in the fund's asset portfolio. The funding ratio is the present value of fund liabilities discounted by the risk-free rate. Debt to state income is measured as debt at date *t* divided by state income at date *t*. For further details see Section 2 of the text.

Figure 5: Pension Fund Risk vs Risk-Free Rate and Debt to State Income when State Debt is Risky



Notes: For the theoretical model in Section 2, when states can choose to default on their debt, the Figure presents the relationship between pension fund asset risk, the risk free rate, and the ratio of state debt to state income. Risk in the figure is measured as the proportion of risky asset in the fund's asset portfolio. Debt to state income is measured as debt at date *t* divided by state income at date *t*. For further details see Section 2 of the text.

Figure 6. VaR-based measure of PPF's asset-risk



Notes: For the Public Pension Funds in our data sample, the figure presents time-series of 5% annual Value-at-Risk (VaR) for each fund' asset-portfolio (gray lines), when the variance-covariance matrix (Sigma) of returns for different types of assets varies through time. The figure also presents the average of funds' 5% VaR (solid black line), as well as averages of VaR for the sample intervals pre- and post- the global financial crisis, i.e., 20024-2008 and 2010-2016 (dashed red lines).

Figure 7. The impact of rediscounting liabilities on funding ratios in 2015

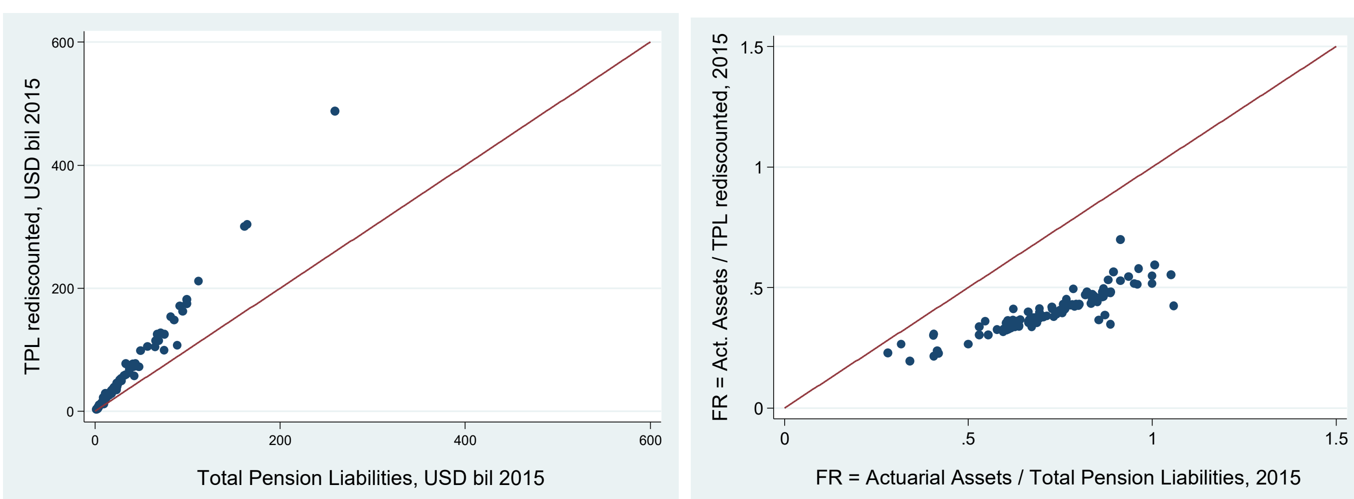


Figure 8. VaR for assets vs. lagged funding ratio, cross-sectional relation for 2016

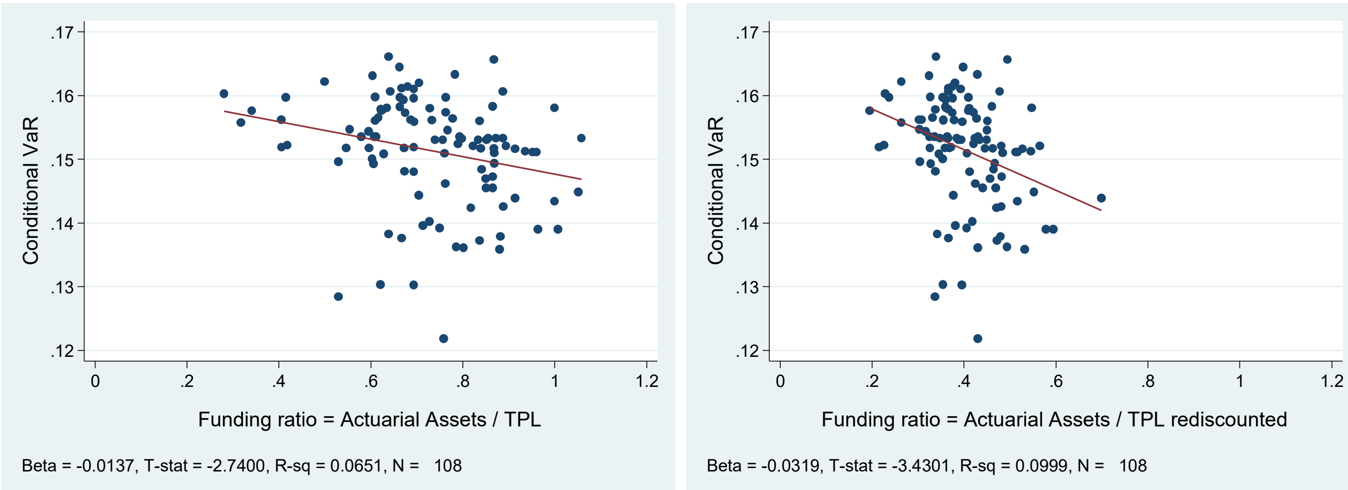
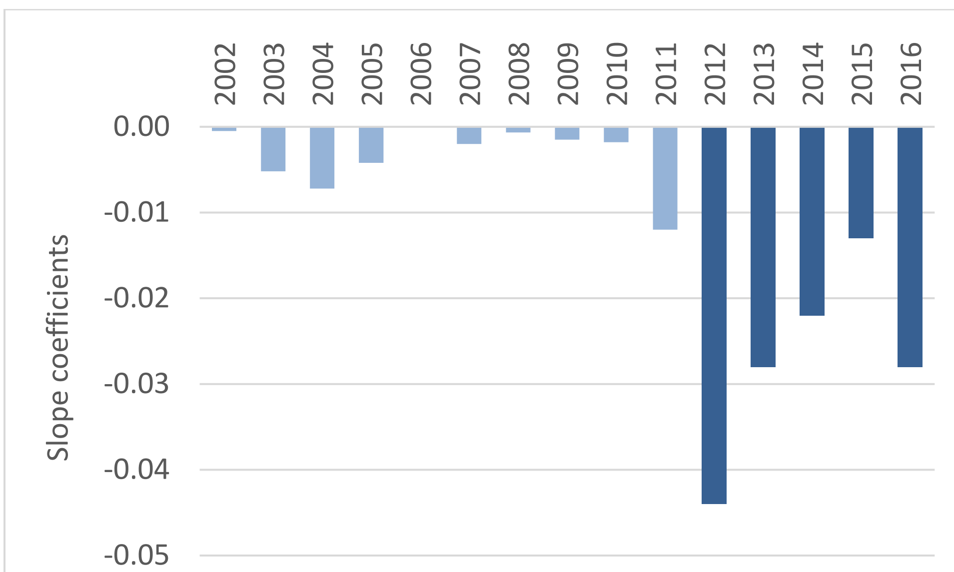


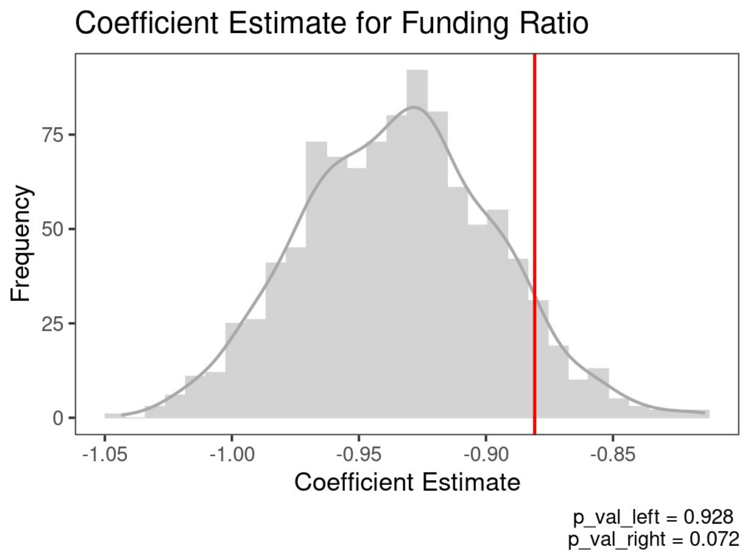
Figure 9. Asset-risk and lagged funding ratios, cross-sectional link for 2002-2016



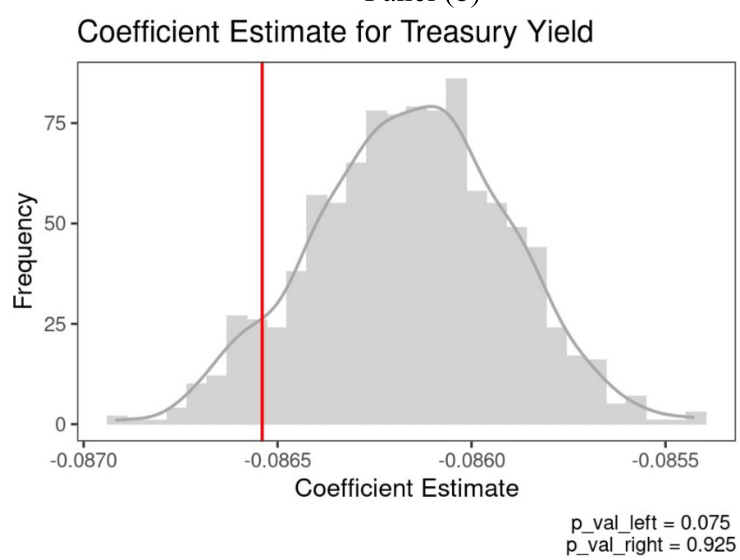
Notes: The chart shows the link between PPFs' risk in their asset portfolio and underfunding based on univariate cross-sectional regressions for each year. The bars show the slope coefficient for each year, with the darker bars showing statistical significance as reported in the Appendix Table A1, panel a. In each regression, the dependent variable is the VaR measure of asset portfolio risk, and the explanatory variable is the lagged time-varying funding ratio computed as actuarial assets divided by actuarial liabilities rediscounted using duration-matched, zero-coupon Treasury yields.

Figure 10. Real-data coefficient estimates and mechanical relation between net-worth risk, funding ratios, and risk-free rates

Panel (a)



Panel (b)



Notes: In the charts, the red lines show the real-data coefficient estimates from regressing net-worth risk on funding ratios and, alternatively, on Treasury yields, corresponding to the results in Table 7, columns (5) and (6). The shaded areas and grey liens show the histogram and kernel density of the corresponding coefficient estimates resulting from 1,000 data simulations as explained in Section 5.1. For each coefficient estimate, the cumulative distribution to the left and the right of the real-data estimates correspond to the p-values reported under each coefficient estimate in Table 7. If the real-data coefficient estimates rest toward the left (right) tail of the histogram, the real-data coefficient estimate is lower (higher) than implied by the mechanical relation between net-worth risk and each regressor, respectively.

Appendix A – Additional Tables and Figures

Table A.1. Asset-risk vs. underfunding, cross-sectional link over 2002-2016

(a) Time-varying funding ratio

	(1) 2002	(2) 2003	(3) 2004	(4) 2005	(5) 2006	(6) 2007	(7) 2008	(8) 2009
Dependent Variable:	VaR assets							
FR flex, rediscounted	-0.00050	-0.0052	-0.0072	-0.0042	-0.0034	-0.0020	-0.00066	-0.0015
	(0.016)	(0.016)	(0.013)	(0.0063)	(0.0050)	(0.0054)	(0.0093)	(0.023)
Observations	96	102	101	104	108	109	109	109
R^2	0.000	0.001	0.003	0.004	0.004	0.001	0.000	0.000

	(9) 2010	(10) 2011	(11) 2012	(12) 2013	(13) 2014	(14) 2015	(15) 2016
Dependent Variable:	VaR assets						
FR flex, rediscounted	-0.0018	-0.012	-0.044**	-0.028**	-0.022***	-0.013*	-0.028**
	(0.020)	(0.014)	(0.017)	(0.011)	(0.0068)	(0.0076)	(0.011)
Observations	108	110	110	110	108	108	104
R^2	0.000	0.007	0.056	0.053	0.087	0.027	0.062

Standard errors in parentheses, *** p<0.01, ** p<0.05, * p<0.1

(b) Fixed funding ratio

	(1) 2002	(2) 2003	(3) 2004	(4) 2005	(5) 2006	(6) 2007	(7) 2008	(8) 2009
Dependent Variable:	VaR assets							
FR fixed, rediscounted	-0.013	-0.0098	-0.012	-0.010	-0.010*	-0.013*	-0.024**	-0.080***
	(0.023)	(0.025)	(0.016)	(0.0081)	(0.0057)	(0.0069)	(0.011)	(0.028)
Observations	96	102	101	104	108	109	109	109
R^2	0.000	0.001	0.003	0.004	0.004	0.001	0.000	0.000

	(9) 2010	(10) 2011	(11) 2012	(12) 2013	(13) 2014	(14) 2015	(15) 2016
Dependent Variable:	VaR assets						
FR fixed, rediscounted	-0.057**	-0.038***	-0.054***	-0.025***	-0.018***	-0.016**	-0.032***
	(0.023)	(0.014)	(0.018)	(0.0093)	(0.0062)	(0.0071)	(0.0093)
Observations	108	110	110	110	108	108	104
R^2	0.000	0.007	0.056	0.053	0.087	0.027	0.062

Standard errors in parentheses, *** p<0.01, ** p<0.05, * p<0.1

Notes: The tables show the link between PPFs' risk in their asset portfolio and underfunding based on univariate cross-sectional regressions for each year. The constant term coefficients are not reported. In each regression, the dependent variable is the VaR measure of asset portfolio risk depicted in Figure 6. The explanatory variable is the lagged funding ratio. Panel (a) uses the funding ratios fixed at their 2015 levels, obtained as the ratio of actuarial assets to total pension liabilities, while panel (b) uses time-varying funding ratios computed with actuarial liabilities. In each case, liabilities are rediscounted using duration-matched, zero-coupon Treasury yields.

Table A.2. Determinants of PPFs' asset-risk: underfunding and risk-free rates, using fixed funding ratios

	(1)	(2)	(3)	(4)
VARIABLES	VaR assets			
FR	-0.026***	-0.045***	-0.026***	-0.014***
	(0.0026)	(0.0034)	(0.0021)	(0.00096)
TRY 5yr	-0.021**	-0.021**		
	(0.0077)	(0.0077)		
FR * TRY 5yr		0.0078***		
		(0.00078)		
Dummy post-GFC			0.054*	0.054*
			(0.030)	(0.030)
FR * Dummy post-GFC				-0.020***
				(0.0027)
Constant	0.21***	0.21***	0.13***	0.13***
	(0.034)	(0.034)	(0.011)	(0.011)
FR	Fixed	Fixed	Fixed	Fixed
FR rediscounted	Yes	Yes	Yes	Yes
Fixed effects	No	No	No	No
Observations	1,296	1,296	1,296	1,296
Funds	108	108	108	108
R-squared	0.208	0.208	0.183	0.183

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Notes: The regressions examine the link between PPFs' asset-risk, underfunding, and risk-free rates, using the following panel specification: $VaR_{i,t} = \alpha + \beta * FR_i + \gamma * TrYield_t + \delta * FR_i * TrYield_t + \epsilon_{i,t}$. The data are annual. The dependent variable is the VaR measure of asset portfolio risk depicted in Figure 6. The funding ratios are fixed at their 2015 levels and computed with total pension liabilities measured either as reported by PPFs (columns 5 to 8) or rediscounted as in Section 3.3 (columns 1 to 4). They are demeaned relative to the sample mean. The measure of risk-free rates is given by the 5-year Treasury yield. In columns 3, 4, 7, and 8, the Treasury yield is replaced by a post-GFC dummy variable that takes the value of 1 for years 2010-2016 and zero otherwise. The sample excludes years 2002, 2003, and 2009. Standard errors are double-clustered at the fund and year level.

Table A.3. Determinants of PPF's net-worth risk (normalized by assets): underfunding and risk-free rates cross-sectional relation over time

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	2002	2003	2004	2005	2006	2007	2008	2009
Dependent variable:				VaR				
FR rediscounted	-0.5264 ***	-0.6110 ***	-1.0348 ***	-0.5970 ***	-0.6024 ***	-0.3290 ***	-0.4127 ***	-0.9834 ***
	(0.982, 0.018)	(0.984, 0.016)	(0.995, 0.005)	(0.992, 0.008)	(0.943, 0.057)	(0.973, 0.027)	(0.794, 0.206)	(0.478, 0.522)
Observations	96	102	101	104	108	109	109	109
R^2	0.6631	0.6919	0.7793	0.7962	0.8188	0.7827	0.7795	0.8242

Standard errors in parentheses. *** p < 0.01; ** p < 0.05; * p < 0.1

	(9)	(10)	(11)	(12)	(13)	(14)	(15)
	2010	2011	2012	2013	2014	2015	2016
Dependent variable:				VaR			
FR rediscounted	-1.1846 ***	-1.2965 ***	-1.2809 ***	-1.4744 ***	-0.9945 ***	-1.0080 ***	-1.1898 ***
	(0.828, 0.172)	(0.186, 0.814)	(0.440, 0.560)	(0.063, 0.937)	(0.529, 0.471)	(0.006, 0.994)	(0.139, 0.861)
Observations	108	110	110	110	108	108	104
R^2	0.7855	0.6744	0.7267	0.5184	0.7335	0.4824	0.6777

Standard errors in parentheses. *** p < 0.01; ** p < 0.05; * p < 0.1

Notes: The table shows the link between PPFs' net-worth risk and underfunding based on univariate cross-sectional regressions for each year. The constant term coefficients are not reported. In each regression, the dependent variable is the VaR measure of net worth (normalized by assets). The explanatory variable is the lagged rediscounted funding ratio, which is the time-varying funding ratio computed with actuarial liabilities rediscounted as in Section 3.3.

Table A.4. Determinants of PPF's net-worth risk (normalized by liabilities): underfunding and risk-free rates, cross-sectional relation over time

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	2002	2003	2004	2005	2006	2007	2008	2009
Dependent variable:				VaR				
FR rediscounted	0.1209 ***	0.1799 ***	0.1170 ***	0.0591 ***	0.0457 ***	0.0557 ***	0.1303 ***	0.3131 ***
	(0.807, 0.193)	(0.823, 0.177)	(0.934, 0.066)	(0.928, 0.072)	(0.767, 0.233)	(0.883, 0.117)	(0.719, 0.281)	(0.514, 0.486)
Observations	96	102	101	104	108	109	109	109
R^2	0.5051	0.6676	0.3744	0.2482	0.1406	0.306	0.6071	0.7776

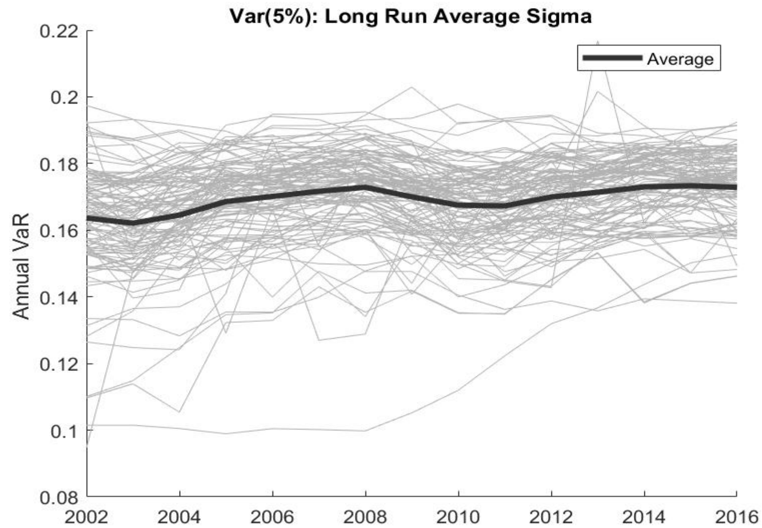
Standard errors in parentheses. *** p < 0.01; ** p < 0.05; * p < 0.1

	(9)	(10)	(11)	(12)	(13)	(14)	(15)
	2010	2011	2012	2013	2014	2015	2016
Dependent variable:				VaR			
FR rediscounted	0.2319 ***	0.1181 ***	0.2031 ***	0.0658 *	0.0512 **	0.0151	0.0688 **
	(0.709, 0.291)	(0.211, 0.789)	(0.531, 0.469)	(0.098, 0.902)	(0.607, 0.393)	(0.004, 0.996)	(0.171, 0.829)
Observations	108	110	110	110	108	108	104
R^2	0.4833	0.1574	0.3981	0.0368	0.0957	0.0038	0.071

Standard errors in parentheses. *** p < 0.01; ** p < 0.05; * p < 0.1

Notes: The table shows the link between PPFs' risk-taking and underfunding based on univariate cross-sectional regressions for each year. The constant term coefficients are not reported. In each regression, the dependent variable is the VaR measure of net worth (normalized by liabilities). The explanatory variable is the lagged rediscounted funding ratio, which is the time-varying funding ratio computed with actuarial liabilities rediscounted as in Section 3.3.

Figure A.1. VaR-based measures of PPF's asset-risk, long run average sigma



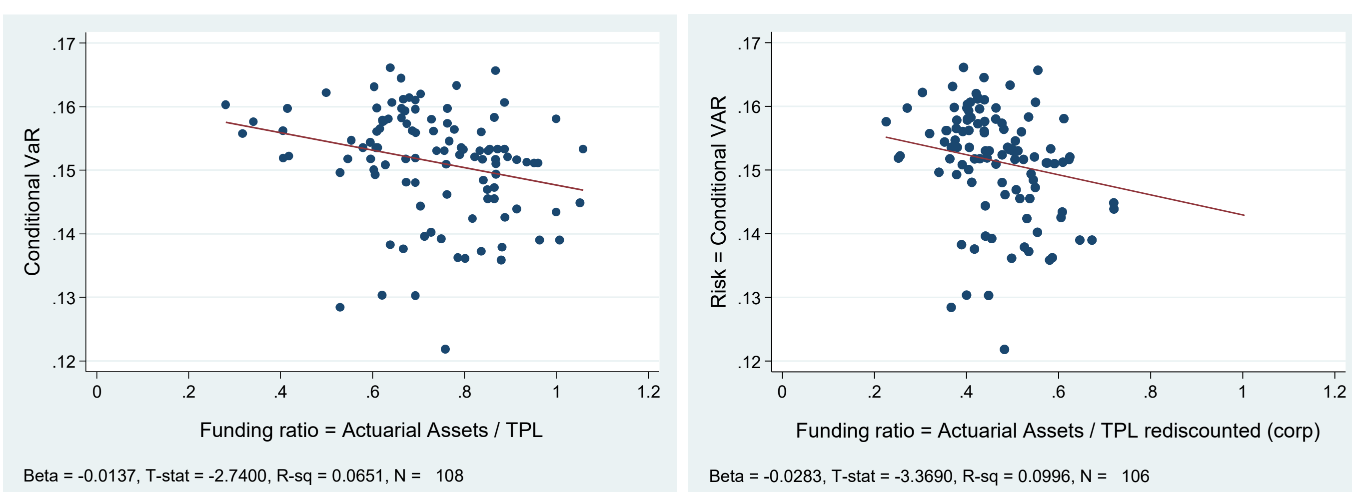
Notes: For the Public Pension Funds in our data sample, the figure present time-series of 5% annual Value-at-Risk (VaR) for each fund' asset-portfolio (gray lines), when the variance-covariance matrix (Sigma) of returns for different types of assets is fixed at its long run average. The figure also presents the average of funds' 5% VaR (solid line).

Figure A.2. Asset-risk vs. lagged funding ratio, cross-sectional link for 2016, robustness test:

Panel (a) "Unconditional" VaR measure of asset risk, time-invariant variance-covariance matrix

Panel (b) VaR measure of asset risk with active portfolio shares adjusted for valuation changes

Panel (c) Liabilities rediscounted with high-quality coprorate bond yields



Appendix B – Economic Significance

This section quantifies the contributions of underfunding and low risk-free rates to the funds' asset risk-taking behavior, and also infers their potential implications for state public finances.

5.1 Magnitude of Asset Risk-Taking Behavior due to Underfunding and Low Interest Rates

To quantify the contributions of key drivers to the increased risk in PPFs' portfolios in recent years, we use our panel regression results from Section 4.3 to decompose the average VaR measure of risk for the post-Global Financial Crisis (GFC) period into two components: one that is related to the change in funding ratios and risk-free rates from the pre-GFC period, and a residual that abstracts from these two factors.

Specifically, using the regression results for equation (17) in Table A.2, column 2, we compute the contribution of lower yields and lower funding ratios in the post-GFC years as:

$$\Delta VaR = (\gamma + \delta * FR_{avrg}) * \Delta TrYield + (\beta + \delta * TrYield_{avrg}) * \Delta FR$$

$$\Delta VaR = (\beta + \delta * TrYield_{avrg}) * \Delta FR$$

where ΔVaR is the change in risk attributable to lower yields and funding ratios; $\Delta TrYield$ and ΔFR are the change in the average 5-year Treasury yield and the average rediscounted time-varying funding ratios from period before to the one after the GFC; FR_{avrg} and $TrYield_{avrg}$ are the average funding ratios and yields over the entire sample period; and the coefficients are β = –0.045, γ = –0.021, and δ = 0.0078.

In Figure B.2, the two bars show the average level of portfolio risk in the pre and post-GFC periods, which increased from less than 13% before the crisis to more than 18% post-crisis. For the post-crisis years, the bar to the right also shows the contributions brought by the lower average Treasury yields and the lower average funding ratios relative to before the crisis, shown by the brown and dark orange layers of the bar. Together, the lower yields and increased underfunding accounted for about 1/4 of the average level of risk in pension funds' portfolios observed in recent years.

In the figure, the change in funding ratios appear to have a small contribution to the change in risk-taking only because the chart shows average portfolio risk in the pre- and post-GFC periods, while collapsing the cross-sectional heterogeneity across funds into averages for each period. After all, funding ratios were already low on average across funds before the crisis (see the rediscounted funding ratios in Figure 1, panel b). However, funding ratios vary dramatically from 20 percent to 60 percent across funds (see Figure 7, panel b), so they have a much larger importance in explaining risk-taking in the cross-section of funds within each time period.

5.2 Implications for State Finances

The pension funds' reach for yield may result in larger portfolio losses in a downturn. In turn, these losses could further strain the public finances of state and local sponsors, which guarantee the pension funds' liabilities.[57] Sponsors of these plans typically have little ability to alter benefit levels or the terms of retirement plans.[58] As a result, most of the downside risks associated with a decline in asset values or lower investment returns is likely to be borne by the taxpayers of the jurisdiction sponsoring the plan.

To measure the potential losses, the 5-percent Value at Risk can be interpreted as the portfolio loss that a pension fund may suffer in a 1-in-20 years, severe stress event. As shown by the formula, the losses under stress would be proportional with the funds' Value at Risk and the size of their assets: $\textit{Losses under Stress}_{2010\text{-}16} = VaR_{2010\text{-}16} * \textit{Net Assets}_{2010\text{-}16}$. To estimate these potential losses and their size relative to the state and local government debts, we use data on: (a) the net assets of the universe of public pension funds from the Federal Reserve's Financial Accounts of the United States; (b) the VaR estimate for the funds in our sample, and (c) state and local debt from the Bureau of Economic Analysis' Survey of State Finances. Thus, we estimate that if all pension funds were to experience their Value at Risk losses in a severe stress event during the post-crisis years, these losses would have been equivalent to almost 20% of the state

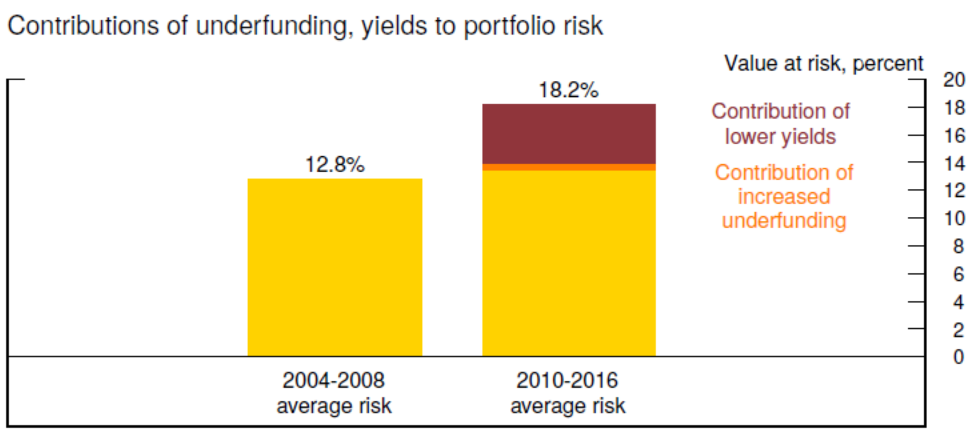
[57] Boyd and Yin (2017).

[58] Munnell and Quimby (2012)

and local debt. In contrast, they would have represented only 13% of debt in the pre-GFC years, when portfolio risk was lower.[59]

While sizeable, the fiscal impact of the pension funds' losses in a downturn would be gradual. In the short term, state and local governments could experience credit rating downgrades and higher funding costs. Most pension funds could continue to pay benefits for some time, but eventually their state and local government sponsors would have to increase contributions. In addition, the impact of funds' risk-taking behavior on state finances is likely to be skewed, with states with weaker finances likely to be hit more. In Section 4.3, we find that funds from the more financially-constrained states are more likely to assume additional levels of risk. That is, risk-taking behavior is most pronounced among funds with sponsors with the least ability to bear additional risk.

Figure B.1. PPFs' asset-risk linked to lower risk-free rates and funding ratios in recent years



[59] The net assets of the U.S. public pension funds amounted to \$3,265 billion on average during 2010-2016 and \$2,776 billion during 2004-2008. The average VaR for the funds in our sample were 18 and 13 percent respectively during the two intervals (see Figure B.2). The state and local government debt at the end of each year was \$3,118 billion on average during 2010-2016 and \$2,753 billion on average during 2004-2008.