Strategic Debt in a Monetary Economy*

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Abstract

We analyze in a New-Monetarist framework how producers improve their bargaining position vis-à-vis heterogeneous consumers by means of issuing debt. Corresponding with empirical evidence, producers negotiate better terms of trade when indebted. In general equilibrium, debt comes with inefficiencies due to a pecuniary externality—debt is too cheap. In absence of a Pigouvian tax, monetary policy improves welfare by deviating from the Friedman rule. Consumers then economize on money holdings, and producers issue less debt in response. Less debt makes trade with low-preference consumers more likely, so that trade accelerates and money is spent faster.

Keywords: bargaining, money search, strategic debt.

JEL Classification: C78, E40, E50, E52.

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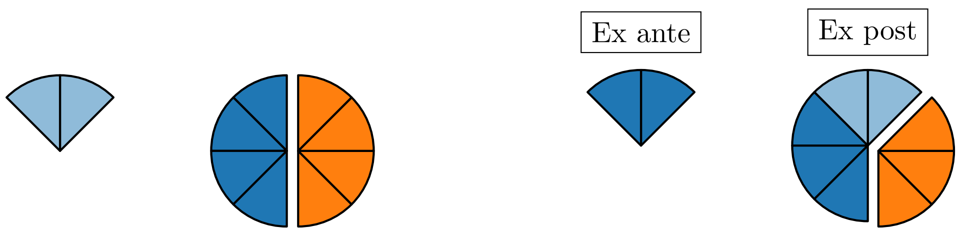
1 Introduction

Why do firms issue debt? Standard corporate-finance theory studies how a firm's capital structure matters for principal-agent problems and taxation, focusing on a firm's relation to its financial stakeholders such as shareholders and managers. We analyze through the lens of a macro-finance model how capital structure affects a firm's non-financial stakeholders. In particular, we study how producers can use limited-liability debt for the sole purpose of improving their bargaining position vis-à-vis their consumers. By casting producer-consumer relationships in a macroeconomic model, we formalize not only the empirically-explored distributive effect of debt within these relationships, but also uncover novel distortionary effects that spill over across relationships. Moreover, we study how fiscal and monetary policy can mitigate these distortions.

Towner (2020) provides empirical evidence for the distributive effect of debt within producer-consumer relationships. He finds that U.S. hospitals with higher debt-to-equity ratios negotiate higher reimbursement rates from health insurers. The theory behind this finding is well-illustrated by Hennessy and Livdan (2009) in how two agents, A and B, bargain over eight pieces of cake. Both agents exert equal bargaining power and receive no piece if they fail to agree. Without any debt issuance, both agents receive four pieces. Now suppose that prior to bargaining, agent A sells a claim on two pieces to some other agent C: in exchange for two pieces today, agent A promises to give the first two pieces from tomorrow's bargaining with agent B to agent C. Agents A and B thus effectively bargain over only six pieces. Bargaining results in five pieces for agent A, i.e., two for agent C and the remaining three for him/herself, and three pieces for agent B. Since agent A has already consumed two pieces ex ante, agent A improves his/her ex-post bargaining position through ex-ante debt issuance. Figure 1 depicts the agents' cake consumption with and without such a *strategic debt* issuance.



(a) Without debt. (b) With debt.

Figure 1: Cake consumption of agents A (●), B (●), and C (●).

Our contribution is to embed strategic debt in a general-equilibrium framework, shedding light not only on the distributive effect of debt but also on potential distortionary effects and policy implications. To this end, we employ the New-Monetarist workhorse model by Lagos and Wright (2005) because it features matching and bargaining in a monetary general-equilibrium model. This allows us to also characterize optimal monetary policy to correct pecuniary externalities in the absence of Pigouvian taxation. In the standard New-Monetarist model, producers and consumers match in a decentralized market and bargain over the terms of trade subject to a liquidity constraint. We enrich this model in essentially two regards: producers can write debt contracts with financiers, and consumers incur idiosyncratic preference shocks.

A debt contract in our model specifies an ex-ante transfer to a producer (borrower) and an ex-post transfer to an financier (lender) contingent on the producer having earned the necessary funds in the decentralized market. Such a claim on future revenues essentially is limited-liability debt.

Producers issue debt neither for real investment needs nor to prepone consumption per se, but rather to skim the consumers' surplus. Particularly, we show that the incentive to issue debt is stronger when producers have less bargaining power—debt and bargaining power are substitutes. Framed differently, producers make up for a lack of bargaining power by using their ability to commit to debt repayment. In the extreme case that consumers make take-it-or-leave-it offers to producers, debt issuance is the only way for producers to appropriate match surplus. In the other extreme case that producers make take-it-or-leave-it offers, producers issue zero debt since they cannot realize any distributive benefits from it. These findings align with Towner's (2020) empirical analysis: hospitals take on less debt when their bargaining power is high.

The fact that producers compensate a lack of bargaining power with debt comes with negative intensive- and extensive-margin effects on trade. By entering a match with more debt, a producer forces the consumer to partially pay for debt repayment. This reduces the consumer's liquid wealth left to pay the producer for production costs. The liquidity constraint tightens and the traded quantity decreases—an intensive-margin effect. Moreover, the producer loses bargaining agreements with consumers who incur low preference shocks—an extensive-margin effect. These consumers either carry too little money balances to pay the producer for debt repayment, or they value the production good so little that the match surplus net of debt repayment would be negative for all production levels. Since debt distorts trade at the intensive and extensive margin, it impairs welfare.

The source of these distortions is that financiers exert a pecuniary externality. They absorb the producers' debt at interest rates that do not capture the full societal cost of debt at the intensive and extensive margin—financiers and producers ignore the utility cost of debt for the consumers. Fiscal policy can impose a Pigouvian tax on debt, e.g., a markup on interest rates, to curb debt issuance. The fiscal authority optimally sets this tax prohibitively high to shut down intermediation.

Whilst fiscal policy can tackle the pecuniary externality of debt directly with a Pigouvian tax, monetary policy can only indirectly mitigate this externality through inflation—

a tax on the consumers' money holdings. The monetary authority determines long-run inflation to maximize the aggregate surplus in all matches in the presence of debt. As a benchmark policy we consider the Friedman rule, which implements a slight deflation to induce a zero opportunity cost of holding money. At the Friedman rule, producers issue positive debt, so that in matches with low-preference consumers, a bargaining agreement cannot be reached. If an agreement is reached, trade is intensively efficient since consumers hold enough money to render the liquidity constraint slack. A deviation from the Friedman rule through an increase of inflation makes carrying real balances expensive. Consumers respond by reducing their money holdings, rendering the liquidity constraint binding when incurring a high preference shock. In turn, producers reduce debt to relax the liquidity constraint in matches with high-preference consumers, also entailing that bargaining agreements with low-preference consumers become more likely. We find that the latter effect is dominant—welfare improves.

In particular, a deviation from the Friedman rule compensates for a lack of coordination among consumers. At the Friedman rule, an individual consumer faces a zero opportunity cost of holding money and thus chooses money holdings that render the liquidity constraint slack for all potential preference shocks. This results in large match surpluses, which the producers skim by entering matches with debt. If consumers could coordinate their money holdings, they would rather jointly reduce money holdings to protect themselves from being skimmed. A deviation from the Friedman rule mitigates this lack of coordination by incentivizing consumers to economize on costly money holdings. This makes bargaining agreements more likely, so that trade accelerates and money changes hands faster.

The remainder of this paper is organized as follows. We relate our work to the existing literature in Section 2, and we develop the model in Section 3. In Section 4, we describe agents' optimal choices and present the bargaining problem. We introduce the equilibrium concept, define the notion of welfare, and discuss the long-run transmission of monetary policy in Section 5. We conclude the analysis in Section 6. Proofs and derivations are in the appendix.

2 Literature

We relate to a recent literature on how leverage impacts a firm's relationship with its non-financial stakeholders like workers, suppliers, and customers (e.g., Titman, 1984). The use of debt in the firm-worker relationship caught particular interest (e.g., Dasgupta and Sengupta, 1993; Matsa, 2010; Perotti and Spier, 1993). Bronars and Deere (1991) show how debt protects the firms' surplus from extraction by workers' unions—debt mitigates the *threat of unionization*. By issuing debt, a firm diverts future cash flows into current cash flows, reducing the future surplus that a union can extract without driving the firm into bankruptcy. Bronars and Deere (1991) confirm their theoretical analysis: they find a positive relationship between unionization and debt-to-equity ratios across U.S. firms.

Hennessy and Livdan (2009) consider bargaining between upstream firms and downstream firms, where the downstream firms issue debt to improve their bargaining position. In contrast to our model, it is thus the consumers and not the producers who issue debt. Similar to our model, though, optimal debt trades off distributional benefits and efficiency costs. On the one hand, the downstream firm undertakes a leveraged recapitalization before bargaining and pays a dividend to its shareholders. Shareholders thus prepone consumption to extract more match surplus. On the other hand, debt reduces operational efficiency since it distorts the incentives of the downstream firm. As in our model, agents issue less debt when exerting more bargaining power.

Dasgupta and Nanda (1993) construct a model in which producers, by issuing debt, skim surplus from consumers in a regulated product market. The authors portray the regulator as a bargaining protocol: the regulator distributes surplus between producers and consumers by setting prices. Dasgupta and Nanda (1993) confirm their model by looking at U.S. electricity firms that operate in states where public utility commissions set prices. They find that firms issue more debt when regulated by commissions that are more generous towards consumers. Our model replicates this finding: the smaller the bargaining power, the larger is the incentive to issue debt. We contribute to the literature by going beyond the analysis of this distributive effect: we pin down the distortions caused by strategic debt in general equilibrium, and derive policy implications.

We also relate to the literature on how inflation accelerates trade. Li (1994) models this insight by endogenizing the matching rate in a first-generation New-Monetarist model in the style of Kiyotaki and Wright (1993). Lagos and Rocheteau (2005) incorporate endogenous matching in a third-generation New-Monetarist model with divisible goods and money, and find that inflation reduces the matching rate. Approaches to reconcile third-generation models with the accelerating effect of inflation on trade focus primarily on consumers' incentives to spend money faster as its value depreciates amid inflation (see, e.g., Dong and Jiang, 2014; Ennis, 2009; Liu, Wang and Wright, 2011; Nosal, 2011, on this *hot-potato* effect). In contrast, Althanns, van Buggenum and Gersbach (2023) study how producers increase their economic activity to borrow more when inflation increases. Specifically, producers exploit a borrowing discount—the difference between their rate of time preference and the interest rate. When inflation increases, the borrowing discount increases as well, so that producers commit to more future economic activity to increase their future income against which they borrow. Ultimately, the frequency at which money is spent increases. We contribute to this literature by stressing how inflation accelerates trade through making bargaining agreements more likely.

Finally, we address a broader literature on debt in New-Monetarist models. In Aruoba, Waller and Wright (2011), Altermatt (2022), and Altermatt, van Buggenum and Voellmy

(2022), debt is necessary to fund real investments. In Althanns *et al.* (2023), agents issue debt to prepone consumption. Our paper contrasts these approaches by investigating the strategic role of debt in bargaining. We also show that debt is the optimal financial contract in our framework if state-verification costs à la Townsend (1979) are large. This finding confirms the benchmark optimality result of Williamson (1987).

3 Model

Time $t \in \{0, 1, \dots\}$ is discrete and goes on forever. Each period is divided into two consecutive subperiods: DM_t and CM_t (see Figure 2). The economy starts in CM_0. There are two types of perfectly divisible and non-storable goods: *DM goods* and *CM goods* (treated as the numéraire). The economy is populated by a unit mass of infinitely-lived agents called *consumers*. In each CM_t, a unit mass of agents called *producers* is born, and these producers die at the end of CM_{t+1}. Moreover, there is a unit mass of infinitely-lived *financiers* born in CM_0. All agents have the same time-discount factor $\beta \in (0, 1)$.

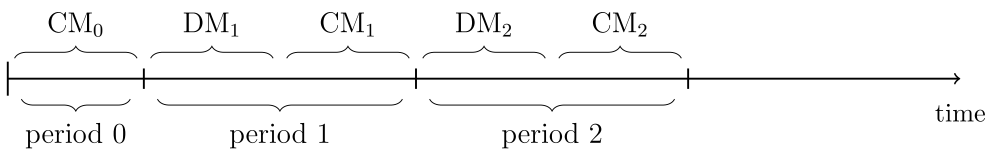


Figure 2: Alternation of DM_t and CM_t.

In DM_t, producers can produce DM goods but cannot consume them, whereas consumers wish to consume DM goods but cannot produce them. Producers and consumers trade DM goods in a decentralized market (hence, DM), in which a unit mass of bilateral matches between consumers and producers is randomly arranged. In a match, a consumer and a producer determine the terms of trade through proportional Kalai (1977) bargaining. In CM_t, all agents can produce and consume CM goods, and they trade them in a centralized Walrasian market (hence, CM).

3.1 Preferences

The periodic utility of a consumer is

$$U_t^c = \epsilon_t u(q_t) + x_t, \tag{1}$$

where $q_t \geq 0$ is DM-goods consumption and $x_t \in \mathbb{R}$ is CM-goods net consumption. The function u is continuously differentiable on $(0, \infty)$ and fulfills $u' > 0$, $u'' < 0$, $u(0) = 0$, $\lim_{q \to 0} u'(q) = \infty$, and $\lim_{q \to \infty} u'(q) = 0$. The consumer incurs preference shock $\epsilon_t \overset{\text{i.i.d.}}{\sim} G$

at the beginning of DM_t. G is the cumulative distribution function of a continuous probability law, G has compact support $[0, \bar{\epsilon}] \subset [0, \infty)$, and we write $g = G'$. A consumer's lifetime utility is $\sum_{t=0}^{\infty} \beta^t U_t^c$.

Lifetime utility of a producer born in CM_t is

$$U_t^p = x_t + \beta[-c(q_{t+1}) + x_{t+1}], \tag{2}$$

where $q_{t+1} \geq 0$ is DM-goods production and $x_t, x_{t+1} \in \mathbb{R}$ is CM-goods net consumption. The function c is continuously differentiable on $[0, \infty)$ and fulfills $c' > 0$, $c'' \geq 0$, and $c(0) = 0$.

The periodic utility of a financier is $U_t^{fi} = x_t$, where $x_t \in \mathbb{R}$ is CM-goods net consumption, and its lifetime utility is $\sum_{t=0}^{\infty} \beta^t U_t^{fi}$.

3.2 Money and Debt

The consumers' anonymity in DM matches necessitates a payment instrument. To this end, the government issues *fiat money*: a perfectly divisible, intrinsically useless, and storable asset. Money supply at the beginning of period t is M_t, and it grows at gross rate γ between periods through lump-sum injections to consumers in the CM. The CM_t injection in real terms equals $\phi_t(M_{t+1} - M_t)$, where ϕ_t is the CM_t price of money.

Financiers are perfectly competitive, so we consider a representative financier. The financier can write financial contracts over one period. We argue when turning to the equilibrium that the producers borrow from the financier to increase their match surpluses. We focus on limited-liability debt because it is the optimal financial contract if Townsend (1979) state-verification costs of the borrower's wealth are prohibitively high, as we show in Appendix A. The financier and the borrower specify a payment $b_{t+1}/R_t(b_{t+1})$ in CM_t from the financier to the borrower as well as a repayment b_{t+1} subject to limited liability in CM_{t+1}. Limited liability implies that the repayment cannot exceed the borrower's wealth. The gross borrowing rate is determined by pricing kernel $R_t : \mathbb{R}_+ \to \mathbb{R}_+ \cup \{\infty\}$. The notion $R_t(b) = \infty$ means that debt is not feasible for b. The kernel will internalize the dependency of the borrower's default probability on the contracted repayment b_{t+1}, as we show later. The financier keeps its claims on debt repayment on its balance sheets.

4 Optimal Choices and Bargaining

4.1 Value Functions

Let $V_t^c(m|\epsilon)$ be a consumer's value of entering DM_t with real balances m and incurring preference shock ϵ. Consumers lack commitment and thus cannot write financial contracts contingent on their trading history. Therefore, they can only carry non-negative amounts of real balances. Let $W_t^c(m)$ be a consumer's value of entering CM_t, so that

$$W_t^c(m) = \max_{m'_{t+1}\geq 0} \left\{ x + \beta \int_0^{\bar{\epsilon}} V_{t+1}^c(m'_{t+1}|\epsilon_{t+1}) G(\mathrm{d}\epsilon_{t+1}) \right\}, \tag{3}$$

$$\text{s.t.} \quad x = m - \frac{\phi_t m'_{t+1}}{\phi_{t+1}} + \tau_t. \tag{4}$$

The consumer chooses next-period real money balances m'_{t+1} to maximize the sum of current CM-goods consumption and the time-discounted expected utility from entering DM_{t+1}. Constraint (4) captures that his/her current CM-goods consumption is equal to the gains from adjusting his/her money holdings plus the government transfer τ_t. Note that W_t^c is quasi-linear in m, eliminating wealth effects.

Let $V_t^p(b|\epsilon)$ be the producer's value of entering DM_t with contracted debt repayment b, being matched with a consumer with preference shock ϵ. Producers do not carry money from CMs to DMs, since this would undermine their bargaining position in decentralized trade. A producer's value of being born in CM_t is

$$W_t^{p,0} = \max_{b'_{t+1}\geq 0} \left\{ \frac{b'_{t+1}}{R_t(b'_{t+1})} + \beta \int_0^{\bar{\epsilon}} V_{t+1}^p(b'_{t+1}|\epsilon_{t+1}) G(\mathrm{d}\epsilon_{t+1}) \right\}. \tag{5}$$

S/he contracts repayment b' to maximize the sum of CM_t-goods consumption and the time-discounted expected utility from entering DM_{t+1}. His/her CM_t-goods consumption is equal to the payment that s/he receives from the financier. His/her value of entering CM_{t+1} with real balances m and contracted debt repayment b is

$$W_{t+1}^{p,1}(m, b) = \max\{m - b, 0\}, \tag{6}$$

where the max-operator elucidates the limited-liability nature of debt.

Financiers are perfectly competitive and risk neutral, and we assume without loss that they fully diversify their portfolios. Hence, any debt contract they write must earn the risk-free rate R_t^f between CM_t and CM_{t+1} in expectation. The financier's value of entering CM_t with a portfolio of debt contracts with expected payoff a thus reads

$$W_t^{fi}(a) = \max_{a'_{t+1}\geq 0} \left\{ a - \frac{a'_{t+1}}{R_t^f} + \beta W_t^{fi}(a'_{t+1}) \right\}. \tag{7}$$

The financier writes new debt contracts to maximize the sum of CM_t-goods consumption and the time-discounted expected utility from entering CM_{t+1}. Its CM_t-goods consumption is equal to the difference between its payoffs and the cost of writing new debt contracts. Financiers are passive in the DMs.

4.2 Decentralized Trade

We now characterize the intensive and extensive margin of decentralized trade.

Intensive margin. Once a consumer with real balances m and preference shock ϵ meets a producer with contracted debt repayment b, they negotiate terms of trade (q, p), i.e., DM-goods quantity q, transferred to the consumer, and real payment p, transferred to the producer. Proportional Kalai (1977) bargaining determines

$$(q, p) = \arg\max_{q,p\geq 0}\{\epsilon u(q) + W_t^c(m-p) - W_t^c(m)\},$$
$$\text{s.t.}\quad p \leq m,$$
$$\theta\left[\epsilon u(q) + W_t^c(m-p) - W_t^c(m)\right] = (1-\theta)\left[-c(q) + W_t^{p,1}(p,b) - W_t^{p,1}(0,b)\right], \tag{8}$$

where $\theta \in [0,1]$ denotes the producer's bargaining power and $p \leq m$ is the liquidity constraint. Substituting W_t^c and $W_t^{p,1}$, it follows that

$$(q, p) = \arg\max_{q,p\geq 0}\{\epsilon u(q) - p\},$$
$$\text{s.t.}\quad p \leq m \quad\text{and}\quad \theta[\epsilon u(q) - p] = (1-\theta)[-c(q) + \max\{p-b, 0\}]. \tag{9}$$

Note that it is limited liability which makes debt affect the bargaining set. If there were full commitment in debt contracts, meaning that the producer would repay b regardless of the bargaining outcome, his/her surplus would read as $-c(q) + p$ rather than $-c(q) + \max\{p-b, 0\}$. If there were no commitment at all, meaning that b would never be repaid, his/her surplus would also be $-c(q)+p$. Hence, not only the availability of debt but also the degree of commitment matters for how debt affects bargaining.

Since the terms of trade are fully determined by (ϵ, m, b), we identify a match with the tuple (ϵ, m, b). We write $q(\epsilon, m, b)$ and $p(\epsilon, m, b)$ for the negotiated terms of trade and call a match (ϵ, m, b) *successful* if $q(\epsilon, m, b) > 0$, i.e., if a bargaining agreement is reached. A successful match requires

$$p - c(q) - b \geq 0 \quad\text{and}\quad p = \theta\epsilon u(q) + (1-\theta)[c(q) + b]. \tag{10}$$

In particular, either the match is successful and the producer fully repays his/her debt in the subsequent CM, or the match is unsuccessful and the producer fully defaults. The reason is that with partial default, the producer's payoff would read $-c(q)$, which would be worse than his/her threat point—refraining from any agreement.

We call $\epsilon u(q) - c(q) - b$ the *net match surplus* in a successful match, and we define the consumer's and the producer's net surpluses $v^c(\epsilon, m, b)$ and $v^p(\epsilon, m, b)$ as their surpluses resulting from bargaining. The surpluses read as

$$v^c(\epsilon,m,b) = (1-\theta)[\epsilon u(q) - c(q) - b]_{q=q(\epsilon,m,b)}$$
$$\text{and} \quad v^p(\epsilon,m,b) = \theta[\epsilon u(q) - c(q) - b]_{q=q(\epsilon,m,b)}. \quad (11)$$

Equation (11) transpires that the producer's contracted repayment translates into a fixed cost for which the producer requires compensation.

We define the *gross match surplus* in a successful match as $\epsilon u(q) - c(q)$, and we define the consumer's and the producer's gross surpluses as $\epsilon u(q) - p$ and $-c(q) + p$. Once we characterize welfare, we show that $q^\star_\epsilon$, which maximizes the gross surplus and thus solves $\epsilon u'(q^\star_\epsilon) = c'(q^\star_\epsilon)$, is the *intensively-efficient* level of q. Whilst the consumer's gross surplus coincides with his/her net surplus, the producer's gross surplus reads as

$$[-c(q) + p]_{q=q(\epsilon,m,b)} = \theta[\epsilon u(q) - c(q) - b]_{q=q(\epsilon,m,b)} + b \quad (12)$$
$$= \theta[\epsilon u(q) - c(q)]_{q=q(\epsilon,m,b)} + (1-\theta)b. \quad (13)$$

We will show further below that only the gross surplus is relevant for the producer's lifetime utility—the process of debt issuance and repayment itself is utility neutral.

Extensive margin. Characterizing when a match is successful requires defining

$$\tilde{q}(\epsilon,b) \equiv \inf\{q \in [0,\infty) : \epsilon u(q) - c(q) - b \geq 0\} \quad (14)$$

as the smallest DM-good quantity that allows for a non-negative net match surplus given ϵ and b.[1]

Lemma 1. *A match (ϵ,m,b) is successful if and only if the following conditions jointly hold:*

(i) $\epsilon u(q^\star_\epsilon) - c(q^\star_\epsilon) - b \geq 0$;

(ii) $m \geq c(\tilde{q}(\epsilon,b)) + b$.

Condition (i) requires the non-negativity of net surplus at the intensively-efficient level. This condition translates into an upper bound on the producer's contracted repayment b. Condition (ii) requires that the consumer's real balances are large enough to pay for $\tilde{q}(\epsilon,b)$. Note that the constraints of Conditions (i) and (ii) both relax when ϵ increases for given levels of m and b. We obtain

Corollary 1. *A match (ϵ,m,b) is successful if and only if*[2]

$$\epsilon \geq \inf\{\epsilon \in [0,\bar{\epsilon}] \; : \; q(\epsilon,m,b) > 0\} \equiv \hat{\epsilon}(m,b). \quad (15)$$

[1]Following the convention, we write $\tilde{q}(\epsilon,b) = \infty$ if $\{q \in [0,\infty) : \epsilon u(q) - c(q) - b \geq 0\} = \emptyset$.

[2]Following the convention, we write $\hat{\epsilon}(m,b) = \infty$ if $\{\epsilon \in [0,\bar{\epsilon}] \; : \; q(\epsilon,m,b) > 0\} = \emptyset$.

Corollary 1 implies that if the match (ϵ_1, m, b) is successful, then any match (ϵ_2, m, b) with $\epsilon_2 > \epsilon_1$ is successful as well. We define $\hat{q}(m, b) \equiv q(\hat{\epsilon}(m, b), m, b)$, and we write $\hat{\epsilon} = \hat{\epsilon}(m, b)$ and $\hat{q} = \hat{q}(m, b)$ if no confusion arises.

Lemma 2. *It holds that* $\hat{q} = \tilde{q}(\hat{\epsilon}, b)$.

Lemma 2 says that the net surplus of a match (ϵ, m, b) is zero if $\epsilon = \hat{\epsilon}$. Lemma 3 captures how the threshold $\hat{\epsilon}$ changes in the producer's contracted repayment b.

Lemma 3. *It holds that*

$$\frac{\partial \hat{\epsilon}}{\partial b} = \frac{1}{u(\hat{q})} \left[1 + \frac{\hat{\epsilon} u'(\hat{q}) - c'(\hat{q})}{c'(\hat{q})} \right]. \tag{16}$$

If the producer contracted a higher debt repayment, the likelihood for the match (ϵ, m, b) to be successful would decrease in the sense that Conditions (i) and (ii) in Lemma 1 would tighten.

4.3 Financiers and Debt Pricing

Financiers. Forward iteration of Equation (7) yields

$$W_t^{fi}(a) = \max_{\{a'_{j+1}\}_{j=t}^{T} \geq 0} \left\{ \sum_{j=t}^{T-1} \beta^{j+1-t} \left[-\frac{1}{\beta R_j^f} + 1 \right] a'_{j+1} + \beta^{T-t} \left[-\frac{a'_{T+1}}{R_T^f} + \beta W_{T+1}^{fi}(a'_{T+1}) \right] \right\}. \tag{17}$$

The transversality condition with respect to wealth as well as the boundedness of the CM value function make the second term within the curly brackets vanish in the limit $T \to \infty$. For the financier to write debt contracts with positive and bounded expected value in CM$_t$, it must hold that $R_t^f = 1/\beta$, as we infer from Equation (17). We thus assume without loss that $R_t^f = 1/\beta$ for all t.

4.4 Consumers

We assume without loss that the equilibrium distributions of money holdings m_{t+1} and debt b_{t+1} at the beginning of DM$_{t+1}$ are degenerate.[3,4] A consumer's DM$_t$ value function reads as

[3]Because of the quasi-linearity of $W_t^c(m)$ in m, wealth does not affect the consumers' choice of money holdings. Hence, we can assume the degeneracy of the distribution of m_{t+1} without loss. Since all producers born in CM$_t$ are identically equal ex ante, the same holds true for the distribution of b_{t+1}.

[4]To be precise, we denote a consumer's individual choice of real balances as m'_{t+1} (as in Equation (3)) and denote the equilibrium level as m_{t+1}. We analogously use b'_{t+1} and b_{t+1}.

$$V_t^c(m|\epsilon_t) = m + v^c(\epsilon_t, m, b_t)\mathbb{1}_{\{\epsilon_t \geq \hat{\epsilon}(m,b_t)\}} + \tau_t + \max_{m'_{t+1}\geq 0}\left\{-\frac{\phi_t m'_{t+1}}{\phi_{t+1}} + \beta\int_0^{\bar{\epsilon}} V_{t+1}^c(m'_{t+1}|\epsilon_{t+1})]G(\mathrm{d}\epsilon_{t+1})\right\}. \quad (18)$$

By repeated forward iteration, we have

$$V_t^c(m|\epsilon_t) = m + v^c(\epsilon_t, m, b_t)\mathbb{1}_{\{\epsilon_t \geq \hat{\epsilon}(m,b_t)\}} + \tau_t + \max_{\{m'_{j+1}\}_{j=t}^T \geq 0}\left\{\begin{array}{l}\sum_{j=t}^{T-1}\beta^{j+1-t}\left[\begin{array}{l}-\left[\frac{\phi_j}{\beta\phi_{j+1}}-1\right]m'_{j+1}\\ +\int_{\hat{\epsilon}_{j+1}}^{\bar{\epsilon}} v^c(\epsilon_{j+1}, m'_{j+1}, b_{j+1})G(\mathrm{d}\epsilon_{j+1}) + \tau_{j+1}\end{array}\right]\\ +\beta^{T-t}\left[-\frac{\phi_T m'_{T+1}}{\phi_{T+1}} + \beta\int_0^{\bar{\epsilon}} V_{T+1}^c(m'_{T+1}|\epsilon_{T+1})G(\mathrm{d}\epsilon_{T+1})\right]\end{array}\right\}, \quad (19)$$

where we write $\hat{\epsilon}_t \equiv \hat{\epsilon}(m'_t, b_t)$. The transversality condition with respect to wealth as well as the boundedness of the CM value function again make the last term within the curly brackets vanish in the limit $T \to \infty$. The necessary first-order condition for the consumer's money demand m'_{t+1} reads as

$$0 \geq -\iota_{t+1} + \int_{\hat{\epsilon}_{t+1}}^{\bar{\epsilon}} \mathcal{L}(\epsilon_{t+1}, m'_{t+1}, b_{t+1})G(\mathrm{d}\epsilon_{t+1}), \qquad \text{with "=" if } m'_{t+1} > 0, \quad (20)$$

and with *liquidity premium*

$$\mathcal{L}(\epsilon, m, b) \equiv \left[\frac{(1-\theta)[\epsilon u'(q) - c'(q)]}{\theta\epsilon u'(q) + (1-\theta)c'(q)}\right]_{q=q(\epsilon,m,b)} \quad (21)$$

and *Fisher rate* $\iota_{t+1} \equiv \phi_t/\beta\phi_{t+1} - 1$. The Fisher rate is the opportunity cost of holding money. It is the hypothetical nominal interest rate that compensates for both inflation and time-discounting. The consumer chooses his/her real balances to equalize this opportunity cost with the expected liquidity premium induced by m'_{t+1}. Note that $\mathcal{L}(\epsilon, m, b)$ is the Lagrange multiplier of the liquidity constraint in match (ϵ, m, b), i.e., the shadow price of an additional unit of liquidity in this match. The Fisher rate cannot be negative, as this would imply an infinite demand for real balances.

4.5 Producers

A producer born in CM_t repays b'_{t+1} in CM_{t+1} if and only if s/he is matched with a consumer with preference shock $\epsilon_{t+1} \geq \hat{\epsilon}(m_{t+1}, b'_{t+1})$ in DM_{t+1}—s/he repays if and only if the match $(\epsilon_{t+1}, m_{t+1}, b'_{t+1})$ is successful. Whilst the consumer-specific ϵ_{t+1} is not yet known when debt is contracted, the equilibrium real balances m_{t+1} are. Debt contracts yield the risk-free rate R_t^f in expectation, so that the pricing kernel R_t fulfils

$$R_t(b'_{t+1}) \int_0^{\bar{\epsilon}} \mathbb{1}\{\epsilon_{t+1} \geq \hat{\epsilon}(m_{t+1}, b'_{t+1})\} G(\mathrm{d}\epsilon_{t+1}) = R_t^f, \qquad \forall\, b'_{t+1} \geq 0. \tag{22}$$

Equation (22) captures a no-arbitrage condition: the borrowing rate times the probability of debt repayment is equal to the risk-free rate.[5]

A producer's value of being born in CM_t simplifies as

$$W_t^{p,0} = \max_{b' \geq 0} \left\{ \beta \int_{\hat{\epsilon}_{t+1}}^{\bar{\epsilon}} \left[\theta[\epsilon_{t+1} u(q) - c(q) - b']_{q=q(\epsilon_{t+1}, m_{t+1}, b')} + b' \right] G(\mathrm{d}\epsilon_{t+1}) \right\}, \tag{23}$$

where we use the pricing kernel in Equation (22) and exploit that $R_t^f = 1/\beta$. Equation (23) demonstrates a key point: the producer takes on debt to maximize his/her expected gross surplus as defined in Equation (12). Particularly, the process of debt issuance and repayment on its own is irrelevant for the producer's lifetime utility. The expected repayment of debt in CM_{t+1} after successful matches namely cancels with the utility from consuming the financier's payment in CM_t. This is because the effective borrowing rate is equal to the rate of time preference $1/\beta$, as captured by the pricing kernel.

Lemma 4. *The necessary first-order condition for the producer's optimal debt repayment b'_{t+1} reads as*

$$0 \geq (1-\theta)[1 - G(\hat{\epsilon}_{t+1})] - \theta \int_{\hat{\epsilon}_{t+1}}^{\bar{\epsilon}} \mathcal{L}(\epsilon_{t+1}, m_{t+1}, b'_{t+1}) G(\mathrm{d}\epsilon_{t+1}) - b'_{t+1} g(\hat{\epsilon}_{t+1}) \frac{\partial \hat{\epsilon}(m_{t+1}, b'_{t+1})}{\partial b}, \tag{24}$$

with " $=$ " *if* $b'_{t+1} > 0$.

Condition (24) stresses three channels through which an increase of b'_{t+1} affects the producer's lifetime utility. The first term on the right-hand side of Condition (24) captures the change in the distribution of the gross surplus for each successful match: the producer increases the negotiated payment by $1-\theta$ when entering the match with one additional unit of real debt, increasing his/her own gross surplus while keeping the total gross surplus unchanged. The smaller the producer's bargaining power θ, and the larger the probability $1 - G(\hat{\epsilon}_{t+1})$ of a successful match, the stronger is the producer's incentive to shift the gross surplus in his/her advantage by means of issuing debt.

The second term on the right-hand side of Condition (24) captures the reduction of the producer's gross surplus in each successful match—the intensive-margin effect of debt. By entering a match with more debt, the producer forces the consumer to compensate him/her for the additional debt repayment. This reduces the consumer's liquid wealth that can effectively be used to pay for production costs. The producer thus tightens

[5] We define $R_t(b'_{t+1}) = \infty$ if $\int_0^{\bar{\epsilon}} \mathbb{1}\{\epsilon_{t+1} \geq \hat{\epsilon}(m_{t+1}, b'_{t+1})\} G(\mathrm{d}\epsilon_{t+1}) = 0$ for a $b'_{t+1} \geq 0$.

the liquidity constraint, which shrinks the gross surplus if the liquidity constraint binds. Debt issuance in this sense has a similar effect on the gross surplus as a reduction of real balances, which is why the expected liquidity premium matters.

The third term captures a negative extensive-margin effect of debt, i.e., the larger the contracted debt repayment, the higher must the preference shock ϵ of a matched consumer be to allow for a successful match, as discussed at Lemma 3. The examples below illustrate the importance of bargaining power θ for how producers trade off the positive distributive effect of debt against the negative intensive- and extensive-margin effects.

Example 1. If consumers make take-it-or-leave-it offers to producers ($\theta = 0$), producers realize zero net surplus in each successful match. However, by issuing debt, they appropriate gross surplus by forcing the consumers to compensate them for debt repayment. Realizing zero net surplus anyways, producers do not account for how debt issuance affects the liquidity constraint. They only account for the negative extensive-margin effect.

Example 2. If producers make take-it-or-leave-it offers to consumers ($\theta = 1$), debt does not directly affect the negotiated payment p since p is equal to the consumer's DM-goods consumption utility. In particular, a producer's gross surplus in successful matches does not increase with contracted debt. Debt issuance thus makes producers only worse off through the negative intensive- and extensive-margin effects. They consequently issue zero debt, so that bargaining yields the same outcome as in a standard money-search model without any debt.

The discussion of Condition (24) and the subsequent examples highlight why producers issue debt: they do not gain or lose anything from preponing consumption through debt issuance *per se*, but they use debt for the sole purpose of changing the bargaining outcome in their favor. The examples moreover show that producers issue more debt when having less bargaining power. We propose two interpretations of this observation. As Towner (2020) puts it, producers compensate a lack of bargaining power by issuing debt to change the division of gross surplus to their advantage. In this sense, debt and bargaining power are substitutes in a producer's optimization problem. When focusing on the producers' ability of commitment to debt repayment rather than on debt issuance itself, commitment appears as a complement for the producers' bargaining power. Commitment empowers producers beyond their bargaining power to improve their bargaining position. However, if their bargaining power is high already, they can deploy their ability of commitment less profitable than when their bargaining power is low, which explains that producers issue less debt when having more bargaining power.

The way in which debt and commitment affect bargaining outcomes are in line with the empirical literature. In particular, the substitutability of debt and bargaining power—or the complementarity of commitment and bargaining power—align with the work of Bronars and Deere (1991), Dasgupta and Nanda (1993), and Hennessy and Livdan (2009). Our contribution is to explore this mechanism in a monetary general equilibrium and to derive policy implications, as we do next.

5 Equilibrium and Policy

We define symmetric monetary equilibria in

Definition 1. *A symmetric monetary equilibrium is a process of real money holdings* $\{m_t\}_{t=1}^{\infty}$*, contracted debt repayments* $\{b_t\}_{t=1}^{\infty}$*, money prices* $\{\phi_t\}_{t=0}^{\infty}$*, pricing kernels* $\{R_t\}_{t=0}^{\infty}$*, risk-free rates* $\{R_t^f\}_{t=0}^{\infty}$*, and government transfers* $\{\tau_t\}_{t=0}^{\infty}$*, so that:*

(i) m_{t+1} *solves the consumer's optimization problem in* (3) *s.t.* (4)*;*

(ii) b_{t+1} *solves the producer's optimization problem in* (5)*;*

(iii) R_t *solves Equation* (22) *for* m_{t+1} *and* $R_t^f = 1/\beta$*;*

(iv) the government's budget is balanced: $\tau_t = \phi_t(M_{t+1} - M_t)$*;*

(v) the money market clears: $\phi_t M_t = m_t$*;*

(vi) the equilibrium is monetary: $m_t > 0$*.*

We define utilitarian welfare $\mathcal{W} \equiv W_0^{fi} + W_0^c + \sum_{t=0}^{\infty} \beta^t W_t^{p,0}$ as the aggregate utility of financiers, consumers, and producers.

Lemma 5. *Welfare reads as*

$$\mathcal{W} = \sum_{t=1}^{\infty} \beta^t \int_{\hat{\epsilon}_t}^{\bar{\epsilon}} [\epsilon_t u(q) - c(q)]_{q=q(\epsilon_t, m_t, b_t)} G(\mathrm{d}\epsilon_t), \tag{25}$$

given equilibrium real balances $\{m_t\}_{t=1}^{\infty}$ *and debt* $\{b_t\}_{t=1}^{\infty}$*.*

Lemma 5 shows that the aggregate expected gross surplus of successful matches determines welfare. The quasi-linearity of preferences and the equal discounting by all agents make all CM trade irrelevant for welfare. Note that welfare is maximized when agents trade $q_\epsilon^\star$ in any match with preference shock ϵ. This confirms our previous statement that $q_\epsilon^\star$ is the intensively-efficient level of DM-goods traded in match (ϵ, m, b). DM trade is *extensively efficient* if $\hat{\epsilon}_t = 0$—every match should be successful—and we define $(\hat{\epsilon}_t, m_t, b_t)$ as the *marginally-successful* match at time t.

Debt issuance impairs welfare at the intensive margin through a reduction of the gross match surplus in a successful match, and at the extensive margin through a reduction of the mass of successful matches (recall the discussion at Condition (24)). The economics underlying these inefficiencies involve a pecuniary externality: the financiers' debt provision drags the borrowing rate below the full societal cost of debt. Fiscal policy can correct this pecuniary externality through a Pigouvian tax on debt, e.g., through a direct markup on borrowing rates. In any case, the Pigouvian tax curbs debt. The fiscal authority optimally sets this tax prohibitively high to shut down debt.

In the absence of Pigouvian taxation, monetary policy could mitigate the effects of the pecuniary externality through inflation—a tax on the consumers' money holdings rather than a direct tax on debt. We focus on this role of monetary policy in the steady state. Real money balances are constant in steady state and hence gross inflation satisfies $\phi/\phi_+ = \gamma$. In turn, the Fisher rate reads as $\iota = \gamma/\beta - 1$ from which we can determine allocations and negotiated terms of trade. In what follows, we thus think of monetary policy as controlling the Fisher rate directly.

We consider the *Friedman rule*, defined as $\iota = 0$, as our benchmark policy. The economy's efficient outcome is that all matches are successful, and that agents trade the intensively-efficient quantity in every match. If debt were prohibited or ruled out by Pigouvian taxation, the monetary authority could implement the efficient outcome by adhering to the Friedman rule, as it is standard in many monetary models. In the absence of Pigouvian taxation, however, the Friedman rule is suboptimal in our model. We show this by deriving the effects of a slight deviation from the Friedman rule on the intensive and extensive margin of DM trade.

Proposition 1. *Let $\theta < 1$. At the Friedman rule, we have $b > 0$. It holds for the intensive margin that $q(\epsilon, m, b) = q^\star_\epsilon$ for all $\epsilon \in [\hat{\epsilon}, \bar{\epsilon}]$, as well as*

$$\frac{\mathrm{d}q(\epsilon, m, b)}{\mathrm{d}\iota} = 0, \quad \forall\, \epsilon \in [\hat{\epsilon}, \bar{\epsilon}), \qquad \textit{and} \qquad \frac{\mathrm{d}q(\bar{\epsilon}, m, b)}{\mathrm{d}\iota} < 0. \tag{26}$$

It holds for the extensive margin that $\hat{\epsilon} > 0$ with

$$\frac{\mathrm{d}\hat{\epsilon}}{\mathrm{d}\iota} < 0. \tag{27}$$

Since the Friedman rule removes the opportunity cost of holding money, consumers can costlessly render the liquidity constraint slack in every successful match, facilitating intensively-efficient trade in these matches. Nevertheless, DM trade is not extensively efficient. This is because producers use debt to overcome their lack of bargaining power unless they exert full bargaining power already ($\theta = 1$). Corollary 2 characterizes the response of producers and consumers to a deviation from the Friedman rule.

Corollary 2. *At the Friedman rule, it holds that*

$$\frac{\mathrm{d}b}{\mathrm{d}\iota} < 0 \qquad \textit{and} \qquad \frac{\mathrm{d}m}{\mathrm{d}\iota} < 0. \tag{28}$$

By deviating from the Friedman rule, i.e., by increasing the Fisher rate, the monetary authority makes carrying real balances costly. Consumers consequently reduce their money holdings and the liquidity constraint becomes binding in matches with ϵ close to $\bar{\epsilon}$, which reduces the traded quantity in these matches. This is a standard result in money-search models. Note that a marginal increase of ι affects only the liquidity constraint in the match with $\epsilon = \bar{\epsilon}$—the liquidity constraints in all other matches remain slack.

Over and above this intensive-margin effect, there is an extensive-margin effect. Producers reduce debt issuance in response to the binding liquidity constraints in matches with large ϵ. They in essence relax these binding liquidity constraints (recall the discussion at Condition (24)) in order to expand their bargaining sets, increasing gross surpluses. While reducing their debt, producers simultaneously increase the net surpluses in all other successful matches. Particularly, the net surplus in the match that was marginally successful at the Friedman rule becomes positive. Hence, the reduction of debt allows for more successful matches—$\hat{\epsilon}$ decreases.

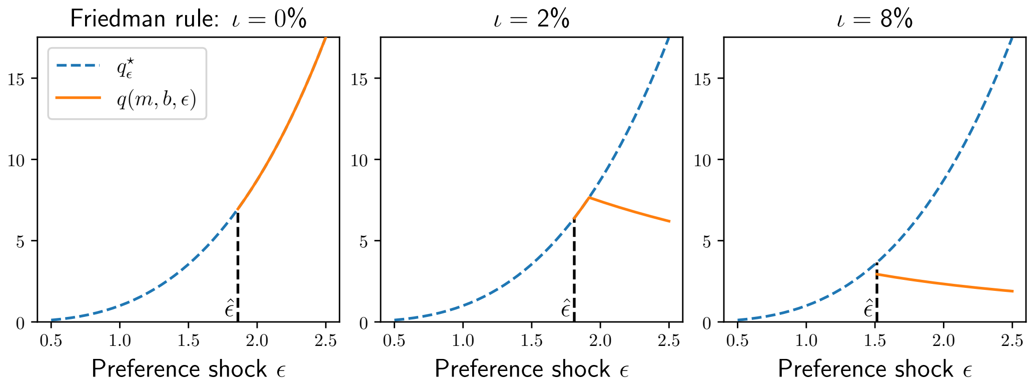
The effect of monetary policy on the incentives to issue debt becomes formally clear by substituting the consumers' money demand from Equation (20) into the producers' FOC for debt in Equation (24), yielding[6]

$$0 = (1-\theta)[1 - G(\hat{\epsilon})] - \theta\iota - bg(\hat{\epsilon})\frac{\partial \hat{\epsilon}}{\partial b}. \tag{29}$$

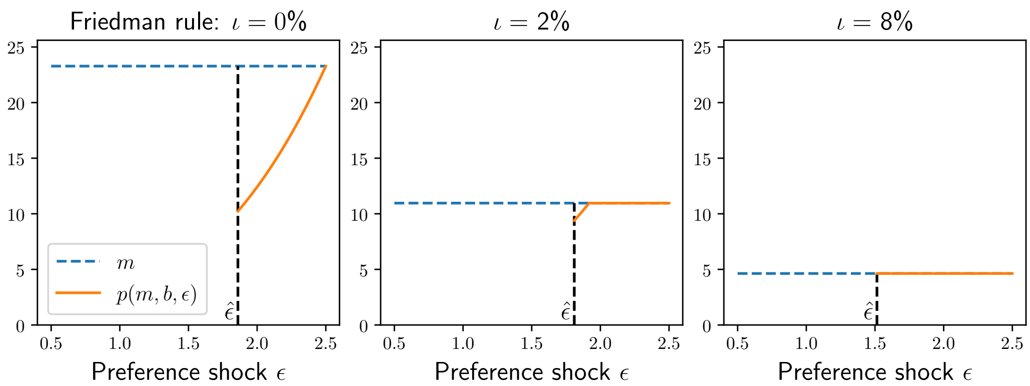
A higher ι increases the producers' marginal disutility of debt issuance at the intensive margin, reflected by the second term, since debt issuance tightens the liquidity constraints in matches with large ϵ.

We evaluate the intensive- and extensive-margin effects of monetary policy away from the Friedman rule by means of graphical illustration for a toy calibration (see Table 1a). Figure 3a shows how monetary policy affects the intensive and extensive margin of DM trade. It displays the bargained quantities $\{q(\epsilon, m, b)\}_{\epsilon \in [\hat{\epsilon}, \bar{\epsilon}]}$ and the preference shock $\hat{\epsilon}$ of the marginally-successful match for different monetary-policy regimes. At the Friedman rule, agents trade intensively-efficient quantities in all successful matches. For $\iota = 2\%$, the liquidity constraint binds for large ϵ (see Figure 3b), driving a wedge between the negotiated quantity and the efficient quantity. For $\iota = 8\%$, consumers carry so little real

[6] We refer to Condition (20) as an equation since it must hold with equality in a monetary equilibrium. We moreover refer to Condition (24) as an equation because debt issuance is positive at the Friedman rule according to Proposition 1.



(a) Quantities.



(b) Payments.

Figure 3: Negotiated terms of trade for parameters in Table 1a.

balances that the liquidity constraint binds whenever a match is successful.[7]

The cut-off preference shock $\hat{\epsilon}$ decreases with the Fisher rate, so that the mass of successful matches $1 - G(\hat{\epsilon})$ increases. The theoretical finding in Proposition 1 thus generalizes away from the Friedman rule—extensive-margin efficiency improves when the opportunity cost of money increases. Figure 3b informs about what renders the net match surplus zero in the marginally-successful match for the three policy regimes. For $\iota = 0\%$ and $\iota = 2\%$, the liquidity constraint is slack at $\hat{\epsilon}$ and therefore the net match surplus at the intensively-efficient level is zero. Cast in the context of Lemma 1, Condition (i) binds and Condition (ii) is slack. For $\iota = 8\%$, the net surplus at the the intensively-efficient level is positive in the marginally-successful match, but the liquidity constraint binds to the extent that the net realized surplus becomes zero. Cast in the context of Lemma 1, Condition (i) is slack and Condition (ii) binds.

The discussed intensive- and extensive-margin effects of monetary policy affect welfare. The welfare implications of a deviation from the Friedman rule are characterized in

[7]Note that $q(\epsilon, m, b)$ declines in ϵ once the liquidity constraint binds. This is because of the bargaining protocol. Since a consumer with a high ϵ commands of the same money holdings as all other consumers, the matched producer appropriates the utility gains of this consumer by producing less and thus facing lower production costs.

Proposition 2. *At the Friedman rule, it holds that*

$$\frac{\mathrm{d}\mathcal{W}}{\mathrm{d}\iota} > 0. \tag{30}$$

The intensive-margin effect of a deviation from the Friedman rule, established in Proposition 1, has a welfare effect of second-order importance because trade is intensively-efficient at the Friedman rule. On the other hand, the extensive-margin effect, i.e., the reduction of $\hat{\epsilon}$, has a positive first-order effect on welfare because it adds matches with positive gross surplus. The Friedman rule is therefore a suboptimal policy.

In contrast to fiscal policy, monetary policy cannot directly solve the problem that debt is too cheap. But it can mitigate the extensive-margin inefficiency that results from this pecuniary externality. Monetary policy does so by compensating for a lack of coordination of money holdings among consumers. At the Friedman rule, an individual consumer faces a zero opportunity cost of holding money and thus chooses money holdings that render the liquidity constraint slack in all successful matches. This results in large gross match surpluses, which the producers skim by entering matches with contracted debt repayments. If consumers could coordinate their money holdings, they would rather jointly reduce money holdings to protect themselves from being skimmed. A deviation from the Friedman rule mitigates this lack of coordination by incentivizing consumers to economize on costly money holdings, as captured by Corollary 2. Whilst fiscal policy counters debt issuance directly, here, monetary policy in essence creates inefficiency at the intensive margin through tightening the liquidity constraint. The producers, in turn, counter this by issuing less debt, which improves welfare at the extensive margin. The utility implications of a deviation from the Friedman rule for producers and consumers are characterized in

Proposition 3. *At the Friedman rule, it holds that*

$$\frac{\mathrm{d}W^{p,0}}{\mathrm{d}\iota} = 0 \qquad \textit{and} \qquad \frac{\mathrm{d}^2W^{p,0}}{\mathrm{d}\iota^2} < 0, \tag{31}$$

as well as

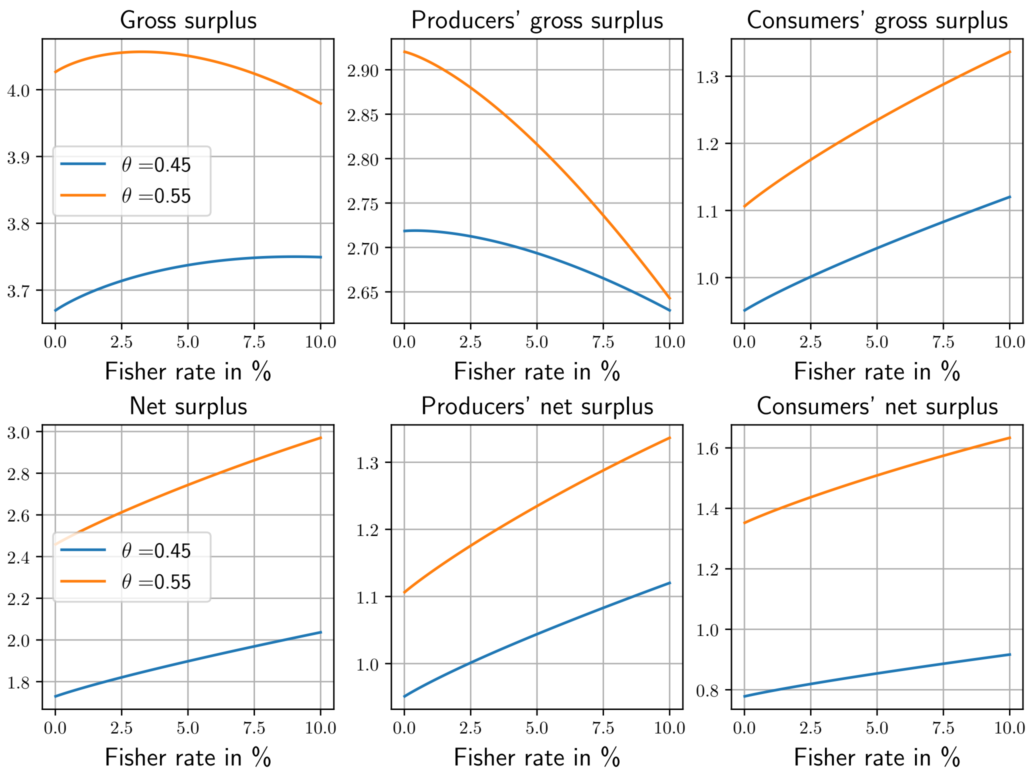
$$\frac{\mathrm{d}W^{c}}{\mathrm{d}\iota} > 0. \tag{32}$$

The positive first-order effect of a deviation from the Friedman rule on consumers' lifetime utility follows from our discussion above. Unsurprisingly, producers are negatively affected from this change in monetary policy as there is less surplus to skim through debt issuance when consumers hold less money. Less obvious, this effect is only of second-order importance. This stems from the fact that a deviation from the Friedman rule induces a tightening of the liquidity constraint only for the match with $\epsilon = \bar{\epsilon}$ by reducing m, as characterized in Corollary 2. Of course, the simultaneous change of b affects all successful matches, but this change is only of second-order importance for producers due to the envelope theorem—b is their optimal choice. Hence, the change of m affects a mass zero of matches and the change of b affects a positive mass of matches only to a second-order degree, so that the overall effect is second-order as well. Proposition 3 also makes clear that the government can induce an economy-wide Pareto improvement by slightly changing the lump-sum taxation scheme in the producers' favor.

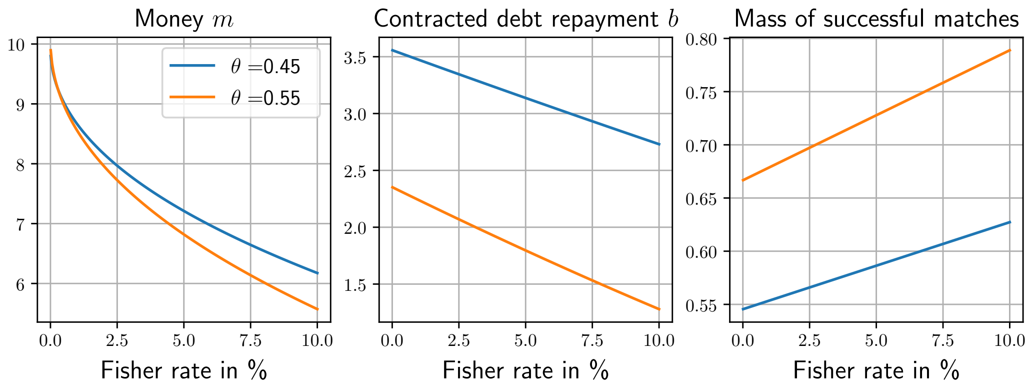
Figure 4a is based on the toy calibration in Table 1b and shows the expected gross and net surpluses of producers and consumers for Fisher rates close to the Friedman rule when producers have bargaining power $\theta \in \{0.45, 0.55\}$. The gross surplus, which determines welfare, is hump-shaped in the Fisher rate, so that the welfare-maximizing Fisher rate is away from the Friedman rule, confirming Proposition 2. Moreover, a smaller θ is associated with lower gross surpluses since the producers' marginal benefit from issuing debt decreases in their bargaining power—Towner's (2020) substitutability of debt and bargaining power. The more debt issuance due to a lack of producers' bargaining power, i.e., due to a small θ, the higher the Fisher rate has to be to reduce money holdings to the socially-optimal level.

Figure 4b shows real balances m, contracted debt repayment b, and the mass of successful matches. The decrease of money holdings m in the Fisher rate reflects the increase in the opportunity cost of holding money. The decrease of b results from the impairment of the producers to skim gross match surplus because consumers carry less real balances. Figure 4c depicts aggregate production $\int_{\hat{\epsilon}}^{\bar{\epsilon}} q(\epsilon, m, b)G(\mathrm{d}\epsilon)$ and GDP $\int_{\hat{\epsilon}}^{\bar{\epsilon}} p(\epsilon, m, b)G(\mathrm{d}\epsilon)$.[8] The lower level of production for $\theta = 0.45$ as compared to $\theta = 0.55$ traces back to the higher debt level for $\theta = 0.45$ and the related distortions. We find that aggregate production and GDP decrease in ι: although there are more successful matches, the reduction of production and payment in every successful match dominates.

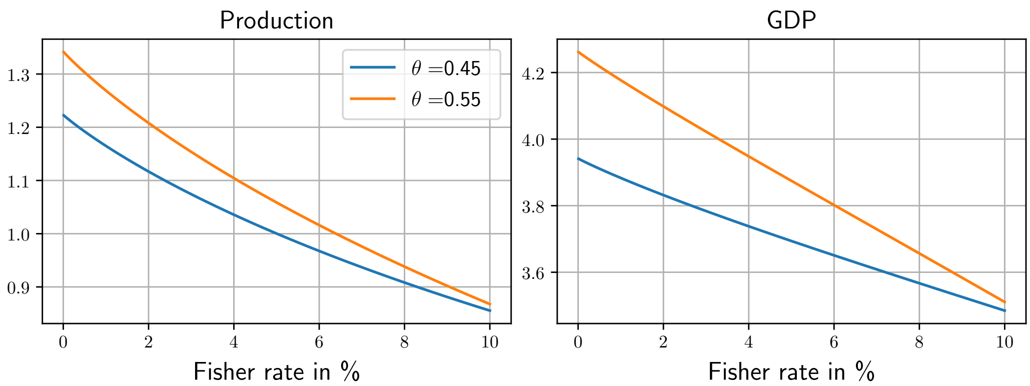
[8] We measure GDP as the value of traded DM-goods in terms of the numéraire payments that consumers transfer to producers in exchange.



(a) Expected surpluses.



(b) Money, debt, and the mass of successful matches.



(c) Aggregate production $\int_{\hat{\epsilon}}^{\bar{\epsilon}} q(\epsilon, m, b)G(\mathrm{d}\epsilon)$ and GDP $\int_{\hat{\epsilon}}^{\bar{\epsilon}} p(\epsilon, m, b)G(\mathrm{d}\epsilon)$.

Figure 4: Equilibrium outcomes for parameters in Table 1b.

6 Conclusion

Our study investigates the role of debt in bargaining, and we argue how fiscal and monetary policy can mitigate the distortions caused by debt. In our money-search model, producers and consumers bargain over the terms of trade in bilateral matches. Producers enter these matches with contracted limited-liability debt repayments, allowing them to negotiate more favorable terms of trade with consumers. This rationale for taking on debt corresponds with empirical evidence by Towner (2020) who argues that U.S. hospitals with more debt negotiate higher reimbursement rates from health insurers. Our theoretical analysis confirms empirical findings in the literature by showing that the producers' incentive to issue debt decreases in producers' bargaining power. In that sense, debt and bargaining power are substitutes.

Debt issuance distorts welfare at the intensive and extensive margin in general equilibrium. The reason for this inefficiency is a pecuniary externality that financiers exert when providing debt to producers: the borrowing rates do not capture the full societal cost of debt. Optimal fiscal policy shuts down debt through a Pigouvian tax. Optimal monetary policy cannot do so directly, but it can steer the consumers' money holdings, by controlling inflation, to a level at which consumers are better protected from being skimmed by producers. We show analytically that a deviation from the Friedman rule improves welfare through a positive extensive-margin effect of inflation. Bargaining agreements are more likely to be reached, so that trade accelerates and money changes hands faster. We view the mitigation of frictions in bargaining through affecting debt issuance as a novel transmission channel of monetary policy.

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A Optimality of Limited-Liability Debt Contracts

We establish that limited-liability debt contracts are optimal loan contracts in our framework. To this end, we make

Definition 2. *A loan contract $\mathcal{C}$ is a vector $(z, r, \mathcal{S})$ with $z \geq 0$, $r : \mathbb{R}^+ \to \mathbb{R}^+$, and $\mathcal{S} \subseteq [0, \infty)$, so that $r(p) + \gamma p \leq p$ for $p \in \mathcal{S}$ and $r(p) \leq p$ for $p \in \mathcal{S}^\complement$.*

A loan contract $\mathcal{C} = (z, r, \mathcal{S})$ has a straight-forward interpretation. The borrower receives transfer z from the lender today. Tomorrow, s/he sends a signal p^s of his/her revenues to the lender. If $p^s \in \mathcal{S}^\complement$, the borrower pays $r(p^s)$ to the lender. If $p^s \in S$, the borrower verifies his/her true revenues p at cost γp, where $\gamma \in (0, 1]$, and pays $r(p)$ to the lender. Limited liability of the borrower implies that $\gamma p + r(p) \leq p$ for $p \in \mathcal{S}$, and $r(p) \leq p$ for $p \in \mathcal{S}^\complement$. We say that a borrower reports truthfully if $p^s = p$.

We call a contract $\mathcal{C}$ *incentive compatible* if for all $p \in [0, \infty)$, the borrower optimally reports truthfully: $p^s = p$. We write $b \equiv \inf_{p \in \mathcal{S}^\complement} r(p)$.

Lemma 6. *A contract $\mathcal{C}$ is incentive compatible if and only if the follwoing conditions jointly hold:*

(i) $\forall\ p \in \mathcal{S}^\complement\ :\ r(p) = b$;

(ii) $\forall\ p \in \mathcal{S}\ :\ \gamma p + r(p) \leq b$.

Proof. Let $\mathcal{C}$ be incentive compatible. Suppose that property (i) does not hold true. Then there are $p_1, p_2 \in \mathcal{S}^\complement$ with $r(p_1) > r(p_2)$. Having realized revenues p_1, the borrower would untruthfully report p_2. Suppose that property (ii) does not hold true. Then there is a $p \in \mathcal{S}$, so that $\gamma p + r(p) > b$. For this p, the borrower is strictly better off by untruthfully reporting a $\tilde{p} \in \mathcal{S}^\complement$—it does not matter which since $r(\tilde{p}) = b$ for all $\tilde{p} \in \mathcal{S}^\complement$ because of property (i). Hence, properties (i) and (ii) are sufficient for $\mathcal{C}$ to be incentive compatible.

Now assume that $\mathcal{C}$ fulfils properties (i) and (ii). If $p \leq b$, untruthful reporting cannot be a strictly dominant strategy because of limited liability—truthful reporting yields return zero at least, whilst untruthful reporting is infeasible or yields zero return. If $p > b$ and $\gamma p + r(p) \leq b$, reporting p yields a weakly smaller disutility than reporting any other $\tilde{p} \in \mathbb{R}_+$. Finally, if $p > b$ and $\gamma p + r(p) > b$, property (ii) implies that $p \in \mathcal{S}^\complement$, and property (i) implies that $r(p) \leq r(\tilde{p})$ for all $\tilde{p} \in \mathcal{S}^\complement$. Hence, it is weakly dominant for the borrower to report p. □

Definition 3. *An incentive compatible contract $\mathcal{C}$ is actuarially fair if*

$$\int_A r(p_\epsilon) G(\mathrm{d}\epsilon) + \int_{A^\complement} b G(\mathrm{d}\epsilon) = R^f z, \tag{33}$$

where $A \equiv \{\epsilon \in [0, \bar{\epsilon}] : p_\epsilon \in \mathcal{S}\}$ *with*

$$(q_\epsilon, p_\epsilon) = \arg\max_{q,p\geq 0}\{\epsilon u(q) - p\} \quad s.t. \quad p = \theta\epsilon u(q) + (1-\theta)[c(q) + r(p) + \gamma p \mathbb{1}\{p \in \mathcal{S}\}]. \tag{34}$$

Let $\mathcal{I}$ be the set of all incentive compatible and actuarially fair contracts. Note that a contract $\mathcal{C} \in \mathcal{I}$ implicitly determines a schedule of terms of trade $\{(q_\epsilon, p_\epsilon)\}_{\epsilon\in[0,\bar{\epsilon}]}$.

The borrower's problem. The borrower solves

$$\max_{(z,r,\mathcal{S})\in\mathcal{I}} \left\{ z + \beta \left(\int_A [-c(q_\epsilon) + (1-\gamma)p_\epsilon - r(p_\epsilon)]\, G(\mathrm{d}\epsilon) + \int_{A^\complement} [-c(q_\epsilon) + p_\epsilon - b] G(\mathrm{d}\epsilon) \right) \right\}. \tag{35}$$

Proposition 4. *If* $\gamma = 1$*, an optimal contract* $\mathcal{C} = (z, b, r(p), \mathcal{S})$ *has expected state-verification costs zero:*

$$\int_A p(\epsilon) G(\mathrm{d}\epsilon) = 0. \tag{36}$$

Proof. Suppose not, and that $\mathcal{C}' = (z', b', r'(p), \mathcal{S}')$ solves the borrower's problem in (35) with

$$\int_{A'} p'(\epsilon) G(\mathrm{d}\epsilon) > 0. \tag{37}$$

Recall that the producer's limited liability implies $r'(p) \leq p$. For monitoring to be credible, it must hold that $r'(p) \geq \gamma p = p$. We infer that $r'(p) = p$. The producer's net surplus is non-negative, so that for $\epsilon \in A'$

$$0 \leq -c(q'(\epsilon)) + p'(\epsilon) - r'(p'(\epsilon)) = -c(q'(\epsilon)) \qquad \Rightarrow \qquad q'(\epsilon) = 0. \tag{38}$$

Hence, $p'(\epsilon) = 0$, which contradicts the supposition. □

If $\gamma = 1$, the optimal contract $\mathcal{C}$ features zero revenues if $p \in \mathcal{S}$, so that state-verification costs are zero as well. Hence, $\mathcal{C}$ is equivalent to a limited-liability debt contract.

B Toy Calibrations

	$u(q)$	$c(q)$	θ	β	$[0, \bar{\epsilon}]$	G
Values	$q^{0.68}/0.68$	q	0.5	0.98	$[0, 2.5]$	$U(0, 2.5)$

(a) Toy calibration inducing Figure 3.

	$u(q)$	$c(q)$	β	$[0, \bar{\epsilon}]$	G
Values	$4q^{1/4}$	q	0.98	$[0, 2.5]$	$U(0, 2.5)$

(b) Toy calibration inducing Figure 4.

Table 1: Toy calibrations.

C Proofs

C.1 Proof of Lemma 1

If a match is successful, there must exist a $q > 0$ for which the net surplus is non-negative. This must definitely hold for $q^\star_\epsilon$, which implies Condition (i). Moreover, the consumer's real balances m must weakly exceed the payment corresponding with the smallest q that guarantees a non-negative surplus, which is $\tilde{q}(\epsilon, b)$. Hence,

$$m \geq \theta\epsilon u(\tilde{q}(\epsilon, b)) + (1-\theta)[c(\tilde{q}(\epsilon, b)) + b] = c(\tilde{q}(\epsilon, b)) + b, \tag{39}$$

as $\epsilon u(\tilde{q}(\epsilon, b)) - c(\tilde{q}(\epsilon, b)) - b = 0$. This implies Condition (ii).

Conversely, Condition (i) implies that $\tilde{q}(\epsilon, b) \in (0, q^\star_\epsilon]$. Condition (ii) then yields that $q(\epsilon, m, b) \geq \tilde{q}(\epsilon, b) > 0$.

C.2 Proof of Lemma 2

It is clear that $\tilde{q}(\hat{\epsilon}, b) \leq \hat{q}$, as the net surplus would be negative otherwise. Suppose that $\tilde{q}(\hat{\epsilon}, b) < \hat{q}$. Because of continuity, there is an $\epsilon < \hat{\epsilon}$ close to $\hat{\epsilon}$, so that

$$\tilde{q}(\hat{\epsilon}, b) < \tilde{q}(\epsilon, b) < \hat{q}. \tag{40}$$

It then holds that

$$\begin{aligned}\theta\epsilon u(\tilde{q}(\epsilon, b)) + (1-\theta)(c(\tilde{q}(\epsilon, b)) + b) &< \theta\epsilon u(\hat{q}) + (1-\theta)(c(\hat{q}) + b) \\ &< \theta\hat{\epsilon} u(\hat{q}) + (1-\theta)(c(\hat{q}) + b) \leq m,\end{aligned} \tag{41}$$

so that a match for ϵ would be successful. This contradicts the definition of $\hat{\epsilon}$.

C.3 Proof of Lemma 3

We distinguish two cases: $\hat{q} < q^{\star}_{\hat{\epsilon}}$ and $\hat{q} = q^{\star}_{\hat{\epsilon}}$. Let $\hat{q} < q^{\star}_{\hat{\epsilon}}$. Hence, the liquidity constraint binds at the margin for all b' in a (small) neighborhood of b. We infer that $\hat{\epsilon}$ and $\hat{q}$ are differentiable, so that

$$m = \theta\hat{\epsilon}u(\hat{q}) + (1-\theta)(c(\hat{q})+b) \quad \Rightarrow \quad 0 = \theta u(\hat{q})\frac{\partial\hat{\epsilon}}{\partial b} + [\theta\hat{\epsilon}u'(\hat{q}) + (1-\theta)c'(\hat{q})]\frac{\partial\hat{q}}{\partial b} + 1 - \theta. \tag{42}$$

Since $\hat{q} = \tilde{q}(\hat{\epsilon}, b)$ according to Lemma 2, it moreover holds that

$$\hat{\epsilon}u(\hat{q}) - c(\hat{q}) - b = 0 \quad \Rightarrow \quad u(\hat{q})\frac{\partial\hat{\epsilon}}{\partial b} + [\epsilon u'(\hat{q}) - c'(\hat{q})]\frac{\partial\hat{q}}{\partial b} - 1 = 0. \tag{43}$$

Combining the equations above, we obtain

$$0 = \theta + [-\theta[\epsilon u'(\hat{q}) - c'(\hat{q})] + [\theta\hat{\epsilon}u'(\hat{q}) + (1-\theta)c'(\hat{q})]]\,\frac{\partial\hat{q}}{\partial b} + 1 - \theta \tag{44}$$

$$\Leftrightarrow \qquad 0 = 1 + c'(\hat{q})\frac{\partial\hat{q}}{\partial b} \tag{45}$$

$$\Leftrightarrow \qquad \frac{\partial\hat{q}}{\partial b} = -\frac{1}{c'(\hat{q})}. \tag{46}$$

With Equation (43), Equation (16) follows.

Let $\hat{q} = q^{\star}_{\hat{\epsilon}}$. We immediately infer from $\hat{\epsilon}u(\hat{q}) - c(\hat{q}) - b = 0$ and $\hat{\epsilon}u'(\hat{q}) - c'(\hat{q}) = 0$ that $\hat{\epsilon}$ is differentiable—but not necessarily $\hat{q}$—with $\partial\hat{\epsilon}/\partial b = 1/u(\hat{q})$.

C.4 Proof of Lemma 4

The first-order derivative of the producer's objective function in Equation (23) w.r.t. limited-liability debt repayment b' reads as

$$\frac{\partial}{\partial b}\left\{\int_{\hat{\epsilon}(m_{t+1},b')}^{\bar{\epsilon}} [v^p(\epsilon_{t+1}, m_{t+1}, b') + b']\, G(\mathrm{d}\epsilon_{t+1})\right\} \tag{47}$$

$$= \int_{\hat{\epsilon}(m_{t+1},b')}^{\bar{\epsilon}} \left[\frac{\partial v^p(\epsilon_{t+1}, m_{t+1}, b')}{\partial b} + 1\right] G(\mathrm{d}\epsilon_{t+1}) \tag{48}$$

$$- [v^p(\hat{\epsilon}(m_{t+1}, b'), m_{t+1}, b') + b']\, g(\hat{\epsilon}(m_{t+1}, b'))\frac{\partial\hat{\epsilon}(m_{t+1}, b')}{\partial b}. \tag{49}$$

Note that $v^p(\hat{\epsilon}(m_{t+1}, b'), m_{t+1}, b_{t+1}) = 0$ with Lemma 2, and that

$$\frac{\partial v^p(\epsilon, m, b)}{\partial b} = \theta \frac{\partial}{\partial b}[\epsilon u(q) - c(q) - b]_{q=q(\epsilon,m,b)} \tag{50}$$

$$= \theta \left[[\epsilon u'(q) - c'(q)] \frac{\partial q(\epsilon, m, b)}{\partial b} - 1 \right]_{q=q(\epsilon,m,b)} \tag{51}$$

$$= -\theta \left[\frac{(1-\theta)[\epsilon u'(q) - c'(q)]}{\theta \epsilon u'(q) + (1-\theta)c'(q)} + 1 \right]_{q=q(\epsilon,m,b)} \tag{52}$$

$$= -\theta \left[\mathcal{L}(\epsilon, m, b) + 1 \right]. \tag{53}$$

Together, this proves Lemma 4.

C.5 Proof of Lemma 5

Note that $W_0^{fi} = 0$ as $R_t^f = 1/\beta$ for all t. Hence,

$$\mathcal{W} \equiv W_0^c + \sum_{t=0}^{\infty} \beta^t W_t^{p,0} = -\frac{\phi_0 m_1}{\phi_1} + \tau_0 + \beta \int_0^{\bar{\epsilon}} V_1^c(m_1|\epsilon) G(\mathrm{d}\epsilon) + \sum_{t=0}^{\infty} \beta^t W_t^{p,0} \tag{54}$$

$$= \tau_0 - \frac{\phi_0 m_1}{\phi_1} - \beta a_1 + \beta \left[m_1 + \tau_1 + \int_{\hat{\epsilon}_1}^{\bar{\epsilon}} v^c(\epsilon_1, m_1, b_1) G(\mathrm{d}\epsilon_1) \right] \tag{55}$$

$$+ \sum_{t=1}^{\infty} \beta^t \left[-\frac{\phi_t m_{t+1}}{\phi_{t+1}} + \beta \left[\int_{\hat{\epsilon}_{t+1}}^{\bar{\epsilon}} v^c(\epsilon_{t+1}, m_{t+1}, b_{t+1}) G(\mathrm{d}\epsilon_{t+1}) + m_{t+1} + \tau_{t+1} \right] \right] \tag{56}$$

$$+ \sum_{t=0}^{\infty} \beta^t \left[\beta \int_{\hat{\epsilon}_{t+1}}^{\bar{\epsilon}} [v^p(\epsilon_{t+1}, m_{t+1}, b_{t+1}) + b_{t+1}] G(\mathrm{d}\epsilon_{t+1}) \right] \tag{57}$$

$$= \tau_0 - \frac{\phi_0 m_1}{\phi_1} + \sum_{t=1}^{\infty} \beta^t \left[m_t - \mathbb{P}[\epsilon_t \geq \hat{\epsilon}_t] b_t - \frac{\phi_t m_{t+1}}{\phi_{t+1}} + \tau_t + \beta \mathbb{P}[\epsilon_t \geq \hat{\epsilon}_t] b_t \right] \tag{58}$$

$$+ \sum_{t=1}^{\infty} \beta^t \int_{\hat{\epsilon}_t} [\epsilon_t u(q) - c(q)]_{q=q(\epsilon_t, m_t, b_t)} G(\mathrm{d}\epsilon_t) \tag{59}$$

$$= \sum_{t=1}^{\infty} \beta^t \int_{\hat{\epsilon}_t} [\epsilon_t u(q) - c(q)]_{q=q(\epsilon_t, m_t, b_t)} G(\mathrm{d}\epsilon_t), \tag{60}$$

where we used that $m_t - \phi_t m_{t+1}/\phi_{t+1} = -\tau_t$.

C.6 Proof of Proposition 1

For $\iota = 0$, consumers choose real balances that render the liquidity constraint slack in every successful match (see Equation (20)). Suppose that $b = 0$. Then, all matches are successful and $\hat{\epsilon} = 0$. Substituting the consumers' FOC for money in Equation (20) into the producers' FOC for debt in Equation (24) yields

$$0 = (1-\theta)[1-G(\hat{\epsilon})] - \theta\iota - bg(\hat{\epsilon})\frac{\partial\hat{\epsilon}}{\partial b} = 1-\theta, \tag{61}$$

which is a contradiction. We infer that $b > 0$ and thus $\hat{\epsilon} > 0$. Before proving Proposition 1, we show

Lemma 7. *There is a (small) neighborhood around $\iota = 0$, so that for all ι in this neighborhood, it holds that $\hat{q}|_\iota = q^\star_{\hat{\epsilon}|_\iota}$. It thus holds in this neighborhood that*

$$\mathrm{d}\hat{q} = -\left[\frac{u'(\hat{q})}{\hat{\epsilon}u''(\hat{q}) - c''(\hat{q})}\right]\mathrm{d}\hat{\epsilon} \tag{62}$$

and

$$\mathrm{d}b = u(\hat{q})\,\mathrm{d}\hat{\epsilon}. \tag{63}$$

Proof. The slackness of the liquidity constraint in all successful matches at the Friedman rule implies that

$$\theta\epsilon u(q^\star_\epsilon) + (1-\theta)[c(q^\star_\epsilon) + b] < \theta\bar{\epsilon}u(q^\star_{\bar{\epsilon}}) + (1-\theta)[c(q^\star_{\bar{\epsilon}}) + b] \leq m \tag{64}$$

for all $\epsilon \in [\hat{\epsilon}, \bar{\epsilon})$. Since the equilibrium objects m and b are continuous functions of ι, $\hat{q}$ and $\hat{\epsilon}$ are as well continuous in ι. In particular, it holds that

$$m|_\iota > \left[\theta\hat{\epsilon}u(q^\star_{\hat{\epsilon}}) + (1-\theta)[c(q^\star_{\hat{\epsilon}}) + b]\right]_\iota \tag{65}$$

in a small neighborhood around $\iota = 0$. Since the bargaining solution maximizes the net surplus, and since $q^\star_{\hat{\epsilon}|_\iota}$ is feasible in the match $(\hat{\epsilon}, m, b)|_\iota$, it holds that $\hat{q}|_\iota = q^\star_{\hat{\epsilon}|_\iota}$.

It thus holds that $0 = \mathrm{d}\left[\hat{\epsilon}u'(\hat{q}) - c'(\hat{q})\right]$ around the Friedman rule, from which we infer Equation (62). Since the liquidity constraint is slack for $\hat{\epsilon}$ in a small neighborhood around the Friedman rule, we conclude with Lemma 1 that $\hat{\epsilon}u(\hat{q}) - c(\hat{q}) - b = 0$ around the Friedman rule, which implies Equation (63) □

Lemmas 3 and 7 together imply that $\partial\hat{\epsilon}/\partial b = 1/u(\hat{q})$ in a small neighborhood around the Friedman rule. Equation (24) thus reads as

$$0 = (1-\theta)[1-G(\hat{\epsilon})] - \theta\iota - \frac{g(\hat{\epsilon})b}{u(\hat{q})}, \tag{66}$$

recalling that $\iota = \phi/\beta\phi_+ - 1$. The differential of Equation (66) reads as

$$\begin{aligned} & 0 = -(1-\theta)g(\hat{\epsilon})\,\mathrm{d}\hat{\epsilon} - \theta\,\mathrm{d}\iota - \frac{g(\hat{\epsilon})\,\mathrm{d}b}{u(\hat{q})} + \frac{g(\hat{\epsilon})bu'(\hat{q})\,\mathrm{d}\hat{q}}{u(\hat{q})^2} - \frac{g'(\hat{\epsilon})b\,\mathrm{d}\hat{\epsilon}}{u(\hat{q})} \\ \Leftrightarrow \quad & \theta\,\mathrm{d}\iota = -(1-\theta)g(\hat{\epsilon})\,\mathrm{d}\hat{\epsilon} - g(\hat{\epsilon})\,\mathrm{d}\hat{\epsilon} - \frac{g(\hat{\epsilon})bu'(\hat{q})}{u(\hat{q})^2}\left[\frac{u'(\hat{q})}{\hat{\epsilon}u''(\hat{q}) - c''(\hat{q})}\right]\mathrm{d}\hat{\epsilon} - \frac{g'(\hat{\epsilon})b\,\mathrm{d}\hat{\epsilon}}{u(\hat{q})} \\ \Leftrightarrow \quad & \theta\,\mathrm{d}\iota = -\left[(1+(1-\theta))g(\hat{\epsilon}) + \frac{u'(\hat{q})^2 g(\hat{\epsilon})b}{u(\hat{q})^2[\hat{\epsilon}u''(\hat{q}) - c''(\hat{q})]} + \frac{g'(\hat{\epsilon})b}{u(\hat{q})}\right]\mathrm{d}\hat{\epsilon}. \end{aligned} \tag{67}$$

We consider the second-order condition of the producers to sign the right-hand side of this equation. Differentiating the right-hand side of Equation (24) for a given Fisher rate, the producers' second-order condition w.r.t. b reads as

$$0 > -(1-\theta)g(\hat{\epsilon})\frac{\partial\hat{\epsilon}}{\partial b} - \left[g(\hat{\epsilon}) + bg'(\hat{\epsilon})\frac{\partial\hat{\epsilon}}{\partial b}\right]\frac{\partial\hat{\epsilon}}{\partial b} - bg(\hat{\epsilon})\frac{\partial^2\hat{\epsilon}}{\partial b^2} \tag{68}$$

$$= -\frac{(1-\theta)g(\hat{\epsilon})}{u(\hat{q})} - \left[g(\hat{\epsilon}) + \frac{bg'(\hat{\epsilon})}{u(\hat{q})}\right]\frac{1}{u(\hat{q})} - \frac{bg(\hat{\epsilon})u'(\hat{q})^2}{u(\hat{q})^3[\hat{\epsilon}u''(\hat{q}) - c''(\hat{q})]} \tag{69}$$

$$= -\frac{1}{u(\hat{q})}\left[(1+(1-\theta))g(\hat{\epsilon}) + \frac{bg(\hat{\epsilon})u'(\hat{q})^2}{u(\hat{q})^2[\hat{\epsilon}u''(\hat{q}) - c''(\hat{q})]} + \frac{bg'(\hat{\epsilon})}{u(\hat{q})}\right], \tag{70}$$

where we used $\partial\hat{\epsilon}/\partial b = 1/u(\hat{q})$ and

$$\begin{aligned} 0 = \hat{\epsilon}u'(\hat{q}) - c'(\hat{q}) \quad \Rightarrow \quad & 0 = u'(\hat{q})\frac{\partial\hat{\epsilon}}{\partial b} + [\hat{\epsilon}u''(\hat{q}) - c''(\hat{q})]\frac{\partial\hat{q}}{\partial b} \\ \Rightarrow \quad & \frac{\partial\hat{q}}{\partial b} = -\frac{u'(\hat{q})}{u(\hat{q})[\hat{\epsilon}u''(\hat{q}) - c''(\hat{q})]}, \end{aligned} \tag{71}$$

so that

$$bg(\hat{\epsilon})\frac{\partial^2\hat{\epsilon}}{\partial b^2} = -\left[\frac{bg(\hat{\epsilon})u'(\hat{q})}{u(\hat{q})^2}\right]\frac{\partial\hat{q}}{\partial b} = \frac{bg(\hat{\epsilon})u'(\hat{q})^2}{u(\hat{q})^3[\hat{\epsilon}u''(\hat{q}) - c''(\hat{q})]}. \tag{72}$$

Equation (67) yields $\mathrm{d}\hat{\epsilon}/\mathrm{d}\iota < 0$.

C.7 Proof of Corollary 2

Proposition 1 states that $\mathrm{d}\hat{\epsilon}/\mathrm{d}\iota < 0$ at the Friedman rule. Lemma 7 states that $\mathrm{d}b = u(\hat{q})\,\mathrm{d}\hat{\epsilon}$ at the Friedman rule. We infer that $\mathrm{d}b/\mathrm{d}\iota < 0$ at the Friedman rule.

To determine the sign of $\mathrm{d}m/\mathrm{d}\iota$, we consider the total differential of Equation (20) w.r.t. ι, which reads

$$0 = -1 + \int_{\hat{\epsilon}_{t+1}}^{\bar{\epsilon}}\left[\frac{\partial\mathcal{L}(\epsilon,m,b)}{\partial m}\frac{\mathrm{d}m}{\mathrm{d}\iota} + \frac{\partial\mathcal{L}(\epsilon,m,b)}{\partial b}\frac{\mathrm{d}b}{\mathrm{d}\iota}\right]G(\mathrm{d}\epsilon_{t+1}) - \mathcal{L}(\hat{\epsilon},m,b)g(\hat{\epsilon})\frac{\mathrm{d}\hat{\epsilon}}{\mathrm{d}\iota}. \tag{73}$$

Recalling the definition of $\mathcal{L}$ in Equation (21), i.e.,

$$\mathcal{L}(\epsilon, m, b) \equiv \left[\frac{(1-\theta)[\epsilon u'(q) - c'(q)]}{\theta\epsilon u'(q) + (1-\theta)c'(q)}\right]_{q=q(\epsilon,m,b)}, \tag{74}$$

we obtain

$$\frac{\partial\mathcal{L}(\epsilon, m, b)}{\partial m} = \left[\frac{(1-\theta)[\epsilon u''(q)c'(q) - \epsilon u'(q)c''(q)]}{[\theta\epsilon u'(q) + (1-\theta)c'(q)]^2} \times \frac{1}{\theta\epsilon u'(q) + (1-\theta)c'(q)}\right]_{q=q(\epsilon,m,b)} < 0 \tag{75}$$

and

$$\frac{\partial\mathcal{L}(\epsilon, m, b)}{\partial b} = \left[\frac{(1-\theta)[\epsilon u''(q)c'(q) - \epsilon u'(q)c''(q)]}{[\theta\epsilon u'(q) + (1-\theta)c'(q)]^2} \times \frac{-(1-\theta)}{\theta\epsilon u'(q) + (1-\theta)c'(q)}\right]_{q=q(\epsilon,m,b)} > 0. \tag{76}$$

Moreover, it holds that $\mathcal{L}(\hat{\epsilon}, m, b) = 0$, since $\hat{q} = q^{\star}_{\hat{\epsilon}}$ at the Friedman rule according to Lemma 7. It is an immediate consequence that $\mathrm{d}m/\,\mathrm{d}\iota < 0$.

C.8 Proof of Proposition 2

In steady state, it holds that

$$\mathcal{W} = \frac{\beta}{1-\beta}\int_{\hat{\epsilon}}^{\bar{\epsilon}} [\epsilon u(q) - c(q)]_{q=q(\epsilon,m,b)} G(\mathrm{d}\epsilon), \tag{77}$$

with Lemma C.5. Hence,

$$\frac{\mathrm{d}\mathcal{W}}{\mathrm{d}\iota} = \frac{\beta}{1-\beta}\left[\int_{\hat{\epsilon}}^{\bar{\epsilon}} \left([\epsilon u'(q) - c'(q)]_{q=q(\epsilon,m,b)} \frac{\mathrm{d}q(\epsilon,m,b)}{\mathrm{d}\iota}\right) G(\mathrm{d}\epsilon) - [\hat{\epsilon}u(\hat{q}) - c(\hat{q})]\frac{\mathrm{d}\hat{\epsilon}}{\mathrm{d}\iota}\right] \tag{78}$$

$$= -\frac{\beta}{1-\beta}[\hat{\epsilon}u(\hat{q}) - c(\hat{q})]\frac{\mathrm{d}\hat{\epsilon}}{\mathrm{d}\iota} > 0. \tag{79}$$

since

$$[\epsilon u'(q) - c'(q)]_{q=q(\epsilon,m,b)} = 0 \tag{80}$$

at the Friedman rule for all $\epsilon \in [\hat{\epsilon}, \bar{\epsilon}]$, and because of $[\mathrm{d}\hat{\epsilon}/\,\mathrm{d}\iota]_{\iota=0} < 0$ in Proposition 1.

C.9 Proof of Proposition 3

Recall from Equation (23) that a producer's value $W^{p,0}$ of being born does not directly depend on ι. A change of the Fisher rate ι rather indirectly affects $W^{p,0}$ through a change of the equilibrium money holdings m, so that

$$\frac{\mathrm{d}W^{p,0}}{\mathrm{d}\iota} = \frac{\mathrm{d}W^{p,0}}{\mathrm{d}m}\frac{\mathrm{d}m}{\mathrm{d}\iota}. \tag{81}$$

Moreover, we can write

$$W^{p,0} = \hat{W}^{p,0}(m, b^*(m)) \tag{82}$$

with

$$\hat{W}^{p,0}(m,b) \equiv \beta \int_{\hat{\epsilon}}^{\bar{\epsilon}} \left[\theta[\epsilon u(q) - c(q) - b]_{q=q(\epsilon,m,b)} + b\right] G(\mathrm{d}\epsilon) \tag{83}$$

and $b^*(m)$ denoting the solution of the optimization problem in Equation (23). The envelope theorem yields

$$\frac{\mathrm{d}W^{p,0}}{\mathrm{d}m} = \frac{\partial \hat{W}^{p,0}(m,b^*(m))}{\partial m} + \underbrace{\frac{\partial \hat{W}^{p,0}(m,b^*(m))}{\partial b}}_{=0} \frac{\partial b^*(m)}{\partial m} = \frac{\partial \hat{W}^{p,0}(m,b^*(m))}{\partial m}. \tag{84}$$

Note that

$$\frac{\partial \hat{W}^{p,0}(m,b^*(m))}{\partial m} = \frac{\beta\theta}{1-\theta} \int_{\hat{\epsilon}}^{\bar{\epsilon}} \mathcal{L}(\epsilon, m, b^*(m)) G(\mathrm{d}\epsilon) \\ - \beta[\theta[\hat{\epsilon}u(\hat{q}) - c(\hat{q}) - b^*(m)] + b^*(m)] \frac{\partial \hat{\epsilon}}{\partial m}. \tag{85}$$

At the Friedman rule, it holds that $\mathcal{L}(\epsilon, m, b^*(m)) = 0$ for all $\epsilon \geq \hat{\epsilon}$, and that $\partial\hat{\epsilon}/\partial m = 0$, so that $[\mathrm{d}W^{p,0}/\mathrm{d}m]_{\iota=0} = 0$ and thus

$$\left.\frac{\mathrm{d}W^{p,0}}{\mathrm{d}\iota}\right|_{\iota=0} = \left.\frac{\mathrm{d}W^{p,0}}{\mathrm{d}m}\right|_{\iota=0} \left.\frac{\mathrm{d}m}{\mathrm{d}\iota}\right|_{\iota=0} = 0. \tag{86}$$

To obtain the second-order effect of a deviation from the Friedman rule on $W^{p,0}$, we note that

$$\frac{\mathrm{d}^2W^{p,0}}{\mathrm{d}\iota^2} = \frac{\mathrm{d}}{\mathrm{d}\iota}\left[\frac{\mathrm{d}W^{p,0}}{\mathrm{d}m}\frac{\mathrm{d}m}{\mathrm{d}\iota}\right] = \frac{\mathrm{d}}{\mathrm{d}\iota}\left[\frac{\mathrm{d}W^{p,0}}{\mathrm{d}m}\right]\frac{\mathrm{d}m}{\mathrm{d}\iota} + \frac{\mathrm{d}W^{p,0}}{\mathrm{d}m}\frac{\mathrm{d}^2m}{\mathrm{d}\iota^2} \tag{87}$$

$$= \frac{\mathrm{d}^2W^{p,0}}{\mathrm{d}m^2}\left[\frac{\mathrm{d}m}{\mathrm{d}\iota}\right]^2 + \frac{\mathrm{d}W^{p,0}}{\mathrm{d}m}\frac{\mathrm{d}^2m}{\mathrm{d}\iota^2}. \tag{88}$$

It holds that

$$\frac{\mathrm{d}^2W^{p,0}}{\mathrm{d}m^2} = \frac{\mathrm{d}}{\mathrm{d}m}\left[\frac{\partial \hat{W}^{p,0}(m,b^*(m))}{\partial m}\right] \tag{89}$$

$$= \frac{\partial^2 \hat{W}^{p,0}(m,b^*(m))}{\partial m^2} + \underbrace{\frac{\partial^2 \hat{W}^{p,0}(m,b^*(m))}{\partial m \partial b}}_{=0}\frac{\partial b^*(m)}{\partial m} \tag{90}$$

$$= \frac{\beta\theta}{1-\theta}\int_{\hat{\epsilon}}^{\bar{\epsilon}} \frac{\partial \mathcal{L}(\epsilon,m,b^*(m))}{\partial m} G(\mathrm{d}\epsilon) \tag{91}$$

$$- \frac{\partial}{\partial m}\left[\beta[\theta[\hat{\epsilon}u(\hat{q}) - c(\hat{q}) - b^*(m)] + b^*(m)]\frac{\partial \hat{\epsilon}}{\partial m}\right]. \tag{92}$$

Since $\partial\hat{\epsilon}/\partial m = 0$ and $\hat{\epsilon}u(\hat{q}) - c(\hat{q}) - b^*(m) = 0$ at the Friedman rule, it holds that

$$\left.\frac{\mathrm{d}^2 W^{p,0}}{\mathrm{d}m^2}\right|_{\iota=0} = \frac{\beta\theta}{1-\theta}\int_{\hat{\epsilon}}^{\bar{\epsilon}} \frac{\partial \mathcal{L}(\epsilon, m, b^*(m))}{\partial m} G(\mathrm{d}\epsilon) \tag{93}$$

$$= \frac{\beta\theta}{1-\theta}\int_{\hat{\epsilon}}^{\bar{\epsilon}} \left[\frac{(1-\theta)[\epsilon u''(q)c'(q) - \epsilon u'(q)c''(q)]}{[\theta\epsilon u'(q) + (1-\theta)c'(q)]^3}\right]_{q=q(\epsilon,m,b^*(m))} G(\mathrm{d}\epsilon) < 0. \tag{94}$$

We obtain $[\mathrm{d}^2 W^{p,0}/\,\mathrm{d}m^2]_{\iota=0} < 0$ and thus

$$\left.\frac{\mathrm{d}^2 W^{p,0}}{\mathrm{d}\iota^2}\right|_{\iota=0} = \underbrace{\left.\frac{\mathrm{d}^2 W^{p,0}}{\mathrm{d}m^2}\right|_{\iota=0}}_{<0} \left[\underbrace{\left.\frac{\mathrm{d}m}{\mathrm{d}\iota}\right|_{\iota=0}}_{<0}\right]^2 + \underbrace{\left.\frac{\mathrm{d}W^{p,0}}{\mathrm{d}m}\right|_{\iota=0}}_{=0} \left.\frac{\mathrm{d}^2 m}{\mathrm{d}\iota^2}\right|_{\iota=0} < 0, \tag{95}$$

using $[\mathrm{d}m/\,\mathrm{d}\iota]_{\iota=0} < 0$ from Corollary 2.

For the consumers, note that

$$W^c = \frac{\beta}{1-\beta}\int_{\hat{\epsilon}_{t+1}}^{\bar{\epsilon}} (1-\theta)[\epsilon u(q) - c(q) - b]_{q=q(\epsilon,m,b)} G(\mathrm{d}\epsilon), \tag{96}$$

so that

$$\frac{\mathrm{d}W^c}{\mathrm{d}\iota} = \frac{\beta}{1-\beta}\left[\begin{array}{l} \displaystyle\int_{\hat{\epsilon}_{t+1}}^{\bar{\epsilon}} \left[\mathcal{L}(\epsilon, m, b)\frac{\mathrm{d}m}{\mathrm{d}\iota} - (1-\theta)[\mathcal{L}(\epsilon, m, b) + 1]\frac{\mathrm{d}b}{\mathrm{d}\iota}\right] G(\mathrm{d}\epsilon) \\ \displaystyle - (1-\theta)[\hat{\epsilon}u(\hat{q}) - c(\hat{q}) - b]\frac{\mathrm{d}\hat{\epsilon}}{\mathrm{d}\iota} \end{array}\right] \tag{97}$$

$$= -\frac{\beta(1-\theta)[1-G(\hat{\epsilon})]}{1-\beta}\frac{\mathrm{d}b}{\mathrm{d}\iota} > 0. \tag{98}$$