Social Norms and the Rise of Fringe Candidates*

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February 29, 2024

Abstract

Recently, some fringe candidates, previously dismissed, have seen unexpected surges in popularity. We explore this issue by focusing on the dynamic interplay between social norms and elections. For this purpose, we develop a two-period electoral competition model with a mainstream candidate and a fringe candidate. Because the fringe candidate claims an extreme view that contravenes prevailing social norms, voting for her incurs a stigma cost. We show that a sufficient vote share of the fringe candidate in the first period signals wider acceptance of the extreme view, eroding established norms even if the fringe candidate loses in the election. This triggers the rise of the fringe candidate in the second period. To induce the erosion of the social norm, the fringe candidate tries to differentiate from the mainstream candidate on standard policy issues, whereas the mainstream candidate imitates the fringe candidate. Furthermore, heightened social norms in the initial election might paradoxically enhance the success of the fringe candidate in a subsequent election.

Keywords: Stigmatization; Social norm; Dynamic electoral competition; Policy divergence; Fringe candidate

JEL classification: D72, D83, D91

*We would like to thank Yasushi Asako, Shohei Doi, Airo Hino, Shoko Kohama, Ikuo Kume, Ryo Nakai, and participants of Workshop on Political Polarization at Hokkaido University for their helpful comments. The remaining errors are our own. This study was financially supported by the JSPS KAKENHI Grant Number 22K13368.

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1 Introduction

This study investigates the unexpected surges in popularity experienced by fringe parties and candidates in contemporary politics, notably the radical right in Europe and the United States. The primary objective is to reveal the mechanisms driving these surges by highlighting the dynamic interplay between social norms and elections. Throughout this paper, we employ the terms "parties" and "candidates" interchangeably.

Fringe candidates often adopt extreme positions that defy prevailing social norms, resulting in their political stigmatization. For example, the radical right frequently advocates anti-immigration arguments, which run counter to established norms. In competitive authoritarian contexts, opposition parties challenge the regime's endorsed social norm, leading to their stigmatization. Subsequent democratization stigmatizes former regime parties.

This stigmatization prompts voters to conceal their true political preferences (Kuran, 1987, 1989). That is, even voters with extreme views can hide their support for fringe candidates who share similar views, fearing the social repercussions of publicly supporting such positions (Valentim, forthcoming).[1] Despite the secrecy of the voting process, empirical evidence indicates that a significant fraction of voters worry about the confidentiality of their voting choices (Gerber et al., 2013).[2] Hence, preference falsification extends to voting decisions (Karpowitz et al., 2011; Gerber et al., 2013; Lagios, Méon and Tojerow, 2022; Valentim, forthcoming). This falsification persists as long as the dominant social norm remains unchallenged, acting as a deterrent to support for fringe candidates.

However, the stability of social norms is not guaranteed. They can quickly erode in response to electoral outcomes. Bursztyn, Egorov and Fiorin (2020) find that the unexpected rise in popularity of Donald Trump in the presidential election led US citizens to publicly reveal their anti-immigration views as such views became destigmatized. Similarly, the rise of radical right parties in Europe encouraged people to express support for these parties and their anti-immigration views (Valentim, 2021; Gerling and Kellermann, 2022; Hagemeister, 2022).[3]

These empirical findings indicate two-way interactions between social norms and electoral results. Social norms hinder the electoral victory of fringe candidates through stigmatization, but they can swiftly change when substantial support is unexpectedly gained. Motivated by these observations, this study aims to uncover the mechanism by which fringe candidates are

[1] People hide their true preferences in responding to surveys when their true preferences contradict with social norms, which is known as the social desirability bias (Blair, Coppock and Moor, 2020). See Janus (2010) for anti-immigration attitudes, Jiang and Yang (2016) for the disapproval of authoritarian regimes, and Dinas, Martínez and Valentim (forthcoming) for the support for previous regime parties after democratization.

[2] Gerber et al. (2013) find that in the US, more than 80 % of the population believes that either someone will learn their voting choices without permission or that they will share their voting choices with others. Therefore, votes are not expected to be generally secret. Furthermore, they find some evidence suggesting that social norms indeed affect for whom voters vote.

[3] Apffelstaedt, Freundt and Oslislo (2022) also find that electoral results influence social norms in a laboratory experiment.

destigmatized, ultimately leading to electoral victories.

To this end, we develop a two-period electoral competition model consisting of a mainstream candidate, a fringe candidate, and a continuum of voters. The policy space is two-dimensional. The first dimension represents a standard policy issue such as economic policy and national security, where social norms are irrelevant. The second dimension is a controversial issue such as anti-immigration. To maximize the probability of winning, each candidate chooses her policy platform in the first dimension from a binary set of policies. On the contrary, each candidate has a fixed position in the second dimension: the mainstream candidate's position is deemed socially acceptable, whereas the fringe candidate's position is not. The distribution of policy preferences among the electorate is uncertain for all players. After observing each candidate's policy platform in the first dimension, voters cast their votes. This game is repeated over two periods.

A notable feature of our model is that voting for the fringe candidate incurs a stigma cost because she has a socially unacceptable view such as anti-immigration in the second dimension. The stigma cost is positively associated with the expected number of voters who share the socially acceptable view in society. Voters do not know the distribution of voters' policy preferences, but they update it based on the electoral result of the first period. This structure enables us to analyze the two-way interactions between social norms and electoral results.

Using this model, our study seeks to answer:

1. How do prevailing social norms collapse, and how do fringe candidates achieve electoral victories?

2. Taking the identified mechanism into account, what policy platforms do mainstream and fringe candidates strategically choose in electoral competition?

3. Under what circumstances do fringe candidates most likely prevail?

Regarding the first question, we show that the fringe candidate could achieve an electoral victory in the second period if and only if her vote share in the first period exceeds a threshold. Due to the social norm, the fringe candidate has no chance of winning the election in the first period. To win the election in the second period, she must induce the erosion of the social norm, which is possible only when her vote share in the first period is sufficiently large.[4] Such a high vote share serves as a signal that a substantial fraction of voters share the socially unacceptable view. This learning by voters weakens the social norm, creating room for an electoral victory for the fringe candidate in the second period. This mechanism describes a process through which a fringe candidate expands political support over time.

[4]The vote share of the fringe candidate in period 1 is always less than half. The threshold vote share is also less than half; that is, the fringe candidate does not have to obtain the majority of votes to induce the erosion of the social norm.

In the first period, the mainstream candidate always wins the election due to high social pressure. However, policy platforms in the first-period election matter for the second-period election because they affect whether the social norm will be maintained in the future. Therefore, candidates strategically determine their policy platforms considering their effect on the second-period election. This leads to the following answer to our second question.

The erosion of the social norm can only occur when the fringe candidate obtains a substantial vote share in the first-period election. However, when both candidates' positions are the same in the first dimension (i.e., for the standard policy issue), no one votes for the fringe candidate in the first period because voting for the fringe candidate incurs a large stigma cost. As a result, voters cannot infer the fraction of voters with the socially unacceptable view based on the electoral result, implying that the social norm is maintained in the second period. Therefore, whether candidates are differentiated in the first dimension matters for the erosion of the social norm. On the one hand, the mainstream candidate attempts to imitate the fringe candidate's policy platform because it enables her to prevent the erosion of the social norm. On the other hand, the fringe candidate tries to differentiate herself from the mainstream candidate to induce a change in the social norm. This nature of electoral competition results in a mixed-strategy equilibrium.

Specifically, in the equilibrium, both candidates mix two policies, but the mainstream candidate is more likely to propose an ex-ante popular policy than the fringe candidate does. To erode the social norm, the fringe candidate must acquire a large vote share. Given policy differentiation among parties, this is least likely to happen when the mainstream candidate proposes the ex-ante popular policy. Therefore, the mainstream candidate leans towards the ex-ante popular policy. In response to this, the fringe candidate leans toward the ex-ante unpopular policy because she must differentiate from the mainstream candidate.

In the empirical analysis of the rise of niche parties in Europe, Meguid (2005) finds that the mainstream parties' strategies matter for the success of niche parties. Specifically, they have three possible strategies: accommodating (i.e., choosing the same policy as the niche party), adversarial (i.e., increasing the policy distance between parties), and dismissive (i.e., not taking a position on the niche party's issue) strategies. She argues that mainstream parties should choose either the accommodating or dismissive strategy. While her argument concerns the issue owned by niche parties (i.e., the second dimension in our model), it echoes our findings. Our results indicate that mainstream parties should not adopt an adversarial but an accommodating strategy, even for standard issues such as economic policy-making (i.e., the first dimension).[5] This is consistent with the empirical finding by Akkerman (2015). In practice, immigration policies are complex and consist of various sub-issues. She finds that

[5]In addition, our model shows that niche parties' policy platforms in the first dimension matter. Because niche parties are typically regarded as single-issue parties focusing on the second dimension of the policy space, one might think that this is implausible. However, this is not necessarily the case. For example, Ivaldi (2015) finds that the Front National (FN), the major radical right party in France, significantly shifted its economic platform from right-wing to left-wing since the 1980s. This indicates that niche parties strategically choose their positions in the first dimension.

mainstream parties do not co-opt extreme views of radical right parties such as anti-Muslim attitudes, but they imitate policies of the radical right parties for a part of sub-issues on immigration, such as the importance of law and order. Once we interpret the former as the second dimension and the latter as the first dimension, our result provides a rationale for why mainstream parties take such a strategy.

Lastly, we conduct comparative statics on the equilibrium probability that the fringe candidate wins the second-period election. This addresses the third question. Specifically, we focus on the effect of a higher social norm in the first period. Let us denote the fraction of voters with the socially acceptable view as β. Our model assumes that $\beta = \bar{\beta}$ with probability p and $\beta = \underline{\beta}$ with the remaining probability ($\bar{\beta} > \underline{\beta}$). Hence, the higher social norm in the first period stems from either a higher p or a higher $\bar{\beta}$. Interestingly, a higher p prevents the fringe candidate's electoral victory as expected, but a higher $\bar{\beta}$ actually induces the fringe candidate's electoral victory. The reason is that a higher $\bar{\beta}$ enables voters to learn the value of β more easily because the difference between $\bar{\beta}$ and $\underline{\beta}$ expands. Without voters learning about β, the fringe candidate cannot win the second-period election. Therefore, a higher $\bar{\beta}$ paradoxically induces an electoral victory for the fringe candidate. In other words, a higher social norm in the first period may increase the chance of an electoral victory of the fringe candidate in the future.

The remainder of the paper is organized as follows. Below, we summarize the related literature. Section 2 presents the model. Section 3 characterizes equilibrium and conducts comparative statics. Section 4 discusses extensions and Section 5 concludes.

Related literature: This study relates to four strands of literature.

First, the mainstream candidate enjoys an electoral advantage due to the unfavorable position of the fringe candidate in the second dimension. A typical example of electoral advantages is the valence advantage, extensively examined in the literature (e.g., Bernhardt and Ingerman, 1985; Aragonès and Palfrey, 2002; Groseclose, 2001). Diverging from valence advantage, our model introduces heterogeneity in how voters evaluate the fringe candidate's position in the second dimension. Recently, static electoral competition models with differentiated candidates have been developed, where the policy space is two-dimensional, and candidates' positions are exogenously fixed in the second dimension (e.g., Krasa and Polborn, 2010, 2012, 2014; Aragonès and Xefteris, 2017; Matakos and Xefteris, 2017; Karakas and Mitra, 2020; Buisseret and Van Weelden, 2022).[6] Our model is a variant of these models. Although the degree of electoral advantage is exogenously given in these models, it is realistically endogenously determined through past electoral results in our model. Our contribution is to reveal the dynamic interplay between elections and electoral advantage by developing a dynamic model with endogenous stigma cost.

Second, our study is related to the literature on voters' social learning in sequential

[6] Some of them analyze electoral competition between a mainstream candidate and a fringe candidate (e.g., Matakos and Xefteris, 2017; Karakas and Mitra, 2020).

elections. Several studies, focusing on primary elections for the US presidency, explore bandwagon effects in sequential elections, where voters' learning about candidates' popularity or quality induces these effects (e.g., Callander, 2007; Knight and Schiff, 2010; Hummel, 2012; Ali and Kartik, 2012).[7] The study by Callander (2007) is particularly related in that voters have the desire to vote for the candidate to win. Although not centered on stigma, his voters can be viewed as exhibiting a form of conformity bias. The key difference from our study is that candidates are not strategic players in the existing models; which focus instead on interactions across voters. By endowing candidates with strategic decision-making in choosing policy platforms, we uncover how candidates' decisions and voters' learning interact with each other.

Third, there is a growing body of literature analyzing the dynamics of social norms and electoral competition (e.g., Besley and Persson, 2019*a*,*b*, 2023). In their approach, voters' values (social norms) evolve across generations through cultural transmission, and the value yielding a higher payoff to voters becomes more widespread over time. What value yields a high payoff depends on policy determined by electoral competition, which in turn depends on the past distribution of values. As a result, voters' values evolve through interaction with electoral competition. Our model has two distinctive features. First, our focus is on the change of social norms in the short-run rather than the long-run dynamics between generations. Therefore, we model a change in social norms based on Bayesian learning rather than cultural transmission. More crucially, our model features farsighted candidates influencing future social norms, a departure from the myopic candidates in existing models. This enables us to analyze how farsighted strategic decisions of candidates influence social norms.

Lastly, in our model, candidates strategically choose policy platforms in the current election to affect future elections. Eguia and Giovannoni (2019) and Izzo (2020) show that candidates may choose an unpopular policy platform to win the next election. In particular, Izzo (2020) develops a model in which the implemented policy induces voters' learning about the desirable policy, with the extent depending on the location of the implemented policy. She finds that ideological parties may choose a radical policy to enhance voters' learning. In addition to a difference in focus, our model has two distinctive features. First, in our model, a change in social norms drives an endogenous shift in each candidate's popularity, but voters' policy preferences are fixed, whereas voters learn their preferred policy, shifting their policy preferences in her model. Second, her candidates are policy-motivated, which is a key for driving her results, whereas candidates are office-seeking in our model.

2 Model

We present a two-period electoral competition model involving two candidates (M and F) and a continuum of voters with measure one. At the start of period t, each candidate i

[7] The literature has also analyzed other types of voters' learning such as learning from protests (e.g., Lohmann, 1994*a*,*b*) and learning from the policymaking of other countries (e.g., Besley and Case, 1995; Kishishita and Yamagishi, 2021).

simultaneously commits to a policy platform, x_{it}; subsequently, voters cast their votes for either of the two candidates. The candidate who secures the majority of votes wins and implements the policy platform. In case of an equal vote share, each candidate wins with a probability of half. This electoral competition game is played twice by the same set of players.

Candidates: The policy space is two-dimensional. In each election, candidate i proposes a policy platform in the first dimension: $x_{it} \in \{0, 1\}$. This dimension captures a standard policy issue such as economic policymaking and national security policies.

In the second dimension, each candidate has a fixed position: $y_i \in \{0, 1\}$.[8] This dimension captures a controversial issue such as whether to take an anti-immigration stance. Specifically, we assume that $y_M = 1$ and $y_F = 0$. As we introduce later, $y = 1$ is a mainstream attitude that has been accepted as socially desirable, while $y = 0$ represents an extreme attitude regarded as undesirable. In this context, candidate M is a mainstream candidate, whereas candidate F is a fringe candidate with an extreme view. Assuming a single mainstream candidate allows us to highlight each candidate's incentive in a transparent way.[9] For the analysis of the case with multiple mainstream candidates, see Section 4.

Candidate i's payoff is given by $\sum_{t=1}^{2} \mathbf{1_{it}}$, where $\mathbf{1_{it}}$ is an indicator function that takes one if and only if candidate i wins the election in period t. Therefore, each candidate chooses her policy platform x to maximize the sum of the probabilities of electoral victory across two periods.

Policy preferences of voters: In period t, each voter $k \in [0, 1]$ has an ideal point for each issue, denoted by $(\hat{x}_{kt}, \hat{y}_k)$. We allow $\hat{x}_{kt}$ to be different across periods, whereas the attitude on y remains time-invariant. Let the fraction of voters whose $\hat{x}_{kt} = 1$ be α_t and the fraction of voters whose $\hat{y}_k = 1$ be β.

As in the literature on electoral competition under uncertainty, we assume that the distribution of ideal points on x is uncertain for players.[10] Specifically, the value of α_t is drawn from a uniform distribution $U[\underline{\alpha}, \bar{\alpha}]$, where $0 < \underline{\alpha} < \bar{\alpha} < 1$ and $\bar{\alpha} + \underline{\alpha} > 1$. This implies that policy 1 is likely to be popular. The values of α_t are drawn independently across time, reflecting the idea that the salient issue in the political arena differs between the two periods

[8] Such fixed positions in the second dimension have been assumed in the literature of electoral competition with differentiated candidates (e.g., Krasa and Polborn, 2010, 2012, 2014; Aragonès and Xefteris, 2017; Matakos and Xefteris, 2017; Karakas and Mitra, 2020; Buisseret and Van Weelden, 2022).

[9] In addition to the case where literally, there is a single mainstream candidate, the current model is applied to the following situations. First, suppose that there are two mainstream parties (moderately liberal and moderately conservative parties), and two extreme parties enter on both sides of ideology. If the left and right are severely divided, no voters would vote for parties with an opposing ideology. Then, we can separately analyze the competition between the right-wing mainstream party and the radical right party and that between the left-wing mainstream party and the radical left party. Second, our model can be interpreted as a model of preliminary elections such as in the US presidential elections. In this case, it is reasonable to consider a single mainstream candidate.

[10] This type of uncertainty is often introduced in electoral competition models with valence advantage (e.g., Aragonès and Palfrey, 2002; Buisseret and Van Weelden, 2022).

(e.g., the economy in period 1 and national security in period 2). We study the case where α_1 and α_2 are correlated in Section 4.

The objective of this study is to reveal how electoral outcomes change voters' perceptions regarding the extent to which an extreme position has been deemed socially unacceptable. For this purpose, we assume that the value of β is also uncertain for voters and candidates. Specifically, β takes $\underline{\beta}$ or $\bar{\beta}$, where $0.5 < \underline{\beta} < \bar{\beta} < 1$, meaning that $y = 1$ is accepted as a socially desirable position. The prior probability of β being $\bar{\beta}$ is $p \in (0, 1)$. $\underline{\beta} > 0.5$ implies that voters who have an extreme view (i.e., $\hat{y} = 0$) are a minority even if $\beta = \underline{\beta}$. This assumption is plausible in contexts where y represents xenophobic attitudes, but may not be valid in other situations. For example, in elections under competitive authoritarianism, with $y = 1$ representing the view that the current regime is desirable, $\underline{\beta} < 0.5$ could be the case. Refer to Section 4 for the analysis of the case where $\underline{\beta} < 0.5$.

Voting behavior and social stigma: Each voter myopically votes for a candidate that gives the highest pay in the period. Specifically, voter k votes for candidate M in period t if and only if

$$\underbrace{-|x_{Mt} - \hat{x}_{kt}|}_{\text{Utility from } M'\text{s } x} \underbrace{-\theta|y_M - \hat{y}_k|}_{\text{Utility from } M'\text{s } y} \geq \underbrace{-|x_{Et} - \hat{x}_{kt}|}_{\text{Utility from } F'\text{s } x} \underbrace{-\theta|y_F - \hat{y}_k|}_{\text{Utility from } F'\text{s } y} \underbrace{-c\mathbb{E}_{kt}[\beta]}_{\text{Social Stigma}},$$

where $\theta \in (0, 1)$ and $c > 0$.[11] If $\theta > 1$, the majority of voters always vote for the candidate M, making the analysis meaningless. Thus, $\theta < 1$ is assumed. Later, we impose several assumptions on (c, θ).

A notable feature of our model is the term, $c\mathbb{E}_{kt}[\beta]$. Apart from $c\mathbb{E}_{kt}[\beta]$, this is the sincere voting assumed in the standard electoral competition model. In our model, $y = 1$ is a socially acceptable attitude toward issue y. Hence, there exists a social norm stipulating that citizens should not vote for candidate F, who has a socially unacceptable attitude. This may influence voters' behavior. The term $c\mathbb{E}_{kt}[\beta]$ captures this factor. We assume that this stigma cost is increasing in the expected share of voters with $y = 1$, $\mathbb{E}_{kt}[\beta]$. For a further discussion, see Section 4.

Timing of the game: The timing of the game is summarized as follows.

Period 1

1. Each voter k privately observes $(\hat{x}_{k1}, \hat{y}_k)$.
2. Each candidate i simultaneously chooses policy platform x_{i1}.
3. Observing (x_{M1}, x_{F1}), voters vote for one of the candidates. After the election, voters observe the vote share of each candidate.

[11] When the voter is indifferent between two candidates, the voter votes for candidate M. This tie-breaking rule does not affect the results.

Period 2

1. Each voter k privately observes $\hat{x}_{k2}$.
2. Each candidate i simultaneously chooses policy platform x_{i2}.
3. Observing (x_{M2}, x_{F2}), voters vote for one of the candidates.

Equilibrium concept: We characterize the perfect Bayesian equilibrium of this game.

3 Equilibrium

3.1 Voting Behavior

Before characterizing the equilibrium policy platform in each election, we start with deriving voting behaviors. The omitted proofs are contained in the Appendix.

Lemma 1. *(i). Suppose that $x_{Mt} = x_{Ft}$. Then, in the period t election:*

(a). Voter k with $\hat{y}_k = 1$ always votes for candidate M.

(b). Voter k with $\hat{y}_k = 0$ votes for candidate M if and only if $c\mathbb{E}_{kt}[\beta] > \theta$.

(ii). Suppose that $x_{Mt} \neq x_{Ft}$. Then, in the period t election:

(a). Voter k with $\hat{x}_{kt} = x_{Mt}$ always votes for candidate M.

(b). Voter k with $(\hat{x}_{kt}, \hat{y}_k) = (x_{Ft}, 1)$ votes for candidate M if and only if $c\mathbb{E}_{kt}[\beta] > 1 - \theta$.

(c). Voter k with $(\hat{x}_{kt}, \hat{y}_k) = (x_{Ft}, 0)$ votes for candidate M if and only if $c\mathbb{E}_{kt}[\beta] > 1+\theta$.

Due to the social norm, voters tend to be reluctant to vote for candidate F. As a result, candidate M has an electoral advantage. To see this, as a benchmark, suppose that there is no social stigma (i.e., $c = 0$). Because $\theta < 1$ is assumed, when two candidates are differentiated in the first dimension, votes should vote for the candidate who promises their preferred policy in the first dimension. However, this does not necessarily occur in our model due to social stigma. As seen in (ii)-(b) and (ii)-(c) in the above lemma, even voters who prefer x_{Ft} over x_{Mt} may vote for candidate M due to the stigma cost.

To focus on meaningful cases where the social norm is sufficiently large in period 1, but could be substantially reduced in period 2, we assume the following throughout the paper:

Assumption 1. *The following holds:*

$$1 + \theta > c\frac{p(1-\bar{\beta})\bar{\beta} + (1-p)(1-\underline{\beta})\underline{\beta}}{p(1-\bar{\beta}) + (1-p)(1-\underline{\beta})} > \max\{\theta, 1-\theta\} \text{ and } 1-\theta > c\underline{\beta}.$$

The first condition is on the social norm in period 1. Using Bayes' rule, for a voter with $\hat{y}_k = 0$, the expected number of voters with $\hat{y}_k = 1$ is given by [12]

$$\begin{aligned}\mathbb{E}_{k1}[\beta|\hat{y}_k = 0] &= \frac{Pr(\beta = \bar{\beta})Pr(\hat{y}_k = 0|\beta = \bar{\beta})\bar{\beta} + Pr(\beta = \underline{\beta})Pr(\hat{y}_k = 0|\beta = \underline{\beta})\underline{\beta}}{Pr(\hat{y}_k = 0)} \\ &= \frac{p(1-\bar{\beta})\bar{\beta} + (1-p)(1-\underline{\beta})\underline{\beta}}{p(1-\bar{\beta}) + (1-p)(1-\underline{\beta})}.\end{aligned}$$

The first condition ensures that the social norm is neither too large nor too low for them in period 1. In particular, it ensures that voters vote for candidate F in period 1 if and only if they prefer x_{F1} over x_{M1} and have $\hat{y}_k = 0$ (see Lemma 1 (i)-(a) , (ii)-(b), and (ii)-(c)).

The second condition concerns the case where the social norm is eroded in period 2. When voters learn that $\beta = \underline{\beta}$, voters who prefer x_{F1} over x_{M1} should vote for candidate F without caring about stigma costs. The second condition ensures this (see Lemma 1 (ii)-(b)).

3.2 Election in Period 2

We solve the game backwardly. As we prove later in Lemma 5, either $c\mathbb{E}_{k2}[\beta] > \max\{\theta, 1-\theta\}$ for all k or $c\mathbb{E}_{k2}[\beta] = c\underline{\beta} < 1-\theta$ for all k holds. Therefore, we consider these two cases in this subsection. We start by deriving the following lemma.

Lemma 2. *(i). Suppose that* $c\mathbb{E}_{k2}[\beta] > \max\{\theta, 1-\theta\}$ *for all* k*. Then, the vote share of candidate F,* v_{F2}*, is always less than 0.5. Therefore, candidate M wins the election in period 2 for any* (x_{M2}, x_{F2})*.*

(ii). Suppose that $c\mathbb{E}_{k2}[\beta] < 1-\theta$ *for all* k*. Then, the vote share of candidate F,* v_{F2}*, is given as follows:*

$$v_{F2} = \begin{cases} < 0.5 & \text{if } x_{M2} = x_{F2} \\ 1-\alpha_2 & \text{if } (x_{M2}, x_{F2}) = (1,0) \\ \alpha_2 & \text{if } (x_{M2}, x_{F2}) = (0,1) \end{cases}.$$

Therefore, candidate M wins the election if and only if either of the following holds:

- $x_{M2} = x_{F2}$
- $(x_{M2}, x_{F2}) = (1,0)$ *and* $\alpha_2 > 0.5$
- $(x_{M2}, x_{F2}) = (0,1)$ *and* $\alpha_2 < 0.5$

(i) considers the case where voters anticipate a limited fraction of the electorate to hold extreme views, thereby imposing substantial social pressure against supporting candidate F.

[12]If a voter's ideal policy on the second dimension, i.e. $\hat{y}_k$, did not give any information on β, we would simply have $\mathbb{E}_{k1}[\beta] = p\bar{\beta} + (1-p)\underline{\beta}$ for all k.

Therefore, the candidate F cannot obtain the majority of votes, implying that candidate M always wins the election in period 2.

(ii) considers the case where voters believe that the fraction of voters having an extreme view is likely to be non-negligible; thus, social pressure that voters should not vote for candidate F is reduced. In this case, voters vote for candidate F if candidate F's policy on x is better for them. Hence, when $(x_{M2}, x_{F2}) = (1, 0)$, voters preferring policy 0 on the standard policy issue vote for candidate F, implying that candidate F's vote share is equal to $1 - \alpha_2$. Similarly, when $(x_{M2}, x_{F2}) = (0, 1)$, voters preferring policy 1 vote for candidate F, implying that candidate F's vote share is equal to α_2. On the contrary, when both candidates propose the same policy, no one votes for candidate F. Social pressure still exists, even though its magnitude is small. Hence, voters are reluctant to vote for candidate F when both candidates offer the same policy on the standard policy issue. Taken together, candidate M wins the election either when candidate M proposes x supported by the majority or when both candidates propose the same policy.

In case (ii), the electoral outcome depends on policy platforms. Hence, candidates strategically choose policy platforms. The payoff matrix of the game is given by Table 1, where $q := \Pr(\alpha_t \geq 0.5)$.

Table 1: Payoff Matrix in Period 2

M/F	1	0
1	$1, 0$	$q, 1-q$
0	$1-q, q$	$1, 0$

In this game, on the one hand, the candidate M attempts to choose the same policy as chosen by candidate F because social pressure enables candidate M to always win the election as long as both candidates propose the same policy. On the other hand, candidate F tries to differentiate from candidate M. Consequently, there is no pure-strategy equilibrium. Instead, there is a unique mixed-strategy equilibrium.

Let σ_{kt} be candidate k's equilibrium probability of choosing policy 1 in period t. The payoff of candidate F for choosing policy 1 is $(1 - \sigma_{M2})q$, and for choosing policy 0, it is $\sigma_{M2}(1-q)$. For candidate F to be indifferent, it must be that $\sigma_{M2} = q$. Similarly, for candidate M to be indifferent between choosing policies 1 and 0, it must be that $\sigma_{F2} = 1 - q$. Then, in equilibrium, $(\sigma_{M2}, \sigma_{F2}) = (q, 1-q)$. Furthermore, the winning probability of candidate F is

$$q^2(1-q) + (1-q)^2 q = q(1-q),$$

where the first (resp. second) term corresponds to the equilibrium probability of $(x_{M2}, x_{F2}) = (1, 0)$ (resp. $(x_{M2}, x_{F2}) = (0, 1)$) multiplied by the winning probability of candidate F in this case.

This equilibrium outcome is exactly the same as the well-known result obtained in the electoral competition model with a valence advantage (e.g., Aragonès and Palfrey, 2002;

Buisseret and Van Weelden, 2022). The literature has shown that the equilibrium is characterized by a mixed strategy equilibrium in the presence of a valence advantage. In our model, voters vote for candidate M when both candidates propose the same policy platform, but they vote for a candidate proposing the ideal policy when candidates propose different platforms. In this sense, candidate M has an electoral advantage, which is the same as in the electoral competition model with a valence advantage. Therefore, we obtain the same characterization of the equilibrium.

In summary, we obtain the following lemma:

Lemma 3. *(i). Suppose that* $c\mathbb{E}_{k2}[\beta] > \max\{\theta, 1-\theta\}$ *for all* k*. Then, candidate* M *wins the election in period 2 with probability one.*

(ii). Suppose that $c\mathbb{E}_{k2}[\beta] < 1-\theta$ *for all* k*. Then, in equilibrium,* $(\sigma_{M2}, \sigma_{F2}) = (q, 1-q)$*. Furthermore, the winning probability of candidate* F *is* $q(1-q)$*.*

The interpretation of candidates probabilistically selecting policy platforms need not be taken literally. An alternative interpretation of mixed strategies is that they represent a distribution of policy platforms across various issues. In practice, there are various issues that could be salient in elections, yet at the time of choosing policy platforms, what becomes salient in the next election may be uncertain for candidates. Under this assumption, a mixed strategy can be construed as candidate i pledging policy 1 for a fraction σ_{i2} of issues.

3.3 Election in Period 1

Assumption 1 implies that $c\mathbb{E}_{k1}[\beta] > \max\{\theta, 1-\theta\}$ for all k. Therefore, Lemma 3 (i) applies, and candidate M wins the period 1 election with probability one, which is also summarized as follows:

Lemma 4. *In period 1, the vote share of candidate* F*,* v_{F1}*, is given as follows:*

$$v_{F1} = \begin{cases} 0 & \text{if } x_{M1} = x_{F1} \\ (1-\alpha_1)(1-\beta) & \text{if } (x_{M1}, x_{F1}) = (1, 0) \\ \alpha_1(1-\beta) & \text{if } (x_{M1}, x_{F1}) = (0, 1) \end{cases}.$$

Hence, candidate M *wins the election with probability one.*

In this sense, each candidate's vote share in period 1 seems irrelevant. However, this is not the case because the vote share of each candidate in the period 1 election changes voters' views on social pressure, which eventually influences the electoral outcome in period 2.

Dynamics of the social norm: As we know from the analysis of the period 2 election, the only way that candidate F can win the period 2 election is that the electoral outcome in period

1 influences voters' belief on the value of β downward, which in turn decreases the stigma cost of voting for candidate F in period 2. Therefore, we now study how voters update their beliefs on β.

To highlight insightful cases in a transparent manner, we impose an additional assumption. Let $\underline{\gamma}$ be $\gamma < 1$ such that

$$c\frac{p(1-\bar{\beta})\bar{\beta}^2\gamma + (1-p)(1-\underline{\beta})\underline{\beta}^2}{p(1-\bar{\beta})\gamma + (1-p)(1-\underline{\beta})} = \max\{\theta, 1-\theta\}.$$

Assumption 2.

$$\underline{\gamma} < \frac{1-\underline{\beta}}{1-\bar{\beta}}\frac{1-\bar{\alpha}}{1-\bar{\alpha}\frac{1-\bar{\beta}}{1-\underline{\beta}}}.$$

This assumption ensures that when candidate F's vote share is in a medium range, the belief updating about β does not change the effect of the social norm on voting behavior.[13]

Under these assumptions, we obtain the following lemma, which shows how voters update their beliefs on β based on the vote share in the period 1 election.

Lemma 5. *(i). Suppose $x_{M1} = x_{F1}$. Then, $c\mathbb{E}_{k2}[\beta] > \max\{\theta, 1-\theta\}$ for any k with probability one.*

(ii). Suppose $(x_{M1}, x_{F1}) = (1, 0)$.

(a). If $v_{F1} > (1-\underline{\alpha})(1-\bar{\beta})$, $\mathbb{E}_{k2}[\beta] = \underline{\beta}$ for any k.

(b). Otherwise, $c\mathbb{E}_{k2}[\beta] > \max\{\theta, 1-\theta\}$ for any k.

(c). Therefore, the probability of $\mathbb{E}_{k2}[\beta] = \underline{\beta}$ is

$$(1-p)\min\left\{\frac{(1-\underline{\alpha})(\bar{\beta}-\underline{\beta})}{(\bar{\alpha}-\underline{\alpha})(1-\underline{\beta})}, 1\right\}$$

for any k.

(iii). Suppose $(x_{M1}, x_{F1}) = (0, 1)$.

(a). If $v_{F1} > \bar{\alpha}(1-\bar{\beta})$, $\mathbb{E}_{k2}[\beta] = \underline{\beta}$ for any k.

(b). Otherwise, $\mathbb{E}_{k2}[\beta] > \max\{\theta, 1-\theta\}$ for any k.

(c). The probability of $\mathbb{E}_{k2}[\beta] = \underline{\beta}$ is

$$(1-p)\min\left\{\frac{\bar{\alpha}(\bar{\beta}-\underline{\beta})}{(\bar{\alpha}-\underline{\alpha})(1-\underline{\beta})}, 1\right\}$$

[13] If the voter type did not give any information about β after observing the election result, there would not be any belief updating in this case, and therefore we would not need this assumption.

for any k.

First, consider the case where both candidates propose the same policy in the period 1 election. In this case, the vote share of candidate F is zero, regardless of the value of β. Hence, voters do not learn anything about the value of β, implying that social pressure remains high in period 2. Combining this observation with Lemma 3 establishes that candidate F is precluded from winning the period 2 election.

Second, consider the case where $(x_{M1}, x_{F1}) = (1, 0)$. Here, policy differentiation comes into play. Specifically, candidate F obtains votes from voters whose ideal policy on the standard policy issue is policy 0 and who share the extreme view on the controversial issue. Because the vote share depends on the fraction of voters who share the extreme view (i.e., $1-\beta$), voters can infer the value of β from the vote share. When candidate F's vote share exceeds a threshold, $(1-\underline{\alpha})(1-\bar{\beta})$, voters fully learn that $\beta = \underline{\beta}$. Because we assume that $c\underline{\beta} < 1-\theta$, part (ii) of Lemma 3 applies, implying that candidate F wins the period 2 election with probability $q(1-q)$. In contrast, when the share of votes of candidate F falls below the threshold, social stigma has the same effect on period 2 election, implying that candidate F never wins the period 2 election. (c) of the lemma specifies the probability that candidate F's vote share exceeds the threshold. Given $\beta = \underline{\beta}$, which occurs with probability $1-p$, F's vote share exceeds the threshold with probability 1 if any realization of α_1 results in this threshold to be overcome, and it is

$$\frac{(1-\underline{\alpha})(\bar{\beta}-\underline{\beta})}{(\bar{\alpha}-\underline{\alpha})(1-\underline{\beta})}$$

otherwise. Therefore, candidate F's probability of winning the period 2 election is

$$V(1,0) := (1-p)\min\left\{\frac{(1-\underline{\alpha})(\bar{\beta}-\underline{\beta})}{(\bar{\alpha}-\underline{\alpha})(1-\underline{\beta})}, 1\right\} q(1-q).$$

Third, consider the case where $(x_{M1}, x_{F1}) = (0, 1)$. In this case, candidate F obtains votes from voters whose ideal policy on the standard policy issue is policy 1 and who share the extreme view on the controversial issue. As in the above case, voters can infer the value of β from the vote share. When candidate F's vote share exceeds a threshold, $\bar{\alpha}(1-\bar{\beta})$, the voters understand perfectly that $\beta = \underline{\beta}$; thus, candidate F wins the period 2 election with probability $q(1-q)$. Conversely, when candidate F's vote share falls below the threshold, social stigma has the same effect on period 2 election, implying that candidate F never wins the period 2 election. (c) of the lemma specifies the probability that candidate F's vote share exceeds the threshold in a similar way to the case where $(x_{M1}, x_{F1}) = (1, 0)$. Therefore, candidate F's probability of winning the period 2 election is

$$V(0,1) := (1-p)\min\left\{\frac{\bar{\alpha}(\bar{\beta}-\underline{\beta})}{(\bar{\alpha}-\underline{\alpha})(1-\underline{\beta})}, 1\right\} q(1-q).$$

Equilibrium: Taken together, the payoff matrix for the election in period 1 is given by Table 2:

Table 2: Payoff Matrix in Period 1

M/F	1	0
1	$2, 0$	$2 - V(1,0), V(1,0)$
0	$2 - V(0,1), V(0,1)$	$2, 0$

In the period 1 election, candidate M always wins because of the high social pressure against voting for a candidate with an extreme view. Nonetheless, candidates have strategic incentives because policy choices influence social pressure in period 2 through voters' learning. As observed in Lemma 5, learning about the value of β occurs only when candidates adopt divergent positions on the standard policy issue. Without policy differentiation in period 1, no learning occurs, and high social pressure remains in period 2. This property yields the following incentives. On the one hand, candidate M wants to mimic the policy proposed by candidate F to maintain high social pressure and secure a guaranteed victory in the next election. On the other hand, candidate F seeks to propose a policy divergent from candidate M to potentially diminish social pressure and secure victory in the subsequent election.

It is crucial to note that these incentives, while reminiscent of those in the second-period election, differ significantly in their mechanisms. In the period 2 election, incentives for imitation and differentiation arise because candidates attempt to manipulate candidate M's electoral advantage in the current period. On the contrary, in the period 1 election, candidates do not have to manipulate the electoral advantage in the current period because candidate M always wins the election. However, candidates attempt to manipulate candidate M's electoral advantage in the next period due to the endogenous shift in social pressure triggered by policy choices in the current period. This dynamic consideration, accounting for an endogenous change in social pressure, generates incentives for both imitation and differentiation.

Due to the incentives of imitation and differentiation, there is no pure-strategy equilibrium. Instead, there is a unique mixed-strategy equilibrium, characterized as follows.[14] Note that σ_{k1} is the equilibrium probability of candidate k choosing policy 1 in period 1.

Proposition 1. *In period 1, the equilibrium is characterized by*

$$(\sigma_{M1}, \sigma_{F1}) = \left(\frac{V(0,1)}{V(1,0)+V(0,1)}, \frac{V(1,0)}{V(1,0)+V(0,1)} \right).$$

Furthermore, the equilibrium winning probability of candidate F in period 2 is

$$W_F := \frac{V(1,0)V(0,1)}{V(1,0)+V(0,1)}.$$

[14]While we believe that the mixed-strategy equilibrium is reasonable, one way to guarantee the existence of a pure-strategy equilibrium is considering a sequential timing where candidate M proposes a policy first as a Stackelberg leader; then, candidate F proposes a policy. In this setting, candidate M proposes policy 1 and candidate F proposes policy 0 in the equilibrium.

The fringe candidate has no prospect of winning the election in period 1 as in Lemma 4. Nonetheless, she may win the election in the next period because $W_F > 0$. This abrupt change in popularity is not rooted in a change in voters' underlying policy preferences; rather, it is endogenously propelled by candidates' strategic choices of policy platforms in the initial period. Although the empirical literature attributes the rise of radical right parties and candidates to changes in economic or social environments such as globalization and refugee crisis (Guriev and Papaioannou, 2022), our findings suggest that these shifts may not be exclusive drivers. Even in the absence of changes in underlying policy preferences, radical right parties and candidates may experience sudden popularity surges.

As a corollary of the above proposition, we obtain the following property on the equilibrium strategies in period 1:

Corollary 1. *The following holds:*

$$\sigma_{M1} \geq \frac{1}{2} \geq \sigma_{F1},$$

where the strict inequalities hold when

$$\frac{1-\underline{\alpha}}{1-\bar{\alpha}} > \frac{1-\underline{\beta}}{1-\bar{\beta}}.$$

Therefore, candidate M is likely to choose policy 1, the ex-ante popular policy, whereas candidate F is likely to choose policy 0, the ex-ante unpopular policy.

This property can be easily observed when $\min\{\cdot, 1\}$ in $V(0,1)$ and $V(1,0)$ do not take the value of one, i.e. when voters do not learn the value of β with certainty in the case where two candidates propose different policies and $\beta = \underline{\beta}$. The condition in Corollary 1 ensures this by making sure that the spread of $1-\alpha_1$ is sufficiently large relative to $1-\beta$. In such a case,

$$\sigma_{M1} = \frac{\bar{\alpha}}{1+\bar{\alpha}-\underline{\alpha}}; \ \sigma_{F1} = \frac{1-\underline{\alpha}}{1+\bar{\alpha}-\underline{\alpha}}.$$

Because $\bar{\alpha} + \underline{\alpha} > 1$ holds by assumption, $\sigma_{M1} > 1/2 > \sigma_{F1}$ is implied. Only when the condition in Corollary 1 is not satisfied, $\sigma_{M1} > 1/2 > \sigma_{F1}$ does not hold. In that case, the spread of $1-\beta$ is sufficiently large relative to $1-\alpha_1$, which implies that voters learn β with certainty when $\beta = \underline{\beta}$ in either case where $(x_{M1}, x_{F1}) = (1,0)$ or $(x_{M1}, x_{F1}) = (0,1)$. When this happens, $\sigma_{M1} = \sigma_{F1} = 1/2$.

The mechanism is understood as follows. Voters realize that the actual value of β is low only when candidate F's vote share is sufficiently large in the period 1 election. Hence, voter's learning on the value of β is more fostered so that the social pressure is more likely to be reduced when $(x_{M1}, x_{F1}) = (0,1)$ than when $(x_{M1}, x_{F1}) = (1,0)$. As a result, $V(0,1) \geq V(1,0)$. This learning effect indicates that candidate M wants to avoid $(x_{M1}, x_{F1}) = (0,1)$ more than $(x_{M1}, x_{F1}) = (1,0)$, resulting in a higher probability of choosing policy 1. Given that candidate M leans towards policy 1, candidate F prefers choosing policy 0 to obtain platform differentiation. Therefore, $\sigma_{M1} \geq 0.5 \geq \sigma_{F1}$. Note that strict inequalities hold as long as the condition in the corollary is met.

Effect of higher expected social norm: How does the expected social pressure influence the likelihood of a rise of candidate F? Before closing this section, we conduct comparative statics of candidate F's winning probability in period 2, W_F, with respect to changes in p and $\bar{\beta}$. In our model, the ex-ante expected value of β is given by $p\bar{\beta} + (1-p)\underline{\beta}$, which captures the expected social norm at the beginning of period 1. Notably, there are two distinctive determinants of the expected social norm: p and $\bar{\beta}$. While both seem to affect W_F in the same direction, this is not the case. The following corollary of Proposition 1 illustrates this point.

Corollary 2. *(i). W_F is decreasing in p.*

(ii). W_F is weakly increasing in $\bar{\beta}$.

As p, the probability that $\beta = \bar{\beta}$, increases, it becomes less likely that the social norm erodes in period 2. Therefore, candidate F is ex-ante less likely to win the period 2 election (see (i)). Notice also that the period 1 equilibrium strategies $(\sigma_{M1}, \sigma_{F1})$ are not affected by p since these strategies are decided conditional on $\beta = \underline{\beta}$.

However, the situation differs when $\bar{\beta}$ increases. In our model, the erosion of social norms can occur when voters learn that $\beta = \bar{\beta}$. Hence, voters' learning is a key determinant of a rise in the fringe candidate. As $\bar{\beta}$ increases, the difference between $\bar{\beta}$ and $\underline{\beta}$ expands; thus it becomes easier to identify the value of β. As a result of this enhanced learning, candidate F has a higher chance of winning the period 2 election. This is (ii) in the corollary.

Under the presumption that $p > 0.5$, a realistic scenario, a higher p reduces the uncertainty about the value of β, while a higher $\bar{\beta}$ magnifies it. Therefore, the corollary implies that the high social norm coupled with great uncertainty creates room for a rise of fringe candidates.

4 Extensions

In this section, we explore three extensions to our baseline model: the case where $\underline{\beta} < 0.5$, multiple mainstream candidates, and the correlation of α_t across periods. Finally, we provide a microfoundation of social stigma.

4.1 Equilibrium when $\underline{\beta} < 0.5$

Thus far, we have assumed that $\underline{\beta} > 0.5$; that is, only a minority of voters support $y = 0$, and thus, candidate F is always stigmatized. This assumption is plausible for contexts such as xenophobic attitudes, but it may not be valid in other scenarios. For example, in elections under competitive authoritarianism, where $y = 1$ represents the view that the current regime is desirable, $\underline{\beta} < 0.5$ could be plausible. In this subsection, we examine the case where $\underline{\beta} < 0.5$.

Setting: We now assume $\underline{\beta} < 0.5 < \bar{\beta}$. Contrary to the baseline model, voters may believe that the majority considers $y = 0$ desirable. Consequently, candidate M may face stigmatization instead of candidate F. Voter k's decision rule is as follows:
(i) when $\mathbb{E}_{kt}[\beta] > 0.5$, voter k votes for candidate M if and only if

$$-|x_{Mt} - \hat{x}_{kt}| - \theta|y_M - \hat{y}_k| \geq -|x_{Et} - \hat{x}_{kt}| - \theta|y_F - \hat{y}_k| - c\mathbb{E}_{kt}[\beta],$$

(ii) when $\mathbb{E}_{kt}[\beta] < 0.5$, voter k votes for candidate M if and only if

$$-|x_{Mt} - \hat{x}_{kt}| - \theta|y_M - \hat{y}_k| - c\mathbb{E}_{kt}[1-\beta] > -|x_{Et} - \hat{x}_{kt}| - \theta|y_F - \hat{y}_k|,$$

(iii) when $\mathbb{E}_{kt}[\beta] = 0.5$, voter k votes for candidate M if and only if

$$-|x_{Mt} - \hat{x}_{kt}| - \theta|y_M - \hat{y}_k| \geq -|x_{Et} - \hat{x}_{kt}| - \theta|y_F - \hat{y}_k|.$$

In the baseline model, $\mathbb{E}_{kt}[\beta] > 0.5$ always holds so we considered only (i).

To maintain meaningful scenarios, we introduce an additional assumption alongside Assumptions 1 and 2:

Assumption 3.

$$\frac{p(1-\bar{\beta})\bar{\beta} + (1-p)(1-\underline{\beta})\underline{\beta}}{p(1-\bar{\beta}) + (1-p)(1-\underline{\beta})} > \frac{1}{2} \text{ and } \underline{\alpha} > 1 - \frac{1}{2(1-\underline{\beta})}.$$

The first inequality guarantees that in the first period, $\mathbb{E}_{k1}[\beta] > 0.5$ holds for every voter i.e., candidate F is stigmatized in period 1. In addition, if $\underline{\beta}$ is sufficiently small, candidate F may win the period 1 election even if the first inequality is satisfied. To exclude such cases, we assume the second inequality.

Equilibrium in period 2: Based on this setting, we first obtain the characterization of equilibrium in period 2.

Lemma 6. *(i). Suppose that* $c\mathbb{E}_{k2}[\beta] > \max\{\theta, 1-\theta\}$ *for all* k*. Then, candidate* M *wins the election in period 2 with probability one.*

(ii). Suppose that $\mathbb{E}_{k2}[\beta] = \underline{\beta} < 0.5$ *for all* k*.*

(a). If $c(1-\underline{\beta}) < \max\{\theta, 1-\theta\}$*, in equilibrium,* $(\sigma_{M2}, \sigma_{F2}) = (1-q, q)$*. Furthermore, the winning probability of candidate* F *is* $1 - q(1-q)$*.*

(b). If $c(1-\underline{\beta}) > \max\{\theta, 1-\theta\}$*, candidate* F *wins the election in period 2 with probability one.*

The difference from the baseline model is the equilibrium strategy in (ii). In the baseline model, $\underline{\beta} > 0.5$ so that candidate F faces an electoral disadvantage even if $\beta = \underline{\beta}$ is revealed.

In contrast, candidate F faces an electoral advantage in the current setting. On the one hand, when $c(1-\underline{\beta}) < \max\{\theta, 1-\theta\}$, candidates' situations in the second period are just switched from the baseline model. Thus, $(\sigma_{M2}, \sigma_{F2}) = (1-q, q)$, candidate F is more likely to choose the ex-ante popular policy in period 2. On the other hand, when $c(1-\underline{\beta}) > \max\{\theta, 1-\theta\}$, candidate F's electoral advantage is sufficiently large so that she wins the election with probability one.

Equilibrium in period 1: When $c(1-\underline{\beta}) < \max\{\theta, 1-\theta\}$, we replace $q(1-q)$ in $V(1,0)$ and $V(0,1)$ by $[1-q(1-q)]$. Similarly, when $c(1-\underline{\beta}) > \max\{\theta, 1-\theta\}$, we replace them by one. Then, Proposition 1 continues to hold. In particular, it still holds that candidate M is more likely to choose the ex-ante popular policy in period 1.

4.2 Multiple Mainstream Candidates

Thus far, we have considered the model with a single mainstream candidate. In this subsection, we examine how the results change if there exist multiple mainstream candidates.

Setting: There are two mainstream candidates (M and M') and a fringe candidate (F). Each voter votes for a candidate yielding the highest utility. When a mainstream candidate and a fringe candidate are indifferent for voter i, the voter votes for the mainstream candidate. When both mainstream candidates are equally preferred, the voter votes for each mainstream candidate with equal probability. The candidate obtaining the largest number of votes wins the election (i.e., we assume the plurality rule). If multiple candidates obtain the largest number of votes, the winner is randomly determined among them. Each candidate's objective is the same as in our baseline model. Assumption 1 is maintained.

For simplicity, instead of a uniform distribution, we suppose that $\alpha_t \in \{\underline{\alpha}, \bar{\alpha}\}$, where $0 < \underline{\alpha} < 0.5 < \bar{\alpha} < 1$ and $\alpha_t = \bar{\alpha}$ with probability $q \in (0,1)$. Furthermore, we assume that $\alpha_t(1-\beta)$ takes different values for any (α_t, β) and $(1-\alpha_t)(1-\beta)$ takes different values for any (α_t, β). These assumptions ensure perfect learning of β from the period 1 electoral result, given $v_{F1} > 0$.[15] In addition, we assume the following:

Assumption 4.

$$\max\left\{(1-\underline{\alpha}), \bar{\alpha}\right\}(1-\underline{\beta}) < \frac{1}{3}, \ \min\left\{\bar{\alpha}, 1-\underline{\alpha}\right\} > \frac{2}{3}, \ \text{and} \ q > \frac{2}{3}.$$

Suppose that mainstream candidates propose the more popular policy (policy 1 if $\alpha = \bar{\alpha}$ and policy 0 if $\alpha = \underline{\alpha}$), whereas candidate F proposes the less popular policy. In the baseline

[15] In the original model, voters perfectly learn the value of β only when v_{1F} exceeds a threshold. If we change the continuous distribution to the binary distribution for α_t in the original model, the value of β is perfectly learned as long as $x_{M1} \neq x_{F1}$. Hence, $V(1,0) = V(0,1) = (1-p)q(1-q)$ holds so that Corollary 1 and 2 (i) change. Except for them, the same results hold even if we assume the binary distribution in the original model.

model, there is a single mainstream candidate; thus, the mainstream candidate should win the election in such cases. However, now, two mainstream candidates split the votes; thus, candidate F who proposes the unpopular policy may win the election even in such cases. The first two inequalities in the assumption guarantee that as in the baseline model, candidate F cannot win the election in such cases. The role of the last inequality, $q > 2/3$, will be discussed after the subsequent lemma.

Equilibrium in period 2: We start with analyzing the period 2 election.

Lemma 7. *In period 2, the following holds:*

(i). Suppose that $v_{F1} = 0$ or voters learn that $\beta = \bar{\beta}$ at the end of period 1. Then, $(x_{M2}, x_{M'2}, x_{F2}) = (1, 1, 1)$ and $(1, 1, 0)$ constitute an equilibrium. In either case, candidate M or M' wins the election in period 2.

(ii). Suppose that voters learn that $\beta = \underline{\beta}$ at the end of period 1. Then, $(x_{M2}, x_{M'2}, x_{F2}) = (1, 1, 0)$ in equilibrium. Candidate M wins the election in period 2 with probability $\frac{q}{2}$, candidate M' wins the election in period 2 with probability $\frac{q}{2}$, and candidate F wins the election in period 2 with probability $1 - q$.

Therefore, as in the baseline model, candidate F may win the period 2 election only when voters learn that $\beta = \underline{\beta}$.

A feature distinctive from the baseline model is that a pure-strategy equilibrium exists even after voters learn that $\beta = \underline{\beta}$ (see (ii)). In the baseline model, to exploit the electoral advantage stemming from the second dimension, y, candidate M attempts to imitate candidate F's policy platform. In response to this, candidate F tries to differentiate from candidate M, which rules out a pure-strategy equilibrium. In contrast, in the model with two mainstream candidates, candidate M faces competition with candidate M' as well as candidate F. If candidate M deviates from policy 1 to imitate candidate F, candidate M' obtains all votes from the supporters of policy 1. Hence, candidate M has no incentive to imitate candidate F under the last inequality of Assumption 4. This mechanism allows us to guarantee the existence of a pure-strategy equilibrium.

Equilibrium in period 1: Based on the above lemma, the equilibrium in period 1 is characterized as follows:

Proposition 2. $(x_{M1}, x_{M'1}, x_{F1}) = (1, 1, 0)$ *in equilibrium, and candidate F wins the election in period 2 with probability* $(1 - p)(1 - q)$.

When candidate F obtains some votes in period 1 (i.e., $v_{F1} > 0$), voters may learn that $\beta = \underline{\beta}$, creating an opportunity for candidate F to secure victory in period 2. To prevent this learning, at least one mainstream candidate should imitate candidate F's policy platform. If such imitation occurs, all voters cast their ballots for the mainstream candidates, preventing any learning.

While this mechanism is the same as in the baseline model, a notable feature of the current model is that mainstream candidates face an electoral cost when imitating candidate F's policy platform. Without considering the effect on the period 2 election, the equilibrium is the same as (i) in Lemma 7. To prevent candidate F from winning the period 2 election, candidate M has to choose policy 0 chosen by candidate F. However, if candidate M does so, candidate M' who chooses policy 1 obtains all the votes from supporters of policy 1, which reduces candidate M's winning probability in period 1. That is, as a result of competition between mainstream candidates, candidate M faces a trade-off between the current election and the future election. As $q > \frac{2}{3}$, the cost of opting for policy 0 in the current election outweighs the potential benefit in the next election. Consequently, mainstream candidates abstain from imitating candidate F, leading to the rise of candidate F in the subsequent period.

Combining this result with the baseline model yields an important implication for the rise of radical candidates. In the baseline model, there is a probability that mainstream candidates successfully imitate candidate F and prevent a shift in the social norm. In contrast, in the presence of multiple mainstream candidates, mainstream candidates do not imitate candidate F's policy platform. Consequently, they do not prevent a change in the social norm. That is, the existence of competition among mainstream candidates makes the social norm more likely to change, which increases the likelihood of a rise of radical candidates.

4.3 Correlation of α_t

In the baseline model, we assumed that α_1 and α_2 are independently drawn. In this subsection, we examine a more general case where α_1 and α_2 are perfectly correlated with probability $\lambda \in [0, 1)$, but they are independent with the remaining probability. Our baseline model can be regarded as a special case where $\lambda = 0$.

As we have shown, candidates' equilibrium strategy in period 2 after the erosion of the social norm depends on the probability of policy 1 being popular (i.e., $\Pr(\alpha_2 > 0.5)$) (see Lemma 3). While its value is q when $\lambda = 0$, this is no longer the case when $\lambda > 0$. Candidates learn the value of α_1 from the electoral result, impacting their belief about $\Pr(\alpha_2 > 0.5)$.

To highlight the main differences from the baseline model in a transparent way, we assume the following in addition to Assumptions 1 and 2:

Assumption 5.

$$2\min\{\bar{\alpha}, 1 - \underline{\alpha}\} \geq \frac{1 - \underline{\beta}}{1 - \bar{\beta}}.$$

Then, we obtain the following lemma.

Lemma 8. *(i). Suppose that* $c\mathbb{E}_{k2}[\beta] > \max\{\theta, 1 - \theta\}$ *for all* k*. Then, candidate* M *wins the election in period 2 with probability one.*

(ii). Suppose that $c\mathbb{E}_{k2}[\beta] = \underline{\beta} < 1 - \theta$ *for all* k.

(a). Suppose that $(x_{M1}, x_{F1}) = (1, 0)$ *and* $v_{F1} > (1 - \underline{\alpha})(1 - \bar{\beta})$. *Then, in equilibrium,* $(\sigma_{M2}, \sigma_{F2}) = (\underline{q}_2, 1 - \underline{q}_2)$, *where*

$$\underline{q}_2 =: (1 - \lambda)q.$$

Furthermore, the winning probability of candidate F *is* $\underline{q}_2(1 - \underline{q}_2)$.

(b). Suppose that $(x_{M1}, x_{F1}) = (0, 1)$ *and* $v_{F1} > \bar{\alpha}(1 - \bar{\beta})$. *Then, in equilibrium,* $(\sigma_{M2}, \sigma_{F2}) = (\bar{q}_2, 1 - \bar{q}_2)$, *where*

$$\bar{q}_2 := \lambda + (1 - \lambda)q.$$

Furthermore, the winning probability of candidate F *is* $\bar{q}_2(1 - \bar{q}_2)$.

If $(x_{M1}, x_{F1}) = (1, 0)$ and candidate F secures a substantial vote share, signifying the erosion of the social norm ((ii)-(a) in the lemma), then a large share of votes suggests that $1 - \alpha_1$ is substantial. Assumption 5 guarantees that candidates learn that $1 - \alpha_1 > 0.5$, indicating that policy 1 is popular in period 2 only when α_1 and α_2 are independently drawn. Consequently, we obtain $\underline{q}_2$ as candidates' posterior belief regarding $\alpha_2 > 0.5$. Similarly, when $(x_{M1}, x_{F1}) = (0, 1)$ and candidate F secures a significant vote share, candidates learn that $\alpha_1 > 0.5$. Thus, $\bar{q}_2$ represents candidates' posterior belief about $\alpha_2 > 0.5$ ((ii)-(b) in the lemma).

The correlation of α_t would influence results when candidates learn the value of α_1 precisely from the electoral result in period 1. Hence, we impose Assumption 5 ensuring precise learning, though it is not crucial.

Equilibrium in period 1: Having this lemma in hand, it is easy to observe that q in $V(1, 0)$ should be replaced by $\underline{q}_2$ and q in $V(0, 1)$ should be replaced by $\bar{q}_2$. After this modification, Proposition 1 continues to hold.

An interesting feature distinct from the baseline model is observed regarding Corollary 1. In the baseline model, $V(0, 1) \geq V(1, 0)$; candidate M prefers avoiding $(x_{M1}, x_{F1}) = (0, 1)$ more than $(1, 0)$, resulting in $\sigma_{M1} \geq \sigma_{F1}$. However, this mechanism can be reversed when α_1 and α_2 are correlated. As depicted in the lemma, $\bar{q}_2 > \underline{q}_2$. Hence, $V(0, 1) < V(1, 0)$ may hold. If so, we would have $\sigma_{M1} < \sigma_{F1}$.

The intuition is understood as follows. When $(x_{M1}, x_{F1}) = (1, 0)$, candidates think that policy 1 is highly likely to be popular in period 2. This reduction of uncertainty reduces the room for candidate F's winning in period 2. If this effect is dominant, candidate M may want to avoid $(x_{M1}, x_{F1}) = (1, 0)$ more than $(0, 1)$. Hence, $\sigma_{M1} < \sigma_{F1}$ may hold; that is, candidate F could be more likely to choose policy 1, the ex-ante popular policy, than candidate M.

4.4 Microfoundation of Social Stigma

In the main analysis, we have assumed that voting for candidate F in period t incurs the (expected) cost of social stigma, $c\mathbb{E}_{kt}[\beta]$. As Bénabou and Tirole (2006) propose, a major driver of stigma cost is reputational cost, which undermines one's social image. Let us consider a scenario where voter k's policy preference $(\hat{x}_{kt}, \hat{y}_k)$ is unobservable to other players. Given that the voter votes for candidate i in period t, his probability of having $\hat{y}_k = 1$ is updated based on the Bayes rule: $r_{kt}(i) \in [0, 1]$.[16] Then, the reputational payoff from voting for candidate i is given by

$$c\beta f(r_{kt}(i)),$$

where $c > 0$ represents the weight of the reputation and f is a weakly increasing function. As only a fraction β of voters internally consider that $y = 1$ is desirable, a higher $rkt(i)$ leads to a better image of voter k only among them. Therefore, we have β in the above setting. Combining this with the original setup implies that voter k votes for candidate M if and only if

$$\begin{aligned} & -|x_{Mt} - \hat{x}_{kt}| - \theta|y_M - \hat{y}_k| + c\mathbb{E}_{kt}[\beta] f(r_{kt}(M)) \\ \geq & -|x_{Et} - \hat{x}_{kt}| - \theta|y_F - \hat{y}_k| + c\mathbb{E}_{kt}[\beta] f(r_{kt}(F)). \end{aligned}$$

Under this setting, we impose two additional assumptions. First, when no one votes for candidate i, $r_{kt}(i)$ cannot be calculated based on the Bayes rule. For such off-path belief formations, we assume the following: when no one votes for candidate M (resp. F), $r_{kt}(M) = 1$ (resp. $r_{kt}(F) = 0$). This is reasonable as voters with $\hat{y}_k = 1$ share the same position on y as candidate M.

In addition, we adopt the following function as f:

$$f(r) = \begin{cases} 1 & \text{if } r \geq p\bar{\beta} + (1-p)\underline{\beta} \\ 0 & \text{otherwise} \end{cases}.$$

Because $p\bar{\beta} + (1-p)\underline{\beta}$ is the prior probability of one having $\hat{y}_k = 0$, this formulation implies that the social image is one if the reputation is upwardly updated, but zero if it is updated downwards.

Under these assumptions, $r_{kt}(M) \geq p\bar{\beta} + (1-p)\underline{\beta} > r_{kt}(F)$ holds in any case. Therefore, voter k votes for candidate M if and only if

$$-|x_{Mt} - \hat{x}_{kt}| - \theta|y_M - \hat{y}_k| + c\mathbb{E}_{kt}[\beta] \geq -|x_{Et} - \hat{x}_{kt}| - \theta|y_F - \hat{y}_k|,$$

which is equivalent to the original setting. Thus, the stigma cost of our model can be

[16] See footnote 2 for empirical evidence that voters have a concern that voting may not be secret. We also assume that the voter's voting choice in period 1 is unobservable in period 2 due to limited memory.

interpreted as the reputational cost.

5 Concluding Remarks

Fringe candidates often adopt extreme positions that defy the prevailing social norms, subjecting them to political stigmatization in society. Consequently, individuals with extreme views may conceal their support for fringe candidates to align with established social norms. This study aims to analyze how fringe candidates gain widespread support by examining the reciprocal interactions between social norms and electoral outcomes.

For this purpose, we developed a two-period electoral competition model consisting of a mainstream candidate, a fringe candidate, and a continuum of voters. The policy space is two-dimensional. In the first dimension, each candidate chooses her policy platform. In contrast, in the second dimension, each candidate has a fixed position: the mainstream candidate's position is socially acceptable, whereas the fringe candidate's position is not. Voting for the fringe candidate incurs stigma cost, the size of which depends on the expected number of voters who share the socially acceptable view in society.

Our analysis yielded three key findings. Firstly, the erosion of the social norm and the potential victory of the fringe candidate in period 2 are contingent upon the fringe candidate's vote share surpassing a threshold in period 1. Consequently, electoral outcomes in period 1 exert an influence on period 2 through the degradation of the social norm. Secondly, the erosion of the social norm occurs only when policy platforms differ between the candidates in the first dimension during period 1. This prompts the mainstream candidate to imitate the fringe candidate, whereas the fringe candidate seeks to differentiate herself from the opponent. This results in a unique mixed-strategy equilibrium, despite the mainstream candidate consistently winning the period 1 election. Thirdly, a higher expected social norm in period 1 may enhance the likelihood of the fringe candidate's rise. These results underscore the importance of understanding the two-way interactions between social norms and electoral results in the analysis of fringe candidates.

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A Omitted Proofs

A.1 Proof of Lemma 1

(i). Voters with $\hat{y}_k = 1$ vote for candidate M if and only if

$$0 \geq -\theta - c\mathbb{E}_{kt}[\beta],$$

which always holds.

Similarly, voters with $\hat{y}_k = 0$ vote for candidate M if and only if

$$0 - \theta \geq -c\mathbb{E}_{kt}[\beta] \Leftrightarrow c\mathbb{E}_{kt}[\beta] \geq \theta.$$

which always holds.

(ii). Voters with $\hat{x}_{kt} = x_{Mt}$ vote for candidate M if and only if

$$-\theta|y_M - \hat{y}_k| \geq -1 - \theta|y_F - \hat{y}_k| - c\mathbb{E}_{kt}[\beta],$$

which always holds because $\theta < 1$.

Voters with $(\hat{x}_{kt}, \hat{y}_k) = (x_{Ft}, 1)$ vote for candidate M if and only if

$$-1 \geq -\theta - c\mathbb{E}_{kt}[\beta] \Leftrightarrow c\mathbb{E}_{kt}[\beta] \geq 1 - \theta.$$

Voters with $(\hat{x}_{kt}, \hat{y}_k) = (x_{Ft}, 0)$ vote for candidate M if and only if

$$-1 - \theta \geq -c\mathbb{E}_{kt}[\beta] \Leftrightarrow c\mathbb{E}_{kt}[\beta] \geq 1 + \theta.$$

□

A.2 Proof of Lemma 2

(i). Assume that $c\mathbb{E}_{k2}[\beta] > \max\{\theta, 1-\theta\}$.

Case 1: $x_{M2} = x_{F2}$. From Lemma 1, voters with $\hat{y}_k = 1$ vote for candidate M. Since a majority of voters have $\hat{y}_k = 1$, $v_{F2} < 0.5$.

Case 2: $(x_{M2}, x_{F2}) = (1, 0)$. From Lemma 1, voters with $(\hat{x}_{k2}, \hat{y}_k) = (1, 1), (1, 0)$ or $(0, 1)$ vote for candidate M. Hence, $v_{F2} < 0.5$.

Case 3: $(x_{M2}, x_{F2}) = (0, 1)$. From Lemma 1, voters with $(\hat{x}_{k2}, \hat{y}_k) = (0, 1), (0, 0)$ or $(1, 1)$ vote for candidate M. Hence, $v_{F2} < 0.5$.

(ii). Assume that $c\mathbb{E}_{k2}[\beta] < 1 - \theta$.

Case 1: $x_{M2} = x_{F2}$. From Lemma 1, voters with $\hat{y}_k = 1$ vote for candidate M. Since a majority of voters have $\hat{y}_k = 1$, $v_{F2} < 0.5$.

Case 2: $(x_{M2}, x_{F2}) = (1, 0)$. From Lemma 1, voters with $\hat{x}_{kt} = 1$ (resp. $\hat{x}_{kt} = 0$) vote for candidate M (resp. F). Hence, $v_{F2} = 1 - \alpha_2$.

Case 3: $(x_{M2}, x_{F2}) = (0, 1)$. From Lemma 1, voters with $\hat{x}_{kt} = 0$ (resp. $\hat{x}_{kt} = 1$) vote for candidate M (resp. F). Hence, $v_{F2} = \alpha_2$.

□

A.3 Proof of Lemma 4

Before examining each case, we observe that

$$\mathbb{E}_{k1}[\beta|\hat{y}_k = 0] = \frac{p(1-\bar{\beta})\bar{\beta} + (1-p)(1-\underline{\beta})\underline{\beta}}{p(1-\bar{\beta}) + (1-p)(1-\underline{\beta})} < \mathbb{E}_{k1}[\beta|\hat{y}_k = 1] = \frac{p\bar{\beta}^2 + (1-p)\underline{\beta}^2}{p\bar{\beta} + (1-p)\underline{\beta}}.$$

Combining this with Assumption 1 implies that

$$1 + \theta > c\mathbb{E}_{k1}[\beta|\hat{y}_k = 0] > \max\{\theta, 1-\theta\};$$

$$c\mathbb{E}_{k1}[\beta|\hat{y}_k = 1] > \max\{\theta, 1-\theta\}.$$

Case 1. $x_{M1} = x_{F1}$. From Lemma 1, no one votes for candidate F; that is, $v_{F1} = 0$ independently of the value of β.

Case 2. $(x_{M1}, x_{F1}) = (1, 0)$. From Lemma 1, voters with $(\hat{x}_{k2}, \hat{y}_k) = (1,1), (1,0)$ or $(0,1)$ vote for candidate M, but voters with $(\hat{x}_{k2}, \hat{y}_k) = (0,0)$ vote for candidate F. Hence, $v_{F1} = (1-\alpha_1)(1-\beta)$.

Case 3. $(x_{M1}, x_{F1}) = (0, 1)$. From Lemma 1, voters with $(\hat{x}_{k2}, \hat{y}_k) = (0,0), (0,1)$ or $(1,1)$ vote for candidate M, but voters with $(\hat{x}_{k2}, \hat{y}_k) = (1,0)$ vote for candidate F. Hence, $v_{F1} = \alpha_1(1-\beta)$.

$\square$

A.4 Proof of Lemma 5

(i). $x_{M1} = x_{F1}$. $v_{F1} = 0$ independently of the value of β. Hence, there is no learning on β i.e., $\mathbb{E}_{k1}[\beta] = \mathbb{E}_{k2}[\beta]$. Combining this with Assumption 1 implies that $\mathbb{E}_{k2}[\beta] > \max\{\theta, 1-\theta\}$.

(ii). $(x_{M1}, x_{F1}) = (1, 0)$

In this case, $v_{F1} = (1-\alpha)(1-\beta)$. When $\beta = \bar{\beta}$, $v_{F1} \sim U[(1-\bar{\alpha})(1-\bar{\beta}), (1-\underline{\alpha})(1-\bar{\beta})]$. When $\beta = \underline{\beta}$, $v_{F1} \sim U[(1-\bar{\alpha})(1-\underline{\beta}), (1-\underline{\alpha})(1-\underline{\beta})]$.

- Suppose that $v_{F1} \in [(1-\bar{\alpha})(1-\bar{\beta}), (1-\bar{\alpha})(1-\underline{\beta})]$. The lower bound of v_{F1} is $(1-\bar{\alpha})(1-\underline{\beta})$ given $\beta = \underline{\beta}$. Thus, when v_{F1} is lower than this value, it implies that $\beta = \bar{\beta}$. Hence, $\mathbb{E}_{k2}[\beta] = \bar{\beta} > \mathbb{E}_{k1}[\beta|\hat{y}_k = 0]$. Combining this with Assumption 1 implies that $\mathbb{E}_{k2}[\beta] > \max\{\theta, 1-\theta\}$.
- Suppose that $v_{F1} \in [(1-\underline{\alpha})(1-\bar{\beta}), (1-\underline{\alpha})(1-\underline{\beta})]$. The upper bound of v_{F1} is $(1-\underline{\alpha})(1-\bar{\beta})$ given $\beta = \bar{\beta}$. Thus, when v_{F1} is greater than this value, it implies that $\beta = \underline{\beta}$. Hence, $\mathbb{E}_{k2}[\beta] = \underline{\beta}$.

- Suppose that $v_{F1} \in [(1-\bar{\alpha})(1-\underline{\beta}), (1-\underline{\alpha})(1-\bar{\beta})]$. We prove that

$$\mathbb{E}_{k2}[\beta | v_{F1} = v, \hat{x}_{k1}, \hat{y}_k] > \max\{\theta, 1-\theta\}$$

holds under Assumption 2. In particular, since $\mathbb{E}_{k2}[\beta | v_{F1} = v, \hat{x}_{k1}, \hat{y}_k = 0] \leq \mathbb{E}_{k2}[\beta | v_{F1} = v, \hat{x}_{k1}, \hat{y}_k = 1]$ holds, it suffices to focus on the case where $\hat{y}_k = 0$. Let $p_{k2}(\beta | v_{F1}, \hat{x}_{k1}, \hat{y}_k)$ be the probability of β being $\bar{\beta}$ conditional on $(\beta | v_{F1}, \hat{x}_{k1}, \hat{y}_k)$. From Bayes rule, $p_{k2}(\beta | v_{F1}, \hat{x}_{k1}, 1)$ is given by

$$\frac{p\Pr(v_{F1}, \hat{x}_{k1}, \hat{y}_k = 0 | \beta = \bar{\beta})}{p\Pr(v_{F1}, \hat{x}_{k1}, \hat{y}_k = 0 | \beta = \bar{\beta}) + (1-p)\Pr(v_{F1}, \hat{x}_{k1}, \hat{y}_k = 0 | \beta = \underline{\beta})}.$$

Note that

$$\Pr(v_{F1}, \hat{x}_{k1}, \hat{y}_k = 0 | \beta) = \Pr(v_{F1} | \beta) \cdot \Pr(\hat{x}_{k1} | \beta, v_{F1}) \cdot \Pr(\hat{y}_k = 0 | \beta),$$

where

$$\Pr(v_{F1} | \beta) = \frac{1}{(\bar{\alpha} - \underline{\alpha})(1-\beta)}; \ \Pr(\hat{x}_{k1} = 1 | \beta, v_{F1}) = 1 - \frac{v_{F1}}{1-\beta}; \ \Pr(\hat{y}_k = 0 | \beta) = 1 - \beta.$$

We first consider the case where $\hat{x}_{k1} = 1$. In this case,

$$\begin{aligned}
\mathbb{E}_{k2}[\beta] &= \frac{\frac{1-\bar{\beta}-v_{F1}}{(1-\bar{\beta})^2} p(1-\bar{\beta})\bar{\beta} + \frac{1-\underline{\beta}-v_{F1}}{(1-\underline{\beta})^2}(1-p)(1-\underline{\beta})\underline{\beta}}{\frac{1-\bar{\beta}-v_{F1}}{(1-\bar{\beta})^2} p(1-\bar{\beta}) + \frac{1-\underline{\beta}-v_{F1}}{(1-\underline{\beta})^2}(1-p)(1-\underline{\beta})} \\
&= \frac{\frac{1-\bar{\beta}-v_{F1}}{1-\underline{\beta}-v_{F1}} \frac{(1-\underline{\beta})^2}{(1-\bar{\beta})^2} p(1-\bar{\beta})\bar{\beta} + (1-p)(1-\underline{\beta})\underline{\beta}}{\frac{1-\bar{\beta}-v_{F1}}{1-\underline{\beta}-v_{F1}} \frac{(1-\underline{\beta})^2}{(1-\bar{\beta})^2} p(1-\bar{\beta}) + (1-p)(1-\underline{\beta}).}
\end{aligned}$$

Because this is decreasing in v_{F1}, it suffices to prove that it is larger than $\max\{\theta, 1-\theta\}$ for $v_{F1} = (1-\underline{\alpha})(1-\bar{\beta})$, which is equivalent to

$$\frac{1-\bar{\beta}-(1-\underline{\alpha})(1-\bar{\beta})}{1-\underline{\beta}-(1-\underline{\alpha})(1-\bar{\beta})} \frac{(1-\underline{\beta})^2}{(1-\bar{\beta})^2} > \underline{\gamma}.$$

This is rewritten as Assumption 2. Therefore, Assumption 2 ensures that

$$\mathbb{E}_{k2}[\beta | v_{F1} = v, \hat{x}_{k1} = 1, \hat{y}_k = 1] > \max\{\theta, 1-\theta\}.$$

We next consider the case where $\hat{x}_{k1} = 0$. In this case,

$$\begin{aligned}\mathbb{E}_{k2}[\beta] =& \frac{\frac{v_{F1}}{(1-\bar{\beta})^2} p(1-\bar{\beta})\bar{\beta} + \frac{v_{F1}}{(1-\underline{\beta})^2}(1-p)(1-\underline{\beta})\underline{\beta}}{\frac{v_{F1}}{(1-\bar{\beta})^2} p(1-\bar{\beta}) + \frac{v_{F1}}{(1-\underline{\beta})^2}(1-p)(1-\underline{\beta})} \\ =& \frac{\frac{(1-\underline{\beta})^2}{(1-\bar{\beta})^2} p(1-\bar{\beta})\bar{\beta} + (1-p)(1-\underline{\beta})\underline{\beta}}{\frac{(1-\underline{\beta})^2}{(1-\bar{\beta})^2} p(1-\bar{\beta}) + (1-p)(1-\underline{\beta})}.\end{aligned}$$

Because

$$\frac{(1-\underline{\beta})^2}{(1-\bar{\beta})^2} > 1 > \underline{\gamma},$$

this implies that

$$\mathbb{E}_{k2}[\beta | v_{F1} = v, \hat{x}_{k1} = 0, \hat{y}_k = 1] > \max\{\theta, 1-\theta\}.$$

Therefore, we conclude that

$$\mathbb{E}_{k2}[\beta | v_{F1} = v, \hat{x}_{k1}, \hat{y}_k] > \max\{\theta, 1-\theta\}.$$

The above analysis together proves (a) and (b). (c) is easily obtained by calculating the probability that $v_{F1} > (1-\underline{\alpha})(1-\bar{\beta})$.

(iii). $(x_{M1}, x_{F1}) = (0, 1)$

In this case, $v_{F1} = \alpha(1-\beta)$. When $\beta = \bar{\beta}$, $v_{F1} \sim U[\underline{\alpha}(1-\bar{\beta}), \bar{\alpha}(1-\bar{\beta})]$. When $\beta = \underline{\beta}$, $v_{F1} \sim U[\underline{\alpha}(1-\underline{\beta}), \bar{\alpha}(1-\underline{\beta})]$.

- Suppose that $v_{F1} \in [\underline{\alpha}(1-\bar{\beta}), \underline{\alpha}(1-\underline{\beta})]$. This implies that $\beta = \bar{\beta}$. Hence, $\mathbb{E}_{k2}[\beta] = \bar{\beta} > \mathbb{E}_{k1}[\beta | \hat{y}_k = 0]$. Combining this with Assumption 1 implies that $\mathbb{E}_{k2}[\beta] > \max\{\theta, 1-\theta\}$.
- Suppose that $v_{F1} \in [\bar{\alpha}(1-\bar{\beta}), \bar{\alpha}(1-\underline{\beta})]$. The upper bound of v_{F1} is $\bar{\alpha}(1-\bar{\beta})$ given $\beta = \bar{\beta}$. Thus, when v_{F1} is greater than this value, it implies that $\beta = \underline{\beta}$. Hence, $\mathbb{E}_{k2}[\beta] = \underline{\beta}$.
- Suppose that $v_{F1} \in [\underline{\alpha}(1-\underline{\beta}), \bar{\alpha}(1-\bar{\beta})]$.

  We prove that

  $$\mathbb{E}_{k2}[\beta | v_{F1} = v, \hat{x}_{k1}, \hat{y}_k] > \max\{\theta, 1-\theta\}$$

  holds under Assumption 2. In particular, since $\mathbb{E}_{k2}[\beta | v_{F1} = v, \hat{x}_{k1}, \hat{y}_k = 0] \leq \mathbb{E}_{k2}[\beta | v_{F1} = v, \hat{x}_{k1}, \hat{y}_k = 1]$ holds, it suffices to focus on the case where $\hat{y}_k = 0$.

  Let $p_{k2}(\beta | v_{F1}, \hat{x}_{k1}, \hat{y}_k)$ be the probability of β being $\bar{\beta}$ conditional on $(\beta | v_{F1}, \hat{x}_{k1}, \hat{y}_k)$.

From Bayes rule, $p_{k2}(\beta|v_{F1}, \hat{x}_{k1}, 1)$ is given by

$$\frac{p\Pr(v_{F1}, \hat{x}_{k1}, \hat{y}_k = 0|\beta = \bar{\beta})}{p\Pr(v_{F1}, \hat{x}_{k1}, \hat{y}_k = 0|\beta = \bar{\beta}) + (1-p)\Pr(v_{F1}, \hat{x}_{k1}, \hat{y}_k = 0|\beta = \underline{\beta})}.$$

Note that

$$\Pr(v_{F1}, \hat{x}_{k1}, \hat{y}_k = 0|\beta) = \Pr(v_{F1}|\beta) \cdot \Pr(\hat{x}_{k1}|\beta, v_{F1}) \cdot \Pr(\hat{y}_k = 0|\beta),$$

where

$$\Pr(v_{F1}|\beta) = \frac{1}{(\bar{\alpha} - \underline{\alpha})(1-\beta)}; \ \Pr(\hat{x}_{k1} = 1|\beta, v_{F1}) = \frac{v_{F1}}{1-\beta}; \ \Pr(\hat{y}_k = 0|\beta) = 1 - \beta.$$

We first consider the case where $\hat{x}_{k1} = 0$. In this case,

$$\begin{aligned}
\mathbb{E}_{k2}[\beta] =& \frac{\frac{1-\bar{\beta}-v_{F1}}{(1-\bar{\beta})^2} p(1-\bar{\beta})\bar{\beta} + \frac{1-\underline{\beta}-v_{F1}}{(1-\underline{\beta})^2}(1-p)(1-\underline{\beta})\underline{\beta}}{\frac{1-\bar{\beta}-v_{F1}}{(1-\bar{\beta})^2} p(1-\bar{\beta}) + \frac{1-\underline{\beta}-v_{F1}}{(1-\underline{\beta})^2}(1-p)(1-\underline{\beta})} \\
=& \frac{\frac{1-\bar{\beta}-v_{F1}}{1-\underline{\beta}-v_{F1}} \frac{(1-\underline{\beta})^2}{(1-\bar{\beta})^2} p(1-\bar{\beta})\bar{\beta} + (1-p)(1-\underline{\beta})\underline{\beta}}{\frac{1-\bar{\beta}-v_{F1}}{1-\underline{\beta}-v_{F1}} \frac{(1-\underline{\beta})^2}{(1-\bar{\beta})^2} p(1-\bar{\beta}) + (1-p)(1-\underline{\beta})}.
\end{aligned}$$

Because this is decreasing in v_{F1}, it suffices to prove that it is larger than $\max\{\theta, 1-\theta\}$ for $v_{F1} = (1-\underline{\alpha})(1-\bar{\beta})$, which is equivalent to

$$\frac{1-\bar{\beta}-(1-\underline{\alpha})(1-\bar{\beta})}{1-\underline{\beta}-(1-\underline{\alpha})(1-\bar{\beta})} \frac{(1-\underline{\beta})^2}{(1-\bar{\beta})^2} > \underline{\gamma}.$$

This is rewritten as Assumption 2. Therefore, Assumption 2 ensures that

$$\mathbb{E}_{k2}[\beta|v_{F1} = v, \hat{x}_{k1} = 1, \hat{y}_k = 1] > \max\{\theta, 1-\theta\}.$$

We next consider the case where $\hat{x}_{k1} = 1$. In this case,

$$\begin{aligned}
\mathbb{E}_{k2}[\beta] =& \frac{\frac{v_{F1}}{(1-\bar{\beta})^2} p(1-\bar{\beta})\bar{\beta} + \frac{v_{F1}}{(1-\underline{\beta})^2}(1-p)(1-\underline{\beta})\underline{\beta}}{\frac{v_{F1}}{(1-\bar{\beta})^2} p(1-\bar{\beta}) + \frac{v_{F1}}{(1-\underline{\beta})^2}(1-p)(1-\underline{\beta})} \\
=& \frac{\frac{(1-\underline{\beta})^2}{(1-\bar{\beta})^2} p(1-\bar{\beta})\bar{\beta} + (1-p)(1-\underline{\beta})\underline{\beta}}{\frac{(1-\underline{\beta})^2}{(1-\bar{\beta})^2} p(1-\bar{\beta}) + (1-p)(1-\underline{\beta})}.
\end{aligned}$$

Because

$$\frac{(1-\underline{\beta})^2}{(1-\bar{\beta})^2} > 1 > \underline{\gamma},$$

this implies that

$$\mathbb{E}_{k2}[\beta | v_{F1} = v, \hat{x}_{k1} = 0, \hat{y}_k = 1] > \max\{\theta, 1-\theta\}.$$

Therefore, we conclude that

$$\mathbb{E}_{k2}[\beta | v_{F1} = v, \hat{x}_{k1}, \hat{y}_k] > \max\{\theta, 1-\theta\}.$$

The above analysis together proves (a) and (b). (c) is easily obtained by calculating the probability that $v_{F1} > \bar{\alpha}(1-\bar{\beta})$. □

A.5 Proof of Proposition 1

It is straightforward that there is no pure-strategy equilibrium. Below, we characterize the mixed-strategy equilibrium. If candidate F chooses policy 1, its expected payoff is $(1-\sigma_{M1})V(0,1)$. If candidate F chooses policy 0, its expected payoff is $\sigma_{M1}V(1,0)$. For candidate F to be indifferent, it must be that $\sigma_{M1} = \frac{V(0,1)}{V(1,0)+V(0,1)}$. The equilibrium value of σ_{F1} is found similarly.

Candidate F wins the period 2 election with probability $V(0,1)$ when $(x_{M1}, x_{F1}) = (0,1)$, and with probability $V(1,0)$ when $(x_{M1}, x_{F1}) = (1,0)$. Therefore, the overall probability is

$$(1-\sigma_{M1})\sigma_{F1}V(0,1) + \sigma_{M1}(1-\sigma_{F1})V(1,0) = \frac{V(1,0)V(0,1)}{V(1,0)+V(0,1)}.$$

This is equal to

$$\frac{(1-p)(\bar{\beta}-\underline{\beta})\bar{\alpha}(1-\underline{\alpha})q(1-q)}{(\bar{\alpha}-\underline{\alpha})(1-\underline{\beta})(\bar{\alpha}+1-\underline{\alpha})}.$$

□

A.6 Proof of Corollary 1

First, by the construction of $(\sigma_{M1}, \sigma_{F1})$,

$$\sigma_{M1} \geq \frac{1}{2} \geq \sigma_{F1} \Leftrightarrow V(0,1) \geq V(1,0).$$

Here, $V(0,1) \geq V(1,0)$ holds because

$$V(0,1) \geq V(1,0) \Leftrightarrow \frac{\bar{\alpha}(\bar{\beta}-\underline{\beta})}{(\bar{\alpha}-\underline{\alpha})(1-\underline{\beta})} \geq \frac{(1-\underline{\alpha})(\bar{\beta}-\underline{\beta})}{(\bar{\alpha}-\underline{\alpha})(1-\underline{\beta})} \Leftrightarrow \bar{\alpha} \geq 1-\underline{\alpha}$$

which holds by the assumption that $\bar{\alpha} + \underline{\alpha} > 1$.

Furthermore, $V(1,0) = V(0,1)$ holds if and only if

$$\frac{(1-\underline{\alpha})(\bar{\beta}-\underline{\beta})}{(\bar{\alpha}-\underline{\alpha})(1-\underline{\beta})} > 1.$$

This does not hold if and only if

$$\frac{1-\underline{\alpha}}{1-\bar{\alpha}} > \frac{1-\underline{\beta}}{1-\bar{\beta}}.$$

□

A.7 Proof of Lemma 6

(i). As before, the vote share of candidate F, v_{F2}, is given as follows:

$$v_{F2} = \begin{cases} 0 & \text{if } x_{M2} = x_{F2} \\ (1-\alpha)(1-\beta) & \text{if } (x_{M2}, x_{F2}) = (1,0) \\ \alpha(1-\beta) & \text{if } (x_{M2}, x_{F2}) = (0,1) \end{cases}.$$

Since $\underline{\alpha} > 1 - 1/(2(1-\underline{\beta}))$ by assumption, M wins with probability one when $(x_{M2}, x_{F2}) = (1,0)$. Therefore, M can guarantee winning by choosing $x_{M2} = 1$.

(ii). (a). Suppose that $c(1-\underline{\beta}) < \max\{\theta, 1-\theta\}$. As before, the vote share of candidate F, v_{F2}, is given as follows:

$$v_{F2} = \begin{cases} 1 & \text{if } x_{M2} = x_{F2} \\ 1-\alpha_2 & \text{if } (x_{M2}, x_{F2}) = (1,0) \\ \alpha_2 & \text{if } (x_{M2}, x_{F2}) = (0,1) \end{cases}.$$

This implies the switch of the roles between M and F with respect to the case where $\underline{\beta} > 1/2$. Therefore, the result follows.

(b). Suppose that $c(1-\underline{\beta}) > \max\{\theta, 1-\theta\}$. As before, the vote share of candidate F, v_{F2}, is given as follows:

$$v_{F2} = \begin{cases} 1 & \text{if } x_{M2} = x_{F2} \\ 1-\alpha_2\underline{\beta} & \text{if } (x_{M2}, x_{F2}) = (1,0) \\ 1-(1-\alpha_2)\underline{\beta} & \text{if } (x_{M2}, x_{F2}) = (0,1) \end{cases}.$$

Therefore, candidate F wins the election for any (x_{M2}, x_{F2}). □

A.8 Proof of Lemma 7

(i). In this case, $c\mathbb{E}_{k2}[\beta] > \max\{\theta, 1-\theta\}$ holds. Hence, voter k does not vote for candidate F if and only if $\hat{x}_{k2} \in \{x_{M2}, x_{M'2}\}$, $\hat{y}_k = 1$, or $x_{F2} \in \{x_{M2}, x_{M'2}\}$. Having this in hand, we derive each candidate's winning probability for each triple $(x_{M2}, x_{M'2}, x_{F2})$. Let P_k be candidate k's winning probability.

(a). $x_{M2} = x_{M'2} = x_{F2}$. In this case, candidates M and M' split the voters. Hence, $(P_M, P_{M'}, P_F) = (1/2, 1/2, 0)$.

(b). $(x_{M2}, x_{M'2}, x_{F2}) = (1, 1, 0)$. In this case, voters with $(\hat{x}_{k2}, \hat{y}) = (0, 0)$ vote for candidate F. Other voters' votes are split by candidates M and M'. Hence, $v_{F2} = (1-\alpha_2)(1-\beta)$, whereas $v_{M_2} = v_{M'2} = (1-v_{F2})/2$. Here, from Assumption 4, $v_{F2} < 1/3$. Hence, $(P_M, P_{M'}, P_F) = (1/2, 1/2, 0)$.

(c). $(x_{M2}, x_{M'2}, x_{F2}) = (0, 0, 1)$. In this case, voters with $(\hat{x}_{k2}, \hat{y}) = (1, 0)$ vote for candidate F. Other voters' votes are split by candidates M and M'. Hence, $v_{F2} = \alpha_2(1-\beta)$, whereas $v_{M_2} = v_{M'2} = (1-v_{F2})/2$. Here, from Assumption 4, $v_{F2} < 1/3$. Hence, $(P_M, P_{M'}, P_F) = (1/2, 1/2, 0)$.

(d). $(x_{M2}, x_{M'2}, x_{F2}) = (1, 0, 1)$. In this case, no one votes for candidate F. Every voter with $\hat{x}_{k2} = 1$ (resp. 0) votes for candidate M (resp. M'). Hence, $v_{M2} = \alpha_2$ and $v_{M'2} = 1-\alpha_2$, implying that $(P_M, P_{M'}, P_F) = (q, 1-q, 0)$.

(e). $(x_{M2}, x_{M'2}, x_{F2}) = (1, 0, 0)$. As in (d), $(P_M, P_{M'}, P_F) = (q, 1-q, 0)$.

(f). $(x_{M2}, x_{M'2}, x_{F2}) = (0, 1, 0)$. As in (d), $(P_M, P_{M'}, P_F) = (1-q, q, 0)$.

(g). $(x_{M2}, x_{M'2}, x_{F2}) = (0, 1, 1)$. As in (d), $(P_M, P_{M'}, P_F) = (1-q, q, 0)$.

Based on these payoffs, we can easily verify that $(1, 1, 0)$ and $(1, 1, 1)$ constitute an equilibrium. Furthermore, $(P_M, P_{M'}, P_F) = (1/2, 1/2, 0)$.

(ii). In this case, $c\mathbb{E}_{k2}[\beta] < 1-\theta$ holds. Hence, voter k does not vote for candidate F if and only if $\hat{x}_{k2} \in \{x_{M2}, x_{M'2}\}$ or $x_{F2} \in \{x_{M2}, x_{M'2}\}$. Having this in hand, we derive each candidate's winning probability for each triple $(x_{M2}, x_{M'2}, x_{F2})$.

(a). $x_{M2} = x_{M'2} = x_{F2}$. In this case, candidates M and M' split the voters. Hence, $(P_M, P_{M'}, P_F) = (1/2, 1/2, 0)$.

(b). $(x_{M2}, x_{M'2}, x_{F2}) = (1, 1, 0)$. In this case, voters with $\hat{x}_{k2}$ vote for candidate F. Other voters' votes are split by candidates M and M'. Hence, $v_{F2} = 1-\alpha_2$, whereas $v_{M_2} = v_{M'2} = (1-v_{F2})/2$. Here, from Assumption 4, $v_{F2} < 1/3$ if and only if $\alpha_2 = \bar{\alpha}$. Hence, $(P_M, P_{M'}, P_F) = (q/2, q/2, 1-q)$.

(c). $(x_{M2}, x_{M'2}, x_{F2}) = (0, 0, 1)$. In this case, voters with $\hat{x}_{k2} = 1$ vote for candidate F. Other voters' votes are split by candidates M and M'. Hence, $v_{F2} = \alpha_2$, whereas $v_{M_2} = v_{M'2} = (1-v_{F2})/2$. Here, from Assumption 4, $v_{F2} < 1/3$ if and only if $\alpha_2 = \underline{\alpha}$. Hence, $(P_M, P_{M'}, P_F) = ((1-q)/2, (1-q)/2, q)$.

(d). $(x_{M2}, x_{M'2}, x_{F2}) = (1, 0, 1)$. In this case, no one votes for candidate F. As in (i)-(d), $(P_M, P_{M'}, P_F) = (q, 1-q, 0)$.

(e). $(x_{M2}, x_{M'2}, x_{F2}) = (1, 0, 0)$. As in (d), $(P_M, P_{M'}, P_F) = (q, 1-q, 0)$.

(f). $(x_{M2}, x_{M'2}, x_{F2}) = (0, 1, 0)$. As in (d), $(P_M, P_{M'}, P_F) = (1-q, q, 0)$.

(g). $(x_{M2}, x_{M'2}, x_{F2}) = (0, 1, 1)$. As in (d), $(P_M, P_{M'}, P_F) = (1-q, q, 0)$.

Because $q > 2/3$ is assumed under Assumption 4, $(x_{M2}, x_{M'2}, x_{F2}) = (1, 1, 0)$ in equilibrium. Furthermore, $(P_M, P_{M'}, P_F) = (q/2, q/2, 1-q)$. □

A.9 Proof of Proposition 2

From Assumption 1, $c\mathbb{E}_{k1}[\beta] > \max\{\theta, 1-\theta\}$ holds. Hence, the electoral outcome in the period 1 election is given as in Lemma 7 (i). Let the discounted sum of the expected winning probability of candidate k across two periods be V_k.

(a). $x_{M1} = x_{M'1} = x_{F1}$. In this case, candidates M and M' split the votes and $v_{F1} = 0$. Hence, the equilibrium in period 2 is (i) in Lemma 7. Taken together, $(V_M, V_{M'}, V_F) = (1, 1, 0)$.

(b). $(x_{M1}, x_{M'1}, x_{F1}) = (1, 1, 0)$. In this case, voters with $(\hat{x}_{k1}, \hat{y}) = (0, 0)$ vote for candidate F. Hence, with probability p, voters learn that $\beta = \bar{\beta}$ so that the equilibrium in period 2 is (i) in Lemma 7. With the remaining probability, voters learn that $\beta = \underline{\beta}$ so that the equilibrium in period 2 is (ii) in Lemma 7. Taken together,

$$(V_M, V_{M'}, V_F) = \left(\frac{1}{2} + p\frac{1}{2} + (1-p)\frac{q}{2}, \frac{1}{2} + p\frac{1}{2} + (1-p)\frac{q}{2}, (1-p)(1-q)\right).$$

(c). $(x_{M1}, x_{M'1}, x_{F1}) = (0, 0, 1)$. In this case, voters with $(\hat{x}_{k1}, \hat{y}) = (1, 0)$ vote for candidate F. Hence, with probability p, voters learn that $\beta = \bar{\beta}$ so that the equilibrium in period 2 is (i) in Lemma 7. With the remaining probability, voters learn that $\beta = \underline{\beta}$ so that the equilibrium in period 2 is (ii) in Lemma 7. Taken together,

$$(V_M, V_{M'}, V_F) = \left(\frac{1}{2} + p\frac{1}{2} + (1-p)\frac{q}{2}, \frac{1}{2} + p\frac{1}{2} + (1-p)\frac{q}{2}, (1-p)(1-q)\right).$$

(d). $(x_{M1}, x_{M'1}, x_{F1}) = (1, 0, 1)$. In this case, no one votes for candidate F. Hence, the equilibrium in period 2 is (i) in Lemma 7. Taken together, $(V_M, V_{M'}, V_F) = (q + 1/2, 1 - q + 1/2, 0)$.

(e). $(x_{M1}, x_{M'1}, x_{F1}) = (1, 0, 0)$. As in (d), $(V_M, V_{M'}, V_F) = (q + 1/2, 1 - q + 1/2, 0)$.

(f). $(x_{M1}, x_{M'1}, x_{F1}) = (0, 1, 0)$. As in (d), $(V_M, V_{M'}, V_F) = (1 - q + 1/2, q + 1/2, 0)$.

(g). $(x_{M1}, x_{M'1}, x_{F1}) = (0, 1, 1)$. As in (d), $(V_M, V_{M'}, V_F) = (1 - q + 1/2, q + 1/2, 0)$.

Now,

$$\frac{1}{2} + p\frac{1}{2} + (1-p)\frac{q}{2} > 1 - q + \frac{1}{2}$$

holds because $q > 2/3$ is assumed. Therefore, in the equilibrium, $(x_{M1}, x_{M'1}, x_{F1}) = (1, 1, 0)$ holds. Furthermore, candidate F wins the period 2 election with probability $(1-p)(1-q)$.

□

A.10 Proof of Lemma 8

(i) is straightforward. Hence, we focus on (ii).

(a). As in Proposition 1, candidate F has a chance of winning the period 2 election if and only if $v_{F1} > (1-\underline{\alpha})(1-\bar{\beta})$. Here, because $\beta = \underline{\beta}$,

$$v_{F1} > (1-\underline{\alpha})(1-\bar{\beta}) \Leftrightarrow (1-\alpha_1)(1-\underline{\beta}) > (1-\underline{\alpha})(1-\bar{\beta}) \Leftrightarrow 1-\alpha_1 > \frac{(1-\underline{\alpha})(1-\bar{\beta})}{1-\underline{\beta}}.$$

Since $\frac{(1-\underline{\alpha})(1-\bar{\beta})}{1-\underline{\beta}} > 1/2$ holds by the assumption, $\Pr(\alpha_1 > 1/2) = 0$. Hence, $\Pr(\alpha_2 > 1/2) = \underline{q}_2$.

(b). As in (a), we obtain the desired result. □