Ambiguity and Unemployment Fluctuations*

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Abstract

We analyze the consequences of ambiguity aversion in the Diamond-Mortensen-Pissarides (DMP) search and matching model. Our model features a cross-section of workers whose productivity contain a match-specific component. Firms are ambiguity averse towards match-specific productivity. Our model delivers two insights. First, we show that ambiguity aversion substantially amplifies unemployment rate volatility. Second, we show that a part of the high value of leisure required by the canonical DMP model to generate realistic unemployment rate volatility can arise from fitting a model missing ambiguity aversion to data generated in an environment where agents are ambiguity averse.

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1 Introduction

Our central premise is that firms are unsure about the model describing the productivity process of individual workers and seek robustness against this concern. We show that this model misspecification concern, henceforth ambiguity aversion, has a quantitatively large effect on the dynamics of aggregate labor market variables. Our model delivers two insights. First, we show that ambiguity aversion substantially increases the volatility of the unemployment rate, thereby improving upon a well-known shortcoming of the canonical Diamond-Mortensen-Pissarides (DMP) model (Shimer, 2005). Second, we show that a part of the high value of leisure required by the canonical DMP model to generate realistic unemployment rate volatility (see Hagedorn and Manovskii 2008) can arise from fitting a model that is missing ambiguity aversion to data generated in an environment where agents are ambiguity averse.

We illustrate and quantify our channel by introducing ambiguity aversion in the canonical Diamond-Mortensen-Pissarides (DMP) model. Our model features a cross-section of workers whose productivity is the sum of an aggregate and a match-specific component. While firms and workers know the process for aggregate productivity, they are ambiguous regarding the model for match-specific productivity. Specifically, firms and workers assume the conditional mean growth rate of match-specific productivity to lie within a range of values surrounding a reference value. They seek robustness against model misspecification by choosing policies assuming the worst case model for productivity (for justifications of this choice of the worst case model in decision making, see Hansen and Sargent 2008, chapter 1). To evaluate the quantitative effect of ambiguity aversion, we discipline the range of the conditional mean growth rate of match-specific productivity using a standard likelihood ratio test for discriminating between the reference model and the worst-case model (i.e., detection error probabilities).

To understand the effect of ambiguity on unemployment dynamics, we derive an equivalence result that relates two economies: (i) the economy in our model, which we call $\mathcal{E}_{ambiguity}$, and (ii) the economy in the canonical DMP model altered to feature a time-varying value of leisure, which we call $\widehat{\mathcal{E}}^{no}_{ambiguity}$. Like the DMP model, worker productivity in $\widehat{\mathcal{E}}^{no}_{ambiguity}$ consists of only the aggregate component; there is no match-specific productivity component and no ambiguity. The value of leisure $\widehat{z}$ in $\widehat{\mathcal{E}}^{no}_{ambiguity}$ is related to the constant value of leisure z in $\mathcal{E}_{ambiguity}$ through the relation

$$\widehat{z} = z + \mathscr{L}S - \mathscr{L}^{wc}S.$$

The quantities on the right-hand side of the above equation refer to the economy $\mathcal{E}_{ambiguity}$; specifically, S is the surplus of the worker firm match, $\mathscr{L}S$ is the expected growth rate of this surplus under the reference model, and $\mathscr{L}^{wc}S$ is the expected growth rate of the surplus under the worst-case model. We refer to $\widehat{z}$ as the effective value of leisure. The two economies $\mathcal{E}_{ambiguity}$ and $\widehat{\mathcal{E}}^{no}_{ambiguity}$ are observationally equivalent in the sense that the two economies have identical paths of the unemployment rate and labor market tightness.

The relation above shows that the difference between the effective and the actual value of leisure depends on the difference in the expected growth rate of the surplus under the reference and the worst-case model. We show that this difference in the expected growth rates is an increasing function of the amount of ambiguity (specifically, the range of values of the conditional mean growth rate of match-specific productivity). Furthermore, the effect of ambiguity is more potent when the volatility of match-specific productivity is high. In our quantitative exercises, we find that in the presence of ambiguity, the effective value of leisure $\widehat{z}$ is greater than the actual value z such that ambiguity concerns have a first order effect on unemployment fluctuations.

The high effective value of leisure provides a new interpretation for the high value of leisure proposed by Hagedorn and Manovskii (2008) as a potential way for the DMP model to generate realistic unemployment rate volatility. Our analysis implies that a significant portion of this value of leisure arises from fitting the canonical DMP model that abstracts away heterogeneity and ambiguity aversion to data generated in a richer environment with heterogeneity and ambiguity aversion. This provides an explicit example of the view in Hagedorn and Manovskii (2008, p. 1692) that the DMP model is an "approximation of a richer model with heterogeneity and curvature in utility and technology".

The observational equivalence of the unemployment rate and tightness dynamics between $\mathcal{E}_{ambiguity}$ and $\widehat{\mathcal{E}}^{no}_{ambiguity}$ implies that the economies $\mathcal{E}_{ambiguity}$ and $\widehat{\mathcal{E}}^{no}_{ambiguity}$ have identical elasticities of tightness with respect to productivity. This allows us to build on the recent analysis of Ljungqvist and Sargent (2017) (henceforth "LS"). We show that the semi-elasticity of tightness with respect to aggregate productivity y is approximately equal to $\Upsilon e^y / (e^y - \widehat{z})$ where Υ is a constant whose value is tightly constrained by the data.[1] We see that the semi-elasticity of tightness is inversely proportional to $e^y - \widehat{z}$, a quantity LS call the "fundamental surplus". Note that with ambiguity aversion, the fundamental surplus is determined by the effective value of leisure $\widehat{z}$ and not the actual value of leisure z. In a comparative static sense, a higher amount of ambiguity or a higher volatility of match-specific productivity is associated with a higher value of effective leisure, a lower fundamental surplus, and hence a higher elasticity of tightness.

The existing literature that has analyzed the impact of ambiguity aversion on business cycle dynamics has focused on ambiguity regarding the model describing an aggregate shock (see, e.g., Ilut and Schneider 2014, and Bhandari et al. 2019).[2] In contrast, we focus on ambiguity regarding match-specific productivity because compared to ambiguity regarding aggregate productivity, ambiguity regarding match-specific productivity amplifies labor

[1]The constant Υ is the same constant Υ used by LS. Our notation differs from LS—we use y to denote log output per worker while LS use y to denote output per worker.

[2]For a review of papers that analyze the effect of ambiguity aversion on asset prices, see, for example, references in Hansen and Sargent (2008). Nishimura and Ozaki (2007) analyzes the effect of ambiguity on a worker's reservation wage.

market fluctuations by a significantly larger amount. This is because: (a) the quantitative impact of ambiguity aversion is an increasing function of the volatility of the shock and (b) existing estimates in the literature suggest that the volatility of match-specific productivity is much larger than the volatility of aggregate labor productivity (see, e.g., Nagypal 2007 and Borovickova 2016 for estimates of the volatility of match-specific productivity). Indeed, we find that, all else equal, ambiguity regarding match-specific productivity amplifies the volatility of tightness by 3.5 times (see Section 4.2) versus 1.13 times when the ambiguity is with regards to aggregate productivity (see Appendix A).

2 Two period model

We use a two period setting to show that ambiguity aversion implies a high elasticity of labor market tightness to productivity shocks.

Ambiguity regarding match productivity. There are two periods, $t \in \{0, 1\}$. There is no production at $t = 0$. At $t = 1$, each employed worker produces output $\exp(y_1)$ where productivity y_1 is random. Workers and firms face ambiguity towards productivity y_1.

We model ambiguity following the robust control literature—agents consider a set of distorted models around a reference model for y_1 (see, e.g., Hansen and Sargent 2008). Here, a model refers to a probability measure which describes the distribution of y_1. We use $\mathcal{P}$ to denote the (probability measure of the) reference model, and $\mathcal{P}^\gamma$ to denote a distorted model.

For concreteness, suppose that $y_1 \overset{\mathcal{P}}{\sim} \mathcal{N}(\mu_y - \frac{1}{2}\sigma_y^2, \sigma_y^2)$ where $\overset{\mathcal{P}}{\sim}$ refers to the distribution under $\mathcal{P}$. That is, log productivity is normally distributed with a mean of $\mu_y - \frac{1}{2}\sigma_y^2$ and a standard deviation of σ_y under the reference model. Consider distorted models $\mathcal{P}^\gamma$ defined by the change of measure

$$d\mathcal{P}^\gamma / d\mathcal{P} = \exp\left(-\frac{1}{2}\gamma^2 - \frac{\gamma}{\sigma_y}\left(y_1 - \mu_y + \frac{1}{2}\sigma_y^2\right)\right).$$

That is, $\mathcal{P}^\gamma$ is defined by $\mathcal{P}^\gamma(A) = \int_A \exp\left(-\frac{1}{2}\gamma^2 - \frac{\gamma}{\sigma_y}\left(y_1 - \mu_y + \frac{1}{2}\sigma_y^2\right)\right) d\mathcal{P}$ for an event A. Then $y_1 \overset{\mathcal{P}^\gamma}{\sim} \mathcal{N}(\mu_y - \frac{1}{2}\sigma_y^2 - \gamma\sigma_y, \sigma_y^2)$; that is, under $\mathcal{P}^\gamma$, productivity is still normally distributed with a standard deviation of σ_y, but its mean is $\mu_y - \frac{1}{2}\sigma_y^2 - \gamma\sigma_y$ instead.[3] The expected output of a worker is $\mathbb{E}_0^\gamma[\exp(y_1)] = \exp(\mu_y - \gamma\sigma_y)$ under the distorted model where $\mathbb{E}_0^\gamma$ is the expectation under $\mathcal{P}^\gamma$.

Agents restrict attention to distorted models in the set

$$\Gamma \equiv \{\gamma : \ \gamma \in [-\overline{\gamma}, \overline{\gamma}]\} \tag{1}$$

where $\overline{\gamma} \geq 0$. Here, $\gamma = 0$ corresponds to the reference model and $\gamma = \overline{\gamma}$ is interpreted as the worst-case model under which expected output is the lowest.

Labor market environment. Matching between firms and workers occur at $t = 0$. There is a unit mass of workers of which $u_0 \in [0, 1]$ is unemployed and searching for work at $t = 0$. There is free entry of firms and firms attempt to hire workers at $t = 0$ by posting v_0 vacancies at a cost of c per vacancy. An unemployed worker becomes employed when matched to a firm. A total of $m(u_0, v_0) = Au_0^\alpha v_0^{1-\alpha}$ matches are successfully formed at $t = 0$, where we have assumed the commonly used Cobb-Douglas matching function. The job finding and vacancy filling rates, $f(\theta_0) \equiv m(u_0, v_0)/u_0 = A\theta_0^{1-\alpha}$ and $q(\theta_0) \equiv m(u_0, v_0)/v_0 = A\theta_0^{-\alpha}$, respectively, are then functions of labor market tightness $\theta_0 \equiv v_0/u_0$.

A fraction $u_1 = u_0 - f(\theta_0)\, u_0$ of workers remain unemployed at $t = 1$ while the remaining workers $1 - u_1$ are employed. Employed workers are paid wages w_1, with the residual profit $\exp(y_1) - w_1$ flowing to the matched firm. Wages w_1 are determined at $t = 1$ according to a generalized Nash bargaining rule in which workers have bargaining power $\beta \in (0, 1)$. Unemployed workers obtain a value of leisure z.

[3] To see this, note that the moment generating function of y_1 under $\mathcal{P}^\gamma$ is $\mathbb{E}_0^\gamma[\exp(\phi y_1)] = \mathbb{E}_0^{\mathcal{P}}[(d\mathcal{P}^\gamma/d\mathcal{P})\exp(\phi y_1)] = \exp\left(\phi(\mu_y - \frac{1}{2}\sigma_y^2 - \gamma\sigma_y) + \frac{1}{2}\phi^2\sigma_y^2\right)$, which corresponds to the moment generating function of a random variable with a $\mathcal{N}(\mu_y - \frac{1}{2}\sigma_y^2 - \gamma\sigma_y, \sigma_y^2)$ distribution.

Ambiguity and firms' hiring incentives. Let J and V denote the value of a filled and unfilled vacancy to the firm, respectively, and W and U denote the value of employment and unemployment to a worker, respectively. Their values at $t = 1$ are

$$J_1 = \exp(y_1) - w_1, \quad V_1 = 0, \quad W_1 = w_1, \quad U_1 = z. \tag{2}$$

The match surplus at $t = 1$ is $S_1 \equiv J_1 - V_1 + W_1 - U_1 = \exp(y_1) - z$ where we have used equation (2) in the second equality. Nash bargaining implies that the firm's and worker's share of the surplus is $J_1 - V_1 = (1 - \beta)S_1$ and $W_1 - U_1 = \beta S_1$, respectively. From the worker's share of the surplus, we see that wages equal

$$w_1 = \beta \exp(y_1) + (1 - \beta)z, \tag{3}$$

while the value of a filled vacancy to the firm is $J_1 = (1 - \beta)(\exp(y_1) - z)$ at $t = 1$.

Firms' hiring incentives at $t = 0$ depend on the value of a filled vacancy to the firm at that date J_0. The latter is the expected present value of J_1,

$$J_0 = \min_{\gamma \in \Gamma} \frac{1}{1 + r_f} \mathbb{E}_0^{\gamma}[(1 - \beta)(\exp(y_1) - z)], \tag{4}$$

where r_f denotes the risk-free rate and Γ is the set of distorted models under consideration (1). Equation (4) embeds the effect of ambiguity—the valuation for J_0 is made under a robust control principle whereby the firm assumes nature chooses from Γ the model that gives the lowest valuation for J_0. The minimizer in equation (4) is

$$\gamma = \overline{\gamma}. \tag{5}$$

Equilibrium. In equilibrium, free entry of firms implies that the cost of posting a vacancy c is equal to the expected benefit of posting the vacancy:

$$c = A\theta_0^{-\alpha} \frac{1}{1 + r_f}(1 - \beta)\mathbb{E}_0^{\overline{\gamma}}[\exp(y_1) - z]. \tag{6}$$

In equation (6), the first term on the right-hand side $q(\theta_0) = A\theta_0^{-\alpha}$ is the probability that a posted vacancy actually gets filled. The remaining terms correspond to the value of a filled vacancy (4) where we have imposed the worst-case model (5).

The solution to equation (6) gives the equilibrium tightness θ_0 at $t = 0$. The equilibrium unemployment rate at $t = 1$ is $u_1 = u_0 - f(\theta_0) u_0$, and equilibrium wages are given by equation (3).

Elasticity of tightness. Let us define the elasticity of labor market tightness with respect to productivity in this static two-period model to be $\epsilon_{\theta,y_1} \equiv d \log \theta_0 / d\mu_y$ where the derivative is evaluated at $\mu_y = 0$. Applying this definition to the equilibrium condition (6) implies that

$$\epsilon_{\theta,y_1} = \frac{1}{\alpha \left(1 - z \exp\left(\overline{\gamma}\sigma_y\right)\right)}. \tag{7}$$

The relation (7) shows that the elasticity is an increasing function of the amount of ambiguity in productivity $\overline{\gamma}$. We also see that the effect of ambiguity is more potent when the volatility of productivity σ_y is higher (i.e., $\partial^2 \epsilon_{\theta,y_1} / (\partial\overline{\gamma}\partial\sigma_y) > 0$). We show that these results carry over to a dynamical setting.

3 Dynamic model

In this section, we extend the two period model from Section 2 to a continuous time dynamical setting to evaluate the quantitative importance of ambiguity in labor market dynamics.

3.1 The environment

Production and match-specific productivity. There is a unit mass of workers and a large mass of firms who hire workers to produce output. The log output of worker $i \in [0, 1]$ is $y_t + \varepsilon_{it}$. The first component y_t is aggregate productivity; it follows the mean reverting process

$$dy_t = -\kappa_y y_t dt + \sigma_y dB_t^y, \tag{8}$$

where $\kappa_y > 0$ is the speed of mean reversion of y_t to its long-run mean (which we have normalized to be zero), σ_y is the volatility of shocks to y_t, B_t^y is a standard Brownian motion under the probability measure of the reference model $\mathcal{P}$.

The second component ε_{it} is "match-specific productivity"; independent from y_t and also across matches. All workers i have an initial match-specific productivity of

$$\varepsilon_{i0} = \varepsilon_0 \equiv 0 \tag{9}$$

at the start of each new match, where we normalize the initial value to $\varepsilon_0 \equiv 0$. Subsequently, match-specific productivity follows

$$d\varepsilon_{it} = -\kappa_\varepsilon \varepsilon_{it} dt + \sigma_\varepsilon dB_{it} + dL_{it}, \tag{10a}$$

$$dL_{it} = \begin{cases} -\min\left\{-\kappa_\varepsilon \varepsilon_{it} dt + \sigma_\varepsilon dB_{it}, 0\right\} & \text{if } \varepsilon_{it} = \underline{\varepsilon}^{ref}(y_t), \\ 0 & \text{if } \varepsilon_{it} > \underline{\varepsilon}^{ref}(y_t), \end{cases} \tag{10b}$$

where $\underline{\varepsilon}^{ref}(y_t)$ is a reflecting boundary that is common across matches. The reflecting boundary depends on y_t in a manner that we describe later in equations (15) and (16). Ignoring the dL_{it} term, we see that the law of motion (10a) is analogous to that of aggregate productivity (8)—ε_{it} has speed of mean reversion κ_ε, volatility σ_ε, and is driven by changes in B_{it} which is a standard Brownian motion under $\mathcal{P}$ (B_{it} is independent from B_t^y and also across matches). The dL_{it} term modifies the match-specific productivity process to make $\underline{\varepsilon}^{ref}(y_t)$ a reflecting boundary—equation (10b) shows that dL_{it} neutralizes further negative productivity changes when ε_{it} reaches $\underline{\varepsilon}^{ref}(y_t)$ so that ε_{it} never goes below $\underline{\varepsilon}^{ref}(y_t)$. The reflecting boundary makes it easier to compare the results of our model to that of the canonical DMP model. We discuss this issue in detail towards the end of Section 3.3.1.

Ambiguity. We focus on ambiguity regarding match-specific productivity. Firms and workers restrict attention to distorted models obtained through distorted measures $\mathcal{P}^{\gamma_i}$ of the form

$$\mathcal{P}^{\gamma_i}(A) = \int_A \exp\left(-\frac{1}{2}\int_0^t \gamma_{iu}^2\, du - \int_0^t \gamma_{iu}\, dB_{iu}\right) d\mathcal{P} \tag{11}$$

where A is an event in the time-t information set and $\gamma_i = (\gamma_{it})_{t\geq 0}$ is a given stochastic process, and $\mathcal{P}$ is the reference probability measure under which match-specific productivity follows the process (10a). The integrand in equation (11) corresponds to the change of measure $d\mathcal{P}^{\gamma_i}/d\mathcal{P}$ (projected onto the time-t information set). Different choices of γ_i give rise to different distorted measures $\mathcal{P}^{\gamma_i}$ through equation (11). In particular, the Brownian motion B_{it} which has zero drift under the reference measure $\mathcal{P}$, has a drift of $-\gamma_{it}$ under the distorted measure $\mathcal{P}^{\gamma_i}$ (the reason is similar to that outlined in footnote 3 for the discrete time model). For this reason, we define

$$B_{it}^{\gamma} \equiv B_{it} + \int_0^t \gamma_{iu}\, du, \tag{12}$$

which is a standard Brownian motion under the distorted measure $\mathcal{P}^{\gamma_i}$. Substituting equation (12) into equation (10a), we see that for a given process γ_i, match-specific productivity behaves according to

$$d\varepsilon_{it} = (-\sigma_\varepsilon\gamma_{it} - \kappa_\varepsilon\varepsilon_{it})dt + \sigma_\varepsilon dB_{it}^{\gamma} + dL_{it} \tag{13}$$

under the distorted model. Compared to the reference model (10a), the drift of the distorted model (13) contains an additional $-\sigma_\varepsilon\gamma_{it}$ term; larger values of γ_{it} or σ_ε lead to distorted models with lower drifts of match-specific productivity.

At any time t, decision makers restrict attention to distorted models $\gamma_i = (\gamma_{is})_{s\geq t}$ that fall within the set

$$\Gamma_{it} \equiv \{\gamma_i :\ \gamma_{is} \in [-\overline{\gamma}, \overline{\gamma}] \text{ for all } s \geq t\}\,. \tag{14}$$

That is, after having observed the history of ε_{it} up to time t, decision makers consider distorted models (13) for the future dynamics of ε_{it} that have a γ_{it} process that lies between $-\overline{\gamma}$ and $\overline{\gamma}$. Here, $\overline{\gamma}$ parameterizes the extent to which alternative models can deviate from the reference model. There is no ambiguity when $\overline{\gamma} = 0$ and larger values of $\overline{\gamma}$ encompass distorted models with larger deviations from the reference model. In our quantitative applications in Section 4, we use detection error probabilities to discipline $\overline{\gamma}$.

Labor market environment. At time-t, a mass $u_t \in [0, 1]$ of workers are unemployed. Firms hire unemployed workers by posting vacancies v_t. The flow cost for maintaining a vacancy is c. Firms can freely enter for the purposes of vacancy creation. An unemployed worker i becomes employed when matched to a firm and produces output $\exp(y_t + \varepsilon_{it})$.

The instantaneous rate at which new firm-worker matches are formed is given by a Cobb-Douglas matching function $m(u_t, v_t) = A u_t^{\alpha} v_t^{1-\alpha}$. The job finding rate $f(\theta_t) = m(u_t, v_t)/u_t = A\theta_t^{1-\alpha}$ and the vacancy filling rate $q(\theta_t) = m(u_t, v_t)/v_t = m(1/\theta_t, 1) = A\theta_t^{-\alpha}$ then depend on labor market tightness $\theta_t \equiv v_t/u_t$.

Employed workers are paid wages at rate w_{it} with the residual profit flowing to the matched firm at rate $\exp(y_t + \varepsilon_{it}) - w_{it}$. Wages are determined according to a generalized Nash bargaining rule in which workers have bargaining power $\beta \in (0, 1)$. Matches exogenously separate at rate s. Employed workers become unemployed following separations and obtain a flow value of leisure z.

3.2 Equilibrium

Choice of reflecting boundary. Let S_{it} denote the surplus from match i. In a Markov equilibrium, which we describe below, the match surplus $S_{it} = S(y_t, \varepsilon_{it})$ is a function of y_t and ε_{it}. This defines the separation threshold

$$\underline{\varepsilon}(y_t) \equiv \max\{\varepsilon : \; S(y_t, \varepsilon) = 0\} \tag{15}$$

whereby matches with $\varepsilon_{it} < \underline{\varepsilon}(y_t)$ separate while matches with $\varepsilon_{it} \geq \underline{\varepsilon}(y_t)$ continue. We show in Appendix B.1 that this endogenous separation policy is optimal from both the worker's and the firm's perspective.

We choose the reflecting boundary for the match-specific productivity process (10) to coincide with the separation threshold (15),

$$\underline{\varepsilon}^{ref}(y_t) = \underline{\varepsilon}(y_t). \tag{16}$$

This choice removes the possibility of endogenous match separations and makes it easier to compare the results of our model with ambiguity to the canonical DMP model which also does not feature endogenous match separations; see the discussion following Proposition 1 towards the end of Section 3.3.1 for details. Absent any endogenous separation, the law of motion for the unemployment rate is

$$\frac{du_t}{dt} = s(1 - u_t) - f(\theta_t)u_t.$$

Markov equilibrium. We show in Appendix B.1 that the equilibrium surplus for match i is given by $S_{it} = S(y_t, \varepsilon_{it})$ where $S(y, \varepsilon)$ solves the following boundary value problem:

$$r_f S(y, \varepsilon) = \min_{\gamma \in [-\overline{\gamma}, \overline{\gamma}]} e^{y+\varepsilon} - z - \frac{\beta c}{1-\beta}\theta(y) + \mathscr{L}^{\gamma}_{y,\varepsilon} S(y, \varepsilon) - sS(y, \varepsilon), \tag{17a}$$

$$c = (1 - \beta)q(\theta(y))S(y, \varepsilon_0), \tag{17b}$$

for $\varepsilon \geq \underline{\varepsilon}(y)$, along with the boundary conditions

$$S(y, \underline{\varepsilon}(y)) = 0, \tag{18a}$$

$$S_{\varepsilon}(y, \underline{\varepsilon}(y)) = 0. \tag{18b}$$

Equation (17a) is the Hamilton-Jacobi-Bellman equation for the surplus in which

$$\mathscr{L}^{\gamma}_{y,\varepsilon} = -\kappa_y y \partial_y + \frac{1}{2}\sigma_y^2 \partial_{yy} + (-\sigma_{\varepsilon}\gamma - \kappa_{\varepsilon}\varepsilon)\partial_{\varepsilon} + \frac{1}{2}\sigma_{\varepsilon}^2 \partial_{\varepsilon\varepsilon} \tag{19}$$

is the infinitesimal generator (see, e.g., Øksendal 2003, chapter 7.3); here, ∂_x and ∂_{xx} denote the first and second partial derivatives with respect to $x \in \{y, \varepsilon\}$. The $-\kappa_y y \partial_y + \frac{1}{2}\sigma_y^2 \partial_{yy}$ term accounts for expected changes in the surplus as a result of aggregate productivity dynamics (8) while the remaining terms account for expected changes due to match-specific productivity dynamics under the distorted model (13). The minimization on the right-hand side of equation (17a) characterizes the worst-case model which corresponds to choosing

$$\gamma(y, \varepsilon) = \overline{\gamma} \text{ for all } y \text{ and } \varepsilon.$$

This follows from the surplus being increasing in ε (i.e., $S_\varepsilon(y, \varepsilon) > 0$ when $\varepsilon > \underline{\varepsilon}(y)$).

Equation (17b) is the free-entry condition for vacancy creation. Equation (18a) is a value matching condition that ensures that the surplus value is zero at the separation threshold (15); equation (18b) is a smooth pasting condition that characterizes the location of the separation threshold.

All equilibrium quantities are a function of the surplus. For example, equilibrium tightness can be recovered from equation (17b), $\theta(y) = q^{-1}\left(c/[(1-\beta)S(y, \varepsilon_0)]\right)$, while wages equal

$$w(y, \varepsilon) = \beta \exp(y + \varepsilon) + (1 - \beta)z + \beta c \theta(y). \tag{20}$$

3.3 Analysis

We analyze the effect of ambiguity regarding match-specific productivity on labor market dynamics in two steps. First, we prove that the economy with ambiguity is observationally equivalent in terms of tightness and unemployment rate dynamics to an economy with neither match-specific productivity nor ambiguity aversion, but an altered value of leisure. Since the equivalent economy does not feature ambiguity, we refer to it as the "economy without ambiguity". We show that the economy without ambiguity is the canonical DMP model extended to feature a time-varying value of leisure. Second, we use comparative static analysis on the economy without ambiguity to gauge the effect of ambiguity for unemployment fluctuations. These analyses help us understand the quantitative implications of our model in Section 4.

3.3.1 An observationally equivalent economy

We begin by formally defining the economy with ambiguity towards match-specific productivity described in Sections 3.1 and 3.2.

Definition 1. *An economy with ambiguity towards match-specific productivity is the tuple*

$$\mathcal{E} \equiv \left(z, \{\gamma_i\}_{i\in[0,1]}, \overline{\gamma}, \varepsilon_0, \kappa_\varepsilon, \sigma_\varepsilon, \{B_{it}, L_{it}\}_{i\in[0,1]}, \Psi_{invariant}\right)$$

where L_{it} is defined to have the boundary (16) and $\Psi_{invariant} = (s, c, A, \alpha, \beta, r_f, \kappa_y, \sigma_y, B_t^y, \mathcal{P})$.

In defining the economy, we have grouped the parameters and shocks into two groups, where the grouping anticipates the existence of the equivalent economy. The first group $\left(z, \{\gamma_i\}_{i\in[0,1]}, \overline{\gamma}, \varepsilon_0, \kappa_\varepsilon, \sigma_\varepsilon, \{B_{it}, L_{it}\}_{i\in[0,1]}\right)$ differs across the economy with ambiguity and the economy without ambiguity which we define in Definition 2 below. The second group $\Psi_{invariant}$ remains the same across the two economies.

Equivalent economy without ambiguity. The environment for the economy without ambiguity is identical to the economy with ambiguity described in Section 3.1 in all but two aspects. The first difference is that there is no heterogeneity in match productivity, that is, $\varepsilon_{it} = \varepsilon_0 = 0$, and the log output of every worker is equal to aggregate productivity y_t. As a result, there is also no ambiguity towards match-specific productivity: $\overline{\gamma} = 0$. The second difference is that the value of leisure $\widehat{z}(y)$ depends on aggregate productivity. The resulting economy, which we denote by $\widehat{\mathcal{E}}$, corresponds to the canonical DMP setting extended to allow for a time-varying $\widehat{z}(y)$. We formally define this economy as follows:

Definition 2. *The DMP economy with a time-varying value of leisure is the tuple*

$$\widehat{\mathcal{E}} \equiv (\widehat{z}(y), \Psi_{invariant})$$

where $\Psi_{invariant}$ is the same as in the economy with ambiguity (defined in Definition 1).

As a matter of notation, we use a "hat" to refer to values from the DMP economy with a time-varying value of leisure. We do not use hat notation for the structural parameters and shocks $\Psi_{invariant}$ since these parameters are identical to those from the economy with ambiguity $\mathcal{E}$.

Markov equilibrium for $\widehat{\mathcal{E}}$. A Markov equilibrium for $\widehat{\mathcal{E}}$ is characterized through the following system for the match surplus $\widehat{S}$ and equilibrium tightness $\widehat{\theta}$,

$$r_f \widehat{S}(y) = e^y - \widehat{z}(y) - \frac{\beta c}{1-\beta}\widehat{\theta}(y) + \mathscr{L}_y \widehat{S}(y) - s\widehat{S}(y), \tag{21a}$$

$$c = (1-\beta) q(\widehat{\theta}(y)) \widehat{S}(y), \tag{21b}$$

where

$$\mathscr{L}_y = -\kappa_y y \partial_y + \frac{1}{2}\sigma_y^2 \partial_{yy} \tag{22}$$

is the infinitesimal generator. The derivation of equation (21) is similar to the derivation for the surplus in the economy with ambiguity (17); see Appendix B.2 for details. The equilibrium tightness and wages are functions of the surplus and equal $\widehat{\theta}(y) = q^{-1}(c/[(1-\beta)\widehat{S}(y)])$ and

$$\widehat{w}(y) = \beta \exp(y) + (1-\beta)\widehat{z}(y) + \beta c \widehat{\theta}(y), \tag{23}$$

respectively. The main result of this section is:

Proposition 1 (Observational equivalence of tightness)**.** *Consider the economy with ambiguity $\mathcal{E}$ from Definition 1, and let $S(y, \varepsilon)$ and $\theta(y)$ denote the equilibrium surplus and labor market tightness in this economy, respectively. Next, consider the DMP economy with a time-varying* *value of leisure* $\widehat{\mathcal{E}}$ *from Definition 2 in which*

$$\widehat{z}(y) = z + \left(\mathscr{L}_y - \mathscr{L}^{\overline{\gamma}}_{y,\varepsilon}\right) S(y, \varepsilon_0) = z + \overline{\gamma}\sigma_\varepsilon S_\varepsilon(y, \varepsilon_0) - \frac{1}{2}\sigma^2_\varepsilon S_{\varepsilon\varepsilon}(y, \varepsilon_0), \tag{24}$$

and let $\widehat{S}(y)$ *and* $\widehat{\theta}(y)$ *be the corresponding equilibrium surplus and tightness, respectively. Then,*

$$\widehat{S}(y) = S(y, \varepsilon_0) \quad \textit{and} \quad \widehat{\theta}(y) = \theta(y). \tag{25}$$

We refer to $\widehat{\mathcal{E}}$ *with* $\widehat{z}(y)$ *set according to condition (24) as the "equivalent DMP economy", and refer to* $\widehat{z}(y)$ *as the "effective value of leisure".*

Proposition 1 follows from evaluating equation (17) for the surplus $S(y, \varepsilon)$ and tightness $\theta(y)$ in the economy with ambiguity at $\varepsilon = \varepsilon_0 \equiv 0$. The resulting equation is identical to equation (21) for the surplus $\widehat{S}(y)$ and tightness $\widehat{\theta}(y)$ in the DMP economy with a time-varying value of leisure from Definition 2 when $\widehat{z}(y)$ satisfies condition (24). Equation (25) follows as a result.

The two economies $\mathcal{E}$ and $\widehat{\mathcal{E}}$ from Proposition 1 are observationally equivalent in the sense that the equilibrium tightness, and hence job finding and vacancy filling rates, are identical across the two economies for all realizations for the path of aggregate productivity y_t. Note also that $\mathcal{E}$ and $\widehat{\mathcal{E}}$ have the same job separation rates because we (i) set the reflecting boundary to coincide with the endogenous separation threshold (see condition (16)), and (ii) choose the same value of the exogenous job separation rate s in both economies. Since $\mathcal{E}$ and $\widehat{\mathcal{E}}$ have identical flow rates into and out of unemployment, the two economies have identical unemployment rate processes. Note, however, that wages in these two economies are not the same: workers in $\mathcal{E}$ earn wages given by equation (20), while all workers in $\widehat{\mathcal{E}}$ earn wages (23).

Proposition 1 shows that the dynamics of tightness and unemployment rates in the economy with ambiguity $\mathcal{E}$ can be determined by analyzing the more familiar DMP model extended to include a value of leisure $\widehat{z}(y)$ given by equation (24). Next, we combine this observation with the analysis in Ljungqvist and Sargent (2017), henceforth LS, to gauge the effect of ambiguity for unemployment fluctuations.

3.3.2 Ambiguity and amplification of productivity shocks

LS show that unemployment fluctuations within a large class of search-based models depend positively on the elasticity of tightness with respect to aggregate productivity shocks. Using steady-state comparative statics, Ljungqvist and Sargent (2017, equation (15)) show that the elasticity of tightness in the canonical DMP model is approximately

$$d\log\theta_{DMP}(y)/dy \approx \Upsilon\frac{e^y}{e^y - z_{DMP}}, \tag{26}$$

where z_{DMP} is the (constant) value of leisure, and $\Upsilon = [r_f + s + \beta\theta_{ss}q(\theta_{ss})]/[\alpha(r_f + s) + \beta\theta_{ss}q(\theta_{ss})]$ with θ_{ss} being the steady-state tightness (see, also, Shimer (2005, p. 36) and Hagedorn and Manovskii (2008, p. 1695) for earlier instances of equation (26)). LS refer to $e^y - z_{DMP}$ as the "fundamental surplus"; they note that a high value of z_{DMP} generates a low fundamental surplus and a high elasticity of tightness. LS show that existing explanations that generate realistic elasticities do so by generating a low fundamental surplus (Υ is tightly constrained by data and do not differ much across models).

The effects of ambiguity in our setting can also be understood through the fundamental surplus. To see this, first adapt equation (26) for the canonical DMP setting to the DMP economy with a time-varying value of leisure $\widehat{\mathcal{E}}$ from Definition 2 by approximating the time-varying $\widehat{z}(y)$ by a constant value: $\widehat{z}(y) \approx \widehat{z}(\mathbb{E}^{\mathcal{P}}[y_t])$, where $\widehat{z}(\mathbb{E}^{\mathcal{P}}[y_t])$ is the effective value of leisure (24) evaluated at the mean productivity value $\mathbb{E}^{\mathcal{P}}[y_t]$. To first-order, the canonical DMP setting with $z_{DMP} = \widehat{z}(\mathbb{E}^{\mathcal{P}}[y_t])$ approximates $\widehat{\mathcal{E}}$ ($\widehat{\mathcal{E}}$ becomes the canonical DMP setting if $\widehat{z}(y) = z_{DMP}$ for all y). Using this approximation along with equation (26), we get $d\log\widehat{\theta}(y)/dy \approx \Upsilon e^y/(e^y - \widehat{z}(\mathbb{E}^{\mathcal{P}}[y_t]))$. Second, we note that Proposition 1 implies $d\log\widehat{\theta}(y)/dy = d\log\theta(y)/dy$. Therefore, $d\log\theta(y)/dy = d\log\widehat{\theta}(y)/dy \approx \Upsilon e^y/(e^y - \widehat{z}(\mathbb{E}^{\mathcal{P}}[y_t]))$, which implies that

$$d\log\theta(y)/dy \approx \Upsilon \frac{e^y}{e^y - \widehat{z}(\mathbb{E}^{\mathcal{P}}[y_t])}. \tag{27}$$

From equation (27) we see that the elasticity of tightness with respect to productivity in the economy with ambiguity is also inversely related to the fundamental surplus. Specifically, comparing expressions (26) and (27), we see that the fundamental surplus in $\mathcal{E}$ is $e^y - \widehat{z}(\mathbb{E}^{\mathcal{P}}[y_t])$ where $\widehat{z}$ is defined in equation (24).

Next, we analyze the quantitative implications of ambiguity in Section 4.

4 Quantitative Analysis

In this section, we investigate the quantitative implications of the dynamic economy with ambiguity described in Sections 3.1 and 3.2. We solve the model numerically using the procedure described in Appendix C.1. We show that ambiguity (i) significantly amplifies the volatility of labor market fluctuations and (ii) is responsible for a part of the high value of leisure required by the canonical DMP model to generate realistic unemployment rate volatility.

4.1 Calibration

We first choose a subset of parameters on an a priori basis before choosing the remaining parameters based on model-implied moments. We summarize our calibration procedure below; further details regarding the calibration are available in Appendix C.2.

Apriori set parameters. Our parameters are based on the sample 1951–2019. We set the risk-free rate $r_f = 0.013$ to be the difference between the average one year nominal treasury rate and average inflation.

We estimate the aggregate productivity process (8) using the quarterly real output per person for the nonfarm business sector series; the resulting estimates are $\kappa_y = 1.09$ and $\sigma_y = 0.018$.

We calibrate the match-specific productivity process (10) as follows. First, we choose $\kappa_\varepsilon = 1.09$ to be equal to the speed of mean reversion for the aggregate productivity process. We then choose σ_ε so that the unconditional volatility of match-specific productivity is equal to the cross-sectional standard deviation of annual earnings growth of workers over 1979-2011 as reported in Guvenen et al. (2014) (see their Table A8). This choice implies $\sigma_\varepsilon/\sqrt{2\kappa_\varepsilon} = 0.53$ for match-specific productivity (for simplicity, we ignore the effect of the reflecting boundary in making this calculation). For our choice of $\kappa_\varepsilon = 1.09$, this implies $\sigma_\varepsilon = 0.78$.

The Cobb-Douglas matching function implies a job finding rate of $f(\theta_t) = A\theta_t^{1-\alpha}$. Hence, we set $\alpha = 1 - std(\log f_t)/std(\log \theta_t) = 0.48$ based on data values of $std(\log f_t) = 0.089$ and $std(\log \theta_t) = 0.171$. Afterwards, we set $A = 8.48$ so that the model-implied job finding rate, evaluated at the mean value of tightness in the sample 0.66, agrees with its sample mean of 6.83. Finally, we set $s = 0.394$ based on the mean separation rate in the data.

Calibrated parameters. There are four remaining parameters: vacancy costs c, the value of leisure z, workers' bargaining power β, and the worst-case model $\overline{\gamma}$. We set $c = 0.298$, $z = 0.865$, and $\beta = 0.106$ so that the model-implied values for average tightness, the volatility of log tightness, and wage elasticity matches their data counterparts of 0.66, 0.171, and 0.4, respectively.

We set $\overline{\gamma} = 1.35$ so that the detection error probability for differentiating between the reference model and the worst-case model is equal to 0.2 (we choose this target of 0.2 following Hansen and Sargent 2008, p. 320). The detection error probability (see, e.g., Hansen and

Sargent 2008, chapter 9) associated with $\overline{\gamma}$ is

$$\text{DEP}(\overline{\gamma}) = \frac{1}{2}\mathcal{P}\left(L(\mathcal{P}^{\overline{\gamma}}, \mathcal{H}) > L(\mathcal{P}, \mathcal{H})\right) + \frac{1}{2}\mathcal{P}^{\overline{\gamma}}\left(L(\mathcal{P}^{\overline{\gamma}}, \mathcal{H}) < L(\mathcal{P}, \mathcal{H})\right), \tag{28}$$

where $L(\mathcal{M}, \mathcal{H})$ denotes the likelihood function, under model $\mathcal{M} \in \{\mathcal{P}, \mathcal{P}^{\overline{\gamma}}\}$, associated with the history of observations for productivity over the life time of a match, $\mathcal{H} = \left\{y_t, \varepsilon_{it} : t \in [\tau^i_{match}, \tau^i_{sep}]\right\}$. The detection error probability (28) can be interpreted as follows. Consider a firm that faces ambiguity regarding the behavior of match-specific productivity ε_{it} for a given match i—it must decide between the reference model (under which ε_{it} behaves according to equation (10)) or the worst-case model (under which ε_{it} behaves according to equation (13) with $\gamma_t = \overline{\gamma}$). The firm applies a likelihood criterion—given an observed history of match productivity $\mathcal{H}$, it decides in favor of the model $\mathcal{M}$ with the higher likelihood function $L(\mathcal{M}, \mathcal{H})$. Equation (28) gives the probability of making a wrong inference under the prior assumption that both models are equally likely. Appendix C.3 provides further details for computing (28).

[Table 1 about here]

Table 1 shows the moments of the baseline model (see column 2) along with their data counterparts (see column 1). We see that the model generates realistic labor market moments.

4.2 Quantitative Implications

4.2.1 Ambiguity amplifies the volatility of tightness

Next, we carry out a comparative static exercise that demonstrate how ambiguity amplifies the volatility of tightness. Specifically, we vary the amount of ambiguity $\overline{\gamma}$ and study the implications for labor market fluctuations.

[Figure 1 about here]

Panel A of Figure 1 shows the volatility of log tightness for various values of the worst-case model $\overline{\gamma}$. To isolate the effect of ambiguity on the volatility of tightness, for each value of $\overline{\gamma}$, we choose c and β to match average tightness and the elasticity of wages in the data. We see that the volatility of tightness increases sharply with $\overline{\gamma}$. For instance, in the absence of ambiguity, that is when $\overline{\gamma} = 0$, the volatility of tightness is 0.049. In comparison, when $\overline{\gamma} = 1.35$ (as in our baseline model), the volatility of tightness matches the data value of 0.171. Therefore, ambiguity towards match-specific productivity amplifies the volatility of tightness by a factor of 3.5. In Appendix A, we show that ambiguity towards aggregate productivity achieves a much smaller amplification factor of 1.13.

Proposition 1 and equation (27) help us understand why tightness volatility increases with $\overline{\gamma}$ using the familiar setting of the canonical DMP model. To this end, we first determine the effect of ambiguity on the effective value of leisure $\widehat{z}$ in the equivalent economy without ambiguity $\widehat{\mathcal{E}}$. The relation between the effective and the actual values of leisure is given by equation (24), which we rewrite as:

$$\widehat{z}(\mathbb{E}^{\mathcal{P}}[y_t]) - z = \underbrace{\overline{\gamma}\sigma_{\varepsilon} S_{\varepsilon}(\mathbb{E}^{\mathcal{P}}[y_t], \varepsilon_0)}_{\text{ambiguity effect}} + \underbrace{-\frac{1}{2}\sigma_{\varepsilon}^2 S_{\varepsilon\varepsilon}(\mathbb{E}^{\mathcal{P}}[y_t], \varepsilon_0)}_{\text{convexity effect}}. \tag{29}$$

We refer to the first term on the right-hand side of equation (29) as the "ambiguity effect"; the solid line in panel B of Figure 1 illustrates its magnitude. This term is positive because the surplus is increasing in match-specific productivity (i.e., $S_{\varepsilon} > 0$). We also see from (29) that the ambiguity effect is expected to become larger when the worst-case model becomes more severe (i.e., for larger values of $\overline{\gamma}$); the solid line in panel B of Figure 1 confirms this.

We refer to the second term on the right-hand side of equation (29) as the "convexity effect"; the dashed line in panel B of Figure 1 illustrates its magnitude. This term is negative and stays roughly constant as we vary $\overline{\gamma}$.

In our calibration, the ambiguity effect is larger than the convexity effect; therefore,

$\widehat{z}(\mathbb{E}^{\mathcal{P}}[y_t]) > z$. Moreover, the effective value of leisure $\widehat{z}(\mathbb{E}^{\mathcal{P}}[y_t])$ increases with $\overline{\gamma}$ because the ambiguity effect increases with $\overline{\gamma}$, while the convexity effect is roughly constant. We show this in Panel C of Figure 1. For example, $\widehat{z}(\mathbb{E}^{\mathcal{P}}[y_t])$ increases from 0.81 to 0.97 when $\overline{\gamma}$ increases from 0 to 1.35.

As discussed in Section 3.3.2, the increase in the effective value of leisure $\widehat{z}(\mathbb{E}^{\mathcal{P}}[y_t])$ associated with an increase in $\overline{\gamma}$ reduces the fundamental surplus $e^y - \widehat{z}(\mathbb{E}^{\mathcal{P}}[y_t])$. Equation (27) then implies that this reduction in the fundamental surplus leads to an increase in the elasticity of tightness with respect to productivity. This heightened elasticity, in turn, leads to an increase in the volatility of tightness.

In Appendix C.4, we report results for a related comparative static exercise in which we quantify the amplification effect of ambiguity for various values of leisure z.

4.2.2 High effective value of leisure from ambiguity aversion

The canonical DMP model requires a high value of leisure to generate empirically realistic volatility of tightness. Hagedorn and Manovskii (2008), henceforth HM, find the necessary value of leisure to be 95.5% of average productivity. In this section we show that a non-negligible portion of this value of leisure in HM can be interpreted as arising due to ambiguity aversion.

To show this, we follow the same calibration strategy in HM for the economy with ambiguity $\mathcal{E}$. We vary $\overline{\gamma}$ and choose c, z, and β to match the level and volatility of tightness and the cyclicality of wages. We fix the remaining parameters to their baseline values.[4]

[Figure 2 about here]

Figure 2 shows the results. The solid line in panel A shows the actual value of leisure z

[4]This exercise is different from the ones in Section 4.2.1 where we chose c and β to match the average tightness and the elasticity of wages in the data. Here, we additionally vary z to match the volatility of tightness.

as we vary $\overline{\gamma}$. We see that higher values of ambiguity $\overline{\gamma}$ require a relatively lower value of z to generate realistic unemployment rate volatility. For instance, when there is no ambiguity (i.e., $\overline{\gamma} = 0$), a value of z slightly greater than 1 is needed to match unemployment volatility, whereas when $\overline{\gamma} = 1.35$ (which is the baseline model in Column (2) of Table 1), the required value of $z = 0.865$.

The dotted line in panel A of Figure 2 shows the effective value of leisure $\widehat{z}(\mathbb{E}^{\mathcal{P}}[y_t])$ computed using equation (24). We see from this plot that $\widehat{z}(\mathbb{E}^{\mathcal{P}}[y_t])$ stays in a narrow range between 0.963 and 0.973, regardless of the level of ambiguity $\overline{\gamma}$. Although $\widehat{z}(\mathbb{E}^{\mathcal{P}}[y_t])$ remains roughly unchanged, its interpretation changes as we vary $\overline{\gamma}$—for example, Panel B of Figure 2 shows that when $\overline{\gamma}$ is high, $\widehat{z}(\mathbb{E}^{\mathcal{P}}[y_t])$ can be interpreted as a combination of a much lower actual value of leisure z along with a significant ambiguity effect.

This observation leads to an interpretation of the value of leisure estimated by HM in which a non-negligible portion of the value of leisure is potentially due to ambiguity aversion towards match-specific productivity. To see this, consider the case of an econometrician who encounters a time series of tightness that has actually been generated in an economy in which $\overline{\gamma} = 1.35$ (as in our baseline model). Further suppose that this econometrician attempts to fit the moments of tightness using a model such as the canonical DMP model which does not account for match-specific productivity and ambiguity aversion. Panel A of Figure 2 shows that the econometrician would estimate the value of leisure to be 0.97, even though the actual value of leisure z is much smaller at 0.865.

5 Conclusion

We illustrate and quantify the consequences of ambiguity aversion on unemployment rate dynamics. To this end, we consider a cross-section of workers in the search and matching framework of the Diamond-Mortensen-Pissarides (DMP) model. The productivity of each worker is the sum of an aggregate and a match-specific component. We focus on model misspecification concerns, or ambiguity aversion, regarding the match-specific component.

We show that this model misspecification concern has a quantitatively large effect on the dynamics of aggregate labor market variables. Our model delivers two insights. First, we show that ambiguity aversion substantially increases the volatility of the unemployment rate, thereby improving upon a well-known shortcoming of the canonical DMP model. Second, we show that a part of the high value of leisure required by the canonical DMP model to generate realistic unemployment rate volatility can arise from fitting a model missing ambiguity aversion to data generated in an environment where agents are ambiguity averse.

Tables and figures

Moment	(1) Data	(2) Baseline	(3) No ambiguity
Tightness: mean, $\mathbb{E}^{\mathcal{P}}[\theta_t]$	0.66	0.66	0.66
volatility, $std^{\mathcal{P}}(\log\theta_t)$	0.171	0.171	0.049
Unemployment: mean, $\mathbb{E}^{\mathcal{P}}[u_t]$	0.058	0.055	0.055
volatility, $std^{\mathcal{P}}(u_t)$	0.0048	0.0046	0.0015
Job finding rate: mean, $\mathbb{E}^{\mathcal{P}}[f(\theta_t)]$	6.83	6.81	6.83
volatility, $std^{\mathcal{P}}[\log f(\theta_t)]$	0.089	0.089	0.026
Elasticity of wages w.r.t. productivity	0.4	0.4	0.4
Detection error probability	–	0.2	–

Table 1: Model moments. Column (2) displays the moments of the baseline model whose parameters are described in Section 4.1; column (1) shows the corresponding data moments. The detection error probability is calculated according to equation (28). Column (3) displays the moments of an economy without ambiguity (i.e., $\overline{\gamma} = 0$) in which we fix $z = 0.865$ to be equal to that of the baseline model; c and β are recalibrated to match the level of tightness and the elasticity of wages.

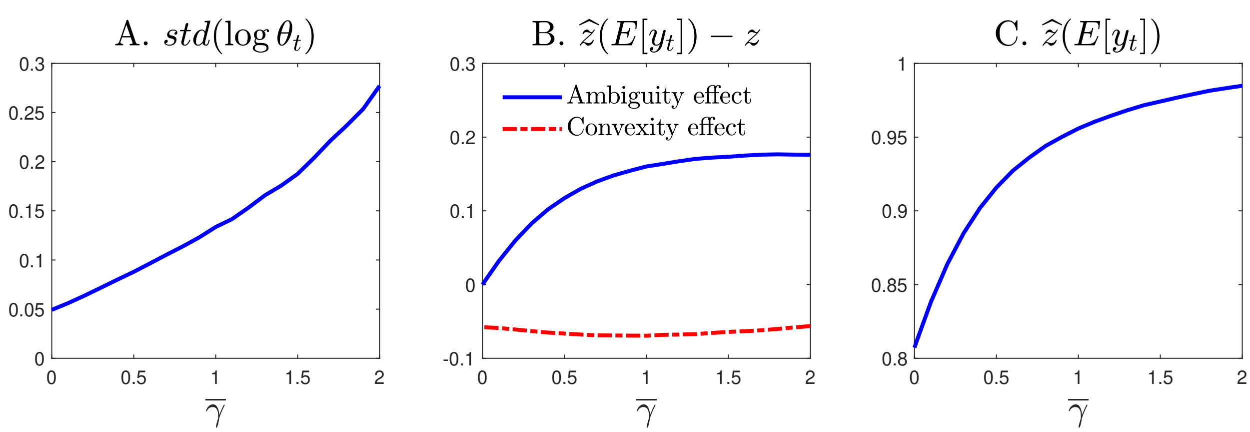

Figure 1: Effect of varying $\overline{\gamma}$. We vary the worst-case model $\overline{\gamma}$ and recalibrate c and β so that the model-implied average tightness and the elasticity of wages match the data. Panel A plots the volatility of tightness as a function of $\overline{\gamma}$. Panel B plots the ambiguity and convexity effects from equation (29) in the solid and dotted lines, respectively. Panel C plots the implied effective value of leisure $\widehat{z}(\mathbb{E}^{\mathcal{P}}[y_t])$.

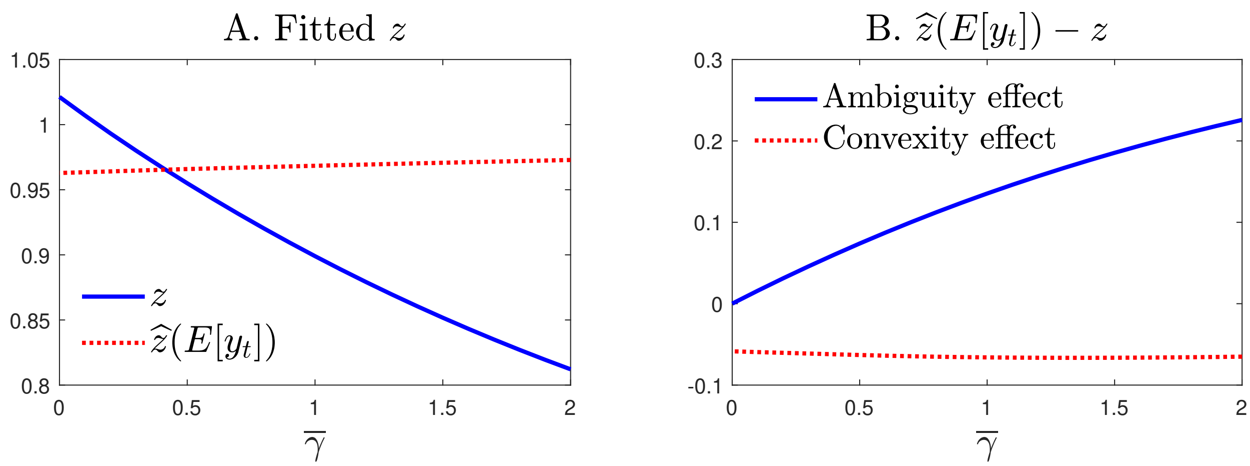

Figure 2: Interpreting a high effective value of leisure as ambiguity aversion. We vary the worst-case model $\overline{\gamma}$ and we recalibrate c, z, and β so that the model-implied level and volatility of tightness and wage elasticity agrees with their data values for each value of $\overline{\gamma}$. Panel A plots the actual and the effective values of leisure. Panel B plots the ambiguity and the convexity effects of ambiguity on the surplus (see equation (29)).

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Appendix

A Ambiguity regarding aggregate productivity

In this section, we show that ambiguity regarding aggregate productivity amplifies unemployment fluctuations by a much smaller amount compared to ambiguity regarding match-specific productivity.

Setting. Consider an environment that is identical to that of our baseline model described in Section 3.1 except for the following two differences. First, there is no match-specific productivity (i.e., $\varepsilon_{it} = 0$ for all i and t); as a result, there is also no ambiguity regarding match-specific productivity. Second, agents have ambiguity regarding the aggregate productivity process (8). Specifically, a distorted model $\mathcal{P}^\gamma$ of the aggregate productivity process is defined through

$$\mathcal{P}^\gamma(A) = \int_A \exp\left(-\frac{1}{2}\int_0^t \gamma_u^2\, du - \int_0^t \gamma_u\, dB_u^y\right) d\mathcal{P} \tag{30}$$

where A is an event in the time-t information set and $\gamma = (\gamma_t)_{t\geq 0}$ is a given stochastic process. Compared to distorted models from the baseline model (11), the distorted model (30) depends on the aggregate Brownian motion B_t^y (instead of the match-specific Brownian motion B_{it}). It then follows that

$$B_t^{y,\gamma} \equiv B_t^y + \int_0^t \gamma_u\, du, \tag{31}$$

is a standard Brownian motion under $\mathcal{P}^\gamma$, and that

$$dy_t = (-\sigma_y\gamma_t - \kappa_y y_t)dt + \sigma_y dB_t^{y,\gamma} \tag{32}$$

is the aggregate productivity process under $\mathcal{P}^\gamma$. We restrict attention to distorted models that fall within the set $\Gamma_t \equiv \{\gamma :\ \gamma_s \in [-\overline{\gamma}, \overline{\gamma}] \text{ for all } s \geq t\}$; the set Γ_t is analogous to specification (14) for the set of models under consideration in the baseline model.

The equilibrium can be derived following the same steps as in Section 3.2. In particular, the equilibrium surplus and tightness satisfies

$$r_f S(y) = \min_{\gamma\in[-\overline{\gamma},\overline{\gamma}]} e^y - z - \frac{\beta c}{1-\beta}\theta(y) + \mathscr{L}_y^\gamma S(y) - sS(y), \tag{33a}$$

$$c = (1-\beta)q(\theta(y))S(y), \tag{33b}$$

where $\mathscr{L}_y^\gamma = (-\kappa_y y - \gamma\sigma_y)\,\partial_y + \frac{1}{2}\sigma_y^2\partial_{yy}$, and the minimizer in equation (33a) is $\gamma(y) = \overline{\gamma}$ for all y. Equation (33) is analogous to the equilibrium characterization for the baseline model (17).

Observational equivalence. The implications of ambiguity regarding aggregate productivity for unemployment fluctuations can be understood through an observational equivalence result similar to Proposition 1. In particular, let $S(y)$ and $\theta(y)$ be the equilibrium surplus and tightness, respectively, in an economy with ambiguity regarding aggregate productivity (i.e., $S(y)$ and $\theta(y)$ are characterized through equation (33)), and let $\widehat{S}(y)$ and $\widehat{\theta}(y)$ be the equilibrium surplus and tightness, respectively, in the DMP economy with a time-varying value of leisure $\widehat{z}(y)$ from Definition 2. Then,

$\widehat{S}(y) = S(y)$ and $\widehat{\theta}(y) = \theta(y)$ if the effective value of leisure is given by

$$\widehat{z}(y) = z + \left(\mathscr{L}_y - \mathscr{L}_y^{\overline{\gamma}}\right) S(y) = z + \overline{\gamma}\sigma_y S'(y). \tag{34}$$

The derivation of this result is analogous to the derivation of Proposition 1.

The implication of equation (34) is that the effect of ambiguity towards aggregate productivity can once again be understood through the arguments of Section 3.3.2. Specifically, the elasticity of tightness is given by equation (27) where the effective value of leisure is given by equation (34). Comparing expression (34) to the term for the ambiguity effect from equation (29), we see that ambiguity increases the effective value of leisure by $\overline{\gamma}\sigma_y S'(y)$ in the context of ambiguity towards aggregate productivity versus an increase of $\overline{\gamma}\sigma_\varepsilon S_\varepsilon(y, \varepsilon_0)$ in the context of ambiguity towards match-specific productivity.

Quantitative analysis. To investigate the amplification effect of ambiguity towards aggregate productivity, we fix $\overline{\gamma} = 1.33$ to target a detection error probability of 0.2,[5] exactly as in our baseline calibration of Section 4.1, and then vary the value of leisure z. To isolate the effect of ambiguity regarding aggregate productivity on the volatility of tightness, we recalibrate c and β for each value of z so as to fix the average value of tightness and the elasticity of wages at their data values. The remaining parameters are taken from the baseline calibration in Section 4.1 (not counting κ_ε and σ_ε, neither of which apply to the context of the model with ambiguity towards aggregate productivity). Figure A.1 shows the results of this exercise.

The magnitude of the amplification effect of (1) ambiguity regarding aggregate productivity versus (2) ambiguity regarding match-specific productivity can be gleamed by comparing Figure A.1 against Figure C.2 in Appendix C.4. The latter figure reports results for an analogous exercise in the context of the baseline model with ambiguity towards match-specific productivity.

Comparing Figure A.1 to Figure C.2, we see that ambiguity regarding aggregate productivity generates a significantly smaller amplification effect compared to ambiguity towards match-specific productivity. For example, the ratio between the volatilities of labor market tightness between economies with and without ambiguity ranges between 1.01 and 1.13 when that ambiguity is over aggregate productivity (see Panel C of Figure A.1) versus between 2.2 and 4.1 when the ambiguity concern is over match-specific productivity (see Panel C of Figure C.2). This difference in amplification can be understood from our earlier discussion in the paragraph following equation (34)—ambiguity towards aggregate and match-specific productivity increase the effective value of leisure by $\overline{\gamma}\sigma_y S'(y)$ and $\overline{\gamma}\sigma_\varepsilon S_\varepsilon(y, \varepsilon_0)$, respectively. The latter increase is substantially larger because match-specific volatility σ_ε is larger than the volatility of aggregate productivity σ_y.

[5] In the context of ambiguity towards aggregate productivity, the detection error probability (28) becomes $\mathrm{DEP}(\overline{\gamma}) = \int_0^\infty \Phi\left(-\frac{1}{2}|\overline{\gamma}|\sqrt{\tau}\right) se^{-s\tau}\, d\tau$ where $\Phi(\cdot)$ denotes the standard normal cumulative density function. Here, $\Phi\left(-\frac{1}{2}|\overline{\gamma}|\sqrt{\tau}\right)$ is the detection error probability associated with distinguishing the process $B_t^{y,\overline{\gamma}}$ (defined in equation (31)) from B_t^y over a fixed horizon τ, and $s\exp(-s\tau)$ is the probability density function over τ given a separation rate of s.

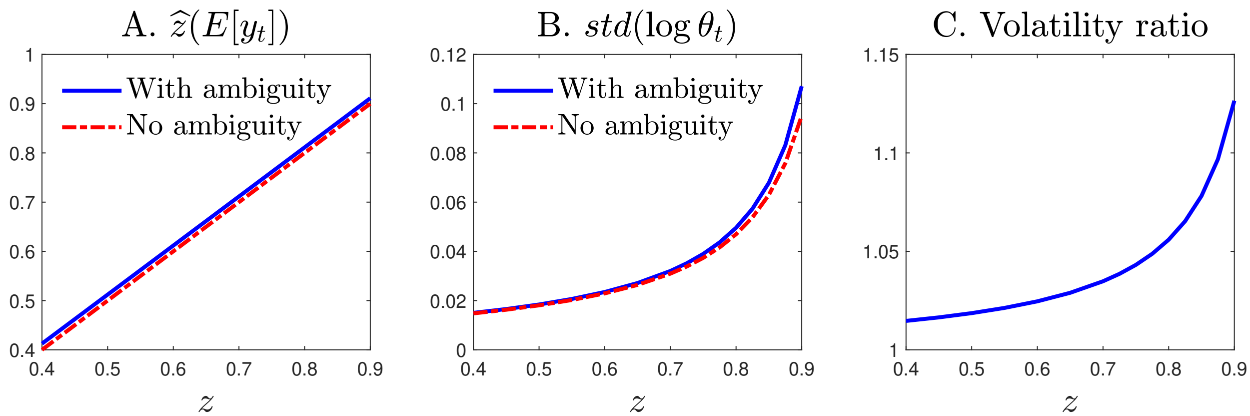

Figure A.1: Amplification: ambiguity regarding aggregate productivity. We vary the value of leisure and compare outcomes in an economy with ambiguity regarding aggregate productivity ($\overline{\gamma} = 1.33$) to an economy without ambiguity ($\overline{\gamma} = 0$); neither of these economies feature match-specific productivity. The solid and dot-dash lines in panel A shows the effective value of leisure $\widehat{z}(\mathbb{E}^{\mathcal{P}}[y_t])$ in the economies with and without ambiguity, respectively. The solid and dash-dot lines in panel B show the volatility of tightness in the economies with and without ambiguity, respectively; panel C shows the ratio of the solid and dot-dash lines from panel B.

B Omitted derivations

B.1 Equilibrium surplus in the economy with ambiguity

Firm and worker value functions. Let J_i and W_i denote the value of a match to the firm and the worker, respectively, when that match is filled by worker i. Their time-t values are given by

$$J_{it} = \min_{\gamma \in \Gamma_{it}} \mathbb{E}^{\gamma}_{it} \left[\int_t^{\tau^i_{sep}} e^{-r_f u} \left(e^{y_u + \varepsilon_{iu}} - w_{iu} \right) du + e^{-r_f(\tau^i_{sep} - t)} V_{\tau^i_{sep}} \right], \tag{35a}$$

$$W_{it} = \min_{\gamma \in \Gamma_{it}} \mathbb{E}^{\gamma}_{it} \left[\int_t^{\tau^i_{sep}} e^{-r_f u} w_{iu} \, du + e^{-r_f(\tau^i_{sep} - t)} U_{\tau^i_{sep}} \right], \tag{35b}$$

where r_f is the risk-free rate, $\mathbb{E}^{\gamma}_{it}$ denotes the time-t conditional expectation taken with respect to the distorted measure (11), and τ^i_{sep} denotes the time when worker i separates from a match. In equation (35a), J_{it} discounts future payoffs to the firm prior to separation $\exp(y_u + \varepsilon_{iu}) - w_{iu}$ and $V_{\tau^i_{sep}}$ is the value of a vacancy upon separation. In equation (35b), W_{it} discounts future wages w_{iu} that the worker earns while matched to the firm and $U_{\tau^i_{sep}}$ is the value of unemployment upon separation. Decision makers apply robust control by assuming that nature is malignant and selects the worst-case model from the set (14) which results in the lowest values for J_{it} and W_{it}; this is reflected in the minimizations in equation (35).

The time-t value of a vacancy to the firm and the value of unemployment for worker i, V_t and

U_t, respectively, are given by

$$V_t = \mathbb{E}_t^{\mathcal{P}}\left[-\int_t^{\tau^i_{match}} e^{-r_f u} c\, du + e^{-r_f(\tau^i_{match}-t)} J_{i,\tau^i_{match}}\right], \tag{36a}$$

$$U_t = \mathbb{E}_t^{\mathcal{P}}\left[\int_t^{\tau^i_{match}} e^{-r_f u} z\, du + e^{-r_f(\tau^i_{match}-t)} W_{i,\tau^i_{match}}\right], \tag{36b}$$

where τ^i_{match} denotes the time at which worker i gets matched to the firm. In equation (36a), V_t is computed by discounting the flow cost of maintaining a vacancy c and $J_{i,\tau^i_{match}}$ is the value to the firm upon successfully being matched to a worker. In equation (36b), U_t is computed by discounting the flow value of leisure z and $W_{i,\tau^i_{match}}$ is the value to the worker upon employment. Note that there is no i subscript associated with V_t and U_t because (i) their flow terms do not depend on ε_{it} prior to τ^i_{match}, (ii) the matching time τ^i_{match} only depends on aggregate conditions, and (iii) the value functions J_{it} and W_{it} are determined under the fixed initial value (9) at the time of the match $t = \tau^i_{match}$. Since both V_t and U_t do not explicitly depend on match-specific productivity, no additional minimizations are present in equation (36); ambiguity does, however, indirectly affect V_t and U_t through its effect on $J_{i,\tau^i_{match}}$ and $W_{i,\tau^i_{match}}$.

The Markovian solution to the system of equations (35) and (36), $J_{it} = J(y_t, \varepsilon_{it})$, $W_{it} = W(y_t, \varepsilon_{it})$, $V_t = V(y_t)$, and $U_t = U(y_t)$, satisfies

$$r_f J(y,\varepsilon) = \min_{\gamma\in[-\overline{\gamma},\overline{\gamma}]} e^{y+\varepsilon} - w(y,\varepsilon) + \mathscr{L}^{\gamma}_{y,\varepsilon} J(y,\varepsilon) + s\left[V(y) - J(y,\varepsilon)\right], \tag{37a}$$

$$r_f V(y) = -c + \mathscr{L}_y V(y) + q(\theta(y))\left[J(y,\varepsilon_0) - V(y)\right], \tag{37b}$$

$$r_f W(y,\varepsilon) = \min_{\gamma\in[-\overline{\gamma},\overline{\gamma}]} w(y,\varepsilon) + \mathscr{L}^{\gamma}_{y,\varepsilon} W(y,\varepsilon) + s\left[U(y) - W(y,\varepsilon)\right], \tag{37c}$$

$$r_f U(y) = z + \mathscr{L}_y U(y) + f(\theta(y))\left[W(y,\varepsilon_0) - U(y)\right], \tag{37d}$$

for $\varepsilon \geq \underline{\varepsilon}(y)$, along with the boundary conditions

$$J(y,\underline{\varepsilon}(y)) = V(y), \quad \text{and} \quad W(y,\underline{\varepsilon}(y)) = U(y), \tag{38a}$$

$$J_\varepsilon(y,\underline{\varepsilon}(y)) = 0, \quad \text{and} \quad W_\varepsilon(y,\underline{\varepsilon}(y)) = 0. \tag{38b}$$

Here, the infinitesimal generators $\mathscr{L}^{\gamma}_{y,\varepsilon}$ and $\mathscr{L}_y$ are given by equations (19) and (22), respectively. All four equations in (37) are a consequence of applying the Feynman-Kac formula (see, e.g., Duffie 2001, Appendix E) to equations (35) and (36).

Optimality of $\underline{\varepsilon}(y)$. To see why the separation threshold $\underline{\varepsilon}(y)$ is optimal from both the firm's and the worker's perspective, consider the match surplus

$$S_{it} \equiv J_{it} - V_t + W_{it} - U_t. \tag{39}$$

Nash bargaining splits this surplus such that the firm obtains $J_{it} - V_t = \beta S_{it}$ and the worker obtains $W_{it} - U_t = (1-\beta)S_{it}$. Hence, both the firm's and the worker's continuation values become equal to their outside options at the threshold (15) where the surplus is zero. The equations in (38a) reflect this value matching condition for the firm and the worker.

The equations in (38b) are smooth pasting conditions that characterize the optimal separation threshold from both the firm's and the worker's perspective. The threshold is the same from the perspectives of both parties since both $J_\varepsilon(y, \underline{\varepsilon}(y)) = 0$ and $W_\varepsilon(y, \underline{\varepsilon}(y)) = 0$ at the same threshold at which $S_\varepsilon(y, \underline{\varepsilon}(y)) = 0$ (note that Nash bargaining implies $J_\varepsilon(y, \varepsilon) = (1-\beta)S_\varepsilon(y, \varepsilon)$ and $W_\varepsilon(y, \varepsilon) = \beta S_\varepsilon(y, \varepsilon)$).

Characterization of the match surplus (17). Equation (17a) follows from plugging the system of equations (37) into the definition of the surplus (39).

To derive equation (17b), note that the free entry condition for vacancy creation implies $V(y) = 0$ and therefore, from equation (37b), $c = q(\theta(y))J(y, \varepsilon_0)$. In combination with equilibrium bargaining, $J(y, \varepsilon_0) = (1-\beta)S(y, \varepsilon_0)$, we obtain $c = (1-\beta)q(\theta(y))S(y, \varepsilon_0)$.

The two boundary conditions (18) for the surplus $S(y, \varepsilon)$ follow from equations (38a) and (38b) after applying the definition of the match surplus (39).

B.2 Equilibrium surplus in the economy without ambiguity

Let $\widehat{J}_t$ and $\widehat{W}_t$ denote the value of a match to the firm and the worker, respectively, when that match is filled by worker i. Let the time-t value of a vacancy to the firm and the value of unemployment for a worker be $\widehat{V}_t$ and $\widehat{U}_t$, respectively. The value functions in this economy without ambiguity are given by

$$\widehat{J}_t = \mathbb{E}_t^{\mathcal{P}}\left[\int_t^{\tau_{sep}} e^{-r_f u}\left(e^{y_u} - \widehat{w}_u\right) du + e^{-r_f(\tau_{sep}-t)}\widehat{V}_{\tau_{sep}}\right], \tag{40a}$$

$$\widehat{V}_t = \mathbb{E}_t^{\mathcal{P}}\left[-\int_t^{\tau_{match}} e^{-r_f u} c\, du + e^{-r_f(\tau_{match}-t)}\widehat{J}_{\tau_{match}}\right], \tag{40b}$$

$$\widehat{W}_t = \mathbb{E}_t^{\mathcal{P}}\left[\int_t^{\tau_{sep}} e^{-r_f u}\widehat{w}_u\, du + e^{-r_f(\tau_{sep}-t)}\widehat{U}_{\tau_{sep}}\right], \tag{40c}$$

$$\widehat{U}_t = \mathbb{E}_t^{\mathcal{P}}\left[\int_t^{\tau_{match}} e^{-r_f u}\widehat{z}(y_u)\, du + e^{-r_f(\tau_{match}-t)}\widehat{W}_{\tau_{match}}\right]. \tag{40d}$$

The system (40) follows from equations (35) and (36) after setting $\mathcal{P}^{\gamma_i} = \mathcal{P}$, $\varepsilon_{it} = 0$, and noting that the value of leisure is now $\widehat{z}(y_t)$.

The Markovian solution to the system (40), $\widehat{J}_t = \widehat{J}(y_t)$, $\widehat{V}_t = \widehat{V}(y_t)$, $\widehat{W}_t = \widehat{W}(y_t)$, and $\widehat{U}_t = \widehat{U}(y_t)$, satisfies:

$$r_f\widehat{J}(y) = e^y - \widehat{w}(y) + \mathscr{L}_y\widehat{J}(y) + s\left[\widehat{V}(y) - \widehat{J}(y)\right], \tag{41a}$$

$$r_f\widehat{V}(y) = -c + \mathscr{L}_y\widehat{V}(y) + q(\theta(y))\left[\widehat{J}(y) - \widehat{V}(y)\right], \tag{41b}$$

$$r_f\widehat{W}(y) = \widehat{w}(y) + \mathscr{L}_y\widehat{W}(y) + s\left[\widehat{U}(y) - \widehat{W}(y)\right], \tag{41c}$$

$$r_f\widehat{U}(y) = \widehat{z}(y) + \mathscr{L}_y\widehat{U}(y) + f(\widehat{\theta}(y))\left[\widehat{W}(y) - \widehat{U}(y)\right], \tag{41d}$$

where we have made use of the fact that equilibrium wages $\widehat{w}_t = \widehat{w}(y)$ and tightness $\widehat{\theta}_t = \widehat{\theta}(y)$ are functions of y. These equations follow from applying the Feynman-Kac formula to the system (40).

Equation (21) for the surplus $\widehat{S}(y)$ follows from plugging the system (41) into the definition of the surplus $\widehat{S} = \widehat{J} - \widehat{V} + \widehat{W} - \widehat{U}$ and using the free entry condition for vacancy creation $\widehat{V}(y) = 0$.

C Details for the quantitative analysis in Section 4

C.1 Solving the model numerically

Solving the model requires us to solve for the boundary value problem (17) and (18). We instead solve the following equivalent problem:

$$(r_f + s)S = \max\left\{\partial_t S + e^{y+\varepsilon} - z - \frac{\beta c}{1-\beta} q^{-1}\left(c/((1-\beta)S)\right) + \mathscr{L}^{\overline{\gamma}}_{y,\varepsilon} S, 0\right\}. \tag{42}$$

Here, the fictitious time derivative $\partial_t S$ equals zero at the final solution; its inclusion conveniently gives rise to an iterative numerical scheme. In addition, equation (42) reformulates the boundary conditions (18) as a variational inequality.

We use finite-difference methods to approximate the viscosity solution for equation (42) (see, e.g., Achdou et al. 2021). We discretize the state space using equally spaced grids for y and ε, and solve for the surplus value using the following unconditionally stable implicit scheme:

$$\mathbf{S}^{n+1} = \max\left\{\left[(1 + (r_f + s)\Delta)\mathbf{I} - \mathbf{L}^{\overline{\gamma}}_{y,\varepsilon}\right]^{-1}\left(\mathbf{a}_n\Delta + \mathbf{S}_n\right), 0\right\}, \tag{43}$$

where the vector $\mathbf{S}^n$ stacks the values of S^n at the grid points, $\mathbf{I}$ is the identity matrix, $\mathbf{L}^{\overline{\gamma}}_{y,\varepsilon}$ is a matrix that discretizes $\mathscr{L}^{\overline{\gamma}}_{y,\varepsilon}$ using an upwind scheme, and the vector $\mathbf{a}_n$ stacks the values of $a_n(y,\varepsilon) = e^{y+\varepsilon} - z - \frac{\beta c}{1-\beta} q^{-1}\left(c/((1-\beta)S^n(y,\varepsilon_0))\right)$ at the grid points, and Δ is a time step.

In our numerical implementation, we set the grids for y and ε to be sufficiently wide so as to cover ± 4 standard deviations of their respective unconditional distributions under both the reference and the worst-case models. In addition, we choose a time step of a month, $\Delta = 1/12$, and iterate equation (43) until $\mathbf{S}^{n+1}$ and $\mathbf{S}^n$ is within 10^{-8} across all grid points.

C.2 Calibration details

This section provides further details for the calibrated parameters described in Section 4.1.

The risk-free rate of $r_f = 0.013$ is set as the difference between the average one year nominal treasury rate and average inflation. The average one year treasury rate is 5.1% over 1962-2019 (based on the DGS1 series from Federal Reserve Economic Data (FRED) which is available starting from 1962) while average CPI inflation is 3.8% per anum over the same period.

We estimate the aggregate productivity process (8) using the quarterly real output per person for the nonfarm business sector series (i.e., PRS85006163 from FRED) which we log and HP-filter using a smoothing parameter of 1600. After fitting an AR(1) model to the resulting series, we find an autoregressive coefficient of 0.761 and a volatility of 0.008 for the innovations. We convert these values to the parameters for the aggregate productivity process (8) through $0.761 = \mathbb{E}^{\mathcal{P}}_t[y_{t+\Delta}]/y_t = \exp(-\kappa_y\Delta)$ and $0.008 = \sqrt{Var^{\mathcal{P}}_t(y_{t+\Delta})} = \sigma_y\sqrt{(1-\exp(-2\kappa_y\Delta))/(2\kappa_y)}$ where $\Delta = 0.25$ for a horizon of a quarter. This results in $\kappa_y = 1.09$ and $\sigma_y = 0.018$.

We obtain (annualized) job-finding rates f_t and (annualized) job-separation rates s_t in the data following the procedure in Elsby et al. (2009). We measure labor market tightness using the updated Barnichon (2010) vacancies series. We find f_t, s_t, and θ_t equal 6.83, 0.394, and 0.66 on average, respectively. We also find $std(\log f_t) = 0.089$ and $std(\log \theta_t) = 0.171$ after we log and HP-filter the monthly f_t and θ_t series using a smoothing parameter of 10000 (as in Shimer 2005).

We follow the procedure in Hagedorn and Manovskii (2008) and estimate the elasticity of wages, measured as the labor share (PRS85006173 from FRED) times labor productivity (OPHNFB from FRED), with respect to labor productivity to be 0.4; both the wage and productivity series are quarterly, and are logged and HP-filtered with a smoothing parameter of 1600. In the model, we measure wage elasticity as the regression coefficient $Cov^{\mathcal{P}}(\log w(y_t, \varepsilon_0), y_t)/Var^{\mathcal{P}}(y_t)$.

C.3 Detection error probabilities

We compute the detection error probability (28) using simulation methods. We describe the procedure below.

Without loss of generality, let $t = 0$ be the time at which match i is formed. The duration of the match $\tau^i_{sep} \sim Exp(1/s)$ follows an exponential distribution with a mean of $1/s$. We simulate discretized processes for y_t and ε_{it} over $t \in [0, \tau^i_{sep}]$. The discretized process for y_t is the same under both the reference model $\mathcal{P}$ and the worst-case model $\mathcal{P}^{\overline{\gamma}}$; it is given by

$$y_{t+\Delta} = \varrho_y y_t + \varsigma_y e_{y,t+\Delta}, \quad e_{y,t+\Delta} \overset{iid}{\sim} N(0,1), \tag{44}$$

where Δ is the step size, $\varrho_y = \exp(-\kappa_y \Delta)$, and $\varsigma_y = \sigma_y \sqrt{(1-\exp(-2\kappa_y\Delta))/(2\kappa_y)}$. The initial value y_0 is drawn according to the steady state distribution for the aggregate productivity process (8), $y_0 \sim N(0, \sigma^2_{y,ss})$, where $\sigma_{y,ss} = \sigma_y/\sqrt{2\kappa_y}$. The law of motion for ε_{it} depends on the model and is

$$\varepsilon_{it+\Delta} = \max\left\{(1-\varrho_\varepsilon)\mu_\varepsilon(\mathcal{M}) + \varrho_\varepsilon \varepsilon_t + \varsigma_\varepsilon e_{\varepsilon,t+\Delta}, \underline{\varepsilon}(y_{t+\Delta})\right\}, \quad e_{\varepsilon,t+\Delta} \overset{iid}{\sim} N(0,1), \tag{45}$$

under model $\mathcal{M} \in \{\mathcal{P}, \mathcal{P}^{\overline{\gamma}}\}$, where the initial value is $\varepsilon_{i0} = \varepsilon_0$, $\mu_\varepsilon(\mathcal{P}) = 0$, $\mu_\varepsilon(\mathcal{P}^{\overline{\gamma}}) = -\overline{\gamma}\sigma_\varepsilon/\kappa_\varepsilon$, $\varrho_\varepsilon = \exp(-\kappa_\varepsilon\Delta)$, $\varsigma_\varepsilon = \sigma_\varepsilon\sqrt{(1-\exp(-2\kappa_\varepsilon\Delta))/(2\kappa_\varepsilon)}$, and $e_{\varepsilon,t}$ is independent from $e_{y,t}$.

Given a history of observations $\mathcal{H} = \{y_{n\Delta}, \varepsilon_{i,n\Delta}\}_{n=0}^{N^i_{sep}}$, where $N^i_{sep} = \tau^i_{sep}/\Delta$ is the number of steps over the life time of the match, the likelihood function under model $\mathcal{M}$ is

$$L(\mathcal{M}, \mathcal{H}) = f_{y,ss}(y_0) \times \Pi_{n=1}^{N^i_{sep}} f_{y'|y}(y_{n\Delta}|y_{(n-1)\Delta}) \times \Pi_{n=1}^{N^i_{sep}} f^{\mathcal{M}}_{\varepsilon'|\varepsilon,y'}(\varepsilon_{n\Delta}|\varepsilon_{(n-1)\Delta}, y_{n\Delta}) \tag{46}$$

where $f_{y,ss}(y_0) = \left(\sigma_{y,ss}\sqrt{2\pi}\right)^{-1}\exp\left(-y_0^2/(2\sigma^2_{y,ss})\right)$, $f_{y'|y}(y'|y) = \left(\varsigma_y\sqrt{2\pi}\right)^{-1}\exp\left(-(y'-\varrho_y y)^2/(2\varsigma_y^2)\right)$, and

$$f^{\mathcal{M}}_{\varepsilon'|\varepsilon,y'}(\varepsilon'|\varepsilon, y') = \begin{cases} \left(\varsigma_\varepsilon\sqrt{2\pi}\right)^{-1}\exp\left(-(y'-(1-\varrho_\varepsilon)\mu_\varepsilon(\mathcal{M})-\varrho_y y)^2/(2\varsigma_\varepsilon^2)\right) & \text{if } \varepsilon' < \underline{\varepsilon}(y'), \\ \Phi\left((y'-(1-\varrho_\varepsilon)\mu_\varepsilon(\mathcal{M})-\varrho_y y)/\varsigma_\varepsilon\right) & \text{if } \varepsilon' = \underline{\varepsilon}(y'), \end{cases} \tag{47}$$

with $\Phi(\cdot)$ denoting the standard normal cumulative density function.

Finally, we average over a large number of paths $\mathcal{H}$ to compute detection error probabilities. For example, to compute the $\mathcal{P}\left(L(\mathcal{P}^{\overline{\gamma}}) > L(\mathcal{P})\right)$ term, we simulate histories $\mathcal{H}_k$, $k =$

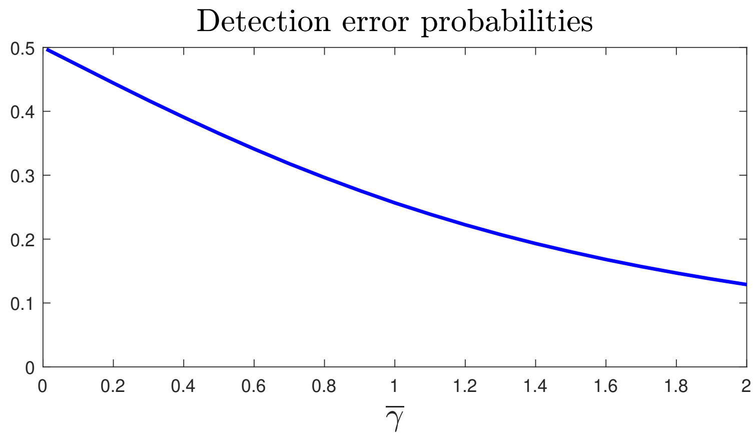

Figure C.1: Detection error probability. We plot the detection error probability defined in equation (28) as we vary the worst-case model $\overline{\gamma}$. For each value of $\overline{\gamma}$, we choose c, z, and β, to match the level and volatility of tightness and the elasticity of wages with respect to productivity. The remaining parameters are equal to those from the baseline calibration described in Section 4.1.

$1, ..., K$, under model $\mathcal{P}$ (i.e., by using equation (44) and equation (45) under $\mathcal{M} = \mathcal{P}$), and approximate $\mathcal{P}\left(L(\mathcal{P}^{\overline{\gamma}}) > L(\mathcal{P})\right) \approx K^{-1}\sum_{k=1}^{K} 1\left\{L(\mathcal{P}^{\overline{\gamma}}, \mathcal{H}_k) > L(\mathcal{P}, \mathcal{H}_k)\right\}$. The computation of the $\mathcal{P}^{\overline{\gamma}}\left(L(\mathcal{P}^{\overline{\gamma}}) < L(\mathcal{P})\right)$ term is analogous. In our simulations, we use a step size of $\Delta = 0.01$ and average over 10^6 paths.

Figure C.1 illustrates the detection error probability (28) as a function of $\overline{\gamma}$.

C.4 Amplification for various values of leisure z.

The comparative statics exercise in Section 4.2.1 considers labor market fluctuations as we vary $\overline{\gamma}$. In this section, we investigate the amplification effect of ambiguity for various values of z. We do so because estimates for the value of leisure z vary widely in the literature. Specifically, we fix $\overline{\gamma} = 1.35$ at its baseline value from Section 4.1 and vary the value of leisure z, starting from a low value of 0.4 (used by Shimer 2005) to a maximum value of 0.9. For each value of z, we recalibrate c and β to hold the average value of tightness and the elasticity of wages at their data values; this helps us isolate the effect of ambiguity on the volatility of tightness. To gauge the amplification effects of ambiguity, we benchmark the results against outcomes in the economy with no ambiguity (i.e., $\overline{\gamma} = 0$).

We begin by showing the effective value of leisure $\widehat{z}(\mathbb{E}^{\mathcal{P}}[y_t])$ as we vary z. The solid and dash-dot lines in panel A of Figure C.2 show the effective value of leisure in economies with and without ambiguity, respectively. We see that the effective value of leisure is substantially higher in the economy with ambiguity, especially at low values of z. For instance, when $z = 0.4$, the effective value of leisure in the economy with ambiguity is two and a half times its counterpart from the economy without ambiguity.

A high value of effective leisure implies a low fundamental surplus $e^y - \widehat{z}(\mathbb{E}^{\mathcal{P}}[y_t])$, which, in turn, implies a high volatility of tightness. We see this in panels B and C of Figure C.2. Panel C shows the amplification factor in the presence of ambiguity—it plots the ratio of the volatility of tightness

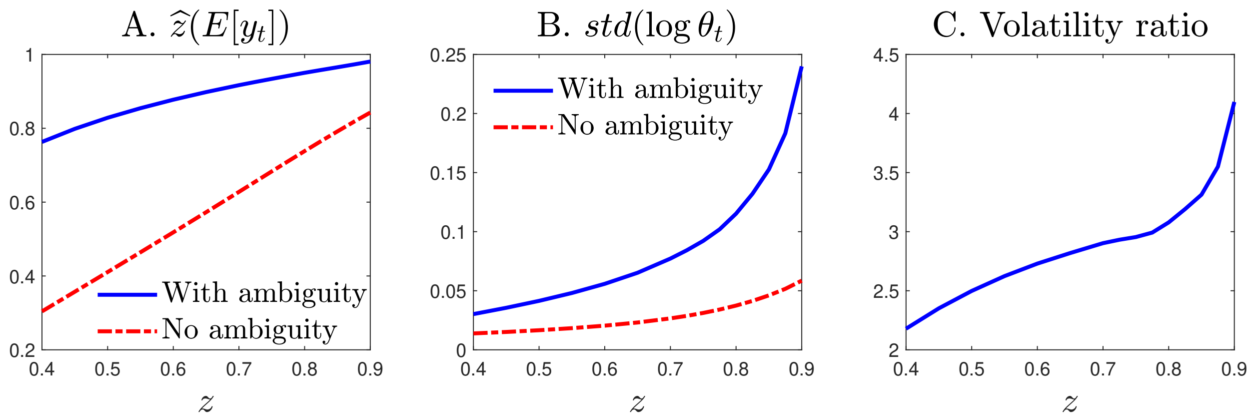

Figure C.2: Amplification for various values of leisure. We vary the value of leisure and compare outcomes in an economy with ambiguity ($\overline{\gamma} = 1.35$) to an economy without ambiguity ($\overline{\gamma} = 0$). The solid and dot-dash lines in panel A shows the effective value of leisure $\widehat{z}(\mathbb{E}^{\mathcal{P}}[y_t])$ in the economies with and without ambiguity, respectively. The solid and dash-dot lines in panel B show the volatility of tightness in the economies with and without ambiguity, respectively; panel C shows the ratio of the solid and dot-dash lines from panel B.

in the economies with and without ambiguity, $std^{\mathcal{P}}(\log\theta_t;\overline{\gamma} = 1.35)/std^{\mathcal{P}}(\log\theta_t;\overline{\gamma} = 0)$. We see that over the range of $z \in [0.4, 0.9]$, ambiguity amplifies tightness volatility by a factor that ranges between 2.2 and 4.1; for the value of leisure of our baseline calibration, $z = 0.865$, this factor is 3.5.

The main takeaway from this exercise is that ambiguity amplifies the volatility of tightness, and the amplification factor is robust across the wide range of values of leisure considered in the search literature. We emphasize the quantitative importance of ambiguity towards match-specific productivity; Appendix A analyzes the case of ambiguity towards aggregate productivity and shows that such an alternative generates smaller amplification for the volatility of tightness.