Asset Safety and Liquidity over the Business Cycle[*]

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Abstract

We decompose the convenience yield of U.S. Treasury bonds into safety and liquidity premia and document a positive relationship of these components with unemployment over the business cycle. This counter-cyclical pattern persists after controlling for monetary policy, the supply of treasuries and uncertainty. In addition, we find that the safety premium is much more volatile than the liquidity premium, especially during recessions. To rationalize these findings, we propose a real business cycle model with labor search frictions, private and public liquid assets and corporate default. We carefully calibrate the model to the U.S. economy and show that it generates endogenous fluctuations in the safety and liquidity premia of U.S. Treasury bonds that are consistent with the data.

Keywords: Treasury bonds; corporate bonds; convenience yield; safety premium; liquidity premium; business cycle.

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1 Introduction

The market for U.S. Treasury securities (also known as U.S. treasuries) is one of the largest financial markets in the world. Not only these public assets are crucial for the functioning of the U.S. economy, they also play a pivotal role in financial markets of other economies. Treasuries are usually considered the safest and most liquid financial asset in the world. Hence they are highly sought after by investors as a safe store of value and for their liquidity services. It is not surprising then that U.S. treasury bonds carry a convenience yield or premium. This premium reflects the additional value that these bonds provide beyond its direct financial return. These benefits arise from the special characteristics or uses of the U.S. treasury bonds, such as their safe haven properties, their collateral use and for their regulatory requirements. Since investors value these benefits, U.S. treasury bonds are more expensive (i.e. offer a lower yield) relative to other debt securities with similar maturity. The difference between the yield on treasuries and the yield on other debt securities captures the value of the treasury premium. Despite its global importance, there is still not much work documenting how this premium behaves over the business cycle. Even fewer papers have developed frameworks to understand the potential channels that deliver such patterns. This paper fills these gaps.

Having a better understanding of what drives variations in the treasury premium is not important in itself, but is also crucial as it can impact monetary policy. This is the case as the Federal Reserve intervenes in the treasury market to conduct monetary policy and to ensure financial stability. Thus, it is important to determine whether abnormal spikes in treasury yields are driven by liquidity or safety considerations. The nature of the intervention is typically going to depend on what is the main driver for such spike. In particular, in the context of quantitative tightening, having a better understanding of the components of the treasury premium is key in assessing how many safe assets are needed for a good functioning of the financial system and how many are needed to effectively conduct open-market operations. In this paper we propose a framework that allows us to decompose the premia of U.S. treasuries and examine their properties over the business cycle.

To disentangle the safety and liquidity components, we first document the fluctuations of the U.S. treasury premium over the business cycle. We then decompose it into the sum of two components: safety and liquidity premia. Following Krishnamurthy and Vissing-Jorgensen (2012), we define the treasury premium as the yield spread between treasury bonds and a risky and illiquid bond of similar maturity. As is standard in the literature, these latter securities are Baa-rated corporate bonds. We then decompose the treasury premium into a spread between the aforementioned bond and an equally illiquid but safe bond (Aaa-rated corporate bonds), which we define as the safety premium. We then define the liquidity premium as the spread between Aaa-rated corporate bonds and the treasury bond.

Using 20-year U.S. treasury bonds and Moody's Aaa and Baa coporate bond indices, we find an average treasury premium of 1.98 percentage points (pp). About 60% of it can be attributed to the safety component (1.17 pp), while the remaining 40% measures the liquidity premia (0.81pp). We also find that during recessions, the treasury premium increases to 2.62pp. This is mainly driven by a jump in the safety premium. We also find that the volatility of the treasury premium is mostly driven by the safety component. This is especially the case during recessions.

To better understand how the components of the treasury premium vary over the business cycle, we estimate a set of regressions for these premia while controlling for the cyclical component of the unemployment rate, which we use as our measure of the business cycle. Our results indicate that the treasury premium and its two components are counter-cyclical, they are positively related to the level of uncertainty in the economy, and they are negatively related to the supply of government bonds. Interestingly, the counter-cyclical nature of the fluctuations in the treasury premium and its components persists even after controlling for the supply of government bonds to GDP, monetary policy and economic uncertainty.

To understand what drives the business cycle fluctuations of these premia, we propose a theoretical framework. The main novelty in our model is that we are able to integrate multiple types of assets, which differ in their safety and liquidity, into a framework that also features equilibrium unemployment and corporate default. These features allow the model to generate endogenous business cycle fluctuations in the liquidity and safety of both corporate and government bonds. Since we want to capture asset fluctuations, we build a real business cycle model with search frictions in the labor market where private and public liquid assets facilitate transactions. Firms hire workers and issue corporate bonds that may default. Both government and corporate bonds can be directly used as collateral or as a means of payment to finance the liquidity needs of households. These consumption opportunities occur in a decentralized market where agents face limited commitment as in Lagos and Wright (2005). As a result, these assets earn a liquidity premium relative to illiquid assets. This is the case as they can be used as collateral.

A key feature of our environment is that we are able to endogenize the safety premium of government bonds through corporate default. In our framework, firms can finance their activity by issuing equity or bonds. As in Gourio (2013) and Gomes, Jermann and Schmid (2016), we use a combination of debt tax benefits and endogenous default risk to get a well-defined corporate financial structure. In each period, the default risk emerges endogenously from idiosyncratic random cost shocks that affect firms. In particular, a firm that suffers high costs will see its equity value turn negative. When this occurs, the firm's shareholders decide to default on its liabilities. Bond investors anticipate this default risk and charge firms a higher borrowing rate relative to safe government bonds. Because the fraction of defaulting firms is state-dependent, this generates fluctuations in the default risk. This in turn affects the price of corporate bonds relative to government bonds over the cycle. In addition, debt issuance and variations in trading opportunities generate endogenous responses that affect the liquidity premium over the business cycle.

Within this frictional environment, we use the model to price an asset that is as liquid as the corporate bond and as safe as the government bond, while keeping constant aggregate liquidity. We define the liquidity component of the treasury premium as the yield difference between this asset and the government bond and the safety component as the yield difference between the corporate bond and this asset. This decomposition gives us an endogenous liquidity and safety premia. These fluctuate in response to changes in aggregate liquidity supply, corporate default and to changes in the demand for liquidity, which arise from trading shocks in the decentralized goods market.

To assess the quantitative predictions of our theoretical framework, we solve our model using a nonlinear global solution methods as in Petrosky-Nadeau, Zhang and Kuehn (2018). We then calibrate it to the U.S. economy using a Simulated Method of Moments approach as inMcFadden (1989); Ruge-Murcia (2012). With this numerical and calibration strategies, we find that the model is able to match qualitatively and quantitatively key aspects of the liquidity and safety premia patterns over the business cycle. Moreover, we study the non-linear and state-dependent short-run dynamics of the model economy using generalized impulse-response functions. Finally, we study the implications for unemployment and welfare when the economy experiences changes in the supply of safe assets.

The rest of the paper is organized as follows. Section 2 summarizes the related literature. Section 3 describes our main empirical results. Section 4 present the model's environment and Section 5 defines and characterizes the corresponding equilibrium. Section 6 describes the calibration procedure and the main quantitative results. Section 7 concludes.

2 Related Literature

Our paper connects and contributes to three different strands of the macro-finance literature. First, we add to the body of work that documents the premium on U.S. Treasury securities and explores the reasons why investors are willing to pay a high price for them. We also relate to and complement the theoretic literature that examines the liquidity properties of different assets. Finally, we also contribute to the finance literature that studies and models corporate default and credit risk.

The seminal paper of Krishnamurthy and Vissing-Jorgensen (2012) initiated a growing literature that studies the U.S. Treasury premium.[1] These authors are the first to study the liquidity and safety components explicitly. They do so by first documenting the spread between assets with different liquidity but similar safety, and then measuring the spread of assets with similar safety but different liquidity. When doing so, Krishnamurthy and Vissing-Jorgensen (2012) find that the supply of treasuries have a substantial effect on different measures of the premium. They show that both changes in the treasury's liquidity and safety attributes drive this variation. Within the same spirit, Nagel (2016) shows that controlling for monetary policy renders the different measures of the treasury premium independent of changes in their supply.[2] While the previous papers document the behavior of the treasury premium, the theoretical framework rationalizing their empirical findings assumes that treasuries are an argument in the investor's utility function. This type of assumption can be problematic as it hard-wires some asset demand aspects to primitives of the environment. This is not innocuous as the demand for treasuries can not fully adjust over the business cycle, impacting the price and premium generated by their framework. Instead, our approach explicitly models the role of different assets in transactions that finance investor's consumption opportunities. As a result, we are able to endogenize both the liquidity and the safety component of the treasury premium. This is a key aspect that helps account, within the context of a structural model, the properties and attributes of the treasury premium over the business cycle.

We also complement to the broader monetary-search literature that studies the liquidity properties of various assets, including money and government bonds.[3] This literature highlights the importance of frictions for the liquidity in asset markets, its impact on asset prices and the underlying market dynamics.[4] Within this literature, the paper that is closest

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[1] We refer the reader to Caramp and Singh (2023); Del Negro, Giannone, Giannoni and Tambalotti (2017b); Krishnamurthy and Li (2023); Krishnamurthy and Vissing-Jorgensen (2012); Nagel (2016) for more details.

[2] This finding overturns some of the results of reported by Krishnamurthy and Vissing-Jorgensen (2012).

[3] We refer to Lagos, Rocheteau and Wright (2017) for a survey of this literature.

[4] We refer the reader to Geromichalos, Licari and Suárez-Lledó (2007); Lagos (2011); Nosal and Rocheteau

to ours is that of Geromichalos, Herrenbrueck and Lee (2023). These authors consider an environment with a safe and risky asset, that are in fixed supply, that can be traded in over the counter (OTC) markets. The risky asset can be exchanged for money (safe asset) to satisfy investors' liquidity needs. They do so in bilateral and frictional OTC markets. In equilibrium, the authors show that usually the safe asset is also the most liquid asset. The authors also identify conditions under which the relationship between safety and liquidity can be reversed. Our paper complements this work by endogenizing the supply and the risk of liquid assets. In addition, we also study the evolution of the equilibrium premia over the business cycle.

Finally, our paper is related to the finance literature that studies corporate default and credit risk. This body of work assesses the factors that contribute to the risk of corporate-bond default. It also investigates credit-spread dynamics and the impact of macroeconomic conditions on default probabilities.[5] To explain the level and volatility of credit spreads, this literature models corporate default. In contrast to this line of work, we consider the credit spread as a measure of the safety premium.[6] We then integrate the approach to corporate default, as in Gourio (2013), Gomes *et al.* (2016) and Bai (2021), into a theory of liquid assets that provides a new perspective on the treasury premium and its two components, as well as on corporate-default risk.

Among the previous work, our paper is closest to Caramp and Singh (2023), Bayer, Born and Luetticke (2023), and Ferrero and Haas (2023), who also study the treasury premia over the business cycle. In particular, Caramp and Singh (2023) study the cyclical fluctuations the spread between Baa and Aaa corporate bonds, which we argue is the safety component of the treasury premium. The authors show that their measure is counter-cyclical, which is in line with our findings. The authors propose a quantitative model to generate an equi-

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(2013); Andolfatto and Williamson (2015); Hu and Rocheteau (2015), among others, for more on this literature.

[5] We refer the reader to Duffie (2011) and Bakshi, Gao and Zhong (2022) for more on corporate default.

[6] Bai (2021) documents a strong, positive relation between unemployment and the credit spread, which is in line with our safety premium results.

librium premium that is consistent with the data. They show that this premium depends on the demand and the supply elasticities of safe assets (including treasuries). The authors find that the demand is inelastic, while the supply is pro-cyclical. This results in safe asset shortages and liquidity traps. Within the same spirit, Bayer *et al.* (2023) estimate an heterogeneous- agent New-Keynesian model with incomplete markets where agents portfolio choices are between liquid deposits offered by banks and illiquid capital based on Bayer, Born and Luetticke (2020). In this economy there is a cost to intermediation, driving a wedge between the policy rate and the interest paid to households. Banks invest household's deposits into long-term government bonds. The authors find in their local projections that a fiscal expansion increases total liquid assets (deposits, stocks, and debt—held directly by households) by up to 0.4 percent. The premia on capital and housing fall by around 20–35 basis points. The convenience yield falls by 2 basis points. The estimated heterogeneous-agent New-Keynesian model is able to reproduce the target evidence from the local projections. Similarly, Ferrero and Haas (2023) propose a medium scale new-Keynesian model with heterogeneous firms and a government that issues perfectly liquid and safe bonds that is based on Del Negro, Eggertsson, Ferrero and Kiyotaki (2017a). In their environment, heterogeneous intermediate goods producers invest in new capital and trade existing capital subject to idiosyncratic investment efficiency shocks. Moreover, firms can only sell an exogenous and time-varying fraction of the existing capital stock. In addition, only the owner of the existing capital knows its quality, which changes over time. Within this environment, the authors find that shocks to asset quality, when there is asymmetric information, are able to explain a large share of fluctuations in investment and financial spreads over the business cycle.

It is important to note that relative to Caramp and Singh (2023), we consider and include the liquidity premium as a second component of the treasury premium. We do so by building on Krishnamurthy and Vissing-Jorgensen (2012) and Del Negro *et al.* (2017b). Contrary to Caramp and Singh (2023), our theoretical framework generates endogenous fluctuations in both the liquidity and safety premium. This allows us to better understand and quantify what is the main component that drives fluctuations over the business cycle in the treasury premium. Finally, in contrast to Bayer *et al.* (2023), and Ferrero and Haas (2023), we consider an environment where defaultable corporate debt is explicitly modelled. Thus, the measures of liquidity and safety premia of U.S treasuries, as measured by Krishnamurthy and Vissing-Jorgensen (2012), directly correspond to those equilibrium counterparts generated by our framework.

3 Empirical Evidence

We decompose the U.S. Treasury bond convenience yield into premia associated with safety and liquidity.[7] Since the U.S. Treasury bonds are a safe asset, we can define the Treasury premium as the average yield difference between Moody's seasoned Baa corporate bonds and U.S. Treasury bonds. We choose the Baa rating since it's the median rating of all corporate bonds in the U.S. and is the lowest possible rating for a bond to still be considered "investment grade". We then define the liquidity premium associated with public debt as the average yield difference between Moody's seasoned Aaa corporate bonds and U.S. Treasury bonds. We consider the Aaa corporate bonds to be a safe and *illiquid* asset as their default risk is extremely small and they are traded much less frequently than public debt.[8] Finally, we define the safety premium as the average yield difference between Baa and Aaa corporate bonds, reflecting the different risk properties that these bonds have.

Given the previous definitions, we focus on the 20-year U.S. Treasury bond. We do so as this public bond has a maturity that better aligns with the privately issued bonds featured in Moody's corporate bond yield indices.[9] In what follows, we build this data series following

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[7] Assets that are considered safe are typically those with low risk of default, such as government bonds issued by stable countries. Liquidity refers to the ease with which an asset can be quickly bought or sold in the market without significantly affecting its price.

[8] In general, Moody's seasoned Aaa corporate bonds are considered almost as safe as government bonds.

[9] Moody's Aaa and Baa indices cover seasoned bonds (circulating for at least a year) issued by non-financial (industrial and utility) companies. The size of these issues must be at least 300 millions USD.

Krishnamurthy and Vissing-Jorgensen (2012).[10] For robustness and completeness, we also present results using 10-year U.S. Treasury bonds that cover the period from 1953 to the present. This analysis can be found in the Appendix.

Table 1 presents the average total, liquidity, and the safety premia over the 1919-2023 period at a quarterly frequency. In particular, we report the average premia over the whole sample as well as the average premia during and outside recessionary periods. When doing so, we use the NBER recession dates and and define the corresponding recession months as those following the peak to the trough.[11]

Table 1: Average size of the premium on 20-year treasury bonds

	Total premium (Baa - TB20Y)	Safety premium (Baa - Aaa)	Liquidity premium (Aaa - TB20Y)
Average	1.99 pp	1.18 pp	0.81 pp
Average during recessions	2.62 pp	1.65 pp	0.97 pp
Average outside of recessions	1.80 pp	1.04 pp	0.76 pp

Notes: Statistics are computed using quarterly averages of monthly U.S. data. Data covers the period 1919Q1-2019Q4.

As we can see from Table 1, the total premium on treasuries is on average equal to 1.99 percentage points (pp). The average safety premium stands at 1.18 pp, about 0.37 pp higher than the average liquidity premium, which is equal to 0.81 pp. The properties of these premia are asymmetric over the business cycle. More precisely, during recessions, the premium on treasuries increases by 0.82 pp relative to normal times. The safety premium increases by about 0.61 pp, while the liquidity premium rises by only 0.21 pp. As a consequence, the difference between the two premia more than doubles during recessions. These results suggest that the safety component is the main driver of the cyclical fluctuations in the convenience yield on treasuries. These features of the various premia are also observed when we consider

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[10] We use data from the Banking and Monetary Statistics, 1914–41, table 128 for the period 1919–24 and the LTGOVTBD series from Federal Reserve's FRED database which covers Treasury bonds over the period 1925-2000. These bonds are due or callable after 8 years for 1919–25, 12 years for 1926–41, 15 years for 1941–51,12 years for 1952, and 10 years for 1953–99 according to Krishnamurthy and Vissing-Jorgensen (2012). From 2000 onward, we use the GS20 series from FRED which covers treasury bonds with a 20-year maturity. The GS20 series stretches back to 1953 but suffers from missing observations from 1986 to 1993.

[11] A recession quarter is a quarter that includes a recession month.

10-year US Treasury bonds over the period 1953-2023. Table 8 in the Appendix presents such findings.

In addition to measuring the average premia, we also document how volatile these premia are. Table 2 summarizes the volatility of the total treasury premium and its two components. As in the business cycle literature, we average and de-trend the data at a quarterly frequency using the Hodrick-Prescott (HP) filter.

Table 2: Standard deviation of the premium on the 20-year treasury bonds

	Total premium (Baa - TB20Y)	Safety premium (Baa - Aaa)	Liquidity premium (Aaa - TB20Y)
STD	0.45 pp	0.34 pp	0.17 pp
STD during recessions	0.68 pp	0.55 pp	0.20 pp
STD outside of recessions	0.31 pp	0.21 pp	0.16 pp

Notes: Statistics are computed using quarterly averages of monthly U.S. data. Data covers the period 1919Q1-2019Q4. Cyclical series are computed as deviations from an HP trend with parameter $\lambda = 1,600$.

Over the 1919Q1-2023Q1 period we find that the total premium has an overall volatility of 0.45 pp relative to the average. The safety premium is substantially more volatile than the liquidity premium with 0.34 pp and 0.17 pp, respectively. As one would expect, all premia measures are more volatile during recessions. In particular, the safety premium becomes much more volatile, increasing from 0.21 pp during normal times to 0.55 pp during recessions. In contrast, the volatility of the liquidity premium barely changes, only exhibiting a modest increase of 0.04 pp during recessions. These patterns over the business cycle can be seen in Figure 1, which plots the cyclical components of the liquidity and safety premia for the 20-year U.S. Treasury bonds.

While for most of our sample the safety and liquidity premia positively co-move, the safety premium exhibits much larger responses during recessions. This feature is what drives the doubling of the volatility of the treasury premium during recessions. To provide further insights, we compute the average cyclical deviation of the safety and liquidity premia from their trend before and after peak unemployment during recessions. The resulting dynamics

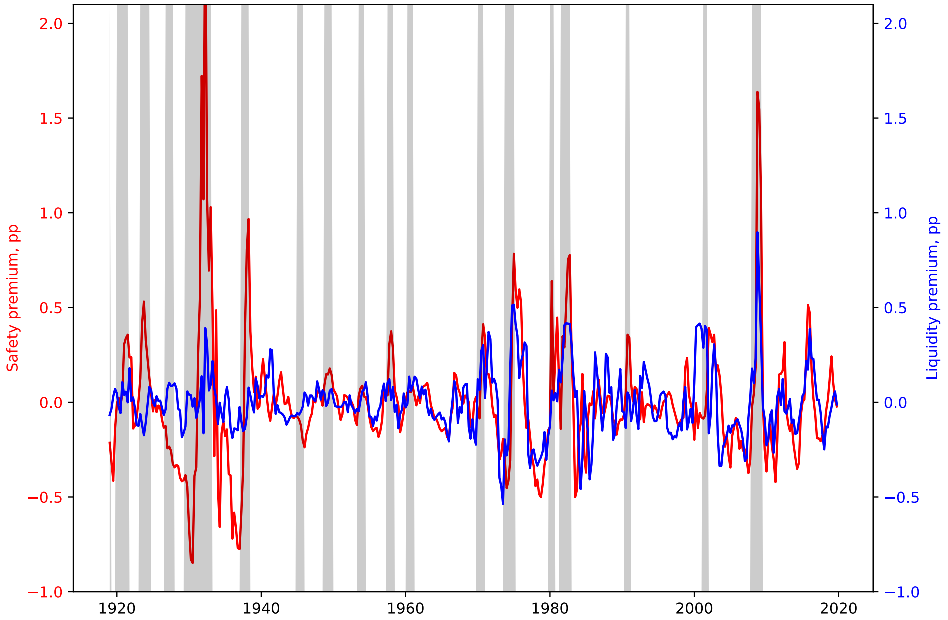

Figure 1: Cyclical component of the safety and liquidity premia measured.

Notes: Quarterly averages of monthly U.S. data. NBER dated recessions are used. Cyclical series are computed as deviations from an HP trend with parameter $\lambda = 1,600$. Data covers the period 1919Q1-2019Q4.

are presented in Figure 2.

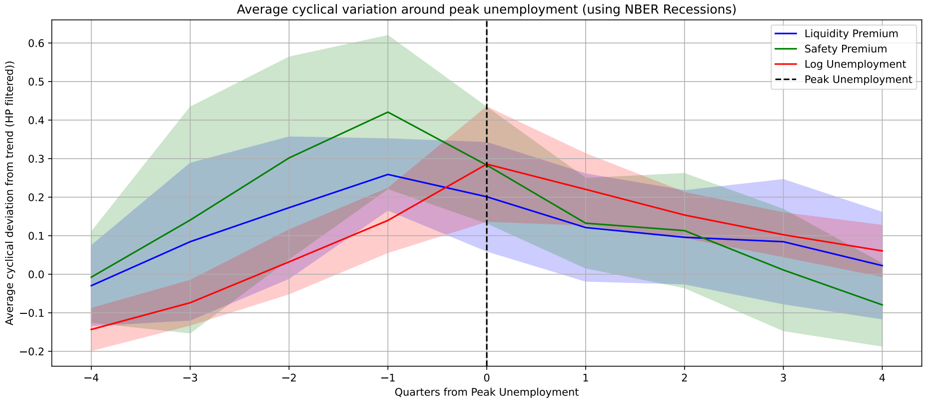

Figure 2: Cyclical variations around peak unemployment during recessions.

Notes: Quarterly averages of monthly U.S. data. Cyclical series are computed as deviations from an HP trend with parameter $\lambda = 1,600$. Average variation before and after peak unemployment across all NBER recessions in the sample. Peak unemployment is measured in level using unfiltered data.

As we can see from Figure 2, the counter-cyclical pattern is clear as the two premia co-move with unemployment. Interestingly, safety and liquidity premia tend to peak one quarter before unemployment. In addition, the safety premium exhibits a faster increase and decrease and a higher peak compared to the liquidity premium in line with our volatility results above.

To have a deeper understanding of how the components of the treasury premium (TP) vary over the business cycle, we conduct a variety of regression analysis that explicitly take into account the state of the economy. More precisely, we perform a regression analysis of these premia while taking into account unemployment, stock market volatility and both fiscal and monetary policies. We consider the unemployment rate as it is an important observable that helps the NBER determine if the U.S. economy is in a recession. Moreover, it is also a good measure of slackness in the private sector. Following the literature, we also include Debt-to-GDP in order to capture the supply of public debt. Moreover, since the premia are expressed in terms of nominal yields, we also consider the Federal Funds rate as a way to control for prices. Finally, we also take into account the volatility of the stock market (VIX) so we can capture uncertainty and risk in the economy. Table 3 summarizes the findings for the 20-year U.S. Treasury premium, which is given by the spread between Moody's seasoned Baa corporate bond index and the yield on 20-year treasury bonds.

As we can see from column (1) from Table 3, the slope coefficient for unemployment is positive and highly significant. This suggests that TP is counter-cyclical and statistically significant. In particular, an increase of 1 pp in unemployment above its mean is associated with an increase in the TP of 0.159 pp relative to its mean. This implies that when economic activity is contracting, the 20-year U.S. Treasury bonds become more expensive relative to the typical corporate bond. To better isolate this cyclical relationship, we consider additional controls that describe the state of the economy. A natural factor to consider is the supply of safe public assets. Column (2) shows the results of controlling for the ratio of public debt to GDP, as measured by Krishnamurthy and Vissing-Jorgensen (2012). The coefficient on unemployment increases to 0.229 and remains highly significant. As expected, the supply of

Table 3: Regression of the 20-year treasury premium on unemployment (HP filtered)

	Dependent variable: Treasury premium (HP filtered)			
	Baa-TB20Y			
	(1)	(2)	(3)	(4)
Unemployment	0.204^{***}	0.265^{***}	0.334^{***}	0.317^{***}
	(0.034)	(0.035)	(0.049)	(0.042)
Debt-to-GDP		-7.535^{***}	-8.965^{***}	-7.534^{***}
		(1.382)	(1.956)	(1.472)
Federal Funds Rate			0.018	0.011
			(0.024)	(0.020)
VIX				0.055^{***}
				(0.013)
Observations	288	288	262	262
R^2	0.156	0.228	0.262	0.500
Adjusted R^2	0.153	0.223	0.253	0.493
Residual Std. Error	0.377	0.361	0.370	0.305
F Statistic	36.545^{***}	32.512^{***}	23.679^{***}	27.300^{***}
Note:			*p<0.1; **p<0.05; ***p<0.01	

public safe assets has a negative and highly significant coefficient. This is consistent with the idea that an increase in the supply of public debt would, ceteris paribus, reduce their price relative to other assets. In column (3), we introduce the effective federal funds rate (EFFR), which helps capture the stance of monetary policy. When all the previous controls are considered, the coefficient on unemployment increases to 0.26 and remains highly significant. However, the coefficient on the EFFR is not statistically significant. Finally, we consider a measure capturing the level of uncertainty and risk in the economy; i,e, the VIX.[12] When doing so, we find a slightly lower but still highly significant coefficient for unemployment. Not surprisingly, the coefficient for the VIX is positive and highly significant. This is consistent with the total treasury premium increasing when there is more risk in the economy.

To summarize, the premium on 20-year U.S. Treasury bonds is counter-cyclical, positively

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[12] Launched in 1993 by the Chicago Board Options Exchange, the VIX captures investors' expectations of stock market volatility. Whaley (2000), Whaley (2009), Bollerslev and Todorov (2011), among others, used the level of the VIX is important for market participants who consider it as a barometer of the equity market volatility

related to the level of uncertainty in the economy and negatively related to the supply of government bonds. Interestingly, the counter-cyclical nature of the fluctuations in the convenience yield persists after controlling for the supply of public safe assets, monetary policy and economic uncertainty.

Next, we explore the cyclical properties of the components of the total treasury premium. We first analyze the safety premium, which we recall is given by Baa-Aaa spread. Table 4 presents the results of the various regressions that control for different aspects of the economy.

Table 4: Regression of safety premium on 20-year treasuries on unemployment (HP filtered)

	Dependent variable: Safety premium (HP filtered)			
	Baa-Aaa			
	(1)	(2)	(3)	(4)
Unemployment	0.129^{***}	0.159^{***}	0.197^{***}	0.187^{***}
	(0.022)	(0.023)	(0.032)	(0.029)
Debt-to-GDP		-3.672^{***}	-4.230^{***}	-3.378^{***}
		(0.867)	(1.266)	(0.994)
Federal Funds Rate			0.010	0.006
			(0.016)	(0.015)
VIX				0.033^{***}
				(0.010)
Observations	288	288	262	262
R^2	0.155	0.197	0.220	0.430
Adjusted R^2	0.152	0.191	0.211	0.421
Residual Std. Error	0.240	0.234	0.242	0.207
F Statistic	34.828^{***}	26.422^{***}	17.740^{***}	18.956^{***}
Note:			*p<0.1; **p<0.05; ***p<0.01	

As we can see from column (1) in Table 4, the unemployment rate has a positive and highly significant coefficient. In particular, a 1pp increase in the unemployment rate above its trend is associated with an increase of 0.098 pp in the safety premium above its trend. This counter-cyclical movement in the safety premium is consistent with flight to safety during recessions.[13] Column (2), (3) and (4) present the results when controlling for the

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[13] We refer to Dong and Wen (2023) for more on this topic.

ratio of public debt to GDP, the EFFR and the VIX, respectively. We find that in all these regressions deliver a positive and highly significant coefficients for unemployment, similar to the convenience yield regressions. Its magnitude, however, is smaller than those reported in the TP analysis. Moreover, as in the findings for the TP, the supply of public safe assets has a negative and highly significant coefficient. We also find that the coefficient on the EFFR is not statistically significant and the coefficient for the VIX is positive and highly significant. The respective coefficients are also smaller in magnitude than in the TP case.

Lastly, we explore the cyclical properties of the liquidity premium, which we recall is the spread between Aaa and treasury bonds. Table 5 presents the results of various regressions that control for various features of the economy.

Table 5: Regression of the liquidity premium on 20-year treasuries on unemployment (HP filtered)

	Dependent variable: Liquidity premium (HP filtered)			
	Aaa-TB20Y			
	(1)	(2)	(3)	(4)
Unemployment	0.075***	0.106***	0.136***	0.129***
	(0.014)	(0.015)	(0.020)	(0.018)
Debt-to-GDP		-3.862***	-4.735***	-4.157***
		(0.672)	(0.897)	(0.749)
Federal Funds Rate			0.008	0.005
			(0.011)	(0.009)
VIX				0.022***
				(0.003)
Observations	288	288	262	262
R^2	0.095	0.181	0.216	0.392
Adjusted R^2	0.092	0.175	0.207	0.383
Residual Std. Error	0.183	0.175	0.179	0.158
F Statistic	26.616***	29.256***	22.195***	33.462***

Note: *p<0.1; **p<0.05; ***p<0.01

The resulting coefficient is equal to 0.061 and is highly significant, as seen in Column (1) of Table 5. Controlling for public debt to GDP, monetary policy and economic uncertainty doesn't overturn this result. However, the coefficient on the liquidity premium is in general lower than the one on the safety premium. This is in line with the volatility results previously discussed.

Summarizing, our analysis of the cyclical properties of the various premia reveal similar and robust patterns. More precisely, the supply of public safe assets is negatively correlated with both components of the total treasury premium. As one would expect, an increase in the supply of treasuries should reduce the scarcity of liquidity in the economy and hence reduce the premium paid for these assets. Similarly, an increase in the supply of treasuries affects the safety premium as there are more safe assets in the market. It is worth noting that our results regarding monetary policy are in contrast with Nagel (2016), who studies deviations rather than levels. Moreover, the unemployment rate is an important factor explaining the cyclical properties of the safety and liquidity premia. This is the case even when monetary policy, changes in the supply of public safe assets and economic uncertainty are also considered. These features of the data motivate our work. In the next sections, we explore whether the observed patterns in the data can be reproduced through the lens of our theoretical model. This will helps us uncover the underlying mechanisms driving the premia features over the business cycle.

4 Model

Our framework has three key components. First, firms match with workers in a frictional labor market, as in the labor search literature. Second, firms sell part of their output in a frictional product market, as in Berentsen, Menzio and Wright (2011). Trade in this goods market requires a medium of exchange as unsecured credit is not feasible. Third, firms issue corporate debt, which, together with treasury bonds, can serve as a medium of exchange in the frictional product market. Firms can default on this debt, as in Gourio (2013), Gomes *et al.* (2016) and Bai (2021).

Time, denoted with $t \geq 0$, is discrete and continues forever. Two perfectly divisible goods—the numeraire good and the special good—are traded in the economy. We express all real prices and quantities in terms of the numeraire good.

Markets. During each period, three markets convene sequentially. First, there is a decentralized goods market (DM) where firms sell the special good to buyers. Second, there is a frictionless centralized market (CM) in which firms pay wages to worker and transform the unsold goods after DM trade, which they sell. This is the numeraire good. In this market households consume this general good and re-adjust their asset holdings. Third, there is a frictional labor market (LM) in which firms hire unemployed workers to produce the numeraire good.

Households. The economy is populated by a unit mass of infinitely lived households. Each household consists of two members—a worker and a buyer—and has preferences described by the following utility function

$$\mathcal{U} = \sum_{t=0}^{\infty} \beta^t \left[h_t + \mathrm{u}\left(x_t\right)\right] \tag{1}$$

where h_t is the time-t net consumption of the numeraire good and x_t denotes the time-t consumption of the special good. The worker and buyer are separated at the beginning of the period and meet only at the end of the period to pool their resources.

Firms. There is a continuum of measure 1 of ex-ante identical large firms indexed by $i \in [0,1]$ that are owned by the households. Each firm hires multiple workers to produce the general good using the following linear technology

$$Y_{i,t} = y_t n_{i,t} \tag{2}$$

where $n_{i,t}$ is the number of workers employed by firm i and y_t represents aggregate productivity. Firms produce the general good at the end of the LM and carry their inventories to the DM. Each worker employed by the firm becomes a seller of the special good in the DM, and sellers are matched with buyers according to a frictional process described in detail below. When a seller is matched with a buyer, they bargain over quantity and price to be traded. The firm transforms part of its production of the general good to be sold to the buyer based on the bargaining outcome. Once the DM closes, firms carry their remaining inventories of the general good to be sold in the CM.

Corporate Default. At the beginning of each period, firms are subject to an idiosyncratic shock to their operating cost, which we denote by $z_{i,t}$ where $z_{i,t} \sim N(0, \sigma_z)$. This cost shock is iid across firms and it is independent of the aggregate productivity shock y_t.

Firms fund their activity using a mix of equity and debt. Firms decide how much to borrow each period and whether to default on their outstanding debt or repay it. Borrowing takes place in the CM while the default decision is taken at the beginning of the period after uncertainty about the current period shocks are resolved. When a firm defaults on its debt, its ownership is transferred to debtholders who resell it to new shareholders and recover its value net of a default cost. This implies that the firm remains in operation and continues to employ workers, produce and sell goods. The only implication when a firm defaults are the costs associated with the transfer of ownership. These will be reflected, in equilibrium, in the price of corporate debt.

Matching and Separation in the LM. At each point in time, each worker is either *unemployed* or *employed.* When the LM convenes, pairwise meetings take place between unemployed workers and vacancy-posting firms. Each firm i employs $n_{i,t}$ workers and posts $v_{i,t}$ vacancies to attract additional workers. The total number of employed workers in the economy is then given by

$$n_t = \int n_{i,t} \mathrm{d}i \tag{3}$$

with unemployment $u_t = 1 - n_t$, while the total number of vacancies is given by

$$v_t = \int v_{i,t} \mathrm{d}i. \tag{4}$$

The matching process in the LM is random and governed by a matching function. In particular, given v_t and u_t, a number $\mathcal{M}(v_t, u_t)$ of meetings take place. Here, $\mathcal{M}(\cdot,\cdot)$ is a constant returns to scale (CRS) matching function satisfying the standard assumptions in the labor search literature. Defining $\theta_t = v_t/u_t$ as the *market tightness*, the job-finding probability for an unemployed worker is given by

$$\mathcal{M}(\theta_t, 1) \equiv f(\theta_t) \tag{5}$$

and the filling probability for a vacancy posted by a firm is then equal to

$$\mathcal{M}(1, 1/\theta_t) \equiv q(\theta_t). \tag{6}$$

During the LM, workers are separated from their employers with exogenous probability $\delta \in (0,1)$. Workers who are separated from their jobs at time t start searching for vacancies at time $t+1$. The mass of employed workers thus evolves according to the following law of motion

$$n_{t+1} = (1-\delta)n_t + q(\theta_t)v_t. \tag{7}$$

Matching and Trade in the DM. In each period there is a mass n_t of sellers seeking matches with a unit mass of buyers in DM. Matching is random and the mass of realized matches is governed by a CRS matching function $\mathcal{N}(n_t, 1)$. The CRS property implies that each buyer is matched to a seller with probability $\alpha\left(n_t\right)$ and that each seller is matched with a buyer with probability $\alpha\left(n_t\right)/(n_t)$, where $\mathcal{N}(n_t, 1) \equiv \alpha(n_t)$, $\alpha' > 0$, $\alpha'' < 0$, and $\alpha\left(n_t\right) \leq n$.

Buyers in the DM want to consume the special good, and firms can produce x_t of it on-demand by using $\mathrm{c}\left(x_t\right)$ numeraire goods as input. Terms of trade in DM matches are determined by proportional bargaining, which we describe below. Information and commitment frictions rule out unsecured credit in the DM. However, agents can use their asset holdings as collateral to obtain goods on secured credit to be repaid in general goods in the CM. We assume buyers can pledge up to a fraction $\chi_g \in (0,1]$ of government bonds, while only a fraction $\chi_c \in (0,1] < \chi_g$ of corporate bonds can be pledged.

Settlement and Re-balancing in the CM. Walrasian market for assets and numeraire goods in the CM. Firms sell their remaining inventories, pay out wages, repay their bonds, issue new ones and post vacancies for the next LM. Households receive their wage or unemployment benefits, repay their DM debt, adjust their asset holdings and pay lump-sum taxes (receive subsidies) collected (distributed) by the government.

Opening up a vacancy for next period's LM costs κ numeraire goods. The wages paid out to the workers by productive firms are determined by Nash bargaining in the previous LM. Unemployed workers receive an exogenous amount ℓ of the numeraire good in the CM, which represents an amalgamation of unemployment benefits, the value of leisure, and home production.

Timing. The timing of the model is as follows. The economy enters the period with a pre-determined stock of corporate debt and employment level. Then, both the aggregate productivity and the idiosyncratic cost shocks realize. Firms then choose whether or not to default on their debt obligations. If the firm defaults, then the current shareholders are wiped out. Then current debt holders take ownership of the firm and sell it immediately to new shareholders for a fraction of its asset value to be paid in the CM. Firms then negotiate wages with their workers. The economy then moves into the DM, where firms sell the special goods to households on-demand, subject to household portfolios of government bonds and corporate bonds net of default. Then, the CM convenes, where previously negotiated wages are paid out, portfolios are adjusted, and firms make borrowing and vacancy posting choices. Finally, labor market matching and separations take place.

5 Stochastic Equilibrium

5.1 Households

A household enters each CM with some portfolio of government and corporate bonds and decides, among other things, on the next period's portfolio. Because of default risk, we need to distinguish between the amount of corporate bonds carried *out* of the CM and the net-of-default amount of wealth carried into the next one. We denote $b_{i,t+1}$ as the amount of bonds issued by firm i that the household carries into period $t+1$. A household purchasing $b_{i,t+1}$ bonds from firm i will enter the period-$t+1$ CM with wealth equal $\bar{b}_{i,t+1} = \mathcal{R}_{i,t+1} b_{i,t+1}$, where $\mathcal{R}_{i,t+1}$ denotes the return on firm i's bonds net of default costs. Combining across firms, we can write the household's end-of-period portfolio as $\mathbf{b}_{t+1} = \{b_{i,t+1}\}_{i\in[0,1]}$ and the next-period portfolio net of default costs as $\bar{\mathbf{b}}_{t+1} = \{\bar{b}_{i,t+1}\}_{i\in[0,1]}$.

CM Value Functions. Consider a household entering the CM. The individual state of this household consists of its employment status $j \in \{0,1\}$, its promised wage w_t (none if unemployed), its portfolio of government and corporate bonds $(b_{g,t}, \bar{\mathbf{b}}_t)$, and the amount d_t owed for DM goods purchased on secured credit. Given the state, the household chooses its next-period portfolio of government and corporate bonds, $(b_{g,t+1}, \mathbf{b}_{t+1})$. The CM value for an employed household is then given by

$$\begin{aligned} W_t^1(b_{g,t}, \bar{\mathbf{b}}_t, d_t, w_t) = \max_{h_t, b_{g,t+1}, \mathbf{b}_{t+1}} h_t + \beta \mathbb{E} \Big\{ (1-\delta) \int_{-\infty}^{\infty} V_{t+1}^1(b_{g,t+1}, \mathbf{b}_{t+1}, w_{t+1}(z)) d\Phi(z) \\ + \delta V_{t+1}^0(b_{g,t+1}, \mathbf{b}_{t+1}) \Big\} \end{aligned} \tag{8}$$

subject to the following budget constraint

$$h_t + d_t + P_{g,t} b_{g,t+1} + \int_0^1 P_{i,c,t+1} b_{i,t+1} di \leq b_{g,t} + \int_0^1 \bar{b}_{i,t} di + w_t + T_t, \tag{9}$$

where $P_{g,t}$ denotes the price of government bonds and $P_{i,t}$ denotes the price of corporate bonds issued by firm i. Similarly, the CM value of an unemployed household is given by

$$\begin{aligned} W_t^0(b_{g,t}, \bar{\mathbf{b}}_t, d_t) = \max_{h_t, b_{g,t+1}, \bar{\mathbf{b}}_{t+1}} h_t + \beta \mathbb{E} \Big\{ & f(\theta_t) \int_{-\infty}^{\infty} V_{t+1}^1 (b_{g,t+1}, \bar{b}_{c,t+1}, w_{t+1}(z)) d\Phi(z) \\ & + (1 - f(\theta_t)) V_{t+1}^0 (b_{g,t+1}, \bar{b}_{c,t+1}) \Big\} \end{aligned} \tag{10}$$

subject to the following budget constraint

$$h_t + d_t + P_{g,t} b_{g,t+1} + \int_0^1 P_{i,c,t+1} b_{i,c,t+1} di \leq b_{g,t} + \int_0^1 \bar{b}_{i,t} di + \ell + T_t. \tag{11}$$

It is straightforward to show that the value functions are linear in asset levels and also in d_t, and that the choice of next period's portfolio does not depend on this period's portfolio — facts that will be used below.[14]

DM Value Functions. Each buyer meets a firm in the product market with probability $\alpha(n_t)$ in the DM. Consider a buyer-household with assets $(b_{g,t}, \bar{\mathbf{b}}_t)$. If meeting a firm, the buyer buys some negotiated amount x_t of the special good in exchange for some payment d_t, which is constrained not to exceed the collateral value of the buyer's assets. Specifically, a fraction χ_g of the government bonds held by the buyer are pledgeable, while a fraction χ_c for corporate debt. The terms of trade x_t and d_t are determined by Kalai bargaining between the buyer and the seller, with $\varphi \in [0, 1]$ being the buyer's bargaining power. As we will verify below, the linearity of the worker's and firm's value functions imply that this

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[14] We refer to Lagos and Wright (2005) for more on this type of portfolio degeneracy.

bargaining solution takes a simple form, which are given by

$$x_t = \min\{\mathrm{v}^{-1}(a_t), x^*\}, \quad d_t = \min\{a_t, \mathrm{v}(x^*)\} \tag{12}$$

where x^* is the solution to $u'(x^*) - 1 = 0$ (the unconstrained bilaterally-efficient trade), the function $\mathrm{v}(x)$ is given by

$$\mathrm{v}(x) \equiv (1-\varphi)\mathrm{u}(x) + \varphi x \tag{13}$$

and a_t denotes the total value of the buyer's collateralizable assets, which is given by

$$a_t = \chi_g b_g + \chi_c \int_0^1 \bar{b}_{i,t} di. \tag{14}$$

An important implication of the optimal terms of trade is that x_t and d_t will depend on the buyer's asset portfolio, but not on the household's employment status nor the characteristics of the firm.

With x_t and d_t characterized by (12), (13), (14), the DM value function of an employed household with promised wage w_t and portfolio $b_{g,t}, \bar{\mathbf{b}}_t$ can be written as follows

$$\begin{aligned} V_t^1(b_{g,t}, \bar{\mathbf{b}}_t, w_t) =& \alpha(n_t)[\mathrm{u}(x_t) + W_t^1(b_{g,t}, \bar{\mathbf{b}}_t, d_t, w_t)] \\ &+ (1-\alpha(n_t))W_t^1(b_{g,t}, \bar{\mathbf{b}}_t, 0, w_t) \\ =& \alpha(n_t)[\mathrm{u}(x_t) - d_t] + W_t^1(b_{g,t}, \bar{\mathbf{b}}_t, 0, w_t) \end{aligned} \tag{15}$$

where the second equality uses the linearity of the CM value function W. Similarly, for an unemployed household, we have that

$$\begin{aligned} V_t^0(b_{g,t}, \bar{\mathbf{b}}_t) =& \alpha(n_t)[\mathrm{u}(x_t) + W_t^0(b_{g,t}, \bar{\mathbf{b}}_t, d_t)] \\ &+ (1-\alpha(n_t))W_t^0(b_{g,t}, \bar{\mathbf{b}}_t, 0) \\ =& \alpha(n_t)[\mathrm{u}(x_t) - d_t] + W_t^0(b_{g,t}, \bar{\mathbf{b}}_t, 0). \end{aligned} \tag{16}$$

Portfolio Choice. We can now derive the household's optimal portfolio choice. Using the linearity of W in asset holdings, as well as the expressions for the DM value function V in (15), equation (16) and the bargaining solution (12)-(14), we can see that the period-t optimal portfolio choice problem is equivalent to the following

$$\begin{aligned}\max_{b_{g,t+1},\mathbf{b}_{t+1}} & -P_{g,t}b_{g,t+1} - \int P_{i,t}b_{i,t+1}di \\ & + \beta\mathbb{E}\Big\{\alpha(n_{t+1})[\mathrm{u}(x) - \mathrm{v}(x)] + b_{g,t+1} + \int \bar{b}_{i,t+1}di\Big\}\end{aligned} \tag{17}$$

subject to the payment constraint

$$\mathrm{v}(x) \leq \chi_g b_{g,t+1} + \chi_c \int \bar{b}_{i,t+1}di, \tag{18}$$

where $\mathrm{v}(x)$ is defined in equation (13), and we recall that $\bar{b}_{i,t+1} = \mathcal{R}_{i,t+1}b_{i,t+1}$ is the value of the corporate bond net of default costs. The first-order condition with respect to government bonds, $b_{g,t}$, then gives us the following equilibrium condition

$$P_{g,t} = \beta\mathbb{E}\left[1 + \chi_g\mathcal{L}_{t+1}\right] \tag{19}$$

where we define *liquidity scarcity* as follows

$$\mathcal{L}_t \equiv \alpha(n_t)\left(\frac{\mathrm{u}'(x_t)}{\mathrm{v}'(x_t)} - 1\right) \tag{20}$$

which measures the marginal value of carrying an additional unit of liquid assets into the DM. This marginal value is the product of two terms: (i) the probability $\alpha(n_t)$ of meeting a seller; and (ii) the buyer's marginal surplus $\left(\frac{\mathrm{u}'(x_t)}{\mathrm{v}'(x_t)} - 1\right)$ generated by an additional unit of liquidity, which in turn depends on the buyer's share φ of the total match surplus.

The household's public debt optimality condition, given by equation (19), states that the price of a government bond must equal the discounted marginal value of carrying that bond across periods, which *includes* its marginal usefulness in exchange in the DM. If an asset is useless in exchange (either because the pledgeability is low, because the probability of meeting a seller is low, or because DM consumption is already nearly satiated), the price of the asset reduces to β, which corresponds to the fundamental value of an asset. Next, we describe obtain the first-order condition with respect to one unit of corporate bonds, which is given by

$$P_{i,t} = \beta \mathbb{E} \left[\left(1 + \chi_c \mathcal{L}_{t+1}\right) \mathcal{R}_{i,t+1} \right]. \tag{21}$$

Note that the pricing of corporate bonds, equation (21), differs from that of public debt, equation (19), in two important ways. First, the term $\mathcal{R}_{i,t+1}$ accounts for the possibility that the corporate bond sometimes defaults. Therefore its dividend payout is less than one-for-one. Second, corporate bonds differ in pledgeability from government bonds, i.e. $\chi_c \neq \chi_g$. These two differences will be the source of the safety and liquidity premia, which we explain below.

In equilibrium all firms will make identical default choices. Hence we have that in equilibrium $\mathcal{R}_{i,t+1} = \mathcal{R}_{t+1}\ \forall i$. However, equation (21) does not restrict the household to symmetric portfolio choices. In particular, the household is free to readjust its portfolio demand if an individual firm changes its behavior. To study this behavioral response, we turn to the firm's hiring, borrowing, and default decisions, which we analyze next.

5.2 Firms

Each firm i has the following value function

$$\begin{aligned} J_{i,t}(n_{i,t}, z_{i,t}, b_{i,t}) = \max_{v_{i,t}, b_{i,t+1}} & \left[\mathcal{O}_t - w_{i,t} - z_{i,t}\right] n_{i,t} - b_{i,t} \\ & - \kappa v_{i,t} + (1+\tau) P_{i,t} b_{i,t+1} \\ & + \beta \mathbb{E} \int \max\{0, J_{i,t+1}(n_{i,t+1}, z', b_{i,t+1})\} \mathrm{d}\Phi(z') \end{aligned} \tag{22}$$

emphasizing that the firm enters the period with some employment level, $n_{i,t}$, realized operating cost $z_{i,t}$ and debt $b_{i,t}$. The firm pays off its debt and obtains per-worker profits $\mathcal{O}_t - w_{i,t} - z_{i,t}$, where $\mathcal{O}_t$ is the firm's revenue, including its proceeds from sales in the DM, and $w_{i,t}$ is the wage it pays, determined below. The firm then chooses vacancies $v_{i,t}$ and next period's debt $b_{i,t+1}$, with the understanding that next period it will default whenever its equity value becomes negative. The firm's maximization is done subject to three constraints. First, vacancies cannot be negative, which implies that

$$v_{i,t} \geq 0. \tag{23}$$

Second, the firm's future employment level $n_{i,t+1}$ evolves according the following law of motion

$$n_{i,t+1} = (1-\delta)n_{i,t} + q(\theta_t)v_{i,t}. \tag{24}$$

Third, the firm internalizes how its choices of vacancies and debt affect the price of its own debt, $P_{i,t}$. In other words, the firm takes *as a constraint* that the demand for its bonds obeys equation (21).

To determine $\mathcal{R}_{i,t+1}$, we first note that the firm will default next period whenever $J_{i,t+1}(n_{i,t+1}, z_{i,t+1}, b_{i,t+1}) < 0$. This is equivalent to a situation when $z_{i,t+1} > z^*_{i,t+1}$, where $z^*_{i,t+1}$ is the unique solution to the following zero value condition:[15]

$$J_{i,t+1}(n_{i,t+1}, z^*_{i,t+1}, b_{i,t+1}) = 0. \tag{25}$$

Using this, the return on a unit of the firm's bonds must satisfy the following condition

$$\mathcal{R}_{i,t+1}b_{i,t+1} = \Phi(z^*_{i,t+1})b_{i,t+1} + \zeta \int_{z^*_{i,t+1}}^{\infty} J_{i,t+1}(n_{i,t+1}, z_{i,t+1}, 0)\mathrm{d}\Phi(z_{i,t+1}). \tag{26}$$

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[15] This argument uses the fact, confirmed below, that the value function defined in equation (22) is monotonically decreasing in $z_{i,t}$.

This latter condition implies that with probability $\Phi(z^*_{i,t+1})$, tomorrow's operating cost is below $z^*_{i,t+1}$, and the firm repays its debt. For realizations of the operating cost above $z^*_{i,t+1}$, the firm defaults and creditors instead recover a fraction ζ of the firm's equity value. This is given by the integral in the second term of (26). The maximization in (22) is thus subject to equations (23), (24), (25), and (26).

5.2.1 Wage Determination

The wage in a worker-firm match is determined by Nash bargaining. The household's surplus from being employed at wage w is given by

$$V_t^1(b_{g,t}, \bar{\mathbf{b}}_t, w) - V_t^0(b_{g,t}, \bar{\mathbf{b}}_t).$$

The firm's surplus from employing an additional worker is captured by

$$\frac{\partial J_t(n_{i,t}, z_{i,t}, b_{i,t})}{\partial n_{i,t}}.$$

The wage offered by firm i is $w_{i,t}$, which is set for each $z_{i,t}$ such that

$$(1-\xi)\left[V_t^1(b_{g,t}, \bar{\mathbf{b}}_t, w) - V_t^0(b_{g,t}, \bar{\mathbf{b}}_t)\right] = \xi \frac{\partial J_t(n_{i,t}, z_{i,t}, b_{i,t})}{\partial n_{i,t}} \tag{27}$$

where the parameter ξ denotes the worker's bargaining power.[16]

5.2.2 Firm's Optimal Choices

Given the firm's problem and condition for the wage determination, given by (27), we can derive the firm's optimality conditions for vacancy posting and debt issuance. From now, since in equilibrium, these choices are the same across all firms, we eliminate the i subscripts.

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[16] Note that, because of linear production and linear utility, it is easy to verify that the Nash-bargained wage does not depend on the firm's employment level, the firm's debt level, nor the worker's asset portfolio.

Start with the optimal vacancy posting choice, which — together with the wage bargaining condition (27) — determines the equilibrium market tightness. It can be shown that the firm's surplus from employing an additional worker, $S_{i,t} \equiv \frac{\partial J_{i,t}}{\partial n_{i,t}} = S_t - (1-\xi)z_{i,t}$, where S_t is the total surplus and is independent of $z_{i,t}$ and satisfies the following recursive expression

$$\begin{aligned} S_t =& (1-\xi)\left[\mathcal{O}_t - \ell + (1-\delta)\left(\frac{\kappa}{q(\theta_t)} - \lambda_t\right)\right] \\ &+ \beta\xi(1-\delta-f(\theta_t))\mathbb{E}S_{t+1} \end{aligned} \tag{28}$$

where λ_t is the multiplier on the non-negativity constraint (23). The market tightness θ_t satisfies the free-entry condition, which is given by

$$\begin{aligned} \frac{\kappa}{q(\theta_t)} - \lambda_t =& \beta\mathbb{E}\Big\{ \left[\Phi(z^*_{t+1}) + \zeta\mathcal{C}_{t+1}(1-\Phi(z^*_{t+1}))\right] S_{t+1} + \Phi(z^*_{t+1})(\mathcal{C}_{t+1}-1)b_{t+1} \\ &+ (1-\xi)(\zeta\mathcal{C}_{t+1}-1)\int_{-\infty}^{z^*_{t+1}} z_{t+1}\mathrm{d}\Phi(z_{t+1})\Big\} \end{aligned} \tag{29}$$

where we $\mathcal{C}_t = (1+\tau)(1+\chi_c\mathcal{L}_t)$ and the equilibrium default cutoff satisfies $z^*_t = \frac{1}{1-\xi}(S_t - b_t)$.

Next, the first-order condition for bond issuance leads to the following condition characterizing the equilibrium corporate debt, b_{t+1},

$$\mathbb{E}\left\{(1-\xi)(\mathcal{C}_{t+1}-1)\Phi(z^*_{t+1}) - (1-\zeta)\mathcal{C}_{t+1}\phi(z^*_{t+1})\frac{b_{t+1}}{n_{t+1}}\right\} = 0. \tag{30}$$

Intuitively, this condition equates the marginal cost of issuing debt to its marginal benefit. Issuing more corporate debt enables the firm to take advantage of both the tax benefit and the premium afforded by corporate debt liqudity. On the other hand, issuing more debt also raises the per-unit price of debt. This is the case as creditors rationally anticipate that the firm will default with a higher probability.

5.3 Market Clearing

Having characterized the optimal decisions, we close the model with the DM market clearing conditions. The firm's revenue per worker must satisfy

$$\mathcal{O}_t = y_t + \frac{\alpha(n_t)}{n_t}(1-\varphi)\left[\mathrm{u}(x_t) - x_t\right] \tag{31}$$

which emphasizes that in each match with a worker produces y_t in the CM. It then meets a buyer in the DM with probability $\frac{\alpha(n_t)}{n_t}$, in which case it generates additional profits in the amount of $\mathrm{u}(x_t) - \mathrm{v}(x_t) = (1-\varphi)\left[\mathrm{u}(x_t) - x_t\right]$. In turn, x_t is determined through (12) and (13).

The equilibrium total level of liquid assets, defined in (14), is determined from (26) together with the firm's equity value and default choices, which, in equilibrium, result in the following condition

$$a_t = \chi_g b_g + \chi_c n_t \left\{ \left[\Phi(z_t^*) + \zeta(1-\Phi(z_t^*))\right] b_t - \zeta(1-\xi)\int_{z_t^*}^{\infty}(z_t - z_t^*)\mathrm{d}\Phi(z_t) \right\}. \tag{32}$$

5.4 Liquidity and Safety Premia

One of our objectives in this paper is to decompose the behavior of corporate bond spreads into safety and liquidity components. Recall from equations (19) and (21) that the price of government bonds and corporate bonds, respectively, are given by

$$P_{g,t} = \beta\mathbb{E}[1 + \chi_g \mathcal{L}_{t+1}] \tag{33}$$

and

$$P_t = \beta\mathbb{E}[(1 + \chi_c \mathcal{L}_{t+1})\mathcal{R}_{t+1}]. \tag{34}$$

Giving these pricing kernels, we can perform the decomposition as follows. Let $\tilde{P}_t$ be the price of a safe (default-free) corporate bond with the same pledgeability as the risky corporate bond. If we introduce a marginal unit of such an asset, a household would price it as follows

$$\tilde{P}_t = \beta \mathbb{E}\left[1 + \chi_c \mathcal{L}_{t+1}\right], \tag{35}$$

where the difference with the risky corporate bond lies only in the absence of the default-related term $\mathcal{R}_{t+1}$. We define the liquidity premium on government bonds as the spread between its price and the price of this hypothetical safe corporate bond. Expressing things in log terms we have the following approximation

$$\log P_{g,t} - \log \tilde{P}_t \approx \left(\chi_g - \chi_c\right) \mathbb{E}\left[\mathcal{L}_{t+1}\right]. \tag{36}$$

We then define the safety premium as the difference in log-prices between the safe corporate bond and the risky corporate bond. As with the public liquidity premium, expressing things in log terms we have the following approximation

$$\log \tilde{P}_t - \log P_t \approx 1 - \mathbb{E}\left[\mathcal{R}_{t+1}\right] - \frac{\chi_c \mathrm{Cov}(\mathcal{L}_{t+1}, \mathcal{R}_{t+1})}{1 + \chi_c \mathbb{E}\left[\mathcal{L}_{t+1}\right]}. \tag{37}$$

Having characterized the equilibrium premiums, equations (36) and (37), we now are able to explore their properties over the business cycle.

6 Calibration and Numerical Results

6.1 Calibration

The model is set-up at a monthly frequency and is calibrated using a mix of monthly and quarterly data covering, whenever possible, the period from January 1919 to December 2019. We calibrate the model to match features of the labor market data and firm's financial characteristics in the United States. We then match the average levels of the safety and liquidity premia observed in the data and evaluate the model on its ability to replicate the empirical volatilities and regression coefficients presented in Section 3.

When parametrizing the model, we proceed in two stages. First, we directly calibrate a small number of parameters. This includes the discount factor, β, the job destruction rate, δ, the recovery rate, ζ, and the government bonds pledgeability, χ_g. Second, we calibrate jointly the remaining parameters to match a set of empirical moments using a Simulated Method of Moments (SMM) approach.[17] In what follows we discuss the calibration procedure and the targeted moments in more detail.

Preferences. The discount factor β is set to match the real interest rate on a safe and illiquid asset. We use the 3-months certificates of deposit (CDs). As pointed out by Krishnamurthy and Vissing-Jorgensen (2012), CDs are insured by the Federal Deposit Insurance Corporation (FDIC) which makes them virtually as safe as treasuries. CDs are also relatively illiquid compared to both treasuries and corporate bonds.[18] The average nominal interest rate on 3-months CDs over the period 1964-2019 is 5.25%. Over the same period, the average CPI inflation was 3.45%. Thus, the real annual rate is 1.75%, which implies a value of 0.9986 for β.

Firms. We assume the aggregate productivity shock, y_t, follows an AR1 process of the form

$$\log y_{t+1} = (1-\rho_y)\log \bar{y} + \rho_y \log y_t + \sigma_y \varepsilon_{y,t+1} \tag{38}$$

where $\varepsilon_y \sim \mathcal{N}(0,1)$ and $\bar{y}$ is normalized to 1. As in Ait Lahcen, Baughman, Rabinovich and van Buggenum (2022), since labor productivity is endogenous and depends on output in the DM, we calibrate the productivity shock process parameters in conjunction with other model parameters to match the observed volatility and persistence of labor productivity. For

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[17] For more details we refer the reader to the appendix.

[18] CDs are designed in such a way that an investor that agrees to leave their money with the financial institution for the duration of the CD's fixed term, which can range from a few months to several years, receives a fixed interest. If they withdraw the funds before the CD matures, they may incur penalties and forfeit some of the interest earned.

our measure of labor productivity, as in Shimer (2005), we use data on the real output per person in the non-farm business sector from the Bureau of Labor Statistics (BLS). We use quarterly log-deviations from the HP filtered trend.

As previously mentioned, the idiosyncratic cost, z_i, is normally distributed with mean 0. Following Bai (2021), its standard deviation σ_z is calibrated to match an average default rate of 0.7% on Baa-rated corporate bonds. The recovery rate, ζ, is set to 42%. The parameter τ summarizes tax benefits of debt issuance. This parameter is calibrated to match the ratio of outstanding corporate bonds to nominal GDP. The average ratio in the data is 17.11%.

Labor Market. We use the matching function proposed by Den Haan, Ramey and Watson (2000), which is given by

$$\mathcal{M}(v,u) = \frac{vu}{(v^{\mu} + u^{\mu})^{1/\mu}} \tag{39}$$

and internally calibrate the parameter μ to match the average job finding rate. The job separation rate δ is directly set to match its empirical counterpart. Both measures are computed using data on short-term unemployment from the BLS following Shimer (2012). As in Hagedorn and Manovskii (2008), the flow value of unemployment, ℓ, is internally calibrated to match the volatility of unemployment.[19]

DM. We assume a CRRA utility over consumption in the DM, which takes the form

$$\mathrm{u}(x) = \frac{x^{1-\gamma}}{1-\gamma} \tag{40}$$

where $\gamma \geq 0$. We set the cost of producing DM goods on-demand to be linear

$$\mathrm{c}(x) = x. \tag{41}$$

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[19] Unemployment volatility is measured as the standard deviation of the quarterly log-deviations of the unemployment rate from its HP-filtered trend using a smoothing parameter of 1600.

Table 6: SMM calibrated parameters

Parameter	**Description**	**Value**	**Moment**	**Frequency**	**Data**	**Model**
κ	Vacancy cost	0.733	Average θ	Monthly	0.655	0.685
ℓ	Flow value of unemployment	0.980	Unemployment volatility	Quarterly	0.138	0.142
μ	Parameter of the LM matching fun.	1.054	Average JFP	Monthly	0.430	0.406
ξ	Worker bargaining weight	0.051	Elast. of wage to labor prod.	Quarterly	0.481	0.472
ρ_y	Persistence parameter of y process	0.956	Autocorr. of labor productivity	Quarterly	0.735	0.750
σ_y	Volatility parameter of y process	0.006	SD of labor productivity	Quarterly	0.013	0.013
γ	Curvature parameter of DM utility	0.120	Average safety premium (annualized)	Monthly	1.179pp	1.173pp
χ_c	Pledgeability of corporate bonds	0.165	Average liquidity premium (annualized)	Monthly	0.810pp	0.809pp
φ	Buyer bargaining weight	0.010	Average DM price markup	Monthly	0.360	0.365
b_g	Real supply of government bonds	0.263	Average treasuries/NGDP	Monthly	0.262	0.270
τ	Tax benefits of issuing corporate bonds	2.6e-4	Average corporate bonds/NGDP	Monthly	0.171	0.169
σ_z	SD of the idiosyncratic cost shock z_i	0.315	Quarterly average default rate on Baa-rated bonds	Monthly	0.059%	0.059%

We define the probability of finding a seller in the DM as

$$\alpha(n) = \frac{n}{1+n}. \tag{42}$$

Assets. The previous functional assumptions leave us with one parameter γ, which we calibrate to match the average safety premium. The real supply of government bonds b_g is set to match the average level of treasuries market value to nominal GDP. Finally, we normalize the peldgeability parameter χ_g to 1 and calibrate χ_c to match the average liquidity premium in the data.

Table 6 summarizes the results of the SMM calibration. Most moments are matched very closely, in particular the average safety and liquidity premia.

6.2 Model Validation

Using simulated data, Table 7 summarizes the regression results of the treasury premium and its components on unemployment. For the sake of having a relevant comparison to the analysis on Section 3, we transform the simulated data using the same procedure. To the extent that in the model there are no monetary shocks and the supply of government bonds is constant, the regression results using simulated data should be comparable to its empirical real data counterpart when these two factors are explicitly taken into account. The coefficients from all three regressions match the positive relationship with unemployment

Table 7: Regression of treasury, safety and liquidity premia on unemployment using simulated data

	Treasury premium (1)	*Safety premium* (2)	*Liquidity premium* (3)
Constant	0.203*** (0.000)	-0.442*** (0.000)	0.645*** (0.000)
Unemployment rate	0.184*** (0.012)	0.168*** (0.009)	0.016*** (0.005)
Observations	288	288	288
R^2	0.986	0.988	0.784

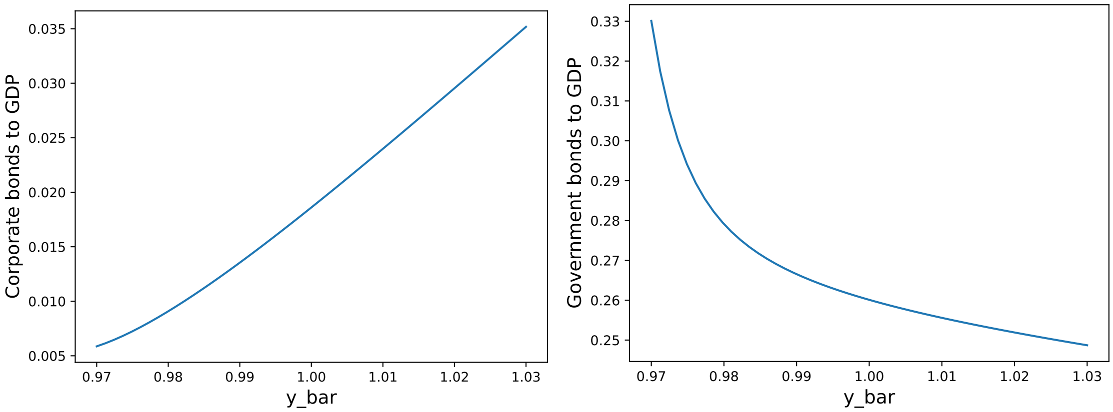
*p<0.1; **p<0.05; ***p<0.01

Notes: The model is simulated for 1,000 runs each of length 1,000 months with a burn-in period of 136 months to obtain the same length as the data. All variables are quarterly averages of monthly simulated data detrended using the HP filter with $\lambda = 1600$. *Regression statistics are averaged over all simulations. Standard errors are in parentheses.*

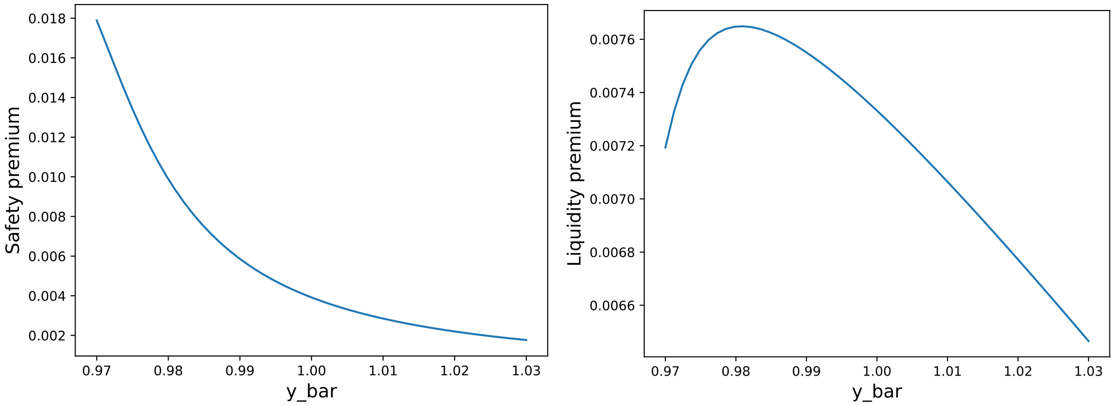
observed in the data. The regression of the treasury premium on unemployment based on simulations results in an average coefficient of 0.184 compared to 0.224 in the data. The model overstates the relationship between the safety premium and unemployment with a coefficient equal to 0.168 compared to 0.126 obtained when using actual data. In contrast, the model's simulated data deliver a coefficient for the liquidity premium of only 0.016, while the the one with actual data is 0.098. The fact that we do not obtain the same magnitude is not too surprising. This is the case as there might be other factors at play that affect the size of the coefficients. Nevertheless, overall, the regression results using simulated data are very encouraging and provide evidence supporting the theoretical framework we have developed.

6.3 Comparative Statics

To better understand the properties and mechanics of the model, we perform some numerical comparative statics in steady state. We mainly focus on the parameters that have first



(a) Corporate bonds to GDP, $\frac{P_c b_c}{GDP}$ (b) Government bonds to GDP, $\frac{P_g b_g}{GDP}$

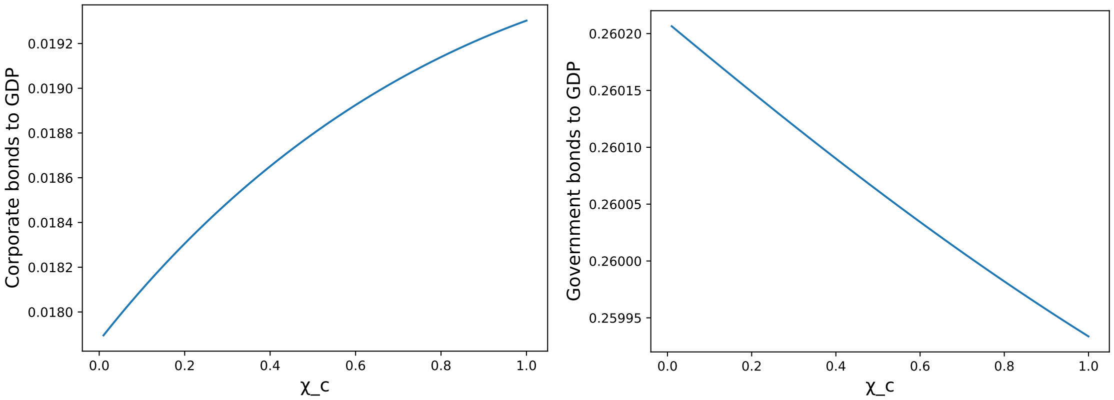


(c) Safety premium (d) Liquidity premium

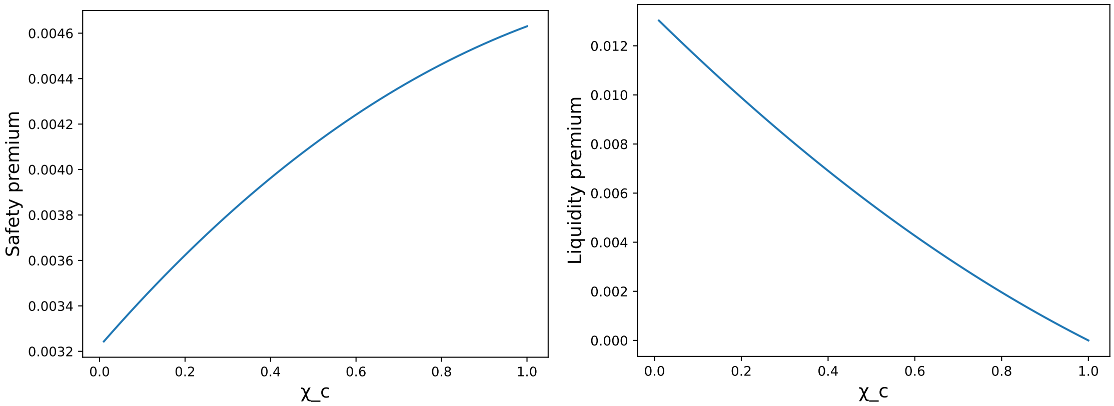
Figure 3: Comparative statics: change in productivity y.

order importance in shaping the equilibrium behavior of the safety and liquidity premia. In particular, we study the effect of changes in productivity, y, the pledgeability of corporate bonds, χ_c, and the real supply of government bonds, b_g.

Productivity. Figure 3 depicts the effect of a change in the productivity parameter y. As productivity increases firms increase their debt issuance. This in turn increases the total amount of liquid assets. Since the supply of government bonds, b_g, is fixed, having more private debt reduces the share of government bonds in the household's portfolio. This is captured by Panels 3a and 3b which show an increase in the ratio of corporate bonds issued to GDP and a decrease in the ratio of government bonds to GDP. Moreover, we also find that the safety premium decreases as firms average productivity increases which translates



(a) Corporate bonds to GDP, $\frac{P_c b_c}{GDP}$ (b) Government bonds to GDP, $\frac{P_g b_g}{GDP}$

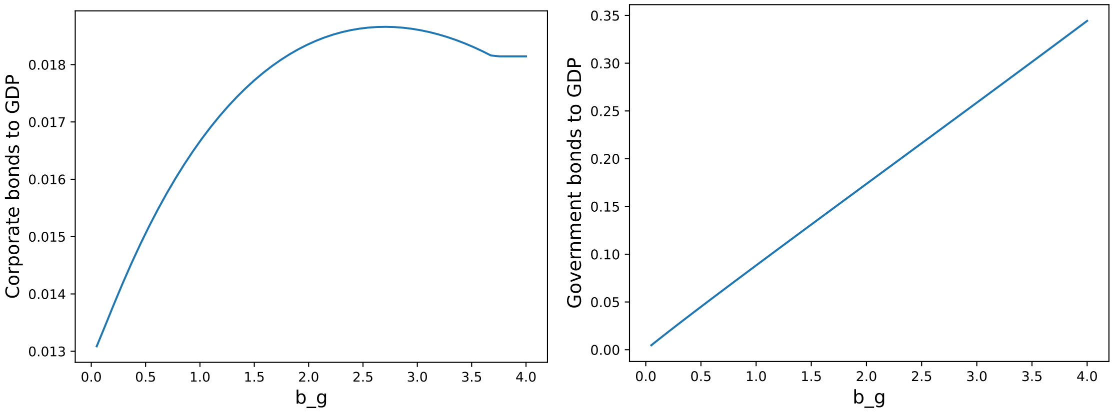


(c) Safety premium (d) Liquidity premium

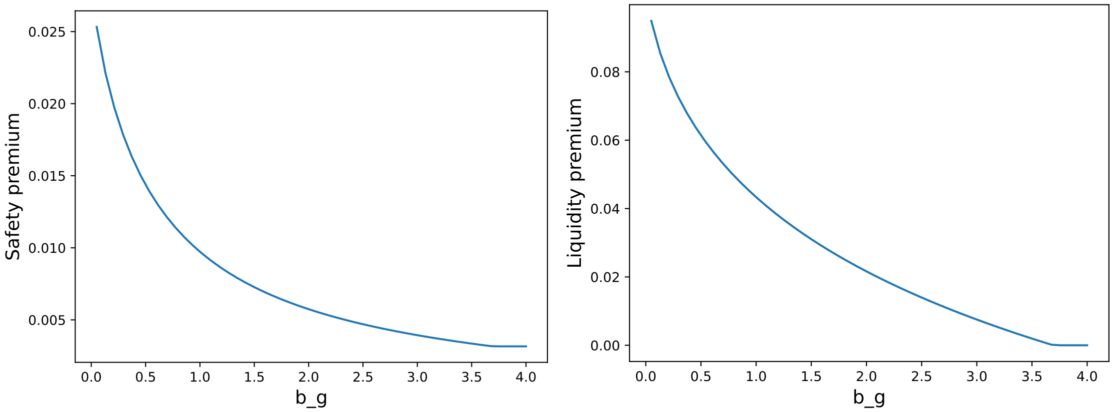
Figure 4: Comparative statics: change in corporate bonds pledgeability χ_c.

into a lower default probability. The liquidity premium increases first and then decreases.

Corporate Bond Pledgeability. The comparative statics with respect to corporate bond pleadgeability, χ_c, are depicted in Figure 4. As we can see, higher χ_c implies an increase in the liquidity of corporate bonds relative to public debt. This makes the former more attractive for buyers and hence reduces the liquidity premium. This effect is captured in Panel 4d. A lower liquidity premium implies cheaper borrowing for firms. As a result, there is higher corporate bonds issuance. Thus, the share of corporate bonds to GDP increases, while that of public debt decreases. But, what happens to the safety premium? The lower liquidity premium increases borrowing but reduces the interest rate paid on debt. These two



(a) Corporate bonds to GDP, $\frac{P_c b_c}{GDP}$ (b) Government bonds to GDP, $\frac{P_g b_g}{GDP}$



(c) Safety premium (d) Liquidity premium

Figure 5: Comparative statics: change in the supply of government bonds b_g.

channels have opposing effects on the default probability and hence the safety premium. In our calibration, the former channel dominates and the safety premium increases as corporate bonds become relatively more risky.

Supply of Government bonds. A change in b_g, has two competing effects on firms' bond issuance. First, b_g affects the supply of liquid assets, which in in turn affects the demand for goods in the DM. When liquidity is scarce, higher b_g increases consumption in the DM. This in turns increases firms' output and profits. This expansion in production and profits implies more debt issuance. However, it also reduces the liquidity premium on all liquid assets. This implies an increase in the cost of borrowing for firms. Panel 5a in Figure 5 depicts a nonlinear relationship between b_g and the share of corporate bonds in

GDP, highlighting these two competing forces. Increasing b_g from low levels first increases corporate bonds issuance up to a certain threshold. Then the effect is reversed. Finally, there is a flat region where liquidity becomes abundant. The effect of a higher supply of safe and liquid government bonds is both a reduction in the safety and liquidity premia. The latter is straightforward. The former is a result of the demand channel of liquid government bonds on firms profits which reduces the probability of default.

6.4 Model dynamics

To better understand the nonlinear dynamics of the model and explore its business cycle properties, we analyze the reaction of endogenous equilibrium objects following a productivity shock. For that we compute generalized impulse-response function (GIRF) for observable Y as follows

$$GIRF_Y(k, \varepsilon_t, \Omega_t) = \mathbb{E}[Y_{t+k}|\varepsilon_t, \Omega_t = \omega_t] - \mathbb{E}[Y_{t+k}|\Omega_t = \omega_t] \tag{43}$$

where $\Omega_t = \omega_t$ is the state of the economy at the beginning of period t and ε_t represents the period t shock to aggregate productivity.[20] The unconditional GIRF is obtained by taking the average of the conditional GIRFs, which is given by

$$\mathbb{E}[GIRF_Y(k, \varepsilon_t, \Omega_t)] = \mathbb{E}[Y_{t+k}|\varepsilon_t] - \mathbb{E}[Y_{t+k}]. \tag{44}$$

Unconditional Impulse Responses. Following a one standard deviation negative productivity shock, Figure 6 depicts the unconditional GIRFs as well as the 95% set of the distribution of conditional GIRFs.

The negative shock leads to a persistent fall in the job creation. Hence we observe a protracted increase in unemployment that reaches its peak during the second quarter after the shock. This is illustrated in the top-left panel. Moreover, after the negative shock, defaults spike on impact as the measure of profitable firms is reduced. As a result, the

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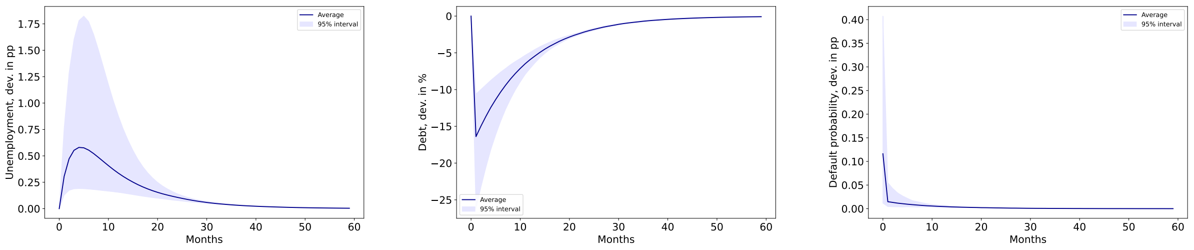
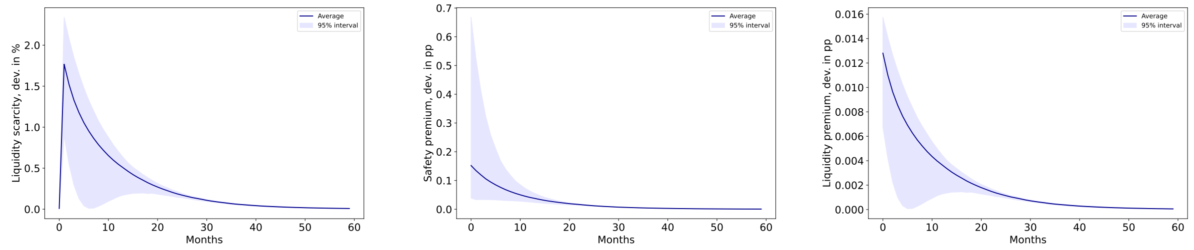
[20] More details of the corresponding computations are presented in Appendix 7.1.



(a) Unemployment, $1 - N$ (b) Corporate debt level, B_c (c) Default probability, $1 - \Phi(z^*)$



(d) Liquidity scarcity, $\mathcal{L}$ (e) Safety premium (f) Liquidity premium

Figure 6: Reaction to a negative productivity shock.

Notes: GIRFs following a 1 standard deviation negative productivity shock. The plots depict the 95% set of the distribution of conditional GIRFs (light blue) and their mean (dark blue). Each conditional GIRF is evaluated at a particular initial state drawn randomly from the ergodic distribution of the calibrated model.

amount of corporate debt issuance decreases very quickly. In particular, following the shock we observed a 15% drop, slowly returning to its ergodic mean.

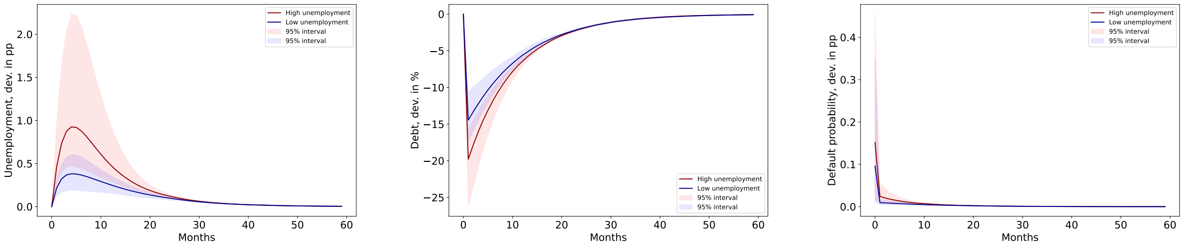
The combined effect of having fewer corporate bonds in circulation and a higher default rate leads to a decrease in the supply of liquid assets. This leads to a sharp increase in liquidity scarcity right after the shock. The sudden increase and then slow reduction in liquidity scarcity tracks the path of debt issuance. The scarce liquidity results in a spike in the liquidity premium followed by a slow decline. The safety premium jumps as well on impact but its reaction is much pronounced. The short run dynamics generated by the model following a negative productivity shock confirm that both the liquidity and safety premia co-move positively with unemployment and that the main driving force is the change in the supply of private liquid assets through both debt issuance and default.

It is important to recall that in our theoretical framework, liquidity scarcity is a function of both supply and demand for liquidity. Since in our environment the supply of government debt is constant, changes in the supply of aggregate liquidity are due to debt issuance as well as corporate default. Both of these equilibrium objects co-move positively with unemployment and their sensitivity to shock increases in unemployment. In contrast, liquidity demand operates through the DM matching function, which is decreasing in unemployment. Simply put, higher unemployment implies a lower probability of finding a seller in the DM. This in turn lowers buyers' demand for liquidity in the CM.

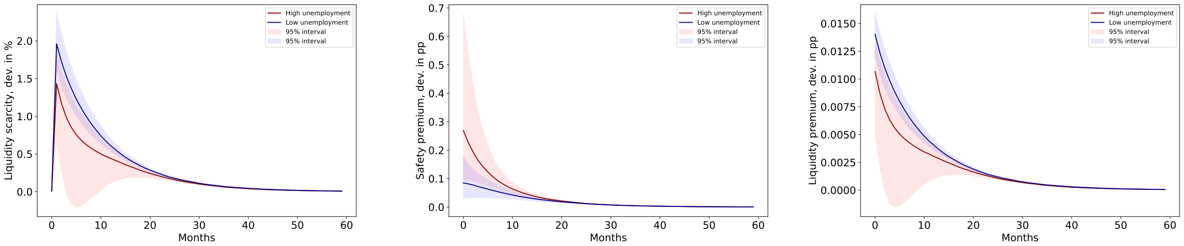
To sum up, an increase in unemployment following a negative productivity shock generates two opposing effects. More precisely, both the liquidity supply and demand curves shift to the left. However, as seen in Figure 6d, the shift in the supply schedule dominates. Thus, liquidity scarcity increases leading to an increase in the liquidity premium.

State-dependent Dynamics. Next we explore potential state-dependence in the model economy's reaction to shocks. The idea here is that the same 1-standard deviation negative productivity shocks. To do so, we compute the GIRFs conditional on the level of unemployment at period 0. Figure 7 presents these results where the red line depicts the reaction of the economy conditional on unemployment above its average. The blue line, on the other hand, illustrates the reaction when unemployment is below its average. The light red and light blue areas depict the 95% interval of the conditional GIRFs' distributions.

The plotted GIRFs in Figure 7 show that the economy's reaction to a negative productivity shock is much more pronounced when unemployment is above average. This is a direct consequence of the nonlinearities present in model. In particular, the features of the labor-search in our economic environment imply nonlinearities. These stem from three main sources: (i) the law of motion of unemployment, (ii) the fundamental surplus fraction of a match, and (iii) the elasticity of the matching function. The first implies that changes in the job finding probability have a stronger effect on unemployment when the pool of job seekers is larger, i.e. unemployment is high. The second one implies that when the share of the match surplus going to job creation is smaller, this results in a higher sensitivity of job



(a) Unemployment, $1 - N$ (b) Corporate debt level, B_c (c) Default probability, $1 - \Phi(z^*)$



(d) Liquidity scarcity, $\mathcal{L}$ (e) Safety premium (f) Liquidity premium

Figure 7: Reaction to a negative productivity shock conditional on unemployment.

Notes: GIRFs following a 1 standard deviation negative productivity shock. The plots depict the 95% set of the distribution of conditional GIRFs and their mean conditional on unemployment above (red) and below (blue) its ergodic mean. Each conditional GIRF is evaluated at a particular initial state drawn randomly from the ergodic distribution of the calibrated model.

creation to changes in productivity. Finally, the third potential source of nonlinearity are due to changes in the elasticity of the matching function to labor market tightness. What's novel in our model is that these nonlinearities transmit to the liquidity in interesting ways. In particular, debt issuance and default probability exhibit a much stronger reaction to the shock when unemployment is already high. This in turn transmits to the safety premium that suddenly increases by almost 0.3pp when unemployment is above average compared to about 0.1pp when unemployment is below average. In contrast, liquidity scarcity and the liquidity premium exhibit the opposite pattern. Exhibiting higher sensitivity to shocks when unemployment is below average. This is probably a result of the interaction between liquidity demand and supply. When unemployment is extremely high, the fall in the demand for liquidity is so strong that it dominates the decrease in supply. As a result, we observe a decrease in liquidity scarcity as seen in Panel 7d.

7 Conclusion

Given the importance of U.S. treasuries in implementing U.S. monetary policy and its prominent role in many portfolios around the world, it is important to empirically disentangle the corresponding safety and liquidity premia over the business cycle as well as have a framework that can rationalize them. To do so, we integrate multiple types of assets, which differ in their safety and liquidity, into a framework that also features equilibrium unemployment and corporate default. These features allow the model to generate endogenous business cycle fluctuations in the liquidity and safety of both corporate and government bonds.

We find an average treasury premium of 1.98 percentage points (pp), with 60% of can be attributed to the safety component, while the remaining 40% measures the liquidity premia. We also find that the volatility of the treasury premium is mostly driven by the safety component. This is especially the case during recessions.

Using our theoretical framework, we perform a regression analysis using simulated data, which yield results that are comparable to its empirical real data counterparts. This provides evidence supporting the theoretical framework we have developed captures the treasury premia and its components over the business cycle.

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Appendix

7.1 Computational details

Solution algorithm.

SMM calibration.

Generalized impulse-response functions. To compute the GIRFs we randomly draw a set of 1,000 states from their ergodic distribution which we obtain by simulating the model for 10,000 runs of 1,000 months each.[21] From each initial state we run 10,000 stochastic simulations with a 1 standard deviation shock to aggregate productivity in the first period and 10,000 simulations without, each simulation lasting 100 months. The conditional GIRF at each initial state is the difference between the average paths of these two simulation sets.

Robustness

Table 8: Average size of the premium on 10-year treasury bonds

	Total premium (Baa - TB10Y)	Safety premium (Baa - Aaa)	Liquidity premium (Aaa - TB10Y)
Average	1.92 pp	0.97 pp	0.95 pp
Average during recessions	2.30 pp	1.33 pp	0.98 pp
Average outside of recessions	1.85 pp	0.91 pp	0.94 pp

Notes: Statistics are computed using quarterly averages of monthly U.S. data. Data covers the period 1953Q1-2023Q1. NBER recession dates are used. Recession months are defined from the month following the peak through the trough. A recession quarter is a quarter that includes a recession month.

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[21] We burn the first 136 months and keep the remaining 864 months to match the length of the data.

Table 9: Standard deviation of the premium on the 10-year treasury bonds (HP filtered)

	Total premium (Baa - TB10Y)	Safety premium (Baa - Aaa)	Liquidity premium (Aaa - TB10Y)
STD	0.45 pp	0.27 pp	0.25 pp
STD during recessions	0.66 pp	0.44 pp	0.28 pp
STD outside of recessions	0.35 pp	0.19 pp	0.23 pp

Notes: Statistics are computed using quarterly averages of monthly U.S. data. Data covers the period 1953Q1-2023Q1. NBER recession dates are used. Recession months are defined from the month following the peak through the trough. A recession quarter is a quarter that includes a recession month. Cyclical series are computed as deviations from an HP trend with parameter $\lambda = 1,600$.

Table 10: Regression of 20-year treasuries premium on unemployment

	Dependent variable: Treasury premium			
	Baa-TB20Y			
	(1)	(2)	(3)	(4)
Unemployment	0.261^{***}	0.260^{***}	0.234^{***}	0.207^{***}
	(0.025)	(0.024)	(0.027)	(0.023)
Debt-to-GDP		-0.941^{***}	0.204	-0.048
		(0.331)	(0.430)	(0.439)
Federal Funds Rate			0.056^{***}	0.049^{***}
			(0.015)	(0.013)
VIX				0.067^{***}
				(0.011)
Observations	288	288	262	262
R^2	0.309	0.334	0.320	0.493
Adjusted R^2	0.307	0.330	0.312	0.485
Residual Std. Error	0.642	0.632	0.629	0.544
F Statistic	106.433^{***}	61.108^{***}	46.418^{***}	43.760^{***}

Note: *p<0.1; **p<0.05; ***p<0.01

Table 11: Regression of liquidity premium on 20-year treasuries on unemployment

	Dependent variable: Liquidity premium			
	Aaa-TB20Y			
	(1)	(2)	(3)	(4)
Unemployment	0.098***	0.098***	0.075***	0.063***
	(0.013)	(0.012)	(0.014)	(0.014)
Debt-to-GDP		-0.442**	0.023	-0.093
		(0.214)	(0.289)	(0.297)
Federal Funds Rate			0.019**	0.016*
			(0.009)	(0.008)
VIX				0.031***
				(0.005)
Observations	288	288	262	262
R^2	0.137	0.154	0.111	0.229
Adjusted R^2	0.134	0.149	0.101	0.217
Residual Std. Error	0.405	0.401	0.402	0.375
F Statistic	57.952***	31.602***	15.393***	20.124***
Note:	*p<0.1; **p<0.05; ***p<0.01			

Table 12: Regression of the safety premium on 20-year treasuries on unemployment

	Dependent variable: Safety premium			
	Baa-Aaa			
	(1)	(2)	(3)	(4)
Unemployment	0.163***	0.163***	0.158***	0.144***
	(0.016)	(0.015)	(0.016)	(0.014)
Debt-to-GDP		-0.499***	0.181	0.045
		(0.162)	(0.185)	(0.185)
Federal Funds Rate			0.037***	0.033***
			(0.008)	(0.007)
VIX				0.036***
				(0.009)
Observations	288	288	262	262
R^2	0.400	0.423	0.449	0.605
Adjusted R^2	0.398	0.419	0.443	0.599
Residual Std. Error	0.329	0.323	0.320	0.272
F Statistic	110.549***	65.569***	60.556***	50.678***
Note:	*p<0.1; **p<0.05; ***p<0.01			