# When do Firms Profit from Wage Setting Power?*

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**Abstract**

In standard models of labor market monopsony, the profits derived from firm monopsony power depends on the firm's labor supply elasticity. There are two puzzles facing these standard models. First, different standard approaches to estimating labor supply elasticities produce dramatically different estimates and hence measures of profits from monopsony power. Second, commonly used low labor supply elasticities imply profit shares of aggregate income that are too high after accounting for price markups and capital income. This paper argues that both of these issues arise from the same limitation - that firms can increase employment only by raising wages. To address this, we develop a tractable model where firms use both higher wages and costly recruiting expenditures to attract workers. Firms have wage setting power due both to search frictions and workers' heterogeneous preferences over workplaces. We show that whether firms profit from their wage setting power depends on the shape of firms' recruiting cost function, and the rents acquired by firms from wage setting power can be dissipated by recruiting costs. In a calibrated quantitative model that also accounts for the strategic behavior of a large firm, profits from wage setting power account for 6% of labor market-wide marginal product and 5% of output. Our findings suggest that wage setting power alone does not imply profits for firms that exploit this power.

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# 1 Introduction

There is a growing consensus that firms have *wage setting power*: firms that choose to lower wages do not lose all of their employees (Card, 2022). In many standard models of labor market monopsony, firm wage setting power implies a finite labor supply elasticity, allowing researchers to infer the markdowns of wages from marginal product, and by extension, the profits that firms earn by exploiting their wage setting power. However, puzzles remain in estimating the labor supply elasticity and its implications for the distribution of income. First, common approaches to estimating labor supply elasticities produce very different conclusions: estimates of firms' labor supply elasticity based on labor market flows are much smaller than estimates implied by the relationship between wages and firm size.[1] Second, commonly used low labor supply elasticities imply implausibly large profit shares of income in the aggregate.[2] Further, many models abstract from recruiting activity and assume that firms' only way to expand is by offering higher wages. By including recruiting expenditures in a setting where firms set wages, we address these puzzles and ask the questions, *under what conditions does wage setting power result in profits for firms, and how large are these profits?*

To answer these questions, we derive a novel and tractable model of dynamic monopsony where workers search on the job, and firms use both higher wages and a recruiting expenditure margin to attract workers. Workers have horizontally differentiated preferences over firms, and the presence of both search frictions and these preferences gives firms wage setting power: firms that choose lower wages do not immediately lose all their workers. We analytically characterize the profit share of marginal product, which is the share of marginal product that is not paid out in wages or spent on recruiting. We show theoretically that whether firms profit from their wage setting power depends primarily on the shape of the recruiting cost function: if recruiting costs are convex in the number of hires, then the profit share of marginal product is positive. If instead greater firm size supports hiring and recruiting costs are a function of hires per incumbent, then in steady state the gap between wages and marginal product is consumed by recruiting costs. We show that the profit share of marginal product is tightly linked to the elasticity of optimal wages to firm size: firms that can elastically scale up employment in the long run without raising wages have low profit shares of marginal product. Thus, whether firms profit from their wage setting power depends on if firms can elastically scale up recruiting in the long run: wage setting power alone is insufficient.

Given the tight link between the profit share of marginal product and the wage-size elasticity, we estimate firms' wage-size elasticity using two methods. First, we leverage the tractable structure of the model and show that the profit share of marginal product can be identified using the empirical distribution of firm sizes and wages. Measuring firm size and estimating firm wage effects using the

[1] Sokolova and Sorensen (2021) provide a meta-analysis of papers these two approaches to estimating labor supply elasticities. Manning (2003) also discusses the different results by estimation method.

[2] Quoting Manning (2021): "The low estimated wage elasticity of the labor supply curve implies that employers have a lot of monopsony power: If this power is exercised it is not clear how it can be reconciled with observed levels of the profit share."

standard AKM (Abowd et al., 1999) method, we infer a profit share of marginal product of .09 for single-establishment firms and .03 for multi-establishment firms.

Second, building on Friedrich et al. (2023), we examine the wages of new hires before and after firm expansion events by medium-size firms in Danish administrative data. Our innovation is to focus on the wage growth of job switchers, as switchers' wages (as opposed to stayers) are the least likely to be affected by rent sharing that is unrelated to long-run upward sloping labor supply. Consistent with findings in Schmieder (2023), we find that switchers who arrive during the expansion event experience greater wage growth, but switchers who arrive after the expansion experience similar wage growth as switchers who arrive before the expansion. These results are consistent with recruiting costs that are a function of hires per incumbent: while firms are growing quickly, firms face diminishing returns to recruiting expenditure and raise wages to accelerate hiring. However, as firms reach their new steady state size, per-worker recruiting costs return to normal, and firms' tradeoff between wage costs and recruiting costs returns to its pre-expansion baseline. These results imply an elastic labor supply in the long run, consistent with a profit share of marginal product close to zero.

We then consider concentration and strategic behavior. We embed our atomistic firms into an equilibrium setting where firms compete for labor, use capital, and sell differentiated goods. We introduce a single granular firm that is large enough to affect labor market aggregates. This large, granular firm can profit from strategically under-hiring and depressing the labor market. The large firm's ability to depress the labor market is constrained by the reaction of its atomistic competitors: atomistic firms respond to greater labor availability by expanding and offsetting the large firm's under-hiring. The extent that atomistic firms can offset the large firm's behavior depends on both how elastically atomstic firms can scale up recruiting and their degree of dimishing returns to labor. In the extreme case of perfectly elastic recruiting and constant returns to labor, atomistic firms can perfectly offset any attempt by the large firm to depress the market, and the large firm derives no additional profit from being large in the labor market. Calibrating the model to our and other existing empirical evidence, we find that firms at the bottom and the top of firm size distribution profit the most from wage setting power. In our preferred calibration, the model-implied profit share of economy-wide marginal product is 6% when the employment Herfindahl-Hirschman index (HHI) is 0.1, and profits from this wage setting power account for 5% of aggregate income.

This paper makes four main contributions. First, we derive a tractable framework that unifies models of on-the-job search and dynamic monopsony. Second, we characterize the conditions under which both atomistic firms and granular firms profit from wage setting power in the presence of a recruiting expenditure margin. Third, we provide novel evidence on firm wage-size elasticities and quantify the profit share of marginal product, including the effects from concentration. Fourth, we provide a resolution to two puzzles in the monopsony literature relating to conflicting estimates of labor supply elasticities and the profits from wage setting power.

Our novel framework unifies key aspects of dynamic monopsony and on-the-job search models. In the dynamic monopsony literature, researchers use highly credible identification strategies to estimate the elasticity of recruits and the elasticity of separations with respect to a firm's wage policy. Our paper is the first to explicitly integrate these recruiting and separation elasticities into an equilibrium search framework. In our setting, the combination of search frictions and workers' heterogeneous preferences creates finite elasticities of recruiting and separations with respect to wages, giving firms wage setting power. This sum of recruiting and separation elasticities is in fact the measure of firm wage setting power in our model. The result is a highly tractable model of on-the-job search that can be easily disciplined by this existing empirical evidence.

We offer a resolution to the two puzzles facing monopsony models. In the first puzzle, estimates of firms' labor supply elasticities differ dramatically based on the estimation method. One common method leverages models of dynamic monopsony, where the elasticity of labor supply to the firm is simply the sum of recruiting and (negative) separation elasticities (Manning, 2003). Based on the empirical evidence, this method typically yields labor supply elasticities between 4-6. In contrast, research that estimates labor supply elasticities using the relationship between firm size and wages imply much larger labor supply elasticities, in the range of 20-100 (Sokolova and Sorensen, 2021). Our model can rationalize both of these results. In our setting, the sum of recruiting and separation elasticities is indeed the firm's labor supply elasticity holding recruiting effort fixed. The presence of a recruiting margin breaks the one-to-one link between size and wages, enabling the firm to grow without raising wages.

In the second puzzle, large markdowns of wages from marginal product in standard monopsony models imply labor shares of income that are too low and profit shares that are too high given estimates from national accounts data in rich countries. In our quantitative section, we show that our model is close to matching aggregate income shares of labor, capital, and profits in a general equilibrium setting when choosing realistic values for price markups and returns to capital. The presence of recruiting margins lowers profits for three reasons. First, some of the gap between wages and marginal product is consumed by recruiting costs. Second, the addition of a recruiting margin that is at least somewhat elastic in the long run flattens firms' long-run marginal cost curve for labor, allowing firms to grow larger and narrow the gap between marginal product and wages. Third, the ability for atomistic firms to scale up recruiting limits the power of the large, granular firm to profit from using its size to depress the labor market.

Lastly, we make an additional theoretical contribution: our environment is one with random on-the-job search and wage posting, but there is a point mass of wages in equilibrium even with ex-ante homogenous firms and workers. This stands in contrast to the famous result in Burdett and Mortensen (1998) that with on-the-job search and ex-ante homogenous workers and firms, the equilibrium must contain wage dispersion. Like Albrecht et al. (2018), we achieve this by workers having horizontally differentiated preferences over workplaces, but unlike much of the existing monopsony literature, we make make these differentiated preferences time-varying. The result is that identical firms choose identical wage policies, and job-to-job mobility occurs even if there is a point mass of wages. This equilibrium without wage dispersion allows us to maintain tractability as we enrich the model with firm heterogeneity and a large, nonatomistic firm.

**Related Literature** This paper is most closely related to the canonical models of dynamic monopsony, nesting and providing a microfoundation for the model of dynamic monopsony in Manning (2003) and a providing a microfounded case of Manning (2006)'s generalized model of monopsony that explicitly integrates worker turnover and on-the-job search. Numerous authors have estimated recruiting and separation elasticities including Azar et al. (2019), Bassier et al. (2022), and Datta (2023). Our study also contributes to a growing literature that emphasizes the importance of concentration in the labor market, including Azar et al. (2020), Berger et al. (2022), Derenoncourt et al. (2021), Jarosch et al. (2021), Naidu et al. (2018), and Schubert et al. (2023), among others.

The importance of non-wage amenities, which is essential in our model, is highlighted in Sorkin (2018) and Hall and Mueller (2018). Heise and Porzio (2022) also have related time-varying horizontal preferences over workplaces. Along with Herkenhoff et al. (2021), we are among the first to jointly consider both concentration and idiosyncratic workplace preferences, which are often believed to be two of the main mechanisms giving firms wage setting power, and we are the first to consider these jointly in a setting of wage posting.

Some authors have directly studied whether recruiting costs are convex. Manning (2006) estimates if hiring costs increase with firm size, finding estimates of diseconomies of scale in recruitment that imply modest markdowns of wages from marginal product. Blatter et al. (2012) find that hiring costs are convex, but this convexity decreases with firm size. Kuhn (2004) discusses the importance of returns to scale in recruiting in monopsony models.

Our estimate of the wage-size elasticity is in line with studies that report a fall in the employer size-wage premium in the US, such as in Cobb and Lin (2017), Bloom et al. (2018), and Song et al. (2019). Earlier studies report firm-size elasticities between .02 and .06, such as Brown and Medoff (1989) and Oi and Idson (1999). Katz (1986) discusses mechanism for higher wages at large firms, such as monitoring costs, reputation and strike avoidance, and rent sharing. Manning (2011) argues this may reflect the labor supply curve to the firm. Bachmann et al. (2022) infer monopsony power from the slope of wages to firm size directly. Other evidence suggests that firms do not need to pay more to grow in the long run. Engbom et al. (2022) find that conditional on the capital stock, wages fall as the number of employees increases. Numerous authors estimate labor supply elasticities using the wages of stayers in response to shocks to firms, such as Lamadon et al. (2022), Trottner (2022), Seegmiller (2021), and Chan et al. (2020).

The paper is organized as follows. Section 2 describes the two puzzles in detail. Section 3 lays out the model of dynamic monopsony. Section 4 presents the data and empirical exercises estimating firm wage-size elasticities. Section 5 derives the quantitative model with labor market concentration. Section 6 concludes.

# 2 Two Puzzles

## 2.1 Puzzle 1: Differing Labor Supply Elasticities by Estimation Method

Sokolova and Sorensen (2021) perform a summary of labor supply elasticities and show that two of the most common methods of estimating of labor supply elasticities produce dramatically different results.[3] The first method of estimating the elasticity of labor supply to the firm leverages the law of motion in a setting of "dynamic monopsony," where the sum of recruiting and (negative of) separation elasticities with respect to the wage add up to the total labor supply of the firm. Researchers use causal methods to estimate the elasticity of recruits (or hires)[4] with respect to the wage $\varepsilon_{R,w}$ or the elasticity of the separation rate with respect to the wage $\varepsilon_{S,w}$. Assumptions on the law of motion of employment yield a labor supply elasticity

$$\varepsilon_{N,w} = \varepsilon_{R,w} - \varepsilon_{S,w},$$

where $\varepsilon_{N,w}$ is the firm's labor supply elasticity. Estimates of the labor supply elasticity inferred this way tend to be quite small, and Sokolova and Sorensen (2021) report a median estimate of the overall labor supply elasticity of 1.4. Accounting for various biases, Sokolova and Sorensen (2021)'s "best-practice" estimate of the labor supply elasticity from this method is around 7, though some recent estimates point to elasticities closer to 4 (Bassier et al. (2022), Datta (2023)).

Another way to estimate the labor supply to the firm is to measure how much higher are wages at large firms, which in elasticity terms $\varepsilon_{w,N}$ is the wage-size elasticity, using various econometric methods. The labor supply elasticity then is just the inverse: $\varepsilon_{N,w} = \varepsilon_{w,N}^{-1}$. Estimates attained this way are much higher, and Sokolova and Sorensen (2021) argue that "best-practice" estimates of the labor supply elasticity from this method to be around 23. Matsudaira (2014) finds in response to regulations that mandate greater staffing levels, employers of nurse aids expand employment without raising wages, pointing to a perfectly elastic labor supply curve.

## 2.2 Puzzle 2: Implausibly Large Profits from Small Labor Supply Elasticities

The second puzzle is that after taking into account capital income and profits from product market power, estimates of low labor supply elasticities imply large profit shares of GDP that are hard to reconcile with observed labor and non-labor shares of income. Rognlie (2016) shows for advanced economies, the labor share of the domestic corporate sector is between 70-80%. This measure excludes depreciation from value added, since payments to capital that pay off depreciation is likely not relevant for welfare considerations (Koh et al., 2020). This measure also excludes housing

[3]Another common method of estimating labor supply elasticities looks at the wage response of stayers to shocks to firm performance. We are concerned about rent-sharing motivations that are unrelated to long-run upward-sloping labor supply, which we discuss further in Section 4.3.

[4]Datta (2023) estimates the elasticity of recruits (i.e., the workers who are willing to work for the firm) and actual hires separately, taking into accounts that firms may reject applicants. For our purposes, the hiring elasticity is more relevant.

income, which is typically not a factor of production. Including housing, the labor share of net income is between 65-73%. We will show that after accounting for product market markups and capital income, the lower estimates of labor supply elasticities around 4 such as in Card et al. (2018) generate too high a profit share and too low a labor share, after accounting correctly for depreciation.

To demonstrate this puzzle, we derive a simple steady-state economy with capital, labor, differentiated goods, and finite labor supply elasticities. Firms operate using a Cobb-Douglas production function with capital share $\beta$. Firms' product demand elasticity $\eta < \infty$ generates a markup of prices over marginal cost $\frac{\eta}{\eta-1}$, and finite labor supply elasticities $\varepsilon_{N,w} < \infty$ generate a wage markdown of wages to marginal product $\frac{\varepsilon_{N,w}}{1+\varepsilon_{N,w}}$. Capital depreciates at rate $\delta$ and demands a return net of depreciation of $r$, with capital rental rate $r_K = r + \delta$.

Calibrating this simple model, we set the demand elasticity $\eta = 7$ (Kline et al., 2019), the Cobb-Douglas capital share $\beta = .3$, the depreciation rate $\delta = .1$, and the return to capital $r = 0.05$ (Jordà et al., 2019). In Table 1, we report these parameters and the share of net value added that is paid out to labor income, net capital income, and profits. We do this separately for two values of labor supply elasticities. In the first row, we report very high labor supply elasticities $\varepsilon_{N,w} = 100$, yielding a wage markdown of .99. This generates a labor share of income within the empirical range reported by Rognlie (2016) of 70-80%. In the second row, we report the labor, capital, and profit shares with the labor supply elasticity $\varepsilon_{N,w} = 4$ as in Card et al. (2018). With this lower elasticity, the profit share of net value added falls to 58%, well outside the empirical range in advanced economies.

Table 1: Profit Puzzle

| $\eta$ | $\beta$ | $\varepsilon_{N,w}$ | $\delta$ | $r$ | $\frac{\text{Labor Income}}{\text{Net VA}}$ | $\frac{\text{Net Capital Income}}{\text{Net VA}}$ | $\frac{\text{Profit}}{\text{Net VA}}$ |
|---|---|---|---|---|---|---|---|
| 7 | .3 | 100 | .1 | .05 | .72 | .10 | .18 |
| 7 | .3 | 4 | .1 | .05 | .58 | .10 | .31 |

This table reports various calibrations. $\eta$ is the product demand elasticity, $\beta$ is the elasticity of output with respect to capital in Cobb-Douglas production, $\varepsilon_{N,w}$ is the labor supply elasticity, $\delta$ is the depreciation rate of capital, and $r$ is the required return on capital.

While the profits puzzle can be resolved with a high labor supply elasticity, this is unsatsifying, as many researchers use well-identified methods and infer low labor supply elasticities. Therefore, we must address both puzzles jointly and will do so by deriving a model that matches all empirical moments - separation and recruiting elasticities, wage-size elasticities, and profit shares - in one model.

# 3 Dynamic Monopsony with Atomistic Firms

In this section, we derive our core model of dynamic monopsony. In Section 3.1, we begin with the most general firm problem in a setting of dynamic monopsony, where a firm has diminishing revenue returns to labor, sets wages, faces a worker separation as a function of wages, and has a hiring cost function. We show that in this very general setting, the firm's marginal product of labor can be analytically decomposed using the elasticity of separations to wages and elasticities of the hiring cost function with respect to number of hires, size of the firm (measured in incumbent workers), and the firm's wage. In Section 3.2, we add more structure to the firm's problem, specifying turnover costs as coming from costly vacancy posting, and we parameterize the firm's recruiting function. This helps us discipline the decomposition of marginal product, as in this particular parameterization, the sum of recruiting and (minus) separation elasticities that has been influential is the measure of firms' wage setting power. The parameterization of the recruiting cost function helps us interpret under what conditions the profit share of marginal product is non-zero, i.e., whether per-worker recruiting costs are constant, increasing in the ratio of hires to incumbents, or increasing in the number of hires. In Section 3.3, we microfound the workers' problem, specifying the frictional labor market and workers' preferences. This allows us to fully specify the equilibrium labor market and draw out additional insights analytically.

## 3.1 Firm's Problem with Nonparametric Recruiting Costs

In this section, we derive a general firm problem of dynamic monopsony with a non-parametric hiring cost function in partial equilibrium, and we show that marginal product can be analytically decomposed into wages, recruiting costs, and profit as a function of the separation elasticity and elasticities of the hiring cost function.

In this general firm problem, the firm maximizes an infinite sum of discounted profits. Firms produce with a diminishing returns to scale production function using only labor $F(N)$, with $F'(N) > 0$ and $F''(N) < 0$. Firms set wages $w_t$ each period and pays cost $\mathcal{C}(H_t, N_{t-1}, w_t)$ to make $H_t$ hires, with a cost that depends on the number of hires $H_t$, the wage $w_t$, and the number of incumbent workers from the prior period $N_{t-1}$. Firms solve:

$$\max_{\{N_t\},\{H_t\},\{w_t\}} \sum_{t=0}^{\infty} \left(\frac{1}{1+\rho}\right)^t \Big(F(N_t) - w_t N_t - \mathcal{C}(H_t, N_{t-1}, w_t)\Big),$$

subject to the law of motion for employment:

$$N_t = (1 - S(w_t))N_{t-1} + H_t,$$

where the rate at which workers separate $S(w_t) \in [0, 1]$ is a decreasing function of the wages this period: $S'(w_t) < 0$.

In steady state, the firm's optimal wage trades off wage costs and recruiting costs. Suppressing the time subscripts and solving for the optimal steady state wage yields:

$$w^* = \frac{\mathcal{C}(H,N,w)}{N}\big( -\varepsilon_{\mathcal{C},w} - \varepsilon_{S,w} \times \varepsilon_{\mathcal{C},H}\big)$$

There are natural restrictions that we apply to the cost function. First, conditional on the number of hires and incumbents, the recruiting cost function should be at least weakly decreasing in the wage: $\mathcal{C}_w(H,N,w) \leq 0$, and so $\varepsilon_{C,w} \leq 0$ as well. Hiring costs should be strictly increasing in the number of hires so $\varepsilon_{C,h} > 0$, and by assumption the separation rate is decreasing in the wage, so $\varepsilon_{S,w} < 0$. The optimal wage finds the point at which lowering wages further yields an equal increase in hiring costs. Letting $\rho \to 0$,[5] we have the ratio of wages to marginal product

$$\frac{w}{F'(N)} = \frac{-\varepsilon_{\mathcal{C},w} - \varepsilon_{S,w} \times \varepsilon_{\mathcal{C},H}}{\varepsilon_{\mathcal{C},N} - \varepsilon_{\mathcal{C},w} + \varepsilon_{\mathcal{C},H}(1 - \varepsilon_{S,w})}$$

Hiring costs per worker as a share of marginal product are

$$\frac{\mathcal{C}(H,N,w)/N}{F'(N)} = \frac{1}{\varepsilon_{\mathcal{C},N} - \varepsilon_{\mathcal{C},w} + \varepsilon_{\mathcal{C},H}(1 - \varepsilon_{S,w})}$$

What is not paid in wages or turnover costs is retained as profit

$$\frac{F'(N) - w - \mathcal{C}(H,N,w)/N}{F'(N)} = \frac{\varepsilon_{\mathcal{C},N} + \varepsilon_{\mathcal{C},H} - 1}{\varepsilon_{\mathcal{C},N} - \varepsilon_{\mathcal{C},w} + \varepsilon_{\mathcal{C},H}(1 - \varepsilon_{S,w})}.$$

This final expression captures what Manning (2006) calls "diseconomies of scale in recruiting." If the sum of the elasticity of hiring costs with respect to hires $\varepsilon_{\mathcal{C},H}$ and the elasticity of hiring costs with respect to incumbents $\varepsilon_{\mathcal{C},N}$ is greater than 1.[6] Another way of looking at this is, if hiring costs are linear in the number of hires, then $\varepsilon_{\mathcal{C},H} = 1$. To the extent that hiring costs are more convex than linear with respect to the number of hires, the hiring cost of function exhibits diseconomies of scale if negative values of $\varepsilon_{\mathcal{C},N}$ does not offset values of $\varepsilon_{\mathcal{C},H}$ above 1; that is, being large does not offset the higher cost of hiring due to convexity in hiring cost.

## 3.2 Parameterizing the Recruiting Cost Function

In this subsection, we parameterize the firm's recruiting cost function and production function, and we analyze the firm's problem in partial equilibrium. We assume that all hiring costs come from vacancy posting, and that firms offering high wages allows the firm to fill vacancies faster. Under this parameterization, we show that the sum of recruiting and (minus) separation elasticities is an

[5] In Appendix A.2, we derive the decomposition of marginal product when firms discount for the parameterized firm problem in Section 3.2. When $\rho > 0$, firms choose lower levels of employment. This raises the firm's marginal product, raising the profit share of marginal product. However, in accordance with the result in Manning (2006) we find that the quantitative effect of discounting is quite small: it adds between 1-2% to the profit share of marginal product. This occurs because the rate at which firms discount future flows from hiring a given worker is the sum of the discount rate and the separation rate. Monthly separation rates are typically around 4% in flexible labor markets like the US and Denmark, whereas typical monthly discount rates are an order of magnitude smaller.

[6] Muehlemann and Pfeifer (2016) estimate these cost elasticities directly in a survey of German firms hiring skilled workers, finding $\varepsilon_{\mathcal{C},H} = 1.3$ and $\varepsilon_{\mathcal{C},N} = 0$.

important term in the decomposition of marginal product. We avoid specifying the worker's problem in this subsection to highlight exactly what assumptions are needed for our results. (We specify the workers' problem and solve for the labor market equilibrium in Section 3.3 to microfound the recruiting and separation elasticities). We also show how the standard model of dynamic monopsony without a recruiting margin is nested as a special case.

**Production** Firms faces diminishing returns to labor $\alpha$, have total factor productivity $A$, and employ $N$ workers, producing $AN^{\alpha}$.

**Recruiting Cost Function** In this section, we assume that all of firms' recruiting costs comes from posting vacancies. We also assume that wages reduce recruiting costs because higher wages help firms fill vacancies faster. In particular we assume that hires every period take the form $H_t = V_t R(w_t)$, where $V_t$ is the number of vacancies posted by a firm, and $R(w)$ is the probability that a vacancy is filled given the firm's wage policy $w$, with $R'(w) > 0$.[7] The firm's problem is now

$$\max_{\{N_t\},\{V_t\},\{w_t\}} \sum_{t=0}^{\infty} \Big(\frac{1}{1+\rho}\Big)^t \Big(AN_t^{\alpha} - w_t N_t - \mathcal{C}(N_{t-1}, V_t)V_t\Big), \tag{1}$$

subject to a new law of motion

$$N_t = (1 - S(w_t))N_{t-1} + R(w_t)V_t, \tag{2}$$

where $S(w)$ is the share of workers who separate from the firm, with $S'(w) < 0$. We leave the recruiting and separation functions general for now, but we maintain the assumptions that the separation rate $S(w)$ is decreasing in the wage and the recruiting rate $R(w)$ is increasing in the wage, with $S(w), R(w) \in [0, 1]$.

The recruiting cost function takes the form

$$\mathcal{C}(N, V) = c \times \Big(\frac{V_t}{N_{t-1}}\Big)^{\chi} N_{t-1}^{\sigma\chi}. \tag{3}$$

For a given level of firm size $N$, the parameter $\chi \in [0, \infty)$ determines the elasticity of the vacancy cost with respect to the number of vacancies posted. $\sigma \in [0, 1]$ governs the elasticity of per-vacancy recruiting costs as firm size increases, holding the ratio of vacancies to incumbents fixed. For example, consider the case when $\sigma = 0$, so the $N^{\sigma\chi}$ term drops out.[8] This means that the vacancy costs are convex in the ratio of vacancies to employment. In practice, this would mean that it's

[7] Note that is definition of the recruiting rate $R(w)$ differs the standard model. In the standard model, the recruiting function is not multiplied by any other term and represents the total number of hires. Here, $R(w)$ is multiplied by the number of vacancies, and represents the probability than any given vacancy results in a hire. In a more general sense, vacancies simply represent some unit of effective recruiting effort, and the recruiting function $R(w)$ captures the rate at which hiring increases with the wage, holding recruiting effort constant.

[8] Potential microfoundations for $N$ in the denominator and $\sigma = 0$ could be social networks as in Caldwell and Harmon (2019).

costly for firms to try to grow *quickly*, but the cost to a 100 worker firm of posting 10 vacancies is proportionately the same as a 500 worker firm posting 50 vacancies. In the case where $\sigma = 1$, the number of incumbent workers in the numerator and denominators cancel, and turnover costs are convex only in the number of vacancies.[9]

If the firm is operating in a stationary environment, then the following three equations characterize the firm's optimal steady state choices of wages $w$, employment $N$, and vacancies $V$. The optimal wage is

$$w = c(1+\chi)\Big(\frac{S(w)}{R(w)}\Big)^{1+\chi}\big(\varepsilon_{R,w} - \varepsilon_{S,w}\big)N^{\sigma\chi}. \tag{4}$$

Define $\mathcal{E}(w) \equiv (\varepsilon_{R,w} - \varepsilon_{S,w})$ as the recruiting elasticity minus the separation elasticity. The optimal steady-state level of employment is

$$N = \left(\frac{\alpha A}{\frac{w}{\mathcal{E}(w)(1+\chi)}\big(1 + \mathcal{E}(w)(1+\chi) + \sigma\chi + \frac{\rho}{1+\rho}\big(\chi(1-\sigma) + (1+\chi)\frac{1-S(w)}{S(w)}\big)\big)}\right)^{\frac{1}{1-\alpha}}. \tag{5}$$

The steady state ratio of vacancies to employment takes the following form:

$$\frac{V}{N} = \frac{S(w)}{R(w)}. \tag{6}$$

As long as firms have diminishing returns to labor in production $\alpha \in (0,1)$, then equations (4), (5), and (6) have multiple implications:

- The optimal wage trades off between wages costs and turnover costs for the firm.
- If $\sigma = 0$, then the optimal wage $w$ is independent of firm size $N$. In steady state, the per-worker employment costs is identical at the margin for firms of any size, consequently, wages are independent of total factor productivity $A$.
- Higher discount rates $\rho$ make the firm choose a smaller optimal size, as higher discount rates discourage from firms investing in acquiring new workers through posting vacancies.
- If $\sigma = 0$, the optimal wage is unaffected by the firm's discount rate.

We show in the Appendix A.2 that when this model is calibrated to standard monthly parameters, the discount factor is quantitatively unimportant. Therefore, for the rest of this section, we will assume $\rho = 0$.

Next we will explore what share of marginal product the firm retains as profits. Recall the definition of the sum of recruiting and (negative) separation elasticities as:

$$\mathcal{E}(w) \equiv \varepsilon_{R,w} - \varepsilon_{S,w}.$$

[9] This creates a recruiting cost function that is quadratic in the number of vacancies. Quadratic vacancy costs are fairly common assumption, for example, see Acemoglu and Hawkins (2014).

For presentation purposes, we will suppress the $(w)$, and simply write this sum of elasticities as $\mathcal{E}$. Table 2 decomposes the share of marginal product that is allocated to wages, recruiting costs, and profits. We consider three key cases of parameters that determine the shape of per-worker recruiting costs: constant ($\chi = 0$), increasing in the ratio of $V/N$ ($\chi > 0$, $\sigma = 0$), and total costs that are infinitely convex in the number of vacancies ($\chi \to \infty$, $\sigma = 1$), the last of which is equivalent to shutting down the number of vacancies as a choice margin.

The first row of Table 2 reports the ratio of the wage divided by the marginal revenue product of labor for our three key parameterizations. Let us first compare the constant case ($\chi = 0$) to the infinitely convexity case ($\chi \to \infty$, $\sigma = 1$). Curiously, in both parameterizations, the ratio is $\mathcal{E}/(1+\mathcal{E})$. However, as we will see, these two cases differ drastically as to whether the gap between wages and marginal revenue product is used to pay for recruiting costs or is retained as profits by the firm. The middle column with $\sigma = 0$ and $\chi > 0$ does not yield the same formula, and in general the wage is closer to marginal revenue product. This is because with a higher value of $\chi$, it is more expensive to be a vacancy-heavy firm, and so firms will pay higher wages to avoid paying increasingly expensive vacancy costs.

The second row reports the share of marginal product that is allocated to recruiting costs in steady state.[10] In the case of per-hire costs that are constant or increasing in the ratio of vacancies to employment (i.e., when $\sigma = 0$), then all of the gap between wages and marginal product is accounted for by recruiting costs.In the infinitely convex case, when the recruiting margin is shut down, firms spend nothing on recruiting costs.

In the third row, we report the labor market profits per worker as a share of marginal product retained by the firm, which we define as the share of marginal product not paid out in wages or recruiting costs. In the cases constant per-vacancy costs or costs that are increasing in the ratio of vacancies to incumbents, then profit share of marginal product is zero. In the case of $\sigma = 1$ and $\chi \to \infty$, we have the standard equation for 1 minus the wage markdown.

In the fourth and final row, we report the wage-size elasticity $\varepsilon_{w,N}$. The wage-size elasticity is a comparative static of wages with respect to firm TFP $A_j$ over employment with respect to $A_j$: $\varepsilon_{w,N} = \frac{\partial \log w_j / \partial \log A_j}{\partial \log N_j / \partial \log A_j}$. This expression captures that how the firm's optimal wage changes relative to changes in optimal employment if the firm's demand for labor increases.

Note that in the denominator of the wage-size elasticity is $\varepsilon_{\mathcal{E},w}$, the elasticity of $\mathcal{E}(w)$ with respect to the wage. This term describes how the sum of the recruiting and (negative) separation elasticities with respect to the wage changes with the wage.[11] If this term is negative, when wages are very high, the sum of recruiting and negative separation elasticities falls. In the case of $\sigma > 0$ where large firms will want to pay more, this mitigates the incentive for large firms to pay higher wages, flattening the wage-size elasticity. More generally, we find that if vacancy costs are linear or convex in the ratio of vacancies to incumbents, then the wage-size elasticity is 0. Again in the final column we replicate the standard model.

[10] Recruiting costs as a share of marginal product is calculated as $\left(c\left((V/N)^{1+\chi} N^{\sigma\chi}\right)/F'(N)\right.$.

[11] We derive the wage-size elasticity in Appendix A.2.

In total, we can see that in the case of $\sigma = 0$, where per-vacancy recruiting costs are constant or increasing in the ratio of vacancies to incumbents, we have already resolved puzzle 1: optimal wages are independent of firm size as shown in equation (4) for any value of recruiting and separation elasticities. The bottom two rows of Table 2 show that the resolution to puzzle 1 and puzzle 2 are linked, as the profit that firms earn on the marginal worker will be tightly linked to the wage-size elasticity. In particular, if $\sigma = 0$ or very low, then both the wage-size elasticity will be zero *and* firms will earn no profit on the margin, regardless of separation and recruiting elasticities as captured by $\mathcal{E}$.

Table 2: Decomposing Marginal Product into Wages, Recruiting Costs, and Profits

| Outcome | General | Constant $\chi = 0$, $\sigma = 0$ | Increasing in $V/N$ $\chi > 0$, $\sigma = 0$ | Infinitely Convex in $V$ $\chi \to \infty, \sigma = 1$ |
|---|---|---|---|---|
| $\frac{w}{MRPL}$ | $\frac{(1+\chi)\mathcal{E}}{1+(1+\chi)\mathcal{E}+\sigma\chi}$ | $\frac{\mathcal{E}}{1+\mathcal{E}}$ | $\frac{(1+\chi)\mathcal{E}}{1+(1+\chi)\mathcal{E}}$ | $\frac{\mathcal{E}}{1+\mathcal{E}}$ |
| $\frac{\text{Recruiting Costs/Worker}}{MRPL}$ | $\frac{1}{1+(1+\chi)\mathcal{E}+\sigma\chi}$ | $\frac{1}{1+\mathcal{E}}$ | $\frac{1}{1+(1+\chi)\mathcal{E}}$ | 0 |
| $\frac{\text{Profit/Worker}}{MRPL}$ | $\frac{\sigma\chi}{1+(1+\chi)\mathcal{E}+\sigma\chi}$ | 0 | 0 | $\frac{1}{1+\mathcal{E}}$ |
| Wage-Size Elasticity $\varepsilon_{w,N}$ | $\frac{\sigma\chi}{1+(1+\chi)\mathcal{E}-\varepsilon_{\mathcal{E},w}}$ | 0 | 0 | $\frac{1}{\mathcal{E}}$ |

This table shows the decomposition of marginal product into wages, recruiting costs, and profits to the firm for three different parameterizations, with assumption that firms do not discount, i.e. $\rho = 0$. These results hold in partial equilibrium and are only a function of the firm's problem. Analogous results for the general $\rho$ case can be found in Appendix A.2. In the final column, $\sigma = 1$ and $\chi \to \infty$ is equivalent to each firm having a fixed number of vacancies for free, so firms maximize $AN^{\alpha} - wN$ subject to $N = R(w)/S(w)$. Recruiting costs as a share of marginal product is calculated as $\big(c(V/N)^{1+\chi}N^{\sigma\chi}\big)/F'(N)$.

**Evidence on Recruiting Costs** One of the benefits of this model is that we can decompose the share of marginal product spent on wages, recruiting costs, and kept as profits as a function only of the firm's problem. Does the model produce realistic levels of recruiting costs, relative to the wage bill?

To get a general sense, we can leverege the fact that some of the terms used in the recruiting cost share of marginal product are fairly well known. For example, we already have a good estimate of the sum of recruiting and separation elasticities $\mathcal{E}$: it is consistently estimated between 4 and 6. We will also have a good guess about the convexity parameter $\chi$: we will assume that $\chi = 1$, which means that conditional on the number of incumbents, total recruiting costs are quadratic in the number of vacancies. Quadratic recruiting costs, in one form or another, are both commonly assumed and generally supported by empirical evidence.[12]

With plausible values of $\mathcal{E} = 5$ and $\chi = 1$, the remaining parameter that we don't know is $\sigma$, and much of the rest of the paper will be dedicated to attaining an estimate of $\sigma$. However, with $\mathcal{E} = 5$, and $\chi = 1$, we already have reasonable bounds for the magnitude of recruiting costs: varying $\sigma$ between 0 and 1, the recruiting costs share of marginal product would be around 8-9% of marginal product and 10% of the wage bill.

How realistic is hiring costs equal to 10% of the wage bill? Earlier survey-based estimates give smaller turnover cost estimates: Manning (2011) surveys evidence and Dube et al. (2010) point to lower recruiting costs as a share of the wage bill, around 2-5%. However, recent papers find higher costs of turnover: Jäger and Heining (2022) and Kline et al. (2019) estimate marginal hiring costs to be 1-3 years of a worker's salary.[13] Using $\chi = 1$, that would imply average hiring costs of 0.5-1.5 years of a workers' salary. As a back-of-the-envelope calculation, using the low end of this range of half a year's salary, and if the monthly separation rate is conservatively 2%[14], this would imply recruiting costs around 12% of the wage bill. Thus, a figure of 10% of the wage bill lands above survey evidence but below what is implied by studies of rent sharing and worker deaths.

Now that we see that $\mathcal{E} = 5$ and $\chi = 1$ gives plausible estimates of the recruiting cost share of marginal product, what do these values imply for the profit share of marginal product? At the low end, if $\sigma = 0$, then the profit share of marginal product is 0. At the high end, the case of $\sigma = 1$ where the presence of more incumbents does not lower recruiting costs at all, the profit share of marginal product is approximately 0.08. Therefore, with reasonable parameter values, the inclusion of a recruiting cost margin at minimum cuts the profit share of marginal product in half relative to the standard model without a recruiting margin.

### 3.3 Microfounding Worker Mobility and Labor Market Equilibrium

In this section, we choose a structure for frictional labor markets and worker preferences. This will be used to derive results for the firm's problem as a function of primitives, including analytical expressions for the optimal wage, and it will allow us to solve for the labor market in equilibrium, even without block recursivity. We will also derive an explicit formula for the firm size-wage

[12] Muehlemann and Pfeifer (2016) estimates approximately quadratic hiring costs in German survey data. Acemoglu and Hawkins (2014) assume quadratic vacancy costs. Gavazza et al. (2018) estimate in a business cycle setting that the most elastic recruiting effort margin has roughly quadratic costs.

[13] Using worker deaths, Bertheau et al. (2021) and Bloesch et al. (2022) find similarly high costs.

[14] The monthly separation rate in the US is 3.6%, and separation rates are similar in the US and Denmark (Caldwell and Harmon, 2019)

premium in terms of primitives and analytically link the wage-size elasticity to the share of marginal product that firms retain as profits.

**Population and Frictional Markets** Workers $i \in [0,1]$ and firms $j \in [0,1]$ both have a unit mass. Firms can post vacancies to recruit workers, and workers can search on the job. The measure of aggregate vacancies is $V^{total} = \int_{j \in J} V_j dj$, and the measure of searchers $\mathcal{S}$ will be all the workers who are enabled to search in period $t$. A worker can be matched with at most one vacancy per period, and vice-versa. Matching occurs according to a constant returns to scale matching function $M(V^{total}, \mathcal{S})$. Define the labor market tightness as $\theta = V^{total}/\mathcal{S}$. Conditional on searching, the probability that a worker encounters a job is $f(\theta) = M(V^{total}, \mathcal{S})/\mathcal{S}$, and the probability that a firm's vacancy encounters a searching worker is $g(\theta) = M(V^{total}, \mathcal{S})/V^{total}$. Workers can search each period with probability $\lambda$.

**Workers** Workers have per-period utility over an aggregate consumption good $C$ and time varying, idiosyncratic preferences over workplaces $\iota$

$$U_{it} = \log(C_{it}) + \iota_{ijt}.$$

Workers inelastically supply one unit of labor and can earn wages. Workers care only about the current period and have no ability to save. The worker's non-wage utility from working at firm $j$ in period $t$ is $\iota_{ijt}$ is drawn i.i.d. each period is distributed type-1 extreme value with scale parameter $1/\gamma$. Workers are always employed, and workers can search for other jobs with probability $\lambda$.

If a worker searches on the job and encounters a vacancy, the worker's choice of whether to stay in the firm or to switch becomes a simple discrete choice problem. For a worker employed at firm $j \in J$ that meets a vacancy of firm $k$, the probability of staying in firm $j$ is

$$P\{\text{worker } i \text{ leaves firm } j \text{ for firm } k\}|_{\text{match with firm } k} = s_{ijk}(w_{ij}, w_{ik}) = \frac{w_{ik}^{\gamma}}{w_{ij}^{\gamma} + w_{ik}^{\gamma}}. \tag{7}$$

Analogously the probability that firm $j$ poaches worker $i$ who currently works at firm $k$, conditional on the worker matching with firm $j$'s vacancy, is

$$P\{\text{firm } j \text{ poaches worker } j \text{ from firm } k\}|_{\text{match with firm } j} = r_{ijk}(w_{ij}, w_{ik}) = \frac{w_{ij}^{\gamma}}{w_{ij}^{\gamma} + w_{ik}^{\gamma}}. \tag{8}$$

Let the cumulative distribution of wage policies wages posted in vacancies be $\Upsilon(w)$, with corresponding density $\upsilon(w)$. Let the cumulative distribution of wages that workers are currently employed at be denoted $\Phi(w)$, with density $\phi(w)$. Then the separation and recruiting functions are defined as

$$S(w_j) = \lambda f(\theta) \int_i s_{ijk}(w_{ij}, w_{ik}) di = \lambda f(\theta) \int_k \upsilon(w_k) \frac{w_k^\gamma}{w_j^\gamma + w_k^\gamma} dw_k \tag{9}$$

$$R(w_j) = \ \ g(\theta) \int_i r_{ijk}(w_{ij}, w_{ik}) di = \ \ g(\theta) \int_k \phi(w_k) \frac{w_j^\gamma}{w_j^\gamma + w_k^\gamma} dw_k, \tag{10}$$

where $w_{ij} = w_j$ and $w_{ik} = w_k$ by the assumption that firms post wage policies that are equal for all workers.

Now that we specified the workers problem and the frictional labor market, we have provided a microfoundation for the firm's recruiting and separation functions $R(w)$ and $S(w)$. Firms solve their maximization problem, maximizing objective function (1) while plugging in the recruiting and separation functions into the law of motion (2). The mass of searchers is simply $\mathcal{S} = \lambda$, as the mass of workers is 1 and workers search on the job with probability $\lambda$.[15] Next we will define an equilibrium and characterize key results about this equilibrium

**Symmetric Equilibrium** A steady-state equilibrium is an employment policy $N^*(A_j, \alpha_j, c_j, \chi_j, \sigma_j, \tilde{w}, \theta)$, wage policy $w^*(A_j, \alpha_j, c_j, \chi_j, \sigma_j, \tilde{w}, \theta)$, and vacancy policy $V^*(A_j, \alpha_j, c_j, \chi_j, \sigma_j, \tilde{w}, \theta)$, a distribution of wages over employed workers $\Phi(w)$ and vacancies $\Upsilon(w)$, a mass of searchers $\mathcal{S}$, labor market tightness $\theta$, and an aggregate wage index $\tilde{w}$ such that (i) firms maximize profits, (ii) workers maximize utility, and (iii) flows of workers into and out of each firm balances.

**Proposition 1** *If there are no exogenous separations, $\gamma$ is positive and finite, and if a steady-state equilibrium exists, then at such an equilibrium*

1. *identical firms choose identical wage, employment, and vacancy policies, and*
2. *the recruiting elasticity minus separation elasticity $\mathcal{E}(w_j) = \varepsilon_{R(w_j),w_j} - \varepsilon_{S(w_j),w_j} = \gamma$ for any choice of wage level $w_j$.*

Proof: see Appendix A.3. Additionally, we prove existence in Appendix A.4 for the case where firms are have identical values for $\alpha$, $\chi$, and $\sigma$ but may differ in $A_j$ and $c_j$.

**Corollary 2** *If $\gamma$ is positive and finite, firms are identical, and there are no exogenous separations, then a steady-state equilibrium exists and there is a point mass of wages in equilibrim.*

This result stands in contrast to the famous result in Burdett and Mortensen (1998), that with on-the-job search and ex-ante homogenous workers and firms, the equilibrium must contain wage dispersion. Like Albrecht et al. (2018), we achieve this by workers having horizontally differentiated

[15]In section 5, we introduce allow for exogenous separations and the ability for workers to voluntarily quit into unemployment.

preferences over workplaces. However, in our setting worker preferences over firms are time-varying rather than permanent. This both means that firms are not limited in their size in the long run by worker preferences, as well as that workers have non-degenerate outside utility distributions even if the outside wage distribution is a point mass. This means that the probability that a worker leaves a given job is a smooth function of their current wage and a competing outside wage, creating a smooth tradeoff for firms between wage costs and turnover probabilities. If identical firms are facing the same smooth tradeoff, then all firms will choose the same wage, generating the point mass of wages in equilibrium.

We now characterize the equilibrium. Given parameters $\alpha_j$, $A_j$, $\chi_j$, $\sigma_j$, and $c_j$, aggregate labor market tightness $\theta$, and aggregate wage index $\tilde{w}$, a firm $j$'s optimal wage is

$$w_j^* = \left( c_j \gamma (1+\chi_j)(\lambda\theta)^{1+\chi_j} \tilde{w}^{\gamma(1+\chi_j)} \left( \frac{\alpha_j A_j \gamma (1+\chi_j)}{1+\gamma(1+\chi_j)+\sigma_j\chi_j} \right)^{\frac{\sigma_j\chi_j}{1-\alpha_j}} \right)^{\frac{1-\alpha_j}{(1-\alpha_j)(1+\gamma(1+\chi_j))+\sigma_j\chi_j}}, \tag{11}$$

where $\tilde{w}$ is a vacancy-weighted index of the distribution of wages: $\tilde{w} = \left( \int_{k\in J} \nu(w_k) w_k^{\gamma} dk \right)^{\frac{1}{\gamma}}$. Taking $\rho \to 0$ for simplicity, optimal employment at firm $j$ is

$$N_j^* = \left( \frac{\alpha_j A_j}{w_j^*} \times \frac{\gamma(1+\chi_j)}{1+\gamma(1+\chi_j)+\sigma_j\chi_j} \right)^{\frac{1}{1-\alpha_j}}. \tag{12}$$

The optimal vacancy policy is simply $V_j^* = V^{total}(\tilde{w}/w_j)^{\gamma} N_j^*$, and labor market tightness is $\theta = (\int_{k\in J} V_k dk)/\lambda$.

As we saw in Table 2, the share of marginal product retained by the firm as profits and the wage-size elasticity have very similar formulas. Now under this microfoundation, we can show that they are explicitly linked. The formula for the wage-size elasticity is now

$$\varepsilon_{w,N}^j = \frac{\sigma_j \chi_j}{1+\gamma(1+\chi_j)}. \tag{13}$$

This is the ratio of the comparative static of wages with respect to firm TFP $A_j$ over employment with respect to $A_j$: $\varepsilon_{w,N} = \frac{\partial \log w_j / \partial \log A_j}{\partial \log N_j / \partial \log A_j}$. As before, if $\sigma = 0$, then firm size will have no effect on wages for firms at their steady state size. This expression is the same as in Table 2, except that the superelasticity term drops out because the sum of the recruiting and negative separation elasticities $\mathcal{E}$ is constant and equal to $\gamma$. The profit share of marginal product is then

$$\frac{\text{Profit}_j}{MRPL_j} = \frac{\varepsilon_{w,N}^j}{1+\varepsilon_{w,N}^j}. \tag{14}$$

The profit share of marginal product that the firm earns will be very close to, and increasing in, the wage-size elasticity. We will next estimate the elasticity of wages to firm size in Section 4.

**An instructive case: identical firms with $\sigma = 0$, $\chi = 0$:** When abstracting from firm heterogeneity, this model permits a very tractable equilibrium of the labor market that yield insights about the nature of wage setting. Consider an equilibrium where all firms are identical and have linear vacancy posting costs. Normalizing the mass of workers and firms to 1 and marginal revenue product $F'(1) = 1$, the equilibrium can be simply characterized by a point mass of wages at $\bar{w}$, tightness $\theta$ and employment $N$

$$\bar{w} = \frac{\gamma}{1+\gamma}, \qquad \theta = \frac{1}{1+\gamma}\frac{1}{c\lambda}, \qquad N = 1.$$

This equilibrium neatly captures how this model nests the competitive model. As workers' mobility decisions become more sensitive to wages and $\gamma \to \infty$, then $\bar{w} \to F'(1)$.

Now suppose that a single firm $j$ differs from all other firms by having its own vacancy cost constant $c_j$ that differs from other firms. This firm's optimal wage is

$$w_j^* = (\gamma\theta\lambda c_j)^{\frac{1}{1+\gamma}} \bar{w}^{\frac{\gamma}{1+\gamma}}.$$

The optimal wage is a geometric average of the market wage $\bar{w}$ and variables relating to the cost of replacing a worker, including the vacancy cost shifter $c_j$, labor market tightness $\theta$, and on-the-job search frequency $\lambda$. The weights in the geometric average are a function of $\gamma$, which is governing the sensitivity of separations and recruits to wages. This expression captures neatly how $\gamma = \mathcal{E}$ captures wages setting power. As $\gamma \to \infty$, a firm's optimal wage is *always* the market wage, because when workers' mobility decisions are highly sensitive to wage differences, and deviating from the market wage is very costly. As $\gamma$ falls, the terms that determine turnover costs $\theta$, $\lambda$, and $c_j$ become more important as the firm uses its wage setting power to optimally trade off wage costs and turnover costs.

**Worker Heterogeneity** In Section 4, we will use Abowd et al. (1999) estimation method to infer firm wage effects, assumes that workers wages are linear in logs of firm and worker effects. Our theoretical model can be straightforwardly extended to generate wages that are linear in log work and firm types: firms would produce with differ skill-types of labor, firms would post skill type-specific vacancies, and workers would direct their search into their relevant submarket.

## 4 Firm Size and Wages: Evidence

In this section, we estimate the wage-size elasticity $\varepsilon_{w,N}$ for firms in Denmark and use the model developed in Section 3 to infer the profit share of marginal product for atomistic firms. These estimates for the wage-size elasticity will be informative for the profit share of marginal product according to the formula $profit/MRPL = \varepsilon_{w,N}/(1 + \varepsilon_{w,N})$.

**Data** We construct an annual merged employer-employee panel dataset using de-identified Danish administrative data from 2008-2019. For workers' employment history and wages, we use the IDAN registry, which reports workers' earnings, hours, occupation, firm, and establishment. For our measures of firm size, we use the FIRM registry, which is an annual dataset that includes data on firm sales, value added, employment (measured in full time equivalents), gross salaries, and gross profits. We include only firms that report value added, which excludes the public sector and financial firms.

### 4.1 Wage-Size Elasticities from Cross-Sectional and Switchers Evidence

To begin, we will establish stylized facts about the relationship between wages and firm size in Denmark. We will do this by running OLS regression of worker wages on dummies for firm size groups, as well as looking at wage changes for workers who switch firms across different size groups.

We group firms into ten groups by firm size, where each group (or decile) contains an equal number of full-time equivalent workers averaged over the sample period. The average firm size in each decile, in full-time equivalents, is 2.5, 6.7, 12.9, 23.6, 43.4, 84.6, 178, 449, 1145, and 5121. Our sample includes workers who switch firms and worked at least 1400 hours in year $t$ at the leaving firm and at least 1400 hours in year $t+2$ at the arrival firm. In the section, we have 6.8 million worker-years, 671,189 switching events, and 92,336 unique firms.

Our model in Section 3.3 yields log wages that are linear in log firm and worker effects. Let worker fixed effects be $\zeta_i$ and firm fixed effects $\psi_j$, with a time varying error term $\epsilon_{ij,t}$. Then wages in levels and first differences are described by

$$\begin{aligned} \log(w_{ij,t}) =& \zeta_i + \psi_j + \epsilon_{ijt} \\ \log(w_{ij',t+k}) - \log(w_{ij,t}) =& \psi_{j'} - \psi_j + \epsilon_{ij',t+k} - \epsilon_{ij,t}. \end{aligned}$$

For this empirical exercise, we assume that firm heterogeneity is discrete: all firms within the same size bin $k$ have the same firm wage effect $\psi$. In levels, we run an OLS regression with dummies for each firm size bin, as well as occupation fixed effects and worker covariates to control for as much observable heterogeneity as possible. For switchers, we utilize the simple version of the discrete heterogeneity framework of Bonhomme et al. (2019). We regress two year changes in job switchers' hourly wages on dummies indicating that a worker $i$ was employed at a firm $j$ in a given firm size bin $k$ in a given year $t$. We also include year dummies and standard demographic controls in $x_{i,t}$. Standard errors are clustered at the firm level.

$$\log(w_{ijt}) = \sum_k \beta_k \{j_t(i) \ \in k\} + \xi x_{i,t} + \tau_{j,t} \tag{15}$$

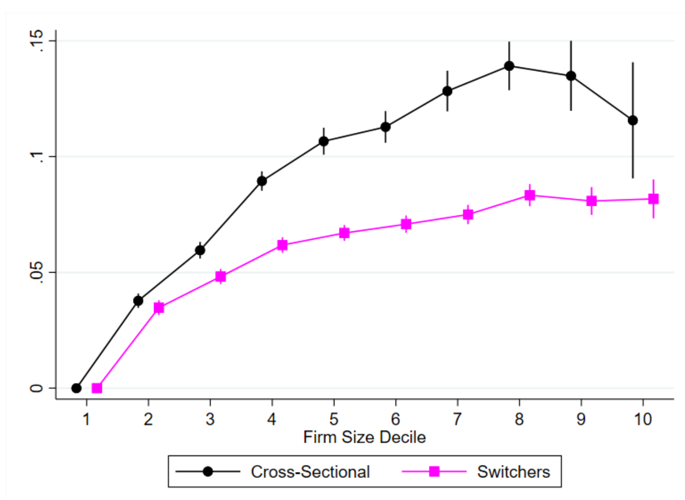
$$\Delta log(w_{i,t,t+2}) = \sum_k \beta_k \big(\mathbb{1}\{j_t(i) \in k\} - \mathbb{1}\{j_{t+2}(i) \in k\}\big) + \xi x_{i,t} + \tau^0_{j,t} + \tau^2_{j,t}, \tag{16}$$

where $x_{i,t}$ are worker controls including age, years of education, and occupation dummies, and $\tau$ captures industry-by-year fixed effects. Under the assumptions of exogenous mobility and no common amenities, the coefficients $\beta_k$ will identify the wage effect of firms in bin $k$.

Figure 1 shows the results. In the cross-sectional regression, wages are increasing at larger firms, with the highest wage effect in 8th decile firms, which on average have 449 full-time equivalent employees. Wages in the top three size deciles are 12 to 14 log points higher than wages at the smallest firms, which have an average size of 3 full-time equivalent workers. For switchers, the wage patterns are similar, but slightly muted. A worker that switches from the bottom size decile firm to a top size decile firm will expect to increase their wages by 8 log points. The gap between the cross-sectional regression and the switchers regressions suggests that higher-wage workers positively sort into large firms.

If interpreted causally, these results would imply very small size-wage elasticities. The average firm size for firms in the smallest decile is 3 FTE, and the average size in the top decile is 5121, yielding an elasticity of $.08/(\log(5121) - \log(3)) \approx .01$.[16] According to equation (14), this would imply that profits retained by firms by exploiting their wage setting power would be approximately 1% of the gross marginal revenue product of labor. However, these cross-sectional and switcher regressions likely bias down the wage-size elasticities: as Manning (2003) argues, firms may have different supply curves. Firms with supply curves shifted out will, all else equal, be larger and pay lower wages, negatively biasing an estimate of the wage-size elasticity. As such, we will need additional strategies to uncover wage-size elasticities in Sections 4.2 and 4.3.

Figure 1: Firm Wage Effects by Firm Size Decile



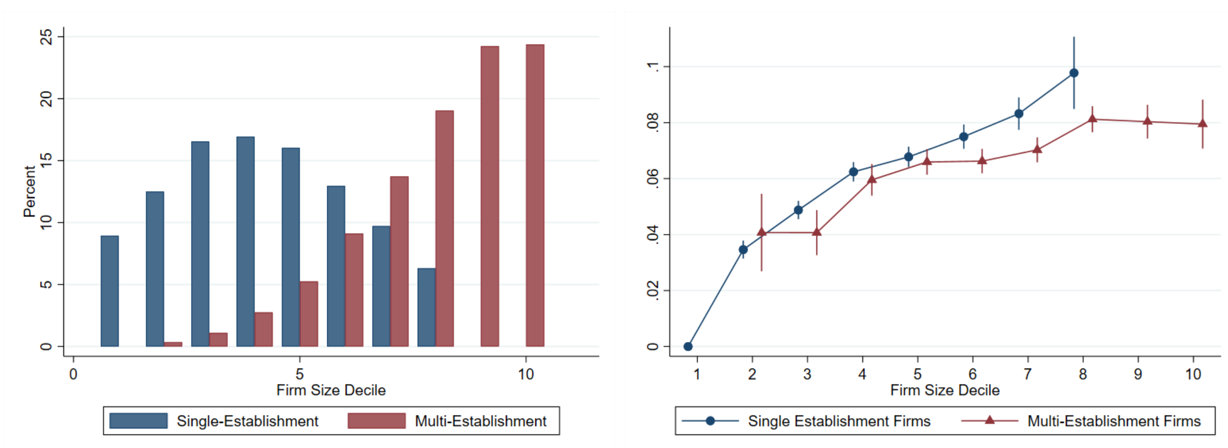

This figure reports the coefficients from equations (15) and (16) estimating the average wage effect for firms in each size decile.

[16] The shape and magnitude of firm wage effects by firm size is very similar to Bloom et al. (2018)'s estimates for the US.

We also estimate the firm wage effects by firm size bin using switchers for single establishment and multi-establishment firms separately. Figure 2a shows the distribution of employment across firms of different sizes and establishment types. Not surprisingly, multi-establishment firms account for the bulk of employment at large firms. Accordingly, single-establishment firms account for the bulk of employment in small firms.[17] Figure 2b reports results for equation (16), where firms are group into firm size-by-establishment type bins. The coefficients are the wage effects for each bin relative to the smallest firm decile of single-establishment firms. The results show that single establishment firms have a slightly steeper slope of wages to firm size, with an elasticity of 0.02. Multi-establishment firms have a flatter wage-size profile with an elasticity of 0.006, and wage effects flatten out for very large multi-establishment firms.[18]

Figure 2: Distribution of Employment and Wage Effects by Firm Size and Establishment Type

(a) Distribution of Employment (b) Firm Wage Effects




The figure shows the distribution of employment across firms in different size bins as well as the estimated wage effects for the corresponding firms, separately for single-establishment and multi-establishment firms. Panel (a) shows how full-time workers are distributed across firms of different sizes, measured by total full-time equivalent workers. Single establishment firms tend to be smaller, and the typical Average size by firm size decile is 3, 7, 13, 24, 43, 85, 178, 449, 1146, and 5121 full-time equivalent workers.

## 4.2 Profit Share of Marginal Product: Indirect Inference

In the last section, we estimated the wage premium paid by firms of different sizes. Our regression, however, is a regression of firm wage effects, which may be influenced by various factors that may vary idiosyncratically across firms, such as variation in hiring costs or firm-specific amenities that

[17] The bins may not be equal size, as firm bins are created on the basis of full-time equivalent workers, while Figure 2a shows the distribution of full-time workers across these size bins.

[18] We estimate this regression excluding multi-establishment firms in the smallest size bin, as well as single-establishment firms in the top two size bins, as there are insufficient number of firms to comply with Danish Statistics' reporting requirements.

are valued in a common way by workers.[19] Therefore, our results in Section 4.1 do not necessarily tell us about the wage-size elasticity that is informative of the profit share of marginal product, which is a counterfactual of when the firm's demand for labor increases, what is the ratio of the log change in wages over the log change of employment: $\varepsilon^j_{wN} = \frac{\partial \log w_j}{\partial \log A_j} / \frac{\partial \log N_j}{\partial \log A_j}$.

To address this bias, we leverage the result from Section 3.3 that firms' steady-state optimal wage and size are a log-linear function of market level variables (labor market tightness $\theta$, the aggregate wage index $\tilde{w}$, worker preferences $\gamma$, and on-the-job search probability $\lambda$) and firm specific parameters (hiring cost parameters $c_j$, $\chi_j$, $\sigma_j$, and output elasticity $\alpha_j$). Taking logs of equilibrium (11) (12), firms' optimal wages and employment size can be expressed as

$$\log(w_j) \propto (1 - \alpha_j) \log(c_j) + \sigma_j \chi_j \log(A_j) \tag{17}$$

$$\log(N_j) \propto \; - \log(c_j) + (1 + \gamma(1 + \chi_j)) \log(A_j) \tag{18}$$

If we knew firms' parameters $\sigma_j$, $\chi_j$, and $\alpha_j$, we could use equations (17) and (18) to point identify firm's TFP $A_j$ and hiring cost $c_j$. However, it is specifically $\sigma$ that we wish to estimate. Therefore, we will leverage these equations to make indirect inference about the parameters of interest $\sigma$. Doing so requires a few assumptions. First, within subsets of similar firms, we will assume that firms have common values of $\chi$, $\sigma$, and $\alpha$. Second, we assume that a firm's productivity or product demand $A_j$ and its hiring cost shifter $c_j$ are uncorrelated $cov(c_j, A_j) = 0$. Under these assumptions, we can show the following result:

**Lemma 3** *If $\chi_j = \chi$, $\sigma_j = \sigma$, and $\alpha_j = \alpha$ $\forall j$, and if $cov(c_j, A_j) = 0$, then*

$$\varepsilon^j_{w,N} = \frac{\sigma\chi}{1 + \gamma(1 + \chi)} = \frac{\sigma^2_w + (1 - \alpha)\sigma_{wN}}{(1 - \alpha)\sigma^2_N + \sigma_{wN}}. \tag{19}$$

The derivations for this formula can be found in Appendix A.6. The above expression says that under the model assumptions that generate the log-linear wage and firm size equations (11) and (12), we can then identify the wage-size elasticity for a set of firms that have identical curvature in their production and recruiting cost functions, but different shifters of labor demand (TFP $A_j$) and supply curves (vacancy cost constant $c_j$). This is the case under the assumption that there is no underlying correlation between total factory productivity $A$ and hiring costs $c$. In economic terms, this assumption says that firms with higher productivity or higher levels of product demand are not endowed with intrinsically more or less productive recruiting technologies. It is reasonable to think that as large, productive firms may be able to pay a fixed cost to invest in better recruiting technologies. However, the ability to increase the efficiency of hiring with firm size is exactly what $\sigma$ captures: how increasing the number of incumbents reduces vacancy costs. Thus, a low value of $\sigma$ (firm size helps reduce recruiting costs) and negative correlation between $c_j$ and $A_j$ (firms with

[19] Manning (2003) discusses how the coefficient in a regression of log wages on log firm size will biased downward in the presence of unobserved supply shocks.

greater product demand have lower baseline hiring costs) are empirically indistinguishable. Given our assumptions, the next step is to estimate the variance of firm wage effects $\sigma_w^2$, firm size $\sigma_N^2$, and their covariance $\sigma_{wN}$. To do this, we use the standard AKM fixed effects regression, recovering an estimate of firm wage effects. We then compute the variance terms for single-establishment and multi-establishment firms separately: based on the evidence in Section 4.1 that single-establishment and multi-establishment firms have different relationships between firm size and wage, it reasonable to assume that these two groups of firms have different recruiting cost parameters. In terms of parameters, this means that we assume that $\chi$, $\sigma$, and $\alpha$ are constant within single-establishment firms and within multi-establishment firms, but the values of $\chi$, $\sigma$ and $\alpha$ may be different between the two groups.

Table 3: Indirect Inference: Profit Share of Marginal Product

| | Single-establishment | Multi-establishment |
|---|---|---|
| $\sigma_w^2$ | .061 | .006 |
| $\sigma_N^2$ | 1.28 | 1.09 |
| $\sigma_{wN}$ | .036 | .015 |
| $\alpha$ | .7 | .7 |
| profit/MRPL | .15 | .03 |
| AKM bias adjusted: | .09 | .03 |

This table reports the variance of AKM wage effects and firm size, as well as their covariance, separately for single-establishment and multi-establishment firms. The table also reports the profit share of marginal product as implied by the wage-size elasticity in equation (19) and the relationship $\frac{profit}{MPRL} = \frac{\varepsilon_{w,N}}{1+\varepsilon_{w,N}}$.

Table 3 reports results for the variance of AKM wage effects and firm size, the covariance terms, and the implied profit share of marginal product. We assume an elasticity of firm revenue with respect to labor of 0.7.[20] The variance of firm wage effects is much larger for single-establishment firms, generating a significantly higher profit share of marginal product: .15 for single-establishment firms but only .03 for multi-establishment firms.

However, due to the well-known issues of limited mobility bias, the profit share of marginal product is likely to be biased up due to the variance of firm wages effects being overestimated. This is a concern particularly for single-establishment firms that experience fewer worker transitions. Using a variety of methods to correct for limited-mobility bias, Bonhomme et al. (2023) show that the variance of AKM firm wage effects is likely double the bias-free estimate. To adjust for this, we assume that the "bias-free" firm wage effects would have the same correlation with firm size, but

[20] Given the product demand elasticity $\eta = 7$ and Cobb-Douglas labor share $1-\beta = .7$ in Section 2.2, if firms' level of capital is fixed, then the correct elasticity of revenue to with respect to labor inputs would be $\frac{\eta-1}{\eta}(1-\beta) = 0.6$. However, in the long-run, firms may be able to adjust their capital. If capital were perfectly elastic, the correct elasticity of revenue with respect to labor inputs would be $\frac{\eta-1}{\eta}(1-\beta)/(1-\frac{\eta-1}{\eta}\beta) \approx 0.8$. We pick $\alpha = .7$ as an intermediate case between the two extremes.

the variance of firm wage effects is only half as large.[21] This brings the estimate of the profit share of marginal product for single-establishment firms to .09. Employment is approximately evenly distribution between single- and multi-establishment firms, which would yield a labor market-wide profit share of marginal product of .06.

Notably, a profit share of marginal product of .09 for single-establishment firms is consistent with our earlier parameterizations of $\mathcal{E} = 5$ and $\chi = 1$ if $\sigma = 1$ discussed at the end of Section 3.2. A profit share of .03 for multi-establishment firms would be consistent with $\sigma = 0.34$. Therefore for atomistic firms, we conclude that a value of $\sigma$ between 0 and 1, and consequently profit shares of marginal product between 0-10%, are consistent both with external evidence to discipline our model as well as evidence on the distribution of firm size and wages effects interpreted through our model.

## 4.3 Firm Expansions Events

Another way to estimate firms' wage-size elasticities is to measure how wages change as firms change size (in terms of employment). To estimate this, we build on Friedrich et al. (2023), who use firm expansion events to estimate the changes in wages as firms increase their size. The wage-size elasticity is then the difference in firm wage effects after the firm has expanded and reach a new steady state size, divided by the change of employment.

To our knowledge, we are the first to look at the wage growth of switchers around firm expansion events.[22] We are primarily interested in the wage changes of new hires, and in particular switchers, because the wages of incumbent workers may respond to firm shocks for reasons unrelated to the long-run labor supply elasticity to the firm (Kline et al. (2019), Garin and Silverio (2023)). For example, if there is any rent sharing due to bargaining or ex-post incentive pay, or if incumbents and new hires are not perfectly substitutable in the short run, then shocks either to firm demand or firm productivity may raise incumbents' wages, even if the supply of new hires is perfectly elastic in the long run.[23]

One difficulty that has discouraged researchers from using the wages of new hires is that the composition of new hires may change in response to a firm shock. Therefore, we focus on job

[21] Using $corr(\psi_j, \bar{N}_j) = \sigma_{Nw}/(\sigma_N \sigma_w)$, we fix $corr(\psi_j, \bar{N}_j)$ and decrease $\sigma_w^2$ by a factor of two, recalculating a bias corrected covariance

[22] Friedrich et al. (2023) estimate the effect of firm expansion events on the wages of both the level of stayers' wages and the level of switchers' wages, controlling for observables. Engbom et al. (2022) estimate time-varying AKM firm wage effects, though their strategy exploits the wage changes of both stayers and switchers over time.

[23] The framework presented in Kline et al. (2019) is a particularly good example of this. The authors show that if there are convex training costs needed to convert new hires into incumbents, then in response to demand or productivity shocks, there is additional surplus in the matches between the firm and incumbent workers. This surplus can be shared if higher wages for incumbents decrease turnover, as these authors find, or if there is bargaining. Bloesch and Weber (2023) present micro-evidence on these convex costs of converting new hires into productive incumbents for workers in software and R&D production. Bloesch et al. (2022) also rationalize passthrough of productivity shocks to wages as rent sharing to retain of costly-to-replace incumbents, rather than an increasing marginal cost of acquiring workers.

switchers, which allows us to substract out worker effects from our estimate of wages. There is yet a further concern that the firms that switchers leave from may change around a firm expansion. For example, if the firm of interest is suddenly expanding, it may intentionally recruit from lower-paying firms where workers are more likely to leave.

To address the concern that the wage of the firms that workers leave may change over the course of a firm expansion, we include a control for the firm wage effects of the firms that workers leave. Specifically, we begin by constructing a sample of firms that are in the treated sample of expanding firms. Then we estimate AKM firm wage effects, *excluding* the treated firms. Then, in our regression that estimates the wage changes of new hires, we include the firm AKM wage effect of the switching worker's prior firm as a covariate.

Our statistical framework continues with the linear-in-logs framework that we developed in Section 3.3, except that we now allow expanding firms to have a time varying firm wage effect $\psi_{jt}$, which captures the wage effect in excess of the time-invariant fixed effect $\psi_j$. The expansion is defined to occur between period $t-1$ and $t$. We follow Friedrich et al. (2023) and set period $t-2$ as the final pre-treatment period, as firms expanding between year $t-1$ and $t$ may have already started raising wages between year $t-2$ and $t-1$. Thus, the final pre-treatment period that a worker can switch into the firm is period $t-2$. Focusing on this final pre-treatment year, taking first differences of wages of a worker who switches from firm $k$ to firm $j$ nets out the worker effect $\zeta_i$

$$
\begin{aligned}
\log(w_{ij,t}) =& \zeta_i + \psi_j + \psi_{jt} + \epsilon_{ijt} \\
\log(w_{ij,t-2}) - \log(w_{ik,t-4}) =& \psi_j + \psi_{j,t-2} - \psi_k + \epsilon_{ij,t-2} - \epsilon_{ik,t-4}
\end{aligned}
$$

As in Section 4, we focus on workers' wages two years apart to allow workers to work full time in both annual observations. Comparing the wage growth of switchers who arrive at firm $j$ in period $t+s$ relative to workers who arrive at firm $j$ in period $t-2$ yields

$$\Delta w_{i'k'j,t+s,t+s-2} - \Delta w_{ikj,t-2,t-4} = \psi_{j,t+s} - \psi_{j,t-2} - \psi_{k'} + \psi_k + \epsilon_{ij,t+s} - \epsilon_{i'k',t+s-2} - \epsilon_{ij,t-2} + \epsilon_{ik,t-4}. \quad (20)$$

This expression captures how much higher wages are when workers arrive in period $t+s$ relative to the final pre-treatment period $t-2$. The size-wage elasticity is then

$$\frac{\sigma\chi}{1+\gamma(1+\chi)} = \frac{\psi_{j,t+S} - \psi_{j,t-2}}{\log N_{j,t+S} - \log N_{j,t-2}}.$$

for some period $S$ that is far enough in the future that the firm expansion is no longer happening.

First we report the path of firm size for firms that expand. A firm expansion event is a firm that grows between 30 and 100 log points within a year. We use a local projections difference-in-difference strategy (Dube et al., 2023) to estimate the path of firm size before and after expansions

$$\Delta \log(N_{j,t+s,t-2}) = \beta_s \mathbb{1}\{\text{expansion in year } t\} + \tau_{jt} + \omega_{j,t-4,t-2} + \xi x_{i,t+s-2} \quad (21)$$

for $s \in \{-4, 4\}$, where $N_{j,t}$ is the number of full-time equivalent workers in firm $j$ at time $t$, $\tau_{jt}$ is an industry-by-year fixed effect, $\omega_{j,t-4,t-2}$ is a dummy for binned firm growth rates between periods $t-4$ to $t-2$. As is standard for local projections, we run as many regressions as there are time periods, with one coefficient for each time horizon.

Our regression specification for the wage growth switchers is

$$\Delta \log(w_{ijk,t+s,t+s-2}) = \sum_s \beta_s \mathbb{1}\{\text{switcher arrives in year } s\} \times \mathbb{1}\{\text{expansion firm}\} + \tau_{jt} + \omega_{j,t-4,t-2} + d_j + \kappa\hat{\psi}_k + \xi x_{i,t+s-2}, \quad (22)$$

where $d_j$ is a firm fixed effect only for treated firms and for observations within $s \in \{-4, 4\}$ years of the expansion,[24] $\hat{\psi}_k$ is the estimated AKM effect of non-expansion firms, and $\xi$ is the coefficient on a vector of worker demographic covariates $x_{ij,t+s-2}$. In both regressions, we include the industry-by-year fixed effects and prior binned growth rates to compare firms who had similar growth trajectories and similar industries prior to the expansion.

**Sample** Our sample of expanding firms is firms that employed between 20 and 200 full-time equivalent workers prior to the expansion. We exclude large firms to avoid firms whose decision to grow may significantly affect the firm's local labor market. We exclude from our expansion sample firms that ever appear to participate in an acquisition, measured as at least 90% of job switchers in one year all moving to the same firm. The full sample of firms that were ever between 20 and 200 workers are 21,154 unique firms, and there are 3,241 unique expansion firms.

Table 4 reports the industry composition of expansion firms and non-expansion firms of similar size. Expansion events occur in all industries with roughly equal proportions.

Table 4: Industry Composition of Expanding Firms

| Industry | All Medium-Sized Firms | Expansion Firms |
|---|---|---|
| Agriculture, Forestry, Fishing | .015 | .011 |
| Manufacturing, Mining, Quarrying | .169 | .138 |
| Construction | .122 | .141 |
| Trade and Transport | .305 | .270 |
| Information and Communication | .060 | .091 |
| Real Estate | .024 | .019 |
| Other Business Services | .148 | .203 |
| Education, Health | .112 | .105 |
| Arts, Entertainment | .038 | .022 |

This table reports the composition of industries for all firms that were between 20-200 full-time equivalent employees (FTE) workers between 2008 and 2019, as well as the industry composition of firms that experienced expansions.

[24] This allows us to compare effects within treated firms over time, using $s = -2$ as the reference year.

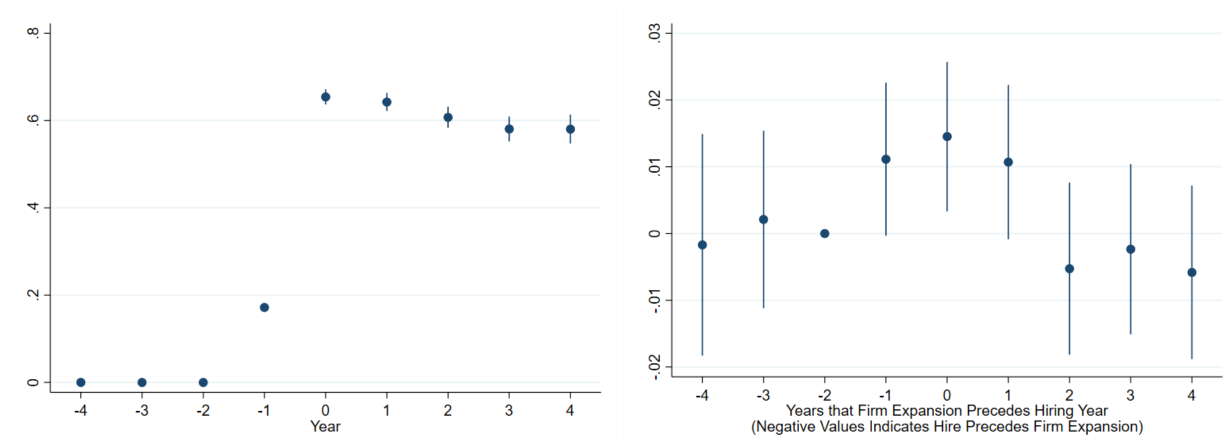
**Results** Figure 3 shows the results from estimating equations (21) and (22). Panel (a) of Figure 3 shows the path of employment growth for expanding firms. By including the dummies for binned growth rates from $t-4$ to $t-2$, we observe the trajectory of expanding firms' size relative to firms with similar prior growth histories. Firms begin their relative expansion between period $t-2$ and $t-1$, and firms grow on average by 44 log points between $t-1$ and $t$. After expansion, expanding firms on average shrink by a small amount over the next three years, stabilizing at an employment level about 57 log points greater employment than prior to the expansion, relative to firm with similar growth trajectories from $t-4$ to $t-2$.

Panel (b) of Figure 3 shows the wage gains of workers who switch into expanding firms, relative to the wage gains of switching into those expanding firms in period $s=-2$, two years prior to the expansion. The peak effect for switchers occurs during the expansion year $s=0$, with a magnitude of approximately 1.5 percentage points. Crucially, two years following the completion of the expansion, switchers no longer attain greater wage gains by switching into firms that have already expanded than switchers who arrived prior to the expansion.

Figure 3: Time-Varying Wage Growth for Switchers as Firms Expand

(a) Firm Size

(b) Wage Growth for Switchers




This figure shows the employment growth path of expanding firms, as well as the wage premium for workers who switch jobs and move into the expanding firm for four years before and after the firm expansion. Expanding firms grow on average about 60 log points relative to firms with similar growth trajectories prior to period $t-2$. Switchers attain larger wage gains in the years around the expansion, but switchers who arrive after the expansion has finished attain wage gains to switchers who arrived before the expansion.

Similar to Acemoglu and Hawkins (2014) and Schmieder (2023), we find that wages are highest while firms are growing quickly. This is consistent with some degree of convexity in recruiting costs or diminishing returns to recruiting effort, i.e. $\chi > 0$. The result that wage growth for switchers returns to the pre-expansion baseline is consistent with $\sigma = 0$: once the firm is finished expanding and has a larger number of incumbents, the cost per vacancy falls, decreasing the firm's incentive to offer high wages.[25] Thus, our evidence from firm expansions would be consistent with profit shares of marginal product close to 0.

**Threats to Identification** There are two main concerns with using firm expansions to identify shocks to firm demand. The first is that this strategy cannot rule out idiosyncratic shocks to a firm's labor supply. If expansion events are a combination of firm demand and supply shocks, this would bias down our estimates for wage changes relative to the true wage-size elasticity. The second is that there is likely selection into the treated sample: firms with elastic recruiting technologies are the most likely to expand quickly.

It is important to clarify what could be an idiosyncratic supply shock to the firm. One circumstance that is *not* an idiosyncratic supply shock is the firm investing in amentities or more efficient recruiting technologies. While the functional forms we described in Sections 3.1 and 3.2 do not directly nest fix costs for investments in improvements in recruiting ability, one way to interpret $\sigma$ is that as firms get larger, there are subsequent amenities or recruiting technologies, each with a fixed cost, that firms can invest in to tap into an additional pool of workers. Thus an idiosyncratic supply shock is necessarily a change in workers' willingness to work for the firm at the same wage, or a change in the ease of the firm recruiting, that was not the result of a choice made by the firm. Understood this way, it is hard to conceive of sudden *idiosyncratic* changes to labor supply that would enable a firm to grow by over 30% in one year.

With that said, even if supply shocks are not a significant driver of firm expansion events, it is still possible that the subsample of firms that happen to cross the threshold of expansions of over 30% in one year experience more favorable supply shocks than firms that do not grow by 30%. If this is the case, this would bias down our estimates of long run wage-size elasticities, biasing down the profit share of marginal product.

The second concern is selection into the treatment sample based on firm heterogeneity, namely that firms with the most elastic recruiting technologies are the firms most likely to cross over growth by 30% in one year. If we are selecting on firms with the most elastic labor supply curves, we will estimate profit shares of marginal product on the subpopulation of firms for which the profit share is the smallest. However, as Table 4 shows, the distribution of industries among expansion firms is quite similar to the industry distribution of all similarly sized firms, alleviating the concern that expansions would occur in only some industries.[26]

[25] In our theory in Section 3, we report results for optimal wages only in steady-state, but we do not report wages during firms' growth trajactories. In Appendix A.7, we numerically solve the firm's dynamic problem and show that firms' optimal wage policy is to pay higher wages when growing. For a firm with $\sigma = 0$ and $\chi = 1$, a firm growing by 30% at an annualized rate would pay wages 10% higher than its optimal steady state wage. However, in our theory, wages are totally flexible, and given downward wage we should expect empirically that firms raises wages less than in the perfectly flexible case.

[26] We tested for different wage effects of expansions at single-establishment and multi-establishment firms, finding no discernable differences.

# 5 Quantitative Model with a Single Granular Firm

In this section, we consider the behavior of a single granular firm in a local labor market that is large enough to affect labor market aggregates. Building on the equilibrium framework developed in Section 3.3, we enrich the model with this single granular firm, finite product demand elasticities, capital in production, and an unemployment state for workers. We show that whether the single granular firm profits from its ability to move the labor market depends on the extent that its atomistic competitors face diminishing returns to labor, either from diminishing returns to scale in production or finite product demand. Calibrating this model, we find that in a local labor market with a Herfindal-Hirschman index (HHI) of employment concentration equal to .1, the labor market-wide profit share of marginal product is 6%, accounting for 5% of aggregate output. We also show that our calibrated model also does a better job of matching the labor share of net value added reported in Section 2.2 than the standard model that imply large markdowns.

## 5.1 Setup

**Firms** As in Section 3.3, we will allow firms to differ in their recruiting technologies. For simplicity, we will only consider discrete heterogeneity: there are three types of firms in this economy. First, there is a mass of ex-ante identical atomistic "small" firms $m_S$, with low productivity $A_S$ and inelastic recruiting technologies $\sigma_S > 0$. These firms will be denoted $S$ because they will be small in equilibrium. Second, there is a mass $m_M$ of ex-ante identical atomistic "medium-sized" firms, denoted $M$, which have higher productivity $A_M > A_S$ and elastic hiring technologies $\sigma_M < \sigma_S$. Finally, we will have a single large firm, denoted $L$, which is granular in the labor market.

**Unemployment** Workers have the same i.i.d. preferences as before over a particular job within firms. In addition, we allow workers to be unemployed and receive income $b$. Workers' utility while unemployed is $\log(b) + \iota_{iut}$, where $\iota_{iut}$ is drawn from the same distribution of idiosyncratic non-wage utilities as workers draw during employment. Workers are allowed to consider quitting into unemployment with probability $\lambda^{EU}$ and are allowed to search on the job with probability $\lambda^{EE}$. Matches also end exogneously at rate $s$. The unemployment rate is $U_t$ and unemployed workers are allowed to search for jobs with probability $\lambda^{UE}$. As before, search is undirected and matching is random. The total mass of searchers is $\mathcal{S}_t = \lambda^{UE} U_t + \lambda^{EE}(1 - U_t)$.

**Atomistic Firms: Product Markets and Production Functions** We assume that atomistic firms produce differentiated products and face downward sloping product demand. We assume that for both small $S$ and medium-sized $M$ firms, product demand is modeled as a nested constant elasticity of substitution demand function, where firms within each industry produce a differentiated good that is bundled into an industry-level good that itself faces a constant demand elasticity. For example, consider the demand for a small firm $j$ in industry $S$. Output of small firms is aggregated into a composite good, with its respective price index:

$$Y_S = \Big(\int_{j\in S} Y_j^{\frac{\epsilon_S-1}{\epsilon_S}} dj\Big)^{\frac{\epsilon_S}{\epsilon_S-1}}, \qquad P_S = \Big(\int_{j\in S} P_j^{1-\epsilon_s} dj\Big)^{\frac{1}{1-\epsilon_s}}.$$

A small firm $j$ then faces the downward sloping product demand, with demand for the composite good is decreasing in its price

$$Y_j^s = \left(\frac{P_j^s}{P_S}\right)^{-\epsilon_s} Y_S. \qquad Y_S = D_S P_S^{-\zeta_s},$$

where $D_S$ is a shifter for the small firms' bundled good. The output of medium-sized firms is nested CES in an analogous way, with parameters $\epsilon_M$ and $\zeta_M$, with aggregate prices $P_M$, aggregate output $Y_M$ and demand shifter $D_M$. We will use $J$ to denote the two sectors: $\{S, M\} \in J$.

Firms produce using a Cobb-Douglas production technology in capital and labor. Firms take the rental rate $r_K$ as given, and capital is elastically supplied. We assume a constant capital share for all firms $\beta$. Atomistic firms solve

$$\max_{N_{jt},w_{jt},V_{jt},K_{jt}} \sum_{t=0}^{\infty} \left(\frac{1}{1+\rho}\right)^t \left(P_{jt}Y_{jt} - w_{jt}N_{jt} - r_K K_{jt} - c_{jt}\Big(\frac{V_{jt}}{N_{jt}^{(1-\sigma_j)}}\Big)^{\chi} V_{jt}\right)$$

subject to:

$$\begin{aligned} N_{jt} =& (1 - S(w_{jt}))N_{j,t-1} + V_{jt}R(w_{jt}), \\ Y_{jt} =& \left(\frac{P_{jt}}{P_{Jt}}\right)^{-\epsilon_J} Y_{Jt} \\ Y_{jt} =& A_j K_{jt}^{\beta} N_{jt}^{(1-\beta)}. \end{aligned}$$

**Large Firm: Product Market and Production Function** The large firm simply faces downward sloping product demand $Y_L = P_L^{-\epsilon_L}$. Otherwise the large firm's problem takes the same form. The large firm solves

$$\max_{\{N_{Lt}\},\{w_{Lt}\},\{V_{Lt}\},\{K_{Lt}\}} \sum_{t=0}^{\infty} \left(\frac{1}{1+\rho}\right)^t \left(P_{Lt}Y_{Lt} - w_{Lt}N_{Lt} - r_K K_{Lt} - c_{Lt}\Big(\frac{V_{Lt}}{N_{Lt}^{(1-\sigma_L)}}\Big)^{\chi} V_{Lt}\right)$$

subject to

$$\begin{aligned} N_{Lt} =& (1 - S(w_{Lt}))N_{L,t-1} + V_{Lt}R(w_{Lt}) \\ Y_{Lt} =& P_{Lt}^{-\epsilon_L} \\ Y_{Lt} =& A_L K_{Lt}^{\beta} N_{Lt}^{(1-\beta)}. \end{aligned}$$

We assume that workers who are currently employed at the large firm can apply and switch to other postings at the large firm.

**Equilibrium** A symmetric steady-state equilibrium is a set of wages $w_L$, $w_M$, $w_L$, employment levels $N_S$, $N_M$, $N_L$, vacancy levels $V_S$, $V_M$, $V_L$, output $Y_S$, $Y_M$, $Y_L$, prices $P_S$, $P_M$, $P_L$, labor market tightness $\theta$, unemployment rate $U$, and mass of searchers $\mathcal{S}$ such that (i) firms maximize profits, (ii) workers maximize utility, (iii) labor market flows into and out of firms balance, and (iv) identical firms make identical choices.

**Analytical Results: Large Firm Strategic Behavior** What differentiates the large firm's problem from an atomistic firm is that the large firm can affect market aggregates. In particular, the aggregate variables that appear in the large firm's maximization problem that the large firm may also potentially influence are its competitors' wages, labor market tightness $\theta$, and the employment and vacancy shares $\phi_J$ and $\upsilon_J$. Assuming for the moment that all its competitors are identical and pay the same wage $\bar{w}$ and setting $\sigma_L = 0$, solving out the large firm's problem non-parametrically yields the following expression

$$\frac{profit_L}{MRPL_L} \approx \frac{(1+\chi)\varepsilon_{\bar{w},N_L}\mathcal{E}_L + \varepsilon_{\theta,N_L}}{1+(1+\chi)\big(\varepsilon_{\bar{w},N_L}\mathcal{E}_L + \varepsilon_{\theta,N_L} + (1-\varepsilon_{\bar{w},w_L})\mathcal{E}_L - \varepsilon_{\theta,w_L}\big)}. \tag{23}$$

In addition to the standard terms that show up in the profit share equation $\mathcal{E}$ and $\chi$, there are four additional terms: $\varepsilon_{\bar{w},N_L}$, $\varepsilon_{\theta,N_L}$, $\varepsilon_{\bar{w},w_L}$, and $\varepsilon_{\theta,w_L}$, which are all the combinations of elasticities of average competitor wages $\bar{w}$ and market tightness $\theta$ to the large firm's choice of employment levels $N_L$ and wages $w_L$. These variables capture the ability of the large firm to influence market level wages and tightness. Notably, and most importantly, this expression is equal to 0 if $\varepsilon_{\bar{w},N_L}$ and $\varepsilon_{\theta,N_L}$, the elasticities of competitor wage and labor market tightness to the large firm's choice of size, respectively, are zero. The response of market variables to $\bar{w}$ and $\theta$ to the large firm's choice of wage $w_L$ show up only in the denominator. This tells us that ultimately what matters for whether the granular firm profits from being large is the extent that its atomistic competitors offset the granular firm's strategic under-employment or under-hiring. As we will see in our quantitative exercises, this will be determined in the model by the extent that there is finite demand elasticity for the atomistic firms' products.

**Calibration** Table 1 presents our choice of parameters values. We set the inverse of workers' preferences over non-wage amenities $\gamma = 5$ to match microeconomic evidence on recruiting and separation elasticities. We set the convexity of recruiting costs with respect to the number of vacancies $\chi = 1$ to yield quadratic costs. We set the parameter governing how the number of incumbents lowers hiring costs for small firms $\sigma_S = 1$. This reflects our evidence in Sections 4.1 and 4.2 that single-establishment firms do appear to face upward-sloping labor supply curves. This combination of parameters will yield a profit share of marginal product of .08 for these atomistic firms. We set $\sigma_M = 0$, reflecting that medium-sized and multi-establishment firms have elastic long-run labor supply curves. We set the on-the-job search probability $\lambda_{EE} = .14$ to match empirical job-to-job flows and quit rates. We set product demand elasticities, the Cobb-Douglas capital share, the depreciation rate of capital, and the rental rate of capital to be the same as in Section 2.2.

Table 5: Calibration

| Parameter | | Value |
|---|---|---|
| $\gamma$ | Inverse variance of non-wage preferences | 5 |
| $\sigma_S$ | Extent that incumbents lower hiring costs, small firms | 1 |
| $\sigma_M$ | Extent that incumbents lower hiring costs, medium firms | 0 |
| $\sigma_L$ | Extent that incumbents lower hiring costs, large firm | 0 |
| $c$ | Baseline hiring cost constant | 3 |
| $\lambda^{EE}$ | On-the-job search probability | .14 |
| $\lambda^{EU}$ | Allowed to quit into unemployment probabiliity | .24 |
| $\lambda^{UE}$ | Search probability for the unemployed | 1 |
| $\epsilon_s$ | Elasticity of substitution, small firm goods | 7 |
| $\epsilon_m$ | Elasticity of substitution, medium firm goods | 7 |
| $\epsilon_L$ | Elasticity of demand, large firm goods | 7 |
| $\beta$ | Cobb-Douglas capital share | .3 |
| $\delta$ | Depreciation rate | .1 |
| $r^K$ | Rental rate of capital | .15 |
| s | Exogenous separation rate | 0 |

**Granular Firm's "Latent" Employment Share $\mathcal{L}$** In our quantitative exercises, employment concentration will be an endogenous outcome: the granular firm will find it profitable to strategically under-hire, so the measure of labor market concentration will ultimately be a function of the granular firm's behavior. Therefore, when performing counterfactual exercises that alter the granular firm's ability to move the labor market, we will want a concept that derives from model primitives of the potential size of the granular firm in the absence of strategic behavior.

To do so, we introduce the concept of *latent employment share*. The measure of latent share $\mathcal{L}$ is what share of employment the granular firm would attain if the granular firm did not act strategically, i.e., chose wages and vacancies per incumbent as if it were atomistic.[27] In our quantitative results, we will compare labor market outcomes for different values of the latent employment share. One of the challenges with such an exercise is that the granular firm's latent share of employment is not a parameter. Instead, it is an outcome in a hypothetical economy that is a function of many other parameters. Therefore, we reverse-engineer the granular firm's latent employment share by varying the granular firm's productivity $A_L$ and the masses of atomistic firms $m_S$ and $m_M$.

[27] Another way to understand latent share is if the granular firm were a bundle of atomistic firms which, prior to being bundled, produced perfectly substitutable output and had common recruiting cost parameters.

**Atomistic Firms' Market Returns to Scale** It will turn out that the extent that atomistic firms have diminishing returns to labor will be an important determinant of the ability of the granular firm to profit from strategic behavior. Firms can have diminishing returns to labor in two ways: either diminishing physical returns or finite product demand. However, finite product demand elasticities also have implications for price markups and ultimately the profit share of output. To allow us to flexibly calibrate price markups but keep constant returns to scale production at the firm level for atomistic firms, we utilize a nested CES demand structure where firms produce differentiated goods that are bundled with a constant elasticity of substitution, and then there is a finite demand elasticity $\zeta_J$ for the bundled good. This generates diminishing returns at the product market level $\frac{\zeta_J - 1}{\zeta_J}$, which we refer to as the "market returns to scale."

## 5.2 Quantitative Exercises

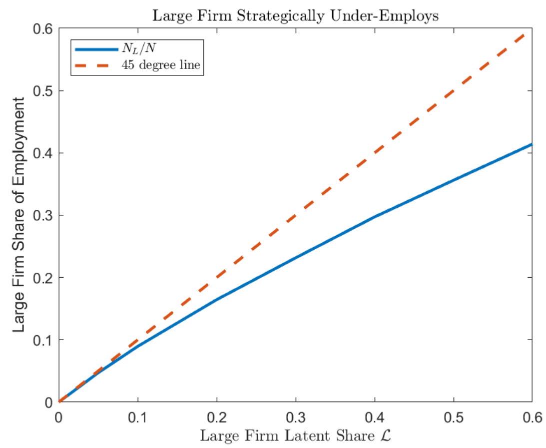
In this section, we report results considering two types of variation. First, we vary the granular firm's latent share of employment, i.e., we vary how large the granular firm is relative to its local labor market. Second, we vary the product demand elasticities for composite goods for the small and medium sized firms $\zeta_S$ and $\zeta_M$, the parameters that govern the "market returns to scale" for atomistic firms.

Figure 4 shows the level of employment, wages, and vacancies as the granular firm's latent share changes, holding small and medium-sized firms market returns to scale fixed (setting $\zeta_L = \zeta_M = 6$, so the market returns to scale is .86). Panel (a) of Figure 4 shows the granular firm's share of employment for different latent employment shares. As the granular firm's latent share increases, the firm under-employs by more relative to a baseline where the granular firm does not behave strategically, represented by the 45 degree line. For example, for a firm that would employ 50% of the labor market if it did not behave strategically, maximizes its profits by employing only 36% of all employed workers. Panel (b) shows the level of wages for the granular firm, medium-size firms, and small firms. By design, and consistent with evidence in Section 4, small firms that face upward sloping labor supplies ($\sigma_S > 0$) pay lower wages than medium-sized firms. As the granular firm's latent share increases and the labor market is increasingly depressed, the wages of both small and medium-sized firms fall. The wage of the granular firm tracks almost exactly with the wage of medium-sized firms, who face similarly shaped recruiting costs functions. This suggests that is very costly for the granular firm to deviate from its competitors' wages. This also supports the analytical result that large firms profit more by under-employing, rather than paying a particularly low wage to drag down the market average.

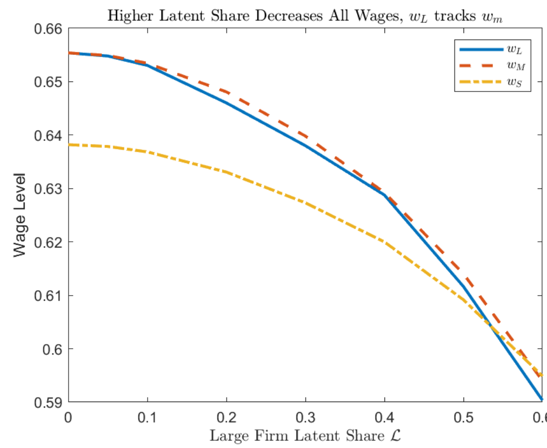
Panels (c) and (d) demonstrate the granular firm's under-hiring and under-recruiting. Panel (c) shows how much firms' employment changes relative to a baseline without strategic behavior. For a firm with a latent share of .5, the granular firm employers approximately 30% fewer workers that it would if it were not behaving strategically. In response to a labor market that is more slack, both small and medium-sized firms absorb some of the additional workers, though the response of medium-sized firms is greater, as these firms are more able to elastically increase their recruiting. Panel (d) shows similar results but for vacancies, showing that small and medium-sized firms partially offset the lower employment of the granular firm, but medium-sized firm's response is greater due to their ability to elastically scale up recruiting.

Figure 4: Employment, Wages, and Vacancies as the Granular Firm's Latent Employment Share Changes

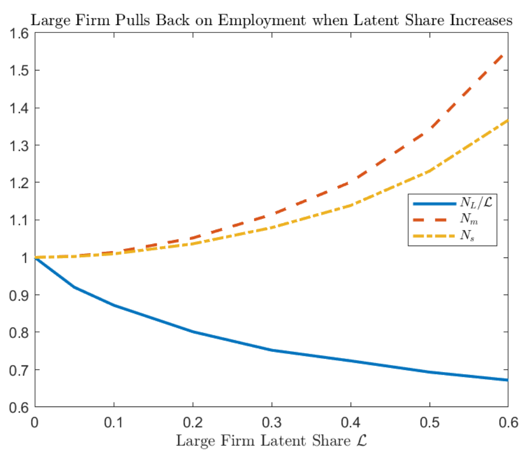
(a) Employment



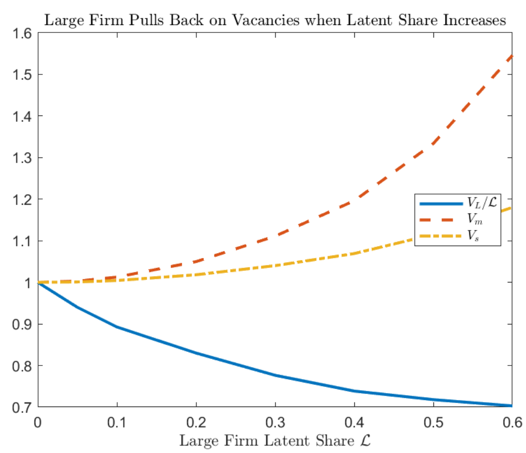

(b) Wages




(c) Relative Employment



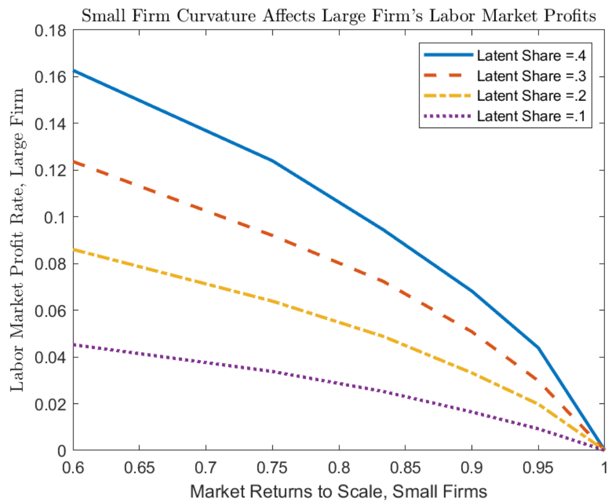

(d) Relative Vacancies




This figure shows the employment share, wage level, employment relative to a non-strategic baseline, and vacancies relative to a non-strategic baseline for the atomistic firm. As the granular firm's latent share of employment increases, the granular firm under-employs, depressing market wages. Atomistic firms respond by increasing employment, but the response is larger among medium-sized firms that have more elasticity recruiting technologies.

Figure 5 shows the granular firm's profit share of marginal product, varying both the large firm's latent share and the market returns to scale for small firms. As the large firm's latent share increases, its rate of labor market profit increases, as the large firm's profitability of strategic behavior increases when it is larger in the market. However, as the market returns to scale of small firms increases, the large firm's profits decrease. At the limit when small firms face linear returns to scale, the large firm makes no profit from its wage setting power.

Figure 5: Profits

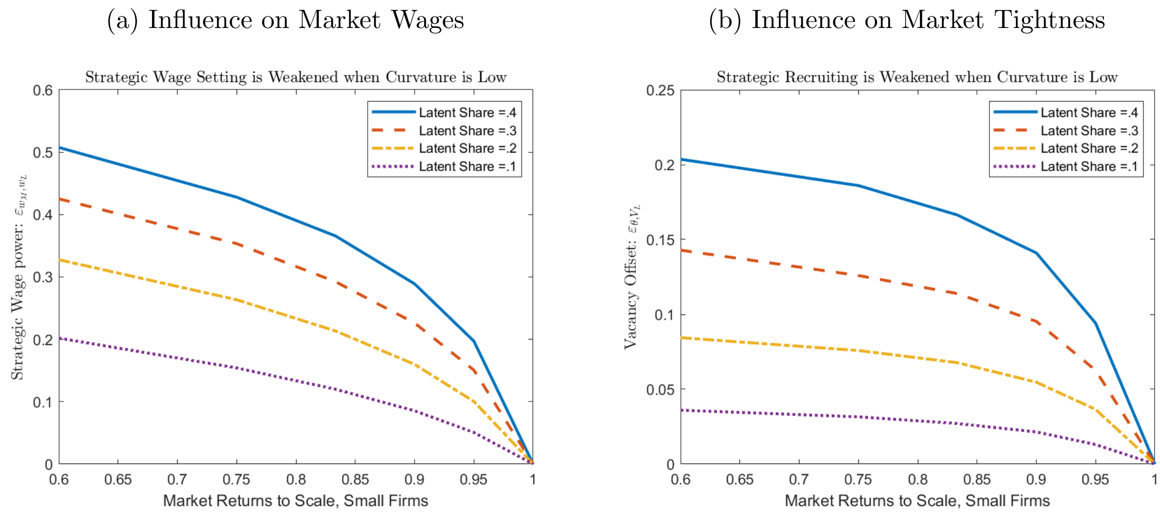



This figure plots the share of marginal product that a large firm retain as profits as a function of the large firm's latent (non-strategic) share of employment and the market returns to scale for small firms. When small firm market returns to scale are below one, a large firms that is larger in the labor market derive higher profits from wage setting power.

Figure 6 shows the magnitude of the large firm's ability to move labor market aggregates. Panel (a) shows the elasticity of the equilibrium wage of medium-sized firms $w_M$ to the large firm's choice of wage $w_L$, holding fixed the large firm's vacancies $V_L$. Naturally, when a firm with a larger latent share of employment changes its wage, this has a larger effect on the optimal wages of its atomistic competitors. However, as is the case for profits, the large firm's ability to move the market weakens as the market returns to scale of atomistic firms rises. This is because when atomistic firms' output is nearly linear, the atomistic firms optimally soak up any available workers not employed by the large firm, restoring the wage level and labor market tightness that would occur if the large firm had a latent share of 0.

Panel (b) of Figure 6 shows the effect of the granular firm's vacancies $V_L$ on labor market tightness $\theta$. As the market returns to scale for atomistic firms approaches 1, the response of tightness $\theta$ to the granular firm's choice of vacancies approaches 0. This means that if the granular firm tries to depress the labor market by under-posting vacancies, the atomistic firms will step in and replace the missing vacancies one-for-one.

Figure 6: Time-Varying Wage Premia for Stayers and Switchers at Expanding Firms




This figure shows the ability of a large firm to strategically move market level wages and labor market tightness. The panel on the left shows the elasticity of the equilibrium wage of medium-sized firms $w_M$ with respect to the large firm's wage. The panel on the right shows how equilibrium tightness responds to vacancies of the large' firm. For both variables, the strategic power of the large firm is increasing in its latent share, i.e., the share of the labor market the large firm would employ if it did not behave strategically. As the market returns to scale for atomistic firms approaches 1, then the large firm's choice of wage has no effect of small firms' wages, and small firms will offset changes in large firm vacancies one-for-one.

**Labor Market Profits and Share of Income** Finally, we report the distribution of the labor, capital, and profit shares of income for our calibrated economy. By choosing as many parameters to be the same as in Section 2.2, we face an upper bound of the labor share of income of .72. By introducing upward-sloping labor supply for small firms and concentration, some of this labor share will be lost to labor market profits.

For our final calibration, we need to take a stance on the market returns to scale. Schubert et al. (2023) find that shifting a labor market from an HHI of .013 to an HHI of .18 lowers aggregate wages by 6%. Our model can match this if the product demand elasticity for the small and medium-sized firms' composite goods are $\zeta_S = \zeta_M = 6$. Then we set the large firm's latent share $\mathcal{L} = .44$ to create a steady-state economy with a HHI of 0.1. With this calibration, the granular firm employs 32% of employed workers and has a profit share of marginal product of 10.7%.

Table 6 decomposes the share of income that goes to wages, capital income net of depreciation, labor market profits, and product market profits. The first row reports the labor market profits of the granular firm. The first column shows that the profit share of marginal product for the granular firm is 10.7%. The second column shows that the profits that the granular firm makes from its labor market power is 2.1% of gross output. The third column shows that these labor market profits of the granular firm account for 2.6% of net output, which subtracts out depreciation.

The second row of Table 6 reports the labor market profits of the small atomistic firms with $\sigma_S = 1$. Their profit share of marginal product is 8.4%, and in the aggregate these profits account for 1.7% of gross output and 2.0% of net output. The third row reports the economy-wide labor market profits. The economy-wide profit share of marginal product in this economy is 6.4%, inclusive of the medium firms whose profit share of marginal product is zero. Total labor market profits account for 3.8% of gross output and 4.6% of net output.

Rows 4, 5, and 6 report the other sources of income in this economy: capital income net of depreciation, product market profits from prices over marginal cost, and labor income. The capital and product market profit shares are identical to the shares in Section 2.2, as the product demand elasticities, Cobb-Douglas parameters, depreciation rate, and return to capital are the same. The differences from Section 2.2 are in the labor share, as some of the wages before are now kept by firms as labor market profits. This drives the labor income share down to 67.8%.

There is a question of how to account for the recruiting costs. One way for accounting for recruiting costs is to assume that they are paid out as labor income: for example, firms may outsource hiring to (unmodeled) third-party party companies, who themselves must pay their employees. The results in Table 6 uses this approach, counting turnover costs as part of labor income. Another approach would be to subtract to turnover costs from output, so that net output subtracts both depreciation and turnover costs. This would drive the labor share of net output even lower to 65.7%.

Table 6: Decomposition of Income in the Calibrated Model with Concentration:

| | Share of MRPL | Share of Gross Output | Share of Net Output |
|---|---|---|---|
| Labor Profits: Concentration | 10.7 | 2.1 | 2.6 |
| Labor Profits: $\sigma_S > 0$ | 8.4 | 1.7 | 2.0 |
| Labor Profits: Total | 6.4 | 3.8 | 4.6 |
| Net Capital Income | - | 8.6 | 10.3 |
| Product Profits | - | 14.3 | 17.2 |
| Labor Income | - | 56.2 | 67.8 |

This table decomposes the share of income that goes to wages, capital income net of depreciation, labor market profits, and product market profits. The labor market is calibrated such that atomistic firms have a market returns to scale of .84, and the granular firm has a latent share of employment of .44, employing 32% of workers and creating HHI of 0.1.

How well does this model match the empirical evidence on the distribution of incomes? Our final number for labor's share of net value added still falls below the empirical range of the labor share of corporate net value added 70-80%, but does fall in the range that includes housing of 65-73%. This is an improvement on models of labor market monopsony that infer markdowns from labor supply elasticities of 4 or lower, which would imply labor shares of net income of 58% at most, and even less for lower labor supply elasticities.

It is remarkable that even with relative modest price markups that we choose relative to common estimates such as De Loecker et al. (2020), combined with the muted labor market profits that we find, we still find labor shares of net income to be slightly too low relative to national accounts data. While the introduction of fixed costs can always reconcile large markups with lower profit levels at the firm level, there remains the issue of how to account for the income that is generated from paying fixed costs. Similarly, firms may pay upfront entry costs and have high markups later, and the income created from paying for those entry costs would need to be accounted for as well. If both entry costs and recruiting costs are in fact primarily labor costs, this may help close the final gap.

## 6 Conclusion

In this article, we derive a model of dynamic monopsony where workers have heterogeneous preferences over firms and can search on the job, and firms can attract workers with higher wages and recruiting expenditures. We use this model to analytical decompose the share of marginal product into wages, recruiting costs, and profits. We show that the profit share of marginal product is tightly linked to firms' size-wage elasticities, which is determined by the shape of the firm's recruiting cost function. We estimate firm size-wage elasticties using indirect inference and with firm expansion events, finding fairly elastic labor supply curves, implying profit shares of marginal product between 0 and 6%. Developing an equilibrium model with granular firms, we show that a granular firm profits from being large in a labor market only if its atomistic competitors have diminishing returns to labor. Calibrating our equilibrium model to our empirical evidence and external evidence, we find an economy-wide profit share of marginal product of 6.4%, bringing our model's estimate of the labor share of net income closer the levels reported in national accounts.

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# A Appendix

## A.1 Labor and Profit Shares of Net Value Added

This section derives the labor, capital, and profit shares of the economy presented in Section 2.2 of the main text. Consider a local economy where ex-ante identical firms produce differentiated goods, rent capital, and compete for workers along a static upward-sloping labor supply curve. Firm $j$'s faces a constant demand elasticity for its output:

$$\frac{Y_j}{\bar{Y}} = \left(\frac{P_j}{\bar{P}}\right)^{-\eta},$$

which we normalize to $Y_j = P_j^{-\eta}$. Revenue is $Y_j P_j = Y_j^{\frac{\eta-1}{\eta}}$. Firms can rent capital at rate $r^K$, and firms' labor supply is:

$$\frac{N_j}{N} = \left(\frac{w_j}{\bar{w}}\right)^{\varepsilon_{N,w}},$$

where $N$ is the total labor supply in the local labor market. Let output be produced with a Cobb-Douglas technology $Y_{jt} = K_{jt}^{\beta} N_{jt}^{1-\beta}$. Per-period profits are

$$\Pi_{jt} = (K_{jt}^{\beta} N_{jt}^{1-\beta})^{\frac{\eta-1}{\eta}} - r^K K_{jt} - w_{jt} N_{jt}.$$

In equilibrium, all firms will choose identical labor $N_j = N \ \forall j$. It is then straightfoward to show that labor share of gross output is:

$$\frac{\text{Labor Income}}{\text{Gross Output}} = \frac{wL}{Y} = \frac{\eta-1}{\eta}(1-\beta)\frac{\varepsilon_{N,w}}{1+\varepsilon_{N,w}}. \tag{24}$$

$(1-\beta)$ is the labor share of income if both labor and product markets were competitive. This number is reduced by $\frac{\eta-1}{\eta}$ due to product markups, and wages are lowered by $\frac{\varepsilon_{N,w}}{1+\varepsilon_{N,w}}$ due to wage markdowns.

The owners of capital get a return net of depreciation $r = r^K - \delta$. Subtracting out depreciation from output, the labor share of net value added is then:

$$\frac{\text{Labor Income}}{\text{Net Value Added}} = \frac{\frac{\eta-1}{\eta}(1-\beta)\frac{\varepsilon_{N,w}}{1+\varepsilon_{N,w}}}{1-\frac{\eta-1}{\eta}\beta\frac{\delta}{r+\delta}}, \tag{25}$$

where depreciation is subtracted from gross value added in the deonominator. The capital share of value added is:

$$\frac{\text{Net Capital Income}}{\text{Net Value Added}} = \frac{\frac{\eta-1}{\eta}\beta\frac{r}{r+\delta}}{1-\frac{\eta-1}{\eta}\beta\frac{\delta}{r+\delta}}.$$

The profit share of net value added is simply the residual:

$$\frac{\text{Profit}}{\text{Net Value Added}} = \frac{\frac{1}{\eta} + \frac{\eta-1}{\eta}(1-\beta)\frac{1}{1+\varepsilon_{N,w}}}{1 - \frac{\eta-1}{\eta}\beta\frac{\delta}{r+\delta}}.$$

The profit share of net value added has two terms in the numerator. The first term $1/\eta$ reflects profits from price markups over marginal cost. The second term $\frac{\eta-1}{\eta}(1-\beta)\frac{1}{1+\varepsilon_{N,w}}$ reflects the profits from wages below the marginal revenue product of labor.

## A.2 Firm's Dynamic Problem

This appendix section derives the decomposition of marginal product in Section 3.2, quantifies the effect of non-zero discounting, and derives the wage-size elasticity in Section 3.2.

If the firm is operating within a stationary environment, (i.e., $S(w)$ and $R(w)$ are not changing), the firm maximizes the present discounted value of profits. Ignoring a firm subscript, the firm's problem is:

$$\max_{\{N_t\},\{w_t\},\{V_t\}} \sum_{t=0}^{\infty} \Big(\frac{1}{1+\rho}\Big)^t (AN_t^{\alpha} - w_t N_t - c\Big(\frac{V_t}{N_{t-1}}\Big)^{\chi} N_{t-1}^{\chi\sigma} V_t)$$
$$\text{s.t. } N_t = (1 - S(w_t))N_{t-1} + R(w_t)V_t.$$

The lagrangian is:

$$\mathcal{L} : \sum_{t=0}^{\infty} \Big(\frac{1}{1+\rho}\Big)^t \Big(AN_t^{\alpha} - w_t N_t - cV_t^{1+\chi} N_{t-1}^{\chi(\sigma-1)} + \lambda_t[(1 - S(w_t))N_{t-1} + R(w_t)V_t - N_t]\Big).$$

The first order conditions are:

$$\begin{aligned}
\mathcal{L}_{N_t} &: \alpha A N_t^{\alpha-1} - w_t - \frac{1}{1+\rho} c\chi(\sigma-1)V_{t+1}^{1+\chi} N_t^{\chi(\sigma-1)-1} - \lambda_t + \frac{1}{1+\rho}\lambda_{t+1}(1 - S(w_{t+1})) = 0 \\
\mathcal{L}_{w_t} &: -N_t - \lambda_t S'(w_t)N_{t-1} + \lambda_t R'(w_t)V_t = 0 \\
\mathcal{L}_{V_t} &: -c(1+\chi)V_t^{\chi} N_{t-1}^{\chi(\sigma-1)} + \lambda_t R(w_t) = 0.
\end{aligned}$$

Rearranging first order condition on $V_t$ yields, solving in steady state, and that $\frac{V}{N} = \frac{S(w)}{R(w)}$:

$$\lambda = \frac{c(1+\chi)(\frac{S(w)}{R(w)})^{\chi}}{R(w)} N^{\sigma\chi}.$$

Using this expression for $\lambda$ and the first order condition on wages yields

$$w = c(1+\chi)\Big(\frac{S(w)}{R(w)}\Big)^{1+\chi}\big(\varepsilon_{R,w} - \varepsilon_{S,w}\big)N^{\sigma\chi}. \tag{26}$$

The steady state value of employment $N$ can be solved for using the previous two expresssions and the first order condition on employment. The choice of firm size $N$ will depend on the discount factor, as growing requires an upfront costs, so firms that discount more steeply will choose to be smaller. Optimal employment is:

$$\alpha A N^{\alpha-1} = \frac{w}{\mathcal{E}(1+\chi)}\Big(1+\mathcal{E}(w)(1+\chi)+\sigma\chi+\frac{\rho}{1+\rho}\big(\chi(1-\sigma)+(1+\chi)\frac{1-S(w)}{S(w)}\big)\Big). \tag{27}$$

In steady state, the level of vacancies is given by

$$V = \frac{S(w)}{R(w)}N. \tag{28}$$

Collectively, equations (26), (27), and (28) characterize the firm's optimal choice of wages, employment and vacancies in steady state.

With these expressions, we can easily solve for what shares of marginal product go to wages, turnover costs, and profits in steady state. Marginal revenue product is $MRPL = \alpha A N^{\alpha-1}$.

$$\frac{w}{MPRL} = \frac{\mathcal{E}(1+\chi)}{1+\mathcal{E}(1+\chi)+\sigma\chi+\frac{\rho}{1+\rho}\big(x(1-\sigma)+(1+\chi)\frac{S(w)}{1-S(w)}\big).}$$

Recruiting costs per worker, in steady state, is $c(V/N)^{1+\chi}N^{1+\sigma\chi}/N$. As a share of marginal product, these costs are:

$$\frac{\text{Recruiting Costs per Worker}}{MPRL} = \frac{1}{1+\mathcal{E}(1+\chi)+\sigma\chi+\frac{\rho}{1+\rho}\big(x(1-\sigma)+(1+\chi)\frac{S(w)}{1-S(w)}\big).}$$

The labor market profits per worker is the gap between marginal product and the sum of wages and per-incumbent recruiting costs.

$$\frac{\text{Labor Market Profits per Worker}}{MPRL} = \frac{\sigma\chi+\frac{\rho}{1+\rho}\big(\frac{1-S(w)}{S(w)}\big)}{1+\mathcal{E}(1+\chi)+\sigma\chi+\frac{\rho}{1+\rho}\big(x(1-\sigma)+(1+\chi)\frac{S(w)}{1-S(w)}\big).}$$

How large is this additional term due to discounting? At a monthly frequency, total monthly separation rates are approximately 0.04, and given an annual discount rate, $\rho$ can be approximated to being equal to .004. Setting $\sigma = 0$ and $\chi = 1$, we have:

$$\frac{\rho}{1+\rho}\big(\chi(1-\sigma)+(1+\chi)\frac{1-S(w)}{S(w)}\big) = \frac{.004}{.996}\times(1+2\times\frac{.96}{.04}) \approx .2.$$

Picking $\mathcal{E} = 5$, the profit per worker as a share of marginal product collected by firms due to discounting is then (suppose $\sigma = 0$):

$$\frac{.2}{1+5(1+1)+.2} \approx .018.$$

This calculation implies that under standard parameters, the additional profits collected on the margin are less than 2% of marginal product. Economically, standard time preferences is quantiatively unimportant because the discounting from time preference rate is an order of magnitude smaller than the separation rate. Because matching with a worker requires an upfront investment with a future flow of payoffs, but the life of that flow of payoffs is affected by the separation rate, the relevant discount rate to the firm is the sum of the separation rate and time preference parameter.

**Deriving the Employer Size-Wage Elasticity in General Form** To derive the wage-size elasticity, we start with the optimal wage equation:

$$w = c(1+\chi)\left(\frac{S(w)}{R(w)}\right)^{1+\chi}\big(\varepsilon_{R,w} - \varepsilon_{S,w}\big)N^{\sigma\chi}.$$

Define $\mathcal{E}(w) = \big(\varepsilon_{R,w} - \varepsilon_{S,w}\big)$, which is the same as before. In logs, this is:

$$\log(w) = \log(c(1+\chi)) + (1+\chi)\log\left(\frac{S(w)}{R(w)}\right) + \log(\mathcal{E}(w)) + \sigma\chi\log(N).$$

Taking the total derivative with respect to $\log(w)$ yields:

$$\begin{aligned} 1 =& (1+\chi)\big(\varepsilon_{S,w} - \varepsilon_{R,w}\big) + \varepsilon_{\mathcal{E},w} + \log(\sigma\chi)\varepsilon_{N,w} \\ =& -(1+\chi)\mathcal{E}(w) + \varepsilon_{\mathcal{E},w} + \log(\sigma\chi)\varepsilon_{N,w}. \end{aligned}$$

Noting that $\varepsilon_{w,N} = \varepsilon_{N,w}^{-1}$ and rearranging yields:

$$\varepsilon_{w,N} = \frac{\sigma\chi}{1 + (1+\chi)\mathcal{E}(w) - \varepsilon_{\mathcal{E}(w),w}}.$$

## A.3 Constant Sum of Recruiting and Separation Elasticities

Suppose that equilibrium exists, characterized by tightness $\theta$, the aggregate wage index $\tilde{w}$, distribution of posted wages $\Upsilon(w)$, distribution of employed wages $\Phi(w)$, and wage, employment, and vacancy policies $w_j^*$, $N_j^*$, and $V_j^*$. We will first show that if there are no exogenous separations $s = 0$, then in any equilibrium the recruiting elasticity $\varepsilon_{R,w_j}$ minus the separation elasticity $\varepsilon_{S,w_j}$ are equal to $\gamma$ for any choice of $w$ in steady state.

Suppose there are $K$ wage levels. Given $w_1, ...w_k$ and $V_1, ..., V_K$, we want to solve for steady state employment shares $\phi_1, ..., \phi_K$. We also want to show that $\varepsilon_{Rw} - \varepsilon_{Sw} = \gamma$ in this steady state. Let us assume that:

$$\frac{\phi_i}{\phi_k} = \frac{\upsilon_i}{\upsilon_k}\left(\frac{w_i}{w_k}\right)^{\gamma}$$

for any two wage levels $i$ and $k$. Then we need (1) to show that the employment share in any wage level is in a steady state, (2) solve for the level of the $\phi_k$'s, and (3) show that $\varepsilon_{Rw} - \varepsilon_{Sw} = \gamma$ for any firm $j$'s wage policity $w_j$.

First let's show that inflows are equal to outflows for any sector $i$. Inflows to sector $i$ are:

$$
\begin{aligned}
&V_i g(\theta) \sum_{k \neq i} \phi_k \frac{w_i^\gamma}{w_i^\gamma + w_k^\gamma} \\
=&V_i \frac{g(\theta)}{f(\theta)} f(\theta) \sum_{k \neq i} \phi_i \frac{\upsilon_k}{\upsilon_i} w_k^\gamma \frac{1}{w_i^\gamma + w_k^\gamma} \\
=&V_i \frac{1}{\theta} f(\theta) \phi_i \frac{1}{\upsilon_i} \sum_{k \neq i} \upsilon_k \frac{w_k^\gamma}{w_i^\gamma + w_k^\gamma} \\
=&V_i \frac{S}{V} f(\theta) \phi_i \frac{V}{V_i} \sum_{k \neq i} \upsilon_k \frac{w_k^\gamma}{w_i^\gamma + w_k^\gamma} \\
=&\lambda N f(\theta) \phi_i \sum_{k \neq i} \upsilon_k \frac{w_k^\gamma}{w_i^\gamma + w_k^\gamma} \\
=&N_i \lambda f(\theta) \sum_{k \neq i} \upsilon_k \frac{w_k^\gamma}{w_i^\gamma + w_k^\gamma},
\end{aligned}
$$

which is the formula for outflows from sector $i$ (using $f(\theta)/g(\theta) = \theta$, $\theta = V/S$, $S = \lambda N$, $\phi_i = N_i/N$). Thus, when $\frac{\phi_i}{\phi_k} = \frac{\upsilon_i}{\upsilon_k}\left(\frac{w_i}{w_k}\right)^\gamma$, this labor market is in steady state.

Next we solve for the values of $\phi_i$. First, define some constant $C$ such that

$$
\frac{\upsilon_i w_i^\gamma}{\phi_i} = \frac{\upsilon_k w_k^\gamma}{\phi_k} = C,
$$

thus

$$
\phi_k = \frac{\upsilon_k w_k^\gamma}{C}, \quad \forall k.
$$

We also know

$$
\sum_{k=1}^{K} \phi_k = 1,
$$

thus

$$
\sum_{k=1}^{K} \frac{w_k^\gamma \upsilon_k}{C} = 1 \longrightarrow \sum_{k=1}^{K} w_k^\gamma \upsilon_k = C.
$$

Thus

$$
\phi_i = \frac{\upsilon_i w_i^\gamma}{C} = \frac{\upsilon_i w_i^\gamma}{\sum_{k=1}^{K} \upsilon_k w_k^\gamma}.
$$

Lastly, we show that for any firm $j$, the firm faces a constant $\varepsilon_{Rw} - \varepsilon_{Sw} = \gamma$ at any value of $w_j$. The firm's ratio of sepration rate to recruiting rate is:

$$
\begin{aligned}
\frac{S(w_j)}{R(w_j)} &= \frac{f(\theta)\lambda\Big(\upsilon_1\frac{w_1^\gamma}{w_1^\gamma+w_j^\gamma}+...+\upsilon_K\frac{w_K^\gamma}{w_K^\gamma+w_j^\gamma}\Big)}{g(\theta)\Big(\phi_1\frac{w_j^\gamma}{w_1^\gamma+w_j^\gamma}+...+\phi_K\frac{w_j^\gamma}{w_K^\gamma+w_j^\gamma}\Big)} \\
&= \lambda\theta\frac{\Big(\upsilon_1\frac{w_1^\gamma}{w_1^\gamma+w_j^\gamma}+...+\upsilon_K\frac{w_K^\gamma}{w_K^\gamma+w_j^\gamma}\Big)}{\Big(\phi_1\frac{w_j^\gamma}{w_1^\gamma+w_j^\gamma}+...+\phi_K\frac{w_j^\gamma}{w_K^\gamma+w_j^\gamma}\Big)}
\end{aligned}
$$

Using that $\upsilon_i w_i^\gamma = \phi_i \sum_{k=1}^K w_k^\gamma \upsilon_k$, we can rewrite the above equation as:

$$
\begin{aligned}
\frac{S(w_j)}{R(w_j)} &= \lambda\theta\frac{\sum_{k=1}^K \upsilon_k w_k^\gamma}{w_j^\gamma}\frac{\Big(\phi_1\frac{1}{w_1^\gamma+w_j^\gamma}+...+\phi_K\frac{1}{w_K^\gamma+w_j^\gamma}\Big)}{\Big(\phi_1\frac{1}{w_1^\gamma+w_j^\gamma}+...+\phi_K\frac{1}{w_K^\gamma+w_j^\gamma}\Big)} \\
&= \lambda\theta w_j^{-\gamma}\sum_{k=1}^K \upsilon_k w_k^\gamma = \lambda\theta\Big(\frac{\tilde{w}}{w_j}\Big)^\gamma,
\end{aligned}
$$

with $\tilde{w} = (\sum_{k=1}^K \upsilon_k w_k^\gamma)^{\frac{1}{\gamma}}$, where the elasticity of this function with respect to $w_j$ is $-\gamma$.

Therefore, when firms solve (1) subject to (2), (9) and (10), the firm's optimal choices follow (4), (5), and (6). Plugging in $\varepsilon_{Rw} - \varepsilon_{Sw} = \gamma$ and $S(w_j)/R(w_j) = \lambda\theta(\tilde{w}/w_j)^\gamma$ into (4) and (5) yield (11) and (12) of the main text.

**Adding Unemployment** In Section 5, we introduce an unemployment state, where workers can voluntarily separate into. Here we show that the previous result about a constant value of $\mathcal{E}$ holds even with unemployment.

Inflows into and outflows from unemployment must balance:

$$
(1-U)\lambda_{EU}\sum_{k=1}^K \phi_k \frac{b^\gamma}{b^\gamma + w_k^\gamma} = U\lambda_{UE} f(\theta)\sum_{k=1}^K \upsilon_k \frac{w_k^\gamma}{w_k^\gamma + b^\gamma}.
$$

Using that

$$
\frac{\upsilon_i w_i^\gamma}{\phi_i} = \frac{\upsilon_k w_k^\gamma}{\phi_k} = C,
$$

the prior equation becomes

$$
(1-U)\lambda_{EU} b^\gamma \sum_{k=1}^K \phi_k \frac{1}{b^\gamma + w_k^\gamma} = U\lambda_{UE} f(\theta)\frac{\upsilon_i w_i^\gamma}{\phi_i}\sum_{k=1}^K \phi_k \frac{1}{w_k^\gamma + b^\gamma}.
$$

Reusing that $\phi_i = \frac{\upsilon_i w_i^\gamma}{\sum_{k=1}^K \upsilon_k w_k^\gamma}$, the previous equation becomes

$$(1-U)\lambda_{EU}b^{\gamma} = U\lambda_{UE}f(\theta)\sum_{k=1}^{K}\upsilon_k w_k^{\gamma}$$

Solving for $U$ yields:

$$U = \frac{\lambda_{EU}b^{\gamma}}{\lambda_{EU}b^{\gamma} + \lambda_{UE}f(\theta)\sum_{k=1}^{K}\upsilon_k w_k^{\gamma}}.$$

The ratio of separations to recruits is:

$$\frac{S(w_j)}{R(w_j)} = \frac{\lambda_{EU}\frac{b^{\gamma}}{b^{\gamma}+w_j^{\gamma}} + f(\theta)\lambda_{EE}\Big(\upsilon_1\frac{w_1^{\gamma}}{w_1^{\gamma}+w_j^{\gamma}} + ... + \upsilon_K\frac{w_K^{\gamma}}{w_K^{\gamma}+w_j^{\gamma}}\Big)}{g(\theta)\Big(\Phi_U\frac{w_j^{\gamma}}{w_j^{\gamma}+b^{\gamma}} + \Phi_E\big(\phi_1\frac{w_j^{\gamma}}{w_1^{\gamma}+w_j^{\gamma}} + ... + \phi_K\frac{w_j^{\gamma}}{w_K^{\gamma}+w_j^{\gamma}}\big)\Big)}$$

Define the share of searchers that are unemployed $\Phi_U$ as:

$$\Phi_U = \frac{\lambda_{UE}U}{\lambda_{UE}U + \lambda_{EE}(1-U)},$$

Plugging in our value for $U$, this expression becomes

$$\Phi_U = \frac{\lambda_{EU}b^{\gamma}}{\lambda_{EU}b^{\gamma} + \lambda_{EE}f(\theta)\tilde{w}^{\gamma}},$$

where $\tilde{w}^{\gamma} = \sum_{k=1}^{K}\upsilon_k w_k^{\gamma}$, and $\Phi_E = 1-\Phi_U$. Plugging in these terms and following familiar algebra yields:

$$\frac{S(w_j)}{R(w_j)} = \frac{\lambda_{EU}\frac{b^{\gamma}}{b^{\gamma}+w_j^{\gamma}} + f(\theta)\lambda_{EE}\tilde{w}^{\gamma}\Big(\phi_1\frac{1}{w_1^{\gamma}+w_j^{\gamma}} + ... + \phi_K\frac{1}{w_K^{\gamma}+w_j^{\gamma}}\Big)}{g(\theta)\frac{w_j^{\gamma}}{\lambda_{EU}b^{\gamma}+\lambda_{EE}f(\theta)\tilde{w}^{\gamma}}\Big(\lambda_{EU}b^{\gamma}\frac{1}{w_j^{\gamma}+b^{\gamma}} + f(\theta)\lambda_{EE}\tilde{w}^{\gamma}\big(\phi_1\frac{1}{w_1^{\gamma}+w_j^{\gamma}} + ... + \phi_K\frac{1}{w_K^{\gamma}+w_j^{\gamma}}\big)\Big)}.$$

As before, large terms on the top and bottom cancel, yielding:

$$\frac{S(w_j)}{R(w_j)} = \frac{\lambda_{EU}b^{\gamma} + \lambda_{EE}f(\theta)\tilde{w}^{\gamma}}{g(\theta)}w_j^{-\gamma}.$$

Nesting the case above when $\lambda_{EU} = 0$ yields

$$\frac{S(w_j)}{R(w_j)} = \lambda_{EE}\theta\tilde{w}^{\gamma}w_j^{-\gamma}.$$

### A.4 Proving Existence for Limited Firm Heterogeneity

In this appendix section, we show that all the endogenous outcomes $\theta$, $\tilde{w}$, $N_j$, $w_j$, and $V_j$ can be solved for as functions of parameters. Using the result from Appendix A.3 that in any equilibrium firms with have a constant value of $\mathcal{E}$, and as such optimal wages and employment will take the form of (11) and (12), we then confirm the existence of such an equilibrium.

**Proposition 4** *If there are no exogenous separations $s = 0$, $\gamma$ is positive and finite, firms have identical $\alpha$, $\chi$, and $\sigma$, then there exists a symmetric, steady-state equilibrium.*

Given labor market tightness $\theta$ and an aggregate wage index $\tilde{w}$, a firm's optimal wage in steady state is:

$$w_j^* = \left(c_j\gamma(1+\chi)(\lambda\theta)^{1+\chi}\tilde{w}^{\gamma(1+\chi)}\right)^{\frac{1-\alpha}{(1-\alpha)(1+\gamma(1+\chi))+\sigma\chi}}\left(\frac{\alpha A_j\gamma(1+\chi)}{1+\gamma(1+\chi)+\sigma\chi}\right)^{\frac{\sigma\chi}{(1-\alpha)(1+\gamma(1+\chi))+\sigma\chi}} \tag{29}$$

Taking $\rho \to 0$ for simplicity, optimal employment at firm $j$ is:

$$N_j^* = \left(c_j\gamma(1+\chi)(\lambda\theta)^{1+\chi}\tilde{w}^{\gamma(1+\chi)}\right)^{\frac{-1}{(1-\alpha)(1+\gamma(1+\chi))+\sigma\chi}}\left(\alpha A_j\frac{\gamma(1+\chi)}{(1+\gamma(1+\chi))+\sigma\chi}\right)^{\frac{1+\gamma(1+\chi)}{(1-\alpha)(1+\gamma(1+\chi))+\sigma\chi}}$$

In steady state, optimal vacancies are:

$$V_j = \lambda_{EE}\theta\left(\frac{\tilde{w}}{w_j}\right)^{\gamma}N_j.$$

**Lemma 5** *Vacancy shares $\upsilon_j$ are a function of only parameters and do not depend on aggregate wages $\tilde{w}$ or tightness $\theta$.*

First we want to show that relative wages are not a function of $\tilde{w}$ or $\theta$. For two firms $j$ and $k$, relative wages are:

$$\frac{w_j}{w_k} = \left(\frac{c_j}{c_k}\right)^{\frac{1-\alpha}{(1-\alpha)(1+\gamma(1+\chi))+\sigma\chi}}\left(\frac{A_j}{A_k}\right)^{\frac{\sigma\chi}{(1-\alpha)(1+\gamma(1+\chi))+\sigma\chi}}$$

Next we show that relative employment is not a function of $\tilde{w}$ or $\theta$. For two firms $j$ and $k$, relative employment is:

$$\frac{N_j}{N_k} = \left(\frac{c_j}{c_k}\right)^{\frac{-1}{(1-\alpha)(1+\gamma(1+\chi))+\sigma\chi}}\left(\frac{A_j}{A_k}\right)^{\frac{1+\gamma(1+\chi)}{(1-\alpha)(1+\gamma(1+\chi))+\sigma\chi}}$$

Thus relative vacancies are a function of parameters only:

$$\frac{V_j}{V_k} = \frac{N_j}{N_k}\left(\frac{w_j}{w_k}\right)^{-\gamma} = \left(\frac{c_j}{c_k}\right)^{\frac{-1-\gamma(1+\alpha)}{(1-\alpha)(1+\gamma(1+\chi))+\sigma\chi}}\left(\frac{A_j}{A_k}\right)^{\frac{1+\gamma(1+\chi)-\gamma\sigma\chi}{(1-\alpha)(1+\gamma(1+\chi))+\sigma\chi}}.$$

Since relative vacancies not function of aggregate wages $\tilde{w}$ and tightness $\theta$, vacancies shares $\upsilon_j$ will not be a function of aggregate wages $\tilde{w}$ or tightness $\theta$.

**Lemma 6** *The ratio of a firm's optimal wage $w_j$ to the index of aggregate wages $\tilde{w}$ is a function of parameters.*

$$\frac{\tilde{w}}{w_j} = \frac{\int_k \upsilon_k w_k dk}{w_j} = \frac{\int_k \upsilon_k \Big(c_k\gamma(1+\chi)(\lambda\theta)^{1+\chi}\tilde{w}^{\gamma(1+\chi)}\Big)^{\frac{1-\alpha}{(1-\alpha)(1+\gamma(1+\chi))+\sigma\chi}} \Big(\frac{\alpha A_k\gamma(1+\chi)}{1+\gamma(1+\chi)+\sigma\chi}\Big)^{\frac{\sigma\chi}{(1-\alpha)(1+\gamma(1+\chi))+\sigma\chi}} dk}{\Big(c_j\gamma(1+\chi)(\lambda\theta)^{1+\chi}\tilde{w}^{\gamma(1+\chi)}\Big)^{\frac{1-\alpha}{(1-\alpha)(1+\gamma(1+\chi))+\sigma\chi}} \Big(\frac{\alpha A_j\gamma(1+\chi)}{1+\gamma(1+\chi)+\sigma\chi}\Big)^{\frac{\sigma\chi}{(1-\alpha)(1+\gamma(1+\chi))+\sigma\chi}}}.$$

All $\gamma$, $\chi$, $\lambda$, $\theta$, and $\sigma$ terms can be factored outside the intregral and cancel on the top and bottom. Therefore, a firm's wage $w_j$ relative to the market index $\tilde{w}$ is:

$$\frac{\tilde{w}}{w_j} = \frac{\int_k \upsilon_k w_k dk}{w_j} = \frac{\int_k \upsilon_k c_k^{\frac{1-\alpha}{(1-\alpha)(1+\gamma(1+\chi))+\sigma\chi}} A_k^{\frac{\sigma\chi}{(1-\alpha)(1+\gamma(1+\chi))+\sigma\chi}} dk}{c_j^{\frac{1-\alpha}{(1-\alpha)(1+\gamma(1+\chi))+\sigma\chi}} A_j^{\frac{\sigma\chi}{(1-\alpha)(1+\gamma(1+\chi))+\sigma\chi}}}.$$

Because the vacancy shares $\upsilon_k$ are also a function of parameters, a firms wage $w_j$ relative to the market $\tilde{w}$ is a function of only parameters and not functions of labor market tightness $\theta$.

Thus, we can write relative wages as a function of parameters only:

$$\frac{\tilde{w}}{w_j} = \frac{\tilde{w}}{w_j}\Big(\mathbb{c}, \mathbb{A}\Big).$$

**Lemma 7** *Given distributions of parameters $\mathbb{A}$ and $\mathbb{c}$, the wage index $\tilde{w}$ is an increasing function of tightness $\theta$.*

Based on equation (29), individual firm wages $w_j$ are increasing in aggregate wages $\tilde{w}$ and tightness $\theta$. Solving out individual wages, and solving $\tilde{w}$ in terms of only $\theta$ and parameters yields:

$$\tilde{w} = \Big(\int_j \upsilon_j w_j^{\gamma} dj\Big)^{\frac{1}{\gamma}}$$

$$\tilde{w} = \Big(\int_j \upsilon_j \Big(\theta^{1+\chi}\tilde{w}^{\gamma(1+\chi)}\Big)^{\frac{\gamma(1-\alpha)}{(1-\alpha)(1+\gamma(1+\chi))+\sigma\chi}} g(c_j, A_j) dj\Big)^{\frac{1}{\gamma}}$$

$$\tilde{w} = \Big(\theta^{1+\chi}\tilde{w}^{\gamma(1+\chi)}\Big)^{\frac{(1-\alpha)}{(1-\alpha)(1+\gamma(1+\chi))+\sigma\chi}} \Big(\int_j \upsilon_j g(c_j, A_j) dj\Big)^{\frac{1}{\gamma}}, \tag{30}$$

with

$$g(c_j, A_j) = \Big(c_j\gamma(1+\chi)\lambda^{1+\chi}\Big)^{\frac{1-\alpha}{(1-\alpha)(1+\gamma(1+\chi))+\sigma\chi}} \Big(\frac{\alpha A_j\gamma(1+\chi)}{1+\gamma(1+\chi)+\sigma\chi}\Big)^{\frac{\sigma\chi}{(1-\alpha)(1+\gamma(1+\chi))+\sigma\chi}}.$$

Grouping all $\tilde{w}$ terms on the left hand side yields:

$$\tilde{w}^{\frac{(1-\alpha)+\sigma\chi}{(1-\alpha)(1+\gamma(1+\chi))+\sigma\chi}} = \theta^{\frac{(1+\chi)(1-\alpha)}{(1-\alpha)(1+\gamma(1+\chi))+\sigma\chi}} \Big(\int_j \upsilon_j g(c_j, A_j) dj\Big)^{\frac{1}{\gamma}}. \tag{31}$$

Simplifying yields:

$$\tilde{w} = \theta^{\frac{(1+\chi)(1-\alpha)}{(1-\alpha)+\sigma\chi}} \Big(\int_j \upsilon_j g(c_j, A_j) dj\Big)^{\frac{(1-\alpha)(1+\gamma(1+\chi))+\sigma\chi}{\gamma((1-\alpha)+\sigma\chi)}}.$$

**Proof of Proposition**

$$V = \int_A \int_c V_j \omega(A, c) dc dA$$

$$V = \int_A \int_c \lambda_{EE} \theta \Big(\frac{\tilde{w}}{w_j}\Big)^\gamma N_j \omega(A, c) dc dA$$

We showed already that $\tilde{w}/w_j$ is a function of parameters:

$$V = \int_A \int_c \lambda_{EE} \theta \Big(\frac{\tilde{w}}{w_j}(\mathbb{c}, \mathbb{A})\Big)^\gamma N_j \omega(A, c) dc dA \tag{32}$$

We can abbreviate the expression for optimal employment as

$$N_j = \left(\theta^{1+\chi} \tilde{w}^{\gamma(1+\chi)}\right)^{\frac{-1}{(1-\alpha)(1+\gamma(1+\chi))+\sigma\chi}} f(c_j, A_j), \tag{33}$$

with

$$f(c_j, A_j) = \left(c_j \gamma(1+\chi) \lambda^{1+\chi}\right)^{\frac{-1}{(1-\alpha)(1+\gamma(1+\chi))+\sigma\chi}} \Big(\frac{\alpha A_j \gamma(1+\chi)}{1+\gamma(1+\chi)+\sigma\chi}\Big)^{\frac{1+\gamma(1+\chi)}{(1-\alpha)(1+\gamma(1+\chi))+\sigma\chi}}.$$

Plugging in (31) and (33) into equation (32) yields:

$$V = \int \int \theta \times \theta^{\frac{-1-\chi}{(1-\alpha)(1+\gamma(1_+\chi))+\sigma\chi}} \times \theta^{\frac{(1+\chi)(1-\alpha)}{(1-\alpha)+\sigma\chi} \frac{-\gamma(1+\chi)(1-\alpha)}{(1-\alpha)(1+\gamma(1+\chi))+\sigma\chi}} H(A_j, c_j, \mathbb{A}, \mathbb{c}) dc dA,$$

with

$$H(A_j, c_j, \mathbb{A}, \mathbb{c}) = \Big(\frac{\tilde{w}}{w_j}(\mathbb{c}, \mathbb{A})\Big)^\gamma f(c_j, A_j) \Big(\int_j \upsilon_j g(c_j, A_j) dj\Big)^{\frac{-1-\chi}{(1-\alpha)+\sigma\chi}}$$

In equilibrium where all workers are employed, and employed workers search with probability $\lambda$, then (normalize the population of workers to 1) tightness $\theta$ is:

$$\theta = \frac{V}{\lambda}.$$

The prior equation then becomes:

$$\lambda\theta = \int \int \theta \times \theta^{\frac{-1-\chi}{(1-\alpha)(1+\gamma(1_+\chi))+\sigma\chi}} \times \theta^{\frac{(1+\chi)(1-\alpha)}{(1-\alpha)+\sigma\chi} \frac{-\gamma(1+\chi)(1-\alpha)}{(1-\alpha)(1+\gamma(1+\chi))+\sigma\chi}} H(A_j, c_j, \mathbb{A}, \mathbb{c}) dc dA.$$

The $\theta$ terms on the right hand side can be pulled out of the integral, and a $\theta$ term on both sides cancels. With some alegbra, we have

$$\theta = \Big(\frac{1}{\lambda} \int \int H(A_j, c_j, \mathbb{A}, \mathbb{c}) dc dA\Big)^{\frac{(1-\alpha)+\sigma\chi}{1+\chi}}.$$

Thus equilibrium labor market tightness can be calculated strictly as a function of parameters. Plugging in our value of $\theta$ in equation (31), we can solve for the aggregate wage $\tilde{w}$ as a function of parameters. Given $\tilde{w}$, we can compute individual firms' $N_j$, $w_j$, and $V_j$.

### A.5 Solving and Calibrating Turnover Costs with Convexity

When solving for equilibrium in the model with convex adjustment costs, we must take one additional step to help with the interpretation of the model parameters. If we were to run comparative statics on results using equation A.5 while changing the convexity parameter $\chi$, we would find strange results. This is because that $\chi$ is an exponent over a variable that is quite a bit below 1: the ratio of vacancies to employment $V/N$ is typically between .04 and .1 for most calibrations. As $\chi$ increases, and holding the constant $c$ fixed, the average cost of vacancy posting becomes cheaper, even if the choice of $V/N$ goes unchanged. Therefore, in order to make results interpretable while holding the constant $c$ fixed, we need to normalize the turnover costs. To do so, we will introduce a normalization parameter $\tau$ in the firm's problem:

$$\max_{N,w} AN^{\alpha} - wN - c\tau^{-\chi}(\lambda\theta)^{1+\chi}\left(\frac{\bar{w}}{w}\right)^{\gamma(1+\chi)} N^{1+\sigma}.$$

Next, we need to calibrate the scaling parameter $\tau$. The least arbitrary answer is to calibrate it such that for a given constant $c$, the per-vacancy cost of posting a vacancy across values of $\chi$ is equal if firms posted as many vacancies as they would under the linear case. What this is implies is that we should calibrate $\tau$ to be equal to the inverse of the optimal choice of $V/N$ when $\chi = 0$, i.e., when costs are linear. To see why, consider the firm's more general problem when $\sigma = 0$ and $\chi \geq 0$:

$$\max_{N,w} AN^{\alpha} - wN - c\tau^{-\chi}\left(\frac{V}{N}\right)^{\chi} V$$
$$AN^{\alpha} - wN - c\left(\frac{1}{\tau}\frac{V}{N}\right)^{\chi} V.$$

If we set $\tau = ((V/N)^*|_{\chi=0}))^{-1}$, then we can see that per-vacancy costs are equal across values of $\chi$ and are exactly equal to $c$ if firms happen to choose $(V/N) = (V/N)^*|_{\chi=0}$.

Now that we have picked a value for $\tau$, we can solve for the equilibrium of this labor market. As a function of parameters, the endogenous outcomes are:

$$N = \frac{P}{m}, \qquad \theta = \frac{1}{\lambda}\left(\frac{\frac{\eta-1}{\eta}A\beta\left(\frac{m}{P}\right)^{1-\beta}}{(\gamma+1)c\tau^{-\chi}}\right)^{\frac{1}{1+\chi}}, \qquad \bar{w} = c(\lambda\theta)^{1+\chi}\gamma(1+\chi).$$

The inverse ratio of vacancies to employment when $\chi = 0$, and also $\tau$, in terms of parameters is:

$$((V/N)|_{\chi=0})^{-1} = \tau = \frac{\frac{\eta-1}{\eta}\beta N^{\beta-1}}{c(\gamma+1)}.$$

Plugging this value into our general formula for $\theta$, we have

$$\theta = \frac{\frac{\eta-1}{\eta}A\beta\left(\frac{m}{P}\right)^{\beta-1}}{\lambda c} \times \frac{1}{(1+\gamma(1+\chi))^{\frac{1}{1+\chi}}(1+\gamma)^{\frac{\chi}{1+\chi}}}.$$

## A.6 Indirect Inference

This appendix section derives the formula for computing the profit share of marginal product based on the variance of firm wage effects and firm size in Section 4.2 of the main text. Beginning by restating equations (11) and (12) of the main text:

$$w_j^* = \left(c_j\gamma(1+\chi_j)(\lambda\theta)^{1+\chi_j}\tilde{w}_j^{\gamma(1+\chi_j)}\right)^{\frac{1-\alpha_j}{(1-\alpha_j)(1+\gamma(1+\chi_j))+\sigma_j\chi_j}}\left(\frac{\alpha_j A_j\gamma(1+\chi_j)}{1+\gamma(1+\chi_j)+\sigma_j\chi_j}\right)^{\frac{\sigma_j\chi_j}{(1-\alpha_j)(1+\gamma(1+\chi_j))+\sigma_j\chi_j}}$$

Taking $\rho \to 0$ for simplicity, optimal employment at firm $j$ is:

$$N_j^* = \left(\frac{\alpha_j A_j}{w_j^*} \times \frac{\gamma(1+\chi_j)}{1+\gamma(1+\chi_j)+\sigma_j\chi_j}\right)^{\frac{1}{1-\alpha_j}}.$$

Plugging in the wage into the expression for employment yields

$$N_j^* = \left(c_j\gamma(1+\chi_j)(\lambda\theta)^{1+\chi_j}\tilde{w}_j^{\gamma(1+\chi_j)}\right)^{\frac{-1}{(1-\alpha_j)(1+\gamma(1+\chi_j))+\sigma_j\chi_j}}\left(\alpha_j A_j\frac{\gamma(1+\chi_j)}{(1+\gamma(1+\chi_j))+\sigma_j\chi_j}\right)^{\frac{1+\gamma(1+\chi_j)}{(1-\alpha_j)(1+\gamma(1+\chi_j))+\sigma_j\chi_j}}$$

Taking logs of the expressions for $w_j^*$ and $N_j^*$, we get

$$\begin{aligned}\log(w_j^*) =& \frac{1-\alpha_j}{(1-\alpha_j)(1+\gamma(1+\chi_j))+\sigma_j\chi_j}\log(c_j) + \frac{\sigma_j\chi_j}{(1-\alpha_j)(1+\gamma(1+\chi_j))+\sigma_j\chi_j}\log(A_j) + \mathcal{C}_w \\ \log(N_j^*) =& \frac{-1}{(1-\alpha_j)(1+\gamma(1+\chi_j))+\sigma_j\chi_j}\log(c_j) + \frac{1+\gamma(1+\chi_j)}{(1-\alpha_j)(1+\gamma(1+\chi_j))+\sigma_j\chi_j}\log(A_j) + \mathcal{C}_N,\end{aligned}$$

where $\mathcal{C}_w$ and $\mathcal{C}_N$ are functions of objects that firm $j$ takes as given.

Let's assume that all firms are identical except in their $c_j$ and $A_j$. Next we take variances and covariances of these expressions. We assume that $Cov(c_j, A_j) = 0$.

$$\sigma_w^2 = \left(\frac{1-\alpha}{(1-\alpha)(1+\gamma(1+\chi))+\sigma\chi}\right)^2\sigma_c^2 + \left(\frac{\sigma\chi}{(1-\alpha)(1+\gamma(1+\chi))+\sigma\chi}\right)^2\sigma_A^2 \tag{34}$$

$$\sigma_N^2 = \left(\frac{1}{(1-\alpha)(1+\gamma(1+\chi))+\sigma\chi}\right)^2\sigma_c^2 + \left(\frac{1+\gamma(1+\chi)}{(1-\alpha)(1+\gamma(1+\chi))+\sigma\chi}\right)^2\sigma_A^2 \tag{35}$$

$$\sigma_{wN} = \frac{-(1-\alpha)}{((1-\alpha)(1+\gamma(1+\chi))+\sigma\chi)^2}\sigma_c^2 + \frac{(1+\gamma(1+\chi))\sigma\chi}{((1-\alpha)(1+\gamma(1+\chi))+\sigma\chi)^2}\sigma_A^2 \tag{36}$$

Now we solve for $\sigma_c^2$ and $\sigma_A^2$ using equations (34) and (35). First we isolate $\sigma_c^2$:

$$((1-\alpha)(1+\gamma(1+\chi))+\sigma\chi)^2\sigma_w^2 = (1-\alpha)^2\sigma_c^2 + (\sigma\chi)^2\left(\frac{((1-\alpha)(1+\gamma(1+\chi))+\sigma\chi)^2}{(1+\gamma(1+\chi))^2}\sigma_N^2 - \frac{1}{(1+\gamma(1+\chi))^2}\sigma_c^2\right)$$

$$\sigma_c^2 = ((1-\alpha)(1+\gamma(1+\chi))+\sigma\chi)^2\frac{\sigma_w^2 - \left(\frac{\sigma\chi}{1+\gamma(1+\chi)}\right)^2\sigma_N^2}{(1-\alpha)^2 - \left(\frac{\sigma\chi}{1+\gamma(1+\chi)}\right)^2}$$

Next we isolate $\sigma_A^2$ using (34) and (35):

$$((1-\alpha)(1+\gamma(1+\chi))+\sigma\chi)^2\sigma_w^2 = (1-\alpha)^2\Big(((1-\alpha)(1+\gamma(1+\chi))+\sigma\chi)^2\sigma_N^2 - (1+\gamma(1+\chi))^2\sigma_A^2\Big) + (\sigma\chi)^2\sigma_A^2$$

$$\sigma_A^2 = ((1-\alpha)(1+\gamma(1+\chi))+\sigma\chi)^2\frac{\sigma_w^2-(1-\alpha)^2\sigma_N^2}{(\sigma\chi)^2-(1-\alpha)^2(1+\gamma(1+\chi))^2}$$

Rearranging (36) gives:

$$((1-\alpha)(1+\gamma(1+\chi))+\sigma\chi)^2\sigma_{wN} = -(1-\alpha)\sigma_c^2 + (1+\gamma(1+\chi))\sigma\chi\sigma_A^2.$$

Plugging our terms for $\sigma_c^2$ and $\sigma_A^2$ and cancelling the $((1-\alpha)(1+\gamma(1+\chi))+\sigma\chi)^2$ term on both sides yields:

$$\sigma_{wN} = -(1-\alpha)\frac{\sigma_w^2-\left(\frac{\sigma\chi}{1+\gamma(1+\chi)}\right)^2\sigma_N^2}{(1-\alpha)^2-\left(\frac{\sigma\chi}{1+\gamma(1+\chi)}\right)^2} + (1+\gamma(1+\chi))\sigma\chi\frac{\sigma_w^2-(1-\alpha)^2\sigma_N^2}{(\sigma\chi)^2-(1-\alpha)^2(1+\gamma(1+\chi))^2}.$$

Rearranging signs in the fist term and multiplying the second term by $\frac{(1+\gamma(1+\chi))^2}{(1+\gamma(1+\chi))^2}$ yields

$$\sigma_{wN} = (1-\alpha)\frac{\sigma_w^2-\left(\frac{\sigma\chi}{1+\gamma(1+\chi)}\right)^2\sigma_N^2}{\left(\frac{\sigma\chi}{1+\gamma(1+\chi)}\right)^2-(1-\alpha)^2} + \frac{\sigma\chi}{1+\gamma(1+\chi)}\frac{\sigma_w^2-(1-\alpha)^2\sigma_N^2}{\left(\frac{\sigma\chi}{1+\gamma(1+\chi)}\right)^2-(1-\alpha)^2}.$$

Pulling the denominator on the right to the left hand side gives:

$$\left(\left(\frac{\sigma\chi}{1+\gamma(1+\chi)}\right)^2-(1-\alpha)^2\right)\sigma_{wN} = (1-\alpha)\left(\sigma_w^2-\left(\frac{\sigma\chi}{1+\gamma(1+\chi)}\right)^2\sigma_N^2\right)+\frac{\sigma\chi}{1+\gamma(1+\chi)}\left(\sigma_w^2-(1-\alpha)^2\sigma_N^2\right).$$

Grouping and factoring terms yields:

$$\left(\left(\frac{\sigma\chi}{1+\gamma(1+\chi)}\right)-(1-\alpha)\right)\left(\left(\frac{\sigma\chi}{1+\gamma(1+\chi)}\right)+(1-\alpha)\right)\sigma_{wN} =$$
$$\sigma_w^2\left((1-\alpha)+\frac{\sigma\chi}{1+\gamma(1+\chi)}\right) - \sigma_N^2\frac{\sigma\chi}{1+\gamma(1+\chi)}(1-\alpha)\left((1-\alpha)+\left(\frac{\sigma\chi}{1+\gamma(1+\chi)}\right)\right).$$

Cancelling $\left(\left(\frac{\sigma\chi}{1+\gamma(1+\chi)}\right)+(1-\alpha)\right)$ from each term yields:

$$\left(\left(\frac{\sigma\chi}{1+\gamma(1+\chi)}\right)-(1-\alpha)\right)\sigma_{wN} = \sigma_w^2 - \sigma_N^2\frac{\sigma\chi}{1+\gamma(1+\chi)}(1-\alpha).$$

Grouping the $\sigma\chi/(1+\gamma(1+\chi))$ yields:

$$\left(\frac{\sigma\chi}{1+\gamma(1+\chi)}\right)\left(\sigma_{wN}+(1-\alpha)\sigma_N^2\right) = \sigma_w^2 + (1-\alpha)\sigma_{wN}.$$

Solving for $\sigma\chi/(1+\gamma(1+\chi))$ yields:

$$\frac{\sigma\chi}{1+\gamma(1+\chi)} = \frac{\sigma_w^2 + (1-\alpha)\sigma_{wN}}{(1-\alpha)\sigma_N^2 + \sigma_{wN}}.$$

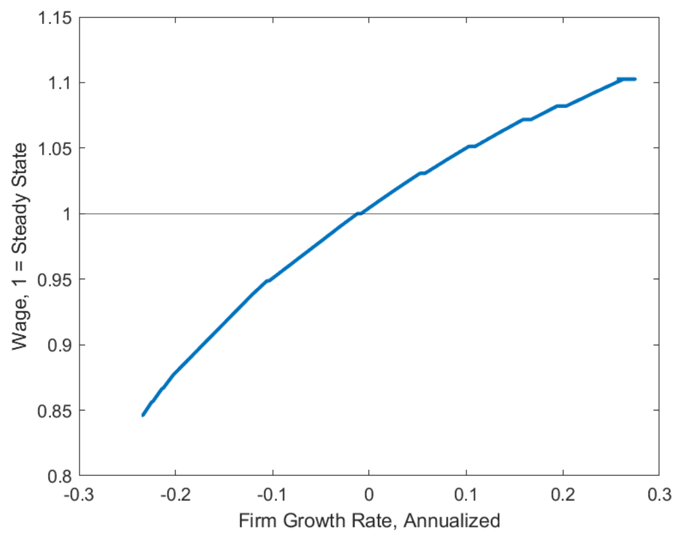
Thus, the wage-size elasticity that comes from shifts in demand for labor $\varepsilon_{w,N}$ is identified off the variances of wages and employment, and their covariance. Coverting this to profit shares of marginal product,

$$\frac{\sigma\chi}{1+\gamma(1+\chi)+\sigma\chi} = \frac{\varepsilon_{w,N}}{1+\varepsilon_{w,N}}.$$

### A.7 Wages and Firm Growth

We recast the firm's problem in equation (1) recursively and solve for the firm's optimal choices of wages, employment, and vacancies out of steady state. With completely flexible wages, a firm that is growing at a 30% annual rate (2.5% per month) would optimally pay wages around 10% above the steady-state wage.

Figure 7: Optimal Wages and Firm Growth




This figure plots a firm's optimal wage given its target growth rate for $\sigma = 0$ and $\chi = 1$.