Disclosure Regulation, Intangible Capital and the Disappearance of Public Firms[†]

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Abstract

Since the mid-1990s, the number of listed firms in the U.S. has halved, and their performance has become increasingly difficult to predict. To analyze the driving forces behind these trends and their macroeconomic implications, we develop a general equilibrium model where the choices of going public or private and the transparency of voluntary disclosure are characterized analytically. In the equilibrium, dispersion in transparency, funding, and public or private status arises endogenously. Going public with transparent disclosure leads to greater funding at the cost of a firm's competitiveness through knowledge spillover. According to the estimation, stricter disclosure regulation and increased intangible capital share are the key drivers of the observed patterns. The increased intangible share has led to significant welfare and productivity loss, while the stricter disclosure regulation improved welfare at the cost of productivity loss. Lastly, we characterize a policymaker's trade-off between welfare and productivity and analyze the optimal disclosure policy.

Keywords: Intangible capital, corporate disclosures, technology diffusion.

JEL codes: D83, E22, G32, G38.

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1 Introduction

"As a smaller private company, Google kept business information closely held, and we believe this helped us against competitors... As a public company, we will of course provide you with all information required by law... But we will not unnecessarily disclose all of our strengths, strategies and intentions."

— Larry Page and Sergey Brin, *A letter from Google founders to the shareholders, 2004*

Since the mid-1990s, the number of listed firms in the U.S. has decreased almost by half. Over the same period, we document that listed firms' performance has become increasingly difficult to predict (Figure 1). What are the driving forces for these changes? What are their macroeconomic consequences? This paper answers these questions through the lens of an estimated general equilibrium model of information disclosure and capital markets, where an analytic solution characterizes a rich set of equilibrium allocations. We then use the model to analyze the optimal disclosure regulation based on the equilibrium.

The U.S. Securities and Exchange Commission (SEC) requires listed firms to publicly reveal their annual and quarterly financial information, business activity and results, and disclose all material events such as transactions involving shareholders and insiders. Moreover, public firms are not allowed to selectively disclose materials to some investors (e.g., Regulation Fair Disclosure of 2000). Disclosure regulation aims to protect investors and facilitate a fair capital market. However, the cost of disclosure is that it may also reveal crucial information to competitors (Bhattacharya and Ritter, 1983). In this paper, we show that stricter disclosure regulation and the increased importance of intangible capital in production are critical factors driving public firms' disappearance.

Support exists for the notion that private firms' ability to avoid public disclosures is an important factor in their decision to stay private.[1] Our key hypothesis is that,

[1]For example, Dambra, Casares Field, and Gustafson (2015) study the effect of Title I of the JOBS Act (Jumpstart Our Business Startups Act), which exempts emerging growth companies from certain disclosure requirements during the IPO process and allows issuers to disclose information exclusively to investors, but not competitors, until the IPO becomes likely to succeed. They find that the act increased the volume of IPOs by 25% compared to their previous level; and this increase is concentrated in firms with a high cost of disclosure, such as firms in the tech sector. Aghamolla and Thakor (2022) exploit a shock to disclosure requirements in the biopharmaceutical industry to show that increased mandatory disclosure requirements for private firms significantly increases their

given its nature, intangible capital is one of the most fragile input factors to the information disclosure process. This is because it can be difficult to establish and enforce exclusive property rights to an intangible: Unlike a physical piece of capital, once information about an intangible is revealed, it can be readily copied or imitated – a property of *limited excludability* explored in Crouzet et al. (2022).[2]

Using an estimated general equilibrium model based on U.S. firm data, we show that disclosure regulation has dual effects on the welfare of risk-averse investors and that its adverse effect has increased over time. On the one hand, mandatory disclosure increases welfare by fostering transparent information disclosure. On the other hand, stricter regulation risks crowding out voluntary disclosure, and in some instances, it may backfire through the extensive margin channel as more firms opt to remain in the private equity market, characterized by a higher level of opacity.[3] As firms adopt more intangible capital, they have a stronger incentive to conceal information, leading to a higher cost of regulation and an increased tendency to remain privately held. One key message of the paper is that the same regulation level has become more binding over time due to the increased importance of intangible capital as an input factor.Finally, we show that the disappearance of public firms and overall greater opacity in financial markets substantially reduce productivity through the mitigated technological diffusion across firms, which partly explains the recently observed macroeconomic phenomena in the U.S. (Akcigit and Ates, 2023). According to our baseline model, it results in a substantial welfare loss in the economy.

In our model, ex-ante homogeneous firms choose whether to go public or private, the level of intangible capital stock, and the transparency of their intangible capital.

propensity of going public. Abuzov, Gornall, and Strebulaev (2023) show that a strengthening of disclosure requirements for public investors in 2002 led many top VCs to exclude these investors from their funds.

[2]We refer to those components of intangible capital whose property rights are not well protected by specific legal institutions and thus not necessarily patentable or patented yet. For example, software, research ideas, early stages innovation and R&D, and also certain novel business methods and organizational innovations, branding and marketing strategies, employee training, information such as some formulas, customer lists, and processes; more in general, firms' strategies and intentions that a public firm cannot selectively disclose.

[3]This is one of the core issues the SEC is concerned about. For example, in a February 2017 speech, SEC Commissioner Kara Stein posed a question regarding additional disclosures and regulation around private market investment: "We also need to understand why more companies are staying private for longer periods of time. Should we apply enhanced disclosure laws to these private companies? Or perhaps they require a unique set of rules." See "The Markets in 2017: What's at Stake?" Commissioner Kara M. Stein, SEC website, https://www.sec.gov/news/speech/stein-secspeaks-whats-at-stake.html

The different levels of transparency of the disclosed information and private firm market are modeled as the submarket under the directed search protocol, following the widely used setup in the macro labor and monetary literature (see Lagos, Rocheteau, and Wright (2017) and Wright et al. (2021) for recent surveys on such protocol). The disclosed intangible capital is subject to diffusion to other firms as an externality in the form of total factor productivity (TFP) gains.[4]

If a firm goes private, transparency is minimal, and there is no technology diffusion to the other firms. However, a private firm must search for an investor and is only guaranteed funding if matched. When a firm chooses to be public, the firm is subject to a disclosure obligation, composed of mandated and voluntary components. The policymaker enforces the minimum mandated portion, and the firm endogenously determines the voluntary portion. As the household prefers transparent firms, which she finds easier to forecast, a more transparent disclosure leads to greater value in the funding market. However, disclosure undermines the firm's profitability, especially for high levels of intangible capital. This trade-off endogenously forms a non-degenerate distribution of firms over the transparency domain and determines the mass of the non-listed market in equilibrium.

One of the advantages of our model is that these decisions have an analytic solution, which allows us to characterize the model and optimal policy globally and cleanly. Despite firms and investors being ex-ante homogenous, the model generates a rich general equilibrium distribution of endogenous objects in analytic form, and as such loosely resembles the one in Burdett and Judd (1983) or Burdett and Mortensen (1998). In the latter, the wage distribution is endogenously determined, as the model captures the endogenous wage postings from the firm side. Similarly, in our model, a risk-averse representative household with CARA utility endogenously chooses the amount of funding for each transparency level.

As discussed, the model predicts that firms with high intangible adoption are associated with lower transparency and are more difficult to forecast. To support the model prediction, we run a panel regression of analysts' forecast errors and different transparency proxies on intangible capital with firm-level controls and fixed effects. We confirm that the inverse of variance and of the value of forecast errors of U.S. analysts are significantly negatively correlated with the firm level of intangible capi-

[4]Similarly, Lagos (2006) develops a model with a frictional labor market where the level of TFP is endogenous and depends on the distribution of idiosyncratic shocks and the job-destruction decision.

tal. We interpret the result in the following way: the negative relationship between intangible and transparency, proxied by analysts' agreement and by forecast accuracy, can be due to two reasons: one, firms with high levels of intangible tend to be less transparent and, therefore, more challenging to forecast. Two, it may be that, given a certain level of disclosure and transparency, firms with high intangible capital are inherently harder to forecast due to their nature (Celentano and Rempel, 2023). In our structural analysis, we are able to disentangle the two forces and their effect.

We then conduct a structural analysis of the macroeconomic effects of the increasing significance of intangible assets and the impact of information regulation policies. We estimate our model using data from two distinct periods. The first period, spanning from 1992 to 1996, serves as our baseline, while the second period, from 2012 to 2016, is considered as the new steady state. Therefore, we compare a period before the dramatic shift in the number of listed firms with a period several years after the change to assume that it has reached a stationary level.

The key structural parameters in the model include intangible capital share and the mandated disclosure rule: the changes in these parameters change the incentive of the voluntary disclosure operating in the listed market. We use the method of simulated moments (MSM) to estimate the parameters, and target moments such as the percentage of listed firms after M&A adjustment, the share of intangible-related expenditures over sales, and the fraction of funded private firms. Moreover, one of the advantages of our model is that it is tightly linked with the data: While the distribution of firms' transparency is not directly observable, the distribution of forecast errors by analysts is both a model output and is observable in our data. Therefore, we discipline our analysis with firm-level data and target several moments of this distribution over the two periods.

Our decomposition analysis reveals that stricter SEC regulation and the rising share of intangible capital accounted for a large part of the decline in listed firms and transparency.[5] We also estimate that the same level of disclosure by firms translates

[5]Some changes in disclosure regulation since 1996, the end of our baseline period, include the implementations in 1997 on Regulation S-K of the recommendations of the Task Force on Disclosure Simplification, available at http://www.sec.gov/rules/final/34-38850.txt and http://www.sec.gov/rules/final/34-38850a.txt, the plain English initiative of 1998, the Regulation Fair Disclosure of 2000, the Sarbanes-Oxley Act of 2002, the newer disclosure requirements introduced by the Dodd-Frank act of 2010 available at https://www.sec.gov/spotlight/dodd-frank/corporategovernance.shtml and https://www.sec.gov/securities-topics/dodd-frank-act. We may also interpret the introduction of machine-readable data on Edgar combined with the ease

into lower information for investors in the more recent period. We interpret this as intangible capital being inherently more opaque and challenging to understand due to its nature, contributing to the decline of listed firms. In line with findings in Ewens and Farre-Mensa (2020), the model also predicts that access to funds by private investor has become easier, contributing to the reduction of public firms. These findings highlight that stricter regulation, increased intangible capital, and greater opacity in financial markets are important and novel channels driving factors behind the reduced transparency, number of listed firms, and productivity.

Finally, we set out to find an optimal disclosure policy. To evaluate the consequences of the information disclosure policy, we provide three criteria: output, productivity, and investors' welfare.[6] A higher mandated transparency level decreases the incentive to go public, leading to more private firms in the equilibrium. However, a stricter policy lowers uncertainty for investors, achieving greater welfare. In the estimated model, a policy change in the neighborhood of the status quo can achieve only higher output and productivity or higher welfare.[7] From the perspective of the protection of investors, we find that the recent regulation has substantially improved welfare. However, we also document that it has led to a loss in productivity in the production sector.

Contribution and literature. Our paper delivers two main contributions to the literature. First, we provide a theoretical and quantitative model framework that analyzes the effect of information disclosure and the rising intangible capital on the firm-level financing decision.[8] Using the estimated model, we show that the stricter regulation on disclosure and rising intangible share have been the key drivers of the disappearing public firms. The qualitative aspect of our model is worth highlighting as it allows the analytic characterization of rich equilibrium allocations, including the distribution of public and private firms. This tractability promotes the transparent illustration of endogenous mechanisms in our model. Also, it enables a fast and

of accessing that data as more transparency through lower frictions to access the same information.

[6]The mission of the SEC is "to protect investors, maintain fair, orderly, and efficient markets, and facilitate capital formation." See "Our Goals," SEC website, https://www.sec.gov/our-goals.

[7]In the global domain of the policy, there are ranges where welfare and productivity increase simultaneously along with the policy change. We discuss this in the policy analysis.

[8]Kahle and Stulz (2017) discussed the possibility of the role of intangible capital in the observed declining trends of listed firms. However, the structural analysis of the channel has been missing in the literature.

accurate quantitative analysis.[9]

Second, we bring to the table a novel policy angle, information regulation, and analyze its macroeconomic trade-off. From the tractable general equilibrium model, we show that a policymaker faces a trade-off between welfare and productivity in a reasonable range of parameters. We believe the analytic closed-form characterization of our model would serve as a useful tool for future research on information regulation policy.

Three strands of the literature are closely related to this paper. The first is the literature that studies the incentive for information disclosure and its real impact. One of the seminal papers in the literature is Hirshleifer (1971), which studies how information disclosure can be incentivized through pecuniary motivation, which is closely related to the firms' incentive for transparent disclosure in our model. On top of this, our model also captures the cost of transparent information disclosure that counterbalances the pecuniary motivation. This side of the incentive has the similarity to the bank's secret-keeping motivation in Dang et al. (2017).[10]

The second is the literature that studies the rising importance of intangible capital. It was only around a decade ago that intangible capital was first recognized as an important macroeconomic factor that affects economic growth and the business cycle. For example, McGrattan and Prescott (2010) and McGrattan (2020) highlight the importance of intangible capital as a key input factor for production and show how mismeasurement of intangible capital may mislead the neoclassical model predictions in terms of economic growth. Relatedly, Atkeson and Kehoe (2005) and Eisfeldt and Papanikolaou (2014) modeled plant-level intangible capital as an important input for production. Mainly, their intangible capital refers to organizational capital that is partly firm-specific and partly embodied in key labor inputs.

We contribute to this literature by analyzing a novel macroeconomic implication of the rising share of intangible capital. Intangible capital has become an important source of competitiveness, leaving firms to put great effort into research and development (R&D) or developing a productive corporate culture. However, intangible

[9]The portion of public firms is often substantially smaller than that of private firms in many countries. Then a computation error of 0.1% in the portion of public firms is a significantly large error. Therefore, a highly-computational model is easily subject to a high approximation error in capturing the portion of large firms.

[10]As noted by Li, Rocheteau, and Weill (2012), disclosure of information regarding firm's characteristics is a reduced form way to model trading frictions, which have been studied in OTC market by Lagos and Rocheteau (2009).

capital has a strong spillover effect, which can benefit competitors as well as the owner firm (Crouzet et al., 2022). Therefore, the rising importance of intangible capital has naturally increased a firm's incentive to stay opaque in its disclosure. Using our model, we theoretically and quantitatively analyze how this change affects the macroeconomy in terms of welfare and productivity.

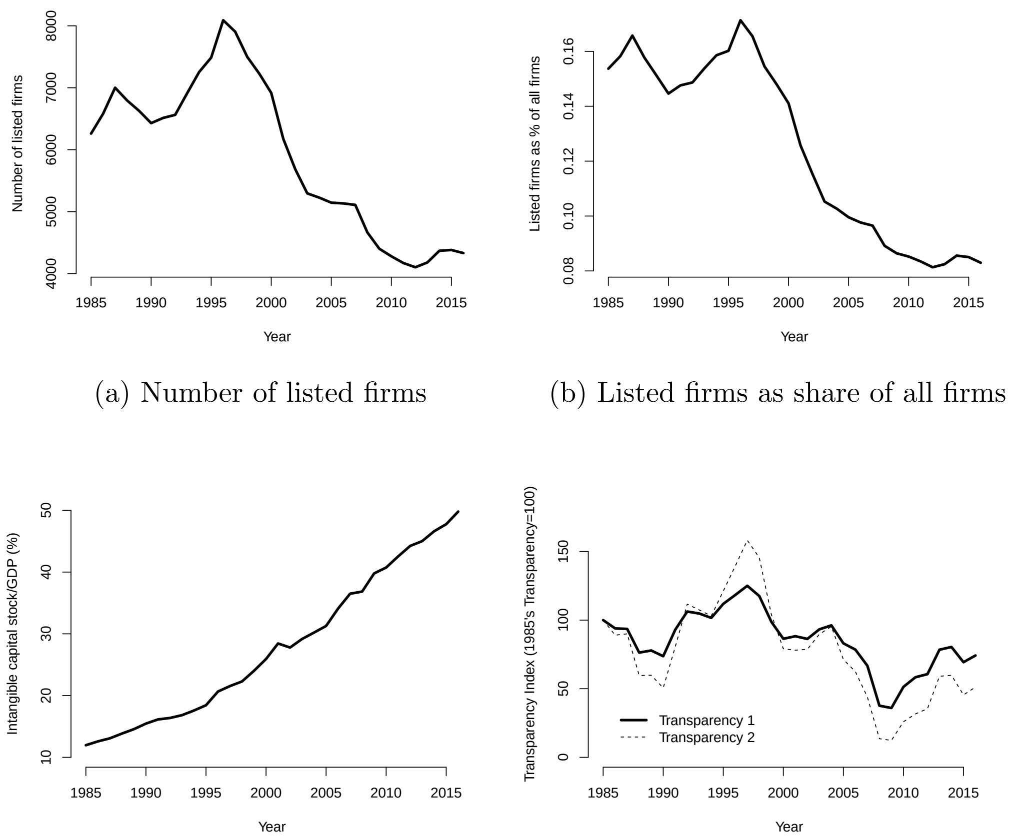
One of the papers closest to ours in this sense is Celentano and Rempel (2023), which finds that the rising share of intangible capital has amplified public CEOs' private information compared to outside investors. This rising informational asymmetry between firm insiders and the general public leads to an increase in CEO compensation due to the design of optimal truth-telling compensation contracts, and a decline in the propensity of going public. We abstract from optimal contracts and the principal-agent problem; instead, we focus on a different and complementary channel: regulation on information disclosure and its interaction with intangible capital and its spillover to competitors as a positive learning externality. Our model allows us to calculate welfare and the optimal level of regulation.

The third literature is about the disappearance of listed firms. Different explanations have been put forward to shed light on this issue. For example, Gao, Ritter, and Zhu (2013) point to the increase in mergers and acquisitions (M&A) among U.S. firms; Doidge, Karolyi, and Stulz (2017) conjecture that as markets have become more globally integrated, the net benefits of going public in the U.S. versus in other markets have decreased; Ewens and Farre-Mensa (2020) argue that the deregulation of securities laws (National Securities Markets Improvement Act of 1996) improved the private equity market, which reduced the incentives for firms to go public.

Our explanation is complementary to the existing literature. We argue that the rise of intangible capital, especially the components of intangible capital that could benefit competitors as well as the owner firm, has increased the cost of disclosing information and made staying private more attractive, which is exacerbated by stricter disclosure requirements. The estimated model also predicts that access to funds by venture capital firms, private equity funds, and other private investors has become easier.

Motivating facts Figure 1 plots the time series of the variables of interest from 1985 until 2015. Panel (1a) plots the number of listed firms in the U.S. The data is

Figure 1: Time series of aggregate variables.



(a) Number of listed firms

(b) Listed firms as share of all firms

(c) Intangible/GDP

(d) Inverse forecast error

Notes: This figure shows the trend in the number and share of listed firms, intangible capital, and the inverse forecast error in the U.S. Data comes from Compustat, I/B/E/S, and the World Development Indicators. See Section Appendix for details on measurement.

from the World Development Indicators (WDI) by World Bank.[11] As shown in the figure, there has been a gradually rising trend in the number of listed firms until the mid-1990s. Then, after the peak in the mid-1990s, the number of listed firms steeply declined to almost half the level at the peak year: 8,090 listed firms in 1996 reduced to 4,102 listed firms in 2012. Panel (1b) shows that listed firms have been declining not only in absolute number, but also as a share of all firms in the U.S.

Panel (1c) shows the time series of the ratio between the total intangible capital

[11]The number of listed firms in WDI is only negligibly different from the one in the Compustat data.

stock of public non-financial corporations and GDP. Over the thirty years, the ratio has dramatically increased from 10% to 50%. This shows how fast intangible capital in the U.S. has grown.

Lastly, panel (1d) shows the time series of the inverse forecast error. The overall patterns of the series closely mimic the one in the number of listed firms: the inverse forecast error has increased until the mid-1990s and decreased after the peak in 1996.[12] The time-series correlation between the inverse forecast error and the number of listed firms is 0.80 for the first measure and 0.61 for the second measure, and all are statistically significant. This co-movement between the number of listed firms and the average transparency is the key motivation of this paper: what drives such co-movements?

Roadmap The rest of the paper is structured as follows. Section 2 outlines the model and the equilibrium. Section 3 describes model predictions. Section 4 describes the supportive empirical evidence. Section 5 estimates the structural model, conducts counterfactuals, and describes the optimal regulation policy. We conclude in Section 6.

2 Theory

In this section, we introduce a general equilibrium model, where a representative household as an investor meets firms at the listed market under the directed search protocol and the non-listed market under the random search protocol. In particular, the listed market is comprised of submarkets indexed by the transparency level q. The funding allocation across the two markets are endogenously determined by the household's preference.

A stand-in household and a continuum of measure one of the ex-ante homogeneous firms are considered. The model is static, but the model incorporates rich firm-level allocations in the equilibrium, including their distribution.[13] A representative

[12]Recessions and especially the Great Recession represent a big shocks to earnings surprises. In order to take that into account, we also measure the inverse forecast error by excluding recession periods as measured by the NBER, and we still find that the time series has been declining.

[13]The model's *one-shot* feature is intended to capture an equilibrium that is formed over long years. Therefore, the dynamic aspect is abstracted. Also, the static setup gives a great degree of tractability in the model, as will be described in the equilibrium analysis.

household decides its asset portfolio and consumes the payouts from the portfolio. A manager of a firm decides in which market the firm operates between the public and private equity markets. If a firm is listed, the manager chooses the disclosure level of the firm's intangible capital to the public, which we define as transparency. On the other hand, a firm does not disclose any intangible capital to the public if the firm is private.

2.1 Household

A stand-in household decides on the asset portfolio and consumes the portfolio return. The household is given a wealth level $a > 0$. The household is risk-averse, and the utility takes the following constant absolute risk aversion form (CARA):

$$u(C) = -e^{-\Lambda C},$$

where $\Lambda > 0$ is the absolute risk aversion parameter.

In the listed market, the household forms a belief about the return $\widetilde{r}(q)$ based on the balance sheet information of a listed firm with transparency level q and the mandated transparency level $\overline{q}$. The belief about the return is as follows:

$$\widetilde{r}(q) \sim_{iid} N\left(\overline{r}(q), \frac{1}{\xi + (\overline{q} + q)\psi}\right)$$
$$\text{s.t.} \quad \overline{r}(q) = \frac{\pi(q)}{P(q)},$$

where $q \in [0, 1 - \overline{q}]$ is the transparency of the balance sheet information.[14] $\overline{q}$ is the mandated transparency required by the policy maker; ξ is the baseline information level a household has about both listed and non-listed firms; $\psi > 0$ is the marginal contribution of transparency to the household's information about the listed firm.[15] $\pi(q)$ is the profit of the firm with transparency q, and $P(q)$ is the price of the firm

[14]The range of transparency is assumed at our convenience. However, the qualitative and quantitative results of this paper are unaffected by this normalization assumption.

[15]We may regard ψ as the functions of a structural parameter θ, the share of intangible capital in the production function. Intuitively, the importance of intangible capital in the production function affects the information quality household can access from the balance sheet. We do not impose any structural assumption on this function. Instead, we identify the level of ψ in our estimation using the firm-level data. Then, in the quantitative analysis, we interpret a change in ψ is affected by the variation in θ.

with transparency q. Note that this belief about the asset return corresponds exactly to the posterior distribution of asset returns after a Bayesian update based on the news about the return. We formally derive this result in Appendix H.

In the private equity market, the household forms the following belief about the non-listed firms:

$$\widetilde{r}^N \sim_{iid} N\left(\overline{r}^N, \frac{1}{\xi}\right)$$
$$\text{s.t.} \quad \overline{r}^N = \frac{\pi^N}{P^N},$$

where π^N and P^N are the profit, and price of a non-listed firm. As non-listed firms do not disclose any information publicly, the household does not distinguish one non-listed firm from another.

Then, the household solves the following portfolio choice problem:

$$\max_{x(q),x^N} \quad \mathbb{E}(-e^{-\Lambda C})$$
$$\text{s.t. } C = \int x(\widetilde{q})\widetilde{r}(\widetilde{q})d\widetilde{q} + x^N\widetilde{r}^N, \quad \int x(\widetilde{q})d\widetilde{q} + x^N = a,$$

where $x(q)$ is the funding supply for firms with transparency level q, and x^N is the funding supply for non-listed firms. As the model does not include the inter-temporal decision of the household, all the payoffs from the equity investment are consumed. We assume the representative household has a large enough wealth a, as our interest is not in the household's constrained portfolio decision.

2.2 Technology

A measure one of the ex-ante homogeneous firms produces output using two inputs: tangible capital (k_T) and intangible capital (k_I). In this economy, there are two types of production technologies. One is listed firms' production technology, and the other is non-listed firms' production technology.

2.2.1 Production function of listed firms

A listed firm i operates using the following production function:

$$f^L(k_i^T, k_i^I, q_i; \overline{q}, \Phi^{ex}) = z(k_i^T)^\alpha(k_i^I(1 - \overline{q} - q_i))^\theta(\Phi^{ex})^\gamma,$$

where $\overline{q}$ is the mandated portion of intangible disclosure imposed by the policy maker, q_i is the voluntarily disclosed portion of intangible, Φ^{ex} is the shared intangible capital from all other firms, z is a constant aggregate productivity level, γ is the scale parameter for the externality, and α and θ are the tangible and intangible capital shares, respectively. We assume $\alpha + \theta + \gamma \leq 1$.

Importantly, we assume the revealed portion of intangible capital disappears from the private intangible stock. This assumption is to let the revealed intangible capital be symmetrically used between the disclosing firms and the free-riding firms without double counting. If this symmetry is not guaranteed, partial knowledge sharing needs to be specified, which requires an additional intensive margin in the shared information. We simplify the model by assuming pure symmetry to avoid such complications.

We assume that a listed firm's disclosed portion of intangibles can range from $\overline{q}$ to 1, which does not rule out the possibility of publicly sharing nearly all intangibles. Therefore, the intangible in this model does not include intellectual properties that are legally protected in terms of ownership. Therefore, we treat these assets as tangible assets.[16]

We assume a firm i's disclosed intangible q_i is perfectly substitutable by the other disclosed intangible. Therefore, the shared intangibles are aggregated in the following additive form:

$$\Phi^{ex} = \int_0^1 1_{\{i \in \text{Listed}\}} \times k_{I,i} \left(\underbrace{\overline{q}}_{\text{Disclosure mandated by the policy maker}} + \underbrace{q_i}_{\text{Voluntary disclosure}} \right) di.$$

A firm chooses first the voluntary disclosure level of the intangible before the operation. The choice problem of voluntary disclosure is elaborated on in the Section 3.[17] The ex-post profit of a firm with voluntary transparency q_i is obtained after taking out the operational costs $rk_i^T + pk_i^I$ from the revenue:

$$\pi(q_i; \overline{q}, \Phi^{ex}) := \max_{k_i^T, k_i^I} \quad z(k_i^T)^\alpha (k_i^I(1 - \overline{q} - q_i))^\theta (\Phi^{ex})^\gamma - rk_i^T - pk_i^I,$$

where r is the capital rental rate, and p is the $R\&D$ cost per unit of intangible capital.

[16] Given these assets are even used as collateral in reality, the exclusion of them from the definition of intangible is desired for the focus of this paper. In our quantitative, we calibrate the intangible share parameter based on the expenditures rather than the stock. Therefore, the protected intellectual assets, such as patent, do not significantly affect the main results.

[17] The assumption of timing is solely for the descriptive purpose. Even if the decision of input levels and the disclosure level occur simultaneously, the model stays unaffected.

For the notational brevity, we assume r and p already include the depreciation rates.

2.2.2 Production function of non-listed (private) firms

If a firm is private, it does not disclose the intangible capital publicly. The production function of a non-listed firm i is as follows:

$$f^N(k_i^T, k_i^I; \Phi^{ex}) = z(k_i^T)^\alpha (k_i^I)^\theta (\Phi^{ex})^\gamma.$$

Except for the disclosure of the intangible capital, the production function is assumed to take the same form and parameters as the one for the listed firms. The profit is also defined similarly to that of listed firms:

$$\pi^N(\Phi^{ex}) := \max_{k_i^T, k_i^I} \quad z(k_i^T)^\alpha (k_i^I)^\theta (\Phi^{ex})^\gamma - rk_i^T - pk_i^I.$$

2.3 Financial markets

In this section, we characterize the financial market in the model. The funding supply is driven by the representative household's portfolio choice problem. The funding demand is determined by each firm's value maximization problem. The listed market is comprised of submarkets indexed by the transparency level q.

Following proposition specifies the funding supplies of the household for listed firms and the non-listed firms.

Proposition 1 (Funding supply)**.**
The household's optimal funding supplies for listed firms with transparency q, $x^*(q)$, and for non-listed firms, x^{N*} are as follows:

$$x^*(q) = \frac{\pi(q)/P(q)}{\Lambda/(\xi + \psi(\overline{q} + q))}, \quad x^{N*} = \frac{\pi^N/P^N}{\Lambda/\xi}. \tag{1}$$

Proof. See Appendix G. ■

A manager of a firm chooses where to operate to maximize the firm's price. The price is interchangeable with the value of a firm. The decision problem of where to operate is characterized as follows:

$$\max\left\{ \max_{q\in[0,1-\overline{q}]} P(q), P^N \right\}.$$

Where $P(q)$ is the price of the firm operating in the listed market with the transparency level at q, and P^N is the price of a non-listed firm. In the equilibrium, firms become indifferent among all the options, which makes the non-listed and listed markets actively co-exist despite their different matching protocols.

In the funding market for the listed firms, the price of a firm, $P(q)$, is determined at the level where funding supply in the number of firms $\frac{x^*(q)}{P(q)}$ meets funding demand in the number of firms $\mathcal{M}(q)$. Thus, the market-clearing condition is as follows:

$$\frac{x^*(q)}{P(q)} = \mathcal{M}(q). \tag{2}$$

Recall that a manager needs to determine the transparency level after going on the listed market. The optimal q is determined at the level where price P is maximized. Using Equations (1) and (2), the price maximization problem can be translated into an ex-ante profit maximization form as in the right-hand side formulation of the following line:

$$\max_{q\geq 0} P(q) \iff \max_{q\geq 0} \sqrt{\frac{\pi(q)}{\Lambda \frac{\mathcal{M}(q)}{\xi+\psi(\overline{q}+q)}}} \iff \max_{q\geq 0} \quad \pi(q)\phi^L(q)$$

where $\phi^L(q) := \frac{\xi+\psi(\overline{q}+q)}{\mathcal{M}(q)}$ is the net funding intensity. The solution to this problem characterizes the funding demand in the listed market.

The price of a non-listed firm, P^N, is determined at the level where funding supply in the number of firms, x^{N*}/P^N, is matched with the demand in a frictional private equity market. Especially, we assume the congestion among non-listed firms generates attrition in the funding opportunity in the following way:

$$\frac{1}{\nu_N}\frac{x^{N*}}{P^N} = M_N,$$

where M_N is the total measure (number) of non-listed firms applying for the non-listed market's funding and $\nu_N > 1$ is a structural parameter that captures the congestion effect in the non-listed financial market. For analytical clarity, we assume that the total measure of matches is determined solely by the funding supply, x^{N*}/P^N. That is, none of the household's funding remains unused in the market in the equilibrium. From the perspective of the standard CRS matching function, this could be interpreted as both the elasticity of the non-listed match with respect to the household's funding supply and the matching efficiency being at unity. In contrast, firms participating in the non-listed market face costly matching due to the matching rate being lower than the unity $1/\nu_N$ caused by the congestion effect.

2.4 Equilibrium

Here we define an equilibrium where the economy is given total intangible capital reserve K^I (fixed aggregate intangible supply). This equilibrium endogenously determines the R&D cost of intangible capital p. The R&D cost is not a price for a trade. Instead, it is a cost that increases if all the other firms increase their spending in R&D. This captures the intuition that developing new knowledge is harder if more firms seek new knowledge. The rental rate for the tangible capital r is exogenously given.

Definition 1. *A collection of functions* $(k_T, k_I, q, \mathcal{M}, M_N, p, P, P^N, x^*, x^{N*}, \Phi^{ex})$ *is an equilibrium if*

1. $\left(x^*, x^{N*}\right)$ *solves the household's problem.*

2. $(k_T(q,\mathcal{M}), k_I(q,\mathcal{M}), q(\mathcal{M}))$ *solves the listed firm's problem.*

3. *The measure of listed firms choosing a transparency level* q *is consistent with* $\mathcal{M}(q)$ *for all* $q \in [0, 1-\overline{q}]$.

4. *The measure of non-listed firms is* M_N *and satisfies*

$$\int_0^{1-\overline{q}} \mathcal{M}(q)dq + M_N = 1.$$

5. *R&D cost of intangible capital* p *is determined by the following equation:*

$$K^I = \int_0^1 k_{I,i}di.$$

6. *Aggregate shared knowledge satisfies*

$$\Phi^{ex} = \int_0^1 1_{\{i \in Listed\}} \times k_{I,i}(\overline{q} + q_i)di.$$

7. *Financial market is cleared:*

$$\frac{x^*(q)}{P(q)} = \mathcal{M}(q) \quad and \quad \frac{1}{\nu_N}\frac{x^{N*}}{P^N} = M_N.$$

8. *Indifference in the extensive-margin decision:*

$$P(q) = P^N, \; for\; \forall q \in [0, 1-\overline{q}].$$

With the endogenously determined distribution $\mathcal{M}$ of firms for each q, we can re-write the market-clearing condition for intangible capital and the externality condition using $\mathcal{M}$. In the definition, each firm is aggregated along with index $i \in [0, 1]$. Instead, we aggregate firms over the distribution of firms at each q. This aggregation is doable since $\mathcal{M}$ is endogenously obtained, and k_I is also a function of q and $\mathcal{M}$. Therefore, we re-write those two conditions in the following way:

$$K^I = \int_0^{1-\overline{q}} k_I(q, \mathcal{M})\mathcal{M}(q)dq,$$

$$\Phi^{ex} = \int_0^{1-\overline{q}} k_I(q, \mathcal{M})(\overline{q} + q)\mathcal{M}(q)dq.$$

Among all possible equilibrium, we are interested in the non-degenerate equilibrium where all the homogeneous firms use mixed strategies over the transparency level q. The mixed strategy leads to the distribution of firms at each level of q. In the equilibrium, this distribution needs to be consistent with the distribution that a firm takes as a given state variable.

In the following section, we analytically characterize the equilibrium allocations in this economy.

3 Theoretical predictions

In this section, we analytically characterize the equilibrium allocations and study the model predictions.

Given a net funding intensity function, ϕ^L and the externality, Φ^{ex}, a listed firm's problem is characterized as follows:

$$\max_{q} \quad \overbrace{\left[\max_{k_T,k_I} \left(zk_T^{\alpha}(k_I(1-\overline{q}-q))^{\theta}(\Phi^{ex})^{\gamma} - rk_T - pk_I\right)\phi^L(q)\right]}^{\text{Interim problem}}$$
$$\text{s.t. } \phi^L(q) = \frac{\xi + \psi(\overline{q}+q)}{\mathcal{M}(q)}.$$

From the optimality conditions of the interim problem, we can derive the relationship among the transparency q, the regulation parameter $\overline{q}$, and the intangible capital k_I. The relationship is formally stated in the following proposition:

Proposition 2. *(Intangibles and the transparency)*
Given $\alpha + \theta < 1$, both q and $\overline{q}$ are negatively associated with intangible input k_I. Specifically,

$$k_I(q, \mathcal{M}; \overline{q}) = \left(\left(\frac{\alpha z(\Phi^{ex})^{\gamma}}{r}\right)^{\frac{1}{1-\alpha-\theta}} \left(\frac{r\theta}{p\alpha}\right)^{\frac{1-\alpha}{1-\alpha-\theta}}\right)(1-\overline{q}-q)^{\frac{\theta}{1-\alpha-\theta}}.$$

Proof.
See Appendix G. ■

If a firm is in a state where the knowledge has to be transparently revealed to the public, it naturally disincentivizes the firm to accumulate less knowledge. Therefore, the marginal increase in voluntary or mandatory transparency leads to a marginal decrease in the deployment of intangible capital stock. This result can be interpreted as positive correlation between the household's forecast error (variance) about the stock return of a firm and the firm's intangible share.

Corollary 1. *(Intangibles and the forecast error)*
Given $\alpha+\theta < 1$, the household's forecast error is positively associated with $k^I(q, \mathcal{M}; \overline{q})$.

Proof. The proof is immediate from Proposition 2, given that the forecast error of the household is $\frac{1}{\xi+(\overline{q}+q)\psi}$. ■

Then, from the optimality condition with respect to the transparency, q, we can characterize an ordinary differential equation (ODE) where the function of interest is the net funding intensity function $\phi(q)$. The ODE is specified in Appendix G. Solving the ODE, we characterize the transparency distribution $\mathcal{M}$ in the analytic form. We state the analytic form of $\mathcal{M}$ in the following proposition:

Proposition 3. *(Transparency distribution)*
The unnormalized probability density function $\mathcal{M}$ of transparency q has the following analytic form:

$$\mathcal{M}(q) = (\xi + \psi(\overline{q} + q))\,(1 - \overline{q} - q)^{\frac{\theta}{1-\alpha-\theta}}\,\frac{1}{\phi^N}.$$

Proof.
See Appendix G. ■

The following corollary establishes that the equilibrium distribution is unique for the given support of the transparency $[0, 1 - \overline{q}]$.

Corollary 2. *(Uniqueness of the transparency distribution)*
Given the support $[0, 1-\overline{q}]$, the equilibrium unnormalized probability density function $\mathcal{M}$ is unique.

Proof. The result is immediate from the uniqueness of the ODE solution that satisfies the boundary condition. ■

In the multiplicative form of the closed-form endogenous distribution in Proposition 3, each component is directly interpretable.[18]

$$\mathcal{M}(q) = \underbrace{(\xi + \psi(\overline{q} + q))}_{\text{funding supply}}\,\underbrace{(1 - \overline{q} - q)^{\frac{\theta}{1-\alpha-\theta}}}_{\text{funding demand}}\,\underbrace{\frac{1}{\phi^N}}_{\text{eq. normalizer}}.$$

The first component is the household's preference for transparent firms. For a higher q, the household is willing to provide greater funding to the firm (funding supply channel). This behavior generates an incentive for a firm to choose high q. In contrast, the second term captures firms' incentive to reveal less information (funding demand channel). This is consistent with the intuition that a greater revelation only benefits competitors at the firm's own cost. The third term is the equilibrium object that balances the measure of listed and non-listed firms. The first two elements in the analytical form are the key trade-offs in information disclosure that lead to a non-degenerate equilibrium distribution, which shares the similar theoretical properties of the equilibrium in Burdett and Judd (1983).

[18]It is worth noting that the endogenous distribution is independent of the productivity level z. Thus, the firm-level productivity heterogeneity does not matter in this setup. In the quantitative analysis, we normalize the productivity z at 1.

In the funding demand channel, the transparency terms interact with the intangible share θ. When the intangible share is higher, the negative effect of transparency on the transparency density through the funding demand channel becomes steeper. Intuitively, a greater reliance on the intangible, which is subject to spillover effect, makes firms more sensitively respond to the disclosure policy variation. This captures the interaction effect of the disclosure policy and the intangible share parameter on the aggregate transparency distribution. In the structural analysis, we analyze how the macroeconomic allocations' sensitivity to the disclosure policy changes over the θ variation around the estimated level of parameters.

The probability density function $\mathcal{M}(q)$ belongs to a variant of a well-known class of density functions: Beta distribution. In the following corollary, we prove that $\mathcal{M}(q)$ follows a shifted truncated beta distribution and provide the closed-form characterization of the net funding intensity of the private firms, ϕ^N. For brevity of notation, we define $B := \theta/(1-\alpha-\theta)$.

Corollary 3. *(Truncated normalized Beta distribution)*
The gross transparency, $y := q + \overline{q}$, follows a truncated normalized Beta distribution where the shape parameters are $B+1$ and 2, and the support is $[\overline{q}, 1]$.

$$q + \overline{q} \sim \frac{\mathbb{I}\{q \in [0, 1-\overline{q}]\}}{1 - M_N} \times Beta\left(B+1, 2\right),$$

where $B := \theta/(1-\alpha-\theta)$.

Proof.
See Appendix G. ■

It is worth noting that the probability density of q depends on the net funding intensity of non-listed firms, ϕ^N. This net funding intensity is determined by the following identity that requires the total measure of firms is unity:

$$\psi \frac{\nu_N}{\xi} M_N \int_0^{1-\overline{q}} \left(\frac{\xi}{\psi} + (\overline{q} + q) \right) (1 - \overline{q} - q)^B dq = 1 - M_N. \tag{3}$$

Equation (3) is the fundamental component of the model, which captures how the total measure of non-listed firms, M_N, behaves when the policy parameter $\overline{q}$ changes. After rearranging the terms, we obtain the analytic form of the measure of non-listed firms as stated in Proposition 4

Proposition 4 (Non-listed firms' measure)**.**
In equilibrium, the measure of non-listed firms M_N is as follows:

$$M_N = \frac{1}{1 + \psi\frac{\nu_N}{\xi}(1+\frac{\xi}{\psi})^{B+2}\mathcal{B}(B+1,2)F\left(\frac{1-\overline{q}}{1+\xi}; B+1, 2\right)} \tag{4}$$

where $\mathcal{B}$ is the beta function, and F is the cumulative distribution function of beta distribution.[19]

Proof.
See Appendix G. ■

Importantly, the analytic form of the non-listed firms' measure and the distribution of listed firms do not include either the price of the intangible or the externality. That is, the firms-level financing decision is independently determined from the general equilibrium effects and externality. The intuition behind this result is that both the productivity shift through the externality and the general equilibrium effect uniformly affect the operating profit of each firm, so they do not affect the decision of how to finance their operating activities.[20] This separation mimics the block-recursive nature of the dynamic equilibrium under the directed search (Menzio and Shi, 2010).

Due to this separation, a measure of private firm M_N is determined directly by Equation (4). M_N determines the funding intensity of private firm ϕ^N. Then, from Proposition 3, the distribution of firms over transparency is also independently determined from the general equilibrium effect and the externality. Therefore, the mandated transparency $\overline{q}$ and the intangible share θ affect the firm-level distribution without any feedback effects in the general equilibrium.[21] We establish the relationship between M_N and the structural parameters $\overline{q}$ and θ in the following proposition.

Proposition 5. *(The relationship between the measure of listed firms and the structural parameters)*
M_N strictly increases in $\overline{q} \in (0,1)$ and $\theta > 0$.

[19]The beta function is defined as follows:

$$\mathcal{B}(a,b) := \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)} = \frac{(a-1)!(b-1)!}{(a+b-1)!} = \int_0^1 x^{a-1}(1-x)^{b-1}dx.$$

[20]For the same logic, the heterogeneous firm-level productivity does not affect the analytic form in the current setup.

[21]Note that in Equation (4), $B = \frac{\theta}{1-\alpha-\theta}$ is the function of θ.

Proof.
See Appendix G. ■

As the importance of the intangible capital as an input factor increases, firms have greater incentive to conceal their competitive resource. Therefore, the increased intangible share leads to the greater portion of non-listed firms. On top of this, we show that the aggregate transparency in the economy shrinks in the intangible share in Proposition 6, which can be intuitively explained by the same reason.

Proposition 6. *(The relationship between the aggregate transparency and the intangible share)*
The aggregate transparency $\mathcal{T}$ decreases in θ, where $\mathcal{T} := \int_0^{1-\overline{q}} (q + \overline{q})\mathcal{M}(q;\theta)dq$.

Proof.
See Appendix G. ■

According to Proposition 5, as the policymaker requires a stricter disclosure regulation on financial information, the measure of non-listed firms increases. The reason is that the aggregate productivity gain from the shared information does not affect the firm's decision on the source of financing. As can be observed from Equation (4), the measure of non-listed firms is independent of the externality effect, Φ^{ex}.

The total measure of listed or non-listed firms, however, cannot solely serve as an objective of the disclosure regulation. The desired objective is stated clearly in the following mission of the SEC in the U.S.: "The mission of the SEC is to protect investors, maintain fair, orderly, and efficient markets, and facilitate capital formation."[22] Consistent with the view of the SEC, we investigate the effect of regulation on the investors' welfare, productivity, and output in the following section. The definitions of welfare, productivity, and output are available in Appendix F.

Despite the sharp theoretical predictions of the model, the magnitude of each channel's economic implication is subject to a quantitative specification. Also, there are interesting aspects of the model that are not analytically doable, such as the interaction between the intangible share and the disclosure policy. In Section 5, we discipline our model using the micro and macro-level data and quantitatively investigate the role of the intangible share and the disclosure policy on the macroeconomy and their interactions using the model.

[22]See "Our Goals," SEC website https://www.sec.gov/our-goals.

4 Cross-sectional empirical evidence

In this section, we describe cross-sectional evidence that links high reliance on intangible capital with the value of transparency and earning surprises. Specifically, we wish to test whether firms with high intangible capital are associated with lower transparency and higher forecast errors, which is the content of the prediction of Proposition 2 and its corollary.

We use firm level data on public U.S. firms from Compustat covering the period from 1985 to 2016 to measure firm-level intangible capital stock and other firm characteristics. We report the details on the measurement of internally generated intangible capital in the appendix. The data on earning surprises come from the I/B/E/S. The dataset collects quarterly estimates made by professional financial analysts on the future earnings of publicly traded companies. From there, we closely follow Dellavigna and Pollet (2009) for the definition and calculation of earnings surprises. Specifically, earnings surprise $ES_{i,j,t}$ is defined as the difference between a firm's announced actual earnings per share $e_{t,i}$ and the earnings forecast per share $\epsilon_{i,j,t}$ made by an analyst for that firm, normalized by the price of a share $P_{i,t}$:

$$ES_{i,j,t} := \frac{\epsilon_{i,j,t} - e_{i,t}}{P_{i,t}}$$

where t is the indicator of a quarter; i and j are firm and analyst indicators, respectively. Thus, the surprise is measured at the analyst-firm level.

Since we do not observe transparency directly, we use our available data on forecasts to define two different proxies for the transparency of the firm. Our idea is to proxy transparency by the dispersion and accuracy of earnings surprises, and is substantiated by research in the accounting literature that finds that analysts' forecast agreement and accuracy are positively related to the levels of disclosure of the company (see, for example, Lang and Lundholm (1996)) and analysts' earnings forecasts are less accurate when firms issue less readable 10-Ks (Lehavy, Li, and Merkley, 2011).

Our first proxy is the inverse of the variance of earnings surprises for a firm in a given quarter:

$$Transparency^1_{i,t} := \frac{1}{var(ES_{i,j,t})}$$

The intuition behind this proxy is that more transparent firms have lower dispersion (disagreement) in the earnings surprise among the analysts, on average.[23]

The second is the inverse of distance between firm earnings from the consensus analyst forecast, i.e. the median forecast among all the analysts:

$$Transparency^2_{i,t} := \frac{1}{median(|ES_{i,j,t}|)}$$

This proxy is based on the hypothesis that more transparent firms have lower absolute earnings surprise, on average.

We then run the following regression on our baseline sample, which includes all firms in Compustat from 1985 to 2016 for which information on earnings forecasts by at least two (for the first proxy) or one (for the second proxy) analysts is available:

$$\log \mathrm{y}_{i,t} = \theta_t + FEs + \beta \times \text{Intangible over total assets}_{i,t} + \gamma \times X_{i,t} + \varepsilon_{i,t}$$

where $y_{i,t}$ is either our first or second transparency measure. θ_t are year fixed effects and FEs include either industry or firm fixed effects. $X_{i,t}$ represents firm controls.[24] The firm-level controls include book-to-market ratio, sales, liquid capital (cash, inventory, and receivables), leverage (total debt over total asset), employment in logs, age (from the IPO year), and the number of analysts. Intangible, sales, and liquid capital are normalized by total asset. Since we wish to rule out that firms are becoming increasingly less transparent or more difficult to forecast over time due to either a gradual worsening of analysts' ability and effort, analysts' coverage, or common changes in idiosyncratic and aggregate risk, we include year fixed effects and control for the number of analysts covering a given firm.

Table 1 reports the results for the coefficient on intangible capital asset ratio. We report the full regression table in Appendix C. The regressions show that indeed intangible capital and transparency are inversely related. Specifically, an increase of one percentage point in the intangible capital over assets ratio decreases the value of the first transparency by 0.64 percent, and the value of the second transparency measure by 0.31 percent, and the effect resists the inclusion of firm fixed effects.

[23]This proxy is therefore calculated only for firms with multiple analysts' forecasts available in the data. In our dataset, the average number of analysts covering a firm is three.

[24]Firm-level controls and regression specifications are based on Li (2010) and Bird, Karolyi, and Ruchti (2017).

Table 1: Regression of transparency proxies on intangibles

	Transparency 1		Transparency 2	
	(1)	(2)	(3)	(4)
Intangible	-.6386	-.3117	-.3191	-.1529
	(.0871)	(.0971)	(.0414)	(.0497)
Year FE	✓	✓	✓	✓
Industry FE	✓		✓	
Firm FE		✓		✓
Adj. R^2	0.295	0.649	0.289	0.634
Observations	78878	77944	76959	76014

Notes: This table reports the estimates of the coefficients from the following regression using our baseline sample, which includes all firms in Compustat from 1985 to 2016 for which information on earnings forecasts by at least two (for the first proxy) or one (for the second proxy) analysts is available:

$$\log y_{i,t} = \theta_t + FEs + \beta \times \text{Intangible capital over total assets}_{i,t} + \gamma X_{i,t} + \varepsilon_{i,t}$$

where $y_{i,t}$ is either the inverse of variance of earning surprises when more than one analyst forecast is present, or the inverse absolute value of earning surprises from the consensus. θ_t are year fixed effects and FEs include either industry or firm fixed effects. $X_{i,t}$ represents firm controls. Standard errors are clustered at the industry and year level.

The corollary is easily testable using our data. We estimate the regression:

$$\log \mathrm{y}_{i,t} = \theta_t + FEs + \beta \times \text{Intangible over total assets}_{i,t} + \gamma \times X_{i,t} + \varepsilon_{i,t}$$

where $y_{i,t}$ is the absolute value of earning surprises for each firm. A positive β indicates that firms with more intangible capital are harder to forecast. We report the results in Table 2 and the full table in the appendix.

Finally, we interpret the result in the following way. Given the inclusion of year fixed effects and the number of analysts covering a given firm, we can exclude the effect of a gradual worsening of analysts' ability and effort, analysts' coverage, and common changes in idiosyncratic and aggregate risk. Therefore, we can directly link the rise in intangible capital with a decline in the ability of the market to forecast a firm. This relationship can be due to two reasons: one, firms with high intangible intensity tend to be less transparent, and, therefore, more difficult to forecast. Two, it may be that, given a certain level of disclosure and transparency, firms with high intangible intensity are inherently more challenging to forecast due to their nature. We include both possibilities in our model and set out to disentangle the two effects

Table 2: Regression of forecast accuracy on intangibles

	Earnings surprises (absolute value)	
	(1)	(2)
Intangible	.3191	.1529
	(.0414)	(.0497)
Year FE	✓	✓
Industry FE	✓	
Firm FE		✓
Adj. R^2	0.289	0.634
Observations	76,959	76,014

Notes: This table reports the estimates of the coefficients from the following regression using our baseline sample, which includes all firms in Compustat from 1985 to 2016 for which information on earnings forecasts by at least one analyst is available:

$$\log y_{i,t} = \theta_t + FEs + \beta \times \text{Intangible capital over total assets}_{i,t} + \gamma X_{i,t} + \varepsilon_{i,t}$$

where $y_{i,t}$ is the absolute value of earning surprises. θ_t are year fixed effects and FEs either industry or firm fixed effects. $X_{i,t}$ represents firm controls. Standard errors are clustered at the industry and year level.

using our structural estimation.

5 Structural analysis

Using the model we developed in the theory section, we conduct a quantitative analysis of the macroeconomic effects resulting from the increasing significance of intangible assets and the impact of information regulation policies. We estimate our model using data from two distinct periods. The first period, spanning from 1992 to 1996, serves as our baseline, while the second period, from 2012 to 2016, is considered as the post-change period. As our model is static, we cannot examine the dynamic response that may have occurred immediately after a change in structural parameters. Therefore, we compare a period just before the year of the dramatic shift in the number of listed firms with a period several years after the change to assume that it has reached a stationary level.

5.1 Estimation

In this section, we elaborate on how we fit the firm-level data into the model. The core parameters to be estimated are the following:

$$\{\overline{q}, \theta, \xi, \psi, \nu_N\},$$

where $\overline{q}$ is the mandated transparency of disclosure; θ is the intangible capital share; ξ is the baseline information level a household has about both listed and non-listed firms, ψ is the transparency's contribution to the household's information about listed firms; and ν_N is the efficiency parameter of the private equity market.

To generate our baseline estimates, we match the average target moments between 1992 and 1996. For the estimates of the post-change periods, we match the average target moments between 2012 and 2016. The target moments and simulated moments are reported in Table 3. The parameter $\overline{q}$ is identified based on the adjusted fraction of listed firms out of the total number of firms with more than 100 employees. To account for mergers and acquisitions (M&As) by another public firm (Doidge, Karolyi, and Stulz, 2017) we adjusted the target fraction of listed firms. Starting from 1975, we sequentially updated the exit rate, which is the number of delisting firms minus M&As, over the number of listed firms, plus new entries and minus M&As. Our adjustments show that the total drop in listed firms was about 52%, but after accounting for M&As, the drop is only 31%. This means that the adjusted fraction of listed firms went from 11.08% in the baseline period to 7.60% in the post-change period. Regarding the share of intangible capital, θ is identified from the intangible to tangible ratio.

Since in the model the households form a belief on a stock return that follows a normal distribution:

$$\widetilde{r}(q) \sim N\left(\overline{r}(q), \frac{1}{\xi + \psi(\overline{q} + q)}\right).$$

Analysts' forecast dispersion is a natural data counterpart to the dispersion in the ex-ante stock return. Specifically, earnings surprise is defined as:

$$ES(q) := \overline{r}(q) - \widetilde{r}(q) \sim N\left(0, \frac{1}{\xi + \psi(\overline{q} + q)}\right).$$

Hence, in our analysis, we identify ψ using the average standard deviation value of the returns of all firms, while ξ represents the equivalent value for the top 1% opaque firms. We assume that opaqueness in non-listed firms is comparable to that of the top 1% of opaque listed firms, which allows us to identify ξ. Lastly, ν_D for the baseline period is identified using the 30% fraction of private firms that get funded, and for the post-change period, we use the 4 percentage points estimate of improvement in the private equity market friction following Ewens and Farre-Mensa (2020).

We use the method of simulated moments to estimate the parameters. The weight matrix is chosen to be an identity matrix. However, the choice of the weight matrix is not an issue in our estimation, as the parameters are exactly identified at the level where the level of moments is exactly matched.

Table 3: Fitted moments

Moments	Data	Model	Reference
Baseline (1992 $\sim$ 1996)			
Fraction of listed after M&A adj. (%)	11.08	11.08	Compustat & BDS
(*cf. without M&A adj. (%)*)	(8.30)		
Intangible Exp./Sale (%)	2.906	2.906	Compustat
Average $sd(\widetilde{r})$ (%)	12.53	12.53	Compustat & I/B/E/S
Average $sd(\widetilde{r})$ of top 1% (%)	25.52	25.52	Compustat & I/B/E/S
Portion of funded non-listed firms (%)	30.30	30.00	Ewens and Farre-Mensa (2020)
Post-change (2012 $\sim$ 2016)			
Fraction of listed after M&A adj. (%)	7.60	7.60	Compustat & BDS
(*cf. without M&A adj. (%)*)	(4.01)		
Intangible Exp./Sale (%)	5.356	5.356	Compustat
Average $sd(\widetilde{r})$ (%)	28.00	28.00	Compustat & I/B/E/S
Average $sd(\widetilde{r})$ of top 1% (%)	84.81	84.81	Compustat & I/B/E/S
Portion of funded non-listed firms (%)	34.30	34.00	Ewens and Farre-Mensa (2020)

Table 4 reports the estimated parameters. In the post-change period, the estimated mandated transparency parameter, $\overline{q}$, slightly increased, indicating that information regulation has become stricter, consistent with the intended direction of the reform. The share of intangible assets, θ, has increased by approximately 50%, reflecting the significant rise in the importance of intangible input. The baseline information level a household has about both listed and non-listed firms, ξ, has decreased substantially, and the transparency's contribution to the household's information about the listed firms, ψ, has decreased, both changes indicating an increase in the return variance on both listed and non-listed markets. Furthermore, the friction parameter ν_N has decreased, indicating an improvement in the private equity market, which reflects the impact of the National Securities Markets Improvement Act of 1996 (Ewens and Farre-Mensa, 2020).

Table 4: Estimated parameters

Param.	Description	Baseline (1992 ∼ 1996)	Post-change (2012 ∼ 2016)
$\bar{q}$	Mandated transparency	0.981	0.995
θ	Intangible share	0.029	0.054
ψ	Transparency's contribution to public info.	38.539	11.394
ν_N	PE market friction	3.300	2.915
ξ	Baseline information level	25.520	1.390

Besides the estimated parameters, we fix the following parameters before the estimation:

$$\{\alpha, \gamma, K^I\}.$$

Capital share, α, is set to be 0.30.[25] The public intangible share, γ, is assumed to be equal to the private intangible share, θ. The total intangible capital stock, K^I, is normalized to 1.

Figure 2 shows the non-normalized distribution of listed firms over the transparency level for the baseline and the post-change periods. The distribution shrinks in the post-change period due to the reduced number of listed firms, and shifts leftward, indicating a decrease in the average transparency level.

[25]Because our model is abstract from a labor input, the capital share in the model needs to be interpreted as an after-labor-adjustment capital share, as in the following formulation:

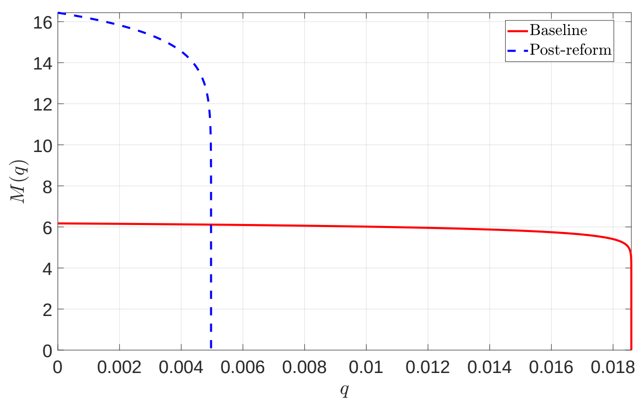
$$\begin{aligned} Ak^{\alpha} &= \max_L \widetilde{A}k^{\widetilde{\alpha}}L^{\epsilon} - wL \\ &= (1-\epsilon)\widetilde{A}^{\frac{1}{1-\epsilon}}\left(\frac{\epsilon}{w}\right)^{\frac{\epsilon}{1-\epsilon}}k^{\frac{\widetilde{\alpha}}{1-\epsilon}} = Ak^{\frac{\widetilde{\alpha}}{1-\epsilon}}, \end{aligned}$$

where $A = (1-\epsilon)\widetilde{A}^{\frac{1}{1-\epsilon}}\left(\frac{\epsilon}{w}\right)^{\frac{\epsilon}{1-\epsilon}}$. Therefore, our model's α is equivalent to a standard model's $\frac{\widetilde{\alpha}}{1-\epsilon}$. We assume $\widetilde{\alpha} = 0.12$, and $\epsilon = 0.6$, leading to $\alpha = 0.30$.

Table 5: Fixed parameters

Parameters	Description	Value
α	Capital share	$0.30 - \theta$
γ	Public intangible share	$= \theta$
r	Rental rate tangible capital plus depreciation	0.10
K^I	Total intangible supply	1
z	TFP level	1

Figure 2: Distribution of listed firms over transparency



5.2 Decomposition analysis

In this section, we calculate the average contributions of each parameter to the decrease in the measure of listed firms and the decrease in the average transparency. We obtained these contributions by first keeping the estimated parameters at their baseline values and changing only one parameter to its post-change value to obtain the counterfactual measure of listed firms and average transparency if only that specific parameter changed. Second, we kept the estimated parameters at their post-change values and changed only one parameter to its baseline value to obtain the counterfactual measure of listed firm and average transparency if only that specific parameter remained at the baseline value. We performed this calculation for all five estimated parameters and then averaged both numbers from each parameter to obtain the average contributions to the decrease in the measure of listed firms and the decrease in the average transparency. Table 6 reports the results of the decomposition analysis in annualized percentage.[26]

Table 6: Decomposition of the channels in the macroeconomic changes

		Contribution to the change (p.a.):			
Param.	Channel	#listed	transparency	productivity	welfare
	Total change	-1.88	-1.85	-0.42	-1.42
$\overline{q}$	SEC regulation	-6.22	-6.18	-0.25	0.20
θ	Rising intangible share	-0.89	-0.89	-0.37	-0.81
ψ	Harder to forecast public firms	-3.72	-3.72	-0.16	0.16
ν_N	PE market friction	-0.56	-0.56	-0.02	-0.59
ξ	Baseline information level	8.62	8.62	0.34	-0.92

Table 6 presents the results of a decomposition analysis examining the factors contributing to the observed decline in the number of listed firms over the past two decades. The analysis reveals that the percentage of listed firms decreased from 11.08% in the baseline period to 7.60% in the post-change period, representing a 31% drop over 20 years, with an average annual change of -1.88. Furthermore, transparency, productivity, and welfare have also exhibited annual changes of -1.85, -0.42, and -1.42, respectively.

The decomposition analysis identifies several factors contributing to the observed decline in the number of listed firms. Specifically, the stricter SEC regulation accounted for the majority of the change, contributing -6.22 percentage points. The rising share of intangible capital contributes to the declining transparency through two channels. One is through the direct effect of the firms' declining willingness for transparent disclosure, and the other is through the transparency's contribution to listed firms' information ψ. Each intangible channel contributed -0.89 and -3.72 percentage points, marking the intangible share as the second most important factor for the observed decline of the listed firms. We obtain similar decomposition outcomes for the observed declining transparency.

On the contrary, the decline in the household's baseline information level about listed and non-listed firms contributed positively to the changes by 8.62 percentage points. This is because the declined information level makes the household provide little funding to the non-listed market, which makes the firms tend to go listed.

[26] The two periods of comparison are 20 years apart from each other. So, we annualized the total change by a division of 20.

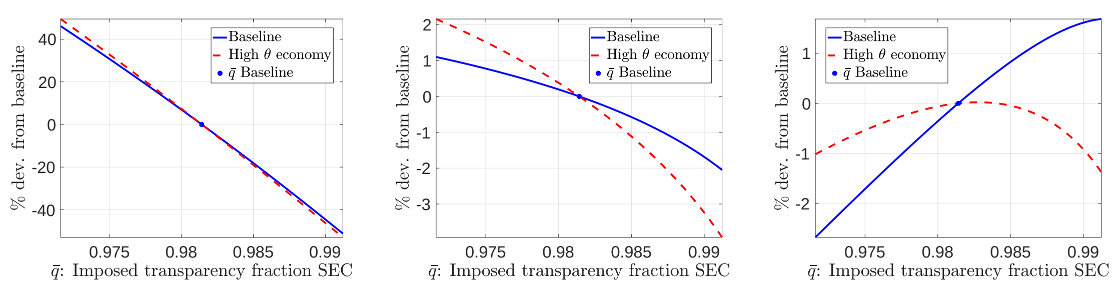
Overall, the results suggest that the key drivers of lowered transparency, productivity, and welfare are the stricter SEC regulation and the increased share of intangible capital. These results provide valuable insights for policymakers and market participants seeking to understand the underlying factors contributing to the decline in the number of listed firms and the associated macroeconomic implications.

5.3 Disclosure policy and the intangible share

In this section, we analyze how the intangible share affects the macroeconomic allocations' sensitivity to the disclosure policy variation. When the intangible share varies, individual firms' incentive to reveal the information and decision to go listed or non-listed are changed (Proposition 5 and 6). On top of this direct effect, the intangible share change affects the macroeconomy through the channel of the effectiveness of disclosure policy.

Figure 3 plots the level of macroeconomic allocations in the vertical axis (in % deviation from the baseline level) at different policy $\bar{q}$ in the horizontal axis for baseline economy and the high intangible share economy: Panel (3a) is for the measure of listed firms, Panel (3b) is for the aggregate productivity, and Panel (3c) is for welfare. The high intangible share economy is based on the post-change estimate of θ, while the other parameters are at the baseline level.[27]

Figure 3: Macro-level sensitivities to the disclosure policy changes: Baseline vs. High intangible share economy



(a) Measure of listed firms (b) Productivity (c) Welfare

When, $\bar{q}$ increases, the measure of listed firms sharply declines, as can be seen from Panel (3a). This negative response becomes more sensitive in a high θ economy,

[27]For the high θ economy, % deviation means % deviation from the economy with the post-change θ and the other parameters at the baseline.

as captured in the steeper downward curve. However, the magnitude of the change in the slope is not as stark as the ones observed in other responses. Productivity significantly declines in $\overline{q}$, and it decreases faster in high θ economy (Panel (3b)). In the high θ economy, the firm-level information disclosure policy more sensitively responds to the policy variable, leading to a significantly more dampened externality effect for the same policy change. Lastly, the sensitivity of welfare to the policy change nonlinearly responds to θ change. In the baseline, welfare strictly increases in $\overline{q}$, locally around the neighborhood of the estimate.[28] However, in the high θ economy, the accelerated productivity loss over the strengthened policy makes the welfare curve bend down even for a small positive variation in $\overline{q}$ than in the baseline. Therefore, the welfare curve displays inverted U shape around the estimated level of $\overline{q}$, so the welfare-maximizing level of the policy becomes significantly closer to the estimated level.

5.4 Optimal Policies

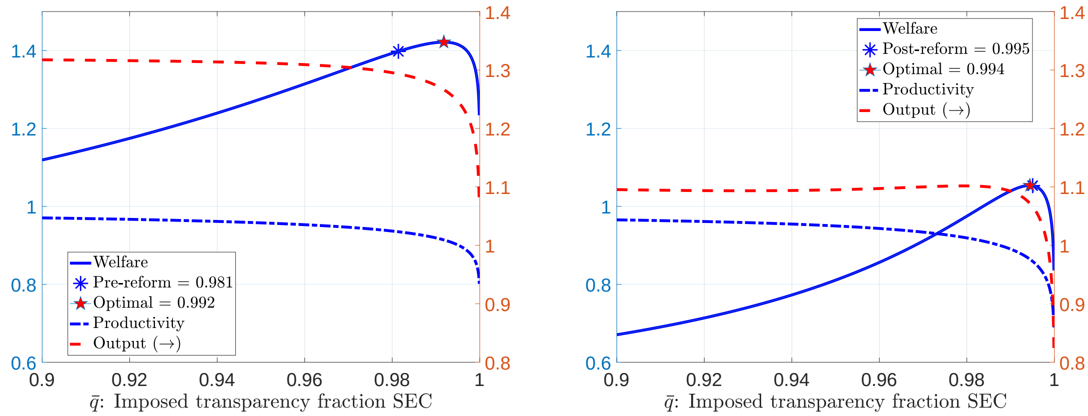
In this section, we use the proposed model to analyze the optimal level of imposed transparency for welfare maximization. As shown in the previous section, the policy maker can choose the imposed transparency level $\bar{q}$. However, since welfare is obtained from the utility maximization problem of the household, $\bar{q}$ will have two effects on welfare. On the one hand, lower imposed transparency increases the measure of listed firms that will have more access to finance relative to private firms, increasing output and consumption. On the other hand, lower imposed transparency also increases the portfolio return's uncertainty, lowering the welfare of the risk-averse household. Hence there is a trade-off between the level of consumption and its uncertainty. In Figure 4, we show the Laffer-type curve for the transparency policy for both periods: Panel 4a is for the baseline period, and Panel 4b is for the post-change period.[29]

The estimated level of transparency in the pre-change period is 0.981 (Table 4) and the optimal level is 0.992, suggesting the mandated transparency was below the optimal level in the pre-change period. In the post-change period estimation, the results suggest that both the estimated and the optimal level of transparency increased to 0.994 and 0.995, respectively (Table 4). It is worth mentioning that output and

[28]Globally, the welfare curve is the inverted U shape.

[29]Note that Panel 4b is based on the post-change parameter estimates, while Panel 3c in Figure 3 is based on the post-change θ estimate and other parameters at the baseline.

Figure 4: Optimal level of mandated transparency



(a) Baseline estimation

(b) Post-change estimation

productivity are also non-monotonic with respect to the imposed transparency level.[30] This property of the model suggests that depending on the value of the estimated parameters, moving $\bar{q}$ towards the welfare-optimal point could increase both output and productivity as well, achieving a *divine coincidence*. With the current estimated parameters, such a *divine coincidence* happens when $\bar{q}$ is above the welfare optimal point: Decreasing $\bar{q}$ toward the optimal would increase welfare, output, and productivity.

6 Concluding remarks

This paper analyzes the driving forces of the disappearing listed firms, and rising opacity of the disclosed balance sheets, and the macroeconomic consequences through the lens of a general equilibrium model. In our model, the household determines the funding level for non-listed and listed markets depending on the transparency of disclosure, and firms determine which market to operate in and the transparency level. The policymaker's disclosure regulation parameter is considered in the model, which allows an equilibrium-based analysis of the disclosure policy. In the equilibrium, a listed market based on directed search and a non-listed market based on random match endogenously co-exist. Notably, the model allows the analytical characterization of a

[30]Figure 4 shows only the region where output and productivity decrease monotonically with respect to $\bar{q}$.

rich set of equilibrium allocations.

Using the model, we theoretically show that a stricter disclosure regulation leads to fewer listed firms. Also, a greater intangible share leads to a lower willingness for transparent disclosure. Using the estimated model, we show that the stricter disclosure regulation and the rising intangible share mainly drive the recently observed macroeconomic trends we document. Then, we quantify the macroeconomic implications of the observed trends. According to the estimated model, overall trends have led to a 0.42 percentage point productivity loss annually due to the reduced knowledge spillover and a 1.42 percentage point annual welfare loss. The stricter regulation has helped mitigate the welfare loss through the transparent information disclosure of the listed firms.

Our approach broadens the scope of structural policy analysis to the regulation of information disclosure. According to our policy analysis, the recent change in the disclosure regulation almost achieved the optimum with respect to the welfare criterion. Still, the policy change has intensified the productivity loss, as the change made it costlier for firms to stay in the listed market.

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Appendix: For online publication

Contents

A Data and measurement

In this section, we explain how we measure the intangible capital stock of public firm.

We use firm level data on public U.S. firms from Compustat covering the period from 1985 to 2016 to measure firm-level intangible capital stock. Our baseline measure of internally generated intangible capital is the sum of two components: (i) estimated knowledge capital, calculated using research and development expenditure (XRD); and (ii) estimated organizational capital, calculated using selling, general, and administrative expenses (XSGA). The measure is constructed using the perpetual inventory method, which aggregates net investment flows over the life of the firm:

$$\begin{aligned}
\text{[Knowledge capital]}: \quad & k^G_{i,t} = (1-\delta_G)\, k^G_{i,t-1} + R\&D_{it}, \\
\text{[Organizational capital]}: \quad & k^O_{i,t} = (1-\delta_O)\, k^O_{i,t-1} + \gamma_O SG\&A_{it},
\end{aligned}$$

where $R\&D$ is research and development expenditure expenditure; $SG\&A$ is selling, general, and administrative expenses. All the intangible flow variables are deflated by the price of intellectual property products from National Income and Product Accounts data (NIPA Table 1.1.9, line 12). δ_G and δ_O are the depreciation rates.[1] γ_O is the fraction of selling,general, and administrative ($SG\&A$) expenditure that adds to the intangible capital stock. We assume $\gamma_O = 0.20$ following Falato et al. (2022). All the empirical results are robust over other reasonable choices of this parameter level.

Then, we calculate the net change in the acquired amount of intangibles from changes in the book values of intangibles after the amortization, using Compustat variables INTAN and AM. We obtain the acquired intangible stock $k^B_{i,t}$, applying the perpetual inventory method to the deflated net change in the intangibles.

[1] We use $\delta_G = \delta_O = 0.15$, which is around the levels estimated in the literature (Corrado, Hulten, and Sichel, 2009).

Our final measure of firm-level intangible capital stock $k^I_{i,t}$ is obtained by combining the internally generated intangible stocks and the acquired intangibles stocks:

$$k^I_{i,t} = k^G_{i,t} + k^O_{i,t} + k^B_{i,t}$$

B Industry-level analysis and additional figures

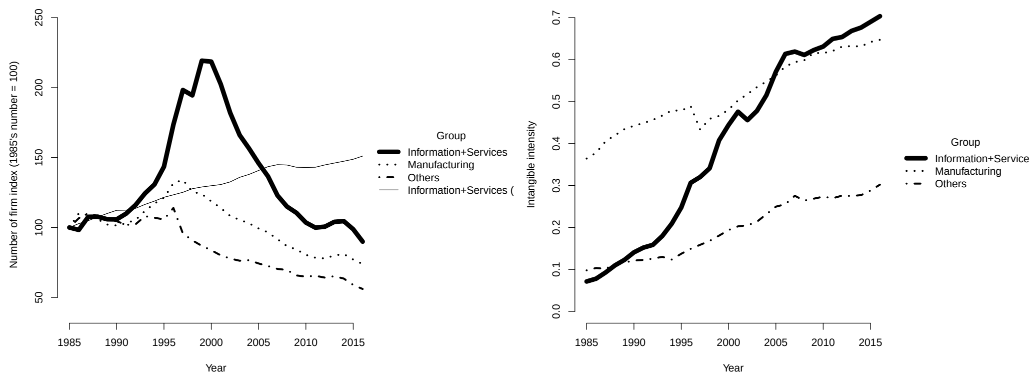
Panel B.1a shows the trend in the number of firms for the Information and Services sector, excluding trade and transportation, Manufacturing, and other sectors (Trade and transportation, Agriculture and Mining, Construction). All sectors show an initial increase and then a decline after the mid-nineties, and the decline is much more pronounced in the information and service sector. We also plot the normalized number of all non-listed firms in the information and service sector: as can be seen, only listed firms are affected by the large decline during the entire period of 2000 and 2010 (well after the dot-com bubble), while the overall number is only slightly affected by the 2001 and 2008 recessions. Panel B.1b shows the *intangible intensity*, defined as the ratio of intangible asset to total intangible and tangible asset values, for the same industries. Manufacturing had historically a higher intangible intensity, which has been taken over in the early 2000s by the service sector.

Finally, Figure B.2 shows our transparency measures for the information and service sector, compared to all other sectors. The information and service sector has a lower transparency over the entire period, and both time series of transparency for all sectors have also declined over time.

Two main take-aways can be taken by the analysis of industry trends: the information and services sector has seen the largest increase in its intangible intensity, and at the same time the largest decline in the number of listed firms and in transparency.

We finally show the trends for more disaggregated industries. We also report the trends in intangible capital using internally generated R&D only, so that the numbers on intangible intensity can be compared to the ones available in the Bureau

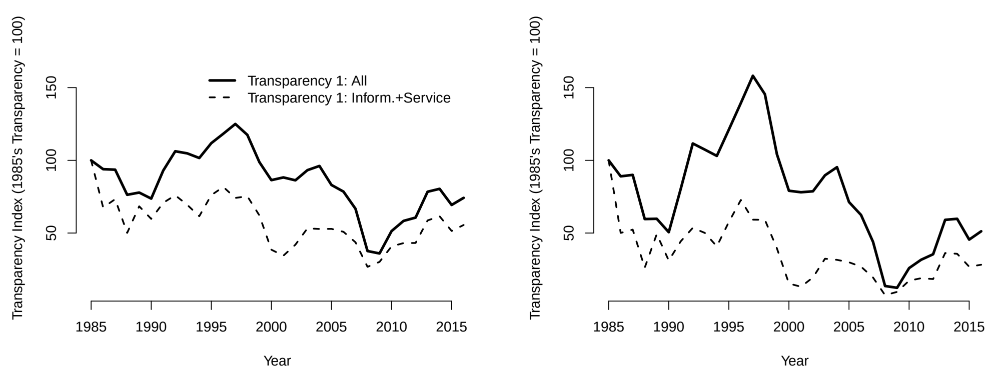
Figure B.1: Number of listed firms and intangible intensity by industry.



(a) Number of listed firms (b) Intangibles intensity

Notes: This figure shows the trend in the number of listed firms and intangible capital intensity in the U.S. Intangible intensity is defined as the ratio of intangible asset to total intangible and tangible asset values. The groups are defined as Information and Services, excluding trade and transportation, Manufacturing,and other sectors (Trade and transportation, Agriculture and Mining, Construction). Data comes from Compustat. See Appendix for details on measurement.

Figure B.2: Time series of transparency for information and service industries.



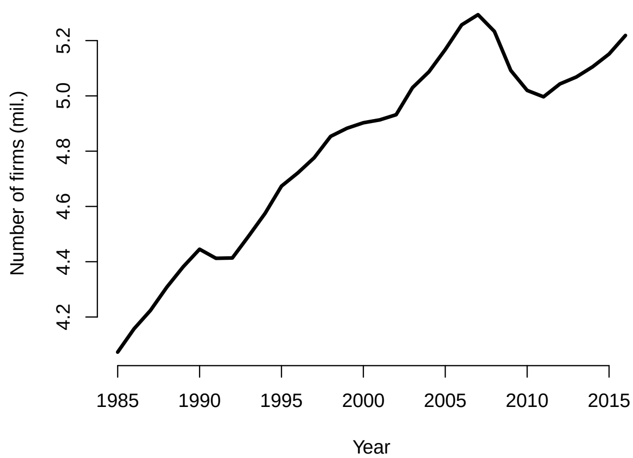
(a) Transparency 1 (b) Transparency 2

Notes: This figure shows the trend in in transparency for information and service industries compared to all other industries. Information and Services excludes trade and transportation. Data comes from Compustat and I/B/E/S. See Appendix for details on measurement.

of Economic Analysis (BEA).

Figure A.1 plots the time series of the total number of firms in the united states. In contrast to the declining trend of the number of listed firms, the total number of firms has been steadily rising.

Figure B.3: Number of all firms

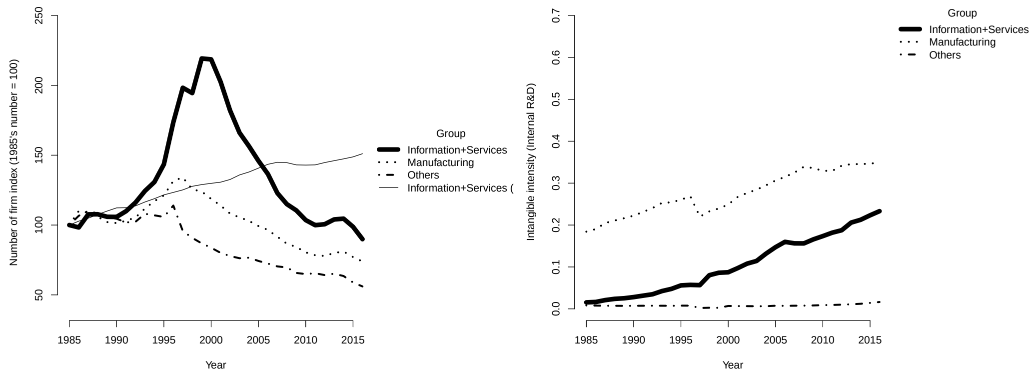


Notes: This figure plots the time series of the number of all firms using the Business Dynamics Statistics (BDS) data from the U.S. Census Bureau.

Figure A.2 plots the time series of (a) the number of listed firms and (b) the intangible intensity measured by using R&D only for multiple industries. Panel (a) shows the industry and service industries have displayed the most dramatic declines in recent years in terms of the number of listed firms. However, the total number of firms in these industries (solid thin line) has steadily risen over the sample period. Panel (b) shows that the information and service industries have shown a fast increase in reliance on intangible assets. However, compared to Figure 2, the difference between the information and service industries and the manufacturing industry is not as drastic.

Figure A.3 plots the time series of the number of listed firms for different industries. Specifically, we report the trends for (a) natural resources and mining, (b)

Figure B.4: Number of listed firms and intangible intensity by industry (internal R&D only).



(a) Number of listed firms (b) Intangibles intensity

Notes: This figure shows the trend in the number of listed firms and intangible capital intensity in the U.S. Intangible intensity is defined as the ratio of intangible asset, excluding acquired intangible and organizational capital, to total intangible (again excluding acquired intangible and organizational capital) and tangible asset values.The groups are defined as Information and Services, excluding trade and transportation, Manufacturing,and other sectors (Trade and transportation, Agriculture and Mining, Construction). Data comes from Compustat. See Section 2.1 for details on measurement.

construction, (c) manufacturing, trade, (d) transportation, and utilities, (e) information, (f) professional and business services, (e) education and health services, and (f) leisure and hospitality industries.

Figure A.4 plots the time series of the intangible intensity of listed firms for different industries. Specifically, we report the trends for (a) natural resources and mining, (b) construction, (c) manufacturing, trade, (d) transportation, and utilities, (e) information, (f) professional and business services, (e) education and health services, and (f) leisure and hospitality industries.

Figure A.5 plots the time series of the transparency measures for all firms and for only survivors. Here, the survivor is ex-post conditioned by the firms of which observations are available at the end of the sample period.

Figure B.5: Trends in the number of public firms by industry

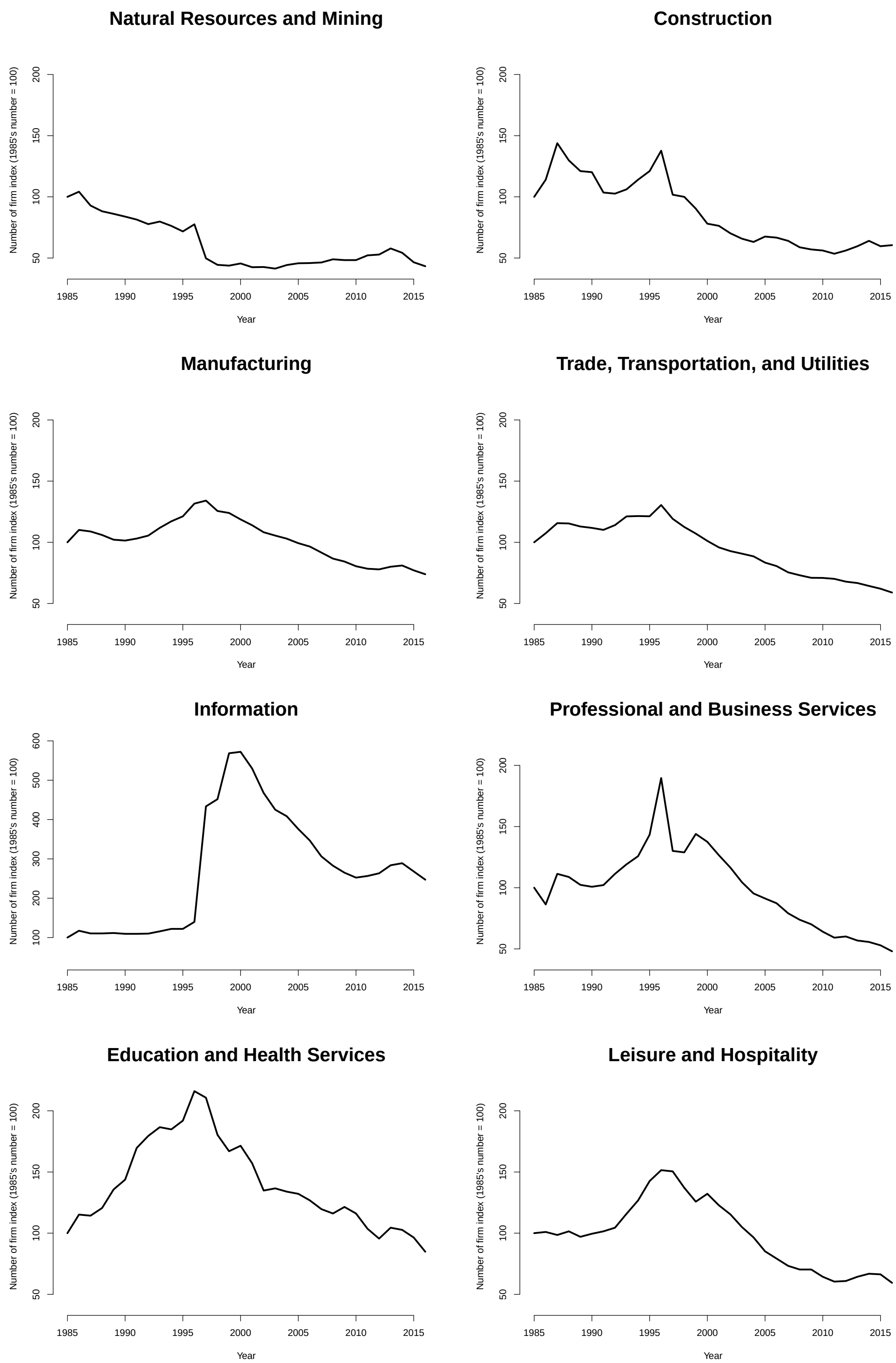


Figure B.6: Trends in intangible intensity by industry

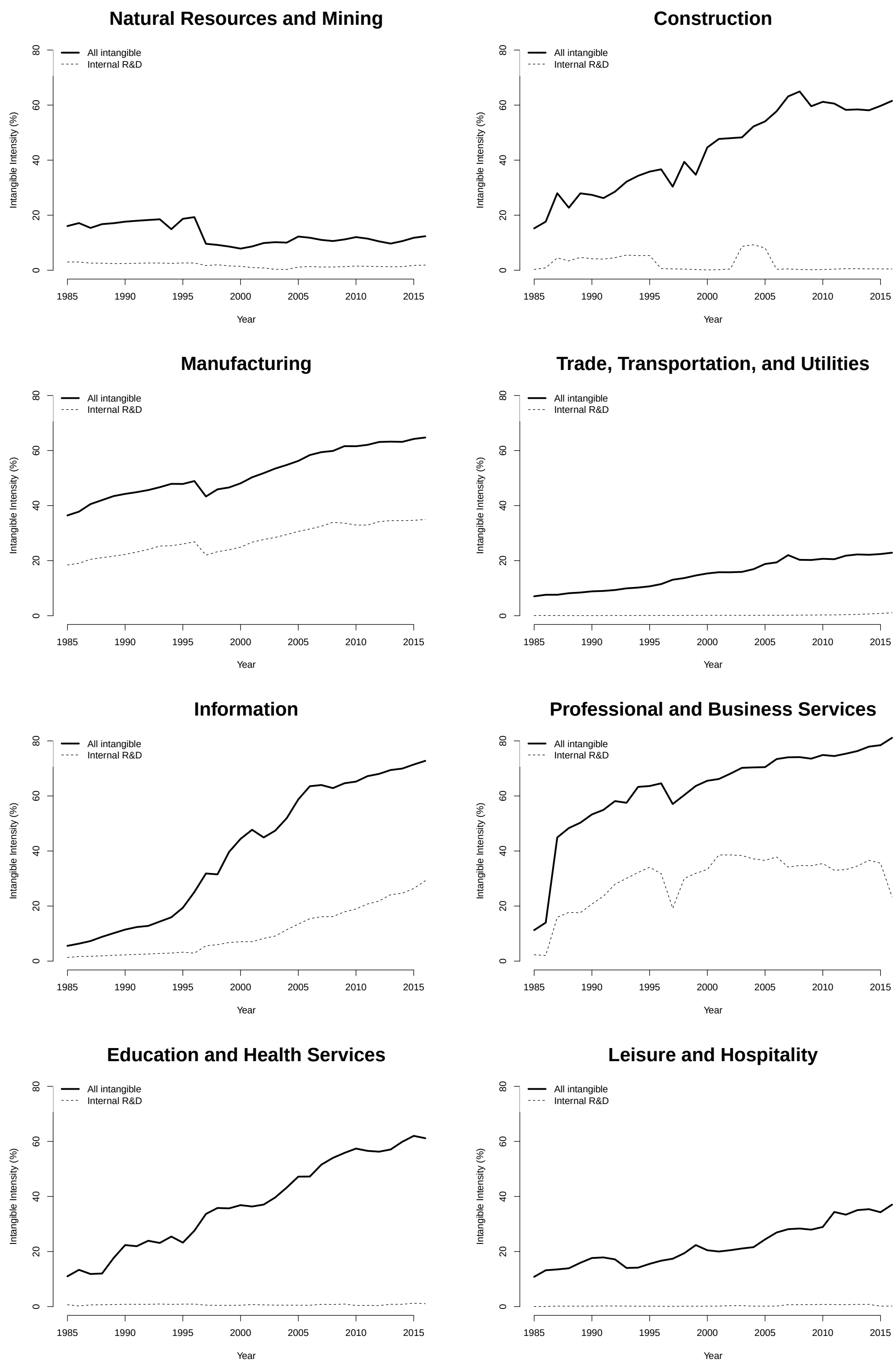
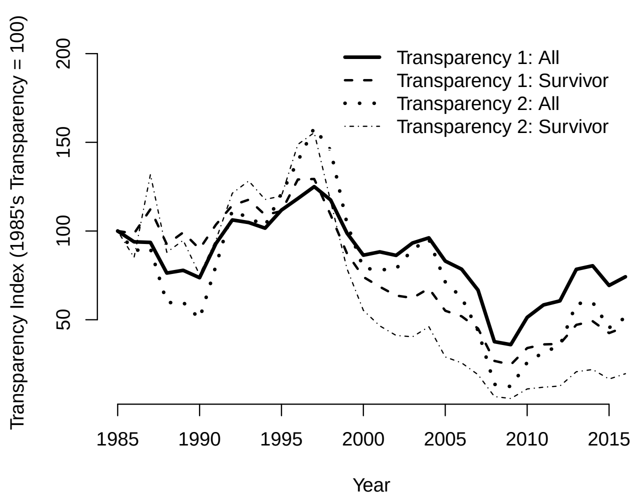


Figure B.7: Time series of transparency: all firms vs. survivors



Notes: This figure shows the trend in in transparency for all firms vs. survivors. Data comes from Compustat and I/B/E/S. See Section 2.1 for details on measurement.

C Cross-sectional evidence: Full tables

Table C.1: Regression of transparency proxies on intangibles

	Transparency 1		Transparency 2	
	(1)	(2)	(3)	(4)
Intangible	-.6386	-.3117	-.3191	-.1529
	(.0871)	(.0971)	(.0414)	(.0497)
Sales	-.2403	-.3077	-.1253	-.148
	(.0632)	(.0746)	(.0319)	(.0362)
Current Assets	-1.493	-.0664	-.7733	-.0608
	(.2679)	(.2543)	(.1373)	(.1238)
Leverage	-3.298	-2.029	-1.599	-.9834
	(.3626)	(.2224)	(.1766)	(.1095)
B/M	-1.993	-1.434	-.9824	-.7053
	(.2257)	(.1669)	(.1134)	(.0861)
log(Employment)	.5757	.1227	.2909	.0791
	(.0438)	(.0309)	(.0223)	(.0179)
Age	-.0017	-.3345	-8.7e-04	-.1775
	(.0029)	(.1635)	(.0014)	(.0803)
#Analysts	.0138	.0162	.0201	.0211
	(.0218)	(.013)	(.0088)	(.0038)
Year FE	✓	✓	✓	✓
Industry FE	✓		✓	
Firm FE		✓		✓
Adj. R^2	0.295	0.649	0.289	0.634
Observations	78878	77944	76959	76014

Notes: This table reports the estimates of the coefficients from the following regression using our baseline sample, which includes all firms in Compustat from 1985 to 2016 for which information on earnings forecasts by at least two (for the first proxy) or one (for the second proxy) analysts is available:

$$\log y_{i,t} = \theta_t + FEs + \beta \times \text{Intangible capital over total assets}_{i,t} + \gamma X_{i,t} + \varepsilon_{i,t}$$

where $y_{i,t}$ is either the inverse of variance of earning surprises when more than one analyst forecast is present, or the inverse absolute value of earning surprises from the consensus. θ_t are year fixed effects and FEs include either industry or firm fixed effects. $X_{i,t}$ represents firm controls. Standard errors are clustered at the industry and year level.

Table C.2: Regression of forecast accuracy on intangibles

	Earnings Surprises (Absolute Value)	
	(1)	(2)
Intangible	.3191	.1529
	(.0414)	(.0497)
Sales	.1253	.148
	(.0319)	(.0362)
Current Assets	.7733	.0608
	(.1373)	(.1238)
Leverage	1.599	.9834
	(.1766)	(.1095)
B/M	.9824	.7053
	(.1134)	(.0861)
log(Employment)	-.2909	-.0791
	(.0223)	(.0179)
Age	8.7e-04	.1775
	(.0014)	(.0803)
#Analysts	-.0201	-.0211
	(.0088)	(.0038)
Year FE	✓	✓
Industry FE	✓	
Firm FE		✓
Adj. R^2	0.289	0.634
Observations	76959	76014

Notes: This table reports the estimates of the coefficients from the following regression using our baseline sample, which includes all firms in Compustat from 1985 to 2016 for which information on earnings forecasts by at least one analyst is available:

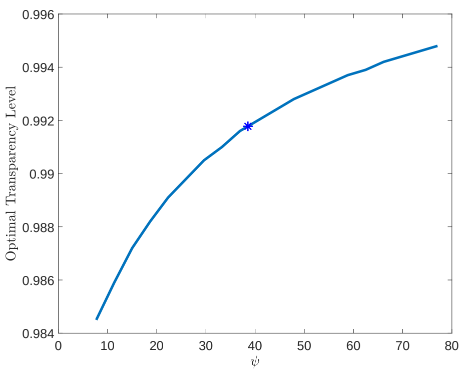
$$\log y_{i,t} = \theta_t + FEs + \beta \times \text{Intangible capital over total assets}_{i,t} + \gamma X_{i,t} + \varepsilon_{i,t}$$

where $y_{i,t}$ is the absolute value of earning surprises. θ_t are year fixed effects and FEs either industry or firm fixed effects. $X_{i,t}$ represents firm controls. Standard errors are clustered at the industry and year level.

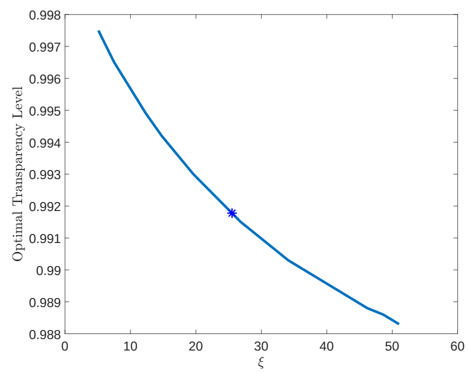
D Comparative statics

Figure C.6 plots the comparative static results of optimal disclosure regulation (vertical axis) for different parameters (horizontal axis). The optimal regulation displays a strict monotone variation with each parameter change.

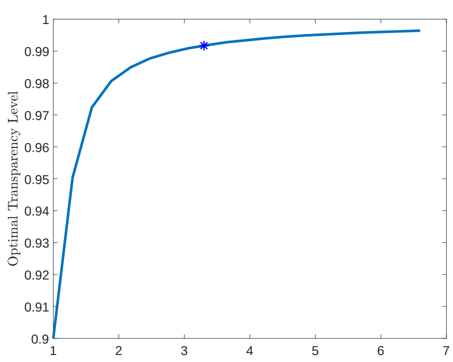
Figure D.8: Comparative statics on optimal transparency level with respect to each parameter



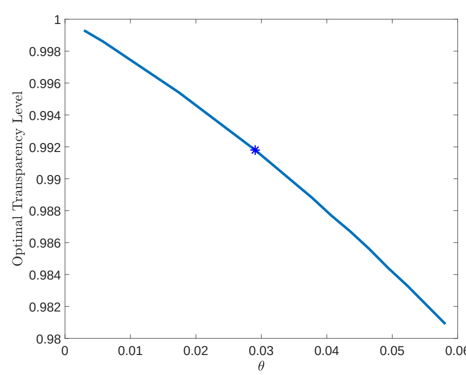
(a) Transparency's contribution to public information ψ



(b) Baseline information level ξ



(c) PE market friction ν_N

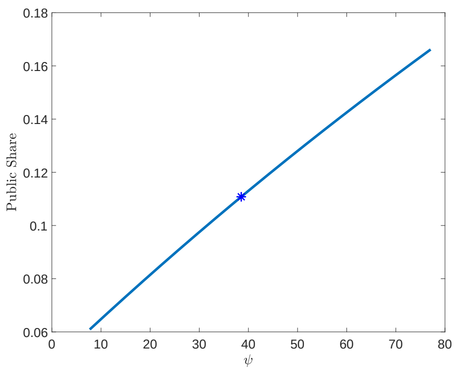


(d) Intangible share θ

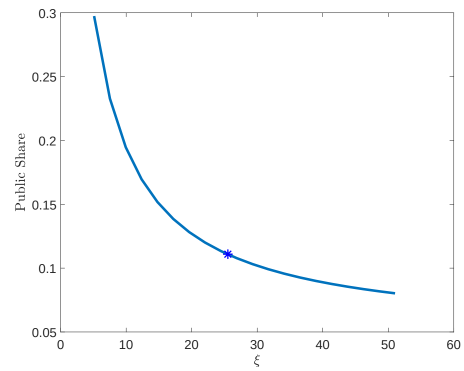
Notes: We change each single parameter, keeping the others constant at their baseline value, and calculate the resulting optimal transparency level.

Figure C.7 plots the comparative static results of the equilibrium mass of listed firms (vertical axis) for different parameters (horizontal axis). The equilibrium mass of listed firms displays a strict monotone variation with each parameter change.

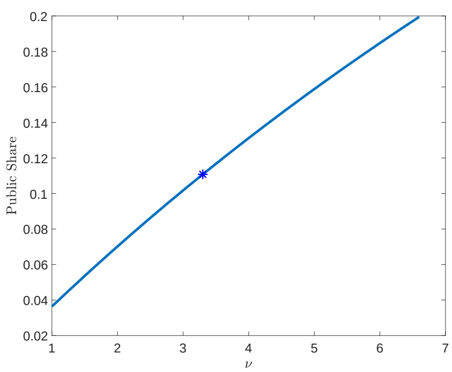
Figure D.9: Comparative statics on fraction of listed firms with respect to each parameter



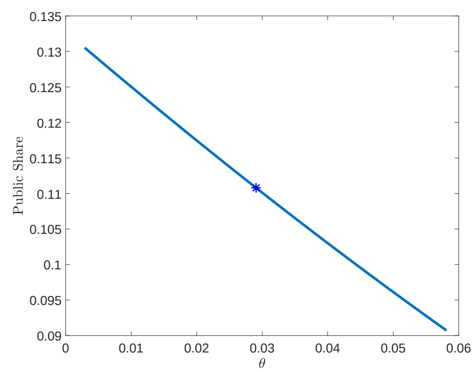
(a) Transparency's contribution to public information ψ



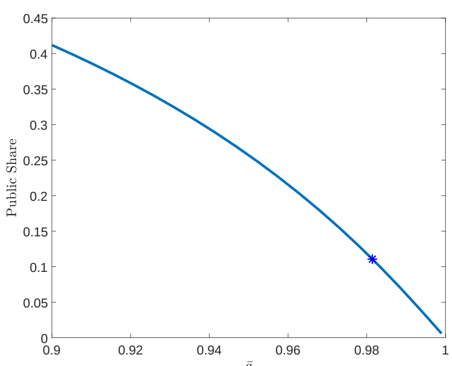
(b) Baseline information level ξ



(c) PE market friction ν_N



(d) Intangible share θ



(e) Mandated minimum transparency $\bar{q}$

Notes: We change each single parameter, keeping the others constant at their baseline value, and calculate the resulting number of listed firms.

E Summary of a firm's problem

A firm's manager decides whether to go listed or non-listed before the operation. If a firm becomes non-listed, the manager does not have to worry about the leakage of their intangibles through disclosure. However, investors penalize the opacity of the non-listed firms by allowing only a low funding intensity.

If a firm becomes public, the manager should decide on the level of transparency $q \geq 0$. If too many firms choose the same transparency level, it will decrease the firm's value in the listed market due to demand-side competition.

A firm's problem could be summarized as follows:

$$
\begin{aligned}
\text{[Entry decision]} \quad & \max\{J^L(\mathcal{M}), J^N(M_N)\}, \\
\text{[Listed firm]} \quad & J^L(\mathcal{M}) := \max_{q} \max_{k_T, k_I} \left(z k_T^{\alpha} (k_I(1-\overline{q}-q))^{\theta} (\Phi^{ex})^{\gamma} - r k_T - p k_I \right) \phi^L(q) \\
& \quad \text{s.t. } \phi^L(q) = \frac{\xi + \psi(\overline{q}+q)}{\mathcal{M}(q)}, \\
\text{[Non-listed firm]} \quad & J^N(M_N) := \max_{k_T, k_I} \left(z k_T^{\alpha} (k_I)^{\theta} (\Phi^{ex})^{\gamma} - r k_T - p k_I \right) \phi^N \\
& \quad \text{s.t. } \phi^N := \xi / (\nu_N M_N).
\end{aligned}
$$

F The scoreboards: Welfare, productivity, and output

In this section, we define the three objectives of the disclosure regulation: welfare, productivity, and output. First, we define the welfare measure. Besides the performance of firms, the investor values the transparency of the disclosed information, as it is helpful for the investor's portfolio. The representative investor's utility can be monotonically transformed into the following mean-variance form:

$$\begin{aligned}
Objective_{welfare} &= \int x(\widetilde{q})\frac{\pi(\widetilde{q})}{p(\widetilde{q})}d\widetilde{q} + x^N\frac{\pi^N}{P^N} - \frac{\Lambda}{2}\int x(\widetilde{q})^2\frac{1}{\xi+\psi(\overline{q}+q)}d\widetilde{q} - \frac{\Lambda}{2}(x^N)^2\frac{1}{\xi} \\
&= \int \mathcal{M}(\widetilde{q})\pi(\widetilde{q})d\widetilde{q} + \nu_N M^N\pi^N - \frac{\Lambda}{2}\int\frac{x(\widetilde{q})P(q)\mathcal{M}(q)}{\xi+\psi(\overline{q}+q)}d\widetilde{q} - \frac{\Lambda}{2}\frac{x^N\nu_N P^N M_N}{\xi} \\
&= \int \mathcal{M}(\widetilde{q})\pi(\widetilde{q})d\widetilde{q} + \nu_N M^N\pi^N - \frac{\Lambda}{2}\int\frac{\frac{\pi(q)/P(q)}{\Lambda/(\xi+\psi(\overline{q}+q))}P(q)\mathcal{M}(q)}{\xi+\psi(\overline{q}+q)}d\widetilde{q} - \frac{\Lambda}{2}\frac{\frac{\pi^N/P^N}{\Lambda/\xi}\nu_N P^N M_N}{\xi} \\
&= \int \mathcal{M}(\widetilde{q})\pi(\widetilde{q})d\widetilde{q} + \nu_N M^N\pi^N - \frac{1}{2}\int \pi(\widetilde{q})\mathcal{M}(\widetilde{q})d\widetilde{q} - \frac{1}{2}\nu_N\pi^N M_N. \\
&= \frac{1}{2}\int \mathcal{M}(\widetilde{q})\pi(\widetilde{q})d\widetilde{q} + \frac{\nu_N}{2}M^N\pi^N. \qquad (1)
\end{aligned}$$

Therefore, the welfare measure is equivalent to the expected profit in equilibrium.

The second measure is the productivity in the production sector that is defined as follows:

$$\begin{aligned}
Objective_{productivity} &= (\Phi^{ex})^\gamma \\
&= \left(\int_0^{1-\overline{q}}(\overline{q}+q)k_I(q,\mathcal{M};\overline{q})\mathcal{M}(q)dq\right)^\gamma,
\end{aligned}$$

The productivity is identical to the externality effect, which is the total shared knowledge in the economy. From the regulator's perspective, there is a trade-off in the productivity measure for increasing the strictness of the disclosure requirement. For higher $\overline{q}$, the amount of shared information is greater, while the pool of listed firms to share the information shrinks due to the firm-level extensive-margin responses. Also, the size of intangible k_I to be shared declines as in Proposition 2.

The third measure is the aggregate output in the economy. The output measure is defined in the following form:

$$Objective_{output} = \int_0^{1-\overline{q}} zk_T(q)^\alpha (k_I(q)(1-\overline{q}-q))^\theta (\Phi^{ex})^\gamma M(q) + zk_{DT}^\alpha k_{DI}^\theta (\Phi^{ex})^\gamma M_N.$$

In our structural structural, we analyze the macroeconomic impact of rising intangible shares and the disclosure policy change, using these three macro-level measures.

G Proofs

G.1 Proof for Proposition 1.

Proposition 1 (Funding supply)**.**

The household's optimal funding supplies for listed firms with transparency q, $x^*(q)$, and for non-listed firms, x^{N*} are as follows:

$$x^*(q) = \frac{\pi(q)/P(q) - \mu}{\Lambda/(\xi + \psi(\overline{q} + q))}$$
$$x^{N*} = \frac{\pi^N/P^N - \mu}{\Lambda/\xi}.$$

Proof.

From the $i.i.d$ assumption of the stock return uncertainty, the consumption (income) satisfies

$$C \sim N\left(\int x(\widetilde{q})\overline{r}(\widetilde{q})d\widetilde{q} + x^N\overline{r}^N, \int x(\widetilde{q})^2 \frac{1}{\xi + \psi(\overline{q} + q)}d\widetilde{q} + (x^N)^2\frac{1}{\xi}\right).$$

Then the investors' expected utility maximization problem is translated into the following form:[2]

$$\max_{\int x(\widetilde{q})d\widetilde{q} + x^N = a} \quad - e^{-\Lambda\left(\int x(\widetilde{q})\frac{\pi(\widetilde{q})}{P(\widetilde{q})}d\widetilde{q} + x^N\frac{\pi^N}{P^N} - \frac{\Lambda}{2}\int x(\widetilde{q})^2\frac{1}{\xi+\psi(\overline{q}+q)}d\widetilde{q} - \frac{\Lambda}{2}(x^N)^2\frac{1}{\xi}\right)}.$$

After a strictly-increasing (log) transformation, the problem reduces down to

$$\max_{\int x(\widetilde{q})d\widetilde{q} + x^N = a} \quad \int x(\widetilde{q})\frac{\pi(\widetilde{q})}{P(\widetilde{q})}d\widetilde{q} + x^N\frac{\pi^N}{P^N} - \frac{\Lambda}{2}\int x(\widetilde{q})^2\frac{1}{\xi + \psi(\overline{q} + q)}d\widetilde{q} - \frac{\Lambda}{2}(x^N)^2\frac{1}{\xi}.$$

The first-order condition with respect to $x(q)$ yields

[2] The derivation of the mean-variance portfolio objective function is as follows: consider a random variable, $y \sim N(\mu_y, \sigma_y^2)$. Then,

$$\mathbb{E}(-e^{-\Lambda y}) = -\mathbb{E}(e^{-\Lambda y}) = -e^{-\Lambda(\mu_y - \frac{\Lambda}{2}\sigma_y^2)}.$$

The last equation is immediate from the moment generating function of the normal distribution.

$$\frac{\pi(q)}{P(q)} - \Lambda x^*(q)\frac{1}{\xi + \psi(\overline{q}+q)} - \mu = 0,$$

where μ is the Lagrange multiplier of the wealth constraint. From this equation, we can derive the following supply curve of funding for the listed market:

$$x^*(q) = \frac{\pi(q)/P(q) - \mu}{\Lambda/(\xi + \psi(\overline{q}+q))},$$

where $x^*(q)$ is the funding supply in a dollar amount for firms with the transparency level q. So, the household is willing to invest $\frac{\pi(q)/P(q)-\mu}{\Lambda\frac{1}{\xi+\psi(\overline{q}+q)}}$ in the firms with transparency level q. As we assume the representative household has a large enough wealth a, $\mu = 0$.

Similarly, from the first-order condition with respect to x^N, the funding supply curve for non-listed firms is characterized as follows:

$$x^{N*} = \frac{\pi^N/P^N}{\Lambda/\xi}.$$

■

G.2 Proof for Proposition 2.

Proposition 2. *(Intangibles and the transparency)*
Given $\alpha + \theta < 1$, $k^I(q, \mathcal{M}; \overline{q})$ decreases in both q and $\overline{q}$. Specifically,

$$k_I(q, \mathcal{M}; \overline{q}) = \left(\left(\frac{\alpha z(\Phi^{ex})^\gamma}{r}\right)^{\frac{1}{1-\alpha-\theta}}\left(\frac{r\theta}{p\alpha}\right)^{\frac{1-\alpha}{1-\alpha-\theta}}\right)(1-\overline{q}-q)^{\frac{\theta}{1-\alpha-\theta}}.$$

Proof.
From FOC

$$[k_T]: \quad z\alpha k_T^{\alpha-1}(k_I(1-\overline{q}-q))^\theta(\Phi^{ex})^\gamma = r$$

$$[k_I]: \quad z\theta k_T^{\alpha}(k_I(1-\overline{q}-q))^{\theta-1}(\Phi^{ex})^\gamma(1-\overline{q}-q) = p$$

$$+ \left(zk_T^{\alpha}(k_I(1-\overline{q}-q))^{\theta}(\Phi^{ex})^\gamma - rk_T - pk_I\right)\phi'^{L}(q) = 0.$$

From the first-order conditions with respect to k_T and k_I, we obtain

$$\frac{r}{p} = \left(\frac{\alpha}{\theta}\right)\frac{k_I}{k_T}.$$

Substituting this relation into the first-order condition with respect to k_T, we get

$$r = \alpha z\left(\frac{\alpha p}{\theta r}\right)^{\alpha-1}(k_I)^{\alpha+\theta-1}(1-\overline{q}-q)^{\theta}(\Phi^{ex})^{\gamma}.$$

Thus,

$$k_I = \left(\left(\frac{\alpha z(\Phi^{ex})^{\gamma}}{r}\right)^{\frac{1}{1-\alpha-\theta}}\left(\frac{r\theta}{p\alpha}\right)^{\frac{1-\alpha}{1-\alpha-\theta}}\right)(1-\overline{q}-q)^{\frac{\theta}{1-\alpha-\theta}} = A(1-\overline{q}-q)^{\frac{\theta}{1-\alpha-\theta}},$$

where $A := \left(\left(\frac{\alpha z(\Phi^{ex})^{\gamma}}{r}\right)^{\frac{1}{1-\alpha-\theta}}\left(\frac{r\theta}{p\alpha}\right)^{\frac{1-\alpha}{1-\alpha-\theta}}\right)$. As $\alpha+\theta<1$, the proposition is immediate from the last equation. ■

G.3 Proof for Proposition 3.

Proposition 3. *(Transparency distribution)*

The probability density function $\mathcal{M}$ of transparency q has the following closed form:

$$\mathcal{M}(q) = (\xi+\psi(\overline{q}+q))(1-\overline{q}-q)^{\frac{\theta}{1-\alpha-\theta}}\frac{1}{\phi^N}.$$

Proof.

We derive the following equations using the first-order condition with respect to q:[3]

$$\begin{aligned}\frac{\phi'^{L}(q)}{\phi^{L}(q)} &= \frac{z\theta k_T^{\alpha}(k_I(1-\overline{q}-q))^{\theta-1}(\Phi^{ex})^{\gamma}k_I}{zk_T^{\alpha}(k_I(1-\overline{q}-q))^{\theta}(\Phi^{ex})^{\gamma}-rk_T-pk_I}\\ &= \frac{z\theta k_T^{\alpha}(k_I(1-\overline{q}-q))^{\theta-1}(\Phi^{ex})^{\gamma}k_I}{(1-\alpha-\theta)zk_T^{\alpha}(k_I(1-\overline{q}-q))^{\theta}(\Phi^{ex})^{\gamma}}\\ &= \frac{\theta}{1-\alpha-\theta}\left(\frac{1}{1-\overline{q}-q}\right).\end{aligned}$$

From $\frac{\partial}{\partial q}log(\phi^{L}(q)) = \frac{\phi'^{L}(q)}{\phi^{L}(q)}$, the solution of the first-order differential equation is as follows:

$$\phi^{L}(q) = (1-\overline{q}-q)^{n}\,\widetilde{C},$$

for some $n \in \mathbb{R}$ and some $\widetilde{C} \in \mathbb{R}$. From the indifference condition in the equilibrium, $\pi^{L}(q)\phi^{L}(q)$ does not depend on q.

$$\pi^{L}\phi^{L}(q) = \left(z(1-\alpha-\theta)\left(\frac{\alpha p}{\theta r}\right)^{\alpha}A^{\alpha+\theta}(1-\overline{q}-q)^{\frac{\theta}{1-\alpha-\theta}}\Phi^{\gamma}\right)(1-\overline{q}-q)^{n}\,\widetilde{C}.$$

Therefore,

$$n = -\frac{\theta}{1-\alpha-\theta}.$$

This leads to $\phi^{L}(q) = (1-\overline{q}-q)^{-\frac{\theta}{1-\alpha-\theta}}\,\widetilde{C}$.

Then, the distribution of listed firms is as follows:

$$\begin{aligned}\mathcal{M}(q) &= (\xi+\psi(\overline{q}+q))/\phi^{L}(q)\\ &= (\xi+\psi(\overline{q}+q))\,(1-\overline{q}-q)^{\frac{\theta}{1-\alpha-\theta}}\,\frac{1}{\widetilde{C}}.\end{aligned}$$

[3]Here the proof is based on the first-order conditions that are simultaneously obtained for k_T, k_I, and q for the brevity of notations. The equilibrium allocations stay unaffected in this problem even if the solution is solved sequentially (interim problem first (k_T and k_I), and then q)

From the indifference condition between listed and non-listed,

$$\begin{aligned}
\phi^N &= \frac{\pi^L(q)\phi^L(q)}{\pi^N} \\
&= \frac{\left(z(1-\alpha-\theta)\left(\frac{\alpha p}{\theta r}\right)^{\alpha} A^{\alpha+\theta}(1-\overline{q}-q)^{\frac{\theta}{1-\alpha-\theta}}\Phi^{\gamma}\right)(1-\overline{q}-q)^{-\frac{\theta}{1-\alpha-\theta}}\widetilde{C}}{\left(z(1-\alpha-\theta)\left(\frac{\alpha p}{\theta r}\right)^{\alpha} A^{\alpha+\theta}\Phi^{\gamma}\right)} \\
&= \widetilde{C}.
\end{aligned}$$

Therefore, $\mathcal{M}(q) = (\xi + \psi(\overline{q}+q))(1-\overline{q}-q)^{\frac{\theta}{1-\alpha-\theta}} \frac{1}{\widetilde{\phi^N}}$.

In the equilibrium, $\phi^N (= \widetilde{C})$ is determined at the level where the following equation holds:

$$\int_0^{1-\overline{q}} \mathcal{M}(q)dq = 1 - M_N.$$

■

G.4 Proof for Corollary 3.

Corollary 3. *(Truncated Beta distribution)*
The gross transparency, $y := q + \overline{q}$, follows a truncated Beta distribution where the shape parameters are 2 and $B+1$, and the support is $[\overline{q}, 1]$.

$$q + \overline{q} \sim \frac{\mathbb{I}\{q \in [0, 1-\overline{q}]\}}{1 - M_N} \times Beta\left(B+1, 2\right),$$

where $B = \frac{\theta}{1-\alpha-\theta}$.

Proof.
We define $M_Y(y)$ as the probability density function of the random variable $y = \frac{1-q-\overline{q}}{1+\xi/\psi}$.

$$M_Y(y) \propto (1-y)y^B \quad \text{and} \quad y \in \left[0, \frac{1-\overline{q}}{1+\xi/\psi}\right].$$

Also, $\int_0^{\frac{1-\overline{q}}{1+\xi/\psi}} M_Y(y)dy = 1 - M_N$. Therefore, $y \sim \frac{\mathbb{I}\{q\in[0,1-\overline{q}]\}}{1-M_N} \times Beta\left(B+1, 2\right)$. ■

G.5 Proof for Proposition 4

Proposition 4 (Non-listed firms' measure)**.**

In equilibrium, the measure of non-listed firms M_N is as follows:

$$M_N = \frac{1}{1 + \psi \frac{\nu_N}{\xi}(1 + \frac{\xi}{\psi})^{B+2}\mathcal{B}(B+1,2)F\left(\frac{1-\overline{q}}{1+\xi}; B+1, 2\right)}$$

where $\mathcal{B}$ is the beta function, and F is the cumulative distribution function of beta distribution.[4]

Proof. We have the following closed-form solution for M_N:

$$M_N = \frac{1}{1 + \psi \frac{\nu_N}{\xi} \int_0^{1-\overline{q}} (\frac{\xi}{\psi} + (\overline{q} + q))(1 - \overline{q} - q)^B dq}. \tag{2}$$

Using Corollary 1, we can integrate out the $M(q)$ in the right-hand side of the equation in the following steps, using $y = \frac{1-q-\overline{q}}{1+\xi/\psi} \in [0, \frac{1-\overline{q}}{1+\xi/\psi}]$:

$$M_N = \frac{1}{1 + \psi \frac{\nu_N}{\xi}(1 + \frac{\xi}{\psi})^{B+2} \int_0^{\frac{1-\overline{q}}{1+\xi/\psi}} (1-y)(y)^B dy}.$$

$$M_N = \frac{1/\mathcal{B}(B+1,2)}{1/\mathcal{B}(B+1,2) + \psi \frac{\nu_N}{\xi}(1 + \frac{\xi}{\psi})^{B+2}\mathcal{B}(B+1,2) \int_0^{\frac{1-\overline{q}}{1+\xi/\psi}} y^B (1-y) dy}.$$

We integrate the denominator using the cumulative distribution function of beta

[4] The beta function is defined as follows:

$$\mathcal{B}(a,b) := \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)} = \frac{(a-1)!(b-1)!}{(a+b-1)!} = \int_0^1 x^{a-1}(1-x)^{b-1} dx.$$

distribution, F:

$$M_N = \frac{1/\mathcal{B}(B+1,2)}{1/\mathcal{B}(B+1,2) + \psi\frac{\nu_N}{\xi}(1+\frac{\xi}{\psi})^{B+2}F\left(\frac{1-\overline{q}}{1+\xi}; B+1, 2\right)}.$$

By multiplying $\mathcal{B}(B+1,2)$ on the numerator and the denominator, we obtain the following analytic form:

$$M_N = \frac{1}{1 + \psi\frac{\nu_N}{\xi}(1+\frac{\xi}{\psi})^{B+2}\mathcal{B}(B+1,2)F\left(\frac{1-\overline{q}}{1+\xi}; B+1, 2\right)}. \tag{3}$$

■

G.6 Proof for Proposition 5.

Proposition 5. *(The relationship between the measure of listed firms and the structural parameters)*
M_N *strictly increases in* $\overline{q} \in (0,1)$ *and* $\theta > 0$.

Proof.
We have

$$M_N = \frac{1}{1 + \psi\frac{\nu_N}{\xi}(1+\frac{\xi}{\psi})^{B+2}\mathcal{B}(B+1,2)F\left(\frac{1-\overline{q}}{1+\xi}; B+1, 2\right)}.$$

F decreases in $\overline{q}$, and M_N decreases in F. Thus, M_N increases in $\overline{q}$.

For the second statement, we write M_N in the following form:

$$M_N = 1 - \int_0^{1-\overline{q}} (\xi + \psi(\overline{q}+q))\,(1-\overline{q}-q)^{\frac{\theta}{1-\alpha-\theta}}\,\frac{1}{\phi^N}dq.$$

Then, taking the partial derivative with respect to θ, we get

$$\frac{\partial M_N}{\partial \theta} = -\underbrace{\frac{\partial}{\partial \theta}\left(\frac{\theta}{1-\alpha-\theta}\right)}_{>0} \int_0^{1-\overline{q}} (\xi + \psi(\overline{q}+q))\,(1-\overline{q}-q)^{\frac{\theta}{1-\alpha-\theta}}\,\frac{1}{\phi^N}\underbrace{log(1-q-\overline{q})}_{<0}\,dq$$

$$> 0.$$

■

G.7 Proof for Proposition 6

Proposition 6. *(The relationship between the aggregate transparency and the intangible share)*
The aggregate transparency $\mathcal{T}$ *decreases in* θ*, where* $\mathcal{T} := \int_0^{1-\overline{q}}(q+\overline{q})\mathcal{M}(q;\theta)dq$*.*

Proof.
We take the partial derivative of $\mathcal{T}$ with respect to θ:

$$\frac{\partial \mathcal{T}}{\partial \theta} = \underbrace{\frac{\partial}{\partial \theta}\left(\frac{\theta}{1-\alpha-\theta}\right)}_{>0} \int_0^{1-\overline{q}} (q+\overline{q})(\xi + \psi(\overline{q}+q))\,(1-\overline{q}-q)^{\frac{\theta}{1-\alpha-\theta}}\,\frac{1}{\phi^N}\underbrace{log(1-q-\overline{q})}_{<0}\,dq$$

$$< 0.$$

■

H A Bayesian-foundation for household's belief on the return

In this section, we elaborate on how the household update the belief on the return based on the canonical Bayesian framework.

Suppose a return of a firm before any information update is the same as the case of the non-listed firm as follows:

$$\widetilde{r} \sim_{iid} N\left(\overline{r}, \frac{1}{\xi}\right)$$
$$\text{s.t.} \quad \overline{r} = \frac{\pi}{P},$$

Then, the noisy information ζ about each firm's return with transparency q arrives:

$$\zeta(q) \sim_{iid} N\left(0, \frac{1}{\psi(q+\overline{q})}\right)$$

where $\psi > 0$ is the marginal contribution of transparency to the information on the listed firm's return. After the information arrival, the household observes the realization y of random variable $Y(q)$ such that $Y(q) = \widetilde{r} + \zeta(q)$.

Then, the posterior distribution of $\widetilde{r}(q)$ given $y(q)$ follows the normal distribution, and the conditional mean and variance are as follows:

$$\mathbb{E}(\widetilde{r}(q)|Y=y) = \overline{q} + \frac{\frac{1}{\psi(q+\overline{q})}}{\frac{1}{\xi}+\frac{1}{\psi(q+\overline{q})}}(y-\overline{q})$$
$$Var(\widetilde{r}(q)|Y=y) = \frac{1}{\xi} - \frac{\frac{1}{\xi^2}}{\frac{1}{\xi}+\frac{1}{\psi(q+\overline{q})}}$$
$$= \frac{1}{\xi+\psi(q+\overline{q})}$$

Then, the unconditional (independent of the specific y realization) posterior distribution of $\widetilde{r}(q)$ is as follows:

$$\widetilde{r}(q) \sim_{iid} N\left(\overline{r}(q), \frac{1}{\xi+\psi(q+\overline{q})}\right),$$

which is the same belief setup as in the main text.

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