Winner-Take-All or Multiple Prizes? The Role of Idea Diversity in Innovation Contests*

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Abstract

Contests are widely used in the procurement of innovative products. Our study of hundreds of real procurement contests reveals that the majority of them offer multiple prizes, particularly for building projects that require diverse and creative ideas. We also find that contests with multiple prizes tend to attract more players. We propose an incomplete-information contest model with the contest organizer valuing both the overall quality of proposals and the diversity of ideas. The model offers a new explanation for the coexistence of different prize structures: when the organizer's preference for diversity is sufficiently strong, offering multiple prizes is optimal, whereas a winner-take-all structure is optimal otherwise.

Keywords: innovation contests, multiple prizes, endogenous entry, diversity, procurement

JEL codes: D47, D82, L74

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1 Introduction

Since the seminal work by Tullock (1980), contests have been widely used to study competitive environments in which players exert costly effort to compete for one or multiple prizes. Contests are a popular mechanism for procuring innovative products, such as architectural designs, inventions, and solutions to scientific and engineering problems. In a typical innovation contest, the contest organizer (buyer or procurer, she) establishes a prize scheme that specifies prizes to be rewarded to players (contestants, he/they) based on the ranking of their performance. Players are invited to submit proposals demonstrating their ideas, which are then evaluated to determine the ranking of players.

In the literature, a winner-take-all prize structure is effort-maximizing in most contest environments with incomplete information and linear costs and prize valuations. Moldovanu and Sela (2001) establishes the unique optimality of devoting all resources to the first prize in contests with incomplete information when the effort cost function is linear or concave. The optimality of the winner-take-all prize structure continues to hold after introducing the possibility of two-stage contests (Moldovanu and Sela, 2006). Liu and Lu (2014) show that, given (potentially) heterogeneous prizes with fixed amounts, an all-pay auction maximizes total effort, which, combined with the result of Moldovanu and Sela (2001), further implies that an all-pay auction with winner-take-all is effort-maximizing given a fixed budget. Liu and Lu (2019) introduce endogenous entry into the model of Moldovanu and Sela (2001) and show that winner-take-all is still optimal despite the new tradeoff between encouraging the entry of potential contestants and eliciting effort from entrants. In fact, the majority of studies on contests focus on the case of winner-take-all (Fu and Wu, 2019).

However, in practice, many innovation contests offer multiple prizes. For example, in 1993, Boeing and Lockheed-Martin were each given compensation of USD 2.2 billion to produce a prototype in competition for the contract for the Joint Strike Fighter (Kaplan et al., 2002; Matros and Armanios, 2009). In 2012, the Google Xprize foundation launched a contest for ideas to develop affordable transportation to the moon. The contest offers a USD 20 million grand prize, a USD 5 million second prize, and several USD 1 million "diversity prizes."[1] In 2007, the Shanghai government launched an international proposal contest for the architectural design of the China Pavilion for EXPO 2010 Shanghai. The contest offered a CNY 120 million prize to the winner and a CNY 0.5 million prize to each of the eight designers who were shortlisted in the final round.[2] In addition to the anecdotal evidence, we obtain a dataset of 327 contests of design proposals from a public procurement platform in China. We find that approximately 70% of these contests offer multiple prizes. Moreover, contests for building projects tend to offer more prizes and attract more players to enter than other types of projects.

Why are multiple prizes popular in innovation contests, and why do some contests use a winner-

[1] See `lunar.xprize.org`.

[2] The contest attracted 344 designers from throughout the world to submit proposals. See `en.wikipedia.org/wiki/China_pavilion_at_Expo_2010`.

take-all prize while others adopt multiple prizes? In this paper, we extend the standard contest model developed by Moldovanu and Sela (2001) and obtain two main results regarding the choice between a winner-take-all prize and multiple prizes. First, the optimality of multiple prizes requires both a positive entry cost and the contest organizer's preference for idea diversity. Second, the preference for diverse ideas needs to be sufficiently strong to justify the adoption of multiple prizes. In other words, the winner-take-all prize is optimal if uncertainty and idea diversity do not play an important role in the contest.

One important feature of innovation contests is the uncertainty of the ideal approach, so the contest organizers usually prefer diverse ideas from many players. There is ample evidence on the importance of diverse ideas and tying various approaches in innovation. Letina and Schmutzler (2019) note that, in many innovation contests, contest organizers face uncertainty over the ideal approach to achieve the goal. For example, there are at least 9 different approaches to the development of COVID-19 vaccines, and many of these approaches may end up in failure (Le et al., 2020). In architectural design contests, the organizers may not be able to specify the ideal design before reviewing the proposals from design firms (Zhu, 2019). For scientific research problems, Lakhani et al. (2007) provide empirical evidence that attracting solvers with diverse scientific backgrounds greatly improves the success rate of the designated problems.

In our model, the contest organizer sets up a rank-order prize scheme subject to a fixed budget and invites players to submit their proposals. Each player is endowed with an idea as his private information and develops his design proposal based on the idea. The proposals are evaluated by a committee of experts, whose ratings or votes determine the ranking of players. It is easier to convince the committee of a proposal based on a more conventional idea than a less conventional idea. To capture this feature of innovation contests, we assume that demonstrating a less conventional idea in a proposal requires a higher cost. Given the prize scheme and private information, each player endogenously decides whether to enter the contest at a cost (Liu and Lu, 2019).

To incorporate the preference for diversity, we assume that the contest organizer aims to maximize a weighted average of the total quality (effort) and diversity of ideas among participating players. Following Letina and Schmutzler (2019), we assume that there exists an ideal design that is unobservable until all proposals are submitted and winners are determined. Hence, a greater diversity of ideas makes it more likely to obtain a better design proposal. In some cases, the organizer can use the ideas from multiple proposals to develop the ideal design in the final plan.

We show that the organizer's preference for diversity explains her choice between winner-take-all and multiple prizes. Specifically, there is a unique and strictly positive value of the weight that she places on idea diversity such that if she cares about diversity more than this value, offering multiple prizes is optimal; otherwise, winner-take-all is uniquely optimal. We emphasize that the cutoff weight is substantially greater than zero, which explains why winner-take-all is still used even in some industries that value diversity. This result provides a rationale for the coexistence of winner-take-all and multiple prizes used in innovation contests. It is also consistent with our findings from the design contest data: multiple prizes are much more common in contests for building projects that require more creativity and diversity.

The remainder of the paper is organized as follows. In Section 2, we review the related literature. In Section 3, we present the data and motivating evidence. We propose our model and provide its analysis and implications in Section 4. Section 5 concludes the paper. Technical proofs are relegated to the Appendix.

2 Related Literature

Our paper contributes to the literature on optimal prize allocation in contests with entry costs. Liu and Lu (2019) show that after introducing an entry cost to the contest model in Moldovanu and Sela (2001), the winner-take-all prize still maximizes total effort. Hammond et al. (2019) study the contest design problem with endogenous entry but allow the contest organizer to charge an entry fee, which is used to augment the prize budget. We contribute to this strand of literature by introducing the contest organizer's preference for diversity of ideas. We find that the winner-take-all prize scheme is no longer optimal in the presence of both costly entry and a sufficiently strong preference for diversity.

In the literature, there are alternative ways to rationalize the adoption of multiple prizes in contest design. For example, in contests with incomplete information, the shapes of players' effort cost function (Moldovanu and Sela, 2001; Zhang, 2021), prize valuation function (Glazer and Hassin, 1988; Polishchuk and Tonis, 2013), or the distribution of an output shock (Ales et al., 2017) can affect whether the contest organizer should adopt multiple prizes.[3] With complete information, factors such as the relative strength of contestants' abilities (Szymanski and Valletti, 2005) and the distribution of noise in performance (Drugov and Ryvkin, 2020) can also lead to different optimal choices between winner-take-all and multiple prizes. We show that under linear cost function and incomplete information, the preference for idea diversity can be a driving factor of optimal prize structure.

Our paper joins the large and growing literature on innovation contests, such as Taylor (1995), Che and Gale (2003), Ganuza and Hauk (2006), Schöttner (2008), Terwiesch and Xu (2008), Erkal and Xiao (2021), Letina and Schmutzler (2019), Gao et al. (2022), Stouras et al. (2022), and Dong et al. (2023).[4] We emphasize the value of eliciting diverse ideas, which is aligned with Koh (2017), Letina and Schmutzler (2019) and Erkal and Xiao (2021). Koh (2017) shows that contest organizers want to attract more players under higher quality uncertainty. Letina and Schmutzler (2019) study the optimal innovation contest in which the quality of a player's innovation depends on the distance between his approach and an unobservable ideal approach. They show that a simple

[3] Sarne and Lepioshkin (2017) illustrate that a multiple-prize scheme is often optimal when participation is costly and the players' efforts are random variables beyond their control.

[4] See Ales et al. (2019) and Segev (2020) for comprehensive surveys of the management science and operation research literature on innovation contests.

"bonus" tournament with a high prize and a low prize is optimal, which induces the socially optimal variety of approaches.[5] Differing from Letina and Schmutzler (2019) in which each player chooses an approach, in our paper, each player is endowed with a private approach/idea, which determines the marginal cost of preparing his proposal. Moreover, we note that the organizer can approximate the best idea from many proposals, so the quality of all the submitted proposals and the diversity of ideas both matter in our setting. Erkal and Xiao (2021) also assume that each player is endowed with a private idea and characterize the optimal prize in winner-take-all innovation contests. They propose a new stochastic order to rank idea quality distributions and uncover the relationship between the scarcity of high-quality ideas and the optimal prize. Our study emphasizes that, in some innovation contests, the best idea or approach may not belong to any player but is developed from several players' submitted proposals.

The empirical results in our paper also contribute to the small literature on the empirical study of contests. Implementing theoretical insights in practice requires knowledge of the contest environment based on empirical studies. Jia (2008) uses NBA data to estimate the contest success function (CSF) and uses the Bayesian model selection method to compare three popular forms of CSFs. Hwang (2012) and Jia and Skaperdas (2012) use battle data to estimate the CSF and study military conflict technology. Sunde (2003) and Malueg and Yates (2010) use sports data to test some implications of contest theory. Kang (2016) studies lobbying activities in the U.S. energy sector and shows how spending affects voting outcomes. Lemus and Marshall (2021) use data from Kaggle prediction contests to estimate a dynamic model and show that public information disclosure improves average performance. Huang and He (2021) develop a structural estimation procedure for a Tullock contest and adopt it to study how campaign spending affects U.S. House election outcomes. Using data from software contests, Boudreau et al. (2011) find that having a greater number of competitors reduces the incentive to exert effort but makes it more likely that one competitor obtains an extreme-value solution. The latter effect dominates the former for more uncertain problems. We find a similar result: To acquire bold and unconventional designs, the contest organizer should induce more entry by offering compensation for contest losers.

3 Data and Motivating Evidence

We obtain a sample of 327 design proposal contests from the Guangzhou Public Resource Trading Center (`www.gzggzy.cn`). The data cover all public procurement contests that involve the submission of design proposals from 2014 to 2018 in Guangdong, China. These projects belong to five categories: building and architecture (Obs= 195), urban planning and transportation (Obs= 83), landscaping (Obs= 28), electricity and mechanics (Obs= 12), and water conservancy (Obs= 9). Most procurers (contest organizers) of these projects are governments and state-owned enterprises

[5] Letina (2016) also emphasizes the importance of the diversity of research projects and how it interacts with duplication of research effort among firms in the market.

that are legally required to conduct procurement through designated procurement platforms.[6] Table 1 provides the summary statistics of the data.

Table 1: Summary Statistics

	All projects Obs=327		Other projects Obs=195		Building projects Obs=132		t test
Variable	Mean	SD	Mean	SD	Mean	SD	p-value
p.design	1193	1225	918	928	1379	1361	**0.000**
p.construct	42461	89816	34396	33267	47921	112845	0.116
proposal.day	42.584	21.575	42.765	20.798	42.462	22.137	0.900
design.day	122.786	98.504	117.682	58.918	126.241	118.031	0.387
D.model	0.278	0.449	0.008	0.087	0.462	0.500	**0.000**
D.prequalify	0.018	0.134	0.000	0.000	0.031	0.173	0.014
area	74,710	125,217	68,206	44,592	79,136	158,129	0.363
N.prize	4.896	2.919	4.265	3.061	5.323	2.746	**0.002**
D.multi.prize	0.725	0.447	0.583	0.495	0.821	0.385	**0.000**
V	19.711	35.029	16.972	33.698	21.564	35.869	0.240
v_1	5.599	9.738	5.225	10.161	5.852	9.460	0.574
v.loser	14.402	26.027	12.171	24.207	15.913	27.148	0.193
v.loser.ratio	0.508	0.328	0.414	0.359	0.572	0.290	0.000
n	8.419	4.508	7.205	3.951	9.241	4.683	**0.000**

Note: All monetary variables (*p.design*, *p.construct*, V, v_1, and *v.loser*) are in units of CNY 10,000. *D.multi.prize*, *D.model*, and *D.prequalify* are dummy variables. The last column displays the p-value of a t test between building projects and other projects. Numbers in bold indicate that the test is rejected at the 95% significance level.

We explain the data with related industry backgrounds. A contest organizer launches a contest by posting an announcement on the procurement platform with the help of a professional procurement agency. Appendix B is a sample contest announcement. The announcement contains information on the project and invites players to participate in the proposal contest. For each contest, we observe the budget for the design component (*p.design*), the budget for the construction component (*p.construct*),[7] the total work area in square meters (*area*), whether players need to submit a design model (*D.model*), whether players need to satisfy prequalification (*D.prequalify*), the number of days to submit the proposal (*proposal.day*), the number of days for the winner to complete the detailed design work (*design.day*), and the city of the project. The projects are located across 19 cities in Guangdong Province.

After players complete their design proposals, a committee of experts evaluates them and determines a ranking.[8] The player ranked first is offered the design contract. If the offer is not accepted, the design contract will be offered to the other players in rank order. For all contests, we observe

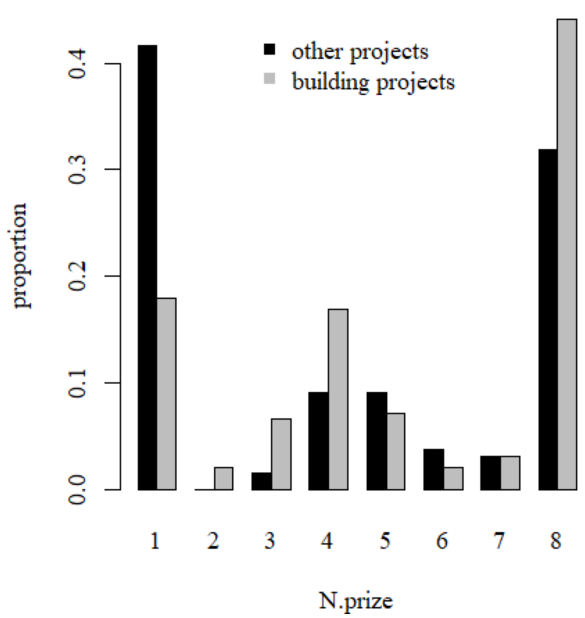
[6] See the Bidding Law of the People's Republic of China (`www.lawinfochina.com/display.aspx?id=27151&lib=law&EncodingName=big5`). We refer to it as "the law" for the rest of the paper.

[7] Players engaged in the design contest only work on the design component and not the construction component. The construction component is carried out by other construction companies.

[8] In practice, this evaluation process may be subject to problems of quality manipulation corruption (Huang, 2019; Huang and Xia, 2019) and uncertainty (Takahashi, 2018). We do not consider these issues in this paper.

the prize scheme, $\mathbf{v} = (v_1, v_2, ..., v_8)$, that specifies the design compensation awarded to the players based on the ranking. These prizes are called design compensation because preparing the design proposal is costly for the players. Note that design compensation is different from *p.design*. The former specifies direct payments to the winner and losers of the proposal contest. The latter is the monetary payment specified in the design contract awarded to the winner to carry out the complete design work after the contest. We compute the total amount of design compensation ($V = \sum_{i=1}^{8} v_i$), the value of prizes for contest losers ($v.loser = \sum_{i=2}^{8} v_i$), and the proportion of loser compensation in the total amount ($v.loser.ratio = v.loser/V = (V - v_1)/V$).

Based on opinions obtained from industry experts, in building and architecture design projects, procurers typically want the design proposals to be more creative and diverse than in other projects (urban planning, transportation, landscaping, electricity, mechanics, and water conservancy). From Table 1, approximately 46% of building projects require the submission of a design model ($D.model$= 1), whereas less than 1% of other projects do. Moreover, *p.design* is also significantly larger for building projects than other projects. This indicates that design work in other projects is relatively standard and does not require a demonstration of innovative ideas. Regarding prize structures, building projects tend to allocate a larger proportion of the compensation budget to losers than other projects. On average, 57.2% of the budget is allocated to the losers in building projects, while in other projects, the figure is 41.4%.



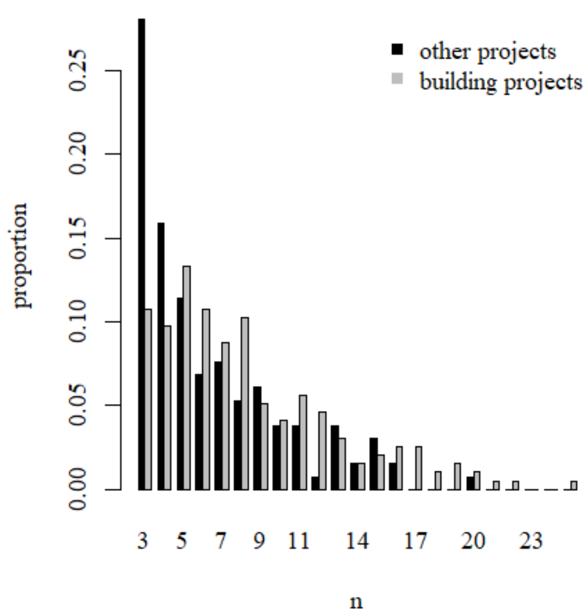


Figure 1: Illustration of Prize Structure

Figure 1-(A) is the histogram of *N.prize*, which is the number of positive prizes offered. In the data, the maximum number of positive prizes is 8. In Table 1, we find that 237 (72.5%) of the 327 contests offer multiple prizes ($D.multi.prize > 1$). This proportion is significantly higher among building projects (82.1%) than other projects (58.3%). On average, a contest offers 4.896

prizes. The average number of prizes of building projects (5.323) is significantly greater than that of other projects (4.265). Hence, the contest organizers of building projects are more inclined to adopt multiple prizes. Figure 1-(B) shows the box plots of prizes by player ranking among all contests offering multiple prizes (Obs= 237). Clearly, the amount of prizes decreases in the ranking. The first runner-up obtains an average compensation of CNY 36,992, while the seventh runner-up obtains an average compensation of CNY 10,434. In fact, all observed prize schemes are weakly decreasing without any exception.

We observe the number of players (n) who submit a complete design proposal before the call-for-proposals deadline. On average, a contest attracts 8.4 players. Figure 2 shows the distribution of n.[9] In general, there are more players participating in building projects than in others.

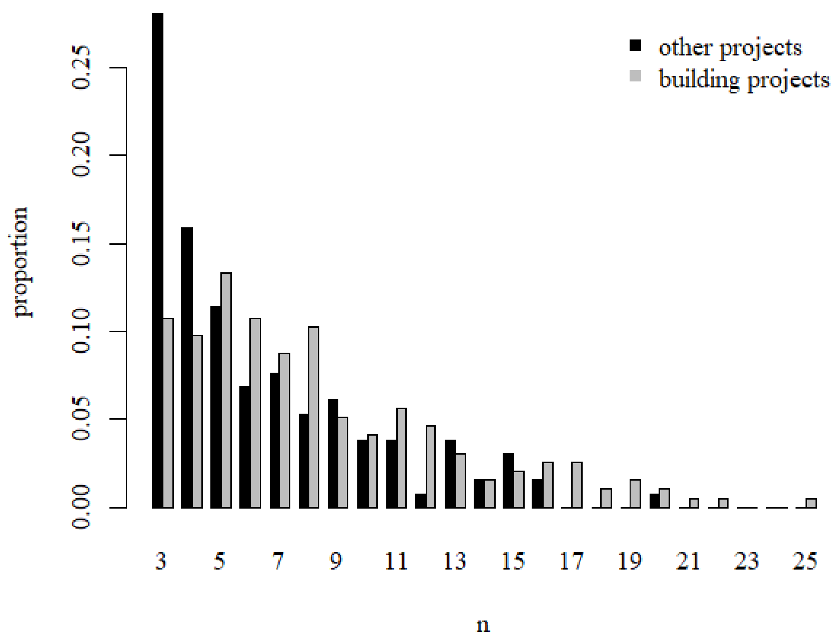


Figure 2: Histogram of Number of Players

Figure 3 depicts the relationship between the prize structure and the number of players entering the contest. There is a clear positive correlation between *v.loser.ratio* and n, which suggests that allocating a larger proportion of the budget to compensate losers attracts more players to submit proposals. This empirical data pattern is consistent with the experimental evidence in Cason et al. (2010), which shows that dividing a fixed prize in proportion to participant contributions induces more entries and greater total effort.

[9] The law requires that at least three players submit bids (proposals) to maintain the competitiveness of the contest. Otherwise, the procurement fails, and the procurer needs to launch another contest.

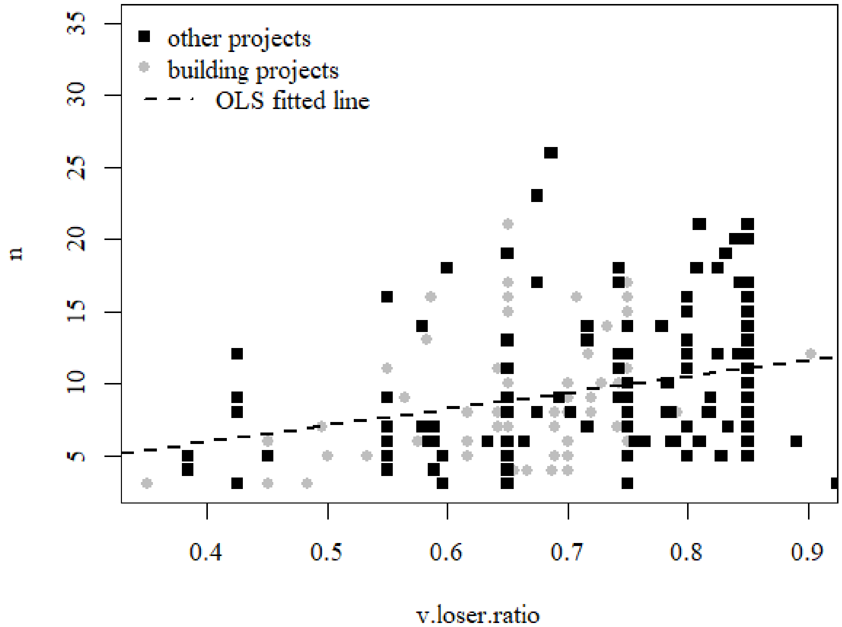


Figure 3: Prize Structure and Player Entry

Table 2: Prize Structure and Player Entry

	Dependent variable: n			
	(1)	(2)	(3)	(4)
V	0.047***	0.032***	0.043***	0.026***
	(0.007)	(0.007)	(0.007)	(0.007)
v.loser.ratio		4.226***		4.361***
		(0.768)		(0.809)
D.building			1.941***	1.114*
			(0.571)	(0.567)
p.design			0.0001	0.0004*
			(0.0002)	(0.0002)
p.construct			−0.00000	−0.00000*
			(0.00000)	(0.00000)
proposal.day			0.001	0.0001
			(0.011)	(0.011)
design.day			0.004	0.006**
			(0.003)	(0.003)
D.model			0.289	0.222
			(0.608)	(0.581)
D.prequalify			−0.816	−1.370
			(1.769)	(1.694)
area			0.004*	0.004
			(0.002)	(0.002)
city FE	✓	✓	✓	✓
Observations	327	327	326	326
R^2	0.160	0.235	0.236	0.304

Note: For all regressions in this paper, * indicates significance at 10%; ** indicates significance at 5%; and *** indicates significance at 1%.

Table 2 further demonstrates the endogenous entry pattern of design contests. Having a large total prize amount (V) attracts players. Regressions (2) and (4) demonstrate that allocating a larger proportion of resources to contest losers significantly increases the number of players. Regressions (1) and (3) do not include *v.loser.ratio*, which is a measure of the prize structure. As a result, their R^2s are lower than those in regressions (2) and (4), respectively.

In Table 3, we use v_1 and *v.loser* as measures of the prize structure. In regressions (2) and (4), we find that the winner's prize does not have a significant impact on n. However, designating more resources for losers significantly increases the number of players.

In summary, many real-world contest organizers choose to offer multiple prizes, and offering multiple prizes can effectively attract more players to enter the contest. Moreover, for projects requiring more innovative and diverse ideas (building and architectural design), contest organizers tend to offer more prizes. These data patterns serve as motivating evidence for our model in which the contest organizer exhibits a preference for the diversity of ideas.

Table 3: Price Structure and Player Entry (Continued)

	Dependent variable: n			
	(1)	(2)	(3)	(4)
v_1	0.151***	0.020	0.137***	0.021
	(0.025)	(0.050)	(0.025)	(0.049)
v.loser		0.056***		0.051***
		(0.019)		(0.019)
D.building			1.991***	1.924***
			(0.578)	(0.573)
p.design			0.0002	0.0001
			(0.0002)	(0.0002)
p.construct			−0.00000	−0.00000
			(0.00000)	(0.00000)
proposal.day			−0.001	0.002
			(0.011)	(0.011)
design.day			0.004	0.004
			(0.003)	(0.003)
D.model			0.324	0.291
			(0.615)	(0.609)
D.prequalify			−0.296	−0.904
			(1.787)	(1.782)
area			0.004*	0.004*
			(0.002)	(0.002)
city FE	✓	✓	✓	✓
Observations	327	327	326	326
R^2	0.136	0.160	0.217	0.236

4 The Model

Motivated by the evidence of endogenous entry and the coexistence of different prize structures, we propose a contest model with an opportunity cost of entry and a contest organizer with a preference for diversity. We show that the preference for diversity explains the organizer's optimal choice between winner-take-all and multiple prizes.

4.1 Model Setup

Consider a contest environment with a contest organizer and N (≥ 2) potential players. The contest organizer and all players are risk-neutral. The contest organizer has a fixed budget $V(> 0)$ for the procurement of an innovative project (product). Retaining the budget does not increase her payoff. Players participate in the contest by submitting proposals that demonstrate their ideas.

For a particular project, players have different ideas due to their backgrounds, expertise, and experiences. A generic player i is endowed with an idea t_i, which is his privately informed type. Assume that t_i's are independently drawn from the distribution function $F(\cdot)$. $F(\cdot)$ has a compact support $[a, b]$ with $a > 0$ and a strictly positive probability density function $f(\cdot)$ on $[a, b]$.

In the innovation contest, there exists an ideal design or the best idea located somewhere in the interval $[a, b]$. Neither the organizer nor the players observe the ideal design until the proposals are submitted and winners are determined. The organizer then utilizes the proposals to approximate the ideal design.[10] Hence, the contest organizer does not necessarily prefer a "lower" or a "higher" idea. In practice, it is common for the final design to incorporate ideas from several proposals collected from the contest. For example, in the EXPO 2010 China pavilion design contest, the final construction plan used the main idea from the winner in conjunction with elements from three other proposals.[11] According to the law, offering design compensation to losers allows the procurer to claim the intellectual property of the losing proposals. In most contest announcements, we observe the following clause: "Except for authorship, the other intellectual property rights belong to the procurer. The procurer has the right to refer to and use them in the project implementation process without paying additional fees." We assume that the organizer can combine the ideas in submitted proposals to approximate the ideal design once it is realized. To successfully combine the ideas, it is important that the ideal design falls into the *idea coverage*, which is the range between the "highest" and the "lowest" possible ideas of entrants. If the ideal design lies outside of the coverage of proposed ideas, the organizer would not be able to combine the ideas in the desired way. Therefore, idea coverage is an important objective in innovation contests.

A player's idea can be conventional or unrestrained. Proposals are usually evaluated by a committee of experts, whose ratings or votes determine the ranking of players. It is easier to convince the committee of a proposal based on a more conventional idea than a less conventional idea. To capture this feature of innovation contests, we assume that demonstrating a less conventional idea in a proposal requires a higher cost. We refer to idea b as the most conventional idea and assume that it is less costly to prepare a proposal for a conventional idea with a given quality. For a more

[10] Letina and Schmutzler (2019) assume that the ideal approach is unknown to all parties. Zhu (2019) assumes that the ideal design is unknown to the players but known to the contest organizer.

[11] See `en.wikipedia.org/wiki/China_pavilion_at_Expo_2010`.

unconventional idea located closer to a, the player needs to incur a higher marginal cost to produce a high-quality proposal to demonstrate it. Specifically, to prepare a proposal of quality $e_i > 0$, player i incurs a cost of e_i/t_i. A player's payoff is the prize he receives minus the cost of preparing the proposal.

Moreover, each player incurs a commonly known opportunity cost $c \in (0, V/N]$ to participate in the contest, which means that a player's payoff is c if he does not enter the contest (Liu and Lu, 2019).[12] Opportunity costs are ubiquitous in innovative contests. For a particular contest, players need to spend time and resources acquiring project-related information before entering the contest and forgo the opportunity to participate in other contests due to limited capacity. With the presence of entry costs, players will selectively enter the contest based on the prize scheme and their private information.

The contest organizer offers an N-vector prize scheme, $\mathbf{v} = (v_1, v_2, \ldots, v_N)$, with $v_1 \geq v_2 \geq \cdots \geq v_N \geq 0$ and $\sum_{i=1}^{N} v_i \leq V$. v_i is the prize for the player with the ith highest quality. Given $\mathbf{v}$, each player decides whether to participate in the contest. When there are n entrants, these entrants win the respective first n prizes, $(v_1, v_2, \ldots, v_n)$, from $\mathbf{v}$ according to the ranking of the quality of their proposals. The winner receives v_1; the first runner-up receives v_2; ...; the player ranked at the bottom receives v_n; and all nonparticipating players obtain 0 prizes. For convenience, we call the scenario with n entrants scenario n.

The timing of the game is as follows.

• Period 0: Each of N players independently draws his idea t from the distribution F.

• Period 1: The contest organizer chooses $\mathbf{v}$ and commits to it.

• Period 2: All potential players simultaneously decide whether to participate in the contest by bearing the opportunity cost c.

• Period 3: Each entrant chooses his quality level after learning the number of entrants n.

• Period 4: The prize allocation is implemented according to $\mathbf{v}$.

The contest organizer's payoff consists of two parts: the total quality from all participating players and the diversity of ideas. The organizer wants all players to spend time and effort developing and demonstrating their ideas in high-quality proposals, so she cares about the total quality instead of the quality of the winning proposal only. This follows from the same reason that the contest organizer maximizes total effort in the literature (e.g., Moldovanu and Sela 2001; Liu and Lu 2019). In addition, the diversity of ideas matters, which is measured by idea coverage - the maximum difference among ideas of all participating players. This is because a larger such range is more likely to include the ideal design *ex ante*, which in turn leads to a higher *ex ante* probability that the organizer is able to combine ideas from submitted proposals to approximate the ideal design.

The contest organizer's goal is to choose a prize allocation rule, $\mathbf{v}$, to maximize the weighted sum of total quality and diversity using her budget V. The weight, $\lambda \in [0, 1]$, measures the contest

[12] $c \leq V/N$ ensures that the contest organizer can induce full entry if she wishes.

organizer's preference for diversity and the extent to which she wishes to, *ex ante*, be able to approximate the ideal design. The exact expression for her objective function is given below in expression (O-λ).

4.2 Equilibrium Analysis

As a first step, we characterize the equilibrium for any given prize structure $\mathbf{v}$. It forms the basis for the search for the optimal prize allocation rule. Since the contest rule is anonymous and players are symmetric, it is natural to focus on symmetric equilibria. Throughout the paper, we restrict our attention to monotone equilibria in which quality increases in t. In equilibrium, proposals of conventional ideas have higher quality and are more likely to win. Unconventional ideas are less likely to win, as it is more costly to write high-quality and convincing proposals for these ideas.

In period 2, we focus on a symmetric Bayesian Nash equilibrium of the entry decision characterized by a threshold type (or idea) t^c. Given F and $\mathbf{v}$, a player will enter the contest if and only if his type $t \geq t^c$; otherwise, he will not participate.[13] The participation constraint requires that the expected payoff is greater than the opportunity cost c. Under monotone equilibria, a player with the threshold type t^c has the most unconventional idea among all entrants, and thus he will be allocated the lowest prize v_n in scenario n, for any n. Moreover, the player with t^c must choose $e = 0$ in equilibrium and has an expected payoff of c from the contest.[14] Therefore, the threshold type t^c is determined by

$$\sum_{n=1}^{N} p_n(t^c)v_n = c, \tag{1}$$

where $p_n(t^c) = \binom{N-1}{n-1}(1 - F(t^c))^{n-1}F^{N-n}(t^c)$ is the probability that there are exactly $n - 1$ rival players entering the contest.

In period 3, given the entry threshold $t^c \in [a, b)$ determined by (1), let n be the realized number of entrants. An entrant knows that each of his rivals has a type independently drawn from the truncated distribution function $G(t, t^c) = \frac{F(t)-F(t^c)}{1-F(t^c)}$ with density function $g(t, t^c) = \frac{f(t)}{1-F(t^c)}$ and support $t \in [t^c, b]$. The equilibrium bidding function and total quality of n entrants are as follows.

Proposition 1. *Suppose that the induced entry threshold is t^c. In scenario $n \geq 1$ with prizes $\mathbf{W}_n \equiv (v_1, v_2, \ldots, v_n)$, we have the following:*
(i) The unique symmetric monotone bidding function $e^{(n)}(t, \mathbf{W}_n, t^c)$ for type $t \in [t^c, b]$ is

$$e^{(n)}(t, \mathbf{W}_n, t^c) = tV^{(n)}(t) - \int_{t^c}^{t} V^{(n)}(s)ds - t^c v_n,$$

[13] The proof is standard based on Samuelson (1985) and McAfee and McMillan (1987).
[14] A player with the threshold type may enjoy a payoff strictly larger than c if the prize structure induces full entry. However, this is clearly not optimal, unless the organizer does not value total quality (i.e., $\lambda = 1$ in (O-λ) below).

where

$$V^{(n)}(t) = \sum_{j=1}^{n} v_{n+1-j} \binom{n-1}{j-1} G^{j-1}(t,t^c)(1-G(t,t^c))^{n-j}$$

is the expected prize that an entrant of type t obtains.
(ii) The corresponding scenario-n total quality is

$$TE^{(n)}(\mathbf{v},t^c) = n\int_{t^c}^{b} J(t)V^{(n)}(t)g(t,t^c)dt - nt^c v_n,$$

where $J(t) = t - \frac{1-F(t)}{f(t)}$.

In period 1, before n is realized, the expected total quality is the weighted average of scenario-n expected total quality across all scenarios. Specifically,

$$TE(\mathbf{v},t^c) = \sum_{n=1}^{N} \binom{N}{n} (1-F(t^c))^n F^{N-n}(t^c) TE^{(n)}(\mathbf{v},t^c), \tag{2}$$

where $TE^{(n)}(\mathbf{v},t^c)$ is given in Proposition 1. Note that whether the player knows the number of rivals does not affect the expected total quality or the results of contest design. If players do not know the number of rival(s), a player of type $t \geq t^c$ will choose to bid $\sum_{n=1}^{N} p_n(t^c)e^{(n)}(t,\mathbf{W}_n,t^c)$, which is the weighted average of the bidding functions in different scenarios. As a result, the expected total quality is the same as that in (2).

Let $G_{(i,n)}(t,t^c)$ denote the CDF of the ith order statistics of n independent random variables, with each following CDF $G(t,t^c)$. The CDF of the ith order statistics is $G_{(i,n)}(t,t^c) = \sum_{j=i}^{n} \binom{n}{j} G^j(t,t^c)(1-G(t,t^c))^{n-j}$, with density function

$$g_{(i,n)}(t,t^c) = n\binom{n-1}{i-1} G^{i-1}(t,t^c)(1-G(t,t^c))^{n-i} g(t,t^c).$$

The following result shows that the expected total quality can be expressed as a linear function in all N prizes, with coefficients related to all these order statistics across different scenarios and different orders.

Lemma 1. *The expected total quality (2) can be rewritten as*

$$TE(\mathbf{v},t^c) = N(1-F(t^c))\left[\sum_{n=1}^{N} \frac{p_n(t^c)}{n} \sum_{j=1}^{n} v_{n+1-j}\left(\int_{t^c}^{b} J(t)g_{(j,n)}(t,t^c)dt\right) - ct^c\right].$$

Lemma 1 implies that we can rewrite the expected total quality as

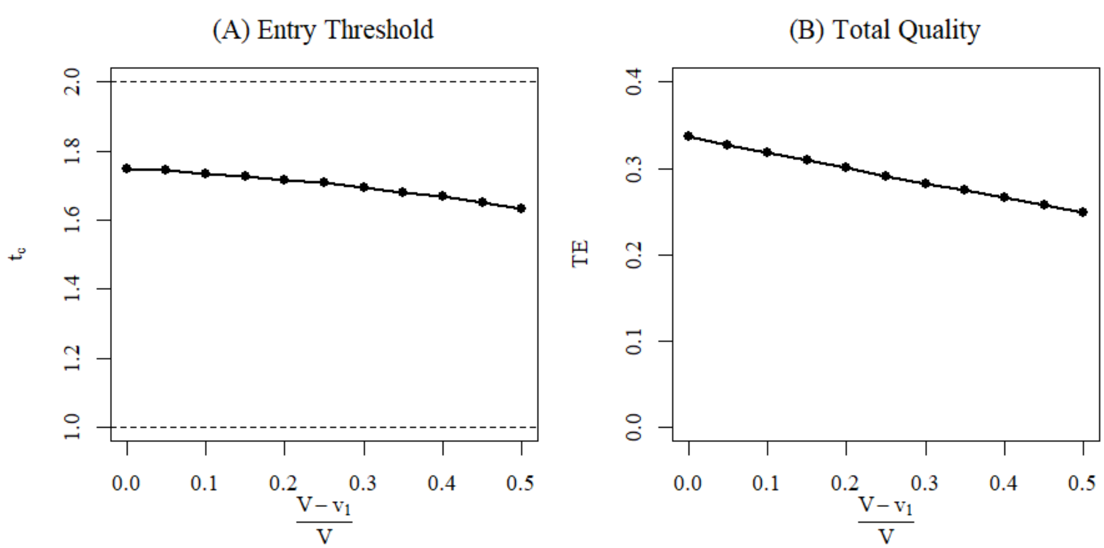
$$TE(\mathbf{v},t^c) = \sum_{k=1}^{N} \beta_k(t^c)v_k - Nc(1-F(t^c))t^c, \tag{3}$$

where $\beta_k(t^c)$ is the coefficient associated with v_k and so

$$\beta_k(t^c) = N(1 - F(t^c)) \sum_{n=k}^{N} \frac{p_n(t^c)}{n} \left(\int_{t^c}^{b} J(t) g_{(n+1-k,n)}(t, t^c) dt \right), \ k = 1, 2, \ldots, N. \tag{4}$$

Although the expected total quality is essentially a linear function, it is not easy to characterize its optimum for each fixed t^c under the participation constraint. As a linear function, it is important to investigate the properties of its coefficients to find its optimum. However, these coefficients are quite complicated because they involve binomials and weighted averages of various order statistics. We will discuss how to resolve this challenge in Section 4.3.

Figure 4 illustrates the equilibrium entry threshold and total quality with a numerical example. In the example, we consider uniformly distributed types and the prize scheme with two positive prizes, $\mathbf{v} = (v_1, V - v_1, 0, \ldots, 0)$. Panel (A) indicates that allocating a larger proportion of the budget to the second prize lowers the entry threshold and thereby encourages entry. However, Panel (B) shows that the total quality decreases in the prize offered to the runner-up. This is consistent with the result in Liu and Lu (2019) that the winner-take-all prize structure maximizes total effort.



Note: $N = 5$, $t \sim$Uniform$[1, 2]$, $V = 0.01$, $c = 0.002$, and $\mathbf{v} = (v_1, V - v_1, 0, 0, 0)$. The horizontal axis, $\frac{V-v_1}{V}$, is the proportion of the budget used for compensating the runner-up.

Figure 4: Entry Threshold and Total Quality

4.3 Preference for Diversity and Contest Design

We are now ready to present the main result: The presence of both winner-take-all and multiple prizes in the data can be explained by the contest organizer's preference for diversity.

The Organizer's Problem

We propose the contest organizer's problem as follows:[15]

$$\text{(O-}\lambda\text{)} \qquad \max_{t^c \in [a,b),\ \mathbf{v}} P(\mathbf{v}, t^c; \lambda) = (1-\lambda) TE(\mathbf{v}, t^c) + \lambda(b - t^c)$$

$$\text{subject to } \sum_{n=1}^{N} v_n \le V \text{ and } \sum_{n=1}^{N} p_n(t^c) v_n = c,$$

where $TE(\mathbf{v}, t^c)$ is given in (2) and $\lambda \in [0,1]$ is a fixed number. The two constraints in the optimization problem are the contest organizer's budget constraint and the participation constraint for the threshold type t^c. The contest organizer's goal is to find a prize scheme $\mathbf{v}$ (and hence t^c) to maximize her payoff $P(\mathbf{v}, t^c; \lambda)$.

The objective function $P(\mathbf{v}, t^c; \lambda)$ is a convex combination of the total quality and the diversity of ideas. As noted above, diversity is measured by the maximum difference between ideas among participating players, which is $b - t^c$ by Proposition 1.[16] The parameter λ captures the degree of preference for diversity. Across different kinds of contests, contest organizers may value diversity differently. We will show that it is precisely such a difference in the degree of preference for diversity that drives the optimality of winner-take-all or multiple prizes observed in the data.

Note that when $\lambda = 0$ and $c = 0$, our model reduces to Moldovanu and Sela (2001); when $\lambda = 0$ and $c > 0$, our model corresponds to the setting of Liu and Lu (2019) except that the prize allocation rule in Liu and Lu (2019) can be contingent on the number of entrants n. However, we follow Moldovanu and Sela (2001) and do not allow this contingency, because most prize allocation rules in reality are not contingent on n. This is also the case for the observed proposal contests in Section 3. Liu and Lu (2019) demonstrate the complexity of analyzing the optimal contest design problem under a larger set of prize structures. As explained and analyzed in Liu and Lu (2023), due to endogenous entry, establishing the optimum in a restricted class of prize schemes can be much more technically challenging because of the presence of more constraints. Together with the preference for diversity, the analysis of problem (O-λ) would be even more complicated.

The Challenge

Given Proposition 1, the expression for the contest organizer's objective function in (O-λ) has various binomials and truncated distribution functions, which is very cumbersome. We analyze the problem by first examining the value function of the organizer's problem for any fixed $t^c \in [a, b]$

[15] When $t^c = b$, it is obvious that $P(\mathbf{v}, b; \lambda) = 0$, which can never be optimal. Additionally, note that in the contest organizer's problem, once $\mathbf{v}$ is given, t^c is pinned down correspondingly by equation (1). Including t^c as a choice variable in the contest organizer's problem is purely for notational convenience.

[16] For tractability, we use $b - t^c$ as the measure of idea diversity. We expect that alternative measures, such as the number of entrants and the expected difference between the highest and the lowest realized ideas, also lead to qualitatively similar results, following a similar intuition - the central tradeoff between idea diversity and quality elicitation.

and any fixed λ. In this case, the contest organizer's problem is the same as

$$\max_{\mathbf{v}} TE(\mathbf{v}, t^c), \text{ subject to } \sum_{n=1}^{N} v_n \leq V \text{ and } \sum_{n=1}^{N} p_n(t^c) v_n = c.$$

Denote the value function of the above problem as $TE^*(t^c)$ and the set of optimal solution(s) as $S^*(t^c)$. Then, the contest organizer's problem can be restated as

$$\max_{t^c \in [a,b]} (1-\lambda) TE^*(t^c) + \lambda(b - t^c).$$

In the second-step analysis, if we can further pin down the optimal entry threshold t^{c*}, then $S^*(t^{c*})$ will be the set of optimal prize schemes.

This two-step analysis relies on the characterization of the optimum in the first step. Note that for any fixed entry threshold t^c, the contest organizer's problem is linear programming in the sense that all functions in the problem—the objective function, the budget constraint, and the participation constraint—are linear in the N prizes. Therefore, the properties of these coefficients are crucial for the first-step analysis.

However, Liu and Lu (2023) note that coefficients in this linear programming problem vary in a highly intractable and nonlinear way as the induced entry threshold t^c changes. The arbitrariness and intractability of coefficients in the linear programming problem cause complications in characterizing the optimum for a fixed entry threshold, which makes the aforementioned two-step procedure quite challenging if not infeasible. Liu and Lu (2023) solve their problem in an indirect way that does not require solving the optimum for any given entry threshold. However, their approach does not apply to our model, because prizes here have to be nonnegative (more constraints) and the objective function has an additional term (the preference for diversity). Therefore, our problem is even more challenging than that in the literature.

In general, the number of positive prizes varies with the entry threshold. This issue is further complicated by the restriction imposed by the participation constraint: For a fixed t^c, only a certain restrictive subset of prize schemes induces it. For example, winner-take-all induces a unique entry threshold, so any other entry thresholds cannot be supported by winner-take-all. Fortunately, although explicitly identifying the optimum is inapplicable, we are able to characterize the essential feature of the solution indirectly as follows.

The Analysis

For a given λ, denote the set of optimal entry thresholds as $t^{c*}(\lambda)$. First, when $\lambda = 1$, the organizer does not value quality at all, so she maximizes $b - t^c$, which can only be achieved by setting $t^c = a$. Therefore, any prize structure $\mathbf{v}$ that supports full entry is optimal. Clearly, winner-take-all cannot induce full entry because for $\mathbf{v} = (V, 0, \ldots, 0)$, $\sum_{n=1}^{N} p_n(a) v_n = v_N = 0 < c$. On the other hand, $\mathbf{v} = (V/N, V/N, \ldots, V/N)$ supports full entry because $\sum_{n=1}^{N} p_n(a) v_n = v_N = \frac{V}{N} \geq c$. Therefore, when $\lambda = 1$, the set of optimal solutions to problem (O-λ) is nonempty and winner-take-all is not optimal. Multiple positive prizes must arise at the optimum when $\lambda = 1$.

Now, consider the case in which $\lambda < 1$. Problem (O-λ) can be analyzed in a two-step way. We first fix the entry threshold t^c to characterize the optimum within the class of prize allocation rules inducing t^c. Then, we vary across all entry thresholds $t^c \in [a, b)$ to pin down the optimum. Note that when $t^c = b$, the induced total quality $TE(\mathbf{v}, b) = 0$ for any $\mathbf{v}$, and the diversity $b - t^c$ is also zero. Therefore, $t^c = b$ can never be optimal.

In the first step, for any fixed $t^c \in [a, b)$, since $\lambda \in [0, 1)$ and t^c is fixed, the contest organizer's problem is equivalent to the following problem:[17]

$$\text{(O1-}\lambda\text{)} \qquad \max_{\mathbf{v}} TE(\mathbf{v}, t^c) \text{ subject to } \sum_{n=1}^{N} v_n \leq V, \text{ and } \sum_{n=1}^{N} p_n(t^c) v_n = c.$$

Denote the value function of problem (O1-λ) as $TE^*(t^c)$ and the set of optimal solution(s) as

$$S^*(t^c) = \{\mathbf{v} \in S(t^c) : TE(\mathbf{v}, t^c) = TE^*(t^c)\}, \tag{5}$$

where $S(t^c)$ is the feasible set of problem (O1-λ), i.e.,

$$S(t^c) = \{\mathbf{v} \in \mathbb{R}^N : v_1 \geq v_2 \geq \cdots \geq v_N \geq 0, \sum_{n=1}^{N} v_n \leq V, \text{ and } \sum_{n=1}^{N} p_n(t^c) v_n = c\}.$$

Because $TE(\mathbf{v}, t^c)$ is continuous in $\mathbf{v}$ and the feasible set is compact, the existence of optimal solutions and the value function are guaranteed by the extreme value theorem. To ease notation, define $TE^*(b) = 0 = \lim_{t^c \to b^-} TE^*(t^c)$.

The following result characterizes the optimal $TE^*(t^c)$ and $S^*(t^c)$, which implies that when ignoring the preference for diversity (because we fix the entry threshold in this step), winner-take-all is the unique optimal prize structure and its corresponding entry threshold is also the unique optimal entry threshold. This result echoes Liu and Lu (2019).

Lemma 2. *$TE^*(t^c)$ reaches its unique optimum over $t^c \in [a, b)$ at $t_1 = F^{-1}\left[\left(\frac{c}{V}\right)^{\frac{1}{N-1}}\right]$. Furthermore, $S^*(t_1) = \{(V, 0, \ldots, 0)\}$, that is, winner-take-all is the unique optimum.*

Now, in the second step, we solve the following problem:

$$\max_{t^c \in [a,b)} P^*(t^c; \lambda) = (1 - \lambda) TE^*(t^c) + \lambda(b - t^c), \tag{6}$$

where $P^*(t^c; \lambda)$ is the value function of problem (O-λ) when fixing the entry threshold at t^c. Note that for any $t^c > t_1$, it cannot be the solution to the above problem, because by Lemma 2,

$$P^*(t^c; \lambda) = (1 - \lambda) TE^*(t^c) + \lambda(b - t^c) < (1 - \lambda) TE^*(t_1) + \lambda(b - t_1) = P^*(t_1; \lambda).$$

[17] When two optimization problems have the same set of solutions, we say that they are equivalent.

Hence, we can restrict our attention to the domain $t^c \in [a, t_1]$. In other words, in the second step, we solve the following problem:

$$\text{(O2-}\lambda\text{)} \qquad \max_{t^c \in [a,t_1]} P^*(t^c; \lambda).$$

Recall that for a given λ, $t^{c*}(\lambda)$ denotes the set of optimal entry thresholds in problem (O-λ). Using (5), the optimal prize scheme for problem (O-λ) can be denoted as $S^*(t^{c*}(\lambda)) = \{\mathbf{v} : \mathbf{v} \in S^*(t^c),\ \forall t^c \in t^{c*}(\lambda)\}$. Note that $S^*(t^{c*}(\lambda))$ may not be a singleton, and any $\tilde{\mathbf{v}} \in S^*(t^{c*}(\lambda))$ satisfies

$$P(\tilde{\mathbf{v}}, t^*(\lambda); \lambda) = P^*(t^*(\lambda); \lambda),\ \forall t^*(\lambda) \in t^{c*}(\lambda),\ \forall \lambda \in [0, 1]. \tag{7}$$

Lemma 2 implies that $t^{c*}(0) = \{t_1\}$ and $S^*(t^{c*}(0)) = \{(V, 0, \ldots, 0)\}$, i.e., winner-take-all is the unique optimum when $\lambda = 0$. The following observation is helpful, which gives an equivalent characterization of the optimality of winner-take-all in problem (O-λ) using the induced entry threshold in problem (O2-λ).

Lemma 3. *The winner-take-all prize allocation is a solution to problem (O-λ) if and only if t_1 is a solution to problem (O2-λ). Furthermore, winner-take-all is the unique solution to problem (O-λ) if and only if t_1 is the unique solution to problem (O2-λ).*

Lemma 3 further implies the following result.

Lemma 4. *(i) If there is some $\lambda_0 \in [0, 1)$ such that winner-take-all is the unique solution to problem (O-λ_0), then winner-take-all remains the unique optimum of problem (O-λ) for any $\lambda \in [0, \lambda_0]$.*
(ii) If there is some $\lambda_1 \in (0, 1]$ such that winner-take-all is not a solution to problem (O-λ_1), then winner-take-all is not a solution to problem (O-λ) for any $\lambda \in [\lambda_1, 1]$.
(iii) If there is some $\lambda_2 \in (0, 1)$ such that winner-take-all is a solution, but not the unique solution, to problem (O-λ_2), then such λ_2 must be unique.

This result reveals the relationship between the winner-take-all prize and inducing diversity. Whenever winner-take-all is optimal for some degree of preference for diversity, the contest organizer should continue using winner-take-all when she values diversity less; on the other hand, whenever using multiple prizes is optimal for some degree of preference for diversity, she should still offer multiple prizes when she values diversity more.

Recall Lemma 2 that when the organizer does not value diversity (i.e., $\lambda = 0$), winner-take-all is the unique optimum (equivalently, t_1 is the unique optimal entry threshold). On the other hand, can winner-take-all still be optimal when the organizer values diversity? Surprisingly, the answer is "Yes." Denote $S_1 = \{\lambda \in [0, 1] : t_1 \text{ is the unique solution to problem (O2-}\lambda)\}$. Lemma 2 implies that $0 \in S_1$. The following lemma reveals that S_1 cannot be a singleton, so winner-take-all can still be optimal even when the organizer values diversity.

Lemma 5. $S_1 \neq \{0\}$.

The Optimal Prize Design

Combining all the above observations, we can characterize the essential feature of the optimal solution indirectly, which brings us the key result as follows.

Proposition 2. *There exists a unique* $\tilde{\lambda} \in (0,1)$*, such that the followings hold:*
(i) when $\lambda \in [0, \tilde{\lambda})$*, the winner-take-all prize structure, that is,* $\mathbf{v} = (V, 0, \ldots, 0)$*, is the unique optimal prize structure;*
(ii) when $\lambda = \tilde{\lambda}$*, the winner-take-all prize structure is optimal but may not be unique; and*
(iii) when $\lambda \in (\tilde{\lambda}, 1]$*, the winner-take-all prize structure is not optimal, and multiple positive prizes must arise at the optimum.*

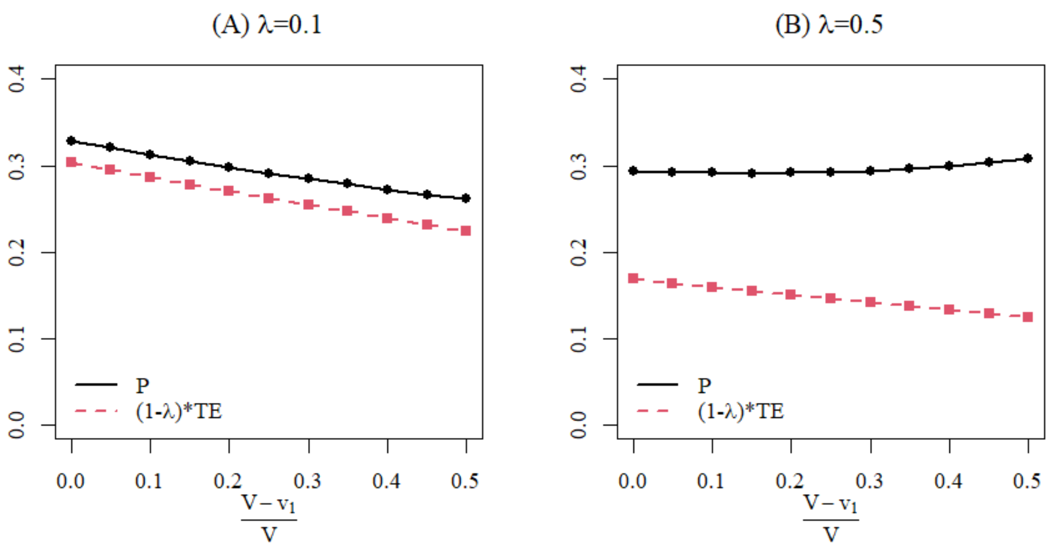


Figure 5: Contest Organizer Objective Function

Figure 5 illustrates Proposition 2 using the same numerical example as in Figure 4. In Panel (A), the preference for diversity is low ($\lambda = 0.1$), and the winner-take-all prize scheme maximizes the objective function in (O-λ). In contrast, Panel (B) shows that when the preference for diversity becomes large ($\lambda = 0.5$), allocating half of the budget as the second prize is optimal.

The Role of Preference for Idea Diversity

Proposition 2 reveals that the degree of preference for idea diversity explains the prize allocation patterns observed in the data. The winner-take-all prize scheme is the unique optimum if and only if the contest organizer is not particularly concerned about the diversity of ideas ($\lambda < \tilde{\lambda}$). However, as long as the preference for diversity is sufficiently strong ($\lambda > \tilde{\lambda}$), the contest organizer should offer multiple prizes. These observations are consistent with the empirical observations that a significantly larger proportion of contests for building projects offer multiple prizes than those for other projects, because procurers of building and architectural design typically value more diverse and creative ideas.

We emphasize that the threshold $\tilde{\lambda} > 0$. This means that a low degree of preference for diversity does not necessarily lead to the multiplicity of prizes. According to the law, the procurer can use the ideas in the losing proposals by paying design compensation to losers. Then, it seems trivial that as long as the contest organizer cares at least a little about diversity, she should always offer at least a sufficiently small prize to all losers to claim their property rights. This is certainly inconsistent with our data that there are also many contest rules that offer a single prize.

Our model nicely explains this: The contest organizer offers multiple prizes if and only if she cares sufficiently enough about diversity ($\lambda > \tilde{\lambda}$). The fact that $\tilde{\lambda} > 0$ implies that she may still offer a single prize even if she cares about diversity, which further explains why winner-take-all is still used even in some industries that value idea diversity. This observation is summarized below.

Corollary 1. *Even if the organizer values the diversity of ideas, winner-take-all can still be the unique optimum, which is the case when* $\lambda < \tilde{\lambda}$.

As a final remark, note that the two factors introduced in the model—opportunity cost (or equivalently, endogenous entry) and preference for diversity—are both essential in justifying the empirical observations in innovation contests. A preference for diversity alone cannot justify the use of multiple prizes. This is obvious: In Moldovanu and Sela (2001), full entry is assumed (i.e., t^c is fixed at a), and the winner-take-all prize scheme maximizes both the total quality and the type difference (diversity). On the other hand, if there is only endogenous entry (i.e., $\lambda = 0$), Liu and Lu (2019) show that winner-take-all is the unique optimum, and thus, it cannot rationalize why contest organizers offer multiple prizes.

5 Conclusion

We gather a dataset of hundreds of contests for the procurement of innovative design works. A majority of these contests adopt prize schemes that compensate not only the winner but also one or several losers. To explain this phenomenon, we establish a contest model with endogenous entry, in which the contest organizer values both the total quality and the diversity of ideas. Multiple prizes can be justified only if an opportunity cost of entry and a sufficiently strong preference for diversity are both present. Our model also explains why winner-take-all is still used even in some industries that value diversity.

In innovation contests, the optimal prize structure depends on several important aspects of the nature of procurement such as the uncertainty of ideal design and the preference for the diversity of ideas. A winner-take-all prize scheme can be optimal in eliciting players' effort in presenting their ideas in high-quality proposals but may discourage players with unconventional ideas. Offering compensation to contest losers can stimulate players to propose unconventional and bold designs.

The application of the new objective function proposed in the paper is not limited to contests for creative designs or innovative ideas. Even for contests where participants are purely vertically differentiated, the prize scheme may not be winner-take-all. For example, in a sales contest, the contest organizer may offer multiple prizes to encourage those who are inexperienced and left behind such that they remain with the job until they become experienced. We expect that our modeling framework of contests with a wider class of objective functions can be applied to the analysis of other competitive environments.

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Appendix

A. Proofs

Proof of Proposition 1. See Lemma 1 in Liu and Lu (2019).

Proof of Lemma 1. By Proposition 1,

$$\begin{aligned}
TE^{(n)}(\mathbf{v}, t^c) &= n\int_{t^c}^{b} J(t)V^{(n)}(t)g(t,t^c)dt - nt^c v_n \\
&= n\int_{t^c}^{b} J(t)[\sum_{j=1}^{n} v_{n+1-j}\binom{n-1}{j-1}G^{j-1}(t,t^c)(1-G(t,t^c))^{n-j}]g(t,t^c)dt - nt^c v_n \\
&= \sum_{j=1}^{n} v_{n+1-j}\left(\int_{t^c}^{b} J(t)g_{(j,n)}(t,t^c)dt\right) - nt^c v_n.
\end{aligned}$$

Therefore,

$$\begin{aligned}
TE(\mathbf{v}, t^c) &= \sum_{n=1}^{N}\binom{N}{n}(1-F(t^c))^n F^{N-n}(t^c)TE^{(n)}(\mathbf{v}, t^c) \\
&= \sum_{n=1}^{N}\binom{N}{n}(1-F(t^c))^n F^{N-n}(t^c)\left[\sum_{j=1}^{n} v_{n+1-j}\left(\int_{t^c}^{b} J(t)g_{(j,n)}(t,t^c)dt\right) - nt^c v_n\right] \\
&= (1-F(t^c))\sum_{n=1}^{N}\frac{Np_n(t^c)}{n}\left[\sum_{j=1}^{n} v_{n+1-j}\left(\int_{t^c}^{b} J(t)g_{(j,n)}(t,t^c)dt\right) - nt^c v_n\right] \\
&= N(1-F(t^c))\sum_{n=1}^{N}p_n(t^c)\left[\frac{\sum_{j=1}^{n} v_{n+1-j}}{n}\left(\int_{t^c}^{b} J(t)g_{(j,n)}(t,t^c)dt\right) - t^c v_n\right] \\
&= N(1-F(t^c))\left[\sum_{n=1}^{N}\frac{p_n(t^c)}{n}\sum_{j=1}^{n} v_{n+1-j}\left(\int_{t^c}^{b} J(t)g_{(j,n)}(t,t^c)dt\right) - ct^c\right],
\end{aligned}$$

where the last equality uses (1). □

Proof of Lemma 2. Liu and Lu (2019) analyze a similar contest design problem with entry costs. The only difference between their model and ours is that the prize allocation rule can be contingent on the number of entrants in their model. Specifically, in their paper, the contest rule is a set of scenario prize vectors $\mathbf{W} = \{\mathbf{W}_1, \mathbf{W}_2, \ldots, \mathbf{W}_N\}$, where in vector $\mathbf{W}_n = (w_{n,1}, w_{n,2}, \ldots, w_{n,n}) \in \mathbb{R}_+^n$, $w_{n,1} \geq w_{n,2} \geq \ldots \geq w_{n,n} \geq 0$ and $\sum_{j=1}^{n} w_{n,j} \leq V$.[18] Here, $w_{n,j}$ is the jth prize in scenario n,

[18] Liu and Lu (2019) normalize the prize budget $V = 1$. It is clear that the contest organizer's problem is linear in V, so their result applies to any $V > 0$, as mentioned in footnote 3 in their paper. Specifically, normalizing every prize and the entry cost by V - i.e., dividing $w_{n,j}$ by V and dividing c by V - returns to their model. The only difference is that the total quality elicitable for budget V is then V times the total quality when the budget is 1 dollar, as is

which is for the jth highest quality. Thus, when fixing the entry threshold at $t^c \in [a,b)$, the contest organizer's problem, as in the first paragraph of Section 3.2 on page 141 of Liu and Lu (2019), can be equivalently written as

$$\text{(O1-}\lambda\text{-contingent)} \qquad \max_{\mathbf{W}} TE(\mathbf{W}, t^c) = \sum_{n=1}^{N} \binom{N}{n} (1-F(t^c))^n F^{N-n}(t^c) TE^{(n)}(\mathbf{W}_n, t^c)$$

$$\sum_{j=1}^{n} w_{n,j} \leq 1,\ w_{n,1} \geq w_{n,2} \geq \ldots \geq w_{n,n} \geq 0, \forall n, \text{ and } \sum_{n=1}^{N} p_n(t^c) w_{n,n} = c.$$

where $TE^{(n)}(\mathbf{W}_n, t^c)$ is characterized in Lemma 1 of Liu and Lu (2019) (or equivalently, Proposition 1 in this paper). We call this problem (O1-λ-contingent) because its objective function is the same as (O1-λ) when restricting the prize allocation rule to be independent of n. In problem (O1-λ-contingent), we ignore the choice variable $\mathbf{V}$ in Liu and Lu (2019). This clearly does not change anything because once the allocation rule $\mathbf{W}$ is fixed, the budget V_n in scenario n is pinned down correspondingly - just as they mention in footnote 7 in Liu and Lu (2019).

Denote the value function, the feasible set, and the set of solutions of problem (O1-λ-contingent) as $TE^{**}(t^c)$, $\tilde{S}(t^c)$, and $\tilde{S}^*(t^c)$, respectively. The feasible set is

$$\tilde{S}(t^c) = \{\{\mathbf{W}_1, \mathbf{W}_2, \ldots, \mathbf{W}_N\} : \sum_{j=1}^{n} w_{n,j} \leq 1, \forall n;\ w_{n,1} \geq w_{n,2} \geq \ldots \geq w_{n,n} \geq 0,\ \forall n; \text{ and } \sum_{n=1}^{N} p_n(t^c) w_{n,n} = c\}.$$

Recall that $S(t^c)$ is the feasible set of problem (O1-λ). Because problem (O1-λ-contingent) allows the allocation rule to be contingent on n, but problem (O1-λ) does not, it is clear that for any $t^c \in [a,b)$, $S(t^c) \subseteq \tilde{S}(t^c)$. Moreover, for any $t^c \in [a,b)$, $TE^*(t^c) \leq V \cdot TE^{**}(t^c)$.[19] Nevertheless, Liu and Lu (2019) show that[20]

$$\text{(8)} \qquad TE^{**}(t_1) > TE^{**}(t^c),\ \forall t^c \in [a,b)\backslash\{t_1\}$$

and $\tilde{S}^*(t_1)$ is a singleton with

$$\tilde{S}^*(t_1) = \{\{\mathbf{W}_1^*, \mathbf{W}_2^*, \ldots, \mathbf{W}_N^*\}\} = \{\{(V),\ (V,0),\ (V,0,0),\ \cdots,\ (V,\ \underbrace{0,\ldots,0}_{N-1 \text{ times}})\},$$

i.e., $\mathbf{W}_n^* = (V,\ \underbrace{0,\ldots,0}_{n-1 \text{ times}})\}$, for all $n \geq 1$.

This optimal prize allocation rule does not depend on n, so it is also feasible in the noncontingent contest design problem in this paper. It is the same as $\mathbf{v}^* = (V, 0, \ldots, 0)$, which implies that

clear from the linear nature revealed in Lemma 1 and Proposition 1 in their paper.

[19]We multiply the value function $TE^{**}(t^c)$ by V, as Liu and Lu (2019) normalize the budget to 1.

[20]Note that t_1 is defined as $F^{-1}(c^{\frac{1}{N-1}})$ in Liu and Lu (2019). As mentioned in footnote 18, the entry cost c in their paper corresponds to c/V in our paper after normalization.

$TE^*(t_1) = V \cdot TE^{**}(t_1)$. As such, given that $TE^*(t^c) \leq V \cdot TE^{**}(t^c)$ for any $t^c \in [a, b]$ and (8), Lemma 2 follows directly. $\square$

Proof of Lemma 3. If winner-take-all is a solution to problem (O-λ), then it must satisfy the participation constraint, $p_1(t^c)V = c$. However, this implies that $t^c = t_1$, which, by (7), further implies that t_1 solves problem (O2-λ). Conversely, if t_1 solves problem (O2-λ), then it means that for problem (O1-λ), when the threshold is t_1, the set of optimal solutions is $S^*(t_1)$. However, Lemma 2 implies that $S^*(t_1)$ is a singleton, that is, $S^*(t_1) = \{(V, 0, \ldots, 0)\}$.

Note that the only possible entry threshold that winner-take-all induces is t_1. Therefore, the above argument for the first statement of Lemma 3 obviously applies to the uniqueness statement of Lemma 3. $\square$

Proof of Lemma 4. We first prove (i). Suppose that there is some $\lambda_0 \in [0, 1)$ such that winner-take-all is the unique solution to problem (O-λ_0). Then, by Lemma 3, t_1 is the unique solution to problem (O2-λ_0). Therefore, $P^*(t_1; \lambda_0) > P^*(t^c; \lambda_0)$, for any $t^c \in [a, t_1)$, which further implies

$$(1 - \lambda_0)\left(TE^*(t_1) - TE^*(t^c)\right) > \lambda_0(t_1 - t^c), \text{ for any } t^c \in [a, t_1).$$

Then, for any $\lambda \in [0, \lambda_0]$ and any $t^c \in [a, t_1)$, since $TE^*(t_1) - TE^*(t^c) > 0$ by Lemma 2,

$$(1 - \lambda)\left(TE^*(t_1) - TE^*(t^c)\right) \geq (1 - \lambda_0)\left(TE^*(t_1) - TE^*(t^c)\right) > \lambda_0(t_1 - t^c) \geq \lambda(t_1 - t^c).$$

Therefore, for any $\lambda \in [0, \lambda_0]$,

$$P^*(t_1; \lambda) > P^*(t^c; \lambda), \text{ for any } t^c \in [a, t_1),$$

which means that t_1 is the unique solution to problem (O2-λ) for any $\lambda \in [0, \lambda_0]$. By Lemma 3, this implies that winner-take-all is the unique solution to problem (O-λ) for any $\lambda \in [0, \lambda_0]$.

Now we turn to (ii). Suppose that there is some $\lambda_1 \in (0, 1]$ such that winner-take-all is not a solution to problem (O-λ_1). By Lemma 3, t_1 is not a solution to problem (O2-λ_1). Therefore, $P^*(t_1; \lambda_1) < P^*(t_2^c; \lambda_1)$, for some $t_2^c \in [a, t_1)$, which further implies

$$(1 - \lambda_1)\left(TE^*(t_1) - TE^*(t_2^c)\right) < \lambda_1(t_1 - t_2^c).$$

Note that $TE^*(t_1) - TE^*(t_2^c) > 0$ by Lemma 2. Then, for any $\lambda \in [\lambda_1, 1]$:

$$(1 - \lambda)\left(TE^*(t_1) - TE^*(t_2^c)\right) \leq (1 - \lambda_1)\left(TE^*(t_1) - TE^*(t^c)\right) < \lambda_1(t_1 - t_2^c) \leq \lambda(t_1 - t_2^c),$$

which further implies that

$$P^*(t_1; \lambda) < P^*(t_2^c; \lambda).$$

This means that t_1 is not a solution to problem (O2-λ) for any $\lambda \in [\lambda_1, 1]$. By Lemma 3, this implies that winner-take-all is not a solution to problem (O-λ) for any $\lambda \in [\lambda_1, 1]$.

For (iii), suppose that there is some $\lambda_2 \in (0, 1)$ such that winner-take-all is a solution, but not the unique solution, to problem (O-λ_2). Then, by Lemma 3, there is some $t_3^c \in [a, t_1)$ such that both t_1 and t_3^c are solutions to problem (O2-λ_2). Since both t_1 and t_3^c are solutions to problem (O2-λ_2), $P^*(t_1; \lambda_2) = P^*(t_3^c; \lambda_2)$, which is equivalent to

$$(1 - \lambda_2)\left(TE^*(t_1) - TE^*(t_3^c)\right) = \lambda_2(t_1 - t_3^c). \tag{9}$$

We first show that for any $\lambda \in (\lambda_2, 1]$, winner-take-all is not a solution to problem (O-λ). Again, by Lemma 3, it suffices to show that t_1 is not a solution to problem (O2-λ) when $\lambda \in (\lambda_2, 1]$. In fact, suppose to the contrary that t_1 is a solution to problem (O2-λ) for some $\lambda' \in (\lambda_2, 1]$. Then, $P^*(t_1; \lambda') \geq P^*(t^c; \lambda')$, for any $t^c \in [a, t_1]$. In particular, $P^*(t_1; \lambda') \geq P^*(t_3^c; \lambda')$, which is equivalent to

$$(1 - \lambda')\left(TE^*(t_1) - TE^*(t_3^c)\right) \geq \lambda'(t_1 - t_3^c).$$

However, since $\lambda' > \lambda_2$,

$$(1 - \lambda')\left(TE^*(t_1) - TE^*(t_3^c)\right) \geq \lambda'(t_1 - t_3^c) > \lambda_2(t_1 - t_3^c) = (1 - \lambda_2)\left(TE^*(t_1) - TE^*(t_3^c)\right),$$

which further leads to $1 - \lambda' > 1 - \lambda_2$ because $TE^*(t_1) - TE^*(t_3^c) > 0$ by Lemma 2; a contradiction.

We next show that for any $\lambda \in [0, \lambda_2)$, winner-take-all is the unique solution to problem (O-λ), which would then complete the proof of (iii). By Lemma 3, this is equivalent to showing that t_1 is the unique solution to problem (O2-λ) when $\lambda \in [0, \lambda_2)$. We first argue that t_1 must be a solution to problem (O2-λ) when $\lambda \in [0, \lambda_2)$. To this end, one needs to show that $P^*(t_1; \lambda) \geq P^*(t^c; \lambda)$, for any $t^c \in [a, t_1]$. This is equivalent to

$$(1 - \lambda)\left(TE^*(t_1) - TE^*(t^c)\right) \geq \lambda(t_1 - t^c), \text{ for all } t^c \in [a, t_1] \text{ and all } \lambda \in [0, \lambda_2). \tag{10}$$

Since t_1 is a solution to problem (O2-λ_2),

$$(1 - \lambda_2)\left(TE^*(t_1) - TE^*(t^c)\right) \geq \lambda_2(t_1 - t^c), \text{ for all } t^c \in [a, t_1]. \tag{11}$$

Thus, for any $\lambda \in [0, \lambda_2)$ and any $t^c \in [a, t_1]$, one has

$$(1 - \lambda)\left(TE^*(t_1) - TE^*(t^c)\right) \geq (1 - \lambda_2)\left(TE^*(t_1) - TE^*(t^c)\right) \geq \lambda_2(t_1 - t^c) \geq \lambda(t_1 - t^c).$$

Therefore, (10) holds.

We next argue that t_1 is the unique solution to problem (O2-λ) when $\lambda \in [0, \lambda_2)$. When $\lambda = 0$, this is obvious, following directly from Lemma 2. Now, suppose, to the contrary, that $t_3' \neq t_1$ is also a solution to problem (O2-λ) for some $\lambda' \in (0, \lambda_2)$. However, this is impossible. We just showed above that if for some $\hat{\lambda} \in (0,1)$ winner-take-all is one, but not the unique, solution to problem (O2-λ), then winner-take-all is not a solution to problem (O2-λ) for any $\lambda \in (\hat{\lambda}, 1]$. This implies that winner-take-all is not a solution to problem (O2-λ) for any $\lambda \in (\lambda', 1]$. However, this contradicts the fact that winner-take-all is a solution to problem (O-λ_2). $\square$

Proof of Lemma 5. Suppose, to the contrary, that $S_1 = \{0\}$. Note that $t^{c*}(0) = \{t_1\}$. Since $t^{c*}(\lambda)$ is upper hemicontinuous in λ and $t^{c*}(0) = \{t_1\}$, there exists a sequence $\{\lambda_k\}_k$ with $\lambda_k \in (0,1)$ and $\lim_{k\to\infty} \lambda_k = 0$, such that there exists some $t^c(\lambda_k) \in t^{c*}(\lambda_k)$ for each k with $\lim_{k\to\infty} t^c(\lambda_k) = t_1$ (note that without loss of generality, one can assume that the sequence $\{t^c(\lambda_k)\}_k$ is convergent, as this sequence is in the compact set $[a, t_1]$). Note further that $t_1 \notin t^{c*}(\lambda_k)$ for each k, by part (i) of Lemma 4 (otherwise, $S_1 \neq \{0\}$). Thus, $t^c(\lambda_k) < t_1$ for all k.

Since $t_1 \notin t^{c*}(\lambda_k)$ for each k, $P^*(t^c(\lambda_k); \lambda_k) > P^*(t_1; \lambda_k)$, $\forall k$, which is equivalent to

$$(1 - \lambda_k)\left(TE^*(t^c(\lambda_k)) - TE^*(t_1)\right) > \lambda_k(t^c(\lambda_k) - t_1), \text{ for any } k.$$

Since $t^c(\lambda_k) < t_1$, we have

$$\frac{TE^*(t^c(\lambda_k)) - TE^*(t_1)}{t^c(\lambda_k) - t_1} < \frac{\lambda_k}{1 - \lambda_k} \text{ for any } k,$$

or equivalently,

$$\frac{TE^*(t_1) - TE^*(t^c(\lambda_k))}{t_1 - t^c(\lambda_k)} < \frac{\lambda_k}{1 - \lambda_k} \text{ for any } k.$$

Recall that in the proof of Lemma 2, we showed that $TE^*(t^c) \leq V \cdot TE^{**}(t^c)$, for any $t^c \in [a, t_1]$, with equality when $t^c = t_1$. Here, $TE^{**}(t^c)$ is the value function of the contest organizer's problem in Liu and Lu (2019). It then follows that

$$\frac{V[TE^{**}(t_1) - TE^{**}(t^c(\lambda_k))]}{t_1 - t^c(\lambda_k)} \leq \frac{TE^*(t_1) - TE^*(t^c(\lambda_k))}{t_1 - t^c(\lambda_k)} < \frac{\lambda_k}{1 - \lambda_k} \text{ for any } k,$$

which implies that,

$$\frac{V[TE^{**}(t_1) - TE^{**}(t^c(\lambda_k))]}{t_1 - t^c(\lambda_k)} < \frac{\lambda_k}{1 - \lambda_k} \text{ for any } k. \tag{12}$$

In their proof of Lemma 2, Liu and Lu (2019) show that $TE^{**}(t^c)$ is differentiable in $t^c \in [t_2, t_1)$, where $t_2 \in (a, t_1)$. In fact, from the proof there, it is clear that the left derivative of $TE^{**}(t^c)$ at $t^c = t_1$ exists and is strictly positive.[21] Recall that, by construction, $t^c(\lambda_k) < t_1$ for all k,

[21] To be clear, we use their notations in their proof of Lemma 2. They show on page 152 that when $t^c \in [t_2, t_1)$ (by letting $n = 2$),

$$\frac{dTE^{**}}{dt^c} = Nf(t^c)[\beta(t^c)(\varphi(2, t^c) - \varphi(1, t^c)) + (c - p_1(t^c))\varphi(2, t^c)].$$

(Note that the notation TE^* in their proof has been changed to TE^{**} to be consistent with the notation used in this paper when referring to their paper.) It is clear from their proof that the left derivative of $TE^{**}(t^c)$ at $t^c = t_1$ exists

$\lim_{k\to\infty} \lambda_k = 0$, and $\lim_{k\to\infty} t^c(\lambda_k) = t_1$. Thus, in (12), ignoring V and letting $k \to \infty$,

$$\lim_{k\to\infty} \frac{TE^{**}(t_1) - TE^{**}(t^c(\lambda_k))}{t_1 - t^c(\lambda_k)} \leq \lim_{k\to\infty} \frac{\lambda_k}{1-\lambda_k} = 0.$$

However, note that the left-hand side of the above inequality is precisely the left derivative of $TE^{**}(t^c)$ at $t^c = t_1$, which is strictly positive. This is clearly a contradiction. □

Proof of Proposition 2. Note that $TE^*(t^c)$, the value function of problem (O1-λ), is continuous in $[a, t_1]$ by the maximum theorem because in problem (O1-λ), the objective function $TE(\mathbf{v}, t^c)$ is continuous in $(\mathbf{v}, t^c) \in \mathbb{R}^N_+ \times [a, t_1]$ and the feasible set $S(t^c)$ is continuous in t^c and compact-valued. Therefore, the objective function of problem (O2-λ),

$$P^*(t^c; \lambda) = (1-\lambda)TE^*(t^c) + \lambda(b - t^c)$$

is continuous in $(t^c, \lambda) \in [a, t_1] \times [0, 1]$. Again, the maximum theorem implies that the optimal solution set $t^{c*}(\lambda)$ of problem (O2-λ) is compact and is upper hemicontinuous in $\lambda \in [0, 1]$.

Denote $S_1 = \{\lambda \in [0,1] : t_1 \text{ is the unique solution to problem (O2-}\lambda)\}$. Lemma 2 implies that $0 \in S_1$. Lemma 5 reveals that S_1 cannot be a singleton. Denote $S_2 = \{\lambda \in [0,1] : t_1$ is a solution, but not the unique solution, to problem (O2-λ)$\}$ and $S_3 = \{\lambda \in [0,1] : t_1$ is not a solution to problem (O2-λ)$\}$. Obviously, $S_1 \cup S_2 \cup S_3 = [0, 1]$. Furthermore, Lemma 4 implies that if $S_2 \neq \emptyset$, then S_2 must be a singleton. Therefore, by Lemmas 4 and 5, there exists some $\tilde{\lambda} \in (0, 1)$ such that $[0, \tilde{\lambda}) \subseteq S_1$ and $(\tilde{\lambda}, 1] \subseteq S_3$. Moreover, $\tilde{\lambda} \in S_1$ or $\tilde{\lambda} \in S_2$. By construction, $\tilde{\lambda}$ is unique. To see why $\tilde{\lambda} \notin S_3$, suppose, to the contrary, that $\tilde{\lambda} \in S_3$. Consider a convergent sequence $\{\lambda_k\}_k$ with $\lambda_k \in S_1$ and $\lim_{k\to\infty} \lambda_k = \tilde{\lambda}$. By definition, $t^{c*}(\lambda_k) = \{t_1\}$ for all k. Thus, $\lim_{k\to\infty} t^{c*}(\lambda_k) = \{t_1\}$. However, $t_1 \notin t^{c*}(\tilde{\lambda})$, which violates the upper hemicontinuity of $t^{c*}(\lambda)$ at $\lambda = \tilde{\lambda}$.

Finally, Proposition 2 then follows by applying Lemma 3. □

B. A Sample Contest Announcement

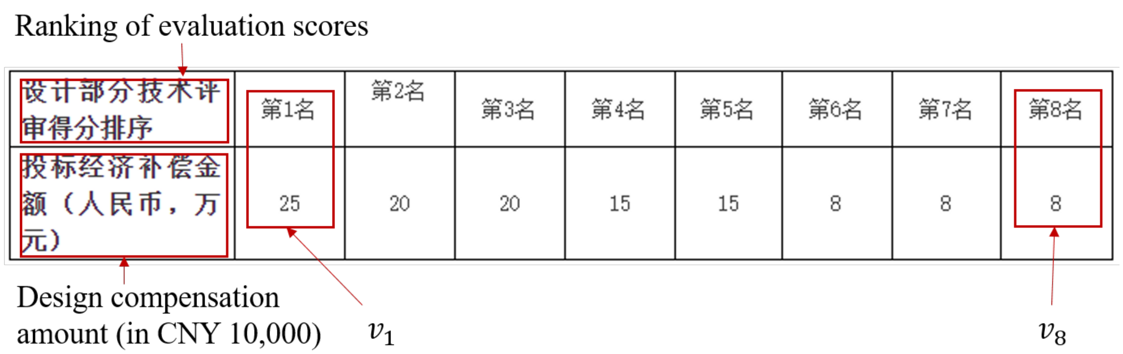
Let us see a contest announcement for a building project from our data. The original announcement is posted on the website of the procurement platform (`www.gzggzy.cn/cms/wz/view/index/layout3/index.jsp?siteId=1&channelId=503&infoId=537554.`). In case the website removes this announcement, we saved a copy of it (`www.dropbox.com/s/mugalp5y780np8i/contest.announcement.ch.pdf?dl=0`) and its English translation (`www.dropbox.com/s/mh4z7py9uuj01jr/contest.announcement.en.pdf?dl=0`).

and can be obtained by letting $t^c \to t_1^-$ in the above equation. Thus, the left derivative at t_1 is equal to

$$Nf(t_1)[\beta(t_1)(\varphi(2, t_1) - \varphi(1, t_1)) + (c - p_1(t_1))\varphi(2, t_1)].$$

By the definition of these notations in their paper, one has $Nf(t_1) > 0$, $c - p_1(t_1) = 0$, $\beta(t_1) = p_2(t_1) + 2p_1(t_1) - 2c = p_2(t_1) > 0$, and $\varphi(2, t_1) - \varphi(1, t_1) > 0$. Therefore, the left derivative at t_1 is strictly positive.

The procurer is Sun Yat-sen University (`www.sysu.edu.cn`) in Guangzhou, China. The contest is intended to procure design proposals for an apartment building for faculty housing. One important part of the announcement is the prize scheme shown in Figure 6. For this contest, the prize scheme is $\mathbf{v} = (25, 20, 20, 15, 15, 8, 8, 8, 0, 0, ...)$.



设计部分技术评审得分排序	第1名	第2名	第3名	第4名	第5名	第6名	第7名	第8名
投标经济补偿金额（人民币，万元）	25	20	20	15	15	8	8	8

Figure 6: Prize Scheme in the Contest Announcement