What is productive investment? Insights from firm–level data for the United Kingdom*

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Abstract

This paper studies the effects of different types of investment and levels of debt on productivity. We first examine the issue empirically using data on listed firms in the UK. Our main finding is that intangibles investments are a good proxy for productivity–enhancing investment, as they have a positive effect on Total Factor Productivity (TFP). On the other hand, we find no consistent evidence of positive TFP effects for tangible investment. In those firms that have high debt and high level of intangibles, the positive TFP effects are even more pronounced. Hence, debt can be "good" if it is associated with productivity–enhancing investments. We then set out a stylised model of a dynamic firm profit–maximisation problem, and augment it with an external financing option in a novel way. Uniquely, we use neural network methods to solve the value function of the model and propose a moments matching approach that allows us to estimate some of the parameters of the model. We use the model to illustrate how productivity–enhancing investment differs from other investments in its effects on TFP, and how these positive effects can be stronger for firms that have higher indebtedness. Applying our model to aggregate TFP dynamics in the UK suggests that around a tenth of the TFP slowdown in the UK since the Global Financial Crisis can be attributed to weaker intangibles investments.

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1 Introduction

In this paper, we study the effects of corporate investment and levels of debt on productivity in the UK, using firm-level data. Given the recent relatively high level of corporate indebtedness in the UK, the topic is highly policy relevant, both in terms of risks for the financial sector and its macroeconomic consequences. At the aggregate level, UK data suggests a strong positive correlation between corporate debt and investment, whereas the correlation between debt and productivity is more tenuous. However, at the firm level, there is strong evidence in the literature suggesting that high corporate debt leads to lower investment, especially in times of crisis, with negative subsequent effects on productivity. In particular, in the existing literature, high corporate leverage has been identified as one of the leading indicators of firm vulnerability. Typically leverage is assumed to be "good" in the boom phase, as it allows firms to invest in their productive capacity. Debt then becomes "bad" in a downturn owing to debt overhang reasons.

We take a somewhat different approach in our analysis. Rather than studying *state*–dependency (i.e., effects of debt depending on the state of the business cycle), we study *type*–dependency (i.e., effects of debt depending on what type of investments the firm undertakes). We hypothesise that one can distinguish between "good" and "bad" leverage more generally, by means of analysing the types of investments that firms undertake. We analyse firms' investment and debt finance decisions to see how well they explain their productivity (measured by total factor productivity (TFP)). In other words, the mechanism through which firm debt should affect firm TFP is through the investments firms undertake.

First, we look at selected stylised facts and relationships in the data. We use a fairly large (unbalanced) panel of financial accounts data for listed firms in the UK. The granular nature of this dataset means that all the variables needed for the analysis are available for a relatively large sample of firms. We use standard panel regressions with firm and year fixed effects to study the relationship between TFP and a selection of relevant explanatory variables. We also introduce an interaction term between different types of investment and debt to analyse whether debt is always "bad" for TFP. The way we mix interactions between continuous and dummy variables in our models is common in recent micro–level panel data literature (for similar approaches in different setting, see, for example, Buera and Karmakar (2021), Chodorow-Reich (2014), Giroud and Mueller (2017) and Joseph et al. (2020)). However, endogeneity is likely to be an issue in the types of models we use; *ex ante*, it is not obvious whether investment and finance structure causes productivity, or the other way round. We aim to mitigate this problem by using lagged values of the explanatory variables.

Our main empirical finding is that intangibles investment are a good proxy for productivity–enhancing investment, as they have a positive effect on TFP. On the other hand, we find no consistent evidence of positive TFP effects for tangible investment, and in fact, this effect is negative. A combination of high levels of debt high level of intangibles tends to have a positive effect on TFP. Hence, leverage can be "good" if it is associated with productivity–enhancing investments.

We then set up a structural model to illustrate the theoretical channels between investment, debt and TFP that we want to focus on. The stylised structural model we use builds on the traditional neoclassical model of Eberly et al. (2008), Warusawitharana (2015) and Levine and Warusawitharana (2021) (see also Moll (2014) and Midrigan and Xu (2014) for similar approaches). The model is necessarily a partial equilibrium model, but it is useful in defining the channels through which investment and debt can effect TFP of a profit–maximising firm, with underlying assumptions that are standard in the literature. Importantly and to our knowledge uniquely, we augment the model with an option for external financing, which enters the model both as a control and a state variable and thus allows for a full characterisation of the solution in a more realistic setting than the more basic approaches. Using neural network methods, we also solve the model numerically to highlight some of the key dependencies in the model. We also apply our model to aggregate TFP dynamics in the UK, and find that around a tenth of the TFP slowdown in the UK since the Global Financial Crisis can be attributed to weaker intangibles investments in large UK firms.

Our main contribution to the literature is in showing that high levels of debt are not necessarily bad for TFP, if the debt is accompanied by high levels of productive investment. We show this with a combination of our structural model and empirical analysis of the UK data. Our evidence suggests that a particular type of investment, namely intangible investment, is a good proxy for productive investment. We show its positive effects on TFP. We also show that a combination of high debt and high intangibles investment can be conducive to high TFP. On the other hand, we find no evidence of positive TFP effects for tangible capital expenditure. Furthermore, in our structural model, we propose a new way to estimate value function parameters based on the observed states and investment choices in the data.

Related literature: Our paper relates to literature on the effects of debt on investment and productivity, channels between different types of investment and productivity and on the definition of productivity–enhancing investment. In the first strand, typically, the literature on corporate indebtedness has found evidence of a negative effect of excessive indebtedness on corporate investment. This points to frictions deviating from the traditional Modigliani-Miller model of the irrelevance of a firm's capital structure for its value. The links between corporate leverage and investment have been studied in a large, well-established strand of literature, tracing back to the financial accelerator theory introduced by Bernanke and Gertler (1989) and the debt overhang theory of Myers (1977). Since then, this link has been examined both at the aggregate as well as the firm level. Typically, firm-level studies find a negative relationship between high levels of corporate debt and subsequent capital expenditure, accentuated by times of financial crises (see, for example, Fernando et al. (2014), Duchin et al. (2010), Almeida et al. (2009), Jaeger (2003), Goretti and Souto (2013), Kalemli-Ozcan et al. (2022) and Buera and Karmakar (2021)). Our paper differs from this literature by concentrating on the links between types of investment and leverage, rather than business cycle properties of this relationship.

With regard to the links between debt, investment and productivity, Duval et al. (2020) study the effects of various financial vulnerabilities on firm-level TFP in advanced economies, finding that firms with weak balance sheets prior to the financial crisis performed worse in terms of TFP since the crisis. In a more structural approach, Gopinath et al. (2017) develop a model that helps to explain how financial frictions can lead to capital misallocation and lower aggregate productivity. Franklin et al. (2015) find that contractions in credit supply after the financial crisis led to lower productivity in a sample of UK firms. Doerr et al. (2018) report negative effects of credit shocks on investment and productivity of Italian firms. Huber (2018) finds evidence for negative aggregate demand effects of lower credit on innovation and productivity for German firms, irrespective of their direct exposure to the credit shocks. Bahaj et al. (2017) show evidence of a U-shaped – and hence non–linear – relationship between UK firms' indebtedness and levels of productivity. The use of external finance for productive/unproductive reasons is also analysed by Anderson et al. (2015) and Bank of England (2016)). However, these papers do not examine the links between different types of investment, debt and productivity, which is what our study aims to do.

Several papers have also shown evidence for a positive endogenous relationship between different types of productivity–enhancing investment, like intangibles investment, and subsequent productivity growth. For example, Aghion et al. (2010) establish a negative causality between credit frictions and long-term, productivity–enhancing investment. However, their empirical analysis concentrates on country–level data and a diffrerent definition of the types of investment compared to ours. Anzoategui et al. (2019) apply an endogenous growth model to US data and find that the long-term productivity slowdown is mainly due to lower R&D investment and spillovers. De Ridder (2018) shows that tighter credit conditions during the financial crisis had a negative effect on productivity–enhancing investment and aggregare growth after the crisis. Moran and Queralto (2018) estimate a strong and persistent effect of R&D shocks on TFP with US aggregate data. And OECD (2016) finds a causality from R&D spending to productivity in a large panel of advanced–economy firms. On the other hand, De Ridder (2022) develops a general–equilibrium model, and also finds empirical evidence, for a negative long-term effect of intangible investment on TFP, as large firms carrying out intangible investment gain market power, which deters entry of new firms and creative destruction. These papers are similar in spirit to our analysis in terms of distniguishing between the effects of different types of investment, but our analysis differs from the existing literature in looking at the relationships between debt, different types of investment and productivity in a unified framework. In particular, our granular firm–level financial account data allows us to define productivity–enhancing investments empirically, with foundations in a structural model.

The rest of the paper is organised as follows. Section 2 sets out our empirical analysis. Section 3 presents the structural model. Section 4 concludes.

2 Empirical analysis

This section sets out our empirical analysis, starting with a description of our empirical model and data, and then moving to the results and selected robustness checks.

2.1 Motivation

The relationship between investment, debt and productivity is not straight-forward. Figure 1 shows aggregate time series on investment growth (measured by capital services), TFP growth and corporate sector debt in the UK. As the chart suggests, there is a relatively strong positive correlation between investment and debt in the past few decades (the contemporaneous correlation coefficient is 0.6) – firms typically use debt to invest. However, the link between investment and debt with TFP growth is much more tenuous (the correlation coefficient between both investment and TFP, and debt and TFP, is 0.1). There are various factors that affect TFP, and there are time lags in those effects. Not all investment may be productivity–enhancing, or driven by technological progress that shows up in higher TFP. We aim to shed on light on these relationships by going beyond the aggregate numbers, and analysing firm-level heterogeneity.

2.2 Empirical model

In our empirical regressions, we have firm–level productivity (measured as TFP) as the left–hand side variable. As explanatory variables, we use two types of investment variables (described in more detail in the Data section below); i) capital expenditure (proxy for tangible investment) and ii) intangible investment. The purpose of making this distinction between different types of investment is to study whether – empirically – different types of investment can have different effects on TFP, and how these investments have interacted with different levels of debt.

Given that a key aim of our exercise is to study the effects of debt on TFP, we also include a measure of debt as an explanatory variable, and its interactions with investment. This interaction can be important in determining how debt affects TFP. This will allow us to distinguish between "good" and "bad" leverage. We hypothesise that leverage per se is not detrimental, if it is used to finance productive investment and therefore, there needs to be a distinction between leverage in different kinds of firms. We also control for various firm characteristics (firm age, size, cash holdings and profits) that can be expected to have an effect on how productive a firm is. All our baseline regressions include firm and time fixed effects.

For our baseline analysis, we use simple OLS panel regressions with lagged explanatory variables[1] of the following form:

$$z_{it} = \alpha + \beta_1 D_{i,t-1} + \beta_2 R_{i,t-1} + \beta_3 D_{i,t-1} R_{i,t-1} + c_i + f_t + X_{i,t-1} + e_{it} \tag{1}$$

where z_{it} is a firm-level measure of TFP for firm i at time t, $R_{i,t-1}$ are the two different investment variables in turn, $D_{i,t-1}$ is the debt ratio, $X_{i,t-1}$ includes firm characteristics (sector, age, size, profits, cash) as controls and e_{it} is an i.i.d. error term. To facilitate the interpreration of the interaction variable, and to focus on the high–investment firms, the investment variables are 0/1 dummies, with value 1 if the firm's investment ratio is in the top quartile of firms for a particular year, and 0 otherwise[2]. We also use industry median corrected values for the debt ratio[3]. The terms c_i and f_t are firm and time fixed effects, respectively.

2.3 Data

The firm–level data used in the analysis comes from Refinitiv Worldscope, which is a proprietary dataset that includes financial account information on large (mainly listed) UK firms since the 1980s. Table 1 introduces the data series we use for our analysis for a

[1]It is worth noting that in our set–up identification of the parameters comes from variation in TFP across firms and time. However, there are obvious endogeneity issues in our framework, in particular related to simultaneity bias. Choices on the uses of funds by a firm are likely to depend on how productive the firm is; for example, decisions on debt dynamics can depend on the level of TFP. In our baseline method, we mitigate these issues by including lagged, rather than contemporaneous values of all RHS variables. This removes the simultaneity bias by definition, as the lagged RHS variables cannot depend on current level of TFP. To make a more econometrically robust correction for these issues, we also estimate a system-GMM version of the model (as introduced by Arellano and Bover (1995) and Blundell and Bond (1998)). We do not report the results of the system-GMM for brevity and clarity, but they are qualitatively in line with our baseline results.

[2]The results we present below are also qualitatively robust to other choices, and we explore some of the non-linearities of the effects with different thresholds for dummies below.

[3]This approach is similar with Levine and Warusawitharana (2021) and provides a way of controlling for time-varying industry-specific effects. The main results below are also robust to not using the median corrections.

sample from 1990 to 2018 (Worldscope identifiers are given in the last column). The key series for our analysis are the dependent variable, productivity (measured by firm–level TFP), and the main explanatory variables, debt, and tangible and intangible investment. The exact definitions of the firm–level financial account data series and the aggregate level deflator data are provided in Appendix A.1.

There are different options for measuring firm–level productivity empirically, none of which is without its challenges. The simplest measure, revenue productivity, divides a firm's revenue (turnover) by its number of employees. As has been suggested in the literature, this is not a satisfactory measure of technological progress at firm level (see, for example, Comin (2010)). For that, one needs a measure of TFP, which is what we use. However, this is not often easy to calculate from firm–level data, because typically, only proxies for capital, labour and other inputs are available in the data, and there is no firm–level data on prices.[4]

For this study, we use an established method based on a production function approach suggested by Ackerberg et al. (2015) (building on the method originally introduced by Olley and Pakes (1996)).[5] To preserve consistency with the set-up in our structural model, where the TFP process depends on (potentially) different types of investment flows, we use both tangible (i_{it}) and intangible (ii_{it}) investments as instruments to yield the following proxy for TFP (ω_{it}) in our empirical TFP calculations:

$$\omega_{it} = h(i_{it}, ii_{it}, \mathbf{x}_{it}) \tag{2}$$

where $\mathbf{x}_{it}$ is the state variable (capital stock) and $h(.)$ is a monotonous, increasing function

[4]Strictly speaking, the measure we are using is revenue TFP (TFPR), and in the absence of firm–level price data, the underlying measure of firm–level technological progress cannot be estimated. This is an important caveat to keep in mind when interpreting our results, and implies i) that the regression coefficients are downward biased, and ii) the TFPR measure may include a demand shock component (see Barterlsman and Wolf (2018)). Nevertheless, using real values (deflated wtih relevant aggregate GDP and investment deflators) for the revenue TFP measure is much closer conceptually to the technological progress than pure revenue productivity, and we will call it TFP for simplicity in the analysis that follows.

[5]Another popular control function approach was introduced by Wooldridge (2012). We have also replicated our main analysis with this TFP measure; results are very similar with our baseline results reported below.

of unknown form.[6] From the point of view of the production function estimation, TFP is assumed to be an exogenous Markov process, while the TFP process in our empirical specifications can potentially depend on variables other than only the ones included in the production function. The inclusion of both tangible and intangible investment in equation (2) helps to mitigate this unavoidable conceptual discrepancy.

Appendix A.1 defines the variables used in the production function estimation. The estimation is carried out at sector level (with 1–digit SIC sectors). We do not report the detailed estimation results for the sake of brevity, but it is worth mentioning that the weighted (by number of firms per sector) average coefficient for the labour input is 0.74, and for the capital input it is 0.17. These coefficients are statistically significant at the 99% level for nearly all sectors, and the assumption of constant returns to scale is rejected for most sectors. We use these average coefficients in our structural model calibrations (see below).

We use the following firm financial account items as measures of different types of investment:

- intangibles assets and change in intangible assets (intans for brevity), as defined in more detail in Appendix A.2. This is the main proxy for intangible investment.
- capital expenditure (tans). This is the main proxy for tangible investment.

Table 2 shows the main features of selected key variables used in the analysis, weighted by firm size. We restrict the sample to firm–year observations for which the main explanatory variables of interest are available. TFP is available for 27,712 observations, which will mainly dictate the sample size for the regressions below. Overall, the dataset is large enough for meaningful analysis to be carried out with the panel data methods we use.

Table 2 reveals that there is a lot of variation in some of the variables; standard devations are high and the differences between the largest and smallest values are large. The mean values of the variables look generally sensible. The average TFP growth is around 0.8% per annum, which is broadly in line compared to estimates of aggregate TFP growth in the UK during the same time. We also find that there is a positive contemporaneous

[6] The role of $h(.)$ in the solution methods, and the Stata prodest package we use is described in more detail in Mollisi and Rovigatti (2017).

correlation between TFP and the intangibles stocks and flows, and a negative relationship between TFP and tangible investment flows (not shown in the Table), but the relationship between TFP and debt is more complicated, also in line with the aggregate data in Figure 1. There are some signs of the U–shaped relationship also discovered by Bahaj et al. (2017) with a larger set of UK firms, although there is a lot of variation in this relationship. Overall, these stylised facts suggest that it will be important to look at the relationships between TFP, investment and debt in different percentiles of the firm distribution.

Figure 2 shows *medians* of selected key variables over time. While this hides firm–level heterogeneity, it is a useful sense–check on the data. Some intuitively appealing facts emerge. First, the debt ratio has tended to increase since the Global Financial Crisis, whereas the tangibles ratio has declined somewhat over time. Third, the share of intangible assets of total assets increased strongly until the financial crisis in the late 2000's, but has been relatively steady since.[7] Fourth, intangible investment flows are lower than tangible flows, and declining over time.

The characteristics of firms with high level of intangibles investment is of special interest to our analysis, so it is worth detailing some of these facts more broadly. Correlations in our sample suggest that intangibles stocks (and flows) are higher in firms that are less indebted, younger, smaller, more cash rich[8] and less profitable than those with smaller intangibles stocks. Figure 3 shows that the TFP distribution of high-intans firms is higher than that of low-intans firms, so evidence clearly points to a positive contemporaneous and unconditional relationship between the level of TFP and intangibles. In terms of industry decompositions, high intan firms are heavily concentrated in the manufacturing and ICT sectors.[9]

In terms of correlations between the sample investment variables with corresponding

[7]The recent weakness in intangible investment growth and its structural macroeconomic consequences has been analysed by Bailey et al. (2022).

[8]This is a common finding in the literature (see, for example, Dao and Maggi (2018)); typically, firms with high intangibles spending tend to hold more cash, as it is harder to use external financing for intangible investment projects due to their less collaterisable nature.

[9]We have also compared our data with data from the ONS Innovation Survey (2017) (https://www.gov.uk/government/statistics/uk-innovation-survey-2017-main-report), which suggests that internal R&D spending covers over half of all innovation–related expenditure in the surveyed UK firms.This underlines the importance of taking into account accumulated R&D spending in our intans measure, as it is a crucial component of innovation and potentially TFP growth. Furthermore, consistent with our industry decompositions, according to the survey, innovation activities are especially important in high–tech manufacturing and knowledge–intensive services.

aggregate macroeconomic variables – to the extent these can be measured – the correlation of R&D spending in our sample, with the relevant ONS aggregate series is positive at around 0.37, and the level of R&D spending is higher in the sample than in the economy more generally. A similar comparison between the sum of capital expenditure spending divided by total sales with the corresponding aggregate ONS series shows a relatively strong positive correlation of around 0.51, emphasising the importance of large firms for driving business investment dynamics in the UK.

2.4 Main Results

Table 3 presents the results of our panel regressions (equation (1)). The effects of the (lagged) intangibles variables on the level of TFP are strongly positive for both intans stocks and flows (columns (1)-(2)). The effect of the tangibles flow is strongly negative. In terms of the debt ratio, the lagged direct effect on TFP is positive, but not significant in all the regressions. To study the full debt effect, we report the p–value of the joint effect of the sum of the direct and the interaction components (last row of Table 3). Interestingly, the total effect with the intangibles variables is significantly positive, whereas the joint effect with tangibles investment is zero. There is also a positive separate effect of debt that is somewhat significant for those firms that are in the highest quartile of intangibles stocks. The effects of the control variables are mostly significant and intuitive; on average, TFP is higher for older, larger and more profitable firms.

Table 4 summarises the economic size of the main effects we are interested in. There is a large and significant effect of 9.7% on TFP for those firms that are in the highest quartile of intangibles stock levels.[10] This effect is somewhat lower, but positive, for intangibles flows, and significantly negative for tangibles flows. It is worth noting that the strongest positive TFP effects accrue from the intangibles stocks rather than flows. This is consistent with the lagged effects of intangibles investment that typically come through after several years documented in the innovation literature (see, for example, Hall et al. (2010)).

Overall, the regression results support the evidence of a positive effect of intangibles

[10]As noted above, due to the nature of our TFP measure, the regression coefficients may underestimate the true size of the effects, and hence the estimates in the table should be seen as conservative.

on TFP. Hence, in this dataset of UK firms, intangible investments are productivity enhancing, with positive spillovers from these investments on TFP, on average. On the other hand, tangible investment flows do not have positive effects on TFP, and in fact, the results suggest these effects are negative. This may appear counter–intuitive; why would firms engage in investments that are not productive? There are two potential reasons for this; 1) these investments may have appeared more positive *ex ante* than *ex post*, or 2) it is possible that these investments have positive longer term effects on firms' revenue, even if the TFP effects are negative or neutral. We will return to this issue in the set-up of our structural model below.

The evidence on the effects of debt are more mixed, but the results do suggest that a combination of high debt and high "productive" investment (as proxied by intangible investment) can be associated with high levels of TFP, even when the direct correlations of debt and TFP are more ambiguous. On average, over the sample period, we do not find that high levels of debt would be associated with poor TFP outcomes. Hence, in this "type–dependent" sense, debt tends to be "good" rather than "bad" in our sample of firms.

2.5 Selected Robustness Checks

A main issue with our baseline results is the question of potential reverse causality (see also Levine and Warusawitharana (2021) (LW) for a discussion of this topic). While our baseline regressions attempt to mitigate endogeneity issues in ways described above, it cannot be excluded that a firm invests in its inputs (like intans) in anticipation of an improvement in its TFP, causing a potential reverse causality issue. To study this effect in more detail, we follow the method used in LW to split the TFP shock into an anticipated and unanticipated component, and then using the unanticipated TFP component as the dependent variable. The intans effects in these regressions are significant and positive (see Table 5, columns 1-2), implying that this effect runs from the intans to TFP, rather than the other way round. Hence, reverse causality of this type does not appear to be an issue in our dataset.

There is also a question on whether our results would look different depending on the type of external financing we include in the models. In other words, we would like to examine whether the effects of equity, rather than debt financing are different. We also run the regressions replacing the debt ratios with equity ratios for all firms. While the effects of the intangibles remain positive as in the baseline, the direct effects of equity financing on TFP are generally more positive than those of debt financing (Table 5, columns 3-4). Hence, to the extent that higher TFP contributes to higher firm value, the predictions of the traditional Modigliani–Miller model do not hold, as the effects of equity financing are more positive than those of debt financing. This result also justifies our focus on the effects of debt in our baseline models, as the debt effects are potentially more diverse and less obviously positive.

We also ran several other robustness checks. These regressions suggest that the results are very robust to different intangibles assumptions[11]; intangibles coefficients remain highly significant, and the joint debt effect remains significantly positive for the high intangibles firms in most cases (Table 5, columns 5-12). We also studied potential non-linearities in the relationships between the key variables by running the regressions with a dummy for the highest decile of the intangibles variables, together with high debt ratios, as explanatory variables. The results show that *given* a firm is in the high debt bucket, being in the top decile in terms of intangibles stocks/flows has a strong positive debt effect on TFP, which suggests there are non-linearities in these effects. Finally, we also study the effects of different deciles of intangibles on TFP by shifting the intangibles dummy decile–by–decile from the 50th to the 90th. The highest effects are at the 90th decile, suggesting that being in the very top deciles of intangibles stocks has a more positive effect on TFP than being closer to the median.

3 Structural model

We use a structural model to set out the mechanisms between debt, investment in tangible as well as intangible capital and productivity that we studied empirically with UK firm–level data in the previous section. The model builds on a standard firm profit maximisation problem, using dynamic optimisation (see Warusawitharana (2015) and Levine and Warusawitharana (2020) (LW) for the basis of this model). The main feature

[11]Following Peters and Taylor (2017), we allow the R&D depreciation rate in the accumulation of the intans stock to vary between 10 and 25%, and the SGA depreciation rate between 10 and 30%. We also vary the share of SGA that is assumed to be intangibles investment between 10 and 50%.

of this partial equilibrium neoclassical model is a firm that maximises future profits, and these profits can be enhanced by two types of investment; i) "tangible" investment that accumulates the physical capital stock and ii) "intangible" investment that enhances productivity (TFP)[12]. Firms also face convex capital adjustment costs related to both types of investment.[13]

In a novel feature of our model, we augment it with an option for external debt financing, which can only be used for either type of investment, and which carries an interest that needs to be paid on the accumulated debt in future periods. The novelty of our approach comes from the way financing stocks and flows are embedded in the model as a result of the control and state variables, rather than as mere residuals of the cash–flows, as in the standard approach (see, for example, Levine and Warusawitharana (2020)). Our approach is arguably more realistic, as it explicitly links investment and financing decisions taken by firms, and it also allows us to study the interactions between finance, different types of investment and TFP directly.

In our proposed model for the firm's value maximisation problem, we capture the positive impact of higher intangible capital on TFP and the negative association of high leverage and a high physical capital stock to TFP. Our model also features a negative effect of higher leverage on the cost of external financing. We assume in our model that a firm maximises profits by choosing investment into physical or intangible capital. Both are inputs into firm output. To the extent that investment cannot be financed by current dividends, it will be financed externally. External financing increases the stock of external debt. Higher leverage, meaning a higher ratio of external debt over tangible and intangible capital, will increase interest payments on external debt. Firms will take this additional cost of external financing into account when choosing the optimal investment expenditure.

We further assume that a higher intangible capital share in total capital has the potential

[12] Our approach assumes a separate investment in intangible capital, which does not add to the tangible capital stock of the firm. In this setting, productivity–enhancing investments, either flows or stocks of them, have a direct positive effect on how efficiently the firm uses its tangible capital stock. This approach is similar to a large number of notable contributions to this strand of the innovation literature (see, for example Klette and Kortum (2004) and Warusawitharana (2015)), and is consistent with our empirical TFP and investment series. Note that we use "intangible" and "productivity–enhancing" as synonyms, as based on our empirical analysis above, intangible investments were strongly productivity–enhancing, while tangible investments were not.

[13] Convex adjustment costs are a standard assumption in these types of models. See Eberly et al. (2008), who provide evidence for this with firm–level data.

to further increase debt cost. We follow here the evidence presented in Falato et al. (2022) that a higher intangible capital share reduces a firm's capacity to provide collateralisable assets.

Our proposed firm value function is;

$$\rho V(k,s,b) - \frac{\delta V}{\delta t} = \pi(k,s,b,i,r) + \frac{\delta V}{\delta k}(-\delta_k k + i) + \frac{\delta V}{\delta s}(-\delta_s s + r) + \frac{\delta V}{\delta b}(-xb + f) \ . \quad (3)$$

The firm's continuous time value function is the sum of the current dividends and the change in the value function with respect to the value function states as implied by Ito's lemma. $\frac{\delta V}{\delta t} = 0$ must hold as the value function is not expected to change with respect to time. The firm maximises this value function for the three state variables k (physical capital) s (intangible capital), and b (debt) by choosing tangible investment i and intangible investment r optimally.

The current dividends at any point in time, $\pi(k,s,b,i,r)$, are a function of the current tangible capital state k, the intangible capital state s, the debt state b, and investment decisions in tangibles i and intangibles r. The parameter ρ captures the discount factor. δ_k and δ_s are the respective depreciation rates of physical and intangible capital. x is the share of external debt a firm pays back in every period, while f is the amount of external financing taken out. We assume that $f \geq 0$, thus firms don't have the ability to pay off their debt after receiving a lot of income. Intuitively the reason is that equity holders are assumed to want to extract any dividends from the firm, which are not optimally invested back into the firm. Any investment back into the firm beyond the share of debt that is required to be returned (x) is assumed not to be optimal for individual firm shareholders. Current dividends are defined as follows;

$$\pi(k,s,b,i,r) = y(k,s) - i - r - \lambda_k \frac{k}{2}(\frac{i}{k} - \delta_k)^2 - \lambda_s \frac{s}{2}(\frac{r}{s} - \delta_s)^2 - b(x + B(k,s,b)) \ . \quad (4)$$

Current dividends consist of output produced (equation 5 below), investment cost in tangible (i) and intangible (r) capital, and convex capital adjustment costs. The functional form of these costs are chosen such that they reflect the typical setup described in Hayashi

(1982). Parameters λ_k and λ_s determine the size of capital adjustment costs for tangible and intangible capital. Current dividends are further decreased by the cost of debt (equation 7 below), which is a function of the leverage and the share of intangible capital in the capital stock. Our main functional choice for the firm output is (assuming labour input=1 for simplicity);[14]

$$y(s,k) = \zeta s^{\gamma-\alpha}(s+k)^{\alpha} \ . \tag{5}$$

Equation (5) is the equivalent of our empirical specification and therefore our preferred functional form. For robustness, we also estimate the model for a more common Cobb-Douglas specification, where tangible and intangible capital are kept separately,

$$y(s,k) = \zeta s^{\gamma}(s+k)^{\alpha} \ . \tag{6}$$

For equation (5) we require $\gamma < 1$ and $\gamma \geq \alpha$ so that the firm faces diminishing returns to scale in the capital stocks. For equation (6) we require $\alpha + \gamma \in (0,1)$ and $\alpha, \gamma \in (0,1)$. All firms in our model have the same, productivity dependent steady state, depending on ζ, which is TFP not explained by intangible capital.[15] Estimated TFP in our empirical section is the residual of both capital and labour used in the production process. Physical and intangible capital have an estimated common coefficient α. Unexplained total factor productivity in the model accounting for this is hence $\zeta s^{\gamma-\alpha}$. Intangible capital $s^{\gamma-\alpha}$ then explains TFP to the extent that $cov(s^{\gamma-\alpha}, TFP) > 0$.

Equation (7) captures the idea that leverage increases external financing costs,

$$B(k,s,b) = \psi_1[\exp(\psi_2(\frac{b}{k+s})) - 1] + \psi_3[\exp(\psi_4(\frac{s}{k+s}\frac{b}{k+s})) - 1] \ . \tag{7}$$

[14]This simplification of the model could be relaxed without affecting the main analytical results to derive a profit–maximisation condition with regard to wages and the labour input. Also note that since we are not focusing on labour markets or the household sector in a general equilibrium framework, there is no condition linking labour productivity to the marginal product or wages in the model. However, for our purposes, this partial equilibrium model is sufficient.

[15]ζ is an exogenous productivity parameter. If one wanted to introduce uncertainty into the firm's decision-making problem, one could turn ζ stochastic autoregressive process, but we abstract from this here for clarity of exposition.

The function is in its essence a penalty function as described in Den Haan and De Wind (2012). It allows us to control the increase of the interest as a result of higher leverage and higher intangible capital by varying four parameters while being bounded in positive real space for any positive leverage ratio. Intuitively, the function captures the idea that debt servicing costs typically increase as a firm becomes increasingly risky and the assets held by the firm compared to the value of its debt decrease. This concept is similar to putting penalty functions on consumers (see Algan et al. (2014)) instead of hard debt constraints to avoid consumer Ponzi schemes. A higher share of intangible capital may lead to a further increase in the leverage cost because intangible capital is harder to liquidate in the event of a default, making the firm more risky for lenders.

We assume two consequences of higher leverage on the firm's value function and dividends, thereby affecting its ability to borrow. First, external debt is more costly when the leverage in physical and intangible capital is higher (captured by the term $\psi_1[\exp(\psi_2(\frac{b}{k+s})) - 1]$). Second, external debt is more costly if a higher fraction of the firm's capital owned is intangible (captured by the term $\psi_3[\exp(\psi_4(\frac{s}{k+s}\frac{b}{k+s})) - 1]$)). These costs will impact the firm's ability to invest in intangible capital, as higher debt burden will be detrimental to the cost of the debt.

The parameters ψ_1 and ψ_2 control the cost of external debt and leverage. The parameters ψ_3, and ψ_4 control the cost of a higher intangible capital share on external debt. The non-linearity of the cost of leverage means that large leverage may make investment in intangible capital particularly costly.

States are subject to the laws of motion in equations (8) - (10):

$$\dot{k} = -\delta_k k + i \ , \tag{8}$$

$$\dot{s} = -\delta_s s + r \ , \tag{9}$$

$$\dot{b} = -xb + f \quad f \geq 0 \ , \tag{10}$$

$$f = \pi \ , \qquad \text{when } \pi < 0 \ , \text{ and} \tag{11}$$

$$f = 0 \ , \qquad \text{when } \pi \geq 0 \ . \tag{12}$$

The debt state shrinks by a partial fixed share of debt x, which has to be paid back in every period. Debt only rises when the firm increases investment beyond the point where self–financing would be sufficient. In this case, the firm relies on external financing. All dividends greater than 0 are assumed to be paid out to equity holders.

The first order conditions for the optimal choice of investment are dependent on whether the firm's profit after the investment choice is positive or negative. If it is positive, the debt state does not increase and the marginal cost of increased debt do not enter the investment decision. If the firm needs to take on debt to invest, and the cost of increasing debt given the current state is high, then the firm will invest less (see Appendix B.1 for detailed derivations of the first order conditions).

i: $[1 + \lambda_k(\frac{i}{k} - \delta_k)](1 - \frac{\delta V}{\delta b}) = \frac{\delta V}{\delta k}$ when $\pi < 0$ and $1 + \lambda_k(\frac{i}{k} - \delta_k) = \frac{\delta V}{\delta k}$ when $\pi \geq 0$

r: $[1 + \lambda_s(\frac{r}{s} - \delta_s)](1 - \frac{\delta V}{\delta b}) = \frac{\delta V}{\delta s}$ when $\pi < 0$ and $1 + \lambda_s(\frac{r}{s} - \delta_s) = \frac{\delta V}{\delta s}$ when $\pi \geq 0$

The steady state for k and s when $b = 0$ can be obtained by solving the frictionless value function. It is the result of solving for the equations 13 and 14 in the case of the baseline output equation 5,

$$s = k\frac{\gamma}{\alpha\frac{\delta_s - \delta_k}{\delta_s + \delta_k} + \gamma} = \psi k \ , \tag{13}$$

$$k = \left(\frac{\alpha\psi^\gamma}{(\rho + \delta_k)(1 + \psi)^{1-\alpha}}\right)^{\frac{1}{1-(\alpha+\gamma)}} \ , \tag{14}$$

and the result of solving for the equations 15 and 16 in the Cobb-Douglas case:

$$s^\gamma k^{\alpha-1} = \frac{\rho + \delta_k}{\alpha\zeta} \tag{15}$$

$$s^{\gamma-1}k^\alpha = \frac{\rho + \delta_s}{\gamma\zeta} \tag{16}$$

The cost of capital adjustment and the negative effect of debt and leverage on period profits specified in equation 7 mean that the value function has to be solved numerically.

3.1 Solving and estimating the value function

We solve the value function for the firm's optimal policies given the firm's states. We approximate the value function of the firm, given these states, with a neural network. Our method is similar to the methods used in Maliar et al. (2021) and Fernández-Villaverde et al. (2023), which presents the use of deep learning in approximating value functions and shows that they are more efficient at approximating and optimising unknown functions when the state space is large. Neural networks have an ability to break the "curse of dimensionality". Thus neural networks can approximate a multi-dimensional unknown function, such as our value function presented above.

We methodologically contribute to the literature by using this feature to propose a new way to estimate value function parameters based on the observed states and investment choices in the data. Our approach is a moments matching approach, where we make the parameters of the value function a part of the state space. We write the states of the problem as $X_1 = [k, s, b]$ and the parameter space as $\Psi = [\rho, \alpha, \gamma, \delta_k, \delta_s, \lambda_k, \lambda_s, \zeta, \psi_1, ...]$. The value function is then solved for $V(X_1|\Psi)$, where X_1 are the model state variables k, s, b.

Usually, we would estimate a subset of parameters $\hat{\Psi} \in \Psi$ of this value function given data Y. We would estimate $\hat{\Psi}$ by solving the value function for a guess of the parameter values to be estimated, compute the likelihood and update our guess. We would then solve the value function for each new guess until a point is found, where the likelihood given the parameters is maximised.[16]

Instead, given the comparative ease with which a neural network can be used to solve the value function for multiple states, we propose the following alternative estimation. Take states $X_1 = [k, s, b]$ and add the parameters $\Psi_X = [\gamma, \psi_1, \psi_2, ...]$ as additional states to solve the value function for. Other, calibrated parameters can be summarised as $\bar{\Psi} = [\alpha, \delta_k, \delta_s...]$. We can then estimate the parameters Ψ_X governing this value function given data Y with Algorithm 1 (below). The advantage of this approach is that we are able to estimate parameters, which would be more difficult to calibrate based on empirics.

The reason this approach works within a reasonable amount of time is that the neural

[16] For comparison, more details on this conventional algorithm are set out in in Appendix B.2.

network will approximate the parameter space, making the computation for matching moments a task of minimising the distance of neural network function to the observable states and policies by varying the estimated parameters, and thereby maximising the likelihood of the model producing the observed data. This is similar to Kase et al. (2022), though in our approach we are approximating the model, rather than the likelihood function, as a function of to–be–estimated parameters.

Algorithm 1 Estimating Value function with by solving for to be estimated parameters

Solve $V([X\Psi_X]|\bar{\Psi})$ producing output $\hat{Y}(\Psi_X)$
Alter Ψ_X such that $\min ||\hat{Y}(\Psi_X) - Y||$ e.g. $\min_{\Psi_X}(\hat{Y}(\Psi_X) - Y)W^{-1}(\hat{Y}(\Psi_X) - Y)'$
When $||\hat{Y}(\hat{\Psi}_X) - Y||$ is minimised $\hat{\Psi}_X$ is the estimate

The advantage of our approach is that once the neural network is trained, evaluating the distance of our model to the empirically observed moments, it is computationally cheap. We can therefore use standard methods to find a global minimum within the approximated parameter space. A further advantage of our method is its coding speed and the higher control provided for avoiding local minima, failures to converge, or impossible parameter shifts in the approximated parameter space.

One can describe our approach as matching moments. Concretely, we are matching the investment policies, external financing choice and productivity explained by intangible capital to different quartiles of the state space found in the data. We do this by choosing the value function among the set of value functions solved in the parameter space that best fits the outcomes in the data for investment and financing behaviour as well as productivity explained by intangible capital. While there has lately been a lot of progress in the literature in using neural networks as universal approximators in economic and econometric problems, this is, to our knowledge, the first time the parameters determining a value function have been estimated in this way.

3.2 Estimation results

We estimate the parameters ψ_1 to ψ_4 and γ with our method and solve the value function for the parameter space. Next, we match the investment choices given the capital and debt states in the empirical data used in the previous section, to the model. Additionally, we explain the observed external financing and productivity given k, s and b. To reduce the dimensionality of the problem, we categorise the states in the data into quartiles and weigh each state by the dispersion observed in the data.

The results of our estimations are shown in Table 6. Our estimated results suggest that in both cases $\gamma > \alpha$. $\gamma - \alpha$ is the elasticity with which intangible capital is able to explain total factor productivity. In both the baseline estimation and the weighted Cobb-Douglas estimation the elasticity is relatively small, at around 0.01 and 0.0012 respectively.[17] Our results in Table 6 are less dependent on the concrete weighting of each moment in the baseline estimation, giving confidence in the baseline results, which maps straightforwardly to our empirical estimates. In the baseline result, the penalty for leverage is estimated to be larger, with both parameter controlling the linear cost of higher leverage (ψ_1), and the parameter controlling the exponential effect of higher leverage (ψ_2) being larger. On the other hand, there is also a significant cost to the firm having a higher leverage based on intangible capital. This cost is mostly driven by the parameter controlling linear cost increases (ψ_3) in relation to the parameter giving an exponential cost to a higher intangible share in the leveraged capital (ψ_4).

Table 7 also shows the calibrated parameter values in our model. These are a combination of convenient choices (adjustment costs and unexplained TFP), values typically used in the literature (discount factor) and values based on UK data (exponent on capital, depreciation rates and share of physical capital).

The fit of our estimation is shown in figure 4. We match the moments by varying the parameters to–be–estimated to find a minimum loss in the parameter space, where we have solved the value function. The top panels show the fit of firm physical and intangible investment, conditional on states K, S, B, respectively. The conditional data moment is represented by the solid blue line, while the standard deviation interval is shown with a blue dashed line. The red line shows the predictions for investment conditional on the states from the fitted model. The fitted model matches investment patterns for both tangible (top left–hand panel) and intangible capital (top right–hand panel) as the size of debt, tangible and intangible capital vary. The bottom left panel shows the equivalent

[17] The lower estimation in the Cobb-Douglas case is owed to the fact that it is less transferable on our empirical estimations. In the Cobb-Douglas case the marginal product of physical capital is higher as intangible capital is not added to it, and hence less explanation is needed to fit total factor productivity and firm investment choices.

conditional moments and model predictions for external financing. Except for very small firms, the model further matches external financing well. The bottom right panel in Figure 4 shows estimated firm productivity. We see in the data that estimated productivity increases conditional on intangible capital and decreases mildly conditional on physical capital. The estimated model matches the increase in intangible capital. It does not match the mild decline in productivity conditional on physical capital size. The reason is that only intangible capital enters the model function explaining total factor productivity, $s^{\gamma-\alpha}$.

For robustness, we also show the estimated fit for the model where the production function has the more common Cobb-Douglas specification in figure 5. The model also matches investment, financing, and productivity patterns well, doing slightly better on matching the external financing decision, but slightly worse on matching the physical capital investment decisions conditional on the firm states.

Our model is able to explain the productivity pattern, investment patterns and financing patterns observed in the data. Intangible capital is able to explain measured total factor productivity of capital as $\gamma > \alpha$. Firms that have taken on external debt to finance investment in intangible capital are more productive, but will refrain from taking on too much debt as the cost of leverage is mildly increasing in intangible capital.

Figure 6 shows firm investment behaviour on the horizontal axes and the TFP state for capital explained by intangible capital on the vertical axis for our preferred baseline model. The plots show, similarly to our empirical estimations, that; 1) higher investment in intangible capital (per total assets) is associated with higher productivity, while higher investment in physical capital is not; and 2) firms that take on higher debt and invest more in physical capital will have lower productivity, while firms that take on more debt and invest to a higher extent in intangible capital have higher productivity. These results are fully in line with our empirical results presented above.[18]

[18]In Figure 7 we show similar surface plots for the Cobb-Douglas model. They show the same broad intuition, confirming that our model specification, which is close to our empirical specification, does not produce counterintuitive results.

3.3 Investment and TFP

The different types of investment flows in the model carry different implications, so it is worth exploring these further in an illustrative example. For intuition, we show in Figure 8 the impact of a 1% permanent increase in investment of either capital type. The upper left panel shows the simulated path for tangible investment and the upper right panel for intangible investment from steady state investment levels, whilst holding the other investment type path constant at their steady state values.[19] As only intangible investment enters the productivity (TFP) equation directly in the form $TFP \equiv \zeta s^{\gamma-\alpha}$, only this type of investment has a *direct* effect on the level of productivity, causing it to be permanently higher (bottom left panel). However, the solved value of the firm increases in both cases, given the investment state dynamics (bottom right panel). In this particular parameterisation, the increase in firm value is somewhat larger for tangible investment than for intangible investment, but as intuition would suggest, both types of investment cause a permanently higher firm value.

The example above suggests that firms may behave optimally and choose to invest, even if the (tangible) investment does not lead to higher TFP. This is because the increase in tangible investment may, in the long run, still produce enough revenue to make the investment *ex ante* profitable. However, this does not need to lead to any effect on TFP; one can think of concrete examples where a firm may acquire a new piece of equipment, which will allow them to produce more output, but this will not necessarily affect the efficiency with which the firm combines its inputs to produce output – i.e., its TFP.

Our empirical results suggest that investment in physical capital that is accompanied by high debt may decrease future investment in TFP. This is clearly also a result of our model when most of current firm income is spent on debt payments and the firm has to turn to external financing to invest further. In this case, the optimality condition for investment in productivity building intangible capital ($r = [\frac{\frac{\delta V}{\delta s}}{1-\frac{\delta V}{\delta b}}\frac{1}{\lambda_s} + \delta_s]s$) means that the marginal cost of additional external debt will reduce the amount of investment in intangible capital.

[19]Note that for the purposes of this illustrative example, the model is not solved for optimal paths of the other investment type. Also, starting from different levels of the state variables would result in different marginal benefits for the firm from investing in the two different types of capital.

3.4 Aggregate TFP effects

We can also use our model to estimate the total contribution of intangibles investments in our sample of firms to aggregate TFP dynamics in the UK economy. In particular, we can calculate the model–implied contribution to the slowdown in UK TFP growth in the pre– and post–GFC data. In our sample of large firms, the total domestic turnover, on average over 1990-2018, accounts for about 30% of UK GDP. Even though the number of firms in our sample is a very small proportion of all firms in the UK economy, the tail of large firms can have significant aggregate–level productivity effects in the UK, as shown in Dacic and Melolinna (2022).

For our sample of firms, the annual average intangible investment growth rate was about 18.7% in the pre-GFC period (1991–2007), and then around 4.3% in the post-GFC period (2010-2018), implying a slowdown or around -14.4pp in this growth rate. At the same time, using a conventional Cobb–Douglas production function (with a 2/3 labour share and 1/3 capital share) and official aggregate data, TFP growth rate in the whole economy slowed down from 1.2% to 0.8%, implying a slowdown of -0.4pp.

In terms of our structural model, TFP growth g_a in our production function can be decomposed into an unexplained and an explained component (g_ζ and $(\gamma - \alpha)g_s$, respectively). The latter is explained by growth in intangibles;

$$g_a = g_\zeta + (\gamma - \alpha)g_s \tag{17}$$

Using the parameters in our model, the model hence predicts that the reduced growth in intangibles accounts for a slowdown in TFP growth of -0.07pp, with the following calculation: $(\gamma - \alpha)$*TFP growth slowdown*share of firms of total GDP, which is $0.0103 * (-14.4pp.) * 0.30 = -0.045pp$. This is 11%, so around a tenth, of the whole economy TFP growth slowdown of -0.4pp. While this is only a suggestive back–of–the–envelope estimate, and will depend on the data and model assumptions, it nevertheless suggests that the weaker intangibles investment growth in this group of large firms have had a meaningful negative effect on UK TFP growth since the GFC. Also note that this estimate is likely to be a lower bound on the *total* effect of intangibles growth slowdown in the whole population of UK firms, provided the dynamics seen in the large firms are indicative of other firms in the economy.

4 Conclusions

This paper studies the effects of different types of corporate investment and uses of funds, as well as levels of debt, on productivity. We combine theoretical propositions of a stylised structural model, featuring a dynamic profit–maximisation problem of the firm, with empirical regression results, using firm-level data. We set out a standard neoclassical model and augment it with an external financing option. The model is then used to illustrate, both analytically and by solving the model numerically, why productivity–enhancing investment differs from other uses of company funds in terms of its positive effects on TFP, and how these effects can be stronger for firms that have higher indebtedness. We also provide some back–of–the– envelope estimates on the contribution of large UK firms on the aggregate TFP growth slowdown in the UK since the financial crisis.

Our results suggest that there is a positive effect of intangible investment on TFP, while there is no such positive effect for tangible capital expenditure. The results also suggest that a combination of high debt and high intangibles investment has a positive effect on TFP. In this sense, in our sample of firms, debt is "good" for TFP, if it is associated with a high intensity of productivity–enhancing investments.

Our paper contributes to the long–standing discussions on the effects of innovation, or intangible investment, as well as of debt, on firm performance. The view emerging from our results is a relatively benign one; accumulation of intangible investments has a positive effect on firm TFP, and a combination of high debt and high intangibles intensity also has a positive effect (albeit economically small). However, our study says little about how these effects could vary in different stages of the business cycles, and especially after crisis periods. We have also only scratched the surface of the non–linearity of the effects of debt in some of our robustness analysis. We leave a closer examination of these issues to other methods and papers, but based on our conjecture and evidence for it, we emphasise the importance of understanding what debt is used for, when analysing its effects. This also strikes us as an important consideration when setting policies that affect or operate through firms' debt.

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A. Data

A.1 Firm–level data

The exact definitions of the variables used in the analysis are as follows:

log_tfp: measure of firm–level real (revenue) TFP, based on the Ackerberg et al. (2015) methodology. The main challenge in estimating firm-level production functions is the endogeneity between inputs (capital, labour and internmediate inputs) and output; this positive correlation causes OLS estimates of the production function coefficients to be biased. As is standard in recent literature, we resort to control–function (CF) approaches to calculating TFP. In particular, we use two common measures: 1) Olley–Pakes (OP) with the Ackerberg–Caves–Frazer correction and 2) Wooldridge. We only report results for 1) in the main text.

intan_stock: measure of intangible stocks (defined in Annex A.2) divided by total assets. The suffix d in our regressions indicates a dummy variable, where the highest quartile is 1 and 0 otherwise.

intan_flow_ratio: year–on–year change in *intan_stock*, divided by total assets. The suffix d in our regressions indicates a dummy variable, where the highest quartile is 1 and 0 otherwise.

debt_ratio: total debt divided by total assets (corrected for industry medians).

tan_flow_ratio: capital expenditure divided by total assets. The suffix d in our regressions indicates a dummy variable, where the highest quartile is 1 and 0 otherwise.

age: number of years since firm incorporation

size: real total assets (log level, deflated by GDP deflator)

profit_ratio: profits (EBITDA) divided by total assets

cash_ratio: cash and short-term investment divided by total assets

iex_ratio: interest expense divided by total debt (used as an instrument in system GMM regressions only)

To mitigate the effects of outliers, all variables are winsorised at the 1st and 99th percentiles, as is conventional in the literature.

The following variables are used as inputs into the TFP production function estimations:

<u>output</u>: Log level of revenue (Worldscope:netsales), deflated by GDP deflator

<u>capital input</u>: Log level of net property, plant and equipment (Worldscope:netppe) + intangibles stocks, deflated by GFCF deflator

<u>labour input</u>: Log level of cost of goods sold (Worldscope:cogs), deflated by GDP deflator

<u>tangible investment</u>: Log level of capital expenditure (Worldscope:capex), deflated by GFCF deflator

<u>intangible investment</u>: Log level of year–on–year change in gross intangible assets (as defined in Annex A.2), deflated by GFCF deflator

In addition to the micro data, we use aggregate GDP and GFCF deflators, published by the Office for National Statistics, to transform some of the variables (inputs for the TFP production function estimation and total assets) from nominal into real space. We do not report any stylised facts on these series for brevity, but from 1990 to 2018, the average annual growth rate of the GDP deflator is around 2.4% and the growth rate of the GFCF deflator is around 2%. We also use aggregate data on real GDP, total employment hours and capital services to calculate aggregate TFP (where we also assume the labour share to be 2/3 and capital share 1/3, as is standard in the literature.). All the aggregate data is available on the Office for National Statistics website (www.ons.gov.uk).

A.2 Definition of intangible capital

There is no uniformly agreed measure of intangibles in the literature. For firm-level data, a particular challenge is the fact that intangible investment flows (like R&D spending) are not capitalised into the balance sheet, as they are recorded as expenses in the income account. However, a recent paper by Peters and Taylor (2017) introduces a methodology for measuring intangible stocks and flows, including a method for accumulating expensed intangibles items into stocks. In our definitions, we follow their methodology, which is explained in more detail in this Appendix.

Intangible capital stock (K_{it}^{int}) is defined as:

$$K_{it}^{int} = INT_{it} + A_{it} + B_{it} \tag{18}$$

where INT_{it} is intangible assets in the balance sheet for firm i at time t, A_{it} is accumulated R&D (RD) spending (defined below) and B_{it} is accumulated sales, general and administrative (SGA) spending (defined below).

Accumulated RD spending is defined as follows:

$$A_{it} = (1 - d_{rd})A_{i,t-1} + RD_{it} \tag{19}$$

where d_{rd} is the R&D depreciation rate (assumed to be 15% economy-wide, following previous literature).

Accumulated SGA spending is defined as follows:

$$B_{it} = (1 - d_{sga})B_{i,t-1} + 0.3 * SGA_{it} \tag{20}$$

where d_sga is the SGA depreciation rate (assumed to be 20% economy-wide, following previous literature), and 30% of SGA is assumed to be related to intangible investment.[20]

A choice needs to be made on what to do about starting RD and SGA stocks (A_{i0} and B_{i0}), as the firms are not usually observed from the year they were formed. We assume these to be zero[21]. There is also a small number of cases, where a firm exits the dataset and then re-entres n number of years later. For the accumulation of RD (and analogously for SGA), we use the following proxy formula to calculate the accumulated stocks in the year of re-entry:

$$A_{it} = (1 - d_{rd})^n A_{i,t-n} + (n-1)\left(\frac{(1 - d_{rd})^n RD_{i,t-n} + RD_{it}}{2}\right) \tag{21}$$

As the intangible investment measures used are necessarily only proxies for "true" intangible investment, we want to test the robustness of our result to a number of different intangibles measures. In particular, there is a question on whether goodwill should be included in the intangibles stock. Hence, we also calculate a measure that excludes goodwill. In the main text, we only report and discuss results on our baseline intangibles measure, but the significance and quantitative effects of a measure excluding goodwill are broadly similar.

[20] In the Results section, we also examine the robustness of our regressions to different assumptions of the depreciation rates as well as the intangibes share of SGA.

[21] Peters and Taylor (2017) make a similar assumption, but also apply a more complicated method of accumulating stocks from non–zero initial stocks. They find their results look very similar with either method, and given the lack of data on the UK, we do not pursue these comparisons in our intangibles data.

B. Details of the Structural Model and Algorithm

B.1 Model first order conditions

The model first order conditions can be derived by taking the first derivative of equation (3) towards physical investment i and intangible investment r,

$$\rho V(k,s,b) = \pi(k,s,b,i,r) + \frac{\delta V}{\delta k}(-\delta_k k + i) + \frac{\delta V}{\delta s}(-\delta_s s + r) + \frac{\delta V}{\delta b}(-xb + f) \ . \tag{22}$$

We then have the following first order condition for physical capital i,

$$\frac{\delta V(k,s,b)}{\delta i} = 0 = -1 - \lambda_k(\frac{i}{k} - \delta_k)] = \frac{\delta V}{\delta k}$$

when $\pi \geq 0$ and

$$\frac{\delta V(k,s,b)}{\delta i} = 0 = -1 - \lambda_k(\frac{i}{k} - \delta_k)] = \frac{\delta V}{\delta k} + [1 + \lambda_k(\frac{i}{k} - \delta_k)]\frac{\delta V}{\delta b}$$

when $\pi < 0$.

Equally we have the first order condition for intangible capital r as,

$$\frac{\delta V(k,s,b)}{\delta r} = 0 = -1 - \lambda_s(\frac{r}{s} - \delta_s)] = \frac{\delta V}{\delta s}$$

when $\pi \geq 0$ and

$$\frac{\delta V(k,s,b)}{\delta r} = 0 = -1 - \lambda_k(\frac{r}{s} - \delta_s)] = \frac{\delta V}{\delta s} + [1 + \lambda_s(\frac{r}{s} - \delta_s)]\frac{\delta V}{\delta b}$$

when $\pi < 0$.

Crucially, we assume that the firm cannot distinguish at any point in time whether external fnancing will flow into tangible or intangible capital. Hence the firm only decides on the specific capital investment after it has chosen whether to use external financing in the particular time increment. The firm will have to rely on external financing for all investment that cannot be financed from current income. As a result, when the optimal investment in intangible and tangible capital exceeds current income, taking into account the consequences of an increased debt level, the firm will choose to rely on external financing.

Thus when optimal $i + r > \pi$ exceed current firm income their optimal level is: $i : 1 + \lambda_k(\frac{i}{k} - \delta_k) = \frac{\delta V}{\delta k}$ and $r : 1 + \lambda_k(\frac{r}{s} - \delta_s) = \frac{\delta V}{\delta s}$. Note that only the marginal benefit of getting more capital of either type enters, which is set equal to the marginal capital adjustment cost. In contrast, when $i + r \leq \pi$ then the optimal investment choice is $[1 + \lambda_k(\frac{i}{k} - \delta_k)](1 - \frac{\delta V}{\delta b}) = \frac{\delta V}{\delta k}$ and $[1 + \lambda_s(\frac{r}{s} - \delta_s)](1 - \frac{\delta V}{\delta b}) = \frac{\delta V}{\delta s}$. Note that in this case the cost of financing enters additionally. As $\frac{\delta V}{\delta b} < 0$ this will mean that opimal invesment is lower than in the first case as the firm optimally accounts for the cost of an additional unit of debt when choosing investment.

B.2 More details on the algorithm

As mentioned in the main text, the usual way to estimate a subset of parameters $\hat{\Psi} \in \Psi$ of a value function given data Y is as described in Algorithm 2. One estimates $\hat{\Psi}$ by solving the value function for a guess for the parameter values to be estimated, computes the likelihood and updates our guess. We solve the value function for each new guess until we believe to have found a point where the likelihood given the parameters is maximised.

Algorithm 2 Estimating Value function standard

```
Solve V(X|Ψ_1) producing output Ŷ (Ŷ can be conditional on X)
while N is even do
    Check weighted distance ||Ŷ − Y||
    Update Ψ given distance e.g. Newton
    Solve V(X_1|Ψ_{N+1}) producing output Ŷ
    if |Ψ_{N+1} − Ψ_N| < ε then Break the Loop
    end if
end whileThen Ψ_N = Ψ̂
```

Compared with this solution method, Algorithm 1 may be computationally more intensive, but it is both easier to code and easier to avoid local minima within the approximated range of the value function. With our approach, once the neural network is trained, evaluating the distance of our model to the empirically observed moments is computationally cheap. We can use standard methods to find a global minimum within the approximated parameter space. To avoid local minima we can use stochastic starting points. We can also avoid failures of the algorithm to converge, and avoid impossible parameters in the approximated parameter space.

C. Tables and Figures

C.1 Tables

Table 1: *Variable definitions*

Variable	Definition	Worldscope code
log_tfp	Ackerberg-Caves-Frazer (2015) measure of TFP (log level)	n/a
intan_stock	intan_stock	n/a
intan_flow_ratio	y/y change of intan_stock, divided by total assets	n/a
debt_ratio	total debt divided by total assets (corrected for industry medians)	totdebt/totass
tan_flow_ratio	capital expenditure divided by total assets	capex/totass
age	years since incorporation	age
size	real total assets (log level, deflated by aggregate GDP deflator)	totass
profit_ratio	profits (EBITDA) divided by total assets	ebitda/totass
cash_ratio	cash and short-term investment divided by total assets	csti/totass
iex_ratio	interest expense divided by total debt	intex/totdebt

Table 2: *Selected key variables - stylised facts*

Variable	Obs	Mean	Std. Dev.	Min	Max
log_tfp	27,712	0.982677	0.506992	-0.965	4.186916
log_tfp_yy	26,011	0.00781	0.236682	-1.46004	1.684025
intan stock	27,712	0.5041	0.561825	0	4.618176
intan flow_ratio	26,677	0.04199	0.219215	-2.5988	2.208064
debt_ratio	27,712	0.201115	0.191611	0	1.364855
tan flow_ratio	27,712	0.053065	0.05665	0	0.372803
age	27,712	33.98318	32.7406	0	164
size (assets)	27,712	4.659566	2.08781	-3.41332	12.36988
profit_ratio	27,566	0.061273	0.242948	-2.63523	0.515761
cash_ratio	27,705	0.134	0.163	0.000	0.906

Notes: All variables winsorised at 1st and 99th percentiles.
Data weighted by firm employment.
For definitions of the variables, see main text and Data Annex.

Table 3: *OLS panel regression results*

DEPENDENT VARIABLE:

log_tfp			
RHS VARIABLES:	(1)	(2)	(3)
debt_ratio (t-1)	0.0072	0.0299	0.056
	(0.0339)	(0.0361)	(0.039)
intan_stock_d (t-1)	0.0972***		
	(0.0129)		
intan_flow_ratio_d (t-1)		0.0241***	
		(0.00673)	
tan_flow_ratio_d (t-1)			-0.0271***
			(0.0072)
debt*RHS variable (t-1)	0.107*	0.0579	-0.0393
	(0.0595)	(0.0392)	(0.0383)
size (t-1)	0.0266***	0.016**	0.0142*
	(0.00781)	(0.00798)	(0.00778)
age (t-1)	0.00325***	0.0043***	0.00377***
	(0.000941)	(0.001)	(0.000954)
profit (t-1)	0.115***	0.112***	0.102***
	(0.0219)	(0.0223)	(0.0223)
cash (t-1)	0.0224	-0.00641	-0.0217
	(0.0391)	(0.0405)	(0.0392)
Constant	0.794***	0.830***	0.874***
	(0.04)	(0.0402)	(0.0386)
Observations	24,302	23,293	24,302
R-squared	0.789	0.788	0.787
Joint debt effect	0.048	0.062	0.653

Notes: Robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1
All regressions include time and firm fixed effects.

Table 4:*Economic effects*

What is the effect on level of TFP at time t of...

...firm being in the highest quartile of...

...variable (t-1):	*effect :*
intan_stock_d	9.7% ***
intan_flow_ratio_d	2.4% ***
tan_flow_ratio_d	-2.7% ***

...a 10pp increase in debt ratio and firm being in the highest quartile of...

...variable (t-1):	*effect :*
intan_stock_d	1.1% **
intan_flow_ratio_d	0.9% *
tan_flow_ratio_d	0.3%

Table 5: *Robustness checks*

DEPENDENT VARIABLE:	log_tfp_eta	log_tfp_eta	log_tfp	log_tfp	log_tfp low sga	log_tfp low sga	log_tfp low d	log_tfp low d	log_tfp high sga	log_tfp high sga	log_tfp high d	log_tfp high d
RHS VARIABLES:	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
debt_ratio (t-1)	0.0431	0.0617*			0.00938	0.0266	0.0152	0.0251	0.0228	0.0306	0.0324	0.0373
	(0.0287)	(0.0324)			(0.033)	(0.0351)	(0.0326)	(0.0356)	(0.0342)	(0.0359)	(0.0341)	(0.0343)
equity_ratio (t-1)			0.0242*	0.0358***								
			(0.0146)	(0.0108)								
intan_stock_d (t-1)	0.0972***		0.0887***		0.0513***		0.0956***		0.106***		0.0887***	
	(0.0096)		(0.0131)		(0.0132)		(0.013)		(0.0139)		(0.0127)	
intan_stock_yy_d (t-1)		0.0116**		0.0259***		0.0144***		0.0267***		0.0295***		0.0222***
		(0.00556)		(0.00619)		(0.00652)		(0.00683)		(0.0072)		(0.00722)
debt*RHS variable (t-1)	0.0556	0.0387			0.109*	0.0740*	0.0754	0.0587	0.055	0.049	0.105*	0.0493
	(0.0533)	(0.0329)			(0.0596)	(0.0395)	(0.0599)	(0.0395)	(0.0596)	(0.0403)	(0.0588)	(0.0415)
equity*RHS variable (t-1)			0.00651	-0.00231								
			(0.014)	(0.00784)								
Observations	24,307	23,298	24,260	23,251	24,302	23,293	24,302	23,293	24,302	23,293	24,302	23,293
R-squared	0.565	0.563	0.802	0.801	0.799	0.799	0.805	0.804	0.803	0.802	0.797	0.796
Joint effect	0.06	0.012	P<0.01	P<0.01	0.045	0.037	0.123	0.074	0.171	0.092	0.048	0.085

Notes:Robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1
Regression specifications for controls (not reported) and fixed effects are similar to baseline OLS models (Table 3).
Columns with "low d" contain results of R%D and SGA depreciation rate of 10%, "high d" rates are 25 and 30%, respectively.
Columns with "low sga" contain results of 10% of SGA spending assumed to be intangibles investment, and for "high sga" the share is 50%.

Table 6: *Estimated model parameters fitted in the baseline case according to inverse standard deviations of the data and in the unweighted case weighing each data moment equally. Results are reported for both the baseline and the C-D output choice.*

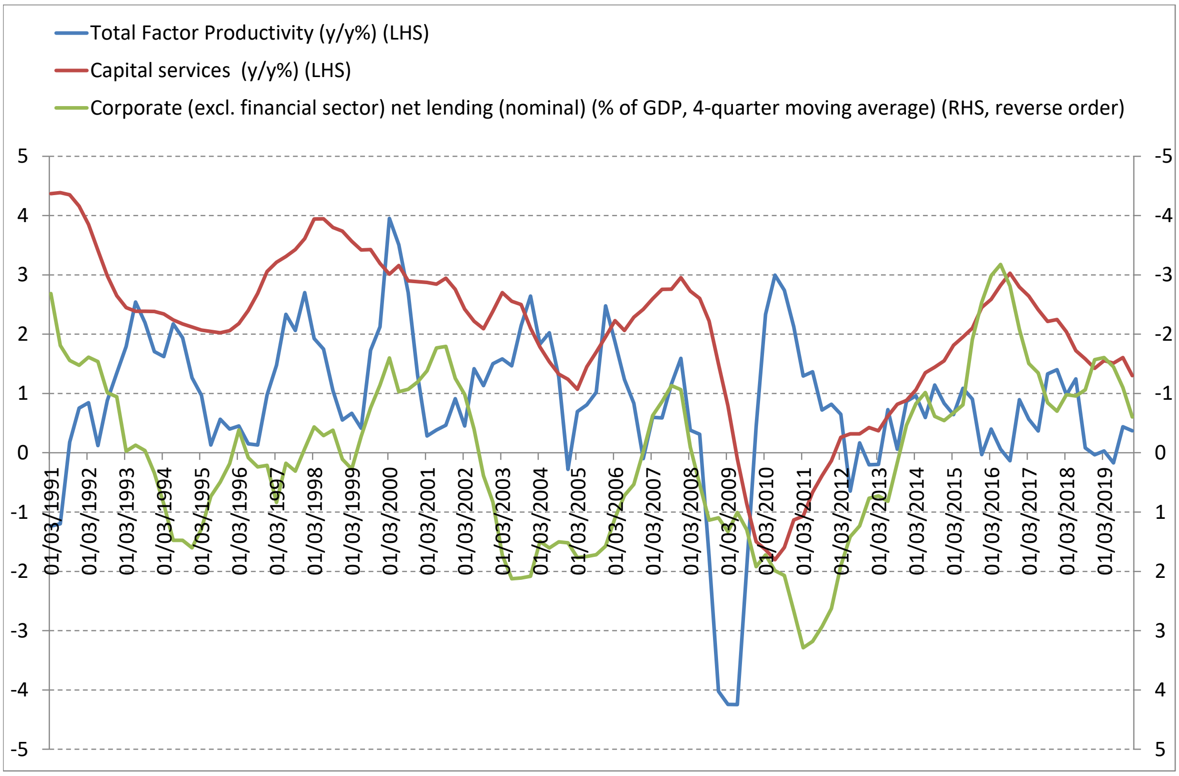
Estimated Parameters	ψ_1	ψ_2	ψ_3	ψ_4	γ
Weighted fit	0.0050	0.0046	0.0309	0.0154	0.1803
Unweighted fit	0.0041	0.0001	0.0452	0	0.1855
Weighted fit C-D	0.0017	0.0096	0.00701	0.0317	0.1712
Unweighted fit C-D	0.00009	0.00006	0.00002	0.00003	0.2228

Table 7: *Model parameter calibration and estimation results*

Parameter	Value	Description
ρ	0.05	Annual discount factor of 0.9512.
λ_k	20	Adjustment cost of physical capital
λ_s	20	Adjustment cost of intangible capital
δ_k	0.07	Depreciation rate of physical capital
δ_s	0.2	Depreciation rate of intangible capital
α	0.17	Exponent on combined (Baseline) or physical (C-D) capital
x	0.1	Principal pay back rate of loans
ζ	1.83 (Baseline) 1.3 (C-D)	Otherwise unexplained TFP (for steady state normalisation)
	Estimated Parameters	
γ	Estimated	Exponent on intangible capital
ψ_1	Estimated	Cost of leverage
ψ_2	Estimated	Elasticity of the cost of leverage
ψ_3	Estimated	Cost of leverage on intangible capital
ψ_4	Estimated	Elasticity of the cost of leverage leverage on intangible capital

C.2 Figures

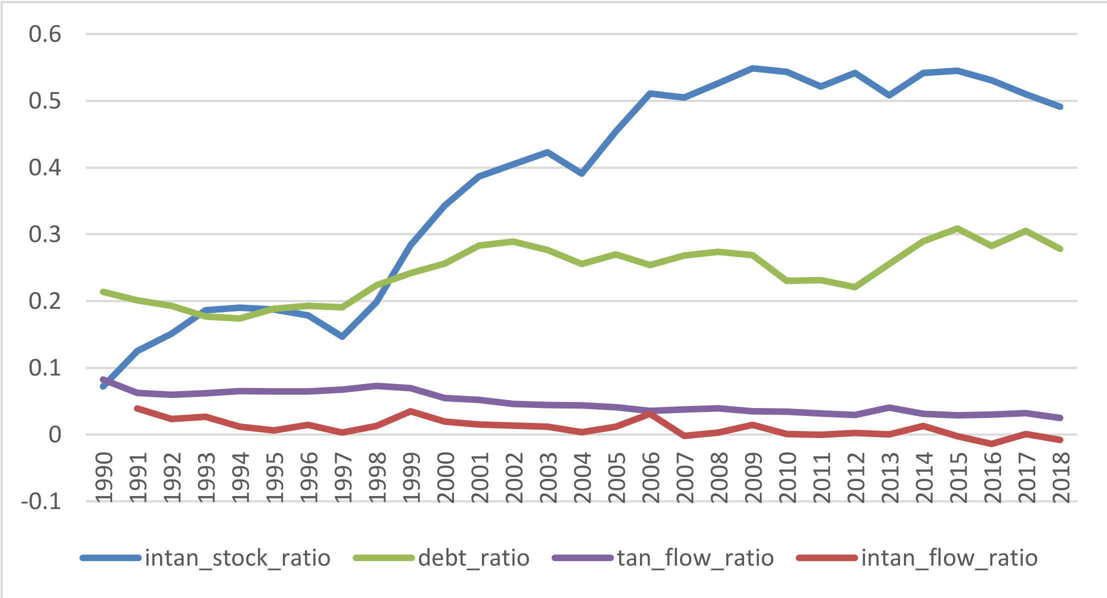
Figure 1: *Aggregate TFP, capital services and corporate debt*



Sources: ONS and own calculations.

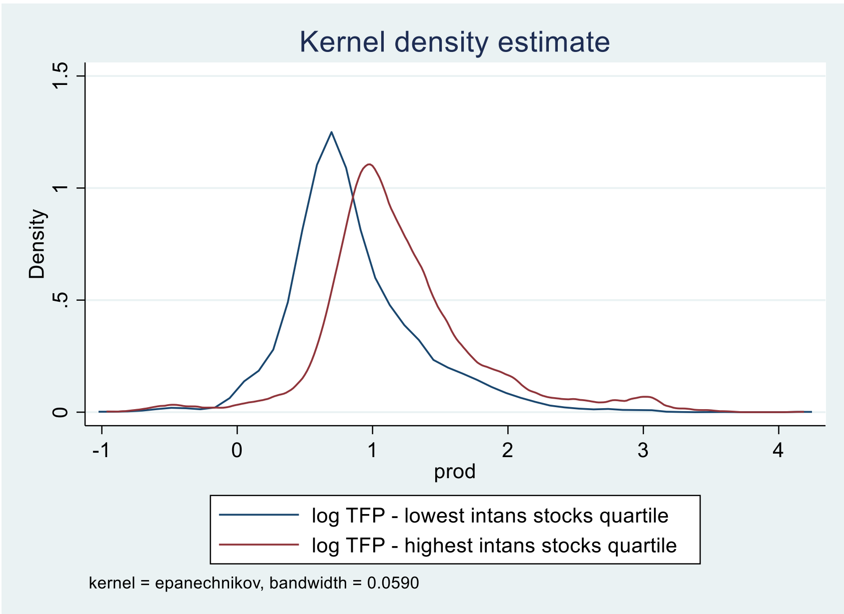
Notes: TFP is calculated from a basic Cobb-Douglas production function, with a labour share of 2/3.

Figure 2: *Selected data series - medians over time*



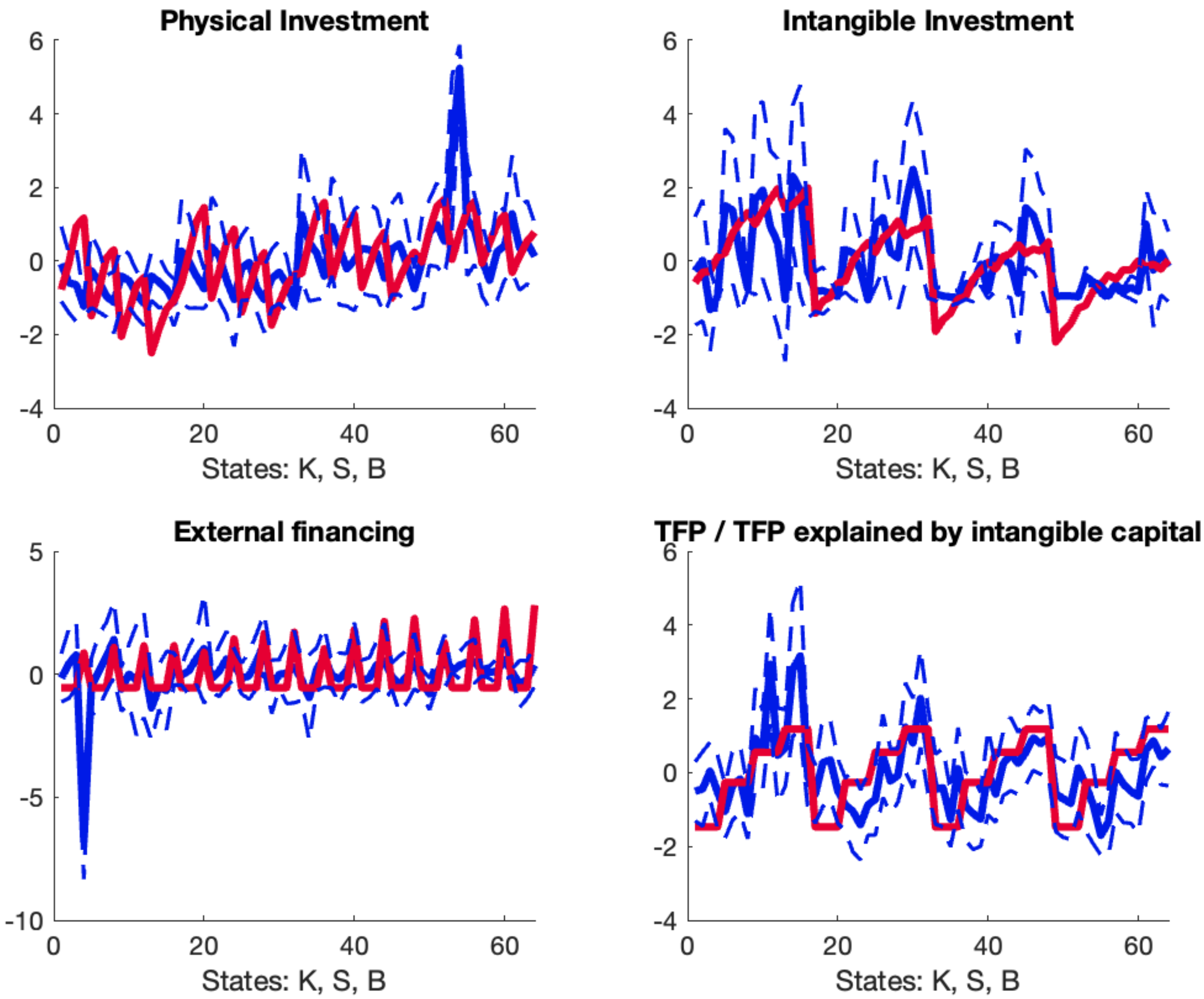
Sources: Worldscope and own calculations.
Notes: Data are employment-weighted medians.

Figure 3: *TFP intan densities*



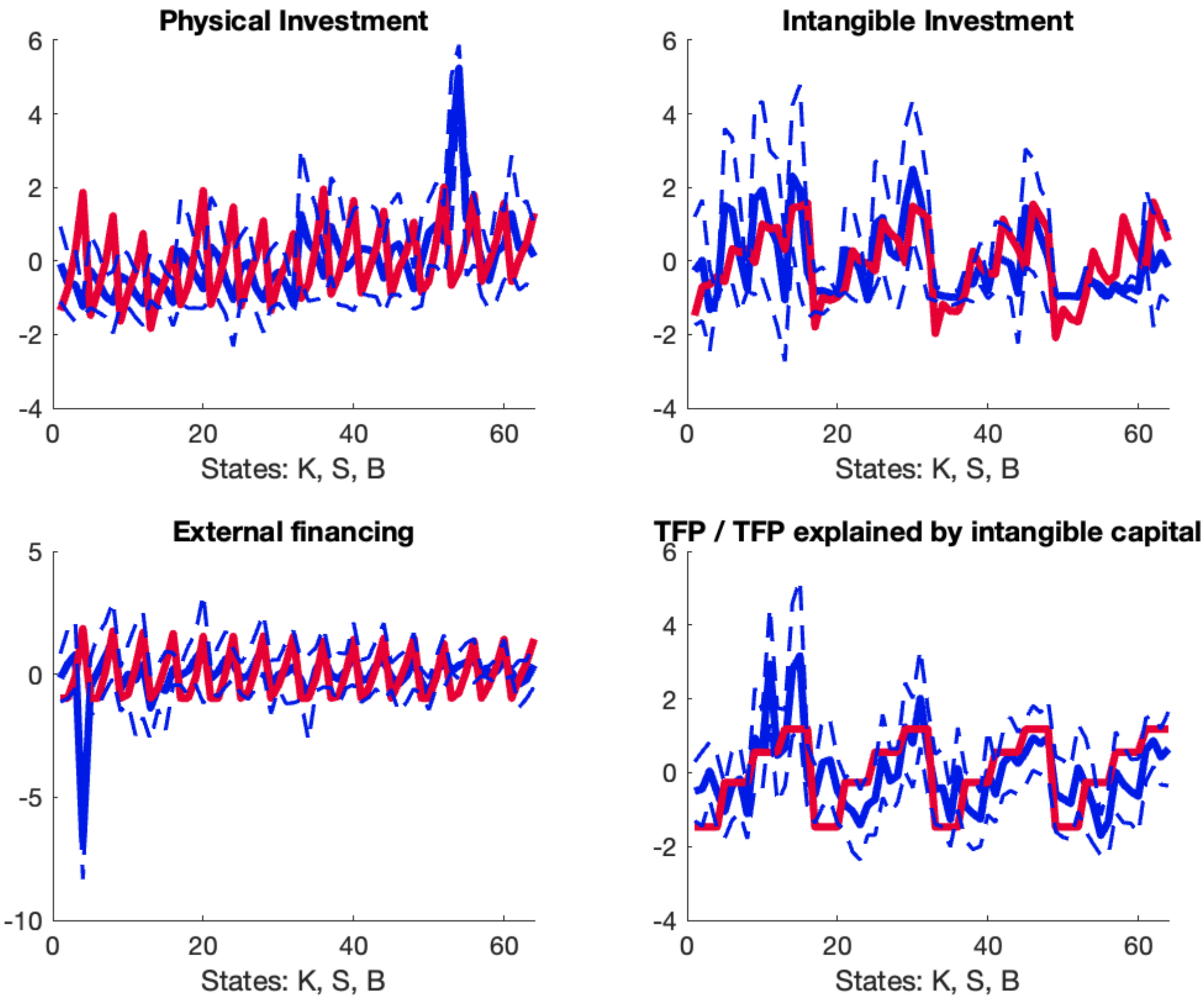
Sources: Worldscope.

Figure 4: Matched moments from the model to the data for the baseline model.



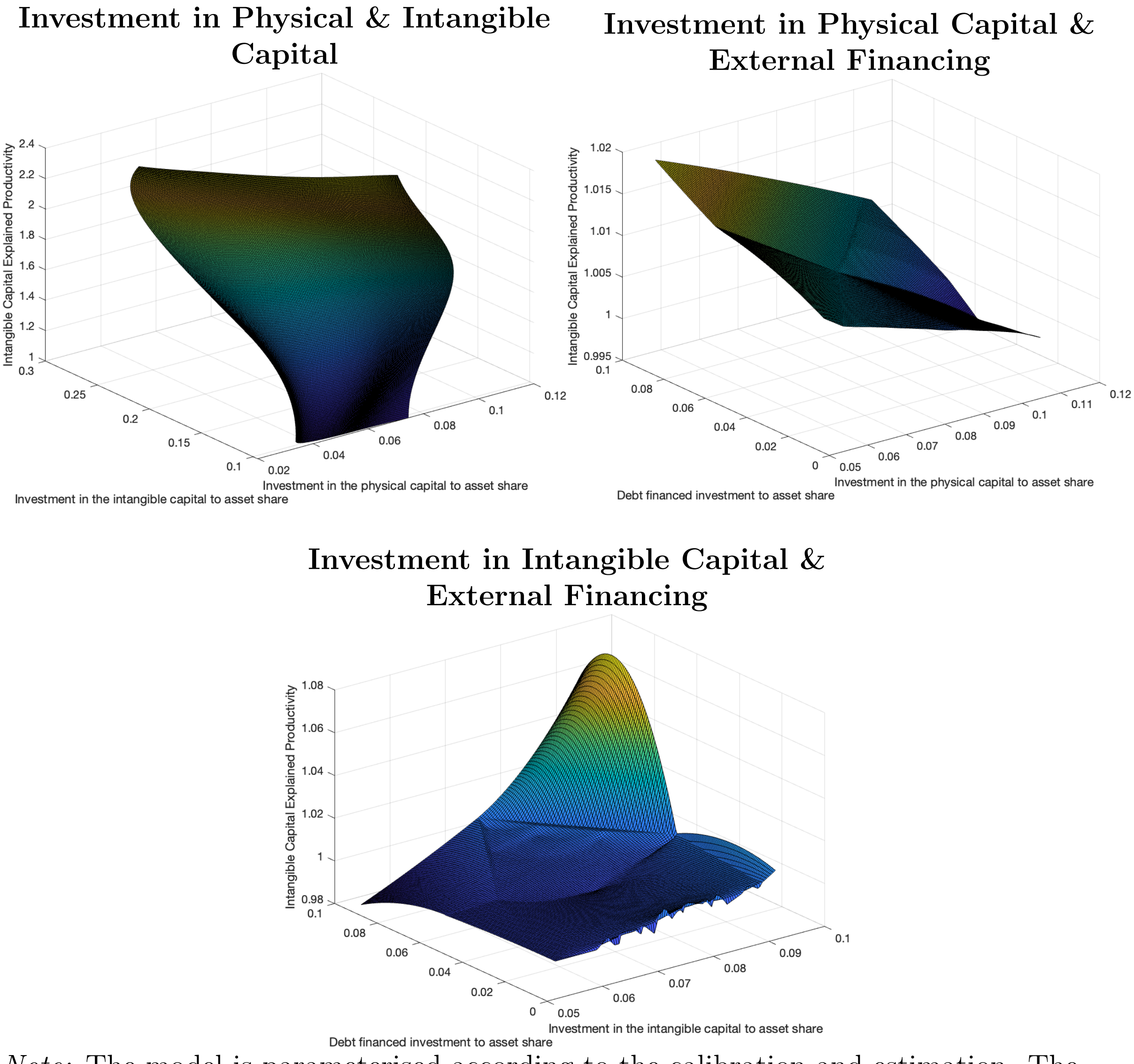
Note: Estimated baseline model results from the data are shown in red for each of physical Capital (K), intangible capital (S), and debt (B) combination. The blue line shows the model moments conditional on the states. The dashed lines show the standard deviation of the moments conditional on the states. The red lines show the predicted model outcomes of the fitted model. The data and model results are normalised. All state combinations are increasing from left to right.

Figure 5: Matched moments from the model to the data for the Cobb-Douglas model.



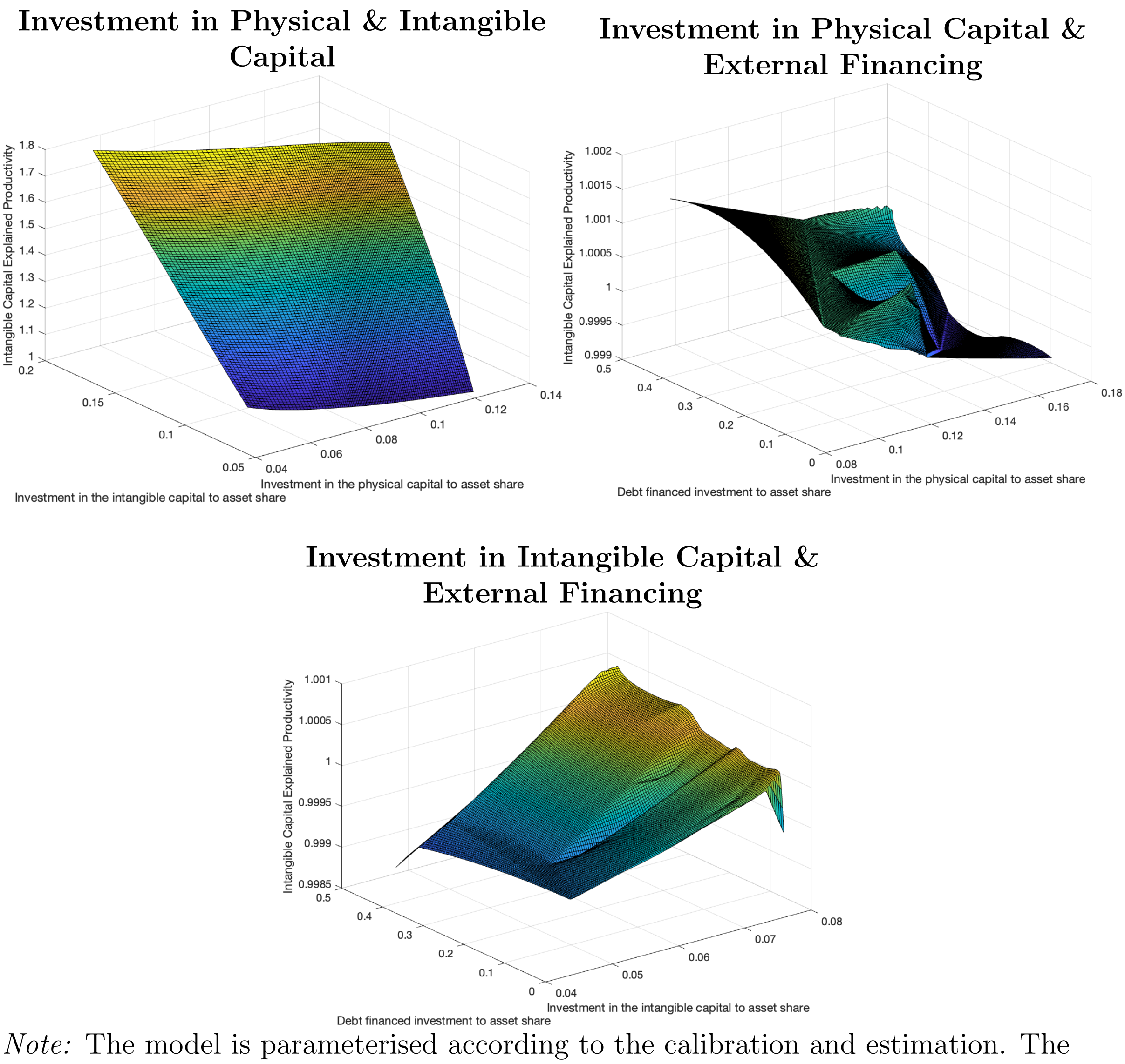
Note: Estimated Cobb-Douglas model results from the data are shown in red for each of physical Capital (K), intangible capital (S), and debt (B) combination. The blue line shows the model moments conditional on the states. The dashed lines show the standard deviation of the moments conditional on the states. The red lines show the predicted model outcomes of the fitted model. The data and model results are normalised. All state combinations are increasing from left to right.

Figure 6: Estimated model results for productivity explained by intangible capital based on investment activity.



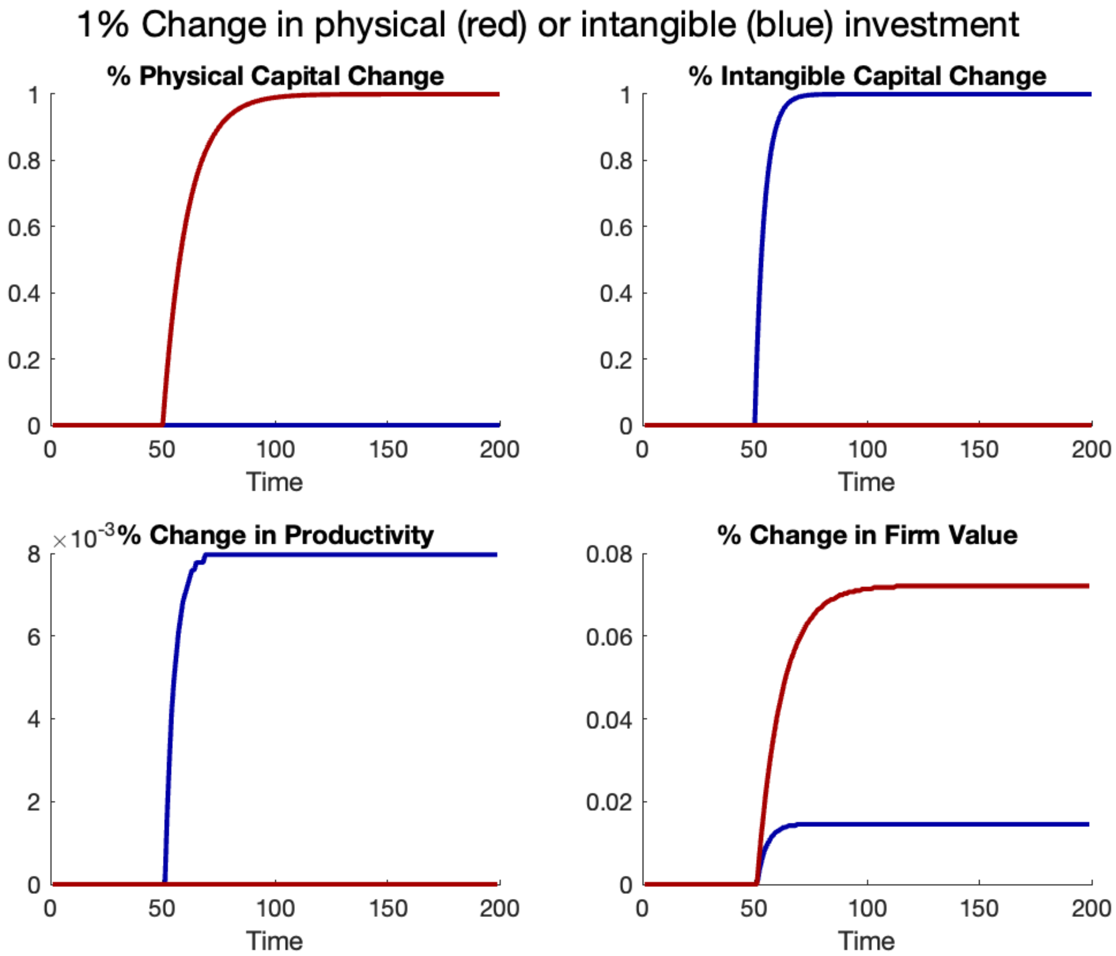
Note: The model is parameterised according to the calibration and estimation. The results are then interpolated based on the model predictions, collapsing the model predictions for investment activity given the three model state variables and the predicted investment outcomes.

Figure 7: Estimated model results for productivity explained by intangible capital based on investment activity for the Cobb-Douglas case.



Note: The model is parameterised according to the calibration and estimation. The results are then interpolated based on the model predictions, collapsing the model predictions for investment activity given the three model state variables and the predicted investment outcomes.

Figure 8: *State paths and firm value paths of a 1% increase in investment categories for the baseline model.*



Notes: The firm value is computed from the relevant starting state assuming optimal firm policy going forward.