Vertical Integration in Auction Markets*

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Abstract

We analyze the effects of a vertical merger between an intermediary and a seller when the intermediary allocates ad positions through auctions. We do so using a symmetric independent private-values model in which the intermediary sets a reserve price and invests in the match quality of the end product. We find that, on average, the intermediary invests more in the integration scenario compared to the separation scenario. Additionally, the integrated seller enjoys a bidding advantage over other sellers ('self-preferencing'). The merging parties always benefit from integration, while non-merging sellers are always worse off. Vertical integration has ambiguous effects on consumer surplus and total welfare. Our results are relevant for vertical integration in ad auctions, platform markets, and procurement. Moreover, they contribute to the ongoing policy debate about self-preferencing by platforms and effective policies to ensure fair competition.

Keywords: Vertical integration; Auctions; Self-preferencing; Competition policy
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1 Introduction

Many digital platforms feature sellers vying for visibility and prominence. However, this competition often unfolds on an uneven playing field due to some sellers being vertically integrated with the intermediary. Such integration can arise from internal research and development or vertical mergers. In such scenarios, the intermediary has an incentive to promote the integrated seller, even when an independent seller might be a better fit for the consumer. This behavior is known as 'self-preferencing'.

There is evidence of self-preferencing in many digital markets. For instance, Google has faced accusations of favoring Places, its proprietary restaurant review service (Edelman, 2011), and meta-search platforms tend to favor affiliated online travel agencies (Cure et al., 2021). Furthermore, Zhu and Liu (2018) show that Amazon tends to sell products on its own marketplace in product markets with high sales and positive reviews, while Farronato et al. (2023) show that Amazon favors its own brands in the search results. This behavior has raised competition concerns as self-preferencing can significantly boost platform profits, to the detriment of consumer welfare and rival sellers. In the United States, Assistant Attorney General Jonathan Kanter expressed concerns about anticompetitive behavior resulting from vertical integration: "We ... see the harms of anticompetitive consolidation across the many dimensions of the modern economy." and "[The vertical merger] guidelines overstate the potential efficiencies of vertical mergers and fail to identify important relevant theories of harm."[1]

The concerns about self-preferencing have resulted in fines and remedies to restore fair on-platform competition. In 2017, the European Commission imposed a €2.42 billion fine on Alphabet, as its subsidiary, Google Search, prioritized Google Shopping over independent price comparison services.[2] The Commission decided that Google Shopping should be treated equally with rival price comparison services, participating fully in the auction for positions.[3] Furthermore, the Digital Markets Act imposes behavioral remedies, such as neutrality, and suggests structural separation as a last resort if behavioral remedies fail to achieve the intended outcomes.

[1] See https://www.justice.gov/opa/speech/assistant-attorney-general-jonathan-kanter-delivers-remarks-modernizing-merger-guidelines, accessed 23 September 2023.

[2] See https://www.nytimes.com/2018/02/20/magazine/the-case-against-google.html, accessed 3 October 2023.

[3] See https://ec.europa.eu/commission/presscorner/detail/en/MEMO_17_1785, accessed 3 October 2023.

In the United States the *Executive Order on Promoting Competition in the American Economy* outlines a policy of rigorous merger enforcement and the potential for retroactive assessments of mergers' compliance with antitrust legislation. Moreover, senator Warren has proposed designating online marketplaces and platforms with revenues exceeding $25 billion as platform utilities, prohibiting them from owning both the platform and its sellers.[4] Furthermore, the proposed *Digital Advertising Act* prohibits advertising firms processing over $20 billion in digital ad transactions from participating in multiple stages of the ad process.

Most existing literature on intermediary-seller integration (e.g., De Cornière and Taylor, 2019; Etro, 2021; Hagiu et al., 2022; Anderson and Bedre-Defolie, 2022) overlooks the fact that intermediaries sometimes use auctions to allocate positions. For instance, Google Search allocates positions for comparison shopping services using an auction, while online travel agencies and hotels bid for positions on meta-search platforms (Cure et al., 2021). In this article, we present a model in which an intermediary uses an auction to allocate positions. We focus on two questions. First, how do integrated firms implement self-preferencing strategies when positions are allocated through an auction? Second, what is the effect of a merger between an intermediary and a seller on consumer surplus, the profits of independent sellers, the profits of the merged firm, and total welfare?

We address these questions theoretically using an independent-private-values auctions framework with a monopolistic platform, vertically differentiated sellers, and consumers. We compare a pre-merger scenario in which all sellers are independent decision makers (the platform is a pure reseller), with a post-merger scenario in which the intermediary is integrated with one of the sellers (a hybrid platform). Our results are also valid for the opposite movement in which the integrated firm divests the seller. The intermediary offers a single advertising position for sale using the second-price sealed-bid auction with a reserve price. Before the merger, if none of the sellers bids more than the reserve price, the ad position remains empty and no sale is made. After the merger, the reserve price depends on the match quality of the integrated seller. If none of the independent sellers bids more than the reserve price, the intermediary places the integrated seller on the ad position. Moreover, in both scenarios, the intermediary can invest in the match quality, affecting all sellers in the

[4] See https://medium.com/@teamwarren/heres-how-we-can-break-up-big-tech-9ad9e0da324c, accessed 3 October 2023.

same proportion.

Our primary contribution is to show that consumers generally prefer integration over separation for two reasons. First, in the integration scenario, part of the investment in match quality by the intermediary is recouped through the integrated seller, leading to increased overall investment levels post-merger. This result, akin to the free-riding problem, holds true for any number of sellers. Second, before the merger, there exists a positive probability that the ad position remains empty, akin to the double-marginalization problem. If the number of sellers is at most three, the likelihood of the ad position remaining empty is high, making consumers prefer integration. If more than three sellers participate in the auction, the outcome depends on whether the intermediary's investment offsets the harm caused by self-preferencing. If investment proves costly, consumers prefer separation over integration. Although the intermediary invests more in the integration scenario, the investment is insufficient to counterbalance the intermediary's bias.

Our second result is that insiders' profits increase after the merger, while outsiders' profits decrease. The profits of the merged firm, in the case of integration, are at least as high as in the case of separation. This is because the merged firm can always perform at least as well by investing the same amount and setting the same reserve price before and after the merger. Furthermore, independent sellers are always worse off after the merger. Despite the increased post-merger investment by the intermediary, there is no investment level that compensates for the loss of profits due to self-preferencing. This finding is reminiscent of the complaints of comparison shopping services alleging harm from Google's favouritism of its own comparison shopping service.[5]

We extend the model in several ways. First, we show that our results generalize to other standard auction formats, such as the first-price, third-price and all-pay auction. Second, we show that our results are robust to a difference in timing, in which the intermediary observes the match value of the integrated seller before investing. Third, we show that mandated neutrality, treating integrated and independent sellers equally, or forcing the integrated seller to participate in the auction, does little to mitigate the adverse effects of self-preferencing. In the case of neutrality, the integrated seller can always bid an amount equal to the reserve price to maximize

[5] See https://curia.europa.eu/jcms/upload/docs/application/pdf/2021-11/cp210197en.pdf, accessed 30 September 2023.

joint profits, rendering the integrated seller's participation or non-participation in the auction inconsequential. The ineffectiveness of mandated neutrality to counteract self-preferencing has previously been documented by De Cornière and Taylor (2019), Ostrovsky (2021), Kittaka and Sato (2022) and Hagiu et al. (2022). Fourth, we show that if the intermediary cannot credibly commit to a reserve price and the integrated seller participates in the auction, consumers and independent sellers prefer integration over separation. The absence of a reserve price leads to an efficient auction in the case of integration and separation, whereas the higher investment level remains in the case of integration. Finally, we study the effect of entry costs, incurred by the sellers to participate in the auction. The presence of entry costs might be an explanation for not observing reserve prices, or substantially lower reserve prices, in online ad auctions in practice (Ostrovsky and Schwarz, 2023). We assume that the integrated seller participates in the auction. In the case of positive entry costs it is optimal for the intermediary to not set a reserve price, both in the case of integration and separation. In that case, total welfare and consumer surplus in the case of integration is always greater than in the case of separation.

The rest of the paper is organized as follows. Section 1.1 discusses the related literature. Section 2 presents the model. Section 3 analyzes investment in match quality and the effect of separation and integration on consumer surplus, producer surplus and welfare. Section 4 presents several extensions. Section 5 concludes.

1.1 Related literature

We combine multiple strands of literature. We study investment incentives in the case of vertical integration. On the one hand, Arrow (1962) describes that vertical integration decreases innovation, because in a competitive market, firms that invest can capture market share of other firms, while the benefit of innovation for a monopolist, with inelastic demand, is limited (the "replacement effect") (Shapiro, 2012; Cai and Spulber, 2022). On the other hand, Schumpeter (1942) argues that innovation incentives are greater in the case of integration, because innovation is fostered by the prospect of market power and profits (Shapiro, 2012). Our result is in line with the latter, because we show that the monopolists' incentive to integrate is greater in the case of integration. However, the welfare effects that drive our results is more related to the elimination of double marginalization (Cournot, 1838; Spengler, 1950), because vertical integration alleviates inefficiencies (i.e., a seller is always displayed to consumers in the case of integration) and vertical integration partially alleviates the free-riding problem of downstream sellers piggybacking on the innovation by the upstream monopolist.

Our results are closely aligned to the positions auctions literature. The sellers in our model participate in a position auction (Athey and Ellison, 2011). Online position auctions that use a VCG-mechanism or (generalized) second-price auction mechanism are efficient (Edelman et al., 2007; Edelman and Ostrovsky, 2013), which is also empirically observed (Varian, 2007; Fukuda et al., 2013). We show that vertical integration can distort the incentives of the auctioneer, which can lead to auction outcomes that are not efficient and favor the integrated seller.

Within the consumer search literature, our model fits within the search framework of consumers with homogeneous match values described in Chen and Zhang (2017) (see also, Chen and He (2011)). Furthermore, Zennyo (2022) assume that there is a positive probability with which the intermediary places an integrated seller in the list of search results. If the intermediary is biased towards the integrated seller this can lead to a lower seller fee, which in turn leads to lower prices and benefits consumers. Janssen and Moraga-Gonzalez (2007) show that mergers are profitable for insiders if search costs are small and mergers can be socially optimal in the case of inelastic demand.

We study endogenous self-preferencing by a hybrid platform (e.g., Hagiu and Spulber, 2013). Calvano and Polo (2021) provide and overview of the literature of biased recommendations. The effect of self-preferencing on consumer surplus is ambiguous. On the one hand, self-preferencing can lead to increased consumers surplus in the case of competition on quality (De Cornière and Taylor, 2019) and in the case of price competition when integration leads to increased on-platform competition and lower prices (Etro, 2021; Hagiu et al., 2022). On the other hand, integration can increase third-party fees, which directs consumers to the integrated seller and leads to less variety, higher prices and lower consumers surplus if sellers are supplying differentiated products (Anderson and Bedre-Defolie, 2022). Moreover, the effect of self-preferencing depends on the level of the fees set by the platform. In the case of low fees, self-preferencing weakens price competition, which harms consumers (Kittaka and Sato, 2022). We show that, in isolation, the effect of self-preferencing is negative for consumers and total welfare. Due to self-preferencing consumers do not always buy from the firm with the highest match value. However, the ability of the intermediary to self-preference also leads to increased innovation incentives, which can alleviate the harm of self-preferencing.

A related literature is that of mergers in auctions markets. Most of the literature focuses on horizontal mergers (e.g., Waehrer, 1999; Dalkir et al., 1999; Klemperer, 2007), which shows that auction markets do not differ significantly from non-auction markets. Frank et al. (2022) consider a market with complete information in which an integrated platform allocates advertising positions using a generalized-second price auction. They show that the integrated seller has an incentive to overbid which raises the costs for competitors. Ostrovsky (2021) shows that Google's choice screen auction is biased in favor of Google Search, due to the auction design (a 'per install' instead of 'per appearance' auction). Arozamena et al. (2014) model a symmetric case in which the auctioneer places a weight on the welfare obtained by all sellers. We study the asymmetric case, in which the intermediary places a full weight on the welfare obtained by one of the sellers and no weight on the welfare of all other sellers. In Carannante et al. (2023) an auctioneer sells objects to sellers in a dynamic game. The repeated nature of the game makes that one of the bidders can win multiple subsequent auctions. The reserve price set by the auctioneer depends on the value of the incumbent bidder. Moreover, if the value of the incumbent bidder is high, it is optimal to set a high reserve price to exclude other bidders. The result corresponds to our inflated reserve price in the case of integration, in which the auctioneer sets a high reserve price in order to let the integrated seller win the auction.

2 Model

We use Athey and Ellison's (2011) position auction model as a starting point. A risk-neutral intermediary offers a single advertising position for sale using the second-price sealed-bid auction with reserve price r. In the auction n sellers, labeled $i = 1, ..., n$, are bidding for the position. The advertising position will match the winning seller to one representative consumer. We will discuss the representative consumer below when discussing consumer surplus. For now, we focus on the interaction between the intermediary and the sellers.

Seller i assigns initial match value $v_i \in [0, \overline{v}]$ to the advertising position, which is private information. Without loss of generality we normalize $\overline{v}$ to 1. The sellers' initial match values are drawn independently from the same, strictly increasing and differentiable cumulative distribution function F on the support $[0, 1]$. Let $f \equiv F'$ denote the continuous density of F and assume that the hazard rate, $f/(1 - F)$, is strictly increasing so that the solution for the optimal reserve price is unique. $F^k(v) \equiv (F(v))^k$ represents the distribution of the highest initial match value $v_k^{(1)} \equiv \max_{i=1,...,k} v_i$ among the first k players, $k = 1, ..., n$. The initial match value describes the expected value of being recommended by the intermediary and encompasses all the relevant variables that affect the expected value, such as the probability of a successful match between seller i's product and the demand of the consumer and the normalized price-cost margin of the product.

We consider two scenarios that the describe the relationship between the intermediary and the sellers.

- Separation [sep]: The intermediary and the sellers are independent decision makers;
- Integration [int]: The intermediary is integrated with seller n. The merged entity acts as single player with joint profits, which we also label as 'the intermediary'. We will call sellers that are not merged with the intermediary, $i \neq n$, 'independent sellers'.

The intermediary has two instruments at its disposal to maximize profits. First, the intermediary sets the reserve price r in the auction. Second, the intermediary can invest an amount $q \geq 0$ at cost $C(q)$ to increase the match quality of all sellers in the same proportion, where C is a strictly increasing and convex function with $C(0) = 0$. For the purpose of analytical tractability, we sometimes assume $F = U[0, 1]$ and $C(q) = aq^2$, with $a > 0$, to derive the results. We study subgame-perfect equilibria of the following 4-stage game, which is solved using backward induction.

- Stage 1: The intermediary delivers match quality q at cost $C(q)$;
- Stage 2: Each seller i is privately informed about her value v_i;
- Stage 3: The intermediary sets the reserve price r;
- Stage 4: The sellers bid in the auction.

In the first stage, the investment by the intermediary increases each seller i's initial match value to $V_i(q) \equiv (1+q)v_i$. In the case of integration, the investment decision is made before the intermediary has learned the match value of the integrated seller. One interpretation is that the intermediary runs many auctions on the same platform, so that investing in the platform once affects the value distribution of all subsequent auctions.

The investment by the intermediary has different interpretations, depending on the way in which the investment leads to a higher expected value of a potential sale. For instance, the intermediary can invest in employees and assets that improve its search algorithm or website accessibility and thereby increase the probability of a successful match between the recommended seller and the consumer. Investment by the intermediary might also lead to lower costs for all sellers, for example because the investment streamlines the process of product returns, which has significant economic impact on sellers. In addition, the intermediary can invest in lobbying efforts to lower local taxes or setup costs (e.g., the intermediary is an online travel agency). Finally, an investment by the intermediary can increase the value of the product, for instance because the intermediary adds value by investing in the software development kit for applications in an online app store. In all these examples, it seems natural that the investment in the platform is made before the intermediary knows the value of integrated seller.

In the fourth stage, sellers submit bids to the intermediary using the second-price sealed-bid auction. Let b_i denote seller i's bid. For $k = 1, ..., n$, the highest and second highest value in the set $\{b_1, b_2, ..., b_k\}$ are denoted $b_k^{(1)}$ and $b_k^{(2)}$, respectively. The intermediary only displays the winning seller if $b_k^{(1)} \geq r$. If two or more firms submit the same highest bid, the intermediary randomly chooses which of these firm to display. The winner is denoted by i^*. As is well known, bidding the true match value, i.e., $b_i = V_i(q)\ \forall i$, is an equilibrium in weakly dominated strategies in the second-price sealed-bid auction (Vickrey, 1961). In the separation scenario, if $b_n^{(1)} < r$ the auction does not have a winner and no product is sold to the consumer. In the integration scenario, if none of the independent sellers, $i = 1, ..., n-1$, bids at least the reserve price the intermediary puts seller n on the advertising position yielding value $V_n(q)$. In both scenarios, sellers that are not displayed by the intermediary will not make a sale. The winning sellers and payments to the intermediary in the case of integration and separation are displayed in Table 1.

Table 1: Seller shown and payments to the intermediary

	Integration		Separation	
Bids	Seller shown	Payment	Seller shown	Payment
$b_k^{(1)} \geq b_k^{(2)} \geq r$	i^*	$b_{n-1}^{(2)}$	i^*	$b_n^{(2)}$
$b_k^{(1)} \geq r > b_k^{(2)}$	i^*	r^{int}	i^*	r^{sep}
$r > b_k^{(1)} \geq b_k^{(2)}$	n	-	-	-

An outcome of this game consists of sellers' bids, the investment in match quality by the intermediary, reserve prices and seller and intermediary profits. The investment in match quality is derived in the next section. The profits of the intermediary and sellers, π_0 and π_i, are displayed in Table 2. The optimal reserve price follows immediately from Myerson's (1981) analysis of optimal auctions and the observation that the investment by the intermediary inflates all values by a factor $1 + q$. Let $r^{sep}(q)$ and $r^{int}(v_n, q)$ denote the optimal reserve price in the case of separation and integration, respectively, given investment level q and if seller n's value equals v_n. Let $r(v_n, 0)$ be the unique solution to $r = v_n + \frac{1-F(r)}{f(r)}$ with respect to the reserve price r. In stage 2, the intermediary optimally sets

$$\begin{aligned} r^{sep}(q) &= (1+q)r(0,0); \\ r^{int}(v_n, q) &= (1+q)r(v_n, 0). \end{aligned} \tag{1}$$

We assume a similar division of surplus as Athey and Ellison (2011), where consumer surplus is proportional to the value of the seller obtaining the advertising position. Consumers receive a benefit of $\gamma V_i(q)$, with $\gamma > 0$, if they buy from seller i. Thus, if the consumer finds seller i, the seller and consumer share the surplus from the resulting transaction, where $\gamma/(1+\gamma)$ determines the share of surplus going to the consumer. In the case seller w is placed on the advertising position and $V_w(q) = 0$ if the ad position remains open, consumer surplus, CS, producer surplus, PS, and

Table 2: Intermediary and seller profits

Separation	Intermediary	$\pi_0^{sep}(q,r) = \begin{cases} \max\{b_n^{(2)}, r\} - C(q), \\ -C(q), \end{cases}$	if $b_n^{(1)} \geq r$ if $b_n^{(1)} < r$
	Seller i	$\pi_i^{sep}(q,r) = \begin{cases} V_i(q) - \max\{b_n^{(2)}, r\}, \\ 0, \end{cases}$	if $b_i = b_n^{(1)} \geq r$ otherwise
Integration	Intermediary	$\pi_0^{int}(v_n,q,r) = \begin{cases} \max\{b_{n-1}^{(2)}, r\} - C(q), \\ V_n(q) - C(q), \end{cases}$	if $b_{n-1}^{(1)} \geq r$ if $b_{n-1}^{(1)} < r$
	Seller $i \neq n$	$\pi_i^{int}(v_n,q,r) = \begin{cases} V_i(q) - \max\{b_{n-1}^{(2)}, r\}, \\ 0, \end{cases}$	if $b_i = b_n^{(1)} \geq r$ otherwise

total welfare, W, are related as follows.

$$
\begin{aligned}
PS &= V_w(q) - C(q); \\
CS &= \gamma V_w(q); \\
W &\equiv PS + CS.
\end{aligned}
\tag{2}
$$

Consumer surplus, producer surplus and welfare are zero if the ad position remains open in the separation scenario.

3 Results

3.1 Investment in match quality

We use Myerson (1981) to solve for the optimal investment in match quality by the intermediary. Let $R^{sep}(q)$ and $R^{int}(v_n, q)$ denote the expected revenue of the intermediary from the optimal auction in the case of separation and integration, respectively, given investment level q and if seller n's value equals v_n. The intermediary's expected revenue from the optimal auction in the case of zero investment in match quality in the two scenarios are the following.

$$
\begin{aligned}
R^{sep}(0) &= n\left(rF^{n-1}(r)(1-F(r)) + \int_r^1 v_i(1-F(v_i))dF^{n-1}(v_i)\right); \\
R^{int}(v_n,0) &= (n-1)\left(rF^{n-2}(r)(1-F(r)) + \int_r^1 v_i(1-F(v_i))dF^{n-2}(v_i)\right),
\end{aligned}
\tag{3}
$$

with $r = r^{sep}(0)$ in the case of separation and $r = r^{int}(v_n, 0)$ in the case of integration.

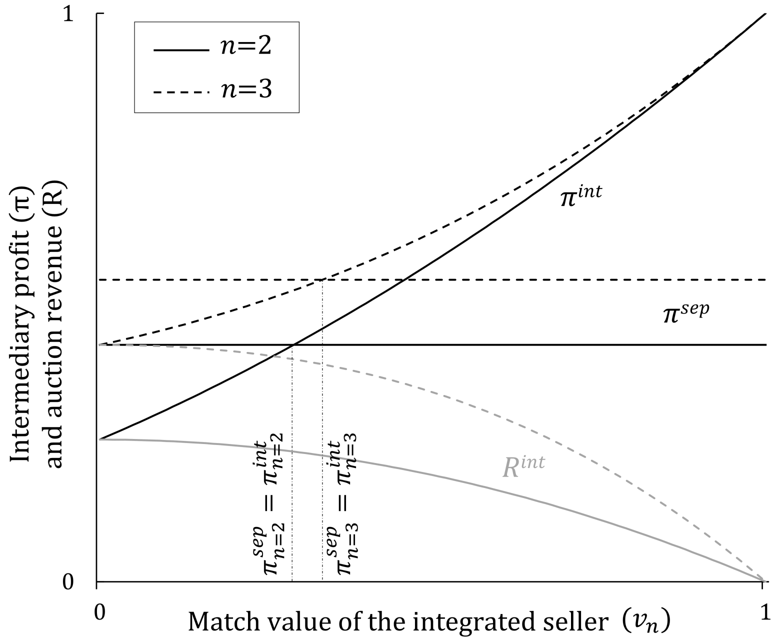
The equilibrium profits of the intermediary can be conveniently expressed in terms of the intermediary's revenue in the case of zero investment in match quality.

$$
\begin{aligned}
\pi_0^{sep}(q) &= (1+q)R^{sep}(0) - C(q); \\
\pi_0^{int}(v_n,q) &= (1+q)\left(\underbrace{v_nF^{n-1}(r(v_n,0))}_{\text{revenue seller } n} + \underbrace{R^{int}(v_n,0)}_{\text{auction revenue}}\right) - C(q).
\end{aligned}
\tag{4}
$$

Figure 1 displays the profits and revenues as a function of the match value of the integrated seller in the case of two and three sellers. In both scenarios, the expected match value of the highest and second-highest seller are greater in the case of more sellers, so that the expected profits of the intermediary increase if the auction is more competitive. Moreover, in the case of integration, the expected revenue from the optimal auction decreases as the match quality of the integrated seller, v_n, increases, because a higher match quality of seller n leads to an increase in the reserve price, less revenue from the auction, and more direct sales by the integrated seller. Furthermore, the match value of the integrated seller for which the intermediary is indifferent between integration and separation increases as the number of sellers goes from two to three, which indicates that merging is especially attractive when the number of sellers is low.

The optimal investment level is obtained by equating the marginal benefits and marginal costs of a unit increase in match quality. The marginal costs is simply the costs of an extra unit of investment. Furthermore, recall that an investment in match quality affects all sellers proportionally, so that the investment leads to an upward shift of the profit curve. Therefore, the marginal benefit of a unit of investment in match quality is the expected revenue of the auction without investment in the case of separation, and the expected revenue of the auction plus the expected revenue of the integrated seller without investment in the case of integration. Let Q^{sep} denote the

Figure 1: Intermediary's profit and revenue as function of the match value of the integrated seller, with $F = U[0, 1]$ and $q = 0$.



investment in match quality in the case of separation and Q^{int} denote the investment in match quality in the case of integration.

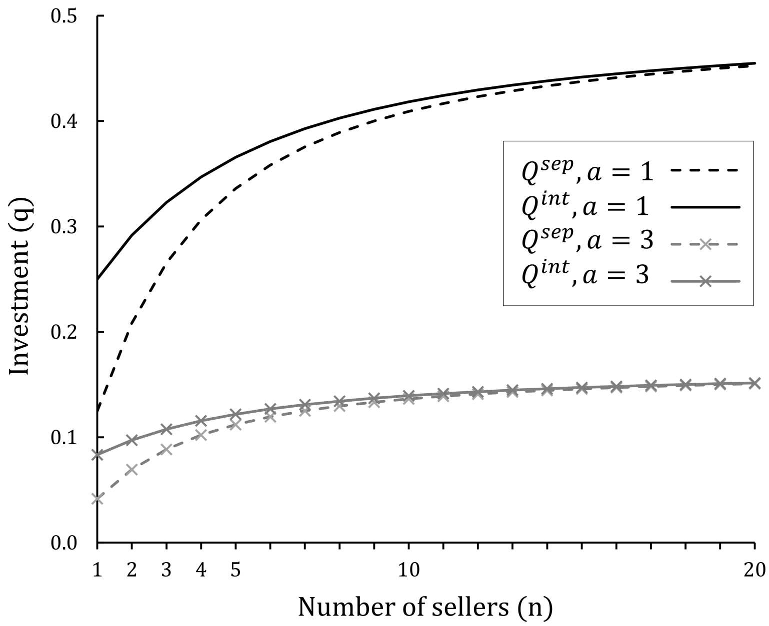
Lemma 1. *After investment by the intermediary the equilibrium match quality levels are implicitly defined by:*

$$C'(Q^{sep}) = \pi_0^{sep}(0) = R^{sep}(0);$$
$$C'(Q^{int}) = E_{v_n}\{\pi_0^{int}(v_n, 0)\} = E_{v_n}\{v_n F^{n-1}(r(v_n, 0) + R^{int}(v_n, 0))\}.$$

Proof. Follows directly from (4). The reserve prices are defined in (1) and the expected revenue from the optimal auction is defined in (3). ■

In the case of integration, part of the investment in match quality by the intermediary is recouped through the integrated seller. In the separation scenario the investment raises the revenue from the auction, but a larger part of the investment leaks away through the independent sellers, which lowers the innovation incentives for the intermediary. As a consequence, the average level of investment is always greater in the integration scenario than in the separation scenario. This result, akin to the

Figure 2: Investment in match quality as a function of the number of sellers, with $F = U[0,1]$, $C(q) = aq^2$ and $a > 0$.



free-riding problem, holds true irrespective of the number of sellers. The following proposition summarizes this result.

Proposition 1. $Q^{int} > Q^{sep}$.

Proof. See Appendix. ■

The investment in match quality as a function of the number of sellers participating in the auction is displayed in Figure 2. The difference in investment converges as the number of sellers increases, because the probability that the integrated seller is shown to the consumer is lower in the case of more sellers.

Now that we have established the outcomes of separation and integration, we are ready for a welfare comparison. Welfare consists of intermediary profits, sellers profits and consumer surplus.

3.2 Intermediary and seller profits

Now, we turn to the question whether a merger between the intermediary and seller n is profitable. Let Π_0^{sep} denote expected intermediary profits in the case of separation and $\Pi_0^{int} \equiv E_{v_n}\{\pi_0^{int}(v_n, q)\}$ denote expected profits of the merged firm in the case of integration. Furthermore, let Π_i^{sep} denote the expected profits of the sellers, $i = 1, ..., n$, in case of separation.

In general, the merger is always profitable for the merged firm. In the integration scenario the intermediary can always copy the investment in match quality optimally set in the case of separation and can set a reserve price $r(v_n) = \max\{r^{sep}, v_n\}$. As a result, the joint profits of the merged firm in the integration scenario are equal or higher than the expected profits of the intermediary and an independent seller in the separation scenario. The following proposition summarizes this result.

Proposition 2. *The takeover is profitable, i.e.,* $\Pi_0^{int} \geq \Pi_0^{sep} + \Pi_n^{sep}$.

Proof. See Appendix. ■

The independent sellers always prefer separation over integration, given the distribution of initial match values $F = U[0, 1]$ and investment cost function $C(q) = aq^2$. Let $\Pi_i^{int} \equiv E_{v_n}\{\pi_i^{int}(v_n)\}$, $i = 1, ..., n-1$, denote the expected profit of independent seller i in the case of integration. Although investment by the intermediary is greater in the case of integration, the additional investment is insufficient to compensate the independent sellers for the foregone profits due to self-preferencing in the case of integration. The following proposition summarizes this result.

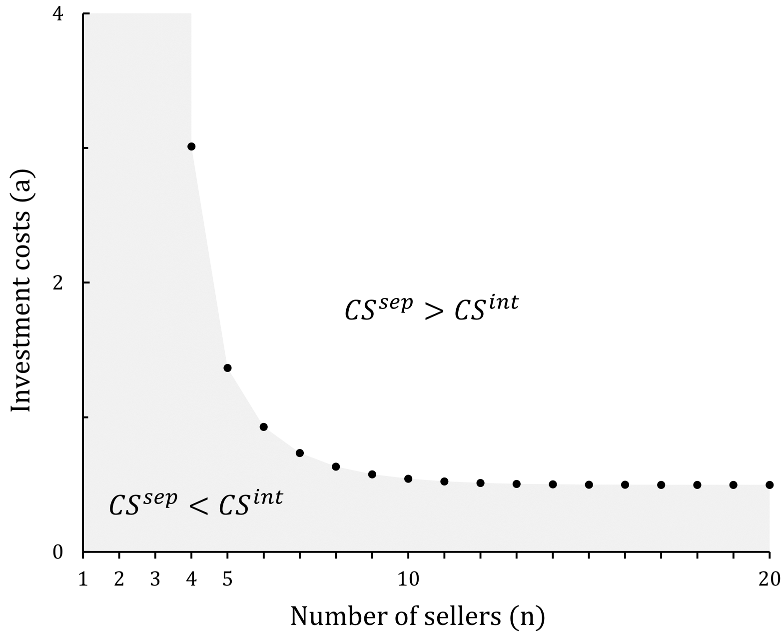
Proposition 3. *Let* $F = U[0, 1]$*. If* $C(q) = aq^2$*, then* $\forall a \geq 0, n > 1$*:* $\Pi_i^{sep} > \Pi_i^{int}$.

Proof. See Appendix. ■

3.3 Consumer surplus

Consumer surplus is the share of the value appropriated by the consumer from the transaction. We analyze expected consumer surplus in the case of separation, CS^{sep}, and expected consumer surplus in the case of integration, $CS^{int} \equiv E_{v_n}\{cs^{int}(v_n)\}$. Recall that the investment q upgrades consumer surplus with a factor $1 + q$ and that consumers appropriate a share $\gamma/(1+\gamma)$ of the surplus resulting from the transaction. Furthermore, assume that the distribution of match values is uniform and $C(q) = aq^2$. Then, we can derive the the critical level of investment in match quality for which the consumer is indifferent between separation and integration, denoted $\overline{a}(n)$.

Figure 3: Consumer surplus, with $F = U[0,1]$ and $C(q) = aq^2$.



Setting $CS^{sep} = CS^{int}$ yields the following expression, which is derived in the proof of proposition 4 and displayed in Figure 3.

$$\overline{a}(n) = \frac{2^{-n-1}(2^n(n-1)+1)(n^3 - 4n + 2^{n+1}(n+2) - 4)}{n(n+1)(2(2^n - 1) - n(n+2))}$$

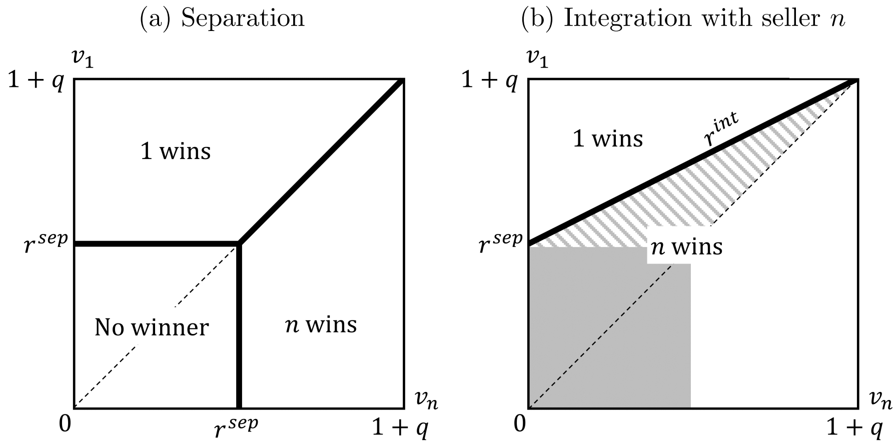
The following proposition states that consumer surplus is always greater in the case of integration in an auction with at most three sellers. In addition, consumer surplus is greater in the case of integration in an auction with more than three sellers if and only if the intermediary invests less than the critical investment level.

Proposition 4. *Let $F = U[0,1]$. Then $CS^{sep} < CS^{int}$ for $n = 2, 3$. If $C(q) = aq^2$, then $CS^{sep} < CS^{int}$ for $n \geq 4$ if and only if $a < \overline{a}(n)$.*

Proof. See Appendix. ■

The intuition behind proposition 4 is as follows. In the case of separation and $n \leq 3$ there is a positive probability that the ad position remains empty if all sellers bid less than the reserve price (the area marked 'No winner' in Figure 4a). In the

Figure 4: Auction outcomes, with $n = 2$.



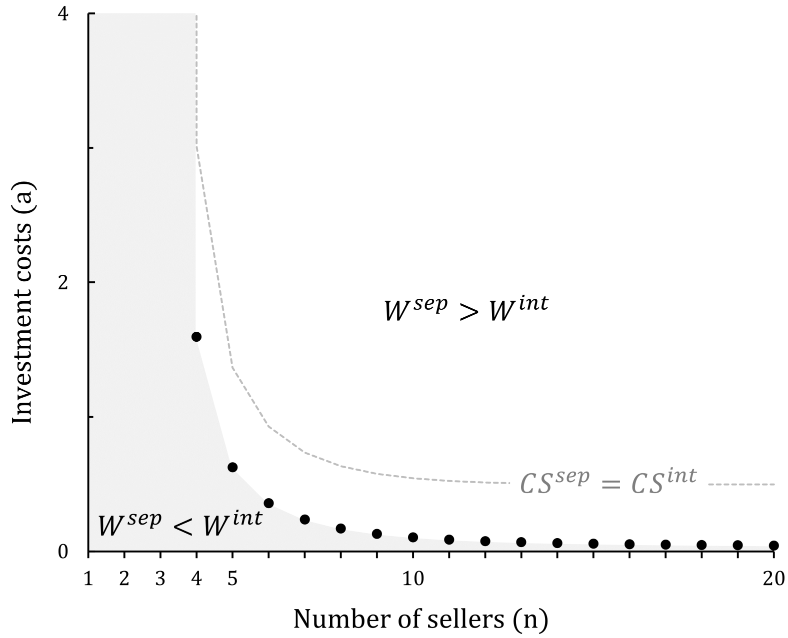
case of integration, a seller is always displayed, which benefits consumers relative to separation (the solid grey area in Figure 4b). However, the reserve price is also greater, which lowers consumer surplus compared to separation if one or more sellers have a match value above r^{sep} and below r^{int} (the shaded grey area in Figure 4b). If $n \leq 3$ the welfare loss due to the advertising position possibly being empty in the case of separation always exceeds the welfare loss due to a higher reserve price in the case of integration. As a result, irrespective of the level of investment by the intermediary, the consumer always benefits from the merger if $n \leq 3$.

As the number of sellers taking part in the auction increases, the probability of the ad position being empty in the case of separation decreases. If $n > 3$, whether consumer surplus is greater in the case of separation or integration depends on the level of investment in match quality. If investment is expensive, $a > \overline{a}(n)$, consumers prefers separation, because the investment by the intermediary is low and does not compensate for self-preferencing.

3.4 Producer surplus and welfare

We now turn our attention to total producer surplus and welfare. Recall that we have assumed that $PS = (1/\gamma)CS - C(q)$ and that the transfer from the winning seller to the intermediary does not affect welfare. Furthermore, let $PS^{sep} \equiv \Pi_0^{sep} + \sum_{i=1}^{n} \Pi_i^{sep}$ and $PS^{int} \equiv \Pi_0^{int} + \sum_{i=1}^{n-1} \Pi_i^{int}$ denote producer surplus in the case of separation and integration, respectively. The following proposition states that integration only

Figure 5: Total welfare, with $F = U[0,1]$, $C(q) = aq^2$ and $\gamma = 1$.



increases welfare relative to separation if consumer surplus increases.

Proposition 5. $CS^{int} < CS^{sep} \implies W^{int} < W^{sep}$.

Proof. Follows directly from the assumption that $W = ((1+\gamma)/\gamma)CS - C(Q)$ and proposition 1. ■

We can now derive the critical investment level for which total welfare is equal in the two scenarios, denoted $\hat{a}(n)$. Setting $W^{sep} = W^{int}$, for $F = U[0,1]$, $C(q) = aq^2$ and $\gamma = 1$, yields the following expression, which is derived in the proof of proposition 6 and displayed in Figure 5.

$$\hat{a}(n) = \frac{2^{-n-2}(n(n(2^{n+1}(n^2-3)+2n+1)+8(4^n-1))-12(2^n-1)^2)}{n(n+1)(2(2^n-1)-n(n+2))}$$

If $n \leq 3$ both consumer surplus and producer surplus is greater in the case of integration. As a result, in the case of a small number of sellers, total welfare is always greater in the case of integration, irrespective of the level of investment by the intermediary. For $n > 3$ the benefit of separation on total welfare increases if the number of sellers increases or in the case of higher investment costs. The following proposition summarizes this result.

Proposition 6. *Let $F = U[0,1]$. If $C(q) = aq^2$, then $W^{sep} < W^{int}$ for $n = 2, 3$ and $W^{sep} < W^{int}$ for $n \geq 4$ if and only if $a < \hat{a}(n)$.*

Proof. See Appendix. ■

The critical investment level for total welfare is lower than for consumer surplus ($\hat{a}(n) < \overline{a}(n)$) due to the welfare decreasing effect of integration on the profits of the independent sellers.

4 Extensions

4.1 Other standard auction formats

Our results on auction revenue and seller profits generalize to other standard auction formats, such as the first-price auction, third-price auction and all-pay auction. Recall that we assume risk-neutral sellers with independently and identically distributed match values. Moreover, the seller with the highest match value, $v_k^{(1)}$, wins the auction if $v_k^{(1)} \geq r$. Sellers with match value $v_i < r$ abstain from bidding and the expected payment of a bidder with match value $v_i = 0$ is zero. Moreover, in the case of separation the ad position remains empty if none of the sellers bids more than the reserve price. In the case of integration, the integrated seller is placed on the position if none of the sellers bids more than the reserve price.

Due to this setup of the model, sellers have symmetric and increasing bidding strategies, both in case of separation and integration. Thus, according to the revenue equivalence principle, the expected payment of sellers and the expected revenue from the auction is the same in any standard auction format (Myerson, 1981). Given that the profit function of the intermediary is the same, the investment by the intermediary is also the same and the results on consumer surplus and welfare are quantitatively similar in any standard auction format chosen by the intermediary.

4.2 Intermediary observes match value before investment

Suppose that, in the case of integration, the intermediary's investment decision, q, at cost $C(q)$, is made after the match value of integrated seller is known. For instance, the intermediary runs multiple unique single-ad auctions on the same platform.

The results are quantitatively similar if we assume that C is a thrice differentiable, convex, and strictly increasing function with $C(0) = 0$ and $C'''(q) \leq 0$. Proposition 1 holds true, because $C''' \leq 0$ implies that C'^{-1} is convex. As a result, investment is greater in the case of integration than separation. In addition, the proof of proposition 2 holds true, as long as the reserve price is set after the match value is known to the intermediary. For the first part of proposition 4 ($n = 2, 3$) it suffices to show that investment is greater in the case of integration than separation. For the second part of proposition 4 ($n \geq 4$), proposition 3 and 6 we assume $C(q) = aq^2$, with $a > 0$, which satisfies the above stated assumptions.

4.3 Mandated neutrality

Our baseline model describes a scenario in which the intermediary puts the integrated seller on the advertising position if none of the independent sellers bids more than the reserve price. The current setup of the model can explain why policy interventions that force the integrated seller to take part in the auction will do little to increase the opportunities of independent sellers and will probably lead to disappointing outcomes. Examples include Google Shopping competing with third-party price comparison websites for positions in Google Search and Android choice screen auctions for browsers and search engines (Ostrovsky, 2021).

Suppose a regulator mandates the integrated seller to take part in the auction. The intermediary sets the optimal reserve price, r^{int}. The integrated seller n observes the reserve price and can always bid an amount equal to this reserve price, $b_n \equiv r^{int}$. As a consequence, whether or not the integrated seller participates in the auction will not affect the results. The reason is that the incentive of the integrated seller is not to maximize individual profits, and thus bid her true valuation $b_n \not\equiv V_n$, but to maximize joint profits, which is done by bidding r^{int}.

4.4 No reserve price

Suppose the intermediary is unable to commit to the reserve price. A reserve price is not always credible or situations might occur in which reserve prices are either prohibited or close to zero. What is the effect of the absence of a reserve price on the profits of the intermediary and sellers and consumer surplus?

Assume $r^{sep} = r^{int} = 0$. Moreover, assume that, in the case of integration, seller n participates in the auction and bids truthfully, so that $b_n \equiv V_n$.[6] Due to the absence of a reserve price a seller will always be displayed in the case of separation. Moreover, there is no bias in the case of integration. As a result, in both scenarios and in all auctions, the seller with the highest match value wins the auction.

In the case of integration, the investment by the intermediary is greater than in the case of separation, because the merged firm recoups part of the investment if the integrated seller wins the auction. Therefore, the auction is efficient in both scenario's and independent sellers and consumers now prefer integration over separation.

4.5 Entry costs

Suppose that, instead of the intermediary investing in the quality of the end product, sellers have to pay entry costs to enter the auction, for instance because an investment has to be made by the sellers to make the product suitable for the platform or because sellers invest in marketing to reach the consumer.

Assume sellers have to pay a fixed entry fee $B > 0$ to take part in the auction. The entry decision is made before the match values are realized, like in McAfee and McMillan (1987) and Engelbrecht-Wiggans (1987, 1993). For example, after the product is placed on the platform the success of branding and marketing may become apparent or there is uncertainty about the compatibility of the product with the platform, which only becomes clear after the investment in made. Moreover, the match value of a seller that does not enter equals zero. Sellers that pay B obtain match value V_i. Furthermore, the integrated seller pays the entry costs if $V_n \geq B$ and takes part in the auction.

We analyse the case $\gamma = 0$, so that all gains from trade are appropriated by the firms and the total welfare function is $W = PS = \pi_0 + \Pi_i - nB$. In the case of positive entry costs, sellers enter the market as long as $E[V_i] \geq B$.[7] As a result, in equilibrium

[6] An alternative setup is that the integrated seller does not participate in the auction. In that case, the integrated seller will only be displayed to consumers if $v_i = 0, \forall i$, with $i = 1, ..., n-1$. In this setting, the auctions are efficient in both scenario's, but there is one less seller participating in the case of integration. As a result, the expected revenue of the intermediary is greater in the case of separation. Thus, the intermediary invests more in the case of separation and consumers prefer separation over integration.

[7] Levin and Smith (1994) study endogenous bidder entry and find that auctioneers should not set reserve prices in the case of positive entry costs. See also Ye (2004) for a more general setup of endogenous entry.

$E[\Pi_i] = 0$ for all seller that enter the auction. Therefore, the total welfare function reduces to $W = PS = \pi_0 - nB$. The intermediary maximizes its own profits, which is done by setting $r^{sep} = r^{int} = 0$ (Samuelson, 1985; McAfee and McMillan, 1987; Engelbrecht-Wiggans, 1987; Levin and Smith, 1994) and the integrated seller bidding $b_n \equiv V_n$ in the case of integration.

The outcome of this game is an efficient auction in both scenario's. Therefore, the expected number of sellers paying the entry costs is equal in both scenario's: $n^{sep} = n^{int}$. Moreover, due to participation of the integrated seller, investment is greater in the integration scenario than in the separation scenario. As a result, in the case of positive entry costs, consumers always prefer integration over separation and total welfare is greater in the integration scenario.

5 Conclusion

In this paper, we have analyzed the effects of a vertical merger between an intermediary and a seller in a setting where the intermediary allocates ad positions using auctions. We compare a pre-merger scenario, in which all sellers are independent decision makers, with a post-merger scenario, in which the intermediary is integrated with one of the sellers. The intermediary offers a single advertising position for sale using the second-price sealed-bid auction with a reserve price. After the merger, the reserve price depends on the match quality of the integrated seller. Moreover, in both scenarios the intermediary can invest in the match quality of all sellers.

Our main results are the following. First, we show that the level of investment of the intermediary is always higher after the merger, because the integrated firm can recoup part of the investment through the integrated seller. Second, the consumer welfare effect of the merger depends on the number of sellers and the cost of investment. Before the merger there is a positive probability that the ad position remains empty, if none of the sellers bid more than the reserve price. After the merger, the level of investment by the intermediary is higher and consumers will always find a seller on the ad position, although this might not be the most efficient seller. As a result, if the number of sellers is low or investment is relatively inexpensive, consumers benefit from the merger. Third, the intermediary and integrated seller benefit from the merger, whereas independent sellers always prefer separation. Although increased post-merger investment by the intermediary increases the profits of independent sellers, there is no investment level that offsets the loss of profits resulting from self-preferencing. Fourth, as the integrated seller can always submit a bid equal to the reserve price, mandated neutrality - treating integrated and independent sellers equally or forcing the integrated seller to take part in the auction - will do little to counter the negative effects of self-preferencing. Fifth, the absence of a reserve price or the introduction of entry costs leads to an efficient auction in both scenarios. Moreover, investment is greater in the case of integration, so that consumer surplus and total welfare are always greater in the case of integration.

Our results speak to competition policy and market regulation in two important ways. First, concerning the way in which vertical mergers in platform market are evaluated, we find that prohibiting mergers or structural remedies can be effective policy measures to increase consumer welfare, but that the outcomes depend on the competitiveness of the auction (i.e., the number of sellers participating) and the innovation costs of the intermediary. In particular, our results show that the merger only reduces consumer surplus and welfare if the number of sellers and the intermediary's innovation costs are both high. Second, only under these conditions does breaking up a vertically integrated intermediary, e.g., as a structural remedy in an abuse-of-a-dominant-position case or in view of digital-market regulation like the Digital Markets Act, produce the desired outcomes.

6 Appendix

Proof of proposition 1

Let, in the integration scenario for $q = 0$, $\tilde{\pi}_0^{int}(v_n, 0)$ be the expected profits of the intermediary if it imposes reserve price $\tilde{r}^{int}(v_n, 0) \equiv \max\{v_n, r^{sep}(0)\}$ instead of the optimal reserve price $r^{int}(v_n, 0)$. Because $\tilde{r}^{int}(v_n, 0)$ is suboptimal,

$$\pi_0^{int}(v_n, 0) \geq \tilde{\pi}_0^{int}(v_n, 0). \tag{5}$$

Let, in the separation scenario for $q = 0$, $\pi_0^{sep}(v_n, 0)$ denote the expected profits of the intermediary given the optimal reserve price $r^{sep}(0)$ and given that seller n's value equals v_n. Notice that the profit of the intermediary under the integration scenario is strictly greater than under the separation scenario if $v_{n-1}^{(1)} < \max\{r^{sep}(0), v_n\}$ and that the profit in the integration scenario is the same as in the separation scenario if

$v_{n-1}^{(1)} \geq \max\{r^{sep}(0), v_n\}$. As a result, for all $v_n > 0$,

$$\tilde{\pi}_0^{int}(v_n, 0) > \pi_0^{sep}(v_n, 0). \tag{6}$$

Inequalities (5) and (6) imply

$$\pi^{int}(v_n, 0) > \pi^{sep}(v_n, 0). \tag{7}$$

By Lemma 1, $Q^{int} = C'^{-1}(E_{v_n}\{\pi_0^{int}(v_n, 0)\})$, where C'^{-1} is the inverse function of C'. As C is strictly increasing and convex, C'^{-1} is increasing. Then,

$$\begin{aligned}
Q^{int} &= C'^{-1}(E_{v_n}\{\pi^{int}(v_n, 0)\}) \\
&> C'^{-1}(E_{v_n}\{\pi^{sep}(v_n, 0)\}) \\
&= C'^{-1}(R^{sep}(0)) \\
&= Q^{sep}.
\end{aligned}$$

The first inequality follows from (7). The second equality follows because $E_{v_n}\{\pi^{sep}(v_n, 0)\} = R^{sep}(0)$ and the third equality follows from Lemma 1.

Proof of proposition 2

In the case of integration the two firms always perform at least as well as under separation if, in the case of integration, the intermediary copies the investment in match quality optimally set in the case of separation (i.e., the intermediary can always set $Q^{int} = Q^{sep}$) and a reserve price $r(v_n) = \max\{r^{sep}, v_n\}$. The resulting joint payoffs are the same in the two scenarios if at least one of the independent sellers has a value greater than $r(v_n)$ and the integrated firm is strictly better off in the case of integration if none of the independent sellers has a value greater than $r(v_n)$. In the latter case, the intermediary places seller n on the advertising position, yielding $V_n(Q^{sep})$ for the integrated firm, which is the same if $V_n(Q^{sep}) \geq r^{sep}$ and strictly greater if $V_n(Q^{sep}) < r^{sep}$.

Proof of proposition 3

Expected seller profits can readily be deduced from auction theory. The profits of sellers in both scenarios are the following.

$$\Pi_i^{sep} = (1 + Q^{sep}) \int_{r^{sep}(0)}^{1} (1 - F(v_i))F^{n-1}(v_i)dv_i,$$
$$\pi_i^{int}(v_n) = (1 + Q^{int}) \int_{r^{int}(v_n,0)}^{1} (1 - F(v_i))F^{n-2}(v_i)dv_i.$$

Suppose $F = U[0,1]$ and $C(q) = aq^2$. Let $\Pi_{i,q=0}^{sep}(n)$ and $\Pi_{i,q=0}^{int}(n)$ denote seller profits without investment by the intermediary in the case of separation and integration, respectively. Furthermore, $Q^{sep} = 1/(2a)g(n)$ and $Q^{int} = 1/(2a)h(n)$, with

$$g(n) = \frac{n - 1 + (\frac{1}{2})^n}{n+1}, \quad h(n) = \frac{2 - 2^{1-n} + n(n-1)}{n(n+1)}.$$

We can derive the level of investment by the intermediary for which an independent seller prefers integration compared to separation as follows.

$$\begin{aligned}
\Pi_i^{sep} &< \Pi_i^{int} \Leftrightarrow \\
(1 + Q^{sep})\Pi_{i,q=0}^{sep}(n) &< (1 + Q^{int})\Pi_{i,q=0}^{int}(n) \Leftrightarrow \\
a &< \frac{1}{2}\left(\frac{h(n)\Pi_{i,q=0}^{int}(n) - g(n)\Pi_{i,q=0}^{sep}(n)}{\Pi_{i,q=0}^{sep}(n) - \Pi_{i,q=0}^{int}(n)}\right), \Leftrightarrow \\
a &< \frac{2^{-n}(n(n^2 + n - 4^{n+1}(n-2) + 2^n(n-1)(n+1)(n+2) - 6) - 12(2^n - 1)^2)}{n(n+1)(4(2^n - 1) - n(n+3))}.
\end{aligned} \tag{8}$$

Notice that

$$\begin{aligned}
\Pi_{i,q=0}^{sep} - \Pi_{i,q=0}^{int} &> 0 \quad \text{for} \quad n > 1, \\
h(n)\Pi_{i,q=0}^{int} - g(n)\Pi_{i,q=0}^{sep} &< 0 \quad \text{for} \quad n > 1.
\end{aligned}$$

It is readily checked that the RHS of (8) is smaller than zero for $n > 1$. As a result, $\forall a \geq 0, n > 1$: $\Pi_i^{sep} > \Pi_i^{int}$.

Proof of proposition 4

Consumer surplus in case of separation and integration are the following.

$$CS^{sep} = \gamma(1+Q^{sep})\int_{r^{sep}(0)}^{1} x dF^n(x),$$
$$cs^{int}(v_n) = \gamma(1+Q^{int})\left(\int_0^{r^{int}(v_n,0)} v_n dF^{n-1}(x) + \int_{r^{int}(v_n,0)}^{1} x dF^{n-1}(x)\right).$$

Assume $F = U[0,1]$ and let $CS^{sep}_{q=0}(n)$ and $CS^{int}_{q=0}(n)$ denote expected consumer surplus without investment in match quality in the case of separation and integration, respectively.

$$CS^{sep}_{q=0}(n) = \gamma\frac{n}{n+1}\left(1-\frac{1}{2^{n+1}}\right),$$
$$CS^{int}_{q=0}(n) = \gamma\frac{n+2^{-n}-1}{n}.$$

We start by showing that consumer surplus is greater in the case of integration compared to separation if $n = 2, 3$. First, we compare consumer surplus without investment by the intermediary.

$$\begin{aligned} CS^{sep}_{q=0}(n) &< CS^{int}_{q=0}(n) \Leftrightarrow \\ \gamma\frac{n}{n+1}\left(1-\frac{1}{2^{n+1}}\right) &< \gamma\frac{n+2^{-n}-1}{n} \Leftrightarrow \\ \frac{n^2(1-\frac{1}{2^{n+1}})-(n+1)(n+2^{-n}-1)}{n(n+1)} &< 0 \Leftrightarrow \\ n(n+2)+2 &> 2^{n+1}. \end{aligned}$$

It is readily checked that the latter inequality holds true for $n = 2, 3$. Moreover, by Proposition 1, $Q^{sep} \leq Q^{int}$. As a result, $CS^{sep} < CS^{int}$ for $n = 2, 3$.

We now derive the critical investment level for which consumer surplus in the case of separation is equal to consumer surplus in the case of integration if $n \geq 4$ and $C(q) = aq^2$. $g(n)$ and $h(n)$ have been defined in proposition 3. The critical level of investment in match quality is derived as follows.

$$
\begin{aligned}
CS^{int} &= CS^{sep}, \\
(1+Q^{int})CS^{int}_{q=0}(n) &= (1+Q^{sep})CS^{sep}_{q=0}(n), \\
a(n) &= \frac{1}{2}\frac{g(n)CS^{sep}_{q=0}(n) - h(n)CS^{int}_{q=0}(n)}{CS^{int}_{q=0}(n) - CS^{sep}_{q=0}(n)}, \\
\overline{a}(n) &= \frac{2^{-n-1}(2^n(n-1)+1)(n^3-4n+2^{n+1}(n+2)-4)}{n(n+1)(2(2^n-1)-n(n+2))}.
\end{aligned}
$$

The function $\overline{a}(n)$ is decreasing, as it is readily checked that $\overline{a}(n+1) < \overline{a}(n)$ for $n = 4, ..., 20$. The limit of $\overline{a}(n)$ is the following.

$$
\lim_{n\to\infty} \overline{a}(n) = \lim_{n\to\infty} \frac{2^{-n-1} \times 2^n \times n \times 2^{n+1} \times n}{n^2 \times 2 \times 2^n} = \frac{1}{2}.
$$

Proof of proposition 6

The critical investment level for which total welfare in the case of separation is equal to total welfare in the case of integration is derived as follows.

$$
\begin{aligned}
W^{sep} &= W^{int}, \\
\frac{1+\gamma}{\gamma}CS^{sep} - C(Q^{sep}) &= \frac{1+\gamma}{\gamma}CS^{int} - C(Q^{int}), \\
\hat{a}(n) &= \frac{g(n)\left(g(n) - 2(1+\gamma)CS^{sep}_{q=0}\right) + h(n)\left(2(1+\gamma)CS^{int}_{q=0} - h(n)\right)}{4(1+\gamma)(CS^{sep}_{q=0} - CS^{int}_{q=0})}.
\end{aligned}
$$

with $g(n)$ and $h(n)$ defined in proposition 3.

Assuming $\gamma = 1$, we find that

$$
\hat{a}(n) = \frac{2^{-n-2}(n(n(2^{n+1}(n^2-3)+2n+1)+8(4^n-1))-12(2^n-1)^2)}{n(n+1)(2(2^n-1)-n(n+2))}.
$$

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