The Rise in Profits and Fall in Firm Entry: A Tale of the Life Cycle of Profits

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Abstract

It is crucial to consider how profits vary over the life cycle of the firm in order to understand why the aggregate profit share has been increasing while firm entry has been declining. All else equal, the more back-loaded profits are, the lower is the value of the firm due to discounting. Therefore, fewer entrepreneurs choose to enter the market, leading to an increase in average profits per firm as market shares are increasing. Under some conditions, this fall in entry also leads to an increase in the aggregate profit share. Empirically, profits have become more back-loaded. Using a quantitative life cycle model of the firm with varying markups I find that this increase in back-loadedness explains between half and all of the rise in profits, and more than fully explains the fall in firm entry.

Keywords: Firm Entry, Profits, Firm Life Cycle, Entrepreneurship
JEL codes: E22, E25, D22, D33, L25

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Recent evidence shows that firms' profits as a share of output have been increasing during the last decades (Barkai, 2020; De Loecker et al., 2020; Van Vlokhoven, 2019), while at the same time firm entry has been declining (Decker et al., 2014). The joint occurrence of these two trends is surprising at first sight as the rise in profits suggests that the benefits of having a firm are large while the decline in entry suggests the opposite. For instance, if the rise in aggregate profits is due to some change in the economic environment that increases profitability for all firms such as an increase in markups for all firms, one would expect firm entry to increase in response as well. In this paper, I argue that to understand these two trends jointly it is crucial to consider how profits vary over the life cycle of the firm. I put forward and provide quantitative evidence for a new hypothesis that can explain both the rise in profits and the fall in firm entry, namely, that profits have become more back-loaded over the life cycle of the firm. Or, put differently, that the relationship between profits and firm age has become steeper.

An entrepreneur enters the market when the value of having a firm is larger than the cost of entry. Thus, in equilibrium, entry costs equal the value of the firm where the value of the firm equals the discounted sum of profits over the life cycle of the firm. Due to discounting, total profits earned over the life cycle generally do not equal entry costs. When profits are back-loaded, total profits exceed the discounted sum of profits and therefore exceed entry costs (Atkeson and Kehoe, 2005). Consider the following thought experiment. Suppose that the economy is initially in equilibrium, but that, over time, profits shift from a young firm age to an old age, in such a way that total profits over the life cycle remain constant. That profits become more back-loaded lowers the value of entering the market as profits that appear later are more heavily discounted than profits that arrive early. Therefore, the entry condition no longer holds as entry costs now exceed the value of the firm. Thus, as a response, firm entry goes down and hence market shares and the profits firms are making increase, until the entry condition holds again. Figure 1 illustrates this graphically. The blue dotted line is an example of what profits over the life cycle might look like initially. Older firms make more profits than younger firms but the profits-age relationship is quite flat. The red dashed line shows what the profits-age relationship might look like when profits become more back-loaded but before the equilibrium response; young firms make less profits and old firms make more profits, but the (undiscounted) sum of profits over the life cycle is the same as in the initial equilibrium. The green solid line shows what profits might look like after the economy has moved to a new equilibrium. The discounted sum of profits is the same in the new as in the initial equilibrium, but profits over the life cycle are larger in the new equilibrium than in the initial equilibrium since entrepreneurs need to be compensated for the fact that profits appear later in life.

The increase in average profits per firm over the life cycle does, however, not necessarily imply that the aggregate profit share is increasing as well. One reason that average profits per firm increase is that firm sizes are increasing due to the decline in entry. It could be that the profit share at the firm level does not change, and that the rise in profits per

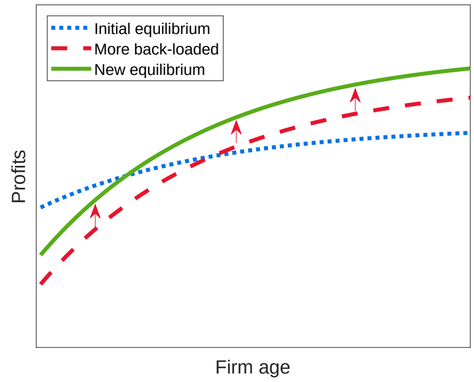


Figure 1: Illustration of profits over the life cycle

firm is entirely driven by firms becoming larger. I show in Section I that if the aggregate profit share is negatively related to the number of firms, then aggregate profits increase if and only if back-loadedness increases. Moreover, independent of the relationship between the number of firms and aggregate profits, if in equilibrium A both the number of firms are smaller and aggregate profits are larger than in equilibrium B then profits must be more back-loaded under equilibrium A than under equilibrium B. The intuition behind this result is straightforward. As the number of firms declines and aggregate profits increase it must be that average profits per firm increase. And by the above, an increase in average profits is associated with an increase in back-loadedness. Thus, that empirically the profit share has been increasing while the number of firms relative to the labor force in the private sector has been declining implies that profits have become more back-loaded over the life cycle of the firm.

In Section II I find that the profits-age relationship has indeed become steeper over time. Before 2000, older firms were only making moderately more profits than young firms, while after 2000 older firms were making much more profits than younger firms. Moreover, young firms after 2000 are making about as much profits as they did during the 1980s and the 1990s. These results are not driven by changes in the industry composition over time and are not driven by the great recession. Also, excluding firms that have been involved in M&A activity does not alter the conclusions. Finally, the results are also not sensitive to outliers as the life cycle of profits has also changed for the median firm. Thus, the hypothesis put forward in this paper does not only apply to the so-called superstar firms.

To study the equilibrium response to a change in the profits-age relationship, I build a quantitative life cycle model of the firm featuring varying markups in Section IV. A firm's level of profits depends positively on its productivity level and on its intangible capital stock. Intangibles represent here product quality or brand value. A larger intangible capital stock shifts out the demand curve and makes consumers less sensitive to prices yielding a higher markup. I vary the depreciation rate of intangibles in order to vary the level of back-loadedness. The lower the depreciation rate the more back-loaded profits become as this favors older firms that have had more time to accumulate intangible capital. Matching the observed increase in back-loadedness, I find that the rise in back-loadedness can explain between half and almost all of the observed rise in profits, depending on which measure of back-loadedness is used. Furthermore, the rise in back-loadedness explains more than fully the fall in the number of firms. It is natural that the rise in back-loadedness more than fully explains the fall in entry given that the rise in back-loadedness cannot fully explain the rise in profits. This is because there must have been another force than a changing profits-age relationship at play which explains the remaining part of the rise in profits. An additional rise in profits that does not affect the profits-age relationship would make it more attractive to become an entrepreneur and would, therefore, lead to a slower decline in entrepreneurship.

That varying the depreciation rate of intangibles leads to a positive relationship between back-loadedness and the profit share, and a negative relationship between back-loadedness and the number of firms does not necessarily imply that these relationships emerge as well when other parameters of the model are varied. For instance, when the sensitivity of consumers to intangibles is varied, a negative relationship between back-loadedness and profits emerges. The reason is as follows. That consumers become more sensitive to intangibles favors older firms as they have had more time to accumulate intangibles. This leads to an increase in back-loadedness of profits and also leads to a decline in the number of firms. This, in turn, increases markups, and if intangible investment would remain constant this would also lead to an increase in the profit share. However, due to the increased sensitivity to intangibles the return to intangible investment increases and, hence, firms increase their intangible investment. This increase in intangibles makes that overhead costs as a share of sales go up, offsetting the rise in markups, and leading to a decline in the profit share. The reason that entrepreneurs are still willing to enter the market despite the increased back-loadedness and fall in aggregate profits is that profits per firm increase as the number of firms drops sharply leading to larger market shares. In general, this example highlights that the change in the economic environment that has led to the rise in back-loadedness cannot have had a large direct negative effect on profits as that would be inconsistent with the rise in profits.

Related literature This paper contributes to the recently emerging literature that studies why market power has been increasing over the last decades. Bornstein (2018) finds that a rise in consumer inertia can explain both the fall in entry and rise in profits, and Akcigit and Ates (2019) argue that a decline in knowledge diffusion between frontier and laggard firms is a powerful explanation.[1] Both Bornstein (2018) and Akcigit and Ates (2019) study a model in which there is a life cycle of the firm, and their mechanisms yield an increase in back-loadedness. In Bornstein (2018), the higher consumer inertia the longer it takes for firms to build a customer base, which means that profits become more back-loaded. In Akcigit and Ates (2019) profits become more back-loaded as well

[1] In another paper, Akcigit and Ates (2021) find as well that a decline in knowledge diffusion leads to a rise in profits, but in that paper they do not study the effect on firm entry.

as due to the decline in knowledge diffusion it takes longer for entrants to catch up with the leader. A potent explanation for the fall in entry and rise in market power seems to be changing demographics. Karahan et al. (2019) find that the fall in firm entry is due to a decline in the growth rate of labor supply, and Engbom (2019) argues that population aging is important for understanding the fall in entry. Hopenhayn et al. (2018) and Peters and Walsh (2021) find that the decline in labor force growth cannot only explain the fall in entry but also the rise in profits. The model in Peters and Walsh (2021) features a life cycle of the firm. As population growth declines, creative destruction declines which shifts economic activity to older firms as they will now accumulate more product lines. Thus, this also features an increased back-loadedness of profits. I add to this literature by demonstrating that a changing life-cycle pattern of profits is crucial for understanding the joint rise in profits and fall in entry. In addition, I provide empirical evidence that profits have indeed become more back-loaded. Furthermore, I show that, in order to explain the trends, the economic forces that have led to the rise in back-loadedness should not be associated with a too large increase in overhead costs.

De Ridder (2019) argues that an increase in the variance of intangible efficiency across firms explains the rise in markups while his calibration targets the fall in entry. However, the rise in markups in his framework is associated with a rise in fixed costs due to a rise in intangibles, making it is unclear what happens to profits. Also De Loecker et al. (2021) focus on markups and not on profits. They find that the rise in markups is due to a fall in the number of potential entrants and a rise in fixed costs as well.

Gutiérrez et al. (2021) argue that a rise in entry costs is causing the rise in markups and fall in entry. Weiss (2020) argues that these trends are due to an increased importance of intangibles, where intangibles can only be purchased in the period when a firm is born. Thus, in his framework, an increased importance of intangibles is effectively an increase in entry costs as well. An increase in entry costs is an alternative explanation of the rise in market power and fall in entry. However, if entry costs are not literally paid upon entry but are paid during the first years of operation then a rise in entry costs makes that profits become more back-loaded. Thus, in this sense is a rise in entry costs not that different from an increase in back-loadedness.

Other explanations for the rise in profits include a decline in interest rates (Liu et al., forthcoming), and IT improvements leading to a fall in the firm-level costs of spanning multiple markets (Aghion et al., 2019). These papers, however, are silent on what happens to firm entry whereas I focus on the joint fall in entry and rise in profits. Chatterjee and Eyigungor (2020) find that a decline in interest rates can also lead to a fall in firm entry and a rise in concentration, but profits are exogenous in their framework.

Finally, Cavenaile et al. (2021) and Edmond et al. (2018) focus on the (dynamic) welfare effects of the rise in market power while I focus on what explains the rise in market power.

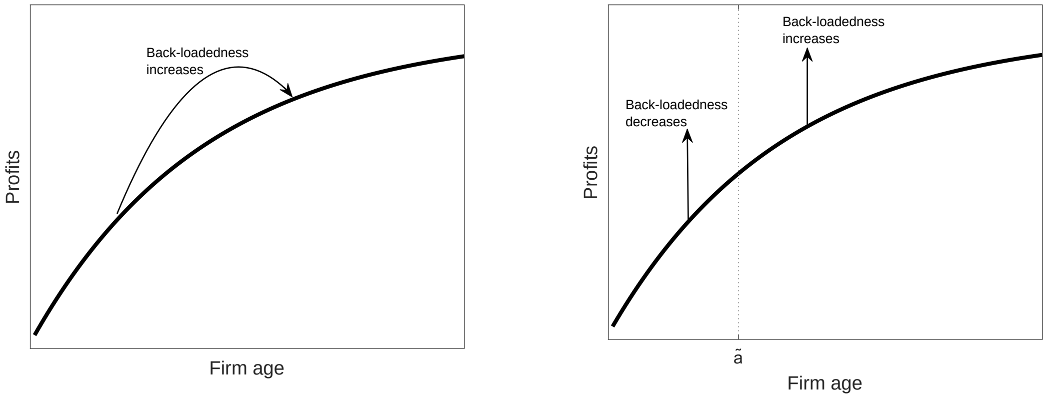


Figure 2: Illustration of changes to back-loadedness

I Back-loadedness

Suppose an entrepreneur can enter the market by paying an entry cost c_e. In an equilibrium with a positive mass of entrants, entry costs equal the value of the firm,

$$c_e = V = \mathbb{E} \sum_a \beta^a \Pi_a \,, \tag{1}$$

where c_e denotes the entry costs. The value of the firm upon entry is denoted by V, which equals the expected discounted sum of profits Π_a over the life cycle of the firm, where firm age is denoted by a and the discount factor is denoted by $\beta < 1$. Naturally, profits might differ over the life cycle, and that is why profits are indexed by a. Also, the firm might eventually exit the market in which case all future profits are zero. The probability that a firm exits is incorporated in the expectation operator.

Denote average expected profits over the life cycle of a firm as $\overline{\Pi} = \frac{\mathbb{E}\sum_a \Pi_a}{T}$ where T denotes the average lifetime of the firm. Due to discounting, an entrepreneur prefers to receive profits sooner rather than later. Compare the value of receiving the actual profit stream with the value of receiving every period a constant level of profits, namely average profits.[2] Then, the more the entrepreneur prefers to receive the constant stream of profits over the actual profit stream, the more back-loaded profits are. This naturally leads to the following definition of back-loadedness.

Definition 1. *Back-loadedness Γ of a profit stream equals $\frac{\sum_a \beta^a \overline{\Pi}}{V}$ where V is the value of the firm and $\overline{\Pi}$ are average profits, as defined in the above.*

This definition of back-loadedness has some properties that seem natural for a measure of back-loadedness. Consider a profit stream and suppose that profits shift to an older

[2] Here, it is assumed that a firm can live potentially forever. When firms live at most until T_{max} it is natural to calculate average profits as $\overline{\Pi} = \frac{\mathbb{E}\sum_a \Pi_a}{T_{max}}$. This has as advantage that total profits over the life cycle are the same when comparing the actual profit stream with receiving every period average profits. This is obviously not the case when considering an infinite horizon. However, the implications for back-loadedness and its relationship with entry and profits are the same across both models.

age, meaning that expected profits at age a_1 decline by x while expected profits at age a_2 increase by x with $a_1 < a_2$. Total expected profits over the life cycle of the firm do not change and therefore average profits do not change. But because x profits now appear at a later moment, the value of the firm goes down due to discounting. This leads to an increase in the measure of back-loadedness. Furthermore, there is a cutoff age,

$$\tilde{a} = \frac{\log\left(V/\mathbb{E}\sum_a \Pi_a\right)}{\log\left(\beta\right)} > 0\,, \tag{2}$$

such that when expected profits increase by 1 for an age larger than this cutoff age, back-loadedness increases. And when expected profits increase by 1 for an age lower than this cutoff age, back-loadedness decreases, or in other words, the profit stream becomes more front-loaded. Both properties of this definition of back-loadedness are graphically illustrated in Figure 2.

Now compare two separate economies with each other and assume that the entry condition holds. If in one economy profits are more back-loaded than in the other economy, average expected profits over the life cycle of the firm are larger in the economy with more back-loaded profits and vice versa. This is because for entrepreneurs to be willing to enter the market, they need to be compensated for the fact that profits appear later in life.

Theorem 1. *In an equilibrium with positive entry, average profits of profit stream b are larger than those of profit stream c ($\overline{\Pi}^b > \overline{\Pi}^c$) if and only if profit stream b is more back-loaded than profit stream c ($\Gamma^b > \Gamma^c$).*

Proof. The entry condition gives that $V^b = V^c$. Using Definition 1 to substitute out firm value gives $\frac{\sum_a \beta^a \overline{\Pi}^b}{\Gamma^b} = \frac{\sum_a \beta^a \overline{\Pi}^c}{\Gamma^c}$. Since $\sum_a \beta^a$ cancels out, this proves the statement. $\square$

Higher average profits over the life cycle of a firm do not necessarily imply that aggregate profits are also larger. This is because there might be fewer firms in the new equilibrium. Denote the number of firms in an economy by N such that aggregate profits equal $\tilde{\Pi} = N\overline{\Pi}$.[3] Suppose that aggregate profits increase while the number of firms declines. This implies that average profits over the life cycle of the firm have risen. And by Theorem 1 this means that profits have become more back-loaded. This is summarized in the next corollary.

Corollary 1. *In an equilibrium with positive entry, both the number of firms are smaller and aggregate profits are larger under profit stream b than under profit stream c ($N^b < N^c$ and $\tilde{\Pi}^b > \tilde{\Pi}^c$) only if profit stream b is more back-loaded than profit stream c ($\Gamma^b > \Gamma^c$).*

Since in the data profits have increased and the number of firms has declined, this corollary implies that back-loadedness must have been increasing. However, Corollary 1 does not imply that a rise in back-loadedness necessarily leads to a rise in profits. It could

[3] To see that this equation holds, note that in a stationary equilibrium life expectancy of a firm is equal to $T = \frac{N}{E}$ where E is the number of firms that enter each year (i.e, the number of 1-year old firms in a stationary equilibrium). Then $N\overline{\Pi} = E\mathbb{E}\sum_a \Pi_a$ which equals aggregate profits.

be that in a new equilibrium in which back-loadedness has increased, aggregate profits are smaller because the number of firms has decreased substantially. In such a case, the rise in average profits over the life cycle of a firm would entirely be due to increases in market shares and not due to increases in firm-level profit shares. However, it turns out that the condition that aggregate profits are decreasing in the number of firms is sufficient to guarantee that aggregate profits are larger the more back-loaded profits are.

Theorem 2. *Suppose $\frac{d\tilde{\Pi}}{dN} < 0$. In an equilibrium with positive entry, aggregate profits are larger for profit stream b than for profit stream c ($\tilde{\Pi}^b > \tilde{\Pi}^c$) if and only if profit stream b is more back-loaded than profit stream c ($\Gamma^b > \Gamma^c$).*

Proof. $\Rightarrow$ Suppose $\tilde{\Pi}^b > \tilde{\Pi}^c$. This implies $N^b < N^c$ by $\frac{d\tilde{\Pi}}{dN} < 0$. This gives $\Gamma^b > \Gamma^c$ by Corollary 1.

$\Leftarrow$ Using the entry condition and substituting out firm value using Definition 1 plus multiplying both sides with $\frac{N^b}{N^c}$ gives after rewriting that $\frac{\tilde{\Pi}^b}{\tilde{\Pi}^c} = \frac{N^b}{N^c}\frac{\Gamma^b}{\Gamma^c}$. Suppose $\Gamma^b > \Gamma^c$. This gives that $\frac{\tilde{\Pi}^b}{\tilde{\Pi}^c} > \frac{N^b}{N^c}$. I continue the proof by means of a contradiction. Suppose that $\tilde{\Pi}^b \leq \tilde{\Pi}^c$ which implies that $N^b \geq N^c$ by $\frac{d\tilde{\Pi}}{dN} < 0$. Hence, it cannot be the case that $\frac{\tilde{\Pi}^b}{\tilde{\Pi}^c} > \frac{N^b}{N^c}$. Therefore, $\tilde{\Pi}^b > \tilde{\Pi}^c$. $\square$

The condition that aggregate profits are decreasing in the number of firms seems mild at first sight. For instance, the presence of more firms implies more competition. An alternative way in which the number of firms affects aggregate profits is through overhead costs. Suppose there is a fixed overhead cost that has to be paid by all firms. Then, the more firms there are, the larger aggregate overhead costs. Hence, this also gives a negative relationship between the number of firms and aggregate profits. Nonetheless, a change in back-loadedness is in practice driven by a change to firm's incentives. For instance, one way in which back-loadedness can have increased is that the return to innovation has increased. Then, it is possible that firms start to spend much more heavily on innovation. If this increase in overhead costs per firm is large enough it could be the case that aggregate profits decrease while the number of firms decreases as back-loadedness increases. Denote the returns to innovation by ψ such that the total derivative with respect to the number of firms becomes

$$\frac{d\tilde{\Pi}}{dN} = \frac{\partial\tilde{\Pi}}{\partial N} + \frac{\partial\tilde{\Pi}}{\partial\psi}\frac{d\psi}{dN}\,.$$

Take the case where $\frac{d\psi}{dN}$ is negative, which is the relevant case since as back-loadedness increases average profits increase (by Theorem 1) which is naturally associated with a decline in the number of firms. When $\frac{\partial\tilde{\Pi}}{\partial\psi}$ is sufficiently negative it could be that $\frac{d\tilde{\Pi}}{dN}$ is positive despite $\frac{\partial\tilde{\Pi}}{\partial N}$ being negative. Conversely, as long as the direct effect on aggregate profits of the change in the economic environment that causes back-loadedness to increase is not largely negative it is the case that increases in back-loadedness are associated with a rise in profits.

Furthermore, I have assumed that the discount rate and cost of entry have not changed. During the last decades, interest rates have declined, and therefore, discount rates might have declined as well. All else equal, this would increase the value of owning a firm, and in equilibrium this would lead to downward pressure on profits as entry increases. Thus, an increase in back-loadedness would not necessary imply a rise in profits if at the same time discount rates fall (i.e., a larger β). However, it is still the case that a rise in profits combined with a fall in entry is associated with profits being more back-loaded. In fact, a fall in the discount rate would imply that back-loadedness would have increased faster compared to when discount rates would not have decreased. Furthermore, entry costs represent partly the opportunity cost of the foregone value of labor earnings. Thus as agents become more patient, the effective entry cost also increases, and therefore, it is not necessarily the case that becoming more patient leads to an increase in firm entry. Finally, a rise in entry costs, for other reasons than a changing discount factor, is an alternative explanation for the rise in profits and fall in firm entry.

To summarize the theory section, as long as entry costs have not increased, back-loadedness must have been increasing to rationalize the joint fall in entry and rise in profits. Moreover, we know that the change in the economic environment that has led to the rise in back-loadedness cannot have had a large direct negative effect on profits as this would be inconsistent with the rise in profits. For instance, a change in the economic environment that not only increases back-loadedness but also increases overhead costs substantially is unlikely to be the reason that back-loadedness has increased.

II Empirical Evidence Profits-Age Relationship

This section provides evidence of how the profits-age relationship has changed over time. I use Compustat data for the United States from 1980 until 2019.[4] Economic profits are equal to nominal output, PY, minus operating expenses, $P^X X$, and minus capital costs, $R \cdot P^K K$,

$$\text{Profits} = PY - P^X X - R \cdot P^K K \,. \tag{3}$$

Nominal output, operating expenses and the nominal capital stock, $P^K K$, are directly observed. However, as the user cost of capital, R, is not directly observed, also economic profits are not directly observed. To estimate the cost of capital and hence profits I use the method developed in Van Vlokhoven (2019). In short, this method regresses firm-level nominal output divided by operating expenses on a constant and on the capital stock divided by operating expenses. The profit accounting identity gives an economic interpretation to the coefficients in this regression. The intercept coefficient gives the price-average cost ratio and the slope coefficient gives the price-average cost ratio times

[4] Compustat data goes back to the early 1950s. However, during the early years there are relatively few firms and hence I focus on the period from 1980 onwards. This is also the relevant period for the increase in profits and fall in firm entry.

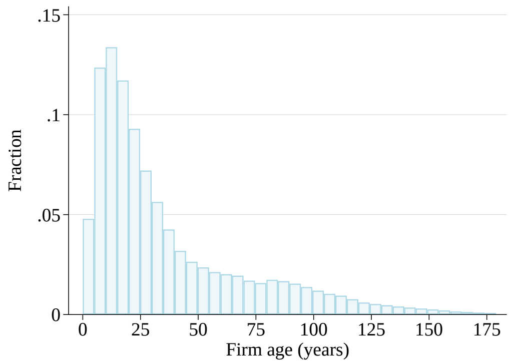


Figure 3: Histogram age distribution

the cost of capital. The cost of capital is then simply the slope coefficient divided by the intercept coefficient. I estimate the cost of capital this way for each industry and year.[5] Alternatively, capital costs can be estimated using the required rate of return approach which uses information on the real interest rate and depreciation rate (Hall and Jorgensen, 1967). Using the required rate of return approach gives similar results for the profits-age relationship. See Appendix A for details on how I estimate economic profits, and for details regarding the data.

In addition to profits, I need data on firm age which is not directly available in Compustat. Therefore, I complement Compustat with data on the year in which a firm was incorporated. This data comes from Field and Karpoff (2002), Loughran and Ritter (2004), Jovanovic and Rousseau (2001) and fundinguniverse.com. Then, the age of the firm in a given year is simply the year of incorporation subtracted from the reporting year. When different sources report a different incorporation year, I take the lowest value among these sources. I observe firm age for around 56% of the observations and I trim the top and bottom percentile of the profits and profit share (of sales) distribution to remove outliers. This leaves me with a sample of 82 thousand firm-year observations and a bit more than 7 thousand unique firms for which I observe profits and firm age.

Compustat data covers mainly publicly listed firms and a disadvantage of this data is thus that the firms included are not a random sample of the universe of US firms. These are firms that tend to be older and larger than the typical firm. Nevertheless, although Compustat tends to overrepresent older firms there are still many young firms in this data set. Figure 3 shows that the firm ages that are most common in this data are the ages from five till twenty years old.[6] However, there are relatively few firms younger than five years old. I mainly use the Compustat data to document how the profits-age relationship has changed over time. Sample selection is therefore not a problem per se. However, it can become problematic when sample selection is changing over time. The number of

[5]When I refer to industries I refer to 2-digit SIC codes. I use the SIC code instead of the NAICS code to classify industries as around 1.5% of observations has a missing NAICS code (mainly in the early years).

[6]Also the distribution of firm age does not change much over time, although in the later period of the sample there are particularly few firms younger than five years old (see Figure 11 in Appendix C).

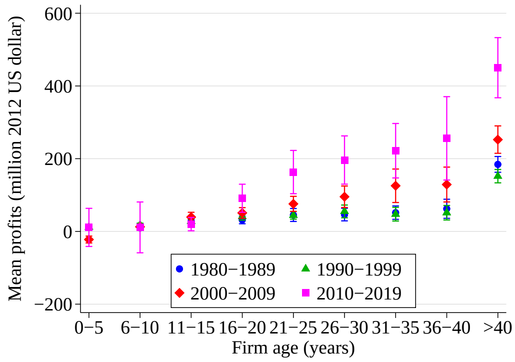


Figure 4: Profits over the life cycle by decade

firms in Compustat displays an inverse U-shape over time with the maximum number of firms occurring in the late 1990s. This can affect the results in several ways. If firms that go public tend to be relatively profitable, then the decline in the number of IPOs would have led to selection into more profitable firms after 2000 compared to before 2000. Thus, this would lead to an upward biased estimate of profits among the relatively young firms that appear in the data for the later periods. Hence, in this case, I would underestimate the increase in the slope of the profits-age relationship. If, alternatively, during the later period the most profitable young firms decided to stay private for a longer time or were more likely to be acquired, I would overestimate the increase in the relationship between profits and firm age. It is important to note here that Compustat also includes some firms that have not gone public. For instance, when a firm has issued corporate bonds it might be required to file reports to the SEC which provides the basis for Compustat.

To calculate profits over the life cycle of the firm I bin firms into age bins of five years and I collapse the data into decades starting with the 1980s and ending with the 2010s. As an example, suppose that I observe a firm that was founded in 1986 from 1990 onward. Then, in order to calculate profits by age bin in the 1990s, that firm is used twice to calculate profits of firms in the 0–5 years age bin (namely the observations for 1990 and 1991). The observations of that firm for the years 1992 until 1996 are used to construct profits among firms six to ten years old in the 1990s. And likewise for the years 1997 until 1999. The observations of this firm for the years 2000 and 2001 are used to construct profits among firms 11–15 years old in the 2000s etc.

Figure 4 shows the resulting average profits by firm age for the four different time periods. Profits are deflated by the GDP deflator and the vertical lines denote 95% confidence intervals in this and all subsequent figures, with standard errors clustered at the firm level. The profits-age relationship has become steeper over time. During the 1980s and the 1990s, old firms were only making moderately more profits than young firms. But after 2000, old firms started to make much more profits while the profits of young firms hardly changed. During the 2000s, firms younger than ten years old made essentially no profits, while firms that were more than twenty years old made more than 100 million

Table 1: Effect of age on profits

	(1) Profits	(2) Profits	(3) Profits	(4) Profits
Age	2.295***	2.149***		
	(0.203)	(0.192)		
Log age			86.36***	63.42***
			(6.179)	(4.896)
1990s × age	-0.264*	-0.220		
	(0.103)	(0.147)		
2000s × age	0.991***	1.133***		
	(0.205)	(0.315)		
2010s × age	3.567***	3.961***		
	(0.493)	(0.837)		
1990s × log age			-3.012*	-9.556*
			(1.279)	(3.747)
2000s × log age			11.71***	50.89***
			(2.344)	(8.677)
2010s × log age			47.41***	180.6***
			(5.684)	(28.65)
Industry × year FE		X		X
Observations	82727	82727	82537	82537
R^2	0.090	0.146	0.065	0.133

Standard errors clustered at the firm level in parentheses. Profits refer to real profits in millions of 2012 US dollars, deflated using the GDP deflator. Age is denoted in years. The left out category for the decade times age interaction is the 1980s.

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

dollar of profits on an annual basis. During the 1980s and 1990s only for firms more than 40 years old there was a substantial increase in profits compared to younger firms, but this increase in profits for the oldest firms is even steeper after 2000. The profits-age gradient was also larger during the 2010s than during the 2000s.

Figure 4 simply plots average profits for each age bin. An alternative method to study the life-cycle pattern of profits is to regress profits on firm age interacted with a dummy for each decade, potentially with industry times year fixed effects. Table 1 shows that in the 2000s and 2010s the relationship between profits and age was significantly larger than in the 1980s and 1990s both when profits depend linearly on age or in a logarithmic fashion. Also, the results are robust to including industry times year fixed effects. In the 1980s and 1990s there was a positive relationship between age and profits. Firms that were 1 year older would have on average 2.1 million larger profits measured in 2012 US dollars, and the profits-age relationship was not significantly different between the 1980s and 1990s. In the 2000s firms that are 1 year older were making on average 3.3 million dollar more profits, and during the 2010s increasing age by 1 year led to a 6.1 million dollars increase in profits on average.[7] These regressions assume profits depend linearly on age or in a logarithmic fashion. These results are not driven by the very youngest and oldest firms. Similar coefficients are obtained when omitting firms younger than 10 years and older than 50 years. Table 4 in Appendix C shows that the coefficient on firm age also becomes larger for the more recent periods when including firm-level fixed effects. Figure 10a in Appendix C shows that also when profits are allowed to non-parametrically depend on age (again by means of 5-year age bins) the profits-age relationship becomes steeper over time when controlling for industry times year fixed effects.

It is not only the case that the slope of profits with respect to age has been increasing over time, but also back-loadedness as defined by Definition 1 has been increasing. Figure 4 shows profits conditional on firm survival, whereas the definition of back-loadedness uses profits expected at the time of entry, including the possibility of exiting the market. To calculate expected profits I obtain the probability of survival up to a certain age from the US census Business Dynamics Statistics. And expected profits is equal to profits conditional on surviving multiplied by the survival probability. Figure 12 in Appendix C shows that also expected profits have predominantly increased for the older firms. Given the results on profits conditional on firm survival this should come at no surprise as the exit rate by firm age has not changed much over time (Hopenhayn et al., 2018).[8] More formally, we can calculate back-loadedness according to Definition 1. Back-loadedness depends on the discount rate. When the annual discount rate is 4% back-loadedness increases by 30% from 8.0 before 2000 to 10.3 after 2000. And when the discount rate is 8% back-loadedness increases from 7.5 to 16.7. Thus, we can conclude that back-loadedness has increased over time. Note that a discount rate of 8% might seem large but one has to take into account that this number refers to a stochastic discount rate. Furthermore, it is relevant to note that the level of back-loadedness depends strongly on profits at a young age and these are the least precisely estimated. However, Figure 4 indicates that also the relationship between profits and age has become steeper over time when the youngest firms are excluded.

It is also interesting to see whether the change in the profits-age relationship is different across industries. Table 2 shows that for both manufacturing firms and non-manufacturing firms the relationship between firm age and profits has become steeper over time, although

[7]One reason that there is a steeper age profile is that aggregate profits have been increasing over time. When profits increase in the same proportion for all ages, the coefficient on age would mechanically increase (it is not possible to regress log profits on age as profits can be negative). During the 2000s the profit share was 27% larger than during the 1980s (Van Vlokhoven, 2019). Thus, by this effect, we would have expected the age coefficient for the 2000s to be 0.58 ($= 0.27 \times 2.15$) larger while instead I find it to be 1.13 larger than in the 1980s. And during the 2010s, the profit share was 58% larger and thus we would expect the age coefficient to be 1.25 ($= 0.58 \times 2.15$) larger, while I find it to be 3.96 larger. Thus, the change in the profits-age relationship exceeds what would have been expected from only an increase in aggregate profits.

[8]When we consider median profits instead of average profits, it becomes more clear that the youngest firms make less profits in expectation than they used to do (see Figure 12).

Table 2: Effect of age on profits for manufacturing and non-manufacturing

	Manufacturing		Non-Manufacturing	
	(1)	(2)	(3)	(4)
	Profits	Profits	Profits	Profits
Age	2.410***	2.052***	2.301***	2.449***
	(0.248)	(0.223)	(0.353)	(0.367)
1990s × age	-0.126	-0.122	-0.707***	-0.527*
	(0.120)	(0.176)	(0.204)	(0.261)
2000s × age	1.220***	1.675***	1.159***	1.300**
	(0.242)	(0.373)	(0.339)	(0.483)
2010s × age	3.920***	5.000***	2.615***	1.407
	(0.619)	(1.011)	(0.788)	(1.445)
Industry × year FE		X		X
Observations	47859	47859	34868	34868
R^2	0.109	0.155	0.057	0.135

Standard errors clustered at the firm level in parentheses. Profits refer to real profits in millions of 2012 US dollars, deflated using the GDP deflator. Age is denoted in years. The left out category for the decade times age interaction is the 1980s.

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

the change has been somewhat stronger for manufacturing. Figure 13 in Appendix C shows that also according to the raw data profits have become more back-loaded for both manufacturing and non-manufacturing firms.

The changing life-cycle pattern of profits is robust to a variety of checks. The profits-age gradient also increases over time when considering median profits within an age-decade bin, and therefore the changing life-cycle pattern of profits is not driven by only a few firms becoming extremely profitable (Figure 10b), although one difference is that the slope for the median firm mainly changes only after 2010. Moreover, as already indicated, the results are robust to using another estimate of the cost of capital. An alternative method to estimate capital costs and economic profits is the required rate of return approach which approximates the cost of capital by the real interest rate plus the depreciation rate. Figure 10c shows that doing so leads to similar results. Also, the results are not driven by the great recession affecting younger firms to a larger extent than older firms since there is also a steeper life-cycle profile of profits for the period 2000–2005 (Figure 10d). Finally, some of the firms in the data set are the result of a merger or of acquiring other firms. Then one reason that older firms make more profits nowadays is that M&A activity has increased over time. Therefore, I use data on M&A deals from SDC platinum and exclude all firms that have been involved in M&A activity. This specification has as downside that this excludes some of the most profitable firms as firms that are involved in M&A activity are likely to be relatively profitable.[9] Figure 10e shows that the profits-age gradient still increases over time when firms with M&A deals are excluded although the rise in the gradient is smaller than in the baseline.

Finally, one possible explanation for the observed change in the life cycle of profits is that young profitable firms are acquired by older firms nowadays. This would lower the observed average profits among young firms compared to when these profitable firms were not acquired and thus observed as a separate entity. This explanation would be consistent with the increased number of M&A deals over time. To rule this possibility out I obtain the M&A deals from 1980 onward from Securities Data Company Platinum when the acquired company is based in the United States. I then correlate the deal value with the firm age of the acquired firm at the time of acquisition (Figure 14 in Appendix C). This exercise leads to two striking results. The first is that the value of firms younger than 20 years is not higher in real terms after 2000 compared to the period before 2000. Thus, the results in Figure 4 are not driven by the most profitable young firms being more likely to exit nowadays. The second striking result is that although Figure 14 shows that the value of the youngest firms has not increased over time, the value of the older firms did increase over time. This is consistent with the hypothesis laid out in this paper. Namely, when entry cost are constant over time, but with profits becoming more back-loaded it is the case that the (average) firm value only increases for older firms.

III Age or Firm Size?

The changing relationship between firm age and profits could be due to a direct effect or it could be mediated through an other variable such as firm size. Older firms tend to be larger and the relationship between firm size and profitability has become stronger over time (Van Vlokhoven, 2019). Then, changes in the relationship between age and firm size and in the relationship between firm size and profits affects the relationship between age and profits. Suppose that age and firm size affect profits in an additive way:

$$\Pi_t(a) = \mathcal{A}_t(a) + \mathcal{S}_t(s_t(a)) \,. \tag{4}$$

The changing relationship between profits and firm age, $\Pi_t(a)$, can be due to a direct effect of changes in $\mathcal{A}_t(a)$, but it could also be due to a changing relationship between firm size and profits as denoted by the function, $\mathcal{S}_t(\cdot)$, or a changing relationship between age and firm size, $s_t(a)$.

Here I decompose the change in the life cycle of profits in these three terms. That is, I

[9]Since I only observe M&A deals from 1980 onward I analyze profits only from 1990 onward as I am likely to overestimate profits in the 1980s (i.e., I would still include firms that did an M&A deal in 1979). That I only observe deals from 1980 onward is less of a problem for the later periods as firms that engage in M&A activity are likely to do so more than once.

regress the following equation

$$Profits_{ijt} = \beta_0 + \sum_k \beta_k^{age,<2000} D_{ijt}^k \mathbb{1}_{t<2000} + \sum_k \beta_k^{age,\geq 2000} D_{ijt}^k \mathbb{1}_{t\geq 2000} + \tag{5}$$
$$\mathcal{S}^{<2000}(s_{ijt})\mathbb{1}_{t<2000} + \mathcal{S}^{\geq 2000}(s_{ijt})\mathbb{1}_{t\geq 2000} + \Theta_{jt} + \varepsilon_{ijt} \,,$$

where D_{ijt}^k denote a dummy variable for whether a firm i in industry j in year t is in age bin k. And $\mathcal{S}$ consists of a third-order polynomial in sales and log sales. Θ_{jt} denote industry times year fixed effect. I use the coefficients of this regression to construct counterfactuals of what would have happened to the profits-age relationship if only the size-age relationship, $s(a)$, would have changed and if the size-age relationship plus the profits-size relationship, $\mathcal{S}(s)$, would have changed. Figure 5 shows the results of such a decomposition. Like before, the blue circles and pink squares shows average profits for each age bin before and after 2000, respectively. These values are identical to the values in Figure 4, except that now the entire time period is split into two periods instead of into four decades. The orange triangles show average profits for each age bin when profits are constructed using $\hat{\beta}^{age,<2000}$ and $\hat{\mathcal{S}}^{<2000}(s)$, but with the size distribution for the period after 2000. Thus, the orange triangles show the effect on profits over the life cycle of only changing the relationship between firm size and firm age. As the orange triangles are not much different from the blue circles, the changing profits-age relationship is not due to a change in the size-age relationship.[10] The black diamonds show average profits for each age bin when profits are constructed using $\hat{\beta}^{age,<2000}$ and $\hat{\mathcal{S}}^{\geq 2000}(s)$. As the black diamonds are close to the pink squares and very different from the orange triangles, the change in the life cycle of profits is to a large extent determined by a changing relationship between firm size and profits as captured by a change in $\hat{\mathcal{S}}^{\geq 2000}(s)$ compared to $\hat{\mathcal{S}}^{<2000}(s)$. However, these results do not mean that almost the entire effect is due to a change in $\mathcal{S}(s)$ only. The black triangles namely also include the interaction between a change in the profits-size relationship and a change in the size distribution. It turns out that this interaction is important. When calculating the life cycle of profits for the size distribution before 2000 but with $\hat{\mathcal{S}}^{\geq 2000}(s)$ we get average profits that are relatively close to the orange triangles. This implies that the changing life-cycle pattern of profits is mainly due to a changing interaction between the size-age relationship and profits-size relationship. Finally, the remaining direct effect of a change in $\mathcal{A}_t(a)$ is limited as the black triangles are close to the pink squares.

IV Model

I build a quantitative model of the firm to study to what extent the changing life-cycle pattern of profits explains the rise in profits. This also allows me to study what change

[10] This does not mean that the size-age relationship has not changed over time. Figure 15 shows that older firms have become on average larger.

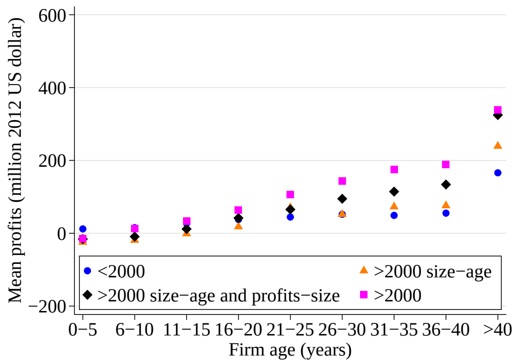


Figure 5: Decomposition of profits over the life cycle

in firms' incentives has led to an increase in profits, fall in entry and increase in back-loadedness at the same time. I consider a model of monopolistic competition with a Kimball (1995) aggregator.

Households Suppose there is a representative household that consumes a level C of the final good, supplies labor, L, inelastically in a competitive labor market for a wage w, and owns the firms. For simplicity, I assume that agents do not save and thus consume all their income within each period. The income of households is labor income plus profits.

Final good sector Suppose there are N varieties indexed by ω and that a competitive firm produces the final good, Y, according to a Kimball aggregator

$$\int_0^N \chi(\omega)^\psi \Upsilon\left(\frac{y(\omega)}{Y}\right) d\omega = 1\,, \tag{6}$$

where $\Upsilon(q)$ is strictly increasing, strictly concave and satisfies $\Upsilon(1) = 1$. $\chi(\omega)$ denotes (perceived) quality of a variety, which I will also refer to as brand value or intangible capital, and the parameter ψ governs to what extent quality is valued. Taking input prices $p(\omega)$ as given, the first-order conditions yield the following demand equation for each variety

$$p(\omega) = \chi(\omega)^\psi \Upsilon'\left(\frac{y(\omega)}{Y}\right) \underbrace{PY\left(\int_0^N \chi(\omega)^\psi y(\omega) \Upsilon'\left(\frac{y(\omega)}{Y}\right) d\omega\right)^{-1}}_{D}, \tag{7}$$

where D is a demand index and P denotes the aggregate price index. An increase in perceived quality shifts the demand curve out.

Firms Each variety ω is produced by one firm using production labor, l_p, according to the production function $y = z l_p^\eta$. Each firm chooses its price and production level in order to solve the following static maximization problem $\max_{p(\omega),y(\omega)} p(\omega)y(\omega) - w\left(\frac{y(\omega)}{z}\right)^{1/\eta}$

subject to demand (7). This gives the following first-order condition

$$p(\omega)=\frac{1}{1+\frac{y(\omega)}{Y}\frac{\Upsilon''\left(\frac{y(\omega)}{Y}\right)}{\Upsilon'\left(\frac{y(\omega)}{Y}\right)}}mc(\omega)\,,$$

where the term in front of the marginal cost, $mc(\omega)$, is the markup. I use the Klenow and Willis (2016) specification for Υ such that the markup becomes[11]

$$\mu(\omega)=\frac{\sigma\left(\frac{y(\omega)}{Y}\right)^{-\varepsilon/\sigma}}{\sigma\left(\frac{y(\omega)}{Y}\right)^{-\varepsilon/\sigma}-1}\,.$$

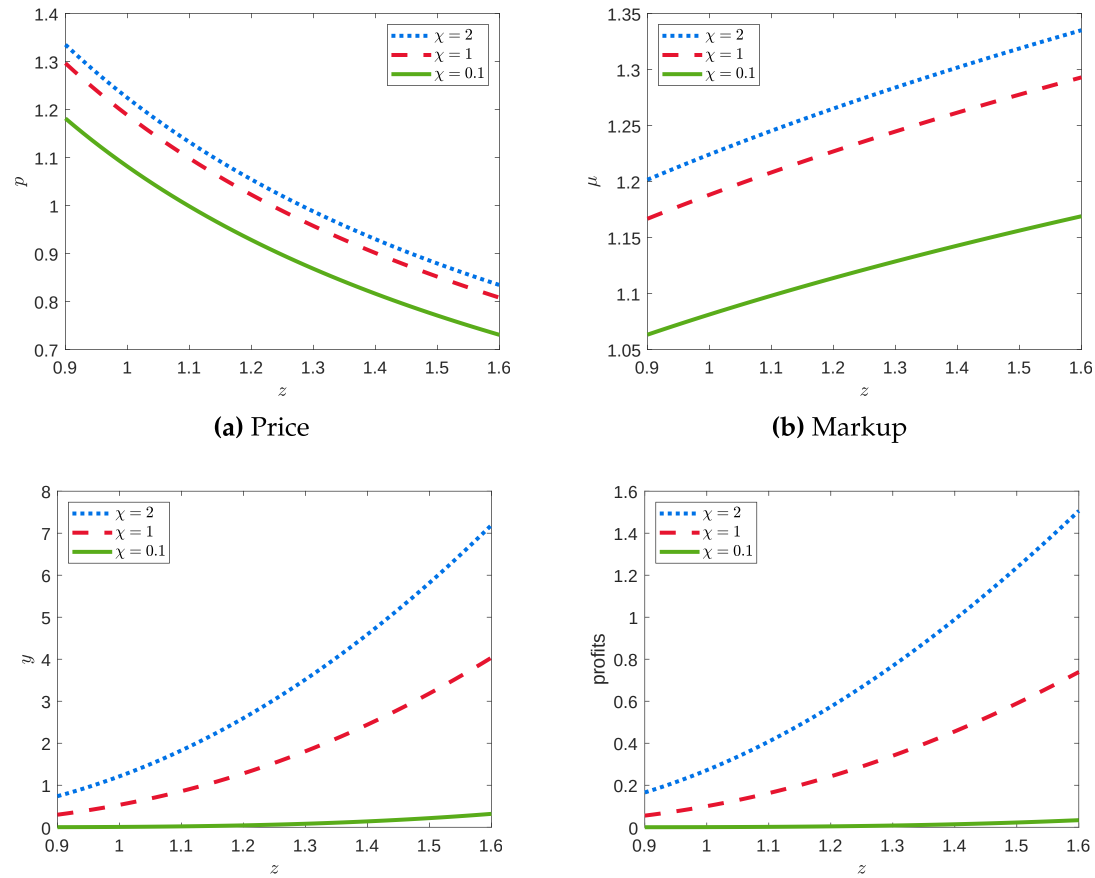
Thus, the price elasticity of demand, $\sigma\left(\frac{y(\omega)}{Y}\right)^{-\varepsilon/\sigma}$, is decreasing in firm size and therefore the markup is increasing in firm size. σ denotes the average elasticity and $-\varepsilon/\sigma$ is the elasticity of the elasticity with respect to firm size (i.e., the super-elasticity). Panel a and b of Figure 6 show how the price and the markup depend on productivity z and intangible capital stock χ. The larger process efficiency the lower the marginal cost and therefore also the lower the price. However, the price does not fall one-to-one with productivity as the markup increases when productivity increases. The markup, and therefore the price, are also increasing in intangibles. The reason is that when intangibles are larger, the demand curve shifts outward leading to a larger quantity bought as displayed in panel c, this in turn leads to a lower elasticity of demand and therefore a higher markup and price. Thus, a larger intangible capital stock makes that consumers want to buy more of the product for a given price and that consumers become less sensitive to the price which implies a larger markup. Panel d shows that both productivity and intangibles have a positive effect on profits. This is because both lead to a larger quantity sold and a higher markup charged.

Panel c and d of Figure 6 also show that the relationship between intangible capital on the one hand and firm size and therefore profits on the other hand is larger when productivity is larger. When productivity is low, the marginal cost is high and therefore the price is relatively high. Given this high price, demand will always be relatively low even when intangibles are large. On the other hand when productivity is large the price is relatively low and a small increase in intangibles will lead to a large increase in demand. This joint relationship between productivity, intangibles, and size and profits means that there is a complementary between productivity and intangible capital. More productive firms want to invest more in intangible capital. They can do so by hiring labor l_χ. I further assume that these investments take time to materialize and that the current intangible

[11]The Klenow-Willis specification is the following

$$\Upsilon(q)=1+(\sigma-1)\exp(1/\varepsilon)\varepsilon^{\frac{\sigma}{\varepsilon}-1}\left[\Gamma\left(\frac{\sigma}{\varepsilon},\frac{1}{\varepsilon}\right)-\Gamma\left(\frac{\sigma}{\varepsilon},\frac{q^{\varepsilon/\sigma}}{\varepsilon}\right)\right],$$

with $\sigma>1$, $\varepsilon\geq 0$ and where $\Gamma(s,x)$ denotes the upper incomplete Gamma function: $\Gamma(s,x)=\int_x^\infty t^{s-1}e^{-t}dt$. This gives that $\Upsilon'(q)=\frac{\sigma-1}{\sigma}e^{\frac{1-q^{\varepsilon/\sigma}}{\varepsilon}}$ and $\Upsilon''(q)=-\frac{\sigma-1}{\sigma^2}e^{\frac{1-q^{\varepsilon/\sigma}}{\varepsilon}}q^{\frac{\varepsilon}{\sigma}-1}$.



(a) Price

(b) Markup

(c) Size measured by output (real)

(d) Profits gross of intangible investment

Figure 6: Relationship of firm-level productivity and intangible capital with indicators of firm performance

capital stock also depends on past investments. This leads to the following dynamic firm problem in which the firm invests in intangibles to maximize the present value of expected profits

$$
\begin{aligned}
V(z,\chi,a) = \max_{l_\chi} \quad & \overbrace{p(z,\chi)y(z,\chi) - wl_p(z,\chi)}^{\text{Gross profits}} - \overbrace{(wl_\chi + wl_o)}^{\substack{\text{Intangible investment}\\ \text{plus fixed overhead}}} + \beta(1-\delta_a)\mathbb{E}_z V(z',\chi',a+1) \\
\text{s.t.} \quad & \chi' = g(\chi, l_\chi)\,.
\end{aligned}
$$

Labor used for intangible investment affects intangibles in the next period through the function $g(\chi, l_\chi)$. In addition to labor used for production and intangible investment, the firm also has to hire a fixed amount of labor, l_o, for overhead purposes each period. The discount factor is denoted by β. I take the exit rate to be exogenous as exit rates by firm age have not changed over time (Hopenhayn et al., 2018). The probability of exit, δ_a, depends on age, a, which makes that age becomes a state variable. Productivity evolves stochastically over time.

Entry In order to enter the market an entrepreneur has to hire an amount of labor

l_e for one period. Only after paying this entry cost the firm becomes aware of its initial productivity level. Thus, the entry condition is

$$wl_e \geq \mathbb{E}V(z, \chi, 1)$$

which holds with equality in case of a positive entry rate.

Equilibrium A stationary recursive equilibrium consists of a measure of firms $H(z, \chi, a)$, a value function $V(z, \chi, a)$, prices $p(z, \chi)$, allocations $y(z, \chi)$, $l_p(z, \chi)$, $l_\chi(z, \chi, a)$, wages w, aggregate output Y, consumption C, price index P, demand index D, labor supply L and aggregate profits Π such that

1. $y(z, \chi)$ and Y solve the optimization problem of the final good sector given $p(z, \chi)$. And $P = \int p(z, \chi, a) \frac{y(z,\chi,a)}{Y} dH(z, \chi, a)$ such that profits of the final good sector are zero;
2. $p(z, \chi)$, $y(z, \chi)$ and $l_p(z, \chi)$ solve the static firm problem given w, D and Y;
3. $V(z, \chi, a)$ and $l_\chi(z, \chi, a)$ solve the dynamic firm problem given w, P, $p(z, \chi)$, $y(z, \chi)$, and $l_p(z, \chi)$;
4. $H(z, \chi, a)$ is consistent with intangible investment $l_\chi(z, \chi, a)$ and the entry condition;
5. Aggregate profits are equal to $\Pi = \int p(z, \chi, a) y(z, \chi, a) - w[l_p(z, \chi, a) + l_\chi(z, \chi, a)] - wl_o] dH(z, \chi, a) - wl_e \int dH(z, \chi, 1)$;
6. Labor markets clear: $\int l_p(z, \chi, a) + l_\chi(z, \chi, a) + l_o \, dH(z, \chi, a) + l_e \int dH(z, \chi, 1) = L$; And good markets clear: $Y = C$.

See Appendix D for the solution algorithm.

Calibration I calibrate the model to the US economy between 1980 and 2000. I assume that a period is one year and set the discount rate to 0.96. Furthermore, I let productivity follow an autoregressive process over the firm's life cycle

$$\log(z_{a+1}) = \rho_a + \rho_z \log(z_a) + \xi_a \, ,$$

where ρ_a denotes changes in average productivity over the life cycle of the firm. This process is parameterized such that average productivity grows at the rate γ_z during the first 25 years and is constant afterwards. γ_z is set to 0.0045 to target the growth rate in profits of firms between the ages 8 and 28. Average log productivity at the time of entry is normalized to zero. The error term, ξ_a, is normally distributed with standard deviation σ_z which is set to 0.05 to target the variance of the log of sales of firms older than 30 years. The autocorrelation of productivity, ρ_z, is set to 0.985 to target the autocorrelation of sales among firms older than 30 years.

The evolution of intangible capital is modeled as follows

$$\chi_{a+1} = (1 - \delta)\chi_a + \nu l_{\chi a}^{\phi} \, .$$

Table 3: Calibration

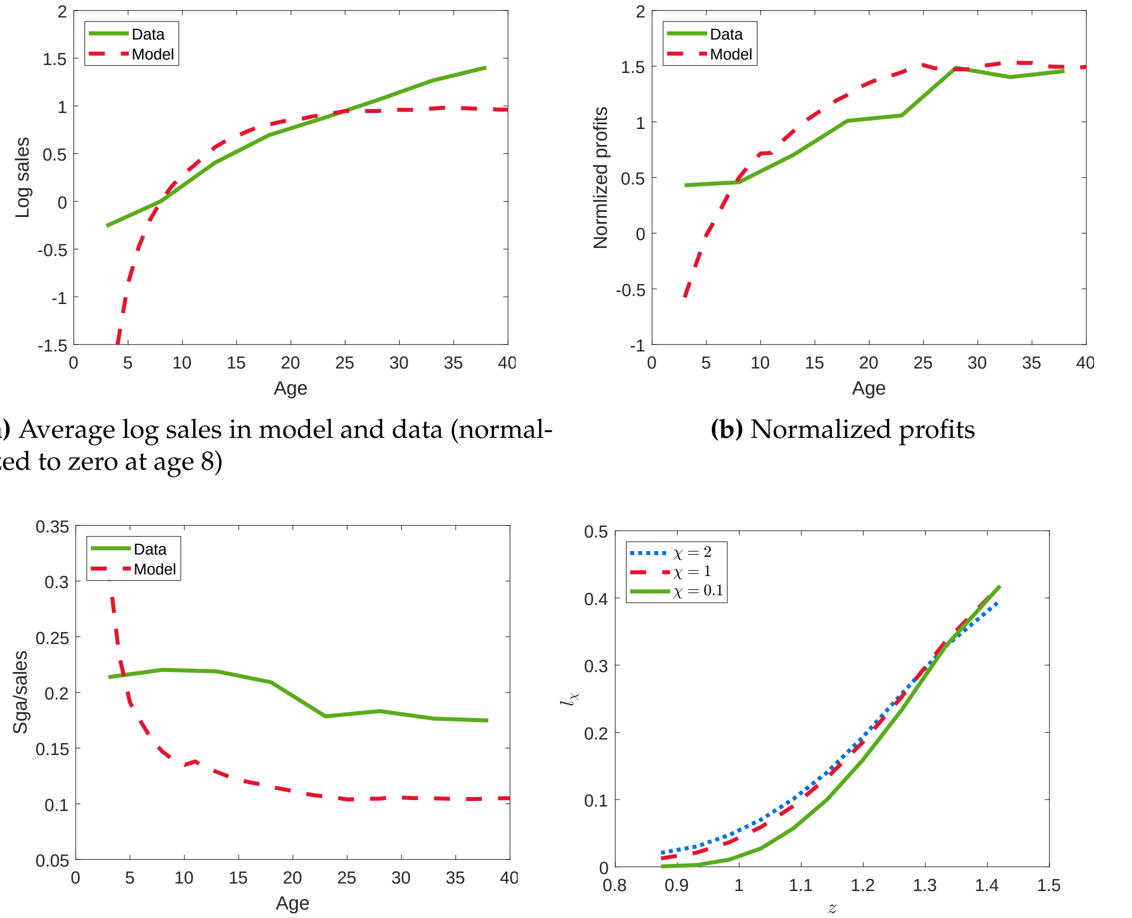
Parameter	Description	Value	Target moment	Data	Model
ψ	Sensitivity to intangible capital	0.245	Difference log of sales between ages 8 and 28	1.06	1.00
σ	Elasticity of demand	5.65	Cost-weighted avg markup (De Loecker et al., 2020)	1.26	1.25
γ_z	Growth rate productivity	0.0045	Difference profits between ages 8 and 28	1.03	1.02
ρ_z	Autocorrelation productivity	0.985	Autocorrelation sales	0.99	0.99
σ_z	Std dev shocks to productivity	0.05	Variance log of sales	2.49	2.47
l_o	Overhead cost	0.0173	Profit share (Van Vlokhoven, 2019)	0.058	0.058
l_e	Entry cost	0.147	Mass of firms	1	1
ν	Productivity marketing	1.036	Output	1	1
δ_a	Probability of exit by age	—	Exit rate (Business Dynamics Statistics)	—	—
β	Discount rate	0.96	Externally calibrated	—	—
δ	Depreciation rate intangibles	0.3	Externally calibrated	—	—
ε	Super-elasticity	1	Externally calibrated	—	—
η	Returns to scale production	1	Externally calibrated	—	—
ϕ	Returns to intangible investment	0.5	Externally calibrated	—	—

Notes: if not indicated otherwise, all moments refer to Compustat data between 1980 and 2000. Sales and profits by age are calculated using a 5-year average in both data and model. Thus, sales and profits at age 8 refer to average sales and profits between the ages of 6 and 10. The autocorrelation and variance of sales are calculated for firms older than 40 years.

Past intangible capital caries over to the current period but depreciates over time at the rate δ. The depreciation rate is exogenously set at 0.3. The returns to scale of the intangible production function are denoted by ϕ and this parameter is exogenously set to 0.5. The efficiency of intangible investment is denoted by ν which is set to 1.04 in order to normalize the size of the economy, Y, to 1. Furthermore, it is assumed that firms enter the market with an intangible capital stock of zero. The extent to which intangible capital or brand value is valued by consumers, ψ, is calibrated targeting the growth of sales between the ages of 8 and 28. The larger ψ, the larger the incentives to investing in intangible capital and therefore the larger older firms are as they have had more time to accumulate intangible capital. If find a ψ of 0.245.

I calibrate the elasticity of demand, σ, to target the cost-weighted average markup in 1990 as estimated by De Loecker et al. (2020). This gives an elasticity of 5.65. As is shown above, the super-elasticity, ε/σ, governs the relationship between markups and firm size measured in real terms. However, as I do not observe real quantities I cannot estimate this parameter directly in this way. An alternative is to use the relationship between the profit share and firm size. However, during the period before 2000 the relationship between the profit share and firm size is limited which would imply a super-elasticity of close to zero, while after 2000 a strong relationship emerges. Therefore, I exogenously set ε to 1. This gives a super-elasticity of 0.18 which is close to the value of 0.16 that is found by Edmond et al. (2018).

Overhead costs are calibrated to target the profit share gross of entry costs as estimated in Van Vlokhoven (2019) and entry costs are such that the mass of firms is equal to 1. This gives that labor needed for fixed overhead each period equals 0.0173 and labor needed to set up a firm equals 0.147. Total labor supply is normalized to 1. The probability of exit is taken from the firm exit rate as reported in the Business Dynamics Statistics. In the first year, the exit rate is 0.19 after which it slowly declines to 0.05 in the eleventh year after



(a) Average log sales in model and data (normalized to zero at age 8)

(b) Normalized profits

(c) Sales-weighted average sga/sales in model and data

(d) Labor used for intangible investment l_χ as a function of productivity z and current intangible stock χ for old firms

Figure 7: Model moments

which I set it constant at 0.05. Finally, the returns to scale of production are set to 1.[12] An overview of the calibration values is given in Table 3. The model matches the targeted variables well.

Figure 7a shows that the model overestimates sales growth for the youngest ages while it underestimates sales growth for older ages. The reason is that the youngest firms that are in Compustat are not representative of the typical young firm. They are likely to be larger than the typical firm. Furthermore, in the model firms stop growing once productivity is no longer increasing exogenously and firms have reached their equilibrium value of intangible capital. Instead, in the data firms continue to grow. One explanation for this difference is that the model captures single-product firms while in the data firms might expand into new product lines as they grow older. Figure 7b shows how profits in the model compare to the data. This figure shows profits normalized such that average

[12]In practice, setting the returns to scale equal to 1 gives some numerical problems as some firms grow extremely large. To bound the firm distribution, I set the returns to scale close to but just below 1: i.e., to $1 - 10^{-12}$.

profits during the first 40 years are equal to 1 in both data and model. In the calibration I target the change in these normalized profits between the ages of 8 and 28. Also here, the youngest firms make less profits in the model than in the data. However, different from sales, profits do not increase for the older firms in the data which is captured by the model. As it is for the purposes here more relevant to match profits than size over the life cycle I choose to let productivity only grow until the age of 25.

Figure 7c shows how overhead expenditure as a share of sales varies over the life cycle of the firm which is non-targeted. The sales-weighted average is plotted. In the model, overhead expenditure refers to fixed overhead costs wl_o plus intangible investments wl_χ while in the data it refers to selling, general and administrative costs (sga). Overhead costs are larger in the data than in the model. This could partly be because firms have freedom in deciding which costs to classify as sga and, therefore, some production costs might be included in sga. However, both model and data display a negative relationship between firm age and overhead expenditure relative to sales, and the relationship becomes flat for older firms. The difference between model and data is that for young firms the model yields much larger overhead expenditure relative to sales. This is mainly due to young firms in the model being relatively small.

Finally, Figure 7d shows the complementarity between productivity and intangible investment. Productive firms invest more in intangibles than unproductive firms. This relationship holds independent of the current level of intangibles. Furthermore, the low-productive firms invest almost nothing in intangibles. As discussed above, this is because the level of profits is more responsive to intangible capital when productivity is large.

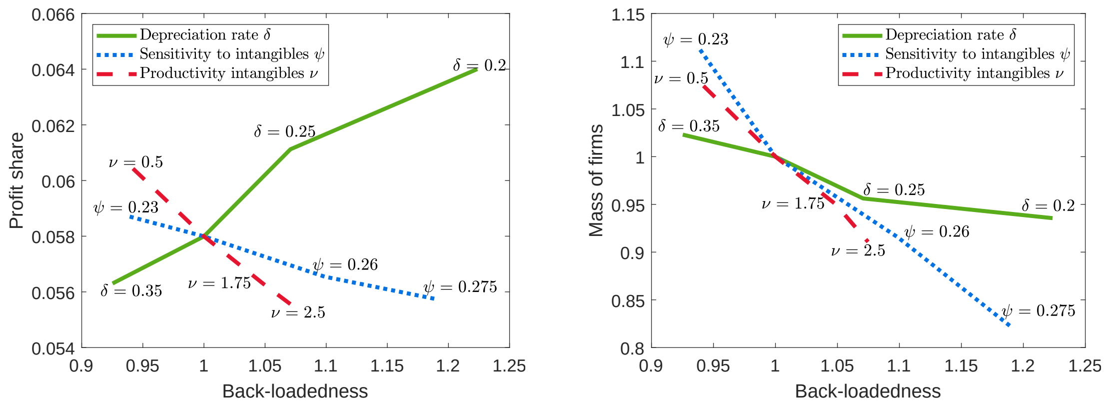
V Results

I will vary three different model parameters that are related to intangibles in order to study the effect of back-loadedness on the profit share and the mass of firms. The first parameter I vary is the depreciation rate of intangible capital. The lower the depreciation rate the higher the stock of intangible capital that firms will accumulate eventually. This favors older firms and therefore leads to profits becoming more back-loaded. The green solid line in Figure 8a shows the relationship between back-loadedness and the profit share when the depreciation rate varies between 0.2 and 0.35, and all other parameter values do not change. When the depreciation rate falls to 0.2, back-loadedness increases by 22% compared to the baseline while the profit share increases from 5.8% to 6.4%. On the other hand, when the depreciation rate increases from 0.3 to 0.35 back-loadedness falls by 7.5% and the profit share drops to 5.6%. Thus, varying the depreciation rate yields a positive relationship between back-loadedness and the profit share as predicted by Theorem 2. However, this relationship flips sign when it is the sensitivity to intangible capital, ψ, in the Kimball aggregator that is changing. The more sensitive consumers are to intangible capital the more firms will invest in intangible capital. A larger sensitivity favors older firms as they have had more time to accumulate intangibles, and therefore, leads to profits becoming more back-loaded. When ψ increases from 0.245 to 0.275, back-loadedness increases by 19%, but the profit share decreases from 5.8% to 5.6%. Also when it is the productivity of intangible investment, ν, that is changing, a negative relationship between back-loadedness and profits emerges. When ν increases from 1.04 to 2.5, back-loadedness increases by 7.5% and the profit share drops to 5.5%. Figure 8b shows that varying each parameter yields a negative relationship between back-loadedness and the number of firms.

Why does a negative relationship emerge between back-loadedness and the profit share when changes in back-loadedness are driven by changes in the sensitivity to intangible capital and productivity of intangible investment? Let's first focus on changes in ψ and for now assume that firms do not change their intangible investment in response. This makes that an increase in ψ is favorable for old firms. Young firms have a relatively low intangible capital stock and therefore their market share and markup will decrease as ψ increases. On the other hand, older firms have a larger intangible capital stock and will therefore increase their market share and markup in response. This shifts profits from a young to an old age and therefore makes that profits become more back-loaded. The intuition behind Theorem 2 is that as a result of this, the value of entry falls below the cost of entry and therefore entry will go down. Due to less competition, the markup and aggregate profits will increase. Also with fewer firms, the fixed overhead costs have to be paid by fewer firms which increases aggregate profits. However, in response to the increased sensitivity to intangible capital firms will also increase their investment in intangibles, which lowers aggregate profits. It turns out that the latter effect dominates the direct effect on profits from a decline in the number of firms. Total expenditure on fixed overhead costs and intangible investment increases by 1.1 percentage points as a share of sales when $\psi = 0.275$ compared to the baseline while the fall in the profit share is only 0.2 percentage points. Thus, the rise in overhead costs is responsible for the fall in profits in this case. Figure 8b shows that entry goes down substantially and the measure of firms decreases by 18% as ψ increases from 0.245 to 0.275. This sharp drop is such that firms can increase their market share and are able to recover their increased overhead expenditure.

Varying ν also leads to a negative relationship between back-loadedness and the profit share but for different reasons. An increased productivity of intangible investment leads to a larger stock of intangibles which in turn changes the firm size distribution. Due to the markup being related to firm size, changes to the firm size distribution affect the aggregate markup. In this case it leads to a lower markup. Thus, despite that there are fewer firms, aggregate markups are declining which leads to a decline in the profit share. Total expenditure on intangibles and fixed overhead as a share of sales does not depend much on ν. Despite the fall in profits and rise in back-loadedness, the entry condition holds because firm are now larger and the average level of profits per firm therefore increases. That firms are larger is because there are fewer firms and total output increases due to the increase in productivity.

Only a fall in the depreciation rate of intangibles can explain the increase in profits



(a) Back-loadedness and the profit share **(b)** Back-loadedness and the measure of firms

Figure 8: Effect of model parameters on back-loadedness, the profit share and the mass of firms. Back-loadedness is normalized to 1 in the baseline

combined with a fall in entry. To study to what extent a change in back-loadedness can explain these trends quantitatively I set the depreciation rate such that the model matches the observed rise in back-loadedness. A depreciation rate of 0.165 matches the observed 33% rise in back-loadedness. In the model, this implies a one percentage point increase in the profit share, from 5.8% to 6.8%, while in the data, the profit share increases to 7.5%. The increase in back-loadedness thus explains a bit more than half of the observed increase in profits. Note, that this is the profit share gross of entry costs. The profit share net of entry costs increases as well; from 4.7% to 5.8%. The rise in back-loadedness also leads to a 10% decline in the number of firms which is a bit more than the observed 8% decline in the number of firms. Thus, the change in back-loadedness can more than fully explain the fall in firm entry.

The measure of back-loadedness is sensitive to the level of profits of the youngest firms and therefore, it is problematic that I only observe relatively few young firms. A more robust measure of the life cycle of profits is by how much do firm-level profits grow between the ages of eight and twenty-eight. In the data, this alternative measure of back-loadedness has grown by 56%. Matching this moment with the model implies that the depreciation rate has fallen from 30% to 13%. This gives that the mass of firms drops with 14% and that the profit share increases to 7.3%. Thus, the changing life-cycle pattern of profits has the potential to explain the rise in profits almost in its entirety.[13]

The model suggests that it is more likely that the depreciation rate of intangibles has declined than that productivity of intangibles has changed or that the sensitivity to intangibles has changed. A natural question to ask is what can explain this fall in the depreciation rate. Some options are given by the literature. For instance, a decrease in knowledge diffusion as studied by Akcigit and Ates (2019) implies that laggards are less

[13]In an earlier version of this paper I have also studied a model in which firms engage in Cournot competition à la Atkeson and Burstein (2008) but without investment in intangibles. Such a model could as well explain more than half of the rise in profits and more than fully the fall in the number of firms. See Van Vlokhoven (2020) for details.

likely to catch up with leaders. From the perspective of the leaders, this implies that their R&D investments depreciate slower. The decline in population growth as studied by Peters and Walsh (2021) suggests there is less creative destruction coming from entrants. Having a lower depreciation rate is a reduced form way of modeling less creative destruction. Finally, an increase in consumer inertia as studied by Bornstein (2018) implies that firms are less likely to lose their customers once they have attracted them. Thus, if intangibles refer to the number of customers captured by a firm, then an increase in consumer inertia refers to a decline in the depreciation rate.

VI Conclusions

This paper puts forward the hypothesis that the rise in profits and fall in entrepreneurship is associated with an increased back-loadedness of profits. Nowadays profits appear at a later stage in the life of the firm than they used to do. Young firms today make as much or less profits than young firms used to do in the past while old firms nowadays make more profits than old firms did in the past. That profits appear later in life lowers the value of the firm and therefore makes entrepreneurs less eager to enter the market. As a result, profits per firm increase such that the entry condition holds.

The increase in average profits per firm does, however, not necessarily imply that aggregate profits increase as well. It could be that the increase in profits per firm is entirely driven by firms becoming larger in terms of sales, and that the profit share does not increase. In the quantitative model that I study, this is the case when the sensitivity to intangibles increase and the productivity of intangible investment increases. When the sensitivity to intangibles increases, expenditure on overhead costs increase which offsets the increase in markups due to the decline in the number of firms. While when the productivity of intangible investment increases, the firm size distribution changes such that the aggregate markup does not change much although the number of firms declines. Only when the depreciation rate of intangibles declines the model is is able to explain both the fall in entry and rise in profits. With a lower depreciation rate, firms will decide to invest more in intangibles but as the number of firms decreases faster it is the case that overhead costs as a share of sales decline in the aggregate. This leads to a rise in aggregate profits.

Quantitatively, the changing life-cycle pattern of profits can account for about half to almost all of the rise in profits, depending on how the life-cycle pattern of profits is measured. Furthermore, it more than fully explains the decline in entrepreneurship. It is reasonable that the model can more than fully explain the fall in entry while less than all of the rise in profits. This implies that there has been another force at play that has led to an additional increase in profits. This could, for instance, be laxer antitrust regulation. An additional rise in profits would make it more attractive to become an entrepreneur and would therefore, lead to a slower decline in entrepreneurship.[14]

[14] For instance, in the monopolostic competition model, this could be modeled as a decline in the elasticitiy

References

Aghion, P., A. Bergeaud, T. Boppart, P. J. Klenow, and H. Li (2019) "A Theory of Falling Growth and Rising Rents," Working Paper 26448, National Bureau of Economic Research.

Akcigit, U. and S. T. Ates (2019) "What Happened to U.S. Business Dynamism?", Working Paper 25756, National Bureau of Economic Research.

——— (2021) "Ten Facts on Declining Business Dynamism and Lessons from Endogenous Growth Theory," *American Economic Journal: Macroeconomics*, Vol. 13, No. 1, pp. 257–298.

Atkeson, A. and A. Burstein (2008) "Pricing-to-Market, Trade Costs, and International Relative Prices," *American Economic Review*, Vol. 98, No. 5, pp. 1998–2031.

Atkeson, A. and P. J. Kehoe (2005) "Modeling and Measuring Organization Capital," *Journal of Political Economy*, Vol. 113, No. 5, pp. 1026–1053.

Barkai, S. (2020) "Declining Labor and Capital Shares," *Journal of Finance*, Vol. 75, No. 5, pp. 2421–2463.

Bornstein, G. (2018) "Entry and Profits in an Aging Economy: The Role of Consumer Intertia," Manuscript.

Cavenaile, L., M. A. Celik, and X. Tian (2021) "Are Markups Too High? Competition, Strategic Innovation, and IndustryDynamics," Available at SSRN 3459775.

Chatterjee, S. and B. Eyigungor (2020) "The Firm Size and Leverage Relationship and Its Implications for Entry and Business Concentration," FRB of Philadelphia Working Paper No. 20–29.

De Loecker, J., J. Eeckhout, and S. Mongey (2021) "Quantifying Market Power and Business Dynamism in the Macroeconomy," Working Paper 28761, National Bureau of Economic Research.

De Loecker, J., J. Eeckhout, and G. Unger (2020) "The Rise of Market Power and the Macroeconomic Implications," *Quarterly Journal of Economics*, Vol. 135, No. 2, pp. 561–644.

De Ridder, M. (2019) "Market Power and Innovation in the Intangible Economy," Cambridge-INET Working Paper Series No: 2019/08.

of substitution, σ, between varieties. This would lead to an increase in the markup for all firms. As a response, firm entry would increase such that market shares decline, until the entry condition holds again. The profit share increases due to the rise in markups.

Decker, R., J. Haltiwanger, R. Jarmin, and J. Miranda (2014) "The Role of Entrepreneurship in US Job Creation and Economic Dynamism," *Journal of Economic Perspectives*, Vol. 28, No. 3, pp. 3–24.

Edmond, C., V. Midrigan, and D. Y. Xu (2018) "How Costly are Markups?", Working Paper 24800, National Bureau of Economic Research.

Engbom, N. (2019) "Firm and Worker Dynamics in an Aging Labor Market," FRB of Minneapolis Working Paper No. 756.

Field, L. C. and J. Karpoff (2002) "Takeover Defenses of IPO Firms," *Journal of Finance*, Vol. 57, No. 5, pp. 1857–1889.

Griliches, Z. and J. Mairesse (1984) "Productivity and R&D at the Firm Level," in *R&D, Patents, and Productivity*: National Bureau of Economic Research, Inc, pp. 339–374.

Gutiérrez, G., C. Jones, and T. Philippon (2021) "Entry Costs and Aggregate Dynamics," Manuscript.

Hall, R. E. and D. W. Jorgensen (1967) "Tax Policy and Investment Behavior," *American Economic Review*, Vol. 57, No. 3, p. 391–414.

Hopenhayn, H., J. Neira, and R. Singhania (2018) "From Population Growth to Firm Demographics: Implications for Concentration, Entrepreneurship and the Labor Share," Working Paper 25382, National Bureau of Economic Research.

Jorgenson, D. W. and M. A. Sullivan (1981) "Inflation and Corporate Capital Recovery," in *Depreciation, Inflation, and the Taxation of Income from Capital* ed. by C. R. Hulten, pp. 171–238, 311–313: Washington: The Urban Institute Press.

Jorgenson, D. W. and K.-Y. Yun (1991) *Tax Reform and the Cost of Capital*: Oxford University Press.

Jovanovic, B. and P. L. Rousseau (2001) "Why wait? A century of life before IPO," *AEA Papers and Proceedings*, pp. 336–341.

Karahan, F., B. Pugsley, and A. Şahin (2019) "Demographic Origins of the Startup Deficit," Working Paper 25874, National Bureau of Economic Research.

Kimball, M. S. (1995) "The Quantitative Analytics of the Basic Neomonetarist Model," *Journal of Money, Credit, and Banking*, Vol. 27, No. 4, pp. 1241–1277.

Klenow, P. J. and J. L. Willis (2016) "Real Rigidities and Nominal Price Changes," *Economica*, Vol. 83, No. 331, pp. 443–472.

Liu, E., A. Mian, and A. Sufi (forthcoming) "Low Interest Rates, Market Power, and Productivity Growth," *Econometrica*.

Loughran, T. and J. R. Ritter (2004) "Why Has IPO Underpricing Changed Over Time?" *Financial Management*, Vol. 33, No. 3, pp. 5–37.

Peters, M. and C. Walsh (2021) "Population Growth and Firm Dynamics," Manuscript.

Van Vlokhoven, H. (2019) "Estimating the Cost of Capital and the Profit Share," Available at SSRN 3509638.

——— (2020) "On the Cost of Capital, Profits and the Diffusion of Ideas," Doctoral dissertation, Department of Economics, Stockholm University.

Weiss, J. (2020) "Intangible Investment and Market Concentration," Manuscript.

Appendix A Estimating Profits

Profits are not directly observed as capital costs are not directly observed. In my baseline estimate I follow the method developed in Van Vlokhoven (2019) to estimate capital costs and hence profits. Rewriting the profits accounting identity gives that nominal output, $P_i Y_i$, of a firm i is equal to the price-average cost ratio, ψ_i, times total costs,

$$P_i Y_i = \psi_i \left(P_i^X X_i + R_i \cdot P_i^K K_i \right) , \tag{8}$$

where total costs consist of operating expenses, $P_i^X X_i$, and capital costs, $R_i \cdot P_i^K K_i$. Nominal output, operating expenses, and the nominal capital stock $P_i^K K_i$ are directly observed. To estimate the user cost of capital, R, I divide both sides of this equation by operating expenses and bring this equation to a regression framework:

$$\frac{P_i Y_i}{P_i^X X_i} = \overline{\psi} + \overline{\psi R} \frac{P_i^K K_i}{P_i^X X_i} + \varepsilon_i \, . \tag{9}$$

ε_i denotes the error term which consists of heterogeneity in the price-average cost ratio and the cost of capital. The intercept coefficient in this equation refers to the average of the price-average cost ratio, and the slope coefficient refers to the average of the price-average cost ratio times the cost of capital. This procedure identifies the cost of capital by dividing the slope coefficient by the intercept coefficient when the cost of capital and price-average cost ratio are uncorrelated with the ratio of the capital stock to operating expenses. Van Vlokhoven (2019) discusses identification in detail. As in Van Vlokhoven (2019), I control for variation in the price-average cost ratio and the cost of capital across firms. In particular, I control for the leverage ratio (i.e., liabilities divided by assets), the amount of long-term debt relative to total liabilities, interest expenses relative to total liabilities, the depreciation rate, lagged sales, growth of sales, and risk measured by the standard deviation of sales within a firm. As these controls are controls for both the intercept and slope coefficient I include them both in levels and interacted with the regressor. In addition, I also allow the cost of capital to depend linearly on the capital-input ratio. This yields the following regression, where Z_i^j refers to the j-th control and $\overline{Z}^j$ to the average of each control:

$$\begin{aligned} \frac{P_i Y_i}{P_i^X X_i} =& \overline{\psi} + \sum_j \upsilon_j \left(Z_i^j - \overline{Z}^j \right) + \\ & \left(\overline{\psi R} + \sum_j \gamma_j \left(Z_i^j - \overline{Z}^j \right) + \zeta \left(\frac{P_i^K K_i}{P_i^X X_i} - \overline{\frac{P^K K}{P^X X}} \right) \right) \frac{P_i^K K_i}{P_i^X X_i} + \varepsilon_i \, . \end{aligned} \tag{10}$$

I run this regression by year and industry (at the two digit level). The coefficients of interest are $\overline{\psi}$ and $\overline{\psi R}$ which denote the price-average cost ratio and the price-average cost ratio times the cost of capital for each industry-year. Dividing these two with each other gives the estimated cost of capital, $\widehat{R}$, by industry and year. Profits are then simply

$$\widehat{\text{Profits}}_i = P_i Y_i - P_i^X X_i - \widehat{R} \cdot P_i^K K_i \,.$$

It is also possible to use the controls to estimate a firm-level cost of capital. Doing so yields similar results for the profits-age relationship.

Required rate of return approach An alternative method to estimate the cost of capital is the required rate of return approach (Hall and Jorgensen, 1967). This method uses that, according to theory, the cost of capital is identical to the expected real interest rate plus the depreciation rate. For instance, Barkai (2020) uses this method to estimate the evolution of the capital share and the profit share. I use this method as a robustness to calculate the profits-age relationship. In particular, I use

$$\widehat{R} = \mathbb{E}\left[\frac{D}{D+E} i^D (1-\tau) + \frac{E}{D+E} i^E + \delta - \pi^k\right] \frac{1 - itc - z\tau}{1-\tau} \,. \tag{11}$$

For the debt cost of capital, i^D, I take the realized yield on AAA corporate bonds and for the equity cost of capital, i^D, I take the realized yield on ten-year government bonds plus a 5% premium. These yields are obtained from FRED, Federal Reserve Bank of St. Louis. D is equal to total liabilities and E (equity) is the number of common shares outstanding times the closing price. Data on the corporate tax rate, τ, is taken from Jorgenson and Yun (1991) for the period until 1986, and from the OECD tax database for the period thereafter. The present value of capital consumption allowances for tax purposes, z, is taken from the tax foundation. I impute the values post-2012 with the 2012 value. Values of the investment tax credit, itc, come from Jorgenson and Sullivan (1981). Expected inflation of the investment good, π^k, is assumed to be equal to realized inflation of the investment good of the non-financial corporate sector (obtained from the BEA). And the depreciation rate, δ, is the depreciation rate as reported in Compustat.[15]

Data

This paper uses Compustat data.[16] To focus on US firms, I keep firms that are incorporated in the United States and report in US dollars. I drop observations for which there is a missing or negative observation for sales, operating expenses or the capital stock. I drop firms in the industries mining, finance or utilities, or for which the industry is not classified. For nominal output I use the variable SALE and for operating expenses the variable XOPR. For the capital stock, I use the book value of the capital stock, namely, property, plant and equipment net of depreciation ("PPENT"). Since this variable is recorded as the end of period stock, I use the lagged value. Furthermore, this variable represents tangible

[15] The depreciation rate is calculated using that under the assumption of a geometric depreciation rate $\delta_t = 1 - \left(\frac{\text{Net capital}_t}{\text{Gross capital}_t}\right)^{1/\text{age}_t}$, where the age of the capital stock is measured as accumulated depreciation divided by the flow value of depreciation.

[16] Downloaded on 17 June 2022.

capital, but some firms also include externally purchased intangible capital in this variable while some other firms report externally purchased intangibles separately. For the latter firms I add these intangibles to the capital stock. Internally developed intangibles do not appear on the balance sheet and the costs of internally developing intangibles are part of operating expenses. However, fully expensing these costs might not be appropriate, and therefore, I also capitalize R&D expenditure and add this to the capital stock in a robustness exercise. To calculate the internally developed intangible capital stock, K^I_{t+1}, I capitalize R&D expenditure as follows

$$K^I_{t+1} = (1 - \delta^I)K^I_t + R\&D_t \,,$$

where $R\&D_t$ is the expenditure on R&D.[17] I set the depreciation rate, δ^I, equal to 15% (Griliches and Mairesse, 1984). Finally, I subtract R&D expenditure from operating expenses to avoid double counting. Including internally developed intangibles in the capital stock leads to similar results for the profits-age relationship.

Appendix B Firm Entry

It is well-known that firm entry has been declining over time. One way of illustrating this is by looking at the number of firms relative to the number of employees in the private sector. Figure 9 shows that the share of entrepreneurs has been declining by around half a percentage point from just below 5% in the early 1980s to just below 4.5% nowadays.

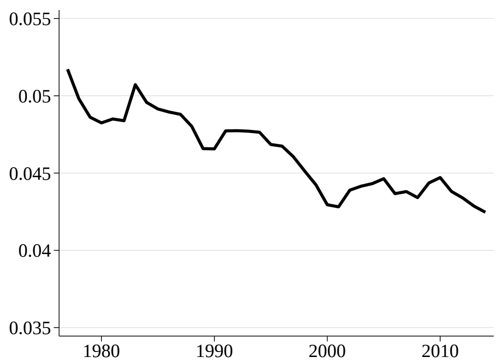
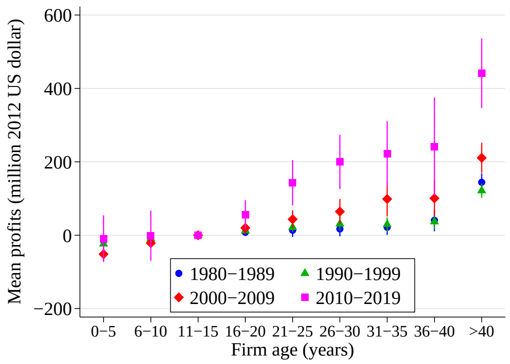


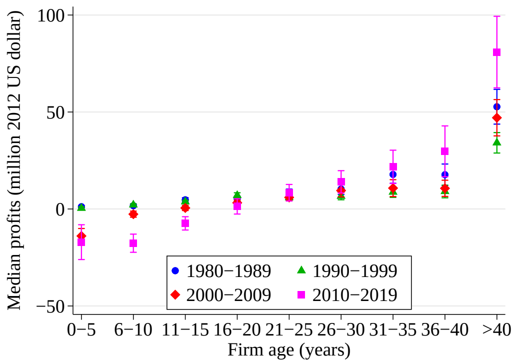
Figure 9: The number of firms relative to the number of employees in the private sector. Source: Business Dynamics Statistics (US Census Bureau).

[17]Given that most of these firms have been founded several years before they enter the Compustat data, it is unlikely that they enter with an intangible capital stock of zero. Therefore, I use as starting value for the intangible capital stock of each firm R&D expenditure divided by δ^I. The underlying assumption is that the firm is in steady state. Furthermore, I set R&D expenditure equal to zero if it is negative in a period, and I interpolate when R&D expenditure is missing.

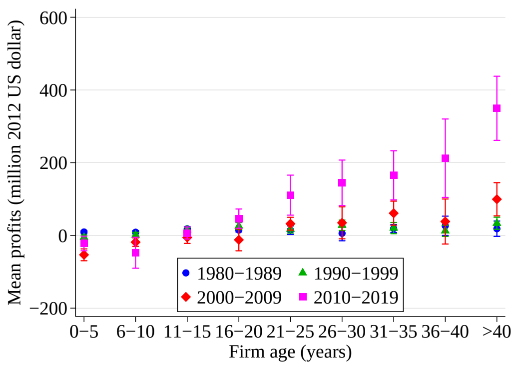
Appendix C Additional Figures



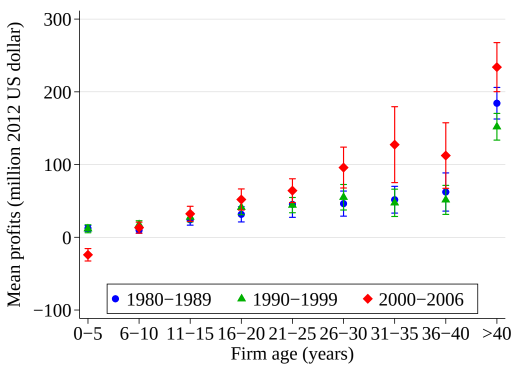
(a) Coefficients of a regression with age, and year times industry fixed effects are plotted. The third age bin is the reference category



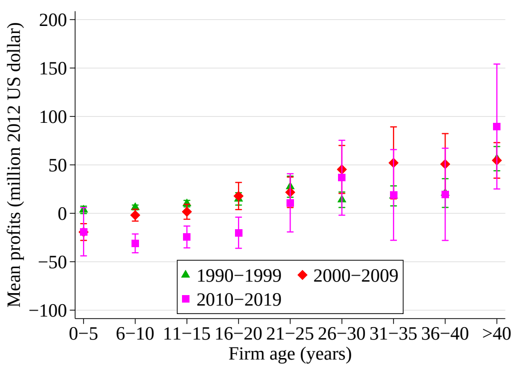
(b) Median



(c) R required rate of return approach

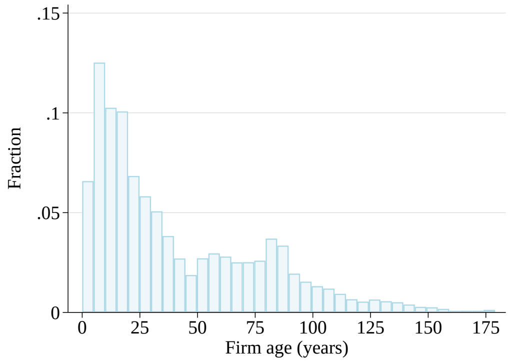


(d) Excluding the great recession

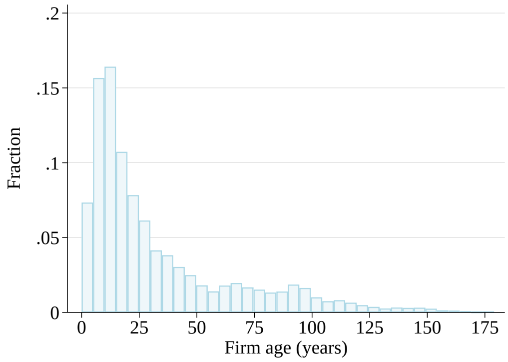


(e) Excluding firms with M&A deals

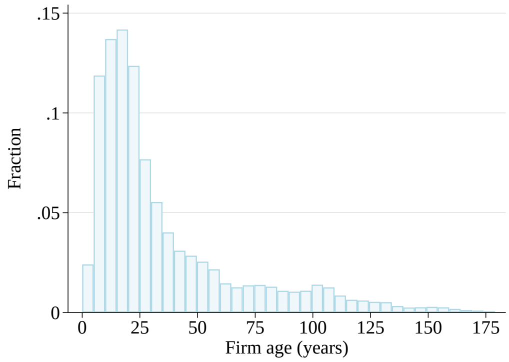
Figure 10: Life cycle of profits - robustness



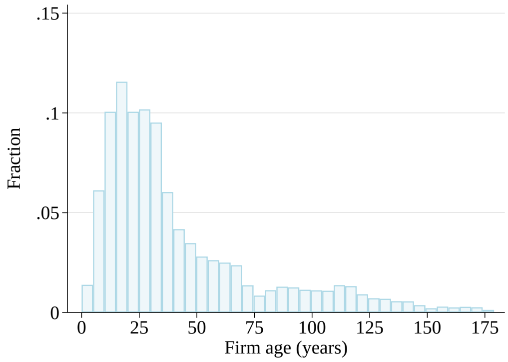
(a) 1980s



(b) 1990s

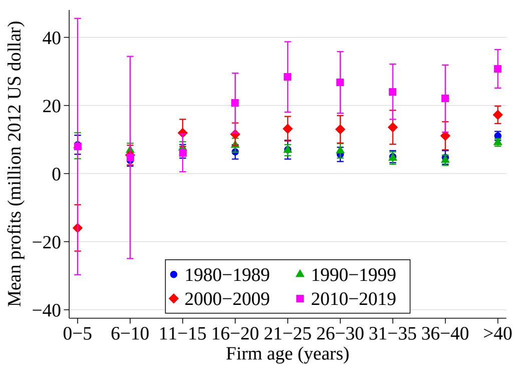


(c) 2000s

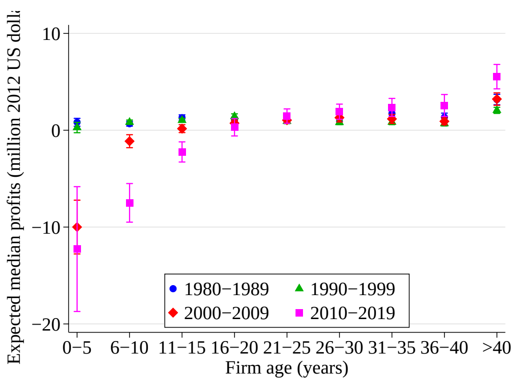


(d) 2010s

Figure 11: Histogram age distribution by decade

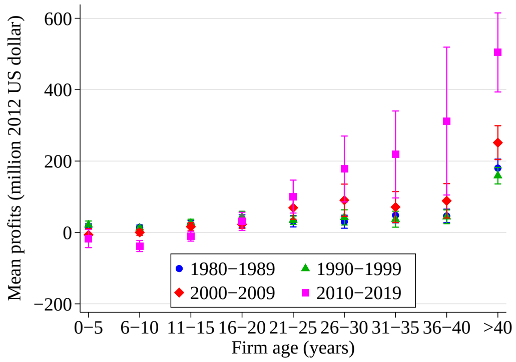


(a) Expected average profits

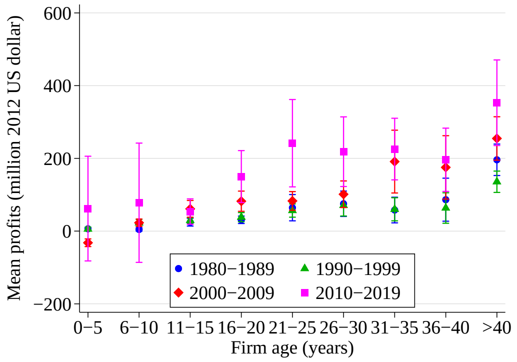


(b) Expected median profits

Figure 12: Expected profits over the life cycle by decade



(a) Manufacturing



(b) Non-Manufacturing

Figure 13: Life cycle of profits by sector

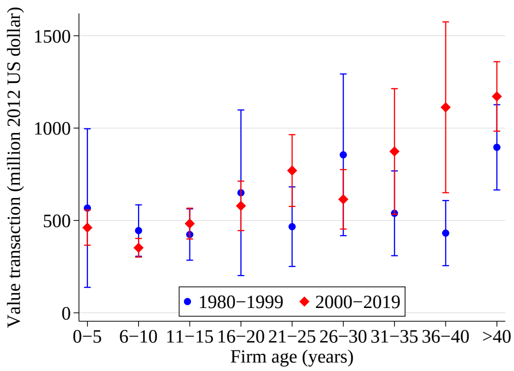


Figure 14: Deal value of acquisitions by age of target firm

Source: Securities Data Company Platinum.
Notes: The plot shows the deal value by age of the target firm. The deal value is calculated as the reported deal value divided by the shares acquired in order to reflect firm values. I only analyze the first transaction for each firm in which more than 50% of the shares of that firm are acquired.

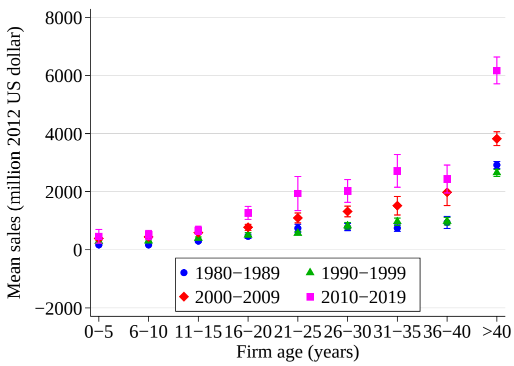


Figure 15: Sales over the life cycle

Table 4: Effect of age on profits with firm fixed effects

	(1) Profits	(2) Profits
Age	4.541***	5.171***
	(1.496)	(1.494)
1990s × age	0.346	0.653*
	(0.267)	(0.275)
2000s × age	1.195*	1.576**
	(0.509)	(0.497)
2010s × age	2.973**	3.350***
	(0.854)	(1.006)
Firm FE	X	X
Year FE		X
Observations	82727	82727
R^2	0.584	0.586

Standard errors clustered at the firm level in parentheses. Profits refer to real profits in millions of 2012 US dollars, deflated using the GDP deflator. Age is denoted in years. The left out category for the decade times age interaction is the 1980s.

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Appendix D Solution Algorithm

I solve for the equilibrium as follows. Suppose we know the joint distribution of z and χ. How to calculate the output and price of each firm? Divide the static firm optimization problem by DY to obtain

$$\max_{q\geq 0} \chi^{\psi}\Upsilon'(q)\, q - \left(\frac{q}{z}\right)^{1/\eta} \underbrace{\frac{w}{D} Y^{\frac{1-\eta}{\eta}}}_{A}, \tag{12}$$

where q equals the market share $\frac{y(z,\chi)}{Y}$ and A is the aggregate state. This leads to the following first-order condition:

$$\chi^{\psi}\Upsilon''(q)\, q + \chi^{\psi}\Upsilon'(q) = \frac{A}{\eta} q^{\frac{1-\eta}{\eta}} z^{-1/\eta}. \tag{13}$$

That the markup equals $\frac{\Upsilon'(q)}{\Upsilon'(q)+q\Upsilon''(q)}$ gives $\Upsilon''(q) = \frac{1-\mu}{\mu q}\Upsilon'(q)$. Plugging this into the first-order condition gives that

$$\chi^{\psi}\Upsilon'(q) = \mu(q)\frac{A}{\eta} q^{\frac{1-\eta}{\eta}} z^{-1/\eta}.$$

And using the Klenow-Willis aggregator gives

$$\frac{\sigma-1}{\sigma}e^{\frac{1-q^{\varepsilon/\sigma}}{\varepsilon}} = \frac{1}{1-\frac{1}{\sigma}q^{\varepsilon/\sigma}}\frac{A}{\eta}q^{\frac{1-\eta}{\eta}}z^{-1/\eta}\frac{1}{\chi^{\psi}}\,.$$

Thus, q and, therefore, μ are a function of $z^{1/\eta}\chi^{\psi}$. Using the above equation we can solve for q as a function of A and $z^{1/\eta}\chi^{\psi}$. Then, we can calculate the equilibrium value of A using the Kimball aggregator

$$\int \chi^{\psi}\Upsilon\left(q(z,\chi,A)\right)dH(z,\chi) = 1\,.$$

This gives us the market share $q(z,\chi) = q(z,\chi,A)$ and the markup. To aggregate this economy let $Y = ZL_p^{\eta}$, where L_p is aggregate labor used in production; $L_p = \int l_p(z,\chi)dH(z,\chi)$.[18] Raising aggregate labor to the power η gives after rewriting the following expression for aggregate productivity

$$Z = \left(\int\left(\frac{q(z,\chi)}{z}\right)^{1/\eta}dH(z,\chi)\right)^{-\eta}. \tag{14}$$

Given that we know $q(z,\chi)$ we can calculate Z using this formula. In addition, if we know how much labor is used for intangible investment we can calculate aggregate output Y using the aggregate production function and how much labor is left for production. Knowing q and Y we can calculate firm-level output and profits.

The above algorithm assumes that we know the joint distribution of z and χ, the amount of intangible labor used and the mass of firms $N = \int dH(z,\chi)$. Given a guess for the aggregate state (i.e., Y, A and N), the policy function $l_\chi(z,\chi)$ can be solved. Given a distribution of productivity and intangible capital at the time of firm entry, this gives the joint distribution of z and χ. I iterate until convergence and the entry condition holds.

[18] Note that I use the notation $H(\cdot)$ interchangeably for the measure $H(z,\chi)$ and $H(z,\chi,a)$.