Strategic use of social media influencer marketing

Manuel Foerster* Tim Hellmann† Fernando Vega-Redondo‡

April 12, 2023

Preliminary draft

Abstract

We set out a model of social media influencer marketing in which a firm may hire influencers to inform consumers about an innovation. Influencers generate sales through purchases of their followers and followers' social networks and set prices for their endorsements. In turn, the firm decides which influencers to hire, which message to convey via the influencers, and sets the retail price of the innovation. In equilibrium, influencers price according to their relative contribution to industry profits. Exploiting naïve consumers, they provide exaggerated endorsements that allow the firm to overprice if product quality is low. Furthermore, we show that the firm may be better off if it could commit to hire fewer influencers.

JEL classification: D21, L11, L15, C72, M30.

Keywords: Strategic product marketing, social media influencer, innovation, social networks, competition.

*Center for Mathematical Economics, Bielefeld University, Germany, email: manuel.foerster@uni-bielefeld.de.

†Department of Economics, University of Southampton, UK, email: t.hellmann@soton.ac.uk.

‡Department of Decision Sciences, Bocconi University, Italy, email: fernando.vega@unibocconi.it.

"People do not buy goods and services. They buy relations, stories, and magic."

Seth Godin

1 Introduction

Over the last decade, social media platforms have become increasingly important, both in people's lives and for society more broadly. While today most adults use social media, these platforms are particularly popular among young people, who often visit them several times a day.[1] This has lead to the emergence of *social media influencers*, who create certain content (videos, podcasts, etc.) often focussed on a specific topic. Many people "follow" influencers they do not know personally, but enjoy to be entertained by their "stories". Influencers' increasing popularity has quickly attracted the interest of the marketing industry, which traditionally has relied on celebrities to advertise new products. Today they often do not just share bits and pieces of their private life, but also advertise or "recommend" products.[2] Many influencers are paid for these recommendations —a fast-growing market whose global market value has more than doubled since 2019, reaching $16.4 billion (Statista Influencer Marketing Hub, 2022).

In this paper, we build a model of social media influencer marketing to understand the effects of competition between influencers on prices and the quality of influencers' endorsements. A firm may hire influencers to inform consumers about and, in particular, endorse an innovation. In equilibrium, influencers price according to their relative contribution to industry profits. Exploiting "naïve" consumers, they provide exaggerated endorsements that allow the firm to overprice if product quality is low.

In our model there is one firm that sells an innovation with unknown quality to a large set of consumers. A subset of consumers follows social media influencers, who may inform their followers of the innovation by publishing a message about the true quality of the innovation; exaggerating quality is possible but comes at the cost of reputation. After the influencers have announced their pricing policies, the firm decides which influencers to hire, which message to convey via the influencers, and sets the retail price of the innovation. Note that a key difference to traditional Bertrand competition is that the firm may hire multiple influencers. Influencers'

[1] According to a recent survey by the Pew Research Center (2021), 72% of US adults use social media (84% for age 18-29). Furthermore, more than half of Instagram and Snapchat users ages 18 to 29 visit the platform several times a day.

[2] See, e.g. Hudders et al. (2021) for an overview.

endorsements generate sales both directly through purchases of their followers and indirectly through subsequent word-of-mouth communication within followers' social networks. We assume that sales are lower if influencers exaggerate quality and the firm accordingly charges a too high price, because sophisticated consumers will not be persuaded into buying the innovation in this case.

We allow influencers to coordinate their pricing policies, which implies that industry profits will be maximized in equilibrium.[3] In a first step, we then show that influencers provide exaggerated endorsements, i.e. overstate product quality, if and only if the true product quality is low. In this case, truth-telling and pricing accordingly would yield low revenue, such that the increase in revenue from exaggeration along with a higher price outweighs its (low) costs. Influencers thus exploit naïve consumers, who are persuaded by exaggerated endorsements into buying the innovation at a too high price.

Second, we refer to an influencer as *essential* if maximal industry profit may be attained only if she is hired. Essential influencers are those with the largest number of followers if reputation costs do not increase too fast in the number of followers, which appears to be a natural assumption.[4] We show that in equilibrium influencers price according to their relative contribution to industry profits, and obtain a positive profit if and only if they are essential. Each influencer can charge at most her marginal contribution to industry profits, which implies that only essential influencers obtain positive profits in equilibrium. In particular, if reputation costs do not increase too fast in the number of followers, then influencers' market power increases in their number of followers. In turn, the firm's market power increases with the intensity of competition between influencers, as more intense competition decreases influencers' marginal contribution to industry profits.

Finally, we illustrate in case of two influencers that the firm may benefit from committing to hire less influencers. Given that hiring both influencers would leave the firm with only a weak outside option (of hiring no influencer at all), the loss in revenue from hiring only one influencer is more than compensated by the stronger outside option, which allows to retain more of the revenue.

Our paper contributes to the recent literature on marketing in social networks. Galeotti and Goyal (2009) have been one of the first to investigate the effects of social networks on firm profits. Subsequently, many papers have studied monopolistic

[3]Formally, we consider strong subgame perfect equilibria, which are stable with respect to coalitional deviations.

[4]Otherwise, if reputation costs would increase fast in the number of followers, influencers with less followers could potentially generate higher profits from exaggerated product endorsements.

pricing in social networks, e.g. Candogan et al. (2012); Bloch and Quérou (2013); Campbell (2013); Chatterjee and Dutta (2016); Fainmesser and Galeotti (2016); Leduc et al. (2017); Ajorlou et al. (2018). Our approach differs from these papers in that we introduce intermediaries, the social media influencers, which the monopolist can hire in order for them to inform consumers in a social network, potentially exaggerating product quality.

This relates our paper to contemporary work by Fainmesser and Galeotti (2021), who consider many marketers and many influencers but neither a social network nor naïve consumers. They focus on the trade-off between paid and organic endorsements, and show that policies that make paid endorsements more transparent may have negative welfare effects, whereas better search technology that matches followers to influencers has positive welfare effects.

Furthermore, social media influencers can be interpreted as media outlets, which relates our paper to the literature on the political economy of mass media and media bias (Prat and Strömberg, 2013; Anderson et al., 2015). Similar to Gentzkow and Shapiro (2006) and Foerster (2023), misrepresenting information comes with reputational costs in our model. Different to these papers, however, we explicitly model a firm that sells a product to consumers and focus on how the firm strategically uses influencers to market the product.

The paper is organized as follows. In Section 2 we present the model and notation. We discuss a micro-foundation with respect to consumer behavior and word-of-mouth communication in Section 2.1 and 2.2, respectively. Section 3 solves the game backwards. We first determine firm behavior in Section 3.1 and then determine influencer pricing in Section 3.2; in Section 3.3, we illustrate the results in case of two influencers. Section 4 concludes.

2 Model

We consider a monopolistic *firm* f, a finite set of *social media influencers* M, with $|M| = m \geq 2$, and a continuous set of consumers $N = [0, 1]$. The firm sells one durable good with quality $\theta \in \Theta = \mathbb{R}_{>0}$, which we call *innovation*. The per unit cost of the innovation is normalized to zero.

Influencers can contribute to sales by advertising the product and they offer that service to the firm. We assume that influencers $i \in M$ are characterized by their share of followers $\eta_i \in (0, 1)$, which may reflect, e.g. the attractiveness of the influencers' content. Each consumer follows influencer i with probability η_i, independent across influencers. Thereby, a subset $M' \subset M$ of social media influencers has a total share of followers

$$\bar{\eta}(M') = 1 - \prod_{i \in M'} (1 - \eta_i). \tag{1}$$

Influencers can advertise the product by sending a message $\tilde{\theta}$ about the true quality θ. Sending a message is costly and deviating from the truth incurs reputation costs, increasing in the amount of deviation. We assume that for any influencer i the total cost of sending the message $\tilde{\theta}$, $c_\theta(\tilde{\theta}, \eta_i)$, consists of a fixed cost $c_0 > 0$ and a separable function of the deviation from the truth $\tilde{\theta} - \theta$ and her share of followers η_i. We therefore write $c_\theta(\tilde{\theta}, \eta_i) = c_0 + c_1(\tilde{\theta} - \theta)c_2(\eta_i)$, where c_1 and c_2 are twice continuously differentiable with $c_1(0) = 0$, $c_1'(0) = 0$, $c_1'' > 0$, $c_2(\eta_i) > 0$ for all $\eta_i > 0$ and $c_2' \geq 0$. The motivation for the fixed cost $c_0 > 0$ is that it takes effort to inform their followers (e.g. a story has to be created on Instagram). Moreover, the more influencers deviate from the truth, the more harmful it may be to reputation and further ability to exaggerate and earn profits. The costs from deviating from the truth may further be larger the more followers they have. To illustrate our model, we will sometimes use costs that are quadratic in the level of exaggeration and linear in the number of followers $c_\theta(\tilde{\theta}, \eta_i) = c_0 + \alpha \frac{\eta_i}{2}(\tilde{\theta} - \theta)^2$ with $\alpha > 0$.

Influencers and firm engage in a two-stage game. In the first stage, both the firm and the influencers learn the quality θ and influencers then simultaneously determine their pricing policies $q_i : [\theta, +\infty) \to \mathbb{R}_+ \cup \{+\infty\}$, where $q_i(\tilde{\theta})$ is the price the firm has to pay influencer $i \in M$ to use the influencer to deliver the message $\tilde{\theta} \geq \theta$. Note first that this entails the assumption that the firm never wants to downplay the quality. Second, influencers may set an infinite price for many (perhaps even all but one) messages, in which case they would not be willing to transmit these messages. Denote the set of possible pricing policies, i.e. the strategy set for each $i \in M$, by $Q_i(\theta)$.

In the second stage, the firm observes the pricing policies $q_M = (q_1, \ldots, q_m)$ and, then, sets a price $p \geq 0$ and chooses which influencers to use, $M_f \subseteq M$, as well as the message $\tilde{\theta}_f \geq \theta$ to deliver to their followers, i.e. the firm's strategy is a function

$$q_f = (p, M_f, \tilde{\theta}_f) : Q_M(\theta) \to \mathbb{R}_+ \times 2^M \times [\theta, +\infty),$$

where $Q_M(\theta) = \prod_{i \in M} Q_i(\theta)$ denotes the set of all possible pricing policies and 2^M denotes the power set of M. Denote the firm's strategy set by $Q_f(\theta)$. Whenever the reference is clear we will subsequently drop dependence of the strategies on θ.

Given influencers' pricing strategies $q_M \in Q_M$ and firm's strategy $q_f = (p, M_f, \tilde{\theta}_f) \in Q_f$ the profits of each influencer $i \in M$ are given by

$$\pi_i(q_M, q_f) = 1_{\{i \in M_f(q_M)\}} \left(q_i(\tilde{\theta}_f(q_M)) - c_\theta(\tilde{\theta}_f(q_M), \eta_i) \right). \tag{2}$$

Profits of the firm depend on generated sales. We do not explicitly model consumer behavior at this stage, but a possible micro-foundation is provided in Section 2.1. Instead, we only assume that by choosing price p, message $\tilde{\theta}_f$, and set of influencers M_f that reaches $\bar{\eta}(M_f)$ followers, expected sales have the following structure:

$$G(p, M_f, \tilde{\theta}_f) = 1_{\{p \leq \theta\}} g_1(\bar{\eta}(M_f)) + 1_{\{\theta < p \leq \tilde{\theta}_f\}} g_2(\bar{\eta}(M_f)) \tag{3}$$

where $g_1, g_2 : [0,1] \to [0,1]$ are functions of the share of followers $\bar{\eta}(M_f)$ generated by the influencers M_f mapping into a share of buyers such that g_1 are the sales generated by setting price below or equal true quality, and g_2 are the sales generated by setting the price strictly above true quality and below the advertised quality $\tilde{\theta}_f$. The idea is that consumers may not be aware of the product and have limited knowledge about the true quality but can be informed by the influencers. In particular, this structure implicitly assumes that sales are insensitive to price as long as price stays below or equal to true quality (e.g. because consumers have valuation $v(\theta) = \theta$) or to the degree of overpricing, as long as the price is below the advertised quality $\tilde{\theta}_f$ (e.g. because a share of "naïve" consumers believes the influencers' message).

We assume that both g_1 and g_2 are strictly increasing and strictly concave in the number of followers, that pricing below true quality yields strictly higher sales than pricing above true quality, and that the former is steeper in the number of informed consumers than the latter. The idea is that sophisticated consumers will not be persuaded into buying the innovation if the price is above the true quality. This is summarized as follows:

Assumption 1. *g_1 and g_2 are twice differentiable, strictly increasing and strictly concave in $\bar{\eta}$, with $g_1(\bar{\eta}) > g_2(\bar{\eta})$ if $\bar{\eta} > 0$, $g_1'(\bar{\eta}) > g_2'(\bar{\eta})$ for all $\bar{\eta} \in [0,1]$, and $g_1(1) = 1$ and $g_2(0) = 0$.*

We introduce one possible micro-foundation of the assumptions on the sales function in the subsequent section. Given the influencers' pricing strategies q_M and the firm's strategy $q_f = (p, M_f, \tilde{\theta}_f)$, the profits of the firm f are given by the total revenue net of prices charged by the hired influencers charge,

$$\pi_f(q_M, q_f) = p(q_M) G\left(p(q_M), M_f(q_M), \tilde{\theta}_f(q_M)\right) - \sum_{i \in M_f(q_M)} q_i\left(\tilde{\theta}_f(q_M)\right). \tag{4}$$

To summarize, the timing of events is as follows:

1. Both the firm and influencers learn the true quality θ.

2. Influencers simultaneously choose their pricing policies $q_M = (q_1, \ldots, q_m)$.

3. After observing q_M, the firm sets a price $p \geq 0$ and chooses the set of influencers to use $M_f \subseteq M$ and the message $\tilde{\theta}_f \geq \theta$.

4. Payoffs realize.

We will study subgame perfect Nash equilibria (SPE) of the game where influencers choose their pricing policies $q_M \in Q_M$ simultaneously in the first stage and the firm chooses its strategy $q_f \in Q_f$ in the second stage. Since there many possible such equilibria, we will restrict attention to SPEs in pure strategies which are also strong Nash equilibria for the most part of this paper. We apply the following notion of strong Nash equilibrium:

Definition 1. *A profile of pricing policies and firm strategy (q_M^*, q_f^*) is a strong Nash equilibrium, if and only if for all $J \subset (M \cup \{f\})$ and for all $q'_J \in \prod_{j \in J} Q_j \backslash q_J^*$ there exists $i \in J$ such that $\pi_i(q'_J, q^*_{-J}) \leq \pi_j(q^*)$ (where $q^*_{-J}) = (q^*_k)_{k \notin J}$).*

In what follows we denote by a strong subgame perfect Nash equilibrium (SSPE) a strategy profile which is both a strong Nash equilibrium and subgame perfect.

2.1 Micro-foundation 1: Consumer behavior

So far we have modelled consumer behavior only indirectly via the structure of the sales functions (3) and Assumption 1. It turns out that this general approach is sufficient to derive the key results in this paper. One possible narrative giving rise to such a structure of the sales function is the following.

Suppose that initially, only a share of consumer $\mu \in (0, 1)$ is aware of the innovation, while no consumer has any information about the quality of the innovation. Consumers follow social media influencers who may make their followers aware of the innovation and send a message $\tilde{\theta} \geq \theta$ about the true quality of the innovation. After becoming aware of the product a share of consumers $\rho \in [0, 1]$ is able to gather costless information about the quality of the product through external sources (verify the quality). This external source always delivers a truthful message $\tilde{\theta} = \theta$. Henceforth we will refer to these consumers as sophisticated consumers, and we simply assume that they always form the belief $\hat{\theta} = \theta$ once becoming aware of the innovation. We assume that the consumers who do not have access to this source, henceforth called naïve consumers, are pessimistic initially such that they form the belief $\hat{\theta}(\emptyset) = 0$ if they do not receive a message, but upon receiving message $\tilde{\theta}_f$ take this at face value and form the belief $\hat{\theta}(\tilde{\theta}_f) = \tilde{\theta}_f$.

Consumers have valuation $v(\theta) = \theta$ for a product of quality θ, have unit demand, and are myopic, i.e. they purchase the product if their belief exceeds the price, $v(\hat{\theta}) \geq p$. After purchase, naïve consumers learn the true quality with probability $\lambda \in [0, 1]$ and otherwise remain with their prior belief. All consumers use word-of-mouth communication to make others aware of the product and communicate their belief. Hence, sophisticated consumers buy the product if and only if they become aware of the product (either initially, through an influencer, or by word-of-mouth communication) and the innovation is not overpriced $p \leq \theta$, while naïve consumers buy the product if and only if they become aware and receive at least one message $\tilde{\theta} \geq p$ (either through an influencer, or by word-of-mouth communication).[5]

Under these assumptions, optimal consumer behavior clearly gives rise to a sales function that has the same structure of our general sales function in (3) and satisfies Assumption 1. Abusing notation, we sometimes write $g_1(\bar{\eta}|\mu, \rho)$ and $g_2(\bar{\eta}|\rho, \lambda)$ when referring to our micro-foundation to clearly point out that the sales generated by pricing below quality g_1 depend on the share of initially aware consumers μ and the share of sophisticated consumers ρ, while the sales generated by overpricing g_2 depend on the share of sophisticated consumers ρ, and the ability to learn after purchase λ.[6]

Optimal consumer behavior further implies that g_1 is increasing in the share of sophisticated individuals if there is a strictly positive share of initially aware consumers since out of these only sophisticated consumers buy the product as no message about quality is communicated. No sales can be created if no individuals are made aware of the innovations, i.e. if no influencers are hired and no consumer is initially aware. Additionally, g_2 is strictly decreasing in the share of sophisticated consumers since they will never buy the product if overpriced and also strictly decreasing in the ability of naïve consumers to learn the true quality after purchase. These are summarized as follows.

Assumption 2.

(i) $g_1(\bar{\eta}|\mu, \rho)$ is differentiable and strictly increasing in ρ if $\mu > 0$ and constant in ρ otherwise, and strictly increasing in μ, with $g_1(0|0, \rho) = 0$,

(ii) $g_2(\bar{\eta}|\rho, \lambda)$ is differentiable and strictly decreasing in ρ and λ.

[5] Note that this assumption is for simplicity and only affects sales quantitatively. Any other rule would also work, e.g. that all messages $\tilde{\theta}$ naïve consumers have received must be such that $\tilde{\theta} \geq p$.

[6] In particular, g_1 does not depend on λ since if the product is not overpriced the capacity of learning the true quality is irrelevant, and g_2 does not depend on μ since only naïve individuals who receive a message above price will purchase the product.

2.2 Micro-foundation 2: Word-of-mouth communication

So far word-of-mouth communication is a general function of initial adopters. Suppose now that there are two periods. Consumer behavior is as described in Section 2.1, and the firm hires influencers that have a total share of $\bar{\eta}$ followers that deliver a message $\tilde{\theta}_f \geq \theta$ about product quality. Consumers are further organized in a social network à la Galeotti and Goyal (2009): each consumer observes k other consumers with probability $P(k) \geq 0$, where $k \in \{1, 2, \ldots, \bar{k}\}$ and $\sum_{k=1}^{\bar{k}} P(k) = 1$. A consumer with *degree* k makes k independent and uniformly distributed draws from N.[7] Consumers' degrees are independently distributed. Hence, following a standard "abuse" of the law of large numbers, there is a fraction $P(k)$ of consumers with degree k. The firm knows the degree distribution P but not the actual network.

In the first period, a share of individuals is aware of the product either because they are followers or because they were initially aware of the product. From our independence assumption, the total share of consumers who are aware of the innovation in the first period can be calculated to be $1-(1-\bar{\eta})(1-\mu)$. If the innovation is not overpriced, then out of the aware consumers, the sophisticated consumers buy the product while naïve consumers only buy it if they follow an influencer. The total share of buyers in the first period is therefore $1-(1-\bar{\eta})(1-\mu\rho)$. If the innovation is overpriced but $p \leq \tilde{\theta}_f$, then the naïve consumers who follow an influencer still buy the product. In this case the total share of buyers in the first period is $\bar{\eta}(1-\rho)$.

In the second period, consumers communicate with their neighbors in the network. As in the first period, sophisticated consumers who are informed learn the true quality, while naïve consumers take the quality judgement of the observed consumers at face value. Consumers who are aware of the product and who did not buy in the first period then decide whether to purchase product. To simplify the exposition, we assume that naïve consumers buy if at least one quality judgement of a neighbor warrants the purchase.

In case the product is not overpriced, the sales function that is generated by this network (i.e. by its degree distribution P), g_1^P, can then be calculated as follows. Out of the share of consumer who did not buy in the first period, $(1-\bar{\eta})(1-\mu\rho)$, each consumer makes k observations with probability $P(k)$ in the second period and ends up buying if one of the k observed consumers purchased in period 1. Hence, a given consumer does not observe a buyer in the second period with probability $\sum_{k=1}^{\bar{k}} P(k)(1-\bar{\eta})^k(1-\mu\rho)^k$, implying that the total share of consumers not buying

[7] Notice that the probability to draw the same consumer twice is zero, as there is a continuum of consumers.

in either period is given by $\sum_{k=1}^{\bar{k}} P(k)(1-\bar{\eta})^{k+1}(1-\mu\rho)^{k+1}$. Thus, total sales for $p \leq \theta$ are given by

$$g_1^P(\bar{\eta}|\mu,\rho) = 1 - \sum_{k=1}^{\bar{k}} P(k)(1-\bar{\eta})^{k+1}\left(1-\mu\rho\right)^{k+1}.$$

Now, if the price exceeds the true quality but not the message communicated by the influencers, i.e. $\theta < p \leq \tilde{\theta}_f$, then the share of individuals communicating message $\tilde{\theta}_f$ in period 2 is given by $\bar{\eta}(1-\rho)(1-\lambda)$. A näive consumer who observes at least one of these consumers in the second period then also buys the product. Hence the sales function g_2^P for $\theta < p \leq \tilde{\theta}_f$ generated by this network is given by

$$g_2^P(\bar{\eta}|\rho,\lambda) = (1-\rho)\left(1-(1-\bar{\eta})\sum_{k=1}^{\bar{k}} P(k)\left(1-\bar{\eta}(1-\rho)(1-\lambda)\right)^k\right).$$

It is straightforward to verify that Assumptions 1 and 2 are satisfied for both sales functions g_1^P and g_2^P emerging from this model of word-of-mouth communication.

3 Equilibrium analysis

We denote *industry profits* under strategy profile (q_M, q_f) by

$$\begin{aligned}\Pi(q_M,q_f) &\equiv \sum_{i\in M\cup\{f\}} \pi_i(p(q_M), M_f(q_M), \tilde{\theta}_f(q_M)) \\ &= p(q_M)G(p(q_M), M_f(q_M), \tilde{\theta}_f(q_M)) - \sum_{i\in M_f(q_M)} c_\theta(\tilde{\theta}_f(q_M), \eta_i). \qquad (5)\end{aligned}$$

Note that (5) depends only indirectly on the influencers' pricing q_M, through the firm's decision q_f. We first establish that any SSPE is such that industry profits are maximized.

Lemma 1. *Let (q_M^*, q_f^*) be a SSPE. Then there does not exist $(q_M', q_f') \in Q_M \times Q_f$ such that $\Pi(q_M', q_f') > \Pi(q_M^*, q_f^*)$.*

Proof of Lemma 1. Suppose to the contrary that (q_M^*, q_f^*) is a SSPE and there exists $(q_M', q_f') \in Q_M \times Q_f$ with $\Pi(q_M', q_f') > \Pi(q_M^*, q_f^*)$. Let $\Delta \equiv \Pi(q_M', q_f') - \Pi(q_M^*, q_f^*) > 0$ and let $p' = p(q_M')$, $M_f' = M_f(q_M')$, and $\tilde{\theta}_f' = \tilde{\theta}_f(q_M')$. Denote by $q_f' = (p', M_f', \tilde{\theta}_f')$ the (constant) strategy of the firm making these choices independently of influencer pricing. Further let $q_i'(\tilde{\theta}) = q_i^*(\tilde{\theta}_f(q_M^*)) - c_\theta(\tilde{\theta}_f(q_M^*), \eta_i) + c_\theta(\tilde{\theta}_f', \eta_i) + \frac{\Delta}{m+1}$ for all $\tilde{\theta} \geq \theta$ and $i \in M_f'$ be the constant pricing of influencers. Then clearly $\pi_i\left((q_{M_f'}', q_{M\setminus\{M_f'\}}^*), q_f'\right) >$

$\pi_i\left(q_M^*, q_f^*\right)$ for all $i \in M_f'$, as these influencers are chosen and charge higher prices net of costs. Further $\pi_f\left((q'_{M_f'}, q^*_{M\setminus\{M_f'\}}), q_f'\right) - \pi_f\left(q_M^*, q_f^*\right) = \Delta - |M_f'|\frac{\Delta}{m+1} > 0$ since $|M_f'| \le m$. Hence the coalition $J = M_f' \cup \{f\}$ has a deviation that strictly benefits all of its members, contradicting that (q_M^*, q_f^*) is a SSPE. $\square$

Lemma 1 implies that any SSPE is efficient in the sense that it maximizes industry profits. The reason is that industry profits $\Pi(q_M, q_f)$ only depend on the realized choices of the firm, as influencer pricing just determines how the pie is divided among the firm and the chosen influencers. If the resulting choices of the firm are not efficient, then we can always find coalitional deviations that benefit all members of the coalition strictly. In what follows, we therefore first determine the firm's behavior under efficiency, i.e. maximizing industry profits. We then solve for the influencers' pricing strategies which give rise to efficient and optimal firm choices.

3.1 Firm behavior

3.1.1 The optimal price

Since sales $G(p, M_f, \tilde{\theta}_f)$ are given by (3), there are at most two candidates for the optimal price (that maximizes industry profits (5)) conditional on choosing message $\tilde{\theta}_f$: if $\tilde{\theta}_f > \theta$, then either $p = \tilde{\theta}_f$ or $p = \theta < \tilde{\theta}_f$ is optimal for the firm while if $\tilde{\theta}_f = \theta$, we must have $p = \theta = \tilde{\theta}_f$. Note that in the first case, choosing $p = \theta < \tilde{\theta}_f$ is not optimal because influencers could save costs by setting prices such that the firm chooses instead $\tilde{\theta}_f' = \theta$. Thus, conditional on choosing $\tilde{\theta}_f$, $p = \tilde{\theta}_f$ maximizes industry profits.

3.1.2 The optimal message

For any choice of the set of influencers M_f, we now determine the message that maximizes industry profits (5). Given $p = \tilde{\theta}_f$, we obtain:

$$\tilde{\theta}^+_{M_f} \equiv \min \underset{\tilde{\theta}_f}{\operatorname{argmax}}\, \tilde{\theta}_f G(\tilde{\theta}_f, M_f, \tilde{\theta}_f) - \sum_{i \in M_f} c_\theta(\tilde{\theta}_f, \eta_i).$$

Note that the maximizer may not be unique in non-generic cases. We break ties by choosing the lowest maximizer, which implies that influencers send a truthful message whenever it maximizes industry profits. The following result characterizes the optimal message for any choice of influencers M_f.

Proposition 1. *For any $M_f \subseteq M$, there exists a threshold*

$$\bar{\theta}((\eta_i)_{i\in M_f}) \equiv \frac{\begin{array}{c}(c_1')^{-1}\left(\left(\sum_{i\in M_f} c_2(\eta_i)\right)^{-1} g_2(\bar{\eta}(M_f))\right) g_2(\bar{\eta}(M_f)) \\ - c_1\left((c_1')^{-1}\left(\left(\sum_{i\in M_f} c_2(\eta_i)\right)^{-1} g_2(\bar{\eta}(M_f))\right)\right)\sum_{i\in M_f} c_2(\eta_i)\end{array}}{g_1(\bar{\eta}(M_f)) - g_2(\bar{\eta}(M_f))}$$

such that the optimal message $\tilde{\theta}^+_{M_f}$ is given by

$$\tilde{\theta}^+_{M_f} = \begin{cases} \theta & \text{if } \theta \geq \bar{\theta}((\eta_i)_{i\in M_f}) \\ (c_1')^{-1}\left(\left(\sum_{i\in M_f} c_2(\eta_i)\right)^{-1} g_2(\bar{\eta}(M_f))\right) + \theta & \text{else} \end{cases}.$$

Moreover, under Assumption 2, $\tilde{\theta}^+_{M_f}$ is strictly decreasing in ρ unless $\tilde{\theta}^+_{M_f} = \theta$.

Proof of Proposition 1. Suppose that $\tilde{\theta}_f > \theta$. Then

$$\begin{aligned}
&\frac{\partial}{\partial \tilde{\theta}_f}\left(G(\tilde{\theta}_f, M_f, \tilde{\theta}_f) - \sum_{i\in M_f} c_\theta(\tilde{\theta}_f, \eta_i)\right) \\
=&\frac{\partial}{\partial \tilde{\theta}_f}\left(\tilde{\theta}_f g_2(\bar{\eta}(M_f)) - |M_f| c_0 - c_1(\tilde{\theta}_f - \theta)\sum_{i\in M_f} c_2(\eta_i)\right) \\
=&g_2(\bar{\eta}(M_f)) - c_1'(\tilde{\theta}_f - \theta)\sum_{i\in M_f} c_2(\eta_i) = 0
\end{aligned}$$

$$\Leftrightarrow \tilde{\theta}^*_f = (c_1')^{-1}\left(\left(\sum_{i\in M_f} c_2(\eta_i)\right)^{-1} g_2(\bar{\eta}(M_f))\right) + \theta, \tag{6}$$

which is strictly greater than θ by definition of c_θ. Hence, we obtain $\tilde{\theta}^+_{M_f} = \theta$ if

$$\begin{aligned}
&\theta G(\theta, M_f, \theta) - \sum_{i\in M_f} c_\theta(\theta, \eta_i) \geq \tilde{\theta}^*_f G\big(\tilde{\theta}^*_f, M_f, \tilde{\theta}^*_f\big) - \sum_{i\in M_f} c_\theta(\tilde{\theta}^*_f, \eta_i) \\
\Leftrightarrow&\theta g_1(\bar{\eta}(M_f)) - |M_f| c_0 \geq \left((c_1')^{-1}\left(\left(\sum_{i\in M_f} c_2(\eta_i)\right)^{-1} g_2(\bar{\eta}(M_f))\right) + \theta\right) g_2(\bar{\eta}(M_f)) \\
&- |M_f| c_0 - c_1\left((c_1')^{-1}\left(\left(\sum_{i\in M_f} c_2(\eta_i)\right)^{-1} \tilde{g}_2(\bar{\eta}(M_f))\right)\right)\sum_{i\in M_f} c_2(\eta_i)
\end{aligned}$$

$$\Leftrightarrow \theta \geq \frac{\begin{array}{c}(c_1')^{-1}\left(\left(\sum_{i\in M_f} c_2(\eta_i)\right)^{-1} \tilde{g}_2(\bar{\eta}(M_f))\right) g_2(\bar{\eta}(M_f)) \\ - c_1\left((c_1')^{-1}\left(\left(\sum_{i\in M_f} c_2(\eta_i)\right)^{-1} \tilde{g}_2(\bar{\eta}(M_f))\right)\right)\sum_{i\in M_f} c_2(\eta_i)\end{array}}{g_1(\bar{\eta}(M_f)) - g_2(\bar{\eta}(M_f))} \tag{7}$$

and $\tilde{\theta}^+_{M_f} = \tilde{\theta}^*_f$ otherwise.

Finally, by Assumption 2, (6) and (7) are both strictly decreasing with respect to ρ. Thus, $\tilde{\theta}^+_{M_f}$ is strictly decreasing in ρ unless $\tilde{\theta}^+_{M_f} = \theta$. $\square$

First, Proposition 1 shows that communication is truthful if the quality of the innovation weakly exceeds the threshold $\bar{\theta}(\bar{\eta}(M_f))$. Otherwise, it is optimal to exaggerate quality. Thus, exaggeration occurs when product quality is low. In this case, the increase in revenue from exaggeration outweighs its (low) costs. Second, the optimal message is decreasing in the share of sophisticated consumers ρ.

The threshold for truthful communication and the degree of exaggeration in Proposition 1 simplify drastically with costs that are quadratic in the level of exaggeration:

Corollary 1. *Suppose that $c_1(\tilde{\theta} - \theta) = \frac{1}{2}(\tilde{\theta} - \theta)^2$. For any $M_f \subseteq M$,*

$$\tilde{\theta}^+_{M_f} = \begin{cases} \theta & \text{if } \theta \geq \bar{\theta}((\eta_i)_{i \in M_f}) = \frac{1}{2\sum_{i \in M_f} c_2(\eta_i)} \frac{\left(g_2(\bar{\eta}(M_f))\right)^2}{g_1(\bar{\eta}(M_f)) - g_2(\bar{\eta}(M_f))} \\ \frac{g_2(\bar{\eta}(M_f))}{\sum_{i \in M_f} c_2(\eta_i)} + \theta & \text{else} \end{cases} . \quad (8)$$

If additionally c_2 is constant and $\frac{g_1'(\bar{\eta})}{g_1(\bar{\eta})} \leq \frac{g_2'(\bar{\eta})}{g_2(\bar{\eta})}$ for all $\bar{\eta} \in (0,1)$, then $\bar{\theta}((\eta_i)_{i \in M_f}) = \bar{\theta}(\bar{\eta}(M_f))$ is strictly increasing in $\bar{\eta}(M_f)$ with $\bar{\theta}(0) = 0$. If, further, $g_2(1) = (1-\rho)$, then $\bar{\theta}(1) = \frac{(1-\rho)^2}{2|M_f|\rho}$.

Proof of Corollary 1. The first part given by (8) follows immediately from Proposition 1 since $(c_1')^{-1}(x) = x$.

Let without loss of generality $c_2 \equiv 1$, then $\bar{\theta}(\bar{\eta}) = \frac{1}{2|M_f|} \frac{(g_2(\bar{\eta}))^2}{g_1(\bar{\eta}) - g_2(\bar{\eta})}$. We can straightforwardly calculate the derivative with respect to $\bar{\eta}$ to get

$$\bar{\theta}'(\bar{\eta}) = \frac{1}{2|M_f|} \frac{g_2(\bar{\eta})}{\left(g_1(\bar{\eta}) - g_2(\bar{\eta})\right)^2} \Big(2g_1(\bar{\eta})g_2'(\bar{\eta}) - g_2(\bar{\eta})(g_1'(\bar{\eta}) + g_2'(\bar{\eta}))\Big) > 0$$

if and only if $2g_1(\bar{\eta})g_2'(\bar{\eta}) > g_2(\bar{\eta})(g_1'(\bar{\eta}) - g_2'(\bar{\eta}))$. Clearly, this condition holds for all $\bar{\eta} \in (0,1)$ if $g_1'(\bar{\eta})/g_1(\bar{\eta}) \leq g_2'(\bar{\eta})/g_2(\bar{\eta})$ for all $\bar{\eta} \in (0,1)$ since $2g_1'(\bar{\eta}) > g_1'(\bar{\eta}) + g_2'(\bar{\eta})$ for $\bar{\eta} \in (0,1)$ by Assumption 1.

Note that we trivially have $\bar{\theta}(0) = 0$ if $g_1(0) > 0$ since $g_2(0) = 0$ by Assumption 1 while if $g_1(0) = 0$, then by L'Hospital's rule we get

$$\lim_{\bar{\eta} \to 0} \bar{\theta}(\bar{\eta}) = \frac{1}{2|M_f|} \lim_{\bar{\eta} \to 0} \bar{\theta}(\bar{\eta}) \frac{2g_2(\bar{\eta})g_2'(\bar{\eta})}{g_1'(\bar{\eta}) - g_2'(\bar{\eta})} = 0$$

since by Assumption 1 we have $g_2(0) = 0$ and $g_1'(0) > g_2'(0)$. Finally note that $\bar{\theta}(1) = \frac{(1-\rho)^2}{2|M_f|\rho}$ if $g_2(1) = 1 - \rho$ since $g_1(1) = 1$ by Assumption 1. $\square$

Note first that the threshold for truthful communication is increasing in the sales created by exaggeration and decreasing in the sales created by truthful communication. Second, the extent of exaggeration is linear in the sales generated by exaggeration.

Further, if c_2 is constant, i.e. if the cost of exaggeration is independent of the number of followers, then with the additional condition that $g_1'(\bar{\eta})/g_1(\bar{\eta}) \leq g_2'(\bar{\eta})/g_2(\bar{\eta})$ we get that the threshold for truthful communication is higher the more followers are reached. Thus, the firm is more tempted to exaggerate the more consumers are reached by the choice of influencers. The condition $g_1'(\bar{\eta})/g_1(\bar{\eta}) \leq g_2'(\bar{\eta})/g_2(\bar{\eta})$ can be interpreted as the ratio of the percentage change in sales created by truthful communication and the percentage change in sales created by exaggeration being smaller than 1, and intuitively holds in our interpretation of sales since influencers play a more important role when exaggerating than when communicating truthfully. We show below in Example 1 that this condition is indeed satisfied when the network of word-of-mouth communication introduced in Section 2.2 is regular.

Example 1 (Regular social network)**.** *Suppose that* $c_1(\tilde{\theta} - \theta) = \frac{1}{2}(\tilde{\theta} - \theta)^2$*, that* c_2 *is constant and consider the model of word-of-mouth communication presented in Section 2.2 with* $P(\bar{k}) = 1$ *for some positive integer* $\bar{k}$*. This implies that all consumers have the same degree, i.e. the implied network of communication is regular. It can be verified that for this degree distribution* P*, the implied sales functions satisfy*

$$\frac{(g_1^P)'(\bar{\eta})}{g_1^P(\bar{\eta})} \leq \frac{(g_2^P)'(\bar{\eta})}{g_2^P(\bar{\eta})} \text{ for all } \bar{\eta} \in (0,1)$$

and, hence, the threshold for truthful communication is strictly increasing by Corollary 1 and can be calculated to be

$$\bar{\theta}(\bar{\eta}) = \frac{1}{2|M_f|} \frac{(1-\rho)^2\big(1 - (1-\bar{\eta})\,(1 - \bar{\eta}(1-\rho)(1-\lambda))^{\bar{k}}\,\big)^2}{\rho(1-(1-\bar{\eta})^{\bar{k}+1}) + (1-\rho)(1-\bar{\eta})\big(\,(1-\bar{\eta}(1-\rho)(1-\lambda))^{\bar{k}} - (1-\bar{\eta})^{\bar{k}}\big)}.$$

3.1.3 The optimal choice of influencers

Finally, it remains to determine the optimal choice of influencers which yield maximal industry profits. Note first that, since industry profits depend only indirectly on the influencers' pricing q_M, and given the optimal message $\tilde{\theta}^+_{M_f}$ and price $p = \tilde{\theta}^+_{M_f}$, maximal industry profits given the choice of influencers $M_f \subseteq M$ are

$$\Pi_{M_f} \equiv \sum_{i \in M \cup \{f\}} \pi_i(\tilde{\theta}^+_{M_f}, M_f, \tilde{\theta}^+_{M_f}) = \tilde{\theta}^+_{M_f} G(\tilde{\theta}^+_{M_f}, M_f, \tilde{\theta}^+_{M_f}) - \sum_{i \in M_f} c_\theta(\tilde{\theta}^+_{M_f}, \eta_i). \qquad (9)$$

For a general cost function, there is no simple relation between the optimal choice of influencers and initial conditions, i.e. number of followers of each influencer. To ease the exposition, we henceforth order influencers according to their "stand-alone" profit:

Assumption 3. $\Pi_1 \geq \Pi_2 \geq \ldots \geq \Pi_m$.

It seems natural that the ranking of the influencers according to their stand-alone profit coincides with that according to their number of followers. As we now show, this will hold in our model if influencers' reputation costs do not increase too fast in the number of followers.

Lemma 2. *There exists $\varepsilon^* > 0$ such that if*

$$c_2'(\eta) < \varepsilon^* \text{ for all } \eta \in (0,1), \tag{C1}$$

we have $\Pi_{M_f \cup \{i\}} \geq \Pi_{M_f \cup \{j\}}$ if and only if $\eta_i \geq \eta_j$ for all $M_f \subset M \backslash \{i,j\}$.

Proof. Let

$$\tilde{\theta}^*_{M_f} \equiv (c_1')^{-1} \left(\left(\sum_{i \in M_f} c_2(\eta_i) \right)^{-1} g_2(\bar{\eta}(M_f)) \right) + \theta$$

and

$$\varepsilon^* \equiv \min_{i,j \in M: i<j} \min_{M_f \subset M \backslash \{i,j\}} \frac{\tilde{\theta}^*_{M_f \cup \{j\}} \big(g_2(\bar{\eta}(M_f \cup \{i\})) - g_2(\bar{\eta}(M_f \cup \{j\})) \big)}{c_1(\tilde{\theta}^*_{M_f \cup \{j\}} - \theta)(\eta_i - \eta_j)}.$$

Fix $i, j \in M$ such that $i < j$ and $M_f \subset M \backslash \{i,j\}$ and note that since $\tilde{\theta}^*_{M_f \cup \{j\}} > \theta > 0$ and $\eta_i > \eta_j$, $\varepsilon^* > 0$ is well defined. If $c_2'(\eta) < \varepsilon^*$ for all $\eta \in (0,1)$, then

$$\begin{aligned}
\Pi_{M_f \cup \{i\}} - \Pi_{M_f \cup \{j\}} \geq & \tilde{\theta}^+_{M_f \cup \{j\}} G(\tilde{\theta}^+_{M_f \cup \{j\}}, M_f \cup \{i\}, \tilde{\theta}^+_{M_f \cup \{j\}}) - \sum_{k \in M_f \cup \{i\}} c_\theta(\tilde{\theta}^+_{M_f \cup \{j\}}, \eta_k) \\
& - \left(\tilde{\theta}^+_{M_f \cup \{j\}} G(\tilde{\theta}^+_{M_f \cup \{j\}}, M_f \cup \{i\}, \tilde{\theta}^+_{M_f \cup \{j\}}) - \sum_{k \in M_f \cup \{j\}} c_\theta(\tilde{\theta}^+_{M_f \cup \{j\}}, \eta_k) \right) \\
= & \tilde{\theta}^+_{M_f \cup \{j\}} \Big(G(\tilde{\theta}^+_{M_f \cup \{j\}}, M_f \cup \{i\}, \tilde{\theta}^+_{M_f \cup \{j\}}) - G(\tilde{\theta}^+_{M_f \cup \{j\}}, M_f \cup \{i\}, \tilde{\theta}^+_{M_f \cup \{j\}}) \Big) \\
& - c_1(\tilde{\theta}^+_{M_f \cup \{j\}} - \theta) \Big(c_2(\eta_i) - c_2(\eta_j) \Big) \\
\geq & \tilde{\theta}^+_{M_f \cup \{j\}} \Big(G(\tilde{\theta}^+_{M_f \cup \{j\}}, M_f \cup \{i\}, \tilde{\theta}^+_{M_f \cup \{j\}}) - G(\tilde{\theta}^+_{M_f \cup \{j\}}, M_f \cup \{i\}, \tilde{\theta}^+_{M_f \cup \{j\}}) \Big) \\
& - c_1(\tilde{\theta}^+_{M_f \cup \{j\}} - \theta) \varepsilon^* (\eta_i - \eta_j).
\end{aligned} \tag{10}$$

If $\tilde{\theta}^+_{M_f \cup \{j\}} = \theta$, then (10) is strictly positive by Assumption 1. Otherwise, if $\tilde{\theta}^+_{M_f \cup \{j\}} = \tilde{\theta}^*_{M_f}$, then (10) is non-negative and the inequality strict by definition of ε^*, which establishes the claim. $\square$

Clearly, influencers with higher number of followers being able to generate higher industry profits requires that reputation costs of deviating from the truth are not exploding for large influencers compared to their smaller counterparts. However, for most results we will not require condition (C1). We immediately get the desired result for the stand-alone values by setting $M_f = \emptyset$ in Lemma 2.

Remark 1. *Suppose that Condition* (C1) *holds. Then* $\Pi_i \geq \Pi_j$ *if and only if* $\eta_i \geq \eta_j$.

We next introduce the notion of an essential influencer:

Definition 2. *An influencer* $i \in M$ *is* essential *(to maximizing industry profit) if* $M_f \in \operatorname{argmax}_{M'_f \subseteq M} \Pi_{M'_f}$ *only if* $i \in M_f$. *Let* M^e *denote the set of essential influencers.*

In a first step, we derive conditions under which the influencers with the highest stand-alone profit are essential. Let $M^k \equiv \{1, 2, ..., k\}$ for any $k \in M$ and $M^0 \equiv \emptyset$.

Proposition 2. $M^e = M^{k^* - l(\eta_{k^*+1})}$ *if either* $m = 2$ *or Condition* (C1) *holds, where* $k^* \equiv \min \operatorname{argmax}_{k \in M \cup \{0\}} \Pi_{M^k}$ *and* $l(\eta_{k^*+1}) = |\{i \in M^{k^*} : \eta_i = \eta_{k^*+1}\}|$.

Proof. First, suppose that $m = 2$. Since $\Pi_1 \geq \Pi_2$, we have that $M^e \neq \{2\}$. Therefore, there exists $k \in M \cup \{0\}$ such that $M^e = M^k$. If $k^* = 0$ or $k^* = 2$, then there is nothing to show. For $k^* = 1$, if $\Pi_1 = \Pi_2$ (i.e. $l(\eta_2) = 1$) neither 1 or 2 are essential since $M_f = \{2\}$ is also a maximizer, hence $M^e = M^0$. If $\Pi_1 > \Pi_2$ (i.e. $l(\eta_2) = 0$) then only 1 is essential, hence $M^e = M^1$.

Second, suppose that Condition (C1) holds. Then, by Remark 1 we have $\eta_i \geq \eta_j$ if $i < j$. Lemma 2 then yields $\Pi_{M^k} \geq \Pi_{M'}$ for all M' such that $|M'| = k$. Thus, $M^e \subset M^{k^*}$. Now, for any influencer $i \in M^{k^*}$ with $\eta_i = \eta_{k^*+1}$ we have $\Pi_{M^{k^*}} = \Pi_{(M^{k^*} \setminus \{i\}) \cup \{k^*+1\}}$, implying $i \notin M^e$, while we get strict inequalities for $i \in M^{k^*}$ with $\eta_i > \eta_{k^*+1}$, proving the claim. $\square$

3.2 Equilibrium pricing

Now we consider the influencers' pricing. Given quality θ, influencers want to charge maximal prices under the two conditions that they will be hired and that they obtain non-negative payoffs (also off-equilibrium because we focus on weakly undominated strategies). Suppose that influencers $M_f \subseteq M$ are hired, and hence $p = \tilde{\theta}_f =$

$\tilde{\theta}^+_{M_f}$. Then each influencer $i \in M_f$ can charge at most her marginal contribution to industry profits, i.e.

$$q_i(\tilde{\theta}^+_{M_f}) \leq \Pi_{M_f} - \max_{M'_f \subseteq M \setminus \{i\}} \Pi_{M'_f} + c_\theta(\tilde{\theta}^+_{M_f}, \eta_i),$$

which yields the following lemma:

Lemma 3. *In any undominated SSPE in which the firm chooses M^*_f, each influencer $i \in M^*_f$ receives a payoff of $\pi^*_i = \bar{\pi}_i(M^*_f) \equiv \Pi_{M^*_f} - \max_{M'_f \subseteq M \setminus \{i\}} \Pi_{M'_f}$.*

Proof. Suppose first that in some SSPE the firm chooses M_f and there exists an influencer $i \in M_f$ with payoff $\pi_i > \bar{\pi}_i(M_f)$. Note that $\sum_{j \in M_f} \pi_j + \pi_f \leq \Pi_{M_f}$ by definition of Π_{M_f}. If $\emptyset \in \text{argmax}_{M'_f \subseteq M \setminus \{i\}} \Pi_{M'_f}$, then

$$\pi_f \leq \Pi_{M_f} - \sum_{j \in M_f} \pi_j \leq \Pi_{M_f} - \pi_i < \Pi_{M_f} - \bar{\pi}_i(M_f) = \Pi_\emptyset,$$

i.e. the firm has a profitable deviation to $M'_f = \emptyset$, a contradiction.

If $\emptyset \notin \text{argmax}_{M'_f \subseteq M \setminus \{i\}} \Pi_{M'_f}$, consider any $M' \in \text{argmax}_{M'_f \subseteq M \setminus \{i\}} \Pi_{M'_f}$ and a deviation to pricing strategies

$$q'_j(\tilde{\theta}) = \begin{cases} \pi_j + c_\theta(\tilde{\theta}, \eta_j) + \frac{\pi_i - \bar{\pi}_i(M_f)}{|M_f|} & \text{if } \tilde{\theta} = \tilde{\theta}^+_{M'} \\ \infty & \text{else} \end{cases} \quad \text{for all } j \in M'.$$

Note that all $j \in M'$ are strictly better off with this deviation. By choosing $M'_f = M'$ and $p' = \tilde{\theta}'_f = \tilde{\theta}^+_{M'}$, the firm obtains

$$\begin{aligned} \pi'_f = \tilde{\theta}^+_{M'} G(\tilde{\theta}^+_{M'}, M', \tilde{\theta}^+_{M'}) - \sum_{j \in M'_f} q'_j(\tilde{\theta}^+_{M'}) =& \Pi_{M'} - \sum_{j \in M'_f} \left(\pi_j + \frac{\pi_i - \bar{\pi}_i(M_f)}{|M_f|} \right) \\ >& \Pi_{M'} - \sum_{j \in M'_f} \pi_j - \pi_i + \bar{\pi}_i(M_f) \\ =& \Pi_{M_f} - \sum_{j \in M_f} \pi_j, \end{aligned}$$

i.e. also the firm obtains a higher payoff, a contradiction. Thus, $\pi_i \leq \bar{\pi}_i(M_f)$ for all $i \in M_f$.

Second, suppose that in some SSPE the firm chooses M_f and there exists an influencer $i \in M_f$ with payoff $0 \leq \pi_i < \bar{\pi}_i(M_f)$. Consider a deviation by i to pricing strategy

$$q'_i(\tilde{\theta}) = \begin{cases} q_i(\tilde{\theta}) + (\bar{\pi}_i(M_f) - \pi_i)/2 & \text{if } \tilde{\theta} = \tilde{\theta}^+_{M_f} \\ \infty & \text{else} \end{cases}.$$

By definition of $q_i'(\tilde{\theta})$ it is still optimal for the firm to hire influencer 1 if all influencers $j \neq i$ play an undominated strategy $q_j \geq c_\theta$, since in this case $\pi_i' = \pi_i + (\bar{\pi}_i(M_f) - \pi_i)/2 = (\pi_i + \bar{\pi}_i(M_f))/2 < \bar{\pi}_i(M_f)$, i.e. $1 \in M_f$. Since further also $\pi_j \leq \bar{\pi}_j(M_f)$ for all $j \in M_f \backslash \{i\}$ by the first part, the firm will still choose M_f, such that i obtain payoff $\pi_i' = \pi_i + (\bar{\pi}_i(M_f) - \pi_i)/2 > \pi_i$, a contradiction. □

Second, it follows from Lemma 1 and 3 that only essential influencers obtain strictly positive profits in equilibrium:

Proposition 3. *Any undominated SSPE is such that* $M_f^* \in \operatorname{argmax}_{M_f' \subseteq M} \Pi_{M_f'}$ *and yields the same payoffs* $\pi_i^* = \bar{\pi}_i(M_f^*) > 0$ *if* $i \in M^e$, $\pi_i^* = 0$ *otherwise, and* $\pi_f^* = \Pi_{M_f^*} - \sum_{i \in M^e} \bar{\pi}_i(M_f^*)$.

Remark 2. *If we define the intensity of competition for influencer* $i \in M$ *via her equilibrium profit as* $(1 + \pi_i^*)^{-1} \in (0, 1]$, *then more intense competition for at least one influencer increases the firm's profits.*

3.3 Two influencers

To derive more concrete results we now consider $m = 2$. Note first that by Proposition 2, we have that $M^{\mathrm{e}} = M^{k^* - l(\eta_{k^*+1})}$. Second, Assumption 1 implies strict concavity of the sales function, which yields:

Remark 3. $\Pi_{\{i\}} - \Pi_\emptyset > \Pi_M - \Pi_{\{i\}}$ *for all* $i \in M$.

We can now establish that:

Corollary 2. *Suppose that* $m = 2$. *Any undominated SSPE yields payoffs*

- $\pi_1^* = \Pi_M - \Pi_2$, $\pi_2^* = \Pi_M - \Pi_1$, *and* $\pi_f^* = \Pi_1 + \Pi_2 - \Pi_M$ *if* $\Pi_M > \Pi_1$,
- $\pi_1^* = \Pi_1 - \max\{\Pi_2, \Pi_\emptyset\}$, $\pi_2^* = 0$, *and* $\pi_f^* = \max\{\Pi_2, \Pi_\emptyset\}$ *if* $\Pi_1 - \Pi_\emptyset > 0 \geq \Pi_M - \Pi_1$,
- $\pi_1^* = \pi_2^* = 0$, $\pi_f^* = \Pi_\emptyset$ *if* $\Pi_\emptyset \geq \Pi_1$.

Proof of Corollary 2. First, note that by Proposition 2, we have that $M^{\mathrm{e}} = M^{k^* - l(\eta_{k^*+1})}$, i.e. either $M^{\mathrm{e}} = M$, $M^{\mathrm{e}} = \{1\}$ or $M^{\mathrm{e}} = \emptyset$. We proceed by case distinction:

(i) $\Pi_M > \Pi_1$. In this case Assumption 3 and Remark 3 yield $\Pi_1 > \Pi_2 > \Pi_\emptyset$, i.e. $M^{\mathrm{e}} = M$. Thus, by Proposition 3, each $i \in M^{\mathrm{e}}$ receives $\pi_i = \bar{\pi}_i(M^{\mathrm{e}}) = \Pi_M - \Pi_j$, where $j \in M \backslash \{i\}$, and $\pi_f = \Pi_{M^{\mathrm{e}}} - \sum_{i \in M^{\mathrm{e}}} \bar{\pi}_i(M^{\mathrm{e}}) = \Pi_1 + \Pi_2 - \Pi_M$.

(ii) $\Pi_1 - \Pi_\emptyset > 0 \geq \Pi_M - \Pi_1$. Then $M^{\mathrm{e}} = \{1\}$, and hence by Proposition 3 $\pi_1 = \bar{\pi}_1(M^{\mathrm{e}}) = \Pi_1 - \max\{\Pi_2, \Pi_\emptyset\}$, $\pi_2 = 0$, and $\pi_f = \max\{\Pi_2, \Pi_\emptyset\}$.

(iii) $\Pi_\emptyset \geq \Pi_1$. In this case Assumption 3 and Remark 3 yield $\Pi_1 > \Pi_2 > \Pi_M$, i.e. $M^{\mathrm{e}} = \emptyset$. Thus, $\pi_1 = \pi_2 = 0$, and $\pi_f = \Pi_\emptyset$.

□

Notably, hiring both influencers weakens the firm's outside option, and thus her market power, which yields:

Remark 4. *If $\Pi_M > \Pi_1$ such that $M^e = M$, then the firm would benefit from committing to hire only one influencer.*

Example 2. *Consider the following pricing functions:*

$$q_1(\tilde{\theta}) = \begin{cases} \Pi_M - \Pi_2 + c_\theta(\tilde{\theta}) & \text{if } \Pi_M - \Pi_1 > 0 \\ \Pi_1 - \max\{\Pi_2, \Pi_\emptyset\} + c_\theta(\tilde{\theta}) & \text{if } \Pi_M - \Pi_1 \leq 0 < \Pi_1 - \Pi_\emptyset \\ c_\theta(\tilde{\theta}) & \text{else} \end{cases} \tag{11}$$

$$q_2(\tilde{\theta}) = \begin{cases} \Pi_M - \Pi_1 + c_\theta(\tilde{\theta}) & \text{if } \Pi_M - \Pi_1 > 0 \\ c_\theta(\tilde{\theta}) & \text{else} \end{cases} \tag{12}$$

Then, provided the firm chooses optimally $\Big(p(q_M), M_f(q_M), \tilde{\theta}_f(q_M)\Big)$ with efficient equilibrium choices, it can easily be checked that $\Big(q_M, (p(q_M), M_f(q_M), \tilde{\theta}_f(q_M))\Big)$ is an SPE. It is also strong Nash since no coalition can deviate. Hence, such an equilibrium always exists.

4 Concluding remarks

We have proposed a model of social media influencer marketing in which a firm may hire influencers to inform consumers about an innovation. After influencers have set prices for their endorsements, the firm decides which influencers to hire, which message to convey via the influencers, and sets the retail price of the innovation. In equilibrium, influencers price according to their relative contribution to industry profits. Exploiting naïve consumers, they provide exaggerated endorsements if product quality is low. Thus, low-quality products yield low consumer welfare, as naïve consumers pay too much for the product. Notably, the extent of exaggeration decreases in the share of sophisticated consumers, so that increasing the share will also increase the welfare of naïve consumers. Finally, we have shown that the firm may benefit from committing to hire less influencers, because doing so would strengthen her outside option.

In the next step, we plan to analyze the welfare effects of asymmetries between influencers in terms of followers, and to use the framework to understand when and how the market for influencers should be regulated. Furthermore, we plan to extend the framework to competition between multiple firms.

Acknowledgements

We would like to thank Philippe Solal and seminar participants at University of Southampton, Bielefeld University, University of Paris 1, GATE Lyon Saint-Etienne and at the 25th CTN workshop in Barcelona for useful comments and discussions. Manuel Foerster gratefully acknowledges financial support from the German Research Foundation (DFG) under grant FO 1272/2-1.

References

Ajorlou, A., A. Jadbabaie, and A. Kakhbod (2018). Dynamic pricing in social networks: The word-of-mouth effect. *Management Science 64*(2), 971–979.

Anderson, S. P., J. Waldfogel, and D. Stromberg (2015). *Handbook of Media Economics*, Volume 1. Elsevier.

Bloch, F. and N. Quérou (2013). Pricing in social networks. *Games and economic behavior 80*, 243–261.

Campbell, A. (2013). Word-of-mouth communication and percolation in social networks. *American Economic Review 103*(6), 2466–2498.

Candogan, O., K. Bimpikis, and A. Ozdaglar (2012). Optimal pricing in networks with externalities. *Operations Research 60*(4), 883–905.

Chatterjee, K. and B. Dutta (2016). Credibility and strategic learning in networks. *International Economic Review 57*(3), 759–786.

Fainmesser, I. P. and A. Galeotti (2016). Pricing network effects. *The Review of Economic Studies 83*(1), 165–198.

Fainmesser, I. P. and A. Galeotti (2021). The market for online influence. *American Economic Journal: Microeconomics 13*(4), 332–72.

Foerster, M. (2023). A theory of media bias and disinformation. SSRN working paper, available at: https://dx.doi.org/10.2139/ssrn.4008492.

Galeotti, A. and S. Goyal (2009). Influencing the influencers: a theory of strategic diffusion. *The RAND Journal of Economics 40*(3), 509–532.

Gentzkow, M. and J. M. Shapiro (2006). Media bias and reputation. *Journal of Political Economy 114*(2), 280–316.

Hudders, L., S. De Jans, and M. De Veirman (2021). The commercialization of social media stars: a literature review and conceptual framework on the strategic use of social media influencers. *Social Media Influencers in Strategic Communication*, 24–67.

Leduc, M. V., M. O. Jackson, and R. Johari (2017). Pricing and referrals in diffusion on networks. *Games and Economic Behavior 104*, 568–594.

Pew Research Center (2021, April). Social media use in 2021.

Prat, A. and D. Strömberg (2013). The political economy of mass media. *Advances in Economics and Econometrics 2*, 135.

Statista Influencer Marketing Hub (2022, March). Influencer marketing market size worldwide from 2016 to 2022 (in billion u.s. dollars). https://www.statista.com/statistics/1092819/global-influencer-market-size/, accessed April 04, 2023.