Sorting, Technology Choice, and Specialization*

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PRELIMINARY DRAFT

Abstract

We study a search model of the labor market with two-side heterogeneity. Firms' technology choice interacts with labor market frictions to determine the level of their specialization. Firms face a tradeoff between efficient specialized technologies and general technologies that can be operated by a wider variety of worker types. To connect theory with data, we define job specialization as the percent loss of output when matched with a random worker in the labor market compared to the best possible match. Using occupational skill requirements from O*NET we construct a measure of occupational specialization and use it to estimate the response of state and MSA specialization to changes in UI benefits and labor market frictions.

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1 Introduction

The extent to which technology enhances labor productivity depends on the level of specialization. Labor market frictions result in imperfect sorting of worker types into job types. Higher levels of specialization increase potential productivity but make it harder for firms to find workers with skills close enough to the ones required to perform the task. In this paper we focus on the trade-off between general technologies that allow workers with different sets of skills to undertake a certain task (e.g. a shovel for digging, pen and drawing paper for the design of buildings by architects) and specialized technologies that make the ideal (or close to ideal) workers a lot more productive but cannot be operated by those with largely different skills (e.g. a small excavator, computer-aided design software). By setting up a labor market model with firm and worker skill heterogeneity we allow a firm's technology choice to interact with labor market frictions. Excessively high levels of specialization can lead to lower productivity as it greatly reduces the productivity of badly matched workers.

Firms can respond to a reduction in frictions or an increase to UI benefits by adopting more or less specialized technologies. Understanding this mechanism is important in the current period where many countries choose to increase and/or extend UI benefits, for example Mitman and Rabinovich (2020), Ganong et al. (2020), European Commission (2020).

Our model builds on Gautier et al. (2010) and Gautier and Teulings (2015) who analyse a search model where the productivity of a filled job depends on the mismatch between the skills required by the job and the actual skills of the worker. They use an exogenously given production technology where worker types have a comparative advantage for certain job types, but no absolute advantage. The curvature of the production function around the optimal job, measured by the specialization coefficient, determines the importance of mismatch in the production process and is related to the elasticity of complementarity between high and low skilled workers in Katz and Murphy (1992).

We extend this model by letting firms choose the degree of specialization of newly created jobs. Firms face a tradeoff between choosing a technology that makes all workers a bit more productive or one that makes the best matches a lot more productive. Specifically, they face a technological frontier in the two dimensional space of the level of productivity of an ideal match and their specialization.

To calibrate the model we use the 2019 O*NET and OES data on occupational skill requirements and employment shares in the US. Our dataset contains 748 different occupations at the 6-digit SOC level, accounting for more than 97% of US employment. For each occupation it includes the "importance", i.e. the relevance in the production process, and the minimum required "level" of 83 different cognitive and manual skills.

To connect our theory with the data we define the specialization of a job as the expected percentage loss of output when it is filled by a random worker in the labor market, relative to being filled by the best possible match. As such, we calculate occupational specialization, which measures the distance of the occupation minimum skill requirements from the skills of the workers in the market, weighted by the importance of each skill in the occupation's production process. The specialization of a market, e.g. a State or an MSA, is given by the employment-share weighted average of the occupational specializations.

[TO DO:] Using an empirical index for the labor market frictions for MSAs from Gautier and Teulings (2003) we derive the elasticity of specialization with respect to frictions. A similar analysis using the UI benefit replacement rate in each state gives us the elasticity with respect to UI benefits. These elasticities are used to calibrate the technological frontier. The effect on wages, unemployment, and total production along the frontier are studied.

Our paper relates to two strands of the literature. The first is studying the effect of changes in benefits policy on specialization. Hassler et al. (2002) compare the EU with the US and discuss how higher levels of UI benefits incentivise workers in the EU to acquire more specific skills. Our work analyses the connection between UI benefits and specialization in terms of the technology choice of firms. Mukoyama and Sahin (2006) consider a search model of worker specialization, by allowing workers to choose to invest in a combination of skills or focus on one of them. They find that increases in UI benefits make unemployment less painful, hence workers become more specialized in the hope of a good match. This type of specialization can be seen as complimentary to our firm specialization. Our work, in comparison, allows for a continuum of skills both from the worker as well as the firm side. Moreover, the acquisition of skills by workers is a much slower process than firm technology specialization, meaning that the former would not suffice to match the macroelasticity of unemployment. Acemoglu (2001) constructs a search model and discusses the effects of higher minimum wage and unemployment benefits on the creation of more "good" high wage jobs. They show that such changes can lead to productivity gains, using exogenously given technologies, by redistributing labor towards more capital-intensive goods. Similarly, we find that changes in UI benefits can result in higher job productivity by jobs becoming more specialized.

The second strand of related literature considers the effects of specialization on the economy. Grigsby (2020) discusses skills specialization and its effect on the implications of shocks. They find that the 1990-91 and 2008-09 recessions had different effects on aggregate wages due to the fact that skills became less transferable between the 1980s and the 2000s. In the context of our model that would indicate an increase in firm specialization. The recent COVID-19 pandemic can be seen as an example where firms altered their technology relatively quickly (e.g. restaurant doing delivery only, theatres giving online performances). Marinescu et al. (2020) study the effect on job applications and postings of the 2020 CARES

Act, which increased unemployment benefits in the US by $600 per week for the first months of the pandemic. They find that higher replacement rates are associated with fewer applications but not with fewer vacancies. Given firms' technology changes, our model can account for this observed pattern. On the one hand, higher UI benefits lead workers to only take up jobs that match their skills better, resulting in fewer applications. On the other hand, it becomes profitable for firms to specialize. The higher expected profits and the amplified unemployment increase caused by specialization lead to an increase in vacancy creation.

The rest of the paper is organized as follows. Section 2 presents our model. Point of departure is Gautier et al. (2010) without on-the-job search. Then we endogenize firms' technology choice. Section 3 discusses the characteristics of the equilibria that arise, along with a theoretical exposition of unemployment elasticities with respect to UI benefits. Section 4 presents our measurement of occupation and labor market specialization. Finally, Section 6 concludes.

2 Model

We model the labor market of a one sector economy following Gautier et al. (2010). Firms post vacancies, meet workers, and, if the worker accepts the job, they match. The number of per period firm-worker meetings rate is given by the following meeting technology:

$$\lambda = \lambda_0 u^b v^a, \tag{1}$$

where v and u are the vacancy and unemployment rates (relative to $L(t)$, the worker population).

Let j be the skills required to undertake the job and i the skills of the worker. Both are uniformly distributed on a unit circle. $x := |j - i|$ is the *skills gap* between a job of type j and a worker of type i, a measure of mismatch between actual and required skills. The probability distribution of the skills gap, x, in a meeting is given by

$$H(x) = P(X < x|\text{meeting}) = 2x, \quad h(x) = 2, \quad 0 \leq x \leq 1/2, \tag{2}$$

where the factor 2 takes into account that a certain x can be due to i lying to the left or to the right of j on the circle.

Production A matched worker-firm pair produces

$$Y(x) = \alpha(\gamma) \left[1 - \frac{1}{2}\gamma x^2\right]. \tag{3}$$

Output drops when the skills of the worker are not aligned with the required skills. Y depends on the skills only via x, hence workers only have relative, not absolute, advantage at jobs. In the absence of labor market frictions the skills are perfectly aligned and each job produces output α. The production function (3) can be seen as a second order Taylor expansion around $x = 0$ of an arbitrary production function (the first order term is zero because output reaches a maximum at $x = 0$).

We call the coefficient α the *potential output* and γ the *specialization coefficient*. Graphically γ corresponds to the curvature of the production function when plotted against x. It indicates the size of the output loss, relative to α, for a given skills gap. The higher γ is, the more important a precise match becomes. We take α to be a function of γ. Firms, before creating a vacancy, can choose their production technology from an (α, γ) menu. Their choice is constrained by the (α, γ) technological frontier, $\alpha = \alpha(\gamma)$, which we assume to be an increasing function of γ.

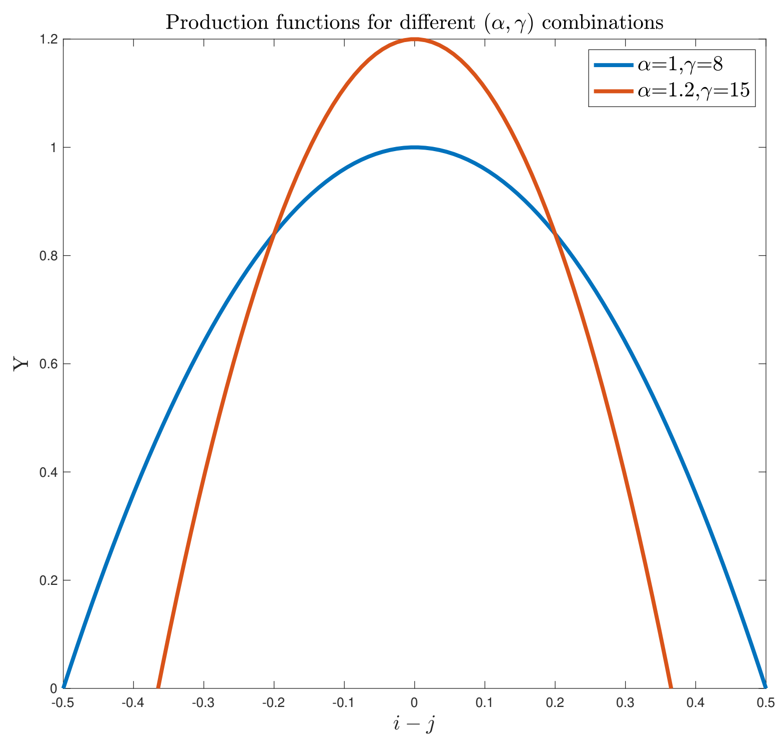

Figure 1: An illustration of two possible production functions

The tradeoff that firms face, illustrated in Figure 1, is between a higher potential output α, which makes good matches more productive, and a lower specialization coefficient, γ, which makes bad matches more productive. In a frictionless economy the firm represented by the orange line is more productive than the blue line firm. In the presence of frictions, though, a large average skills gap can make the productivity of the orange firm to drop below that of the blue firm.

This tradeoff describes situations where firms can choose a specialized, very productive technology, or a less specialised and productive one. A real life example is digging a hole for construction. An excavator is a very efficient machine, but the lack of personnel with the required skills to operate it can reduce its productivity to zero. In such a case, even if more primitive, a shovel can be a much more productive technology to use. Another example are computer-aided design tools that allow architects, designers and engineers to produce and alter a variety of designs for constructions, machines, etc. very efficiently. If the worker is not familiar with the specifics of the software, thought, their productivity drops, making pen and paper, that is in principle less productive, more efficient. As a final example, a new generation of software tools that analyse large amounts of text can massively increase the productivity of young associates at law firms that have to read the documents during the discovery phase of lawsuits. Still, given that the relevant computing skills are not yet in the traditional training of lawyers, it could be rendered unusable, making the traditional reading of boxes upon boxes of files more productive.

The inflow-outflow equation We start by assuming a given equilibrium (α, γ) pair. As in Gautier et al. (2010) we consider an economy on a golden-growth path. That is, the labor force growth rate is equal to the discount rate ρ, $L(t) = e^{\rho t} L_0$. All new workers start from the state of unemployment and jobs are destroyed at rate δ.

On the balanced growth path the inflow and outflow of workers to jobs with skills gap lower than x must be equal to the growth of this subsection of workers according to the golden growth path. New workers start as unemployed, hence

$$\lambda H(x) - \delta(1-u)G(x) = \rho(1-u)G(x), \tag{4}$$

where $1 - u = e$ is the employment rate[1] and $G(x) = P(X < x|\text{match})$ the cumulative distribution function of employed workers with skills gap x. Therefore, the first term on the left-hand-side is the number of people accepting jobs with gap less than x. The second term indicates the existing matches with a skills gap less than x that are destroyed. The right-hand-side indicates that at the balanced growth path the population of all states, including employment at a gap below x, must grow at a rate ρ.

Given that output is decreasing in x, there exists a *reservation skills gap*, $\overline{x}$, at which a firm produces just enough to pay a wage that makes the worker indifferent between accepting and rejecting the job, i.e. the reservation wage $W(\overline{x})$. Workers will accept offers for jobs

[1]The equation holds for values of x that lead to a match

with gap below $\overline{x}$ and reject those above. Hence, $G(\overline{x}) = 1$ and

$$u = 1 - \kappa\overline{x}, \tag{5}$$

$$G(x) = \frac{x}{\overline{x}} \quad \text{and} \quad g(x) = \frac{1}{\overline{x}}, \quad x \leq \overline{x}, \tag{6}$$

where $\kappa := \frac{2\lambda}{\rho+\delta}$.[2]

Value Functions The flow value of employment is given by the wage minus the probability that the job is destroyed times the expected value loss from job destruction:

$$\rho V^E(x) = W(x) - \delta[V^E(x) - V^U]. \tag{7}$$

The flow value of unemployment equals the level of UI benefits and/or home production[3], B, plus the expected gain from finding a job:

$$\rho V^U = B + \frac{2\lambda}{u}\int_0^{\overline{x}}[V^E(x) - V^U]dx := \rho V^E(\overline{x}) = W(\overline{x}). \tag{8}$$

λ/u is the probability for an unemployed worker to meet a vacancy. The second equality is the definition of the reservation skills gap. At $x = \overline{x}$ a worker is indifferent between working and remaining unemployed. The third equality follows using (7).

Combining (8), (7), and (5) gives

$$\begin{aligned} \rho V^U &= uB + (1-u)E_G W = W(\overline{x}), \\ \text{where} \quad E_G W &:= \int_0^{\overline{x}} g(x)W(x)dx. \end{aligned} \tag{9}$$

E_G indicates the expected value over the skills gap distribution of employed workers. Thus the reservation wage is a weighted sum of the flow value of UI benefits and home production, and the expected wage conditional on employment. Given a known $W(x)$ function, the equality in the second line determines the reservation skills gap $\overline{x}$.

Firms are willing to hire workers as long as it is profitable. We normalise the price of the

[2] Note that κ depends on u through λ. Hence (5) does not, in principle, have a closed form solution

[3] We combine both in B

good produced to 1. On the marginal worker, firms spend all production on the wage

$$W(\overline{x}) = Y(\overline{x}) = \alpha \left[1 - \frac{1}{2}\gamma\overline{x}^2\right]. \tag{10}$$

The flow value of a filled job at skills gap x, $V^J(x)$, is given by

$$\rho V^J(x) = Y(x) - W(x) - \delta \left[V^J(x) - V^V\right], \tag{11}$$

where the flow value of an open vacancy, V^V, is given by

$$\rho V^V = -K + \frac{2\lambda}{v}\int_0^{\overline{x}} \left[V^J(x) - V^V\right] dx. \tag{12}$$

K is the cost flow of a vacancy, and λ/v the probability that a vacancy meets a worker. Given free entry, firms will post vacancies up to the point that their value reaches zero, $V^V = 0$, hence

$$vK = (1-u)E_G[Y - W]. \tag{13}$$

$\rho + \delta$ is the effective discounting rate of the firm's profits per match. Thus, all firm profits in the market are spent for the creation of new vacancies.

Wage Formation After meeting, workers and firms bargain for their share of the surplus of the match. Let β be the bargaining power of the worker:

$$W(x) = \text{argmax}_W \left[\hat{V}^E(W) - V^U\right]^\beta \left[\hat{V}^J(W) - V^V\right]^{1-\beta} \tag{14}$$

where the hat indicates that we consider the quantities as a function of the wage, W, rather than x. Due to the monotonicity of the V^E function we can write $\hat{V}^E(W) = V^E(W(x))$[4]. Using (8), (10), (7) and (11) we reach the linear sharing rule

$$W(x) = \beta Y(x) + (1-\beta)Y(\overline{x}). \tag{15}$$

I.e. the wage is a weighted sum of the production and the outside option of the worker, where the bargain power is the weight.

Equations (5), (13), and (9) fully solve for the endogenous variables $\{u, v, \overline{x}\}$. The resulting equilibrium must satisfy $\overline{x} \leq 1/2$. If it does not, (9) is replaced by $\overline{x} = 1/2$

[4]For proof see the appendix of Gautier et al. (2010)

Equilibrium Definition The equilibrium choice of technology for firm i, $(\alpha, \gamma)_i$, satisfies:

1. Given u, v and the technologies of other firms $\{(\alpha, \gamma)_{-i}\}$, firm i maximises $E_H[V^{Ji}((\alpha, \gamma)_i | \{(\alpha, \gamma)_{-i}\})]$

2. If $\max_{\{\alpha_i, \gamma_i\}} E_H[V^{Ji}((\alpha, \gamma)_i | \{(\alpha, \gamma)_{-i}\}] \leq K$ the vacancy is created

The expectation is taken over the distribution of the skills gap, x, in a meeting, taking into account that $V^J = 0$ if no match occurs.

A firm choosing whether to enter the market takes u and v as given. Search is random, hence the probability of meeting a worker is unaffected by the technologies of other firms.[5] The firm chooses the profit maximizing technology, without prior knowledge of the quality of the match. Therefore, the technologies chosen by the other firms do not affect the technology choice of the entering firm, resulting in all vacancies maximising the same profit function and a symmetric equilibrium.

Profit Maximization The equilibrium technology is given by the following constrained maximization problem, where we used (11) and the fact that in equilibrium $V^V = 0$:

$$\max_{\{\alpha_i, \gamma_i\}} E_H[V^{Ji}] = E_H[Y_i(x) - W_i(x)]/(\rho + \delta), \quad \alpha_i = \alpha_i(\gamma_i). \tag{16}$$

In a large market the firm takes the reservation wage of workers, $\overline{W}$, as given. As a result, for a given technology (α_i, γ_i) there is a reservation skills gap that the worker is willing to accept. The firm takes the market unemployment and vacancy levels as given and optimizes its specialization level, taking into account its effect on the reservation skills gap. As discussed before, the actions of other firms do not affect the technology choice decision of the firm. The resulting symmetrical equilibrium, proven in Appendix A, reads

$$\left(1 - \frac{1}{2}\gamma\overline{x}^2\right)\frac{d\alpha}{d\gamma} - \frac{\overline{x}^2}{6}\left[\alpha - 2\gamma\frac{d\alpha}{d\gamma}\right] = 0, \tag{17}$$

$$-\frac{d^2\alpha}{d\gamma^2} > \frac{\alpha\overline{x}^2}{12\gamma}\left[1 + \frac{1}{3}\left(\frac{1 - \frac{1}{2}\gamma\overline{x}^2}{1 - \frac{1}{6}\gamma\overline{x}^2}\right)^2\right] / \left(1 - \frac{1}{6}\gamma\overline{x}^2\right), \tag{18}$$

along with the frontier equation

$$\alpha = \alpha(\gamma). \tag{19}$$

[5] In a directed search model the number of attracted applicants would depend on how many are attracted by other vacancies, i.e. on the technologies chosen by other firms.

This results in an equilibrium $\{u, v, \overline{x}, \alpha, \gamma\}$, which is the solution of a system of five equations: (5), (13), (9), (17), and (19). Similarly to the exogenous model, $\overline{x} \leq 1/2$ must hold.

If the second order condition (18) is not satisfied, the solution $\{u, v, \overline{x}, \alpha, \gamma\}$ represents a a profit minimum, and the equilibrium point is a corner solution with $\gamma \to 0, \infty$. We are not interested in such solutions, as they represent unrealistic scenarios where the skills mismatch is completely irrelevant or where perfectly matched workers produce an infinite amount. We, therefore, consider only points in the parameter space that satisfy (18).

3 Characterization

3.1 Comparative Statics

In this subsection we present numerical results that explore the behaviour of our model by plotting the equilibrium outcomes for a variety of parameter values. The qualitative results are robust to different choices of parameter values, as shown in Appendix D.

Exogenous Technology First we explore the dynamics of the system of a given (α, γ) pair. Figures 2, 3, 4, and 5 present the comparative statics response of the equilibrium $\{u, v, \overline{x}\}$ with respect to α, γ, B, and κ_0.

First, consider the response of equilibrium outcomes to an increase in the potential output α, presented in Figure 2. The worker's incentives, as described by equation (8), are twofold. On the one hand, jobs with a larger skills gap produce enough to pay the reservation wage, pushing for an increase in $\overline{x}$. On the other hand, the value of the optimal $(x = 0)$ match job increases, increasing the reservation wage, and pushing for a decrease in $\overline{x}$. As α increases from a value close to the value of UI benefits, B, the first incentive dominates. When $\alpha = B$ workers do not accept any job offers $(\overline{x} = 0)$. As α increases, firms are able to offer wages above the value of UI benefits for some workers, hence $\overline{x}$ increases. Moreover, expected profits incentivise firms to post more vacancies, increasing the number of matches and amplifying the original drop in unemployment. As α increases further, the second incentive forces $\overline{x}$ to plateau, as the opportunity cost of accepting bad matches increases. According to the free entry condition (13), more meetings and higher expected profits per meeting increase spending in vacancy creation. Unemployment decreases, hence so does the probability of a meeting for a vacancy. But the effect of a higher production, therefore higher expected profits, dominates and vacancy creation rises.

Next, we consider the response of equilibrium outcomes as a function of the specialization coefficient, γ, seen in Figure 3. As γ increases, output decreases and the marginal worker's

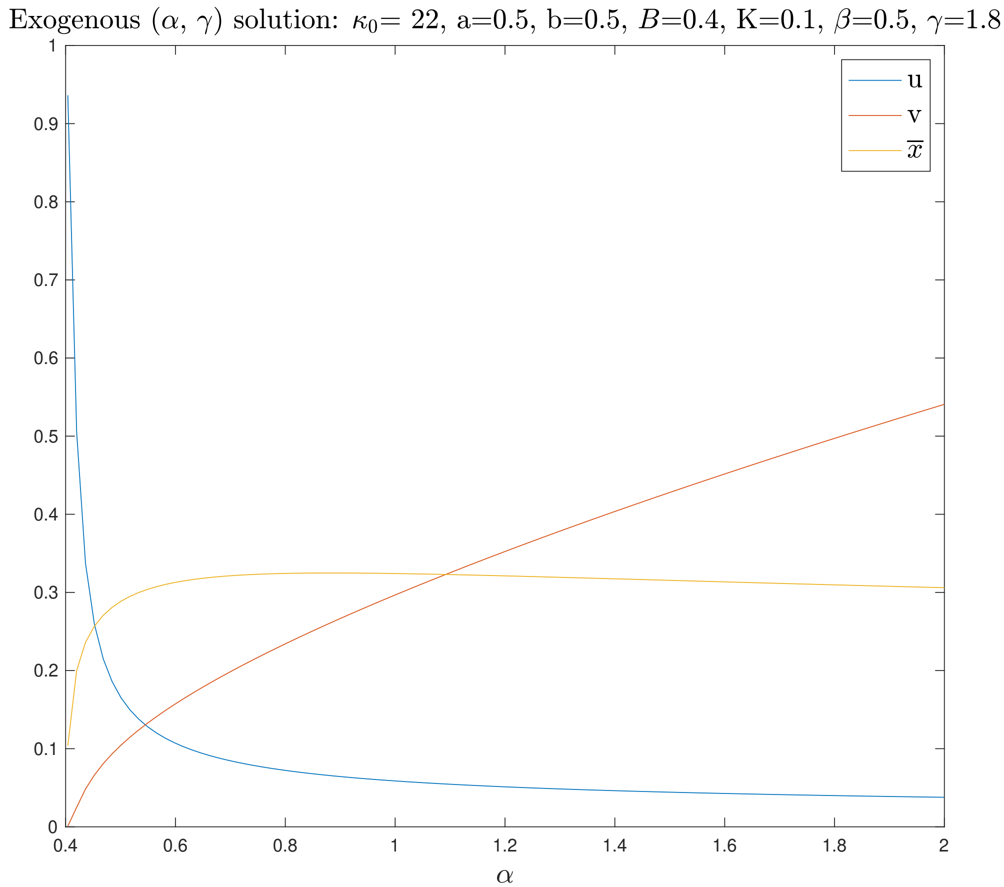

Figure 2: Exogenous technology: Equilibrium outcomes as a function α, for $\gamma = 1.8$.

wage is not enough to induce them to work. Consequently, $\overline{x}$ decreases, as only jobs closer to one's ideal job are accepted. Both expected profits per meeting and unemployment increase, hence, according to the free entry condition (13), vacancies also increase. An increase in v has a negative effect on unemployment, as it increases the probability of a meeting, but the effect of the reduction of $\overline{x}$ (workers becoming more choosy) dominates and unemployment increases slightly.

Next, we present the behavior of equilibrium outcomes for different levels of UI benefits, B. As seen in Figure 4, as B increases, workers have a higher outside option. Therefore, the reservation skills gap, $\overline{x}$, decreases until the new marginal worker is again indifferent between working and being unemployed. The drop in $\overline{x}$ causes fewer matches, and, consequently, an increase in unemployment. As the reservation wage increases, fewer meetings lead to a match, and those that do, result in a higher wage, according to (15). Therefore, expected profits per meeting drop, leading to a decrease in vacancy creation, despite the higher unemployment (i.e. higher probability for a vacancy to meet a worker).

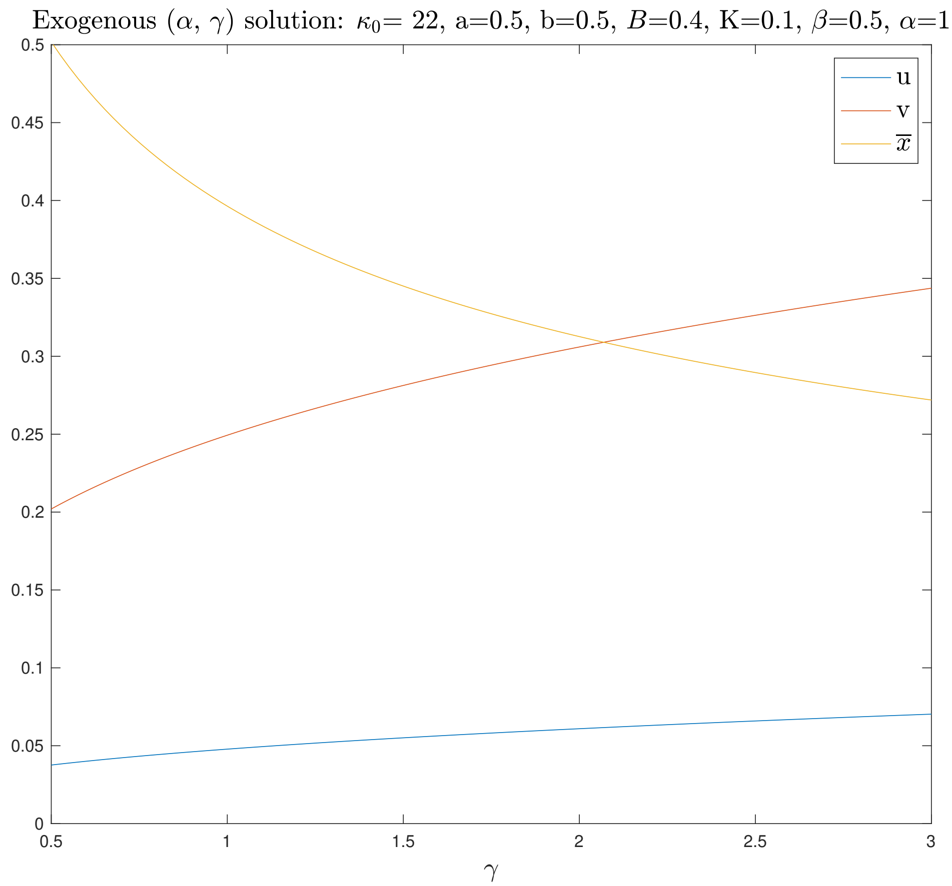

Figure 3: Exogenous technology: Equilibrium outcomes as a function γ, for $\alpha = 1$.

Finally, we present in Figure 5 the response of equilibrium outcomes to changes in the frictions parameter $\kappa_0 = \frac{\lambda_0}{\rho+\delta}$. By equation (1), κ_0 affects the efficiency of the meeting technology. Therefore, a higher κ_0 indicates fewer frictions, where the Walrasian limit is reached for $\kappa_0 \to \infty$. As κ_0 increases, more meetings take place, reducing unemployment. There is a higher probability of a good match, due to the increased meetings rate, increasing the opportunity cost of a bad match. Hence, the reservation skills gap $\overline{x}$ decreases, leading to lower expected profits per meeting, as discussed above. The combination of lower unemployment and lower expected profits results in a drop in vacancy creation, despite the higher meetings technology efficiency.

Endogenous Technology Here we present the comparative statics of the full model, assuming the potential output-specialization frontier has the functional form

$$\alpha(\gamma) = 1 + c\gamma^p, \quad 0 < p < 1. \tag{20}$$

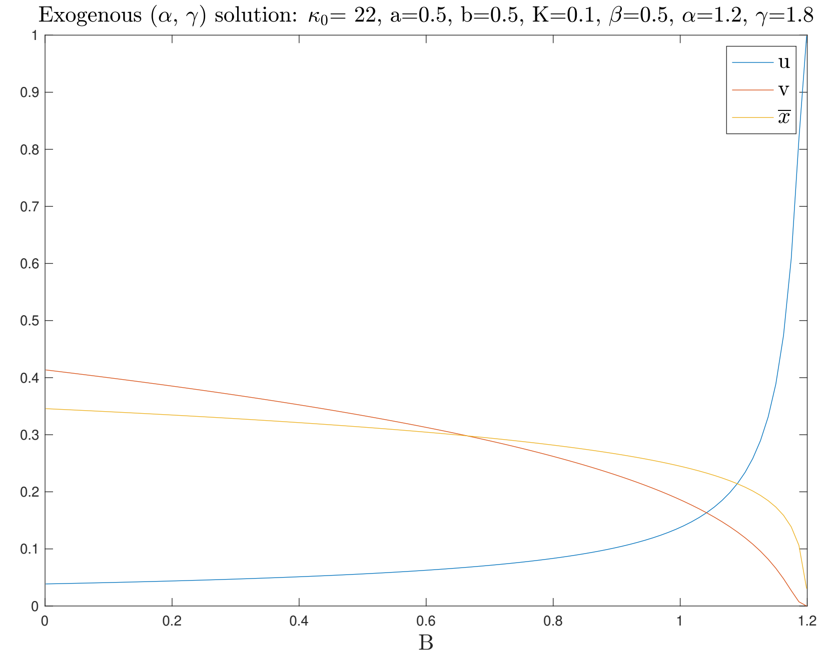

Figure 4: Exogenous technology: Equilibrium outcomes as a function of the size of UI benefits

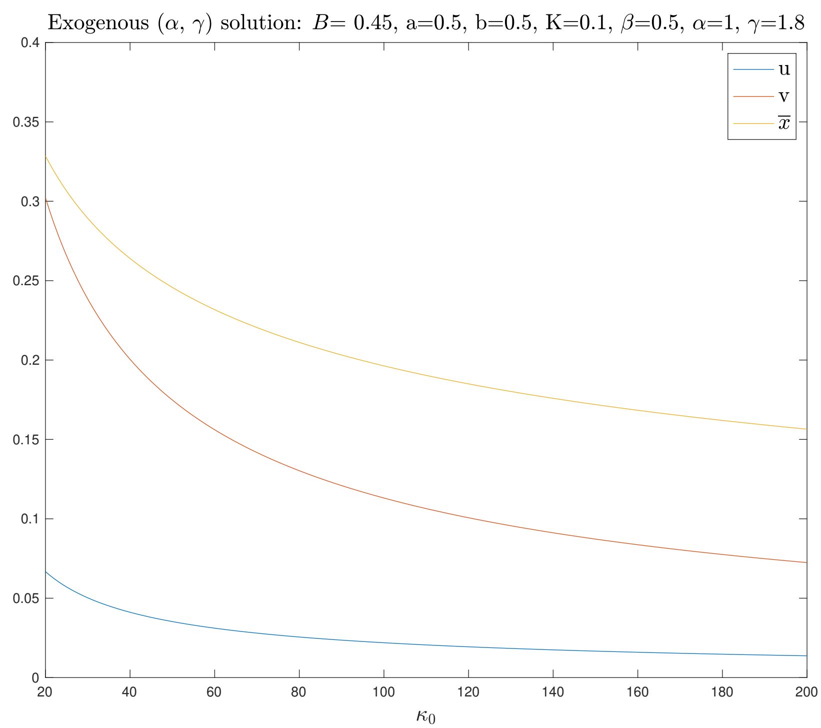

Figure 5: Exogenous technology: Equilibrium outcomes as a function of the frictions parameter κ_0

In Appendix D the comparative statics of various configurations are considered. They all display the same qualitative behavior as the one in the graphs presented below.

Figure 6 displays this behaviour for the comparative statics with respect to $B' = B/\alpha$[6]. Given (20), when γ increases so does α (mechanically, because of the shape of the frontier),

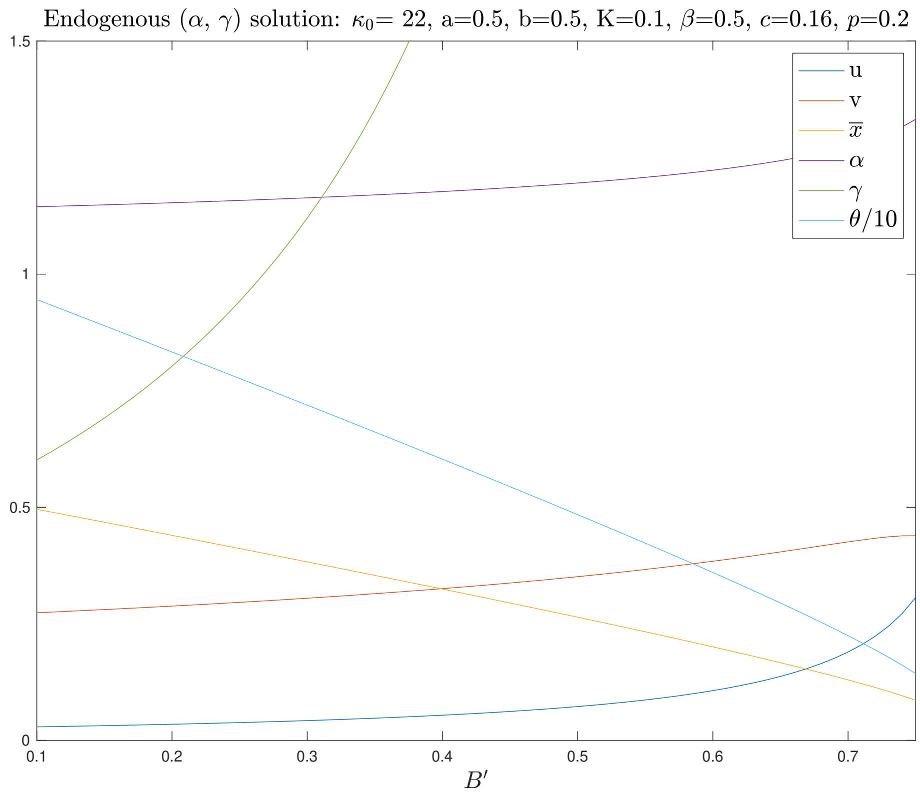

Figure 6: Equilibrium outcomes as a function of the size of UI benefits replacement rate

hence we expect to get a combination of the effects from Figures 2 and 3. As in the exogenous technology case, when B' increases, workers have a higher outside option and $\overline{x}$ decreases. If firms reduce γ, then according to (10) and (9) $\overline{x}$ increases and unemployment falls. However, this reduces the productivity of good matches, as a lower γ corresponds to a lower potential output α on the technology frontier. Firms can also choose to increase γ, meaning that the decrease in $\overline{x}$ is amplified. Fewer workers choose to work, but those that do are more productive and give higher expected profits given a meeting. As we show theoretically in the next subsection, the second option is more profitable for firms in terms of expected profits given a meeting, which is what dictates their technology choice. Therefore, as a response to a higher level of UI benefits firms choose to become more specialized and both unemployment

[6] B' is a proxy of the UI benefits replacement rate, which is a better indicator of the outside option of workers than B (in the exogenous technology model they differ only by a multiplicative constant)

and productivity increase. Due to the technology response, expected profits are also higher compared to the exogenous technology case. As a result, vacancy creation rises, instead of decreasing, as in Figure 4.

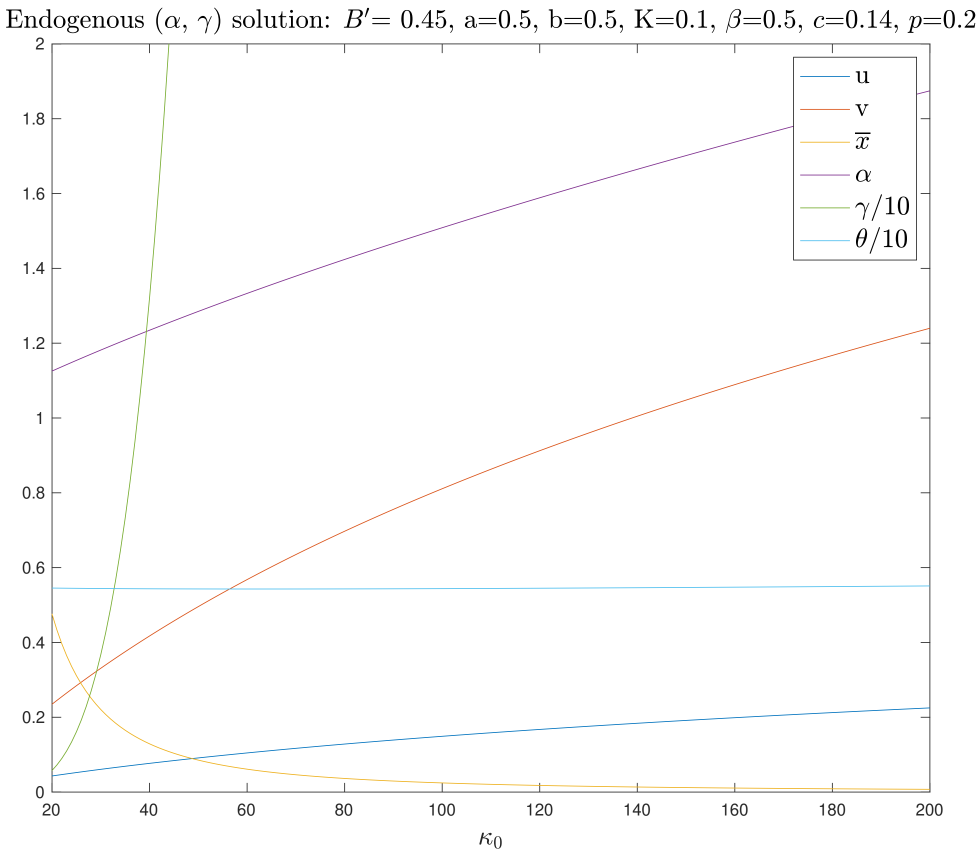

Figure 7: Equilibrium outcomes as a function of the frictions parameter κ_0, holding B' constant

Next, we present in Figure 7 the effect on equilibrium outcomes of a change in the frictions parameter κ_0. Given the technology response of firms, we hold B' constant instead of B. The effect of an increase in κ_0 (fewer frictions) is similar qualitatively to the effect of B' in Figure 6. Workers reduce their reservation skills gap, $\overline{x}$, due to the higher outside option as in the exogenous technology case in Figure 5. Due to the higher probability of a good match, firms increase the mismatch parameter, γ, in order to increase the potential output, α. I.e. firms specialize as they have a higher chance of meeting a suitable candidate. In accordance to our explanation for Figure 6, vacancy creation increases.

The increased specialization amplifies the decrease of $\overline{x}$, resulting to a slight increase in unemployment. I.e. as the meeting rate increases it leads to a higher level of specialization, resulting to a reduced overall rate of matching and higher unemployment. The frontier equation (20) allows firms to infinitely increase their productivity by specializing more (higher γ). In reality, though, there is a physical upper limit to productivity, $\overline{\alpha}$. Hence, at some point, as frictions decrease, firms reach the corner solution $\alpha = \overline{\alpha}$. Thereafter, for higher values of κ_0, the equilibrium behaves as in the exogenous technology scenario, with unemployment decreasing and reaching zero in the frictionless case.

3.2 UI Benefits-Unemployment Elasticity

In this subsection we analyse theoretically the response of the equilibrium outcomes to an increase in the replacement rate $B' = B/\alpha$. The proofs for the expressions presented below can be found in Appendix B. In the context of our model, partial equilibrium microelasticities are identified by the exogenous technology case where vacancy supply is fixed. The general equilibrium macroelasticities are identified when allowing both the vacancy supply and technology to adjust.

First, we present the elasticity of unemployment with respect to the UI benefits replacement rate, B', for the case where technology is exogenous and there is no free entry of new vacancies, $\epsilon^{\text{NF}}_{uB'}$. As the level of UI benefits increases, workers have a better outside option, hence, they accept only job offers with a sufficiently high wage, i.e. a better match, decreasing the reservation skills gap, $\overline{x}$. This leads to fewer matches and, consequently, to an increase in unemployment. The resulting elasticity reads

$$\epsilon^{\text{NF}}_{uB'} = \frac{B'}{\gamma \overline{x}^2 \left(\left[1 - (1-u)\left(1 - \frac{2\beta}{3}\right)\right] \left[\frac{1}{1-u} + \frac{b}{u}\right] + \frac{\beta}{3u}\right)} > 0. \tag{21}$$

Allowing for free entry of new vacancies the elasticity, $\epsilon^{\text{Exog}}_{uB'}$, reads

$$\epsilon^{\text{Exog}}_{uB'} = \epsilon^{\text{NF}}_{uB'} - E_1 \epsilon^{\text{Exog}}_{vB'}, \tag{22}$$

where $\epsilon^{\text{Exog}}_{vB'} = \frac{dv}{dB'}\frac{B'}{v} < 0$ is the elasticity of the vacancy supply and $E_1 > 0$. Both expressions are presented in Appendix B. The intuition is the following: as B' increases firms have a lower incentive to post vacancies as, due to the increased reservation wage, there are lower expected profits from a meeting. Therefore, vacancies drop, reducing the meeting rate, and amplifying the increase in unemployment.

Finally, considering our endogenous technology choice model gives the elasticity

$$\epsilon^{\text{Endog}}_{uB'} = \epsilon^{\text{Exog}}_{uB'} + E_2 \epsilon^{\text{Endog}}_{\gamma B'}, \tag{23}$$

where E_2 and the elasticity of specialization, $\epsilon^{\text{Endog}}_{\gamma B'} > 0$, are shown to be positive for relevant values of the model parameters [In the process: proving analytically]. Intuitively, the increase in the UI benefits replacement rate incentivises firms to specialize (γ increases), which reduces the number of worker types that accept a certain job ($\overline{x}$ decreases), increasing unemployment further.

Therefore,

$$\epsilon_{uB'}^{\text{Endog}} > \epsilon_{uB'}^{\text{Exog}} > \epsilon_{uB'}^{\text{NF}}. \tag{24}$$

4 Measuring Specialization

First, we define the specialization of a job and a labor market below in a way that makes it easy to bring our model to the data.

Definition 1. *The specialization of a job within a given labor market is the expected percentage loss of output when this job is filled by a random worker in this labor market, relative to being filled with the output-maximizing worker for this job in this labor market. The specialization of a labor market is the average specialization of jobs within this labor market.*

This definition is consistent with the notion of specialization in the model, where the strength of specialization is governed by the specialization coefficient γ. Using equation (3), the expected percentage output loss in the definition yields the specialization of a job

$$2\int_{2}^{1/2} \frac{Y(0) - Y(x)}{Y(0)} = \frac{\gamma}{24}. \tag{25}$$

Since this quantity is the same for all jobs, it is also the specialization of the labor market.

In order to make this definition operational we consider a more general setting for the labor market. Let there be J jobs indexed by $j \in \mathcal{J}$, I workers indexed by $i \in \mathcal{I}$, and N skills indexed by $n \in \mathcal{N}$. The skill bundle of worker i is $s^i = \{s_n^i\}_{n \in \mathcal{N}}$. If firm j produces with worker i, output is given by $Y^j(s^i)$ where $Y^j : \mathcal{I} \to \mathcal{R}$ is a production function.

Let

$$Y^{*j} = \max_{i \in \mathcal{I}} Y^j(s^i) \tag{26}$$

denote the output of firm j if it is assigned to the worker that maximizes its output in this labor market. Applying the definition, the specialization of job j is

$$\frac{1}{I} \sum_{i \in \mathcal{I}} \frac{Y^{*j} - Y^j(s^i)}{Y^{*j}} \tag{27}$$

and the specialization of the labor market is

$$\frac{1}{J}\sum_{j\in\mathcal{J}}\frac{1}{I}\sum_{i\in\mathcal{I}}\frac{Y^{*j}-Y^{j}(s^{i})}{Y^{*j}}. \tag{28}$$

Data We use data from the U.S. on occupational skills requirements and employment shares. The former are taken from the 2019 O*NET survey datasets and the latter from the Occupational Employment Statistics (OES) datasets of the U.S. Bureau of Labor Statistics (BLS), which include the occupational employment shares on the U.S., State, and Metropolitan Statistical Area (MSA) level.

The O*NET "Skills" and "Abilities" datasets include information on manual and cognitive skills (henceforth "the skills") required for an occupation, defined at the level of 8-digit SOC codes. For each such skill an "importance" score denotes how important (i.e. relevant) the skill is in the performance of the job. The score has a 1-5 range with "Not Important" at 1 and "Extremely Important" at 5. A "level" score denotes the minimum level at which this skill is required and has a 1-7 range. For example, the skill requirements for "Mathematics" score 2 denotes that a worker should be able to "Count the amount of change to be given to a customer", score 4 "Calculate the square footage of a new home under construction", and score 6 "Develop a mathematical model to simulate and resolve an engineering problem".

The employment data of OES are defined on the 6-digit SOC level and are matched with the occupations in O*NET using a crosswalk provided by the BLS. Therefore, our analysis is on the 6-digit SOC level and includes 748 different occupations, accounting for more than 97% of US employment.

Calculating Specialization Next, we adopt a specification of the production function that makes use of the information from O*NET. From now on, a job is associated with an occupation, and this determines its technology. Let $\{r_n^j\}_{n\in\mathcal{N}}$ denote the O*NET minimum required skill levels, and $\{q_n^j\}_{n\in\mathcal{N}}$ the corresponding importance levels. We normalize the importance indicator to the range $[0, 8]$ and the skill level to the range $[0, 1/2]$[7] and set the production function to

$$Y^{j}(s^{i})=\alpha\prod_{n\in\mathcal{N}}\left[1-\frac{1}{2}q_n^j(r_n^j-s_n^i)^2\right]^{\mathbb{1}(r_n^j>s_n^i)}. \tag{29}$$

This indicates that the productivity of a firm-worker match is reduced whenever the worker's skill level is below the required skill level. The percent reduction in productivity is propor-

[7]The normalization of the skill level follows the normalization of the skills gap x from our model. The normalization of the importance is such that a worker with the maximum skills gap in a skill with the maximum importance produces an output of 0.

tional to the skill gap $r_n^j - s_n^i$ and the importance of that skill in production q_n^j. This is a generalization of our production function (3) for the case of the O*NET multi-dimensional skill requirements . In the case of only one skill, $N = 1$, the production function reduces to equation (3), with $x = r_n^j - s_n^i$ and $\gamma = q_n^j$. Firm specialization then equals the importance it attaches on the skill requirement and the skills gap equal the distance between the minimum skill requirement and the worker's actual skill level. The only difference is the indicator function, which takes into account that minimum skill requirements are vertically ordered in the O*NET data. Hence, a skills gap exists only if the worker's actual skill level is below the required one.

As in our model, we assume that the number of jobs and workers is the same for each skill type. This is theoretically supported by Gautier et al. (2010). They show that under firm free entry the vacancy required skills distribution endogenously matches the workers' skills distribution. We formalize this assumption by requiring that the number of jobs and workers is equal and that there is a one-to-one mapping between jobs and workers $m : \mathcal{J} \to \mathcal{I}$ associating each job with a worker that perfectly matches its minimum skill requirements, that is, $s_n^{m(j)} = r_n^j$ for all $n \in \mathcal{N}$ and for all $j \in \mathcal{J}$. Under this assumption it is clear that $Y^{*j} = \alpha(\gamma^j)$.

Since we assume that technology does not vary within an occupation, the labor market specialization is

$$\sum_{o \in \mathcal{O}} l_o \sum_{o' \in \mathcal{O}} l_{o'} \left[1 - \prod_{n \in \mathcal{N}} \left[1 - \frac{1}{2} q_n^o (r_n^o - r_n^{o'})^2 \right]^{\mathbb{1}(r_n^o > r_n^{o'})} \right], \tag{30}$$

where $o \in \mathcal{O}$ indexes occupations, l_o is the employment share of occupation o in the labor market, $\{r_n^o\}_{n \in \mathcal{N}}$ are minimum required skill levels in occupation o, and $\{q_j^o\}_{n \in \mathcal{N}}$ are the corresponding importance levels. Analogous to the specialization of a job within the labor market, the specialization of occupation o in the labor market is

$$\sum_{o' \in \mathcal{O}} l_{o'} \left[1 - \prod_{n \in \mathcal{N}} \left[1 - \frac{1}{2} q_n^o (r_n^o - r_n^{o'})^2 \right]^{\mathbb{1}(r_n^o > r_n^{o'})} \right]. \tag{31}$$

Intuitively, firms choose their specialization level by choosing the distance of their skill requirements from the workers' skills in the market, for the skills that are important in their production process. For example, pilots are a specialized occupation since in order to be a good pilot it is important that a worker has a high level of both manual and cognitive skills relative to most other workers in the market. This means that an average worker in the labor market will have skills that fall far short of the required skills to be a pilot and, thus, would be a lot less productive compared to a perfect match. In order to be a good secondary school teacher a relatively high level of cognitive skills is important, while manual skills are not very important. Alternatively, to be a good motorcycle mechanic a high level of manual skills is important, while cognitive skills are less important. Thus, secondary school teachers and motorcycle mechanics are medium-specialization occupations. Finally, to be a good dishwasher neither high manual nor cognitive skills are important. Thus, an average worker in the market is approximately as productive as a worker that fully satisfies the skill requirements, hence the dishwasher occupation has a low level of specialization.

The interpretation of our data analysis is that a certain labor market (e.g. a state or a metropolitan area) is specialized because firms have chosen to set up their production process using specialized jobs. For example, a medical centre can hire mostly general practitioners or mostly specialized doctors, such as surgeons. A restaurant can have two cooks and a few waiters and waitresses, or be more specialized by having a chef, a cook, a barista, a bartender, and a smaller number of waiters and waitresses. As such, the specialization of the labor market is a measure of the dispersion of the required skills by firms in the market relative to the actual workers' skills.

5 Quantitative Results (Incomplete)

In this section we examine the behaviour of labor market specialization across 343 US MSAs.[8] Figure 8 presents the specialization distribution of the 748 US occupations and of the jobs in the US labor market. These two differ, as low-specialization occupations have a higher share in the labor market than mid and high-specialization ones.

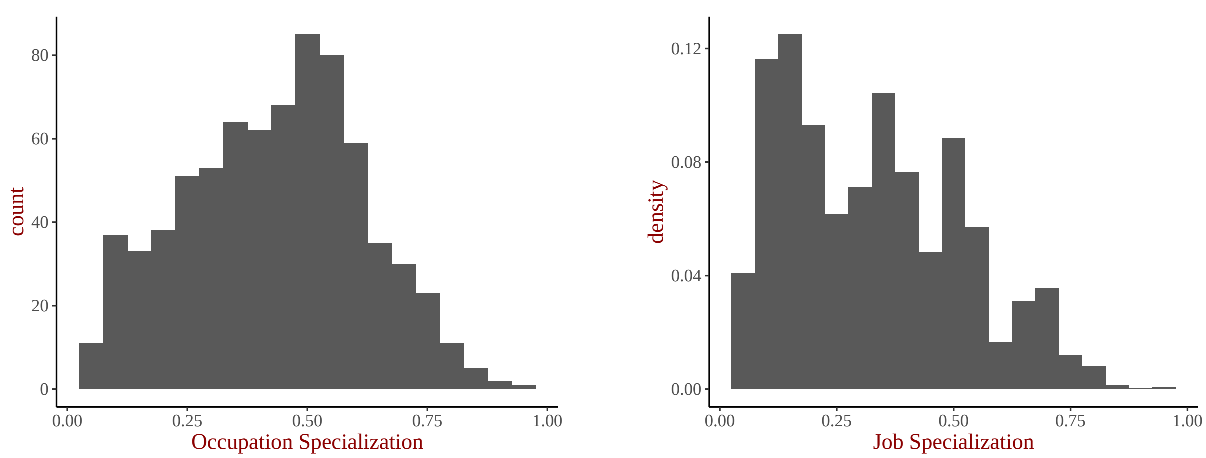

Figure 8: The distribution of the specialization of occupations (left) and jobs (right) in the US labour market

[8]An indirect calibration of the model that infers information about specialization through the micro and macro-elasticities of unemployment with respect to UI benefits is presented in Appendix C.

Airline pilots is the most specialized occupation, followed by physicists and surgeons. A random worker in the US market would have an output of less than 10% of the output of the perfect candidate in these occupations. Flight attendants, budget analysts, and painters and construction maintenance workers are mid-specialization occupations, with a random worker being roughly 65% as productive as the perfect much. Finally, receptionists, locker room attendants, and fast food cooks are low-specialization occupations. A random worker is around 95% as productive as the perfect match in such occupations.

Figure 9 plots the median occupational wages against specialization. The two measures have a positive relationship, but are do not have a one to one correspondence, with their correlation coefficient equal to 0.47. Choreographers, for example, have a specialization similar to that of a microbiologist (around 075) but a median wage roughly 35% lower.

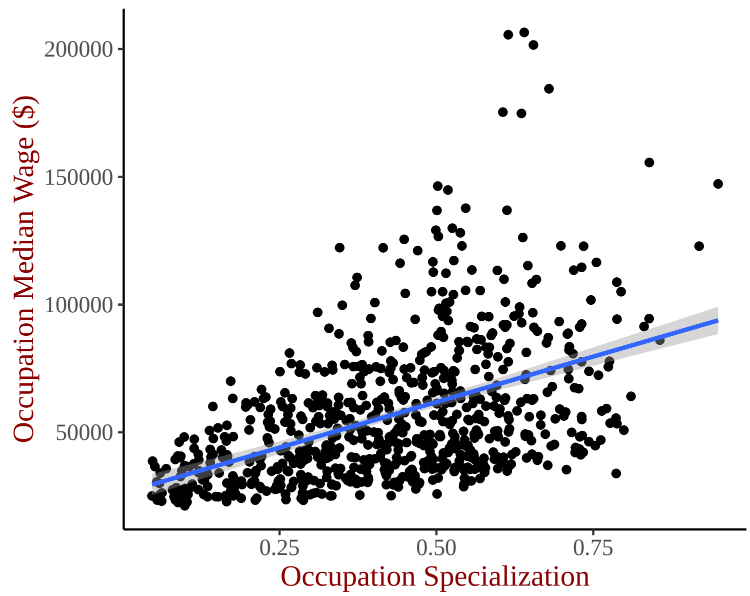

Figure 9: The US-wide median occupational wages plotted against the occupational specialization.

Figure 10 shows the specialization distribution of the US MSAs. Note that, according to our definition, occupations have different specialization in different labor markets. The MSA specialization is the average specialization of these MSA-specific occupational specialisations. The most specialized MSAs are around 30% more specialized than the least specialized ones. As seen in Figure 11 the within-MSA specialization distribution are left-skewed. There is a bunching at low-specialization (mostly low-wage) occupations in all MSAs, with more specialized MSAs having a less skewed and less variant distribution. This indicates that low-specialization MSAs have a more polarized (in terms of specialization) labor market.

Next we examine the dependence of specialization on measures of labor market frictions. We use the 2019 US census estimates for the populations of the MSAs and the 2010 distance-

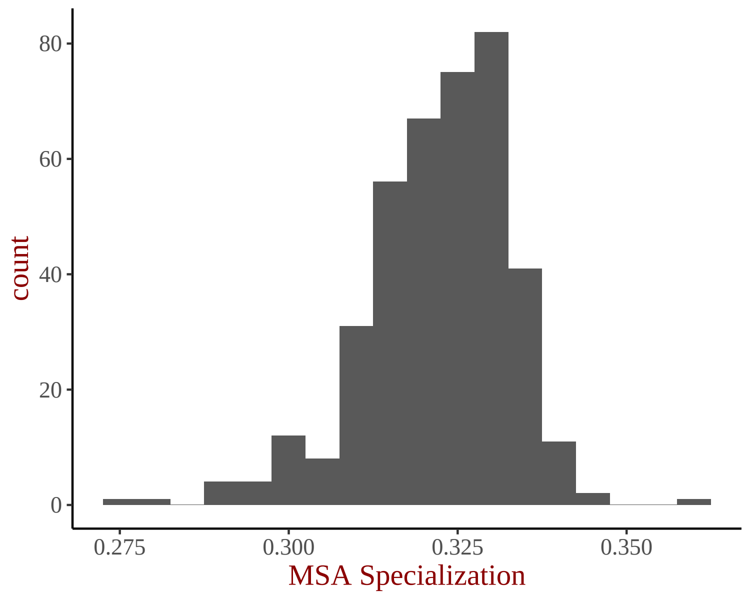

Figure 10: The specialization distribution of US MSAs.

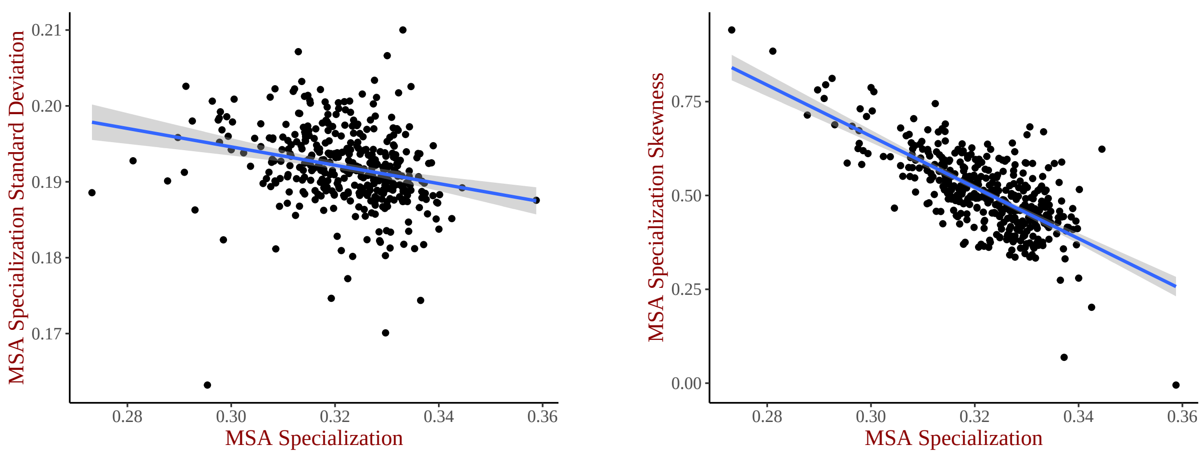

Figure 11: The within-MSA standard deviation (left) and skewness (right) of specialization against the average specialisation of the MSA.

from-the-centre weighted population densities. Figure 12 plots MSA specialization against their population and population density. These two measures are highly correlated. Table 1 shows the result of regressing MSA specialization against these two measures. The effect of population is strongly positive, with the coefficient of the population density insignificantly different than zero.

[In the process: Further work on this point and connection with frictions. Moreover, the dependence of specialization on UI benefits will be explored]

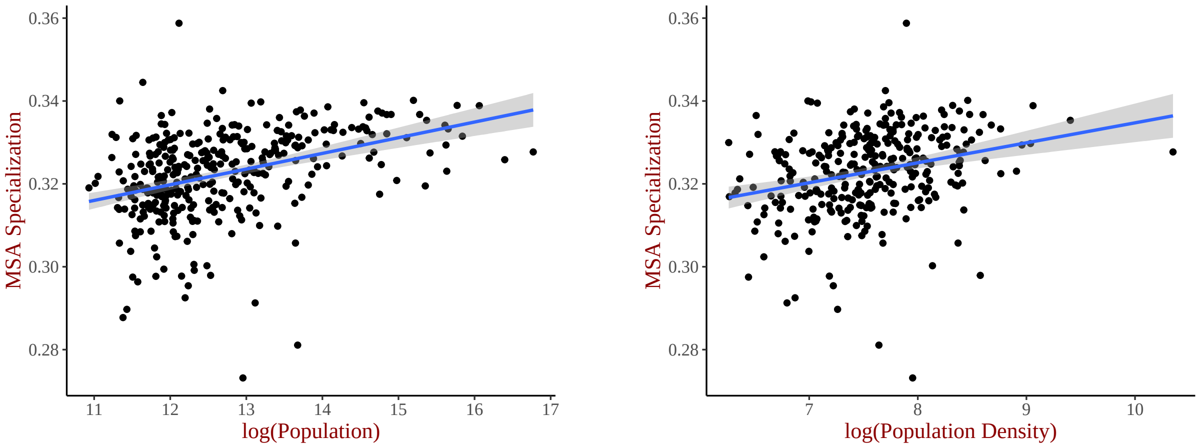

Figure 12: MSA specialization against MSA population(left) and population density (right).

	MSA Specialization
log(Population Density)	0.001
	(0.001)
log(Population)	0.003***
	(0.001)
Constant	0.272***
	(0.007)
Observations	318
R^2	0.155
Note:	*p<0.1; **p<0.05; ***p<0.01

Table 1: The regression of the MSA specialization against population and population density.

6 Final Remarks

In the absence of labor market frictions, firms choose the most productive technology for their production process, as they are able to find workers whose skills perfectly match the ones required to perform the task. Frictions drive a wedge between the actual skills of the workers and the skills required to operate the chosen technology. This gives rise to a tradeoff between very productive specialized technologies and general technologies that can be operated by a wider variety of worker types. We model this tradeoff in terms of a technology frontier from which firms choose their production technology.

In order to connect our theory with data we define a measure for job and labor market specialization. Job specialization is define as the percent loss of output when the job is matched with a random worker in the market, compared to finding its perfect match. Market specialization is the employment-share weighted average of job specialziations within the market. We generalize our production function to account for the multidimensional skill requirements found in O*NET data and calculate occupational specialization for 748 occupations in the US. Finally, we calculate the specialization of the US states and MSAs. We find that specialization increases with population.

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A Endogenous Technology Profit Maximization

In this appendix we prove the first and second order conditions for the firm profit maximization (17) and (18).

The equilibrium technology is given by the following constrained maximization problem, where we used (11) and the fact that in equilibrium $V^V = 0$:

$$\max_{\{\alpha_i,\gamma_i\}} E_H[V^{Ji}] = E_H[Y_i(x) - W_i(x)]/(\rho+\delta), \quad \alpha_i = \alpha_i(\gamma_i). \tag{32}$$

In a large market, the firm takes the reservation wage of workers, $\overline{W}$, as given. Given a match, the firm and the worker bargain over the wage as in (14).

Equations (7) and (8) give

$$W_i(x) = \beta Y_i(x) + (1-\beta)\overline{W}. \tag{33}$$

For a given technology (α_i, γ_i) there is a reservation skills gap that the worker is willing to accept, $\overline{x}_i$, which satisfies $W_i(\overline{x}_i) := \overline{W}$. The expected profit reads

$$\begin{aligned} E_H[Y_i(x) - W_i(x)] &= (1-\beta)\int_0^{\overline{x}_i} \left[Y_i(x) - \overline{W}\right] dH(x) \\ &= (1-\beta)\frac{1}{3}\alpha_i\gamma_i\overline{x}_i^3, \end{aligned} \tag{34}$$

and the first order condition is given by

$$\begin{aligned} \frac{dE_H[\Pi_i]}{d\gamma_i} &= \left.\frac{\partial E_H[\Pi_i]}{\partial\gamma_i}\right|_{\alpha_i} + \left.\frac{\partial E_H[\Pi_i]}{\partial\alpha_i}\right|_{\gamma_i}\frac{d\alpha_i}{d\gamma_i} = 0 \\ \Rightarrow \overline{x}_i\left(\alpha_i + \gamma_i\frac{d\alpha_i}{d\gamma_i}\right) &+ 3\alpha_i\gamma_i\left[\frac{\partial\overline{x}_i}{\partial\gamma_i} + \frac{d\alpha_i}{d\gamma_i}\frac{\partial\overline{x}_i}{\partial\alpha_i}\right] = 0. \end{aligned} \tag{35}$$

Using the definition of the reservation skills gap gives

$$\begin{aligned} Y_i(\overline{x}_i) &= \overline{W} \\ \Rightarrow \frac{\partial}{\partial\alpha_i}\alpha_i(1 - \frac{1}{2}\gamma_i\overline{x}_i^2) &= \frac{\partial\overline{W}}{\partial\alpha_i} = 0 \\ \Leftrightarrow \frac{\partial\overline{x}_i}{\partial\alpha_i} &= \frac{1-\frac{1}{2}\gamma_i\overline{x}^2}{\alpha_i\gamma_i\overline{x}_i}, \end{aligned} \tag{36}$$

and similarly

$$\frac{\partial\overline{x}_i}{\partial\gamma_i} = -\frac{\overline{x}_i}{2\gamma_i}. \tag{37}$$

Hence, the first order condition for the profit maximisation simplifies to

$$\left(1-\frac{1}{2}\gamma_i\overline{x}_i^2\right)\frac{d\alpha_i}{d\gamma_i}-\frac{\overline{x}_i^2}{6}\left[\alpha_i-2\gamma_i\frac{d\alpha_i}{d\gamma_i}\right]=0. \tag{38}$$

From (33) $\overline{x}_i = W_i^{-1}(\overline{W}) = \overline{x}_i(\alpha_i, \gamma_i, \overline{W})$ and the frontier of the technology menu reads $\alpha_i = \alpha_i(\gamma_i)$. Hence, (38) can be expressed in terms of γ as a function of $\overline{W}$, which is the same for all firms. Therefore, all firms choose the same $\gamma_i = \gamma$, resulting in $\alpha_i = \alpha$ and $\overline{x}_i = \overline{x}$. The model, then, collapses to the one sector model of Section 2 with technology given by

$$\left(1-\frac{1}{2}\gamma\overline{x}^2\right)\frac{d\alpha}{d\gamma}-\frac{\overline{x}^2}{6}\left[\alpha-2\gamma\frac{d\alpha}{d\gamma}\right]=0 \tag{39}$$

and

$$\alpha=\alpha(\gamma) \tag{40}$$

This results in an equilibrium $\{u, v, \overline{x}, \alpha, \gamma\}$ which is the solution of a system of five equations: (5), (13), (9), (39), and (40). Similarly to the exogenous model, $\overline{x} \leq 1/2$ must hold.

Second Order Condition

The first order condition (39) describes maximised profit if the second order condition

$$\frac{d^2E_H[\Pi]}{d\gamma^2}<0 \tag{41}$$

holds. The i subscripts have been dropped since this condition holds for all firms. If not, the solution found above is a minimum and the equilibrium point is a corner solution with $\gamma \to 0, \infty$. We are not interested in such solutions, as they represent unrealistic scenarios where the skills mismatch is completely irrelevant or where perfectly matched workers produce an infinite amount. We, therefore, consider only points in the parameter space that satisfy (41).

Similarly to (35), using (36) and (37) we get

$$\begin{aligned} \frac{d^2 E_H[\Pi]}{d\gamma^2} &= \frac{\partial}{\partial\gamma}\left(\frac{dE_H[\Pi]}{d\gamma}\right)\Big|_{\alpha} + \frac{\partial}{\partial\alpha}\left(\frac{dE_H[\Pi]}{d\gamma}\right)\Big|_{\gamma}\frac{d\alpha}{d\gamma} \\ &= (1-\beta)\overline{x}\left[\left(1-\frac{1}{6}\gamma\overline{x}^2\right)\frac{d^2\alpha}{d\gamma^2} + \frac{\alpha\overline{x}^2}{12\gamma}\left[1+\frac{1}{3}\left(\frac{1-\frac{1}{2}\gamma\overline{x}^2}{1-\frac{1}{6}\gamma\overline{x}^2}\right)^2\right]\right] \\ \Rightarrow -\frac{d^2\alpha}{d\gamma^2} &> \frac{\alpha\overline{x}^2}{12\gamma}\left[1+\frac{1}{3}\left(\frac{1-\frac{1}{2}\gamma\overline{x}^2}{1-\frac{1}{6}\gamma\overline{x}^2}\right)^2\right] \Big/ \left(1-\frac{1}{6}\gamma\overline{x}^2\right) > \frac{\alpha\overline{x}^2}{9\gamma}. \end{aligned} \tag{42}$$

B Unemployment Elasticity Proofs

In this appendix we prove the expressions presented in Subsection 3.2 for the response of the equilibrium equilibrium to an increase in the replacement rate $B' = B/\alpha$.

We start by writing the exogenous technology model equations (5), (13), (9) and the endogenous technology model equations (17), (20) in a convenient form using, equations (3) and (10)

$$u = 1 - \kappa\overline{x}, \quad \kappa = \frac{2\lambda}{\rho+\delta}, \quad \lambda = \lambda_0 u^b v^a \tag{43}$$

$$v\frac{3K}{1-\beta} = (1-u)\alpha\gamma\overline{x}^2 \tag{44}$$

$$1 - \frac{1}{2}\gamma\overline{x}^2 = uB' + (1-u)\left[1-\frac{1}{2}\gamma\overline{x}^2\left(1-\frac{2\beta}{3}\right)\right] \tag{45}$$

$$\left(1-\frac{1}{2}\gamma\overline{x}^2\right)p(\alpha-1) = \frac{\gamma\overline{x}^2}{6}\left[\alpha - 2p(\alpha-1)\right] \tag{46}$$

$$\alpha(\gamma) = 1 + c\gamma^p, \tag{47}$$

where in getting (46) from (17) (47) was used.

No Free Entry We take α, γ, v to be fixed and use equations (43) and (45). Using logarithmic differentiation on (43) we get

$$\begin{aligned} \frac{1}{\lambda}\frac{d\lambda}{dB'} &= \frac{b}{u}\frac{du}{dB'} \\ \Rightarrow \frac{1}{\kappa}\frac{d\kappa}{dB'} &= \frac{b}{u}\frac{du}{dB'} \\ \Rightarrow -\left[\frac{1}{1-u}+\frac{b}{u}\right]\frac{du}{dB'} &= \frac{1}{\overline{x}}\frac{d\overline{x}}{dB'}. \end{aligned} \tag{48}$$

Similarly, from (45) we get

$$\gamma\overline{x}\left[1-(1-u)\left(1-\frac{2\beta}{3}\right)\right]\frac{d\overline{x}}{dB'}=\left[\left(1-\frac{1}{2}\gamma\overline{x}^2\left(1-\frac{2\beta}{3}\right)\right)-B'\right]\frac{du}{dB'}-u. \tag{49}$$

Using equation (9) to get $1-\frac{1}{2}\gamma\overline{x}^2\left(1-\frac{2\beta}{3}\right)-B'=\frac{\beta\gamma\overline{x}^2}{3u}$ and combining with (48) gives

$$\begin{aligned}\frac{du}{dB'}&=\frac{u}{\gamma\overline{x}^2\left(\left[1-(1-u)\left(1-\frac{2\beta}{3}\right)\right]\left[\frac{1}{1-u}+\frac{b}{u}\right]+\frac{\beta}{3u}\right)}\\ \Rightarrow \epsilon_{uB'}^{\mathrm{NF}}&=\frac{du}{dB'}\frac{B'}{u}>0.\end{aligned} \tag{50}$$

Exogenous Technology Now we take α, γ to be fixed and use equations (43), (44), and (45). Similarly to the previous calculation we get

$$\begin{aligned}-\left[\frac{1}{1-u}+\frac{b}{u}\right]\frac{du}{dB'}&=\frac{a}{v}\frac{dv}{dB'}+\frac{1}{\overline{x}}\frac{d\overline{x}}{dB'}\\ \gamma\overline{x}\left[1-(1-u)\left(1-\frac{2\beta}{3}\right)\right]\frac{d\overline{x}}{dB'}&=\left[\left(1-\frac{1}{2}\gamma\overline{x}^2\left(1-\frac{2\beta}{3}\right)\right)-B'\right]\frac{du}{dB'}-u\\ \Rightarrow -\gamma\overline{x}^2\left[1-(1-u)\left(1-\frac{2\beta}{3}\right)\right]\left[\left[\frac{1}{1-u}+\frac{b}{u}\right]\frac{du}{dB'}+\frac{a}{v}\frac{dv}{dB'}\right]&=\\ &=\left[\left(1-\frac{1}{2}\gamma\overline{x}^2\left(1-\frac{2\beta}{3}\right)\right)-B'\right]\frac{du}{dB'}-u,\end{aligned} \tag{51}$$

where the last equation came from combining the first two. Using the result in (50) the elasticity reads

$$\begin{aligned}\epsilon_{uB'}^{\mathrm{Exog}}&=\epsilon_{uB'}^{\mathrm{NF}}\\ &-\frac{a\left[1-(1-u)\left(1-\frac{2\beta}{3}\right)\right]}{u\left(\left[1-(1-u)\left(1-\frac{2\beta}{3}\right)\right]\left[\frac{1}{1-u}+\frac{b}{u}\right]+\frac{\beta}{3u}\right)}\epsilon_{vB'}^{\mathrm{Exog}},\end{aligned} \tag{52}$$

where $\epsilon_{vB'}^{\mathrm{Exog}}=\frac{dv}{dB'}\frac{B'}{v}$. Logarithmic differentiation of (44) gives

$$\begin{aligned}\frac{1}{v}\frac{dv}{dB'}+\frac{1}{1-u}\frac{du}{dB'}&=\frac{2}{\overline{x}}\frac{d\overline{x}}{dB'}\\ \Rightarrow \frac{1+2a}{v}\frac{dv}{dB'}+\left[\frac{3}{1-u}+\frac{2b}{u}\right]\frac{du}{dB'}&=0,\end{aligned} \tag{53}$$

where for the second equality the first equation of (51) was used. Combining it with the last equation of (51) gives

$$\begin{aligned}\frac{du}{dB'} &= \frac{u}{\gamma\overline{x}^2\left(\frac{1}{1+2a}\left[1-(1-u)\left(1-\frac{2\beta}{3}\right)\right]\left[\frac{1-a}{1-u}+\frac{b}{u}\right]+\frac{\beta}{3u}\right)} \\ &\Rightarrow \epsilon_{uB'}^{\text{Exog}} > \epsilon_{uB'}^{\text{NF}} \Rightarrow \epsilon_{vB'}^{\text{Exog}} < 0,\end{aligned} \tag{54}$$

where the first inequality comes from the fact that the derivative is the same as in (50) with the first term in the denominator being multiplied by $\frac{1}{1+2a} < 1$ and the $\frac{1}{1-u}$ by $0 < 1-a < 1$. The second inequality comes from (52).

Endogenous Technology Here we allow α, γ to also vary and use equations (43), (44), (45), (46), and (47). From (47) it follows that

$$\begin{aligned}\frac{d\alpha}{d\gamma} &= \frac{p}{\gamma}(\alpha-1) \\ \Rightarrow \frac{d\alpha}{dB'} &= \frac{p}{\gamma}(\alpha-1)\frac{d\gamma}{dB'}.\end{aligned} \tag{55}$$

The second order condition (18) implies that

$$p(1-p)\frac{\alpha-1}{\alpha} > \frac{\gamma\overline{x}^2}{12}\left[1+\frac{1}{3}\left(\frac{1-\frac{1}{2}\gamma\overline{x}^2}{1-\frac{1}{6}\gamma\overline{x}^2}\right)^2\right]\Big/\left(1-\frac{1}{6}\gamma\overline{x}^2\right) > \frac{\gamma\overline{x}^2}{9}. \tag{56}$$

For reasonable values of unemployment we find numerically that this is satisfied for $p \lessapprox 1/3$ and $a+b \leq 1$.

Accordingly, (43), (44), (45) give

$$\begin{aligned}-\left[\frac{1}{1-u}+\frac{b}{u}\right]\frac{du}{dB'} &= \frac{a}{v}\frac{dv}{dB'}+\frac{1}{\overline{x}}\frac{d\overline{x}}{dB'} \\ \frac{1}{v}\frac{dv}{dB'}+\frac{1}{1-u}\frac{du}{dB'} &= \frac{1+p(1-1/\alpha)}{\gamma}\frac{d\gamma}{dB'}+\frac{2}{\overline{x}}\frac{d\overline{x}}{dB'} \\ \left[1-(1-u)\left(1-\frac{2\beta}{3}\right)\right]\left[\frac{\overline{x}^2}{2}\frac{d\gamma}{dB'}+\gamma\overline{x}\frac{d\overline{x}}{dB'}\right] &= \left[\left(1-\frac{1}{2}\gamma\overline{x}^2\left(1-\frac{2\beta}{3}\right)\right)-B'\right]\frac{du}{dB'}-u.\end{aligned} \tag{57}$$

Using the first equation to eliminate the derivative of $\overline{x}$ from the other two, then combining them in order to eliminate the derives of v, and using our result in (52) we get

$$\epsilon_{uB'}^{\text{Endog}} = \epsilon_{uB'}^{\text{Exog}} + \frac{[1-2ap\,(1-1/\alpha)]\left[1-(1-u)\left(1-\frac{2\beta}{3}\right)\right]}{2u\left(\left[1-(1-u)\left(1-\frac{2\beta}{3}\right)\right]\left[\frac{1-a}{1-u}+\frac{b}{u}\right]+(1+2a)\frac{\beta}{3u}\right)}\epsilon_{\gamma B'}^{\text{Endog}}, \tag{58}$$

where the term multiplying $\epsilon_{\gamma B'}^{\text{Endog}}$ is always positive for $a \le 1/2p$, which holds for $p \lessapprox 1/3$ and $a+b \le 1$.

Differentiating (46) and using (47) gives

$$\left[1-\frac{1}{\alpha(1+1/p)-1}\right]\frac{1}{\gamma}\frac{d\gamma}{dB'} = -\frac{2}{\overline{x}}\frac{d\overline{x}}{dB'}. \tag{59}$$

Given that $\alpha > 1$ and $p < 1$ the term in between parentheses on the left-hand-side is positive. Therefore, this equation indicates γ moves in the opposite direction of the $\overline{x}$ movement. If, as a response to an increase in UI benefits, the reservation skills gap decreases, firms will choose to specialize, increasing γ, in response. According to equation (58) this will amplify the unemployment increase.

Using, next, the first equation of (57) to eliminate the derivatives of v from the second equation and combining with (59) yields:

$$\begin{aligned} \frac{du}{dB'} &= \frac{e_1(a,\alpha,p)}{2\gamma\left[\frac{1-a}{1-u}+\frac{b}{u}\right]}\frac{d\gamma}{dB'} \\ \text{where} \quad e_1(a,\alpha,p) &= 1-\frac{p}{1+p\frac{\alpha-1}{\alpha}}\left[\frac{1}{\alpha}+2a\left(1+p\left(\frac{\alpha-1}{\alpha}\right)^2\right)\right] \\ &> 1-p(1+2a) > 0, \end{aligned} \tag{60}$$

where in proving the last inequality we used that $p < 1/3$ and $a+b \le 1$. This equation indicates that unemployment and the specialization coefficient, γ move in the same direction as a response to an increase in B'.

Inserting into (58) gives

$$\begin{aligned} \frac{du}{dB'} &= \frac{u}{\gamma\overline{x}^2\left(\frac{e_2(a,\alpha,p)}{1+2a}\left[1-(1-u)\left(1-\frac{2\beta}{3}\right)\right]\left[\frac{1}{1-u}+\frac{b}{u}\right]+\frac{\beta}{3u}\right)} \\ \text{where} \quad e_2(a,\alpha,p) &= 1-\frac{1-2ap(1-1/\alpha)}{e_1(a,\alpha,p)}. \end{aligned} \tag{61}$$

Given that $u > 0$ we have that $\frac{du}{dB'} \neq 0$. From (60) the derivatives $\frac{du}{dB'}$ and $\frac{d\gamma}{dB'}$ have the same sign. From our numerical analysis, see e.g. Figure 6, we have that $\frac{d\gamma}{dB'} > 0$ for at least one point in the parameter space. Therefore, while changing the parameters continuously in the parameter space the derivatives can become negative only discontinuously. This would indicate a corner solution. Given that we are not interested in such solutions, the derivatives must be positive [9], hence from (58) we have that

$$\epsilon^{\text{Endog}}_{\gamma B'}, \epsilon^{\text{Endog}}_{uB'} > 0 \Rightarrow \epsilon^{\text{Endog}}_{uB'} > \epsilon^{\text{Exog}}_{uB'}. \tag{62}$$

Therefore, we have that $\epsilon^{\text{Endog}}_{uB'} > \epsilon^{\text{Exog}}_{uB'} > \epsilon^{\text{NF}}_{uB'}$. Finally note that equations (50), (52), and (58) depend on γ and $\overline{x}$ only through the product $\gamma\overline{x}^2$, which by equation (45) is determined by u and B'. Moreover, by equation (46) α is a function of $\gamma\overline{x}^2$ and p. Therefore, the elasticities for the three different scenarios are determined by $\{a, b, \beta, u, B'\}$.

C An Indirect Calibration of the Model

In this appendix we calibrate the model indirectly, by inferring the response of specialization to changes in UI benefits through the gap between the macro and micro-elasticities of unemployment with respect to UI benefits. We target the level of unemployment and calibrate the replacement rate, B' so that the exogenous technology elasticity matches the observed microelasticity of unemployment. Then, using the endogenous technology case, the observed macroelasticity and specialization coefficient, γ, are matched to calibrate the technology frontier.

Defining $K' := K/\alpha$, $v' := vK'$, and $\kappa_0' := \frac{\kappa_0}{(K')^a\sqrt{\gamma}}$, the system of equations that solves for the exogenous technology equilibrium $\{u, v, \overline{x}\}$ (5), (13), and (9) can be written, using (3) and (10), as

$$u = 1 - \kappa_0' u^b (v')^a \sqrt{\frac{3v'}{(1-u)(1-\beta)}} \tag{63}$$

$$1 - \frac{1}{2}\frac{3v'}{(1-u)(1-\beta)} = uB' + (1-u)\left[1 - \frac{1}{2}\frac{3v'}{(1-u)(1-\beta)}\left(1 - \frac{2\beta}{3}\right)\right] \tag{64}$$

$$\gamma\overline{x}^2 = \frac{3v'}{(1-u)(1-\beta)}. \tag{65}$$

The endogenous technology model includes, also, equations (17) and (20) which, combined

[9] In the case of disjoint sets of interior solutions the proof holds for the open set of interior solutions around a point where the derivatives are positive. As will be seen in our calibration, below, the derivatives are positive in the region of interest. Moreover, the derivatives are positive in all the robustness checks performed.

with the exogenous model equation (65), give

$$\alpha = 1 + \frac{1}{6p\left(\frac{(1-u)(1-\beta)}{3v'} - \frac{1}{2}\right) - (1-2p)} \tag{66}$$

$$\gamma = \left(\frac{\alpha - 1}{c}\right)^{1/p}. \tag{67}$$

The last five equations jointly determine the equilibrium $\{u, v, \overline{x}, \alpha, \gamma\}$.

Costain and Reiter (2008) calculate unemployment semielasticities with respect to the replacement rate of UI benefits, using cross-country data. The data allows them to estimate macro-semielasticities, in the sense that they compare economies where the general equilibrium effects of different UI benefits are taken into account. In our framework, this corresponds to elasticities where firms can both create new vacancies and adjust their technology. They compare their values with a range of values for the elasticity of unemployment duration from Layard et al. (1991) that identify partial equilibrium effects from within-country estimation methods. Their estimates are derived using microdata and identify the effect of changes in UI benefits on worker choices. In our model these correspond to the case where only workers react to changes in UI benefits by changing their reservation skills gap and firms have fixed vacancies and technology.

Costain and Reiter (2008) explore a variety of estimation methods and find values for the semi-elasticity around 3. In our model we take $B' = B/\alpha$ as the proxy for the replacement rate, as it divides the UI benefits to the potential output which is proportional to wages, as seen in equation (15). At some $B' = B'_1$ and $u = u_1$ the semi-elasticity of unemployment with respect to B' is numerically calculated as

$$\sigma_{uB'} = \frac{(u_2 - u_1)/u_1}{B'_2 - B'_1} = \frac{\epsilon_{uB'}}{B_1}, \tag{68}$$

where B'_2 is close to B'_1 and u_2 is the corresponding unemployment level. $\epsilon_{uB'}$ denotes the elasticity of unemployment with respect to UI benefits.

Layard et al. (1991) report values for the elasticity of unemployment duration, D, ranging between 0.2 and 0.9. Given that

$$\begin{aligned} D &= \frac{1}{\text{matching rate}/u} = \frac{u}{\lambda 2\overline{x}} = \frac{(\rho+\delta)u}{1-u} \\ \Rightarrow \epsilon_{uB'} &= (1-u)\epsilon_{DB'}, \end{aligned} \tag{69}$$

the unemployment of elasticity is numerically very close to that of the duration. For our calibration we target the microelasticity of unemployment at 0.45.

For the calibration we target the unemployment level, u, the specialization coefficient, γ, and the microelasticity, ${\epsilon^{\text{NF}}}_{uB'}$, and macrosemielasticity of unemployment with respect to the replacement rate of UI benefits, ${\sigma^{\text{END}}}_{uB'}$.

We set $a = b = 0.5$ and $\beta = 0.5$. First, we target $u_{\text{target}} = 5\%$, which determines κ_0', as the unemployment level is informative of the frictions present in the labor market. Next, we target ${\epsilon^{\text{NF}}}^{\text{target}}_{uB'} = 0.45$. In the exogenous technology model without free entry the elasticity is determined by the level of the UI benefits, hence targeting the microelasticity fixes B'. As seen in Figure 13 this is easily achieved due to the monotonic relation between $\epsilon^{\text{NF}}_{uB'}$ and B' (yellow line).

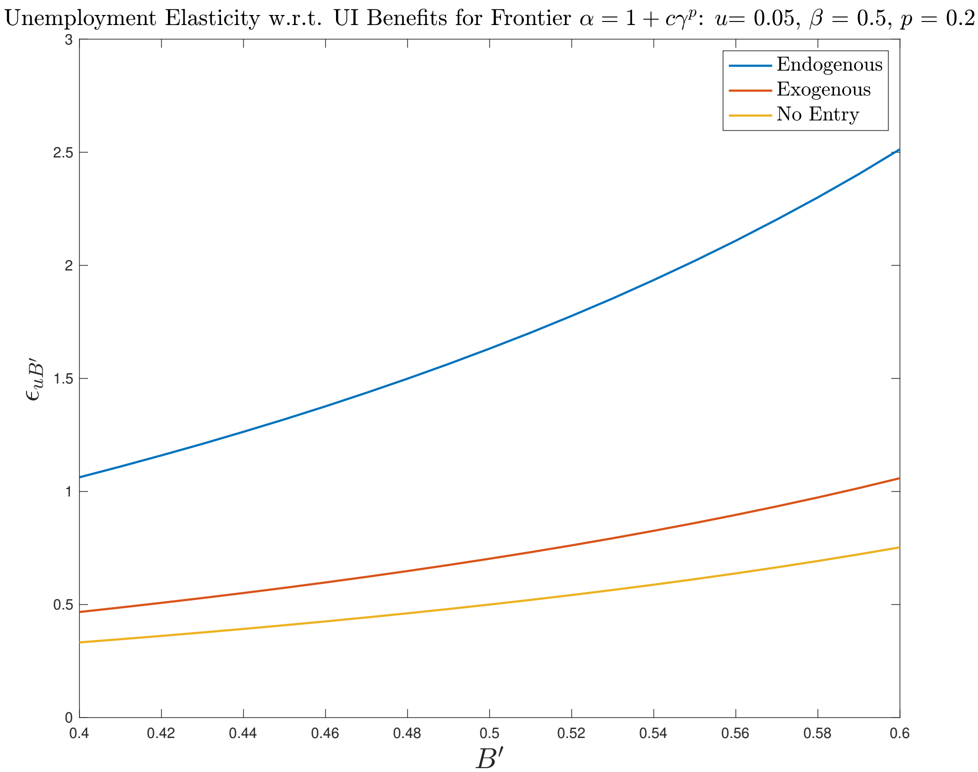

Figure 13: Unemployment elasticity with respect to B' as a function of B' for the three different versions of the model: exogenous technology with fixed vacancies, exogenous technology with free entry, endogenous technology.

Finally, for the calibration of the frontier, we target ${\sigma^{\text{END}}}^{\text{target}}_{uB'} = 3$. The macroelasticity is indicative of the technology response of firms, hence it allows us to calibrate the power, p, of the frontier equation (67). As seen in Figure 14 (blue line) this is easily achieved due to the monotonic relation between $\epsilon^{\text{END}}_{uB'}$ and p. As in Gautier and Teulings (2015), using the elasticity of complementarity between high and low-skilled workers from Katz and Murphy (1992), we target the specialization coefficient $\gamma_{\text{target}} = 1.8$. This allows us to calibrate c.

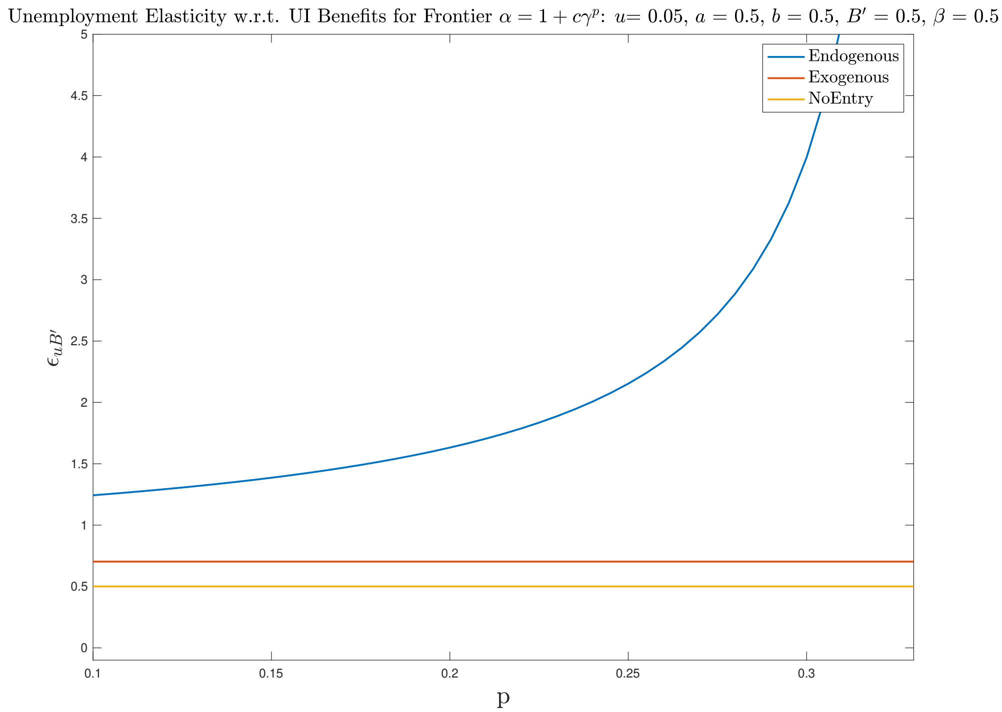

Figure 14: Unemployment elasticity with respect to B' as a function of p. For the exogenous technology with fixed vacancies and the exogenous technology with free entry the elasticity is fixed, determined by u_{target}. For endogenous technology it depends on the power p.

Figure 14 also shows that allowing for free entry of new vacancies (red line) is not enough to explain the gap between the micro (yellow line, targeted at 0.45) and macroelasticity of unemployment (targeted at around 1.5).

The result of the calibration is shown in Table 2, and of the resulting equilibrium outcomes in Table 3. Alternative calibrations and their outcomes are presented in Appendix D.

Table 2: Calibrated Parameters

κ_0'	B'	c	p
74.6460	0.4754	0.1274	0.1953

Aggregate Output Here we present the behavior of the aggregate output per capita to changes in frictions and UI benefits, and compare them with the case of exogenous technology. Then, we study how aggregate output and labor market outcomes are affected by changes in the specialization level of the economy along the technological frontier. Gautier and Teulings (2015) calculate the output loss due to frictions for the exogenous technology case. In the

Table 3: Calibrated Equilibrium

u	vK	$\overline{x}$	α	γ	B
0.0500	0.0226	0.2819	1.1429	1.8000	0.5434

The equilibrium of the calibrated model

endogenous case this measure is not well defined as, according to the frontier equation (20), in a frictionless environment a firm can choose α to reach infinity, as increasing γ has no effect due to perfect matching. Therefore, we calculate the aggregate output per capita, y, instead, and calculate its local response to changes in frictions and UI benefits. This allows us to identify how aggregate output responds in the short run (fixed technology) compared to the long run (endogenous technology).

Aggregate output per capita, y, of the economy is given by

$$y = (1-u)E_G Y + uB - vK, \tag{70}$$

where $G(x)$ is the skills gap distribution of employed workers. The first term denotes the output of employed workers, the second the home production of unemployed workers, and the third the cost of vacancies. The free entry condition, (13), states that all profits are spent on vacancy creation. This simplifies y to

$$\begin{aligned} y &= (1-u)E_G W + uB \\ &= W(\overline{x}) = Y(\overline{x}), \end{aligned} \tag{71}$$

where in going to the second line we use equation (9). The final equality comes from (10). It states that the aggregate output per capita equals the production of the marginal worker. It's elasticity with respect to the friction parameter κ_0 and the home production replacement rate, B', are given by

$$\epsilon_{y\kappa_0} = \frac{dy}{d\kappa_0}\frac{\kappa_0}{y}\bigg|_{B'} \qquad \& \qquad \epsilon_{yB'} = \frac{dy}{dB'}\frac{B'}{y}\bigg|_{\kappa_0}. \tag{72}$$

These can easily be calculated numerically (evaluated at the calibrated equilibrium in Table 3). The results are presented in Table 4.

The results indicate that a doubling of κ_0, roughly translating into a doubling of the meeting rate between workers and firms, increases output by 5%. This increase is slightly larger in the endogenous technology case, as firms react by specializing. For the elasticity with respect to B' this increase is about half in size. The differences between the endogenous

Table 4: aggregate Output and specialization coefficient Elasticities

	Exogenous	Endogenous	γ
κ_0	0.0488	0.0516	4.4560
B'	0.0256	0.0271	2.3378

The first two columns present the aggregate output per capita elasticity with respect to κ_0 and B' at the calibrated equilibrium, for the exogenous and endogenous technology cases. The third columns presents the elasticity of γ with respect to those parameters in the endogenous technology case.

and exogenous cases are driven by the technology response of the firms, which is a factor 2 smaller for a change in B' compared to κ_0. This suggests that the effect of technology choice on aggregate output is rather small.

In order to isolate the effect of a change in the specialization coefficient, γ, we present in Table 5 the elasticities of various equilibrium outcomes with respect to γ along the calibrated frontier, for fixed B', evaluated at the calibrated equilibrium. This gives the percent change of these variables if all firms in the economy used a more specialized technology from the frontier, increasing γ by 1%.

Table 5: Elasticities Along the Calibrated Frontier

u	v	y	$E_G W$	$W(0)/W(\overline{x})$	$E_G[Y]$	α
0.3387	0.3143	0.0006	0.0089	0.0123	0.0168	0.0243

The elasticity of various equilibrium outcomes with respect to γ along the calibrated frontier, evaluated at the calibrated equilibrium.

The elasticity of α indicates how the potential output changes with an increase in γ. The elasticity of the mean production per employed worker, $E_G[Y]$, is lower than the one for the potential output, α, as the output loss due to mismatch increases as well. $W(0)/W(\overline{x})$, the ratio of the maximum to the minimum wage in the economy is a measure of wage inequality. Without on-the-job search, though, the wage distribution cannot be matched well, hence we do not put much emphasis on this value.

Unemployment, u, and vacancies, v, respond strongly to changes in γ. Aggregate output per capita, y, and the average wage of an employed worker, $E_G W$, barely respond. This is also demonstrated by Figure 15, which plots the variables along the frontier.

These elasticities indicate that the different points on the technology frontier are very good substitutes in terms of aggregate production and average wages, but with very different values for unemployment and vacancy supply. The frontier, therefore, represents choices for the firms that result in similar aggregate output per capita and low specialization, unem-

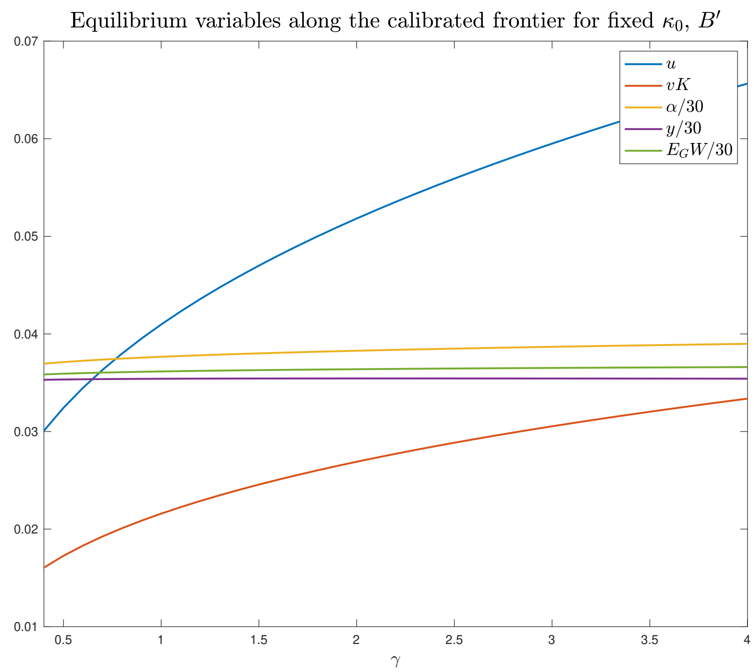

Figure 15: Equilibrium variables along the calibrated frontier. Note that some of them have been rescaled

ployment, and vacancies or high specialization, unemployment, and vacancies. This result is robust to different targeted values for the elasticities, the specialization coefficient, and the unemployment level, as can be seen in Appendix D.

Intuitively, a higher level of specialization increases output and wages of good matches. As a result, workers are less willing to accept bad matches, and unemployment increases. Firms, in turn, increase vacancy creation due to the increased unemployment (i.e. the higher probability of a meeting for a vacancy) and expected profits per meeting. Labor market tightness decreases slightly. Aggregate output remains almost constant, as the gains from higher productivity are lost to higher unemployment and vacancy costs. This indicates that countries with a similar level of GDP per capita can have very different levels of specialization, unemployment and vacancy supply.

D Robustness Checks

In this appendix we present a few robustness checks for the comparative statics of Subsection 3.1, the calibration and the response of equilibrium outcomes to changes in the specialization coefficient along the calibrated frontier of Appendix C. Each part of the Appendix is titled in reference to the figure or table that we perform the robustness checks for. The parameters for which the check is run is shown on the title of the figures, and presented before the tables. All of our checks display the same qualitative behavior as the one presented in the main body of the text.

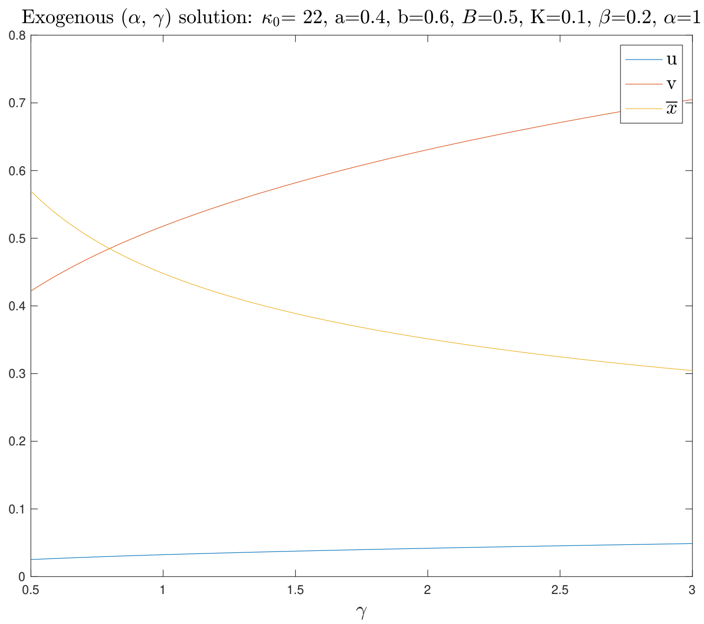

Figure 16: Robustness check for Figure 3

Figure 3

Figure 4

Figure 5

Figure 6

Figure 7

Alternative Calibrations, Tables 2, 3, and 5

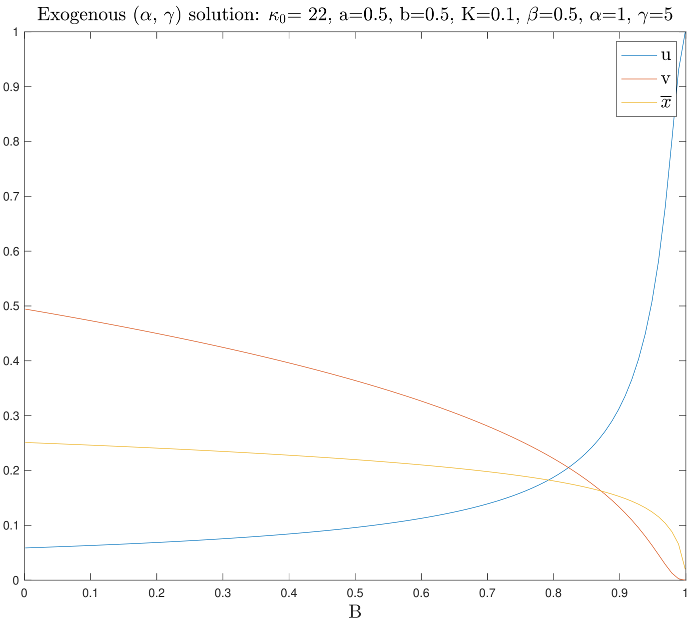

Figure 17: Robustness check for Figure 4

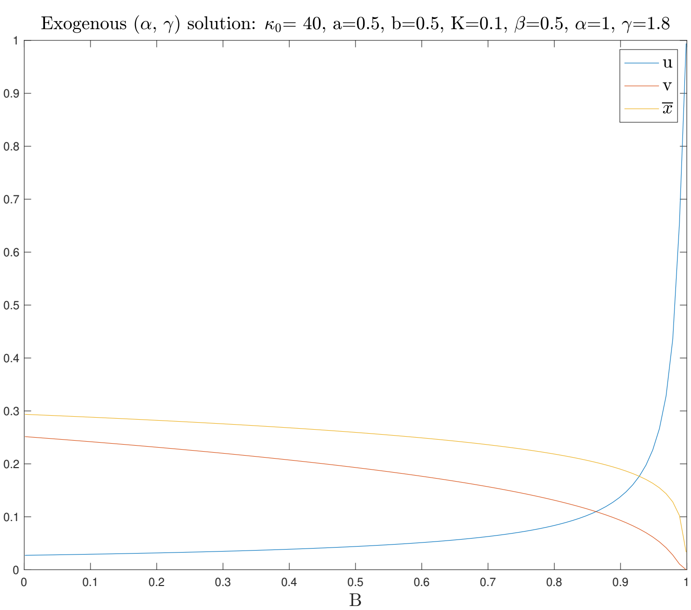

Figure 18: Robustness check for Figure 4

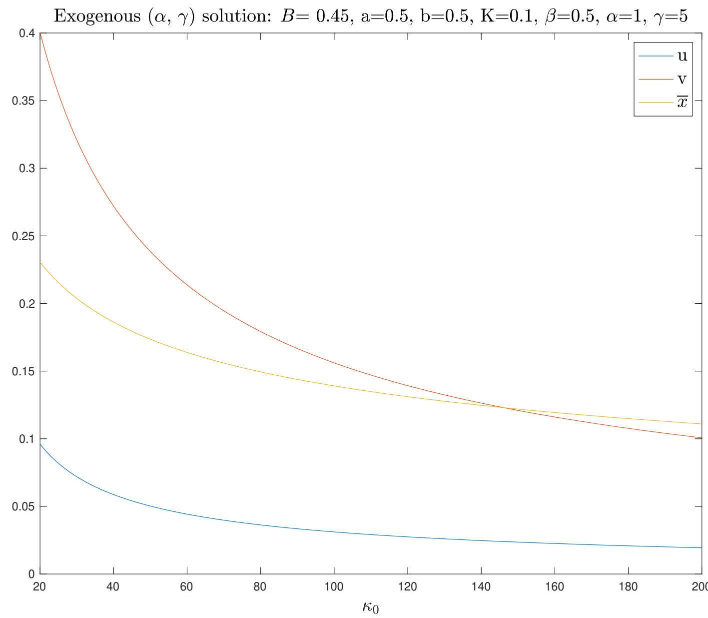

Figure 19: Robustness check for Figure 5

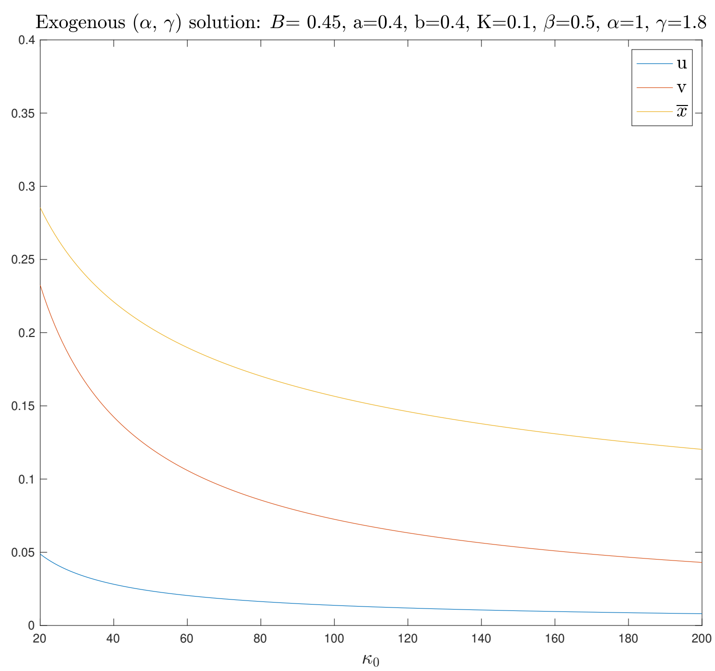

Figure 20: Robustness check for Figure 5

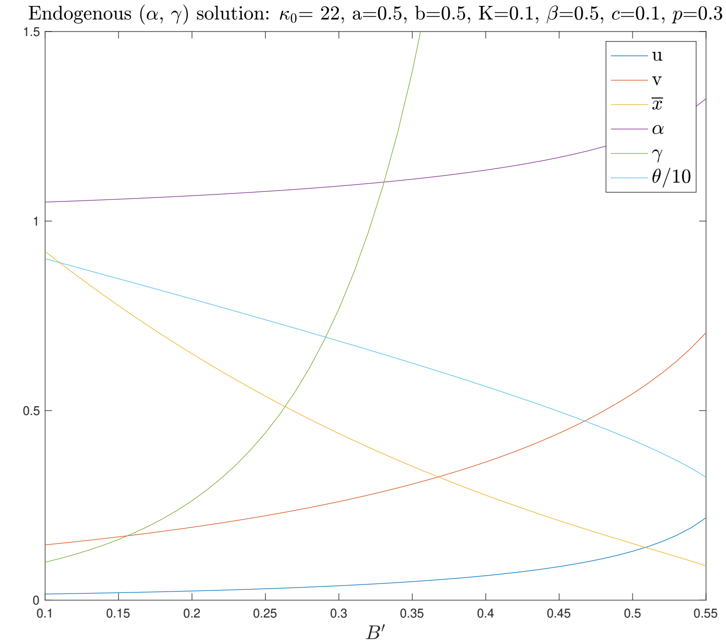

Figure 21: Robustness check for Figure 6

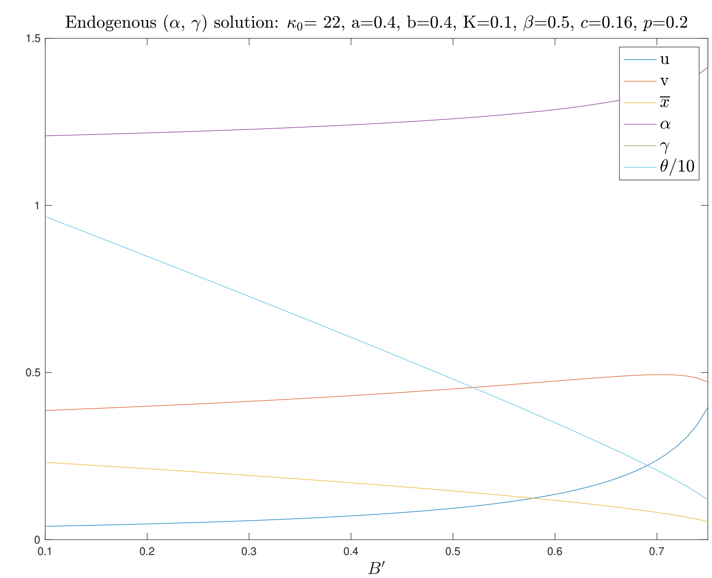

Figure 22: Robustness check for Figure 6

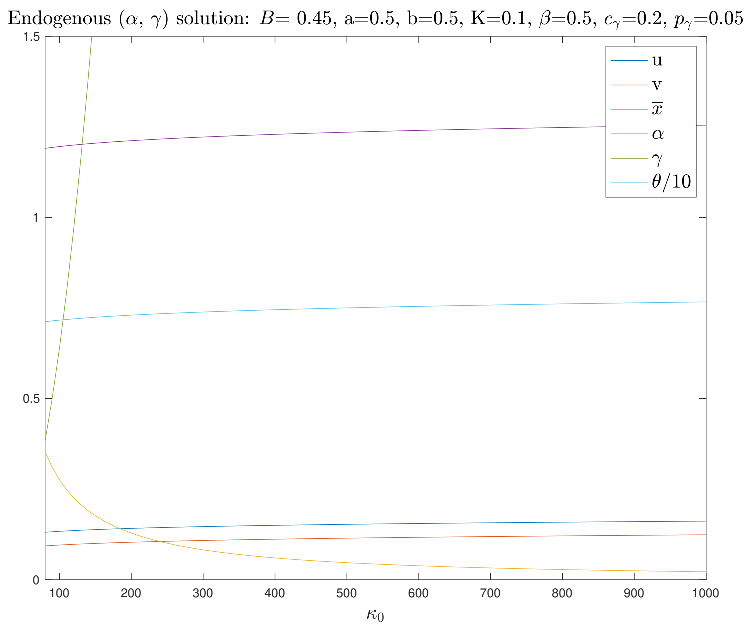

Figure 23: Robustness check for Figure 7

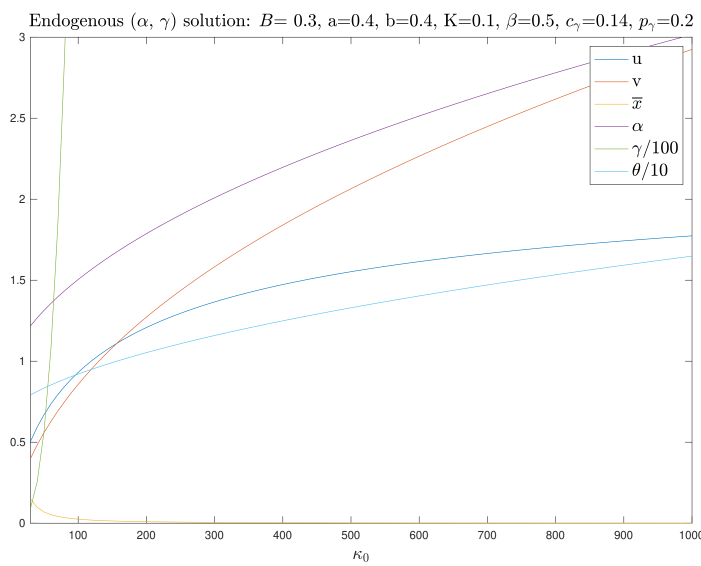

Figure 24: Robustness check for Figure 7

Table 6: Alternative Targets and Parameters

	u_{target}	$\epsilon^{\text{NF}}{}_{uB'}^{\text{target}}$	$\sigma^{\text{END}}{}_{uB'}^{\text{target}}$	γ_{target}	a	b	β
Calibration 1	0.09	0.45	3	1.8	0.5	0.5	0.5
Calibration 2	0.05	0.30	5	1.8	0.5	0.5	0.5
Calibration 3	0.05	0.45	3	0.9	0.5	0.5	0.5
Calibration 4	0.05	0.45	3	1.8	0.4	0.4	0.5
Calibration 5	0.05	0.45	3	1.8	0.5	0.5	0.2

Various targets and parameters used as a robustness check for our calibration

Table 7: Alternative Calibrated Parameters

	κ_0'	B'	c	p
Calibration 1	32.6843	0.4792	0.2725	0.1781
Calibration 2	62.8052	0.3767	0.0931	0.2928
Calibration 3	74.6460	0.4754	0.1459	0.1953
Calibration 4	36.2549	0.4495	0.1029	0.2420
Calibration 5	27.3895	0.4556	0.4648	0.1602

Robustness checks for Table 2

Table 8: Alternative Calibrated Equilibrium

	u	vK	$\overline{x}$	α	γ	B
Calibration 1	0.0900	0.0361	0.3639	1.3026	1.8000	0.6243
Calibration 2	0.0500	0.0269	0.3073	1.1106	1.8000	0.4183
Calibration 3	0.0500	0.0226	0.3987	1.1429	0.9000	0.5434
Calibration 4	0.0500	0.0238	0.2888	1.1186	1.8000	0.5028
Calibration 5	0.0500	0.0781	0.4138	1.5107	1.8000	0.6883

Robustness checks for Table 3

Table 9: Elasticities Along the Alternative Calibrated Frontier

	u	v	y	$E_G W$	$W(0)/W(\overline{x})$	$E_G[Y]$	α
Calibration 1	0.3420	0.2971	0.0020	0.0162	0.0208	0.0292	0.0412
Calibration 2	0.3371	0.3177	0.0006	0.0106	0.0149	0.0201	0.0291
Calibration 3	0.3387	0.3143	0.0006	0.0089	0.0123	0.0168	0.0243
Calibration 4	0.3907	0.3601	-0.0032	0.0069	0.0150	0.0165	0.0256
Calibration 5	0.3681	0.3122	0.0033	0.0111	0.0115	0.0539	0.0389

Robustness checks for Table 5