Public R&D, Private R&D and Growth: A Schumpeterian Approach

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Abstract

This paper introduces public R&D in a tractable Schumpeterian model to study analytically the dynamic effects of changes in public R&D on private R&D, market structure, growth and welfare. While public and private R&D can move in opposite directions in the short run, they move in the same direction in the long run. The tension between the personnel-interaction and knowledge-base effects on one side, and the crowding-out effect on the other, drive these dynamics. The three effects jointly determine firm-level private R&D behavior and thus economic growth in the short run. However, net entry-exit sterilizes the crowding-out effect in the long run, leaving only the first two effects. This difference between short- and long-run behavior rationalizes some of the empirical puzzles documented in the literature. To evaluate quantitatively these analytical insights, we calibrate the model to the USA and feed to it a halving of public R&D that mimics the massive reduction that took place from 1964 to 2021. The economy experiences a long transition characterized by falling productivity of labor in private R&D driven by the falling ratio of public to private knowledge. In the new steady state the growth rate of income per capita falls from 2% to 1.44%. Accounting for the whole transition, welfare falls by about 14%.

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1 Introduction

Public R&D has long been regarded as an important driver of economic performance and the public good nature of its output has long been proposed as a rationale for government support through public finance. There are dissenting views, however, and the debate centers on two closely related questions: (1) does public R&D contribute to productivity growth?; (2) are public R&D and private R&D complements or substitutes? The empirical literature trying to answer them is vast. Yet, multiple rounds of exhaustive review (e.g., Stephan 1996, Forey and Lissoni 2010, Becker 2015, Gersbach et al. 2018) document that it has not reached a consensus on the first question: some studies find that public R&D reduces private production costs and increases productivity growth; others find insignificant or even negative effects. Findings on the second question are also inconclusive, although on closer examination a pattern emerges. Of the earlier firm-level studies that use short-term panel or cross-sectional data, about half support complementarity while the other half support substitutability (David and Hall 2000, David et al. 2000). Recent firm-level studies that use long-term panel data, in contrast, find evidence of complementarity (Becker 2015). Moreover, most aggregate-level studies (industry- and country-level) support complementarity, especially those focused on OECD countries (Prettner and Werner 2016, David and Hall 2000, Ziesemer 2021, Bergeaud et al. 2022, Kantor and Whalley 2023, Fieldhouse and Mertens 2023). The pattern that emerges, thus, is that public R&D has different effects in the short and the long run and also has different effects on firm-level and aggregate private R&D.

To contribute to the debate in light of this evidence, we investigate the effects of public R&D on private R&D, economic growth and welfare in a Schumpeterian model of endogenous growth with endogenous market structure. We organize the paper in two parts. In the first we work with a baseline version of the model that yields the analytical solution for the economy's dynamics. The endogenous market structure allows us to distinguish the effects of public R&D on firm-level variables from its effects on aggregate variables; the analytical solution allows us to identify sharply short- and long-run effects and thus shed new light on the contradictory effects reported in the empirical literature. These two properties allow us to conduct analytical counterfactuals that provide a new perspective on the effects of public R&D. Moreover, they allow us to solve in closed form for the path of household utility and thus solve for the level of welfare generated by the economy's equilibrium path. Consequently, we can investigate analytically the welfare effects of changes in fundamentals and policy variables. The policy variable we focus on is the share of employment in public R&D.

In the second part we generalize these ideas and results in a more ambitious version of the model that allows for cross fertilization between public and private R&D. Specifically, (1) we let the stock of public knowledge enter the private R&D technology and (2) we let the stock of private knowledge enter the public R&D technology. Our specification applies to public knowledge the Peretto-Smulders (2002) mechanism of endogenous technological distance, which says that what matters to the individual firm is knowledge in its own technology domain. Such knowledge comes from two sources: private R&D programs and public R&D programs. The latter is where we deviate from the standard representation of knowledge produced by public R&D as general-purpose, i.e., unstructured and useful equally to all users (see, among many, Anselin et al. 1997, Suen 2013 and Akcigit 2021). In our framework, instead, the government runs domain-specific programs and allocates total public R&D labor across domains. We characterize the dynamics and show that it produces qualitative results in line with those of the analytically solvable version while it produces novel ones. In particular, it exhibits properties that generalize Schumpeterian growth theory beyond this paper's focus on the role of public R&D.

We calibrate the general version of our model to match US data and perform a numerical analysis to investigate the effects of a public R&D cut that mimics what one sees in the data. We find that the economy experiences a long transition characterized by falling productivity of labor in private R&D. The steady-state effect is a reduction of the growth rate of income per capita from 2% to 1.44% while the share of private employment in innovation (both in-house R&D and entry) changes only little. This means that the main margin of adjustment is from public production of knowledge to private production of goods. Moreover, the reduction of the growth rate of income per capita is not due only to the reduction of the growth rate of public knowledge. A major component of the slowdown is that the large reduction of the ratio of public to private knowledge causes a large reduction in the productivity of labor in private R&D. Consequently, even if private R&D per firm remains roughly the same due to the endogenous adjustment in the number of firms, such R&D effort supports a lower growth rate of private knowledge, which falls from 0.248 to 0.145.

As stated, our goal is to link the mechanisms at the heart of modern endogenous growth theory with those identified by the investigation of the contribution of public R&D to economic performance. Summarizing the latter literature, Gersbach et al. (2018) highlights three channels through which public R&D affects private R&D. (1) The contribution of personnel employed in public R&D activity to the productivity of scientist and engineers employed by private firms in their in-house R&D activity. We refer to this channel as the *personnel-interaction effect*.[1] (2) The contribution of knowledge developed by public R&D programs to the productivity of resources employed by private firms in both their production activity and their R&D activity.[2] We refer to this channel as the *knowledge-base effect*. (3) The third channel is the *crowding-out effect*, i.e., public R&D reduces the resources available for private use.[3]

We build on these ideas and incorporate the three channels in a Schumpeterian framework with both vertical and horizontal innovation (Peretto 1998; Dinopoulos and Thompson 1998) that can address several unresolved empirical questions. In the vertical dimension incumbents do R&D to accumulate firm-specific knowledge that reduces their production cost. In the horizontal dimension entrepreneurs develop new products and bring them to market. This process makes market

[1] See Jaffe (1989), Zucker and Darby (1998), Cohen et al. (2002), Jacob and Lefgren (2011), Zucker et. al. (2007), among many others, for specific studies documenting these interactions.

[2] See Jaffe (1989), Mainsfield (1995, 1998), Anselin et al. (1997), Zuker et al. (1998), Rosenberg (2010), and Gersbach et al. (2018) for extensive discussions of this cross-fertilization between public and private R&D and the supporting evidence.

[3] For studies of this effect, see, e.g., Park (1998), Wallsten (2000) and Gullec and van Pottelsberghe (2003).

structure — mass of firms and firm size — endogenous. This class of models has received strong empirical support in recent years (Laincz and Peretto 2006; Ha and Howitt 2007; Madsen 2008; Madsen et al. 2010; Madsen and Ang 2011) and is particularly useful to address our topic.

The interaction between the mass of firms and the allocation of resources across firms delivers rich dynamics for private innovation and productivity growth. In particular, it predicts that the smaller crowding out due to a cut in public R&D boosts firm-level R&D only temporarily. The cut reallocates resources to private activity and thus expands the size of the market for the output of private firms. This expansion induces incumbent firms to produce more and do more R&D. However, it also induces net entry, which over time dilutes the market shares of firms and spreads resources more thinly across them. Therefore, after the initial rise, firm-level R&D and productivity growth start decreasing and converge to lower steady-state levels.

Our first analytical result, therefore, is that a reduction in public R&D reduces the steady-state growth rate even though it can boost growth in the short run. This mechanism provides a plausible explanation of the seemingly contradictory empirical findings concerning public R&D and growth: it suggests that some findings reflect the focus on (1) short-run relationships and (2) aggregate measures of R&D. Our second analytical result is that public R&D can either substitute or complement firm-level R&D in the short run but it surely complements it in the long run. This result is also consistent with the firm-level empirical findings in David et al. (2000) and Becker (2015). The novel implication is that in the short run the knowledge-base and personnel-interaction effects conflict with the crowding-out effect in determining private firm-level R&D and productivity growth and thus produce ambiguous responses. In the long run net entry sterilizes the crowding-out effect, leaving only the knowledge-base and personnel-interaction effects. Consequently, firm-level R&D and growth are monotonically increasing in public R&D.

Our next result is that aggregate private R&D is a hump-shaped function of public R&D both in the short and the long run. Specifically, when public R&D is small the personnel-interaction effect dominates the crowding-out effect and aggregate private R&D rises with public R&D, suggesting a relation of complementarity; when public R&D is large the reverse happens and aggregate private R&D falls with public R&D, suggesting a relation of substitutability. An obvious, perhaps tempting, implication of this result is that there is a level of public R&D that maximizes aggregate private R&D. Such maximization, however, does not maximize economic growth, which is a weighted sum of firm-level knowledge growth and public knowledge growth and is thus monotonically increasing in public R&D. Overall, our result about the relation between public R&D and aggregate private R&D is consistent with the findings from OECD countries summarized in David et al. (2000).

Previous research has incorporated some of the channels discussed above in variety-expansion models (Osano 1992, Gersbach et al. 2018) and in quality-ladder models (Morales 2004, Akcigit et al. 2021, Chu and Lai 2012, Suen 2013). Other contributions of note are Cozzi and Galli (2009, 2017, 2021), which propose models where basic research by public R&D personnel generates fundamental ideas that private agents develop into marketable products. We do not pursue the distinction basic vs. applied in our work, although our cross-fertilization model comes close to capturing the spirit of these models. Gersbach et al. (2018) studies how public R&D affects long-run growth in a model that takes seriously the basic-applied hierarchy, as well as two-way knowledge spillovers between public R&D and private R&D. Gersbach et al. (2019) analyzes public basic research in a multi-country, multi-industry environment with international trade and evaluates the possible inefficiencies that may arise in national investment in basic research. All of these contributions focus on steady states and do not explore how public R&D interacts with private R&D via key market structure variables that are the joint outcome of investment decisions by incumbents and the process of net entry/exit driven by the decisions of entrepreneurs. More importantly, they do not address the potential reason why the empirical literature is inconclusive: the difficulty to sort out short-run and long-run effects.

The paper is organized as follows. Section 2 presents the model. Section 3 characterizes the steady state and transitional dynamics. Sections 4 and 5 analyze the effects of a cut in public R&D on, respectively, private R&D, growth and welfare. Section 6 discusses the general model with cross fertilization. Section 7 carries out a complete quantitative analysis. Section 8 concludes.

2 The model

The economy is populated by a representative household that purchases differentiated consumption goods, supplies labor services inelastically in a competitive labor market and has access to a frictionless financial market. Each consumption good is produced by one monopolistic firm. Firms drive the economy's productivity growth through their decisions concerning two types of innovation: (1) entry, which increases the mass of goods (horizontal innovation), and (2) in-house R&D aimed at reducing unit production costs (vertical innovation). The government hires labor to conduct public R&D and balances the budget via a lump-sum tax. This simple formulation allows us to focus on the role of public R&D abstracting from the potential distortions arising from more complex financing schemes. In this section we present the model's primitives and characterize agents' behavior.

2.1 Household

The representative household has preferences

$$U_0 = \int_0^\infty e^{-\rho t} L\left(t\right) \ln c\left(t\right) dt, \quad \rho > \lambda > 0 \tag{1}$$

where ρ is the discount rate, $L\left(t\right) = e^{\lambda t} L_0$ is the mass of household members (population), λ is the rate of population growth, and c is the consumption per capita index

$$c = N^{\omega - \frac{1}{\epsilon - 1}} \left[\int_0^N \left(\frac{X_i}{L} \right)^{\frac{\epsilon - 1}{\epsilon}} di \right]^{\frac{\epsilon}{\epsilon - 1}}, \quad \epsilon > 1, \tag{2}$$

where ϵ is the elasticity of product substitution, X_i is the household's purchase of good i, N is the mass of goods and ω is the love-of-variety parameter. The associated price index of consumption is

$$P_C = N^{-\omega} \left[\frac{1}{N} \int_0^N P_j^{1-\epsilon} dj \right]^{\frac{1}{1-\epsilon}}, \tag{3}$$

where P_i is the price of good i.

The household maximizes (1) subject to the budget constraint

$$\dot{A} = rA + wL - LE - wL_G, \tag{4}$$

where A is assets holding, r is the rate of return on assets, w is the wage, $E = P_C c$ is consumption expenditure per capita and wL_G is the lump-sum tax that the government levies to finance public R&D. The household's consumption/saving decision is

$$\frac{\dot{E}}{E} = r - \rho. \tag{5}$$

The household's demand curve for good i

$$X_i = LE \frac{P_i^{-\epsilon}}{\int_0^N P_j^{1-\epsilon} dj}. \tag{6}$$

2.2 Firm-level innovation: the interaction of private and public R&D

The typical firm produces with the technology

$$X_i = Z_i^{\theta} D_i^{\gamma} (L_{X_i} - \phi), \quad 0 < \theta < 1, \quad 0 < \gamma < 1, \quad \phi > 0, \tag{7}$$

where X_i is output, L_{X_i} is employment and ϕ is overhead labor. $Z_i^{\theta} D_i^{\gamma}$ represents labor productivity as a function of firm-specific knowledge, Z_i, generated by the firm's in-house R&D, and knowledge generated by public R&D, D_i, that is relevant to the firm's specialized operations.

The two knowledge inputs accumulate as follows. The in-house R&D process is

$$\dot{Z}_i = \alpha f(s_G) \left[\chi K_i^{\eta} + (1-\chi) B_i^{\eta} \right]^{\frac{1}{\eta}} L_{Z_i}, \quad 0 < \eta \leq 1, \; 0 < \chi \leq 1,$$

where $s_G \equiv L_G/L$ is the share of labor employed in public R&D, $K_i = \frac{1}{N} \int_0^N Z_j dj$ and $B_i = \frac{1}{N} \int_0^N D_j dj$. It is convenient to rewrite it as

$$\dot{Z}_i = \chi^{1/\eta} \alpha f(s_G) \left[K_i^{\eta} + \frac{1-\chi}{\chi} B_i^{\eta} \right]^{\frac{1}{\eta}} L_{Z_i} = \alpha f(s_G) K_i L_{Z_i} \cdot \left[\frac{1 + \kappa \left(\frac{B_i}{K_i} \right)^{\eta}}{1+\kappa} \right]^{\frac{1}{\eta}}, \tag{8}$$

where we set $\kappa \equiv (1-\chi)/\chi$. We refer to κ as the cross-fertilization parameter. For $\kappa = 0$ this specification recovers the baseline Schumpeterian model with no public R&D. Similarly, we write the public R&D process as

$$\dot{D}_i = \left[\chi B_i^{\delta} + (1-\chi) K_i^{\delta}\right]^{\frac{1}{\delta}} L_{G_i}, \quad 0 < \delta \leq 1, \; 0 < \chi \leq 1,$$

which we rewrite as

$$\dot{D}_i = \chi^{1/\delta} \left[B_i^{\delta} + \frac{1-\chi}{\chi} K_i^{\delta}\right]^{\frac{1}{\delta}} L_{G_i} = B_i L_{G_i} \cdot \left[\frac{1+\kappa\left(\frac{B_i}{K_i}\right)^{-\delta}}{1+\kappa}\right]^{\frac{1}{\delta}}. \tag{9}$$

Using the same parameter χ for the shares in the CES operators is not necessary but has the desirable property that we capture cross-fertilization in both activities with only one parameter.

This specification proposes several novel elements. The first is the function $f(s_G)$ that describes the interaction of public R&D workers with private R&D workers. We refer to this component of our mechanism as the *personnel-interaction* effect of public R&D and stress that it is a flow concept, i.e., it concerns the contribution to private R&D activity of the flow or resources employed in public R&D at a point in time. When we turn to the stock concepts, our specification applies to public R&D the Peretto-Smulders (2002) mechanism of endogenous technological distance, which says that what matters to individual firm i is knowledge in its own technology domain. Such knowledge comes from two sources: private R&D programs, the term K_i, and public R&D programs, the term, D_i. The latter is where we deviate from the standard representation of knowledge produced by public R&D as general-purpose, i.e., unstructured and useful equally to all users. In our framework, instead, the government runs domain-specific programs and allocates total public R&D labor, $L_G = \int_0^N L_{G_i} di$, across domains. In the quantitative exercise, we work with the functional form $f(s_G) = 1 + \xi s_G$, $0 < \xi < 1$, and use ξ to measure the intensity of this channel.

The interpretation of this representation is that, in principle, all programs benefit from aggregate private knowledge, $\int_0^N Z_j dj$, and aggregate public knowledge, $\int_0^N D_j dj$, because knowledge is non-rival. However, the average usefulness of pieces of knowledge from those two aggregates to the individual program is $1/N$. Thus, our approach abandons the view of public knowledge as purely general-purpose in favor of the view that public knowledge has the same structure as private knowledge, namely, individual pieces of knowledge belong to specialized domains and thus have different usefulness to specialized users. The Peretto-Smulders mechanism of endogenous technological distance rationalizes the representation of the productivity of labor in public R&D as a function of private and public *domain-specific* knowledge. This representation is realistic and produces a scale-invariant Schumpeterian model with public R&D. We stress that in the Peretto-Smulders framework the development of new technology domains is a by-product of private innovative activity, specifically, entry. We assume for simplicity that the government's allocation of public R&D labor across programs is symmetric and that, likewise, the evolution of domain-specific public knowledge over time is symmetric.

One more thing of note is that our CES specification says that neither public R&D employment nor public knowledge are essential for private knowledge production. We prefer it to the popular Cobb-Douglas one because we want to test if upon shutting down the government's intervention by setting $s_G = 0$, the model recovers the market equilibrium dynamics documented in the vast literature that abstracts from public R&D. If not, then the model without public R&D and the model with public R&D are fundamentally different theories of the growth process. We discuss this point further after we present the model's properties and the dynamics that it produces.

We close this subsection with a description of the firm's decisions. The firm maximizes

$$\max_{\{L_{Z_i},P_i\}} V_i(t) = \int_t^{\infty} e^{-\int_t^v [r(s)+\sigma]ds} \Pi_i(v)\, dv, \quad \sigma > 0$$

where $\Pi_i \equiv P_i X_i - wL_{X_i} - wL_{Z_i}$ is the instantaneous profit flow, r is the real interest rate and σ is a death shock.[4] The firm chooses the time path of the price, P_i, and R&D, L_{Z_i}, subject to the demand curve (6), the production function (7) and the R&D technology (8). The detailed solution of this problem is in the Appendix.

2.3 Net entry/exit

Entrepreneurs set up new firms according to the technology

$$\dot{N} = \frac{\beta N}{LE} L_N - \sigma N, \quad \beta > 0$$

where L_N is labor employed in entry and $\beta N/LE$ is its productivity.[5] Accordingly, the cost of setting up a new firm is $LE/\beta N$ and the associated free-entry condition is

$$V_i = \frac{LE}{\beta N}. \tag{10}$$

The appendix shows that allowing public R&D to reduce the entry cost by writing $\beta f(s_G)$ does not change our result in any significant way.

3 A flow view of the private and public R&D interaction

We now turn to the general equilibrium of our economy and its dynamics. To maximize analytical insight, we begin with the simple specification of (8)-(9) that omits the stock components of the interaction between private and public R&D, that is, we set the cross-fertilization parameter at $\kappa = 0$. The reason for doing this is that this version of the model isolates the flow dimension of the interaction between private and public R&D and, moreover, delivers closed-form solutions for

[4]The exit shock guarantees that the model has symmetric dynamics in a neighborhood of the steady state when population growth is not positive.

[5]See Etro (2004, especially p. 299), Barro and Sala-i-Martin (2004, Ch. 6) and Peretto and Connolly (2007) for discussions of this and alternative assumptions on the entry cost that yield similar results.

the dynamics. The resulting structure produces transparent results that illuminate the empirical issues that we discussed in the introduction.

3.1 Firm behavior and the returns to innovation

The equilibrium of the manufacturing sector is symmetric because (1) firms make identical price and R&D decisions, (2) we start them with identical values of initial firm-specific knowledge the equilibrium and (3) we assume that new firms enter at the average level of firm-specific knowledge. The following lemma provides the analytical characterization of firm behavior and the resulting returns to innovation.

Lemma 1. *In symmetric equilibrium, the monopolistic price, the return to in-house R&D,* r^Z*, and the return to entry,* r^N*, are:*

$$P = \frac{\epsilon}{\epsilon - 1} Z^{-\theta} D^{-\gamma}; \tag{11}$$

$$r = r^Z \equiv f(s_G)\alpha \left[\frac{\theta(\epsilon - 1)}{\epsilon} \frac{LE}{N} - \frac{L_Z}{N} \right] - \sigma; \tag{12}$$

$$r = r^N \equiv \left(\frac{LE}{N\epsilon} - \phi - \frac{L_Z}{N} \right) \frac{\beta N}{LE} + \frac{\dot{E}}{E} + \lambda - \frac{\dot{N}}{N} - \sigma. \tag{13}$$

Proof. See the Appendix. □

Equations (12) and (13) reproduce well-known features of this class of models. Both the return to in-house R&D and the return to entry increase with firm size (sales per firm), LE/N. It is useful to highlight the novel elements of this paper's analysis. Equation (11) shows that the typical good's price depends negatively on both private and public knowledge. This feature adds the contribution of public R&D to the standard cost reduction process studied in the literature. Equation (12) shows that the return to in-house R&D increases with public R&D, s_G, because public R&D raises the productivity of labor in private R&D.

3.2 General equilibrium

The economy consists of N goods markets, one labor market and one financial market. We discussed the equilibrium of each differentiated good market wherein the local monopolist sets price P_i given the demand curve for its good. Asset market equilibrium requires (1) equalization of all rates of return, $r = r^Z = r^N$, and (2) that the value of the household's assets equal the total value of existing monopolistic firms, i.e., under symmetry, $A = NV = LE/\beta$. Symmetry allows us to write aggregate employment in production and in-house R&D as $L_X = NL_{X_i}$ and $L_Z = NL_{Z_i}$. Aggregate employment in entry is L_N. The labor market clears when $L = L_G + L_X + L_Z + L_N$, determining the wage w. We use labor as the numeraire and therefore set $w \equiv 1$.

Next, we state a property that greatly simplifies the analysis of the dynamics.

Lemma 2. *Assume $\beta > \rho - \lambda$. At any time t the interest rate is $r = \rho$ and consumption expenditure per capita is*

$$E = E^* \equiv \frac{\beta(1 - s_G)}{\beta - \rho + \lambda}. \tag{14}$$

Proof. See the Appendix. □

This property says that consumption expenditure per capita and the interest rate jump to their steady-state values and remain constant throughout the transition governed by net entry/exit. Equation (14) shows that consumption expenditure falls with public R&D, s_G, capturing the crowding out discussed in the literature. The interpretation is straightforward: to hire workers to perform public R&D the government levies taxes that reduce the household's disposable income.

Finally, we look at key macroeconomic variables. In our economy nominal GDP is equal to consumption expenditure. It follows that at any point in time real GDP per capita and real consumption per capita are

$$y \equiv \frac{Y}{L} = c = \frac{E^*}{P_C} = \frac{\beta(1 - s_G)}{\beta - \rho + \lambda} \frac{\epsilon - 1}{\epsilon} \cdot \underbrace{N^\omega Z^\theta D^\gamma}_{\text{TFP}}. \tag{15}$$

Interpreted as a reduced-form production function, this equation says that aggregate labor productivity, equivalently, output per worker, has two components. One stems from the firm-level markup over the marginal cost of production. The other stems from the endogenous evolution of product variety and firm-level productivity. We denote this component $T \equiv N^\omega Z^\theta D^\gamma$ and call it aggregate TFP. We show below that in steady state the mass of firms per capita, $n \equiv N/L$, is constant and therefore the growth rates of GDP per capita, consumption per capita and TFP are all equal.

3.3 Firm-level innovation

Lemma 2 and equation (12) yield that in-house R&D per firm is

$$\frac{L_Z}{N} = \max\left\{ \frac{LE^*}{N} \frac{\theta(\epsilon - 1)}{\epsilon} - \frac{\rho + \sigma}{f(s_G)\alpha}, \quad 0 \right\}.$$

This equation highlights three important features of the model. First, when positive, in-house R&D per firm is decreasing in the mass of firms, N, because net entry spreads consumption expenditure, LE^*, more thinly and thus reduces the profitability of the individual firm. Second, the typical firm does in-house R&D if and only if its size, LE^*/N, is sufficiently large or, equivalently, the mass of firms per capita, n, is sufficiently small. That is, there exists a threshold value

$$\bar{n} \equiv E^* \frac{\alpha\theta(\epsilon - 1)}{\epsilon(\rho + \sigma)} f(s_G)$$

such that for $n \geq \bar{n}$ in-house R&D per firm is zero because the market is too crowded and the individual firm captures a market share $1/N$ that is too small. Third, public R&D can affect not only the magnitude of R&D per firm but also its activation. Specifically, because public R&D raises the productivity of private R&D, firms are willing to do it for larger values of the mass of firms or, equivalently, at smaller market share.

Recall, moreover, that market size, LE^*, is endogenous and decreasing in public R&D. Together with the behavior of R&D per firm just discussed, this gives us the relation

$$\hat{Z} \equiv \frac{\dot{Z}}{Z} = \max\left\{ f(s_G)\,(1-s_G)\,\frac{\beta\alpha\theta(\epsilon-1)}{(\beta-\rho+\lambda)\,\epsilon}\frac{1}{n} - \rho - \sigma, \quad 0 \right\}, \tag{16}$$

which says that private knowledge growth is a hump-shaped function of public R&D because of the competing crowding-out and personnel-interaction effects.[6] We thus obtain the general-equilibrium result that both the activation threshold and the magnitude of firm-level knowledge growth are non-monotonic functions of public R&D. In particular, we have the condition

$$f(s_G)\,(1-s_G) > \frac{(\beta-\rho)\,\epsilon\,(\rho+\sigma)}{\beta\alpha\theta(\epsilon-1)}n,$$

which says that given the mass of firms per capita, firm-level knowledge growth is positive if and only if public R&D is in the appropriate range, meaning that both too little and too much public R&D can kill firm-level innovation. Another implication of this mechanism is that there exists a level of public R&D that maximizes firm-level knowledge growth. We stress, however, that this relation holds *for given mass of firms per capita* and thus holds only in transition, not necessarily in steady state. In other words, it is exclusively a short-run relation.

3.4 Market structure dynamics

Next, we characterize the process of net entry/exit. To focus on the research question of the paper, namely, the relation between public R&D and private profit-driven innovation, we consider an economy where firm-level R&D is always positive. The interested reader can find the characterization of the full global dynamics in the appendix.

A powerful property of our model is that the general-equilibrium dynamics reduce to a logistic differential equation in the mass of firms per capita. Peretto and Connolly (2007) discusses the intuition for why logistic dynamics arise in this class of models and how it relates to the literature. This property manifests itself in this model as follows.

Proposition 1. *Consider an economy with initial mass of firms per capita, $n_0 \equiv N_0/L_0 < \bar{n}$. Assume:*

$$\nu \equiv \beta\frac{1-\theta(\epsilon-1)}{\epsilon} - \rho - \sigma > 0 \quad \textit{and} \quad \phi - \frac{\rho+\sigma}{f(s_G)\alpha}\left[1 + \frac{v\epsilon}{\beta\theta(\epsilon-1)}\right] > 0. \tag{CG}$$

[6] Technically, this result requires that there exists a value of s_G such that $f'(s_G)\,(1-s_G) = f(s_G)$. With the functional form proposed earlier, we have the solution $s_G = (\xi-1)/2\xi$, which is positive for $\xi > 1$.

The mass of firms per capita follows a logistic process

$$\frac{\dot{n}}{n} = \nu \left(1 - \frac{n}{n^*}\right), \tag{17}$$

with constant coefficients, where ν is the intrinsic growth rate and

$$n^* \equiv \frac{\frac{\nu}{\beta} E^*}{\phi - \frac{\rho+\sigma}{f(s_G)\alpha}} = \frac{\frac{\nu(1-s_G)}{\beta-\rho+\lambda}}{\phi - \frac{\rho+\sigma}{f(s_G)\alpha}} \tag{18}$$

is the carrying capacity. The solution of (17) is

$$n(t) = \frac{n^*}{1 + e^{-\nu t}\left(\frac{n^*}{n_0} - 1\right)}. \tag{19}$$

Proof. See the Appendix. □

Condition (CG) guarantees the existence and stability of the endogenous-growth steady state n^*. Specifically, it guarantees that $n^* < \bar{n}$ so that steady-state firm size is sufficiently large for the typical firm to do in-house R&D. We state this property formally as follows.

Proposition 2. *Assume that condition (CG) holds. In steady state, firm size, firm-level R&D and firm-level knowledge growth are all positive and given by, respectively:*

$$\left(\frac{LE}{N}\right)^* = \left[\phi - \frac{\rho+\sigma}{f(s_G)\alpha}\right]\frac{\beta}{\nu}; \tag{20}$$

$$\left(\frac{L_Z}{N}\right)^* = \left(\frac{LE}{N}\right)^* \frac{\theta(\epsilon-1)}{\epsilon} - \frac{\rho+\sigma}{f(s_G)\alpha}; \tag{21}$$

$$\hat{Z}^* = \left(\frac{LE}{N}\right)^* f(_G)\alpha\frac{\theta(\epsilon-1)}{\epsilon} - \rho - \sigma. \tag{22}$$

Proof. See the Appendix. □

Proposition 2 provides two important results. First, firm-level R&D and firm-level knowledge growth are linear functions of firm size. That is, past a critical activation threshold, firm-level R&D rises in proportion to firm size. Second, the net entry/exit process sterilizes the market size effect. This property is the reason why our Schumpeterian model produces scale-invariant growth and has important implications for the impulse-response exercise that we perform in the next section.

The proposition also highlights an interesting property of our economy that might not be apparent from inspecting the primitives and the main mechanism. If condition (CG) fails, i.e., if $n^* \geq \bar{n}$, our economy converges to a steady state with too many firms that are not profitable enough to support in-house R&D (see the appendix for details). Consequently, firm-level knowledge growth is zero and the economy grows endogenously only because public R&D produces new public knowledge at a constant exponential rate and such public knowledge contributes directly to productivity in manufacturing. In other words, our model nests as a special case the approach to endogenous growth pioneered by Shell (1966, 1967). One of our paper's goals, in fact, is to investigate analytically how that engine of growth interacts with the Schumpeterian one, which emphasizes the role of private firms and entrepreneurs rather than that of the government. As we mentioned earlier, one of the salient aspects of the interaction is that public R&D can activate private in-house R&D by raising its productivity over the threshold identified by equation (16).

4 Public R&D, market structure and economic growth

In this section, we examine the effects of public R&D with an eye to the empirical evidence discussed in the introduction. We begin with comparative statics results for the steady state.

4.1 Steady state effects

To identify sharply the key channels, we begin with a special case: public R&D does not affect private knowledge accumulation and public knowledge does not affect private production.

Proposition 3. *When the knowledge-base and personnel-interaction effects are absent (i.e., $\gamma = \xi = 0$), the comparative statics effects of public R&D, s_G, are:*

$$\partial E^*/\partial s_G < 0 \quad \text{and} \quad \partial n^*/\partial s_G < 0;$$

$$\partial \left(\frac{LE}{N}\right)^* /\partial s_G = \partial \left(\frac{L_Z}{N}\right)^* /\partial s_G = \partial \hat{Z}^*/\partial s_G = \partial \hat{T}^*/\partial s_G = \partial \hat{c}^*/\partial s_G = \partial \hat{y}^*/\partial s_G = 0.$$

Proof. See the Appendix. □

The first line in the proposition shows the core mechanism at work in the model: public R&D crowds out consumption expenditure per capita, see equation (14), and thereby reduces the mass of firms per capita. The next line raises a key question. Why is a change in public R&D unable to affect firm-level variables? The mechanism driving this property is the salient feature of models with endogenous market structure that sterilize the strong scale effect. Consider a *cut* in public R&D that raises consumption expenditure, LE^*. The higher expenditure expands the size of the market and thus attracts entry. Most importantly, the steady-state mass of firms rises one for one with the size of the market so that steady-state firm size, $(LE/N)^*$, does not change. Consequently, firm-level R&D, $(L_Z/N)^*$, does not change. And since we shut down the contribution of public R&D to private R&D, firm-level knowledge growth, $\hat{Z}^*$, also does not change. Finally, because we shut down the contribution of public knowledge to private production, the growth rate of TFP, $\hat{T}^*$, does not change. Consequently, the growth rates of consumption per capita, $\hat{c}^*$, and of GDP per capita, $\hat{y}^*$, do not change.

This result differs from the findings of growth models with only one dimension of innovation, such as Park (1998), Morales (2004) and Chu and Lai (2012), that do not have an endogenous adjustment of firm size via entry/exit. In those models, reducing public R&D only releases resources for private uses and thus expands the market for innovative goods. As a consequence, those models predicts faster private knowledge growth. The only trade-off is that faster private growth comes at the cost of slower public knowledge growth.

When we take into account the contribution of public R&D to private R&D, things are more interesting and the differences of our results from those in the literature even more pronounced.

Proposition 4. *When the knowledge-base and personnel-interaction effects are present (i.e., $\gamma > 0$ and $\xi > 0$), the comparative statics effects of public R&D, s_G, are:*

$$\partial E^*/\partial s_G < 0 \quad \textit{and} \quad \partial n^*/\partial s_G < 0;$$

$$\partial \left(\frac{LE}{N}\right)^* /\partial s_G > 0;$$

$$\partial \left(\frac{L_Z}{N}\right)^* /\partial s_G > 0 \quad \textit{and} \quad \partial \hat{Z}^*/\partial s_G > 0;$$

$$\partial \hat{T}^*/\partial s_G > 0 \quad \textit{and} \quad \partial \hat{c}^*/\partial s_G \quad \textit{and} \quad \partial \hat{y}^*/\partial s_G > 0.$$

Proof. See the Appendix. □

Recall that in the previous analysis a cut in public R&D raises consumption expenditure and the mass of firms in equal proportion, thus leaving firm size the same. Here, instead, firm size decreases in response to a cut in public R&D. The reason is that the cut lowers the productivity of in-house R&D, thereby reducing its rate of return. Accordingly, firms cut down on their in-house R&D and resources flow from in-house R&D to the creation of new firms. This effect raises the steady-state mass of firms more than proportionally to expenditure and thus reduces steady-state firm size, $(LE/N)^*$. The overall effect of the cut in public R&D is thus to reduce firm-level R&D, $(L_Z/N)^*$, and knowledge growth, $\hat{Z}^*$. Moreover, since public knowledge contributes to private production, the cut in public R&D reduces the growth of firm-level labor productivity in production via the term $\hat{D} = \gamma s_G$. Together with the reduction of private knowledge growth, this causes a fall in TFP growth, $\hat{T}^*$, and thus of consumption per capita growth, $\hat{c}^*$, and GDP per capita growth, $\hat{y}^*$. We conclude that a cut in public R&D reduces steady-state economic growth along several margins.

This long-run prediction is consistent with the empirical findings in David and Hall (2000), David et al. (2000), and the summary of the literature in Becker (2015). This prediction also stands in sharp contrast to Park (1998), Morales (2004) and Chu and Lai (2011). In those models higher public R&D lowers the growth rate through the crowding-out effect while it raises it through the faster accumulation of public knowledge. The net effect depends on which force dominates. In our model, instead, the endogeneity of market structure eliminates the crowding-out effect, leaving only the contribution of public R&D to private R&D, the personnel-interactioin effect, and of public knowledge to private production. As a result, in our steady state growth is monotonically increasing in public R&D.

4.2 Transitional effects: an impulse-response exercise

It is useful to cast the characterization of the dynamic effects of public R&D as a policy exercise. Thus, we study the transition of the economy in response to an unanticipated, immediate and permanent cut in public R&D. Enabling one to perform analytically this kind of impulse-response exercise is a major strength of our baseline specification.

Proposition 5. *Consider an economy with public R&D s_G^0 that is in steady state at time $t = 0$. Denote the steady-state values of endogenous variables with the subscript 0, e.g., expenditure per capita and the mass of firms per capita, are respectively, E_0 and n_0. At time $t = 0$ the government implements an unanticipated, immediate and permanent cut in public R&D to $s_G < s_G^0$. The cut yields the percentage change in the steady-state mass of firms per capita*

$$\frac{n^*}{n_0} - 1 = \frac{\frac{1-s_G}{\phi - \frac{\rho+\sigma}{f(s_G)\alpha}}}{\frac{1-s_G^0}{\phi - \frac{\rho+\sigma}{f(s_G^0)\alpha}}} - 1 \equiv \Delta > 0.$$

The resulting transition paths of, respectively, the rate of net entry and the rate of firm-level knowledge growth are:

$$\hat{N}(t) = \frac{ve^{-\nu t}\Delta}{1 + e^{-\nu t}\Delta} + \lambda; \tag{23}$$

$$\hat{Z}(t) = \left[1 + e^{-\nu t}\Delta\right]\left[f(s_G)\alpha\phi - \rho - \sigma\right]\frac{\beta\theta(\epsilon - 1)}{\nu\epsilon} - \rho - \sigma. \tag{24}$$

The transition path of TFP growth is:

$$\hat{T}(t) = \omega\left[\frac{ve^{-\nu t}\Delta}{1 + e^{-\nu t}\Delta} + \lambda\right] + \left[1 + e^{-\nu t}\Delta\right]\left[f(s_G)\alpha\phi - \rho - \sigma\right]\frac{\beta\theta^2(\epsilon - 1)}{\nu\epsilon} - \rho - \sigma + \gamma\frac{1 + e^{-vt}\Delta}{n^*}s_G. \tag{25}$$

Proof. See the Appendix. □

The proposition provides the time path of our endogenous variables caused by a change in policy. According to equation (23) the net entry rate jumps up on impact, at $t = 0$, when our gap operator Δ switches from zero to positive due to the implementation of the permanent cut in public R&D. It then gradually decreases until it returns to the baseline trend dictated by population growth when the economy reaches the new steady state.

Equation (24) says that at $t = 0$ two forces compete: the gap operator Δ tends to push firm-level knowledge growth up; the reduction in the productivity of firm-level R&D caused by the cut in public R&D s_G tends to push it down. The initial jump is thus ambiguous. Afterward, firm-level knowledge growth gradually decreases and converges from above to the new steady-state value

$$\hat{Z}^* = \left[f(s_G)\alpha\phi - \rho - \sigma\right]\frac{\beta\theta(\epsilon - 1)}{\nu\epsilon} - \rho - \sigma.$$

The mechanism is as follows. The cut in public R&D generates an immediate rise of consumption expenditure. Since the mass of firms is pre-determined, at $t = 0$ we have $N(0) = N_0$ and therefore the spending cut raises firm size to $LE^*/N_0 > LE_0/N_0$. This, in turn, raises firm-level R&D. If this rise dominates the fall of private R&D productivity, firm-level knowledge growth accelerates. As the mass of firms rises, it dilutes the size of the market across more and more firms, thereby reducing R&D per firm. Accordingly, firm-level knowledge growth slows down. In the new steady state it is lower than in the old one because the cut in public R&D reduces the productivity of labor in private R&D *and* firms run smaller in-house R&D operations because firm size is smaller.

The mechanism just discussed has a powerful implication. Recall our discussion of equation (16), which says that *given the mass of firms per capita* firm-level knowledge growth, $\hat{Z}$, is a hump-shaped function of public R&D, s_G. Proposition 5 says that eventually the mass of firms becomes proportional to the size of the market. As this happens, the crowding-out effect vanishes from equation (16), rendering firm-level knowledge growth monotonically increasing in public R&D. The closed-form solution for the time path $\hat{Z}(t)$ in Proposition 5 incorporates this mechanism.

Equation (25) shows that TFP growth jumps at $t = 0$ in a direction that depends on the balance of the forces at play. While the net entry rate jumps up, firm-level knowledge growth can jump up or down and, moreover, public knowledge growth jumps down. In this case as well, the initial jump is ambiguous. Along the transition, as firm size shrinks due to net entry, both the net entry rate and firm-level knowledge growth slow down so that TFP growth gradually converges from above to the new steady-state value $\hat{T}^* < \hat{T}_0$. Recall that per capita consumption growth is equal to TFP growth at all times, $\hat{c}(t) = \hat{T}(t)$.

4.3 The dynamic relation between public and private R&D

We now investigate the short- and long-run relation between public R&D and several popular firm-level and aggregate measures of private R&D. Noting that $L_Z/N = \hat{Z}/[f(s_G)\alpha]$, Proposition 5 allows us to write firm-level R&D as

$$\frac{L_Z(t)}{N(t)} = \left[1 + e^{-\nu t}\Delta\right]\left[\phi - \frac{\rho+\sigma}{f(s_G)\alpha}\right]\frac{\beta\theta(\epsilon-1)}{\nu\epsilon} - \frac{\rho+\sigma}{f(s_G)\alpha}.$$

At $t = 0$ the cut in public R&D pushes firm-level R&D down while the gap operator Δ pushes it up. The direction of the initial jump is thus ambiguous. Afterward, firm-level R&D gradually falls and converges from above to the lower steady-state value.

This pattern suggests a positive relation between public R&D and firm-level R&D. In the language of some of the literature on which we build, public R&D "complements" firm-level R&D: when public R&D falls, firm-level R&D follows. This prediction matches the empirical findings based on long-term firm-level panel data summarized in Becker (2015). More generally, our results explain why about half of the firm-level empirical findings document substitutability between public R&D and firm-level R&D and the other half favor complementarity (see David et al. 2000). Our results suggest that those findings reflect short-run dynamics because most of those empirical studies are based on either cross-sectional or short-term panel data.[7]

Next, we multiply firm-level R&D by the mass of firms to obtain the share of the labor force employed in in-house R&D as

$$\frac{L_Z(t)}{L(t)} = \frac{1-s_G}{\beta-\rho+\lambda}\left[\frac{\beta\theta(\epsilon-1)}{\epsilon} - \frac{\nu(\rho+\sigma)}{f(s_G)\alpha\phi-\rho-\sigma}\frac{1}{1+e^{-\nu t}\Delta}\right].$$

At $t=0$ the crowding-out effect pushes the ratio down while the personnel-interaction effect and the gap operator push it up. We thus have an ambiguous initial jump. Afterward, the ratio L_Z/L gradually falls. As $e^{-\nu t}\Delta \to 0$, we are left with a hump-shaped long-run relation between aggregate in-house R&D and public R&D due to the competing crowding-out and personnel-interaction effects. Why is this relation different from the per-firm one, which is monotonically increasing? The answer clearly is that aggregate in-house R&D is the product of the mass of firms times firm-level R&D and our results say that the former falls with public R&D while the latter rises, see Proposition 4. The tension between these forces generates a non-monotonic relation. More insightful is to note that the multiplication by the mass of firms brings into the relation the crowding-out effect, which, as we have discussed at length, counters the personnel-interaction effect.

Finally, note that the entry technology and the results in Proposition 5 yield

$$\frac{L_N(t)}{L(t)} = \left(\frac{\nu e^{-\nu t}\Delta}{1+e^{-\nu t}\Delta}+\sigma\right)\frac{1-s_G}{\beta-\rho+\lambda}.$$

This expression shows that the ratio jumps up at $t=0$. Afterward, it falls gradually. In steady state, only the crowding-out effect remains and the public R&D cut raises employment in entry. The reason is that the mass of firms per capita is larger and more labor is required to replace the mass of exiters with new firms.

If we add up the two measures of aggregate private innovation effort, we obtain

$$\frac{L_Z(t)+L_N(t)}{L(t)} = \frac{1-s_G}{\beta-\rho+\lambda}\left[\frac{\beta\theta(\epsilon-1)}{\epsilon} - \frac{\nu(\rho+\sigma)}{f(L_G)\alpha\phi-\rho-\sigma}\frac{1}{1+e^{-\nu t}\Delta}+\frac{\nu e^{-\nu t}\Delta}{1+e^{-\nu t}\Delta}+\sigma\right]. \quad (26)$$

This measure exhibits the same pattern discussed above: the crowding-out effect yields an initial jump up; the personnel-interaction effect yields a jump down; the gap operator yields a jump up. The initial jump is thus ambiguous. Throughout the transition the ratio falls. As the economy approaches the new steady state and $e^{-\nu t}\Delta \to 0$, the model predicts a hump-shaped relation between public R&D and this measure of aggregate private innovation effort.

Our results in this section provide a plausible explanation for why almost all the long-run empirical findings at the industry and country levels for OECD countries (e.g., David et. al. 2000) support complementarity between public and aggregate private R&D. Namely, in those countries public R&D is a small share of total employment, meaning that those countries are located on the

[7] Refer to the detailed descriptions of those datasets in Tables 2a and 2b of David et al. (2000). Most of the them are based on cross-sectional or short-term panel data.

upward sloping portion of the hump-shaped relation that we uncover.

We also look at some R&D expenditure to GDP ratios popular in the literature. The first is

$$\frac{w\left(t\right)L_G(t)}{P_C\left(t\right)Y(t)}=\frac{s_G}{E^*}=\frac{s_G\left(\beta-\rho+\lambda\right)}{\beta\left(1-s_G\right)}.$$

This expression says that the public R&D to GDP ratio is always constant. Next, we look at the R&D expenditure to GDP ratios for the private sector:

$$\frac{w\left(t\right)L_Z\left(t\right)}{P_C\left(t\right)Y\left(t\right)}=\frac{1}{\beta}\left[\frac{\beta\theta(\epsilon-1)}{\epsilon}-\frac{\nu\left(\rho+\sigma\right)}{f(s_G)\alpha\phi-\rho-\sigma}\frac{1}{1+e^{-\nu t}\Delta}\right];$$

$$\frac{w\left(t\right)L_Z\left(t\right)+w\left(t\right)L_N\left(t\right)}{P_C\left(t\right)Y\left(t\right)}=\frac{1}{\beta}\left[\frac{\beta\theta(\epsilon-1)}{\epsilon}-\frac{\nu\left(\rho+\sigma\right)}{f(L_G)\alpha\phi-\rho-\sigma}\frac{1}{1+e^{-\nu t}\Delta}+\frac{\nu e^{-\nu t}\Delta}{1+e^{-\nu t}\Delta}+\sigma\right].$$

After the initial jump due to the change in s_G, as functions of time t, these ratios have the same properties as those discussed above and thus exhibit the same comovements.

5 Welfare

A powerful implication of the closed-form solution for the model's dynamics is that we can study analytically the welfare effects of public R&D. Our main result is as follows.

Proposition 6. *Consider the economy described in Proposition 1. The welfare generated by the equilibrium path of the economy is*

$$U_0=\frac{\log E^*}{\rho-\lambda}+\frac{\theta\hat{Z}^*+\gamma\hat{D}^*+\omega\lambda}{\left(\rho-\lambda\right)^2}+\frac{\theta\left(\hat{Z}^*+\rho+\sigma\right)+\gamma\hat{D}^*+\nu\omega}{\left(\rho-\lambda\right)\left(\rho-\lambda+\nu\right)}\Delta. \tag{27}$$

Proof. See the Appendix. □

The first two components in this expression are the standard "level" and "growth" effects that the endogenous growth literature has identified and popularized in the 90s. Our structure adds the third component, the "transition" effect, which accounts for the acceleration or deceleration of technological innovation — firm-level productivity growth and product variety expansion — that the economy experiences along the transition to the new steady state. To understand better this component, we divide it in two parts: the multiplier,

$$\frac{\theta\left(\hat{Z}^*+\rho+\sigma\right)+\gamma\hat{D}^*+\nu\omega}{\left(\rho-\lambda\right)\left(\rho-\lambda+\nu\right)},$$

and the gap from the steady state, Δ. A positive gap means that the economy experiences a transition with faster firm-level growth and faster product variety growth than in steady state. The multiplier accounts for this transition: it translates the positive gap into a permanent TFP gain that raises welfare. The multiplier is a function of the steady-state firm-level growth rate, indicating that an economy with growth-favoring fundamentals benefits more from transitions that feature growth accelerations.

Many applications focus exclusively on level and growth effects. Our model, in contrast, emphasizes the transition effect. We show below that this emphasis is supported not only by analytical considerations, but also by the quantitative properties of the calibrated model, which exhibits slow convergence and thus spends a long time out of steady state.

The power of this solution is that the channels through which public R&D affects welfare are transparent. The following proposition states the result formally.

Proposition 7. *Consider the policy exercise in Proposition 5. The change in welfare generated by the equilibrium path of the economy as it converges to the new steady state is*

$$U - U_0 = \frac{\log \frac{E^*}{E_0}}{\rho - \lambda} + \frac{\theta\left(\hat{Z}^* - \hat{Z}_0\right) + \gamma\left(\hat{D}^* - \hat{D}_0\right)}{(\rho - \lambda)^2} + \frac{\theta\left(\hat{Z}^* + \rho + \sigma\right) + \gamma \hat{D}^* + \nu\omega}{(\rho - \lambda)(\rho - \lambda + \nu)} \Delta. \tag{28}$$

Proof. See the Appendix. □

The welfare change in the proposition consists of three components. The first is the change in welfare due to the change in consumption expenditure per capita caused by the cut in public R&D. Since $E^*/E_0 > 1$, this term is positive. Note the weight of this component: if $\rho - \lambda = 0.02$, the weight is 50. The second component is the change in welfare due to the change in steady-state TFP growth. As we discussed, $\hat{Z}^* - \hat{Z}_0 < 0$. Moreover, using the result in Proposition 5, we have

$$\hat{D}^* - \hat{D}_0 = \frac{s_G}{n^*} - \frac{s_G^0}{n_0} = \frac{s_G}{n^*}\left(1 - \frac{s_G^0}{s_G}\frac{n^*}{n_0}\right) = \frac{s_G}{n^*}\left[1 - \underbrace{\frac{s_G^0}{s_G}(1+\Delta)}_{>1}\right] < 0.$$

Thus, this component is negative. Note the weight of this component: if $\rho - \lambda = 0.02$, the weight is $50^2 = 2500$. This simple observation suggests that the growth effect ought to dominate the determination of the welfare effects of changes in policies and fundamentals.

The third component is the welfare change from a temporary acceleration or deceleration of TFP growth. In our case of a public R&D cut, the gap Δ is positive because the new steady-state mass of firms is larger than the initial one, $n^* > n_0$. Therefore, this component is positive. Its interpretation is informative about the role of transitional dynamics. The public R&D cut causes a permanent growth deceleration. However, (1) it takes time to reach the new steady state, which implies that firm-level growth stays above its new, lower steady-state value for some time, and (2) the transition features faster product variety growth than in steady state. The multiplier translates the gap Δ from the new steady state opened up by the policy change into a permanent productivity gain, relative to the new steady state, due to the temporary deviation of both firm-level growth and product variety growth above their new steady-state values.

To summarize the decomposition discussed above, the overall welfare effect of a reduction in public R&D is ambiguous. In Section 7 we support and expand this analytical conclusion with a quantitative exercise in which we investigate the role of parameters that play an important role in determining which component prevails.

6 The general model of private and public R&D interaction

In this section we set $\kappa > 0$ in equations (8)-(9) and allow for cross-fertilization between public and private R&D. As stated, these channels find empirical support in several studies.

6.1 Firm behavior and global dynamics

The cross-fertilization specification of our model gives us the characterization of in-house innovation in symmetric equilibrium

$$r + \sigma = \frac{LE}{N}\frac{\theta(\epsilon - 1)}{\epsilon Z q} + \frac{\dot{q}}{q} \quad \textit{and} \quad \frac{1}{q} = \alpha f(s_G)\left[\frac{1 + \kappa\left(\frac{B}{K}\right)^{\eta}}{1 + \kappa}\right]^{\frac{1}{\eta}} Z.$$

The processes (8)-(9) read:

$$\hat{Z} \equiv \frac{\dot{Z}}{Z} = \alpha f(s_G)\left[\frac{1 + \kappa\left(\frac{B}{K}\right)^{\eta}}{1 + \kappa}\right]^{\frac{1}{\eta}} \frac{L_Z}{N}$$

$$\hat{D} \equiv \frac{\dot{D}}{D} = \left[\frac{1 + \kappa\left(\frac{B}{K}\right)^{-\delta}}{1 + \kappa}\right]^{\frac{1}{\delta}} \frac{L_G}{N} = \left[\frac{1 + \kappa\left(\frac{B}{K}\right)^{-\delta}}{1 + \kappa}\right]^{\frac{1}{\delta}} \frac{L}{N}\frac{L_G}{L}.$$

Recalling $n \equiv N/L$, we define $k \equiv B/K = D/Z$ and, noting that Lemma 2 holds, we write the key equations driving private behavior as:

$$\rho + \sigma = \frac{\theta(\epsilon - 1)}{\epsilon}\frac{E^*}{n}\alpha f(s_G)\left(\frac{1 + \kappa k^{\eta}}{1 + \kappa}\right)^{\frac{1}{\eta}} - \frac{\kappa k^{\eta}}{(1 + \kappa k^{\eta})}\frac{\dot{k}}{k} - \hat{Z};$$

$$\rho + \sigma = \left[\frac{E^*}{\epsilon n} - \phi - \frac{\hat{Z}}{\alpha f(s_G)\left(\frac{1+\kappa k^{\eta}}{1+\kappa}\right)^{\frac{1}{\eta}}}\right]\frac{\beta n}{E^*} - \frac{\dot{n}}{n}.$$

The first describes the return to in-house R&D accounting for the endogenous productivity of labor in in-house R&D due its dependence on the ratio D/Z. The second describes the return to entry expressed in terms of firm-level growth rather than R&D.

Noting that $\dot{k}/k = \hat{D} - \hat{Z}$, we rewrite the first equation as

$$\hat{Z} = (1 + \kappa k^{\eta}) \left\{ \left[\frac{\theta (\epsilon - 1)}{\epsilon} E^{*} \alpha f (s_G) \left(\frac{1 + \kappa k^{\eta}}{1 + \kappa} \right)^{\frac{1}{\eta}} - \frac{\kappa k^{\eta}}{1 + \kappa k^{\eta}} \left(\frac{1 + \kappa k^{-\delta}}{1 + \kappa} \right)^{\frac{1}{\delta}} s_G \right] \frac{1}{n} - \rho - \sigma \right\}. \tag{29}$$

This function has all the qualitative properties of the function (16) discussed in Section 3, plus new ones due to the presence of the term k. At a point in time, firm-level knowledge growth is a linear function of firm size, $LE^{*}/N = E^{*}/n$. The novel property is that firm-level knowledge growth depends through multiple channels on the ratio of public knowledge to private knowledge, k, which drives the productivity of labor both in firm-level R&D and in public R&D. The function also says that firm-level knowledge growth is hump-shaped in s_G and $\hat{Z} = 0$ occurs when s_G is either too small or too large. More generally, the function identifies the boundary in (k, n) space of the region where $L_Z = 0$. Similarly, the equation for the rate of return to entry identifies the boundary of the region where gross entry is zero, $L_N = 0$. We need to take into account these corner solutions in order to characterize the global dynamics. The Appendix provides the details. Here we summarize the key result as follows.

Proposition 8. *Under condition (CG) from Proposition 1, there exists a unique, globally stable steady state (k^{*}, n^{*}). Since the $\dot{n} = 0$ locus shifts up with s_G while the $\dot{k} = 0$ shifts down, the comparative statics with respect to s_G are that $dk^{*}/ds_G > 0$ while the sign of dn^{*}/ds_G is ambiguous.*

Proof. See the Appendix. □

Figure 1 illustrates the dynamics. All initial conditions yield paths that converge to the steady

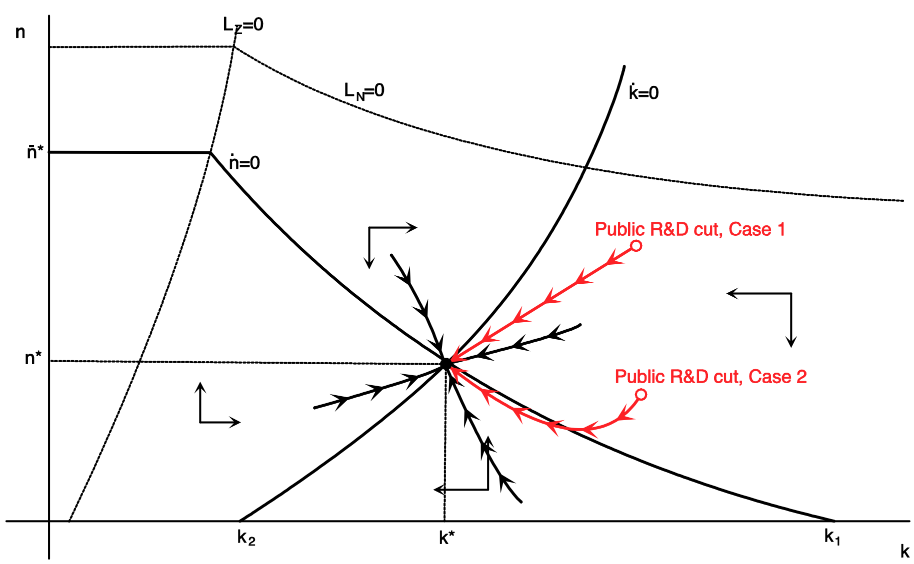

Figure 1: Cross-fertilization model, globally stable dynamics in black and effect of a cut in public R&D, s_G, in red.

state (k^{*}, n^{*}). They are shown as monotonic for simplicity, but the dynamics allow paths that cross one of the isoclines and thus generate non-monotonic behavior of either the mass of firms per capita, n, or the public knowledge to private knowledge ratio, k. One property of note of this system is that it characterizes the dynamics of a scale-invariant endogenous growth model where the productivity of labor employed in R&D by private firms is endogenous and measured by

$$\frac{\dot{Z}/Z}{L_Z/N} = \alpha f\left(s_G\right)\left(\frac{1+\kappa k^{\eta}}{1+\kappa}\right)^{\frac{1}{\eta}}.$$

It thus falls throughout the transition to the steady state when the economy converges to (k^*, n^*) from the right and rises when the economy converges from the left.

The comparative statics imply that a cut in public R&D yields one of two possible paths. Using the terminology set up in Proposition 5, note first that the old steady state (k_0, n_0) must be to the left of the new one since $dk^*/ds_G > 0$. On the other hand, the old steady state is above the new one if the parameters are such that $dn^*/ds_G < 0$, while it is above the new one if the parameters are such that $dn^*/ds_G > 0$. The phase diagram suggests that two cases are possible. These are the two paths in red originating from the hollow circles that denote the old steady state. In Case 1, the economy converges monotonically to the new steady state (k^*, n^*), with both the mass of firms per capita, n, and the public knowledge to private knowledge ratio, k, shrinking throughout. In Case 2, instead, the economy follows a U-shaped path that crosses the $\dot{n} = 0$ locus. The public knowledge to private knowledge ratio, k, converges monotonically from above to k^*, while the mass of firms per capita, n, starts falling, goes below its new steady-state value, n^*, and then starts rising, converging to n^* from below.

In both cases, the transition to the new steady state entails falling productivity of labor employed in R&D by private firms. We can also discuss qualitatively the model-generated comovements among the measures of public and private R&D that we discussed in Section 4. The task is much harder than in that section, however, because we no longer have a closed-form solution for the dynamics. We thus postpone the discussion until Section 7, where we can use the discipline of the quantitative exercise to characterize the dynamic effects of the cut in public R&D.

6.2 Steady-state growth

Next, we solve for the steady state focusing on growth. The appendix shows that in steady state the set of equilibrium relations reduces to the following system in $(k, \hat{Z})$ space:

$$\hat{Z} = \frac{\rho+\sigma}{\frac{\alpha\theta(\epsilon-1)}{\epsilon}E^* f\left(s_G\right)\frac{\left(\frac{1+\kappa k^{\eta}}{1+\kappa}\right)^{\frac{1}{\eta}}}{\left(\frac{1+\kappa k^{-\delta}}{1+\kappa}\right)^{\frac{1}{\delta}}}-1}; \tag{30}$$

$$\hat{Z} = \left[\phi\alpha f\left(s_G\right)\left(\frac{1+\kappa k^{\eta}}{1+\kappa}\right)^{\frac{1}{\eta}} - \rho - \sigma\right]\frac{\beta\theta\left(\epsilon-1\right)}{\nu\epsilon} - \rho - \sigma. \tag{31}$$

We also show that this system has a unique solution $(k^*, \hat{Z}^*)$ with the property that $\hat{Z}^*$ is increasing in s_G because both equations shift up with s_G. The change in k^* looks ambiguous but we know from the phase diagram that k^* rises with s_G. The cross-fertilization model, therefore, confirms the result from Section 4 that a cut in public R&D reduces steady-state firm-level knowledge growth. The novel result here is that it does so, in part, by reducing the public knowledge to private knowledge ratio, k, and thus reducing the productivity of in-house R&D. Recall that in the long run of this model economic growth is a weighted sum of two endogenous components, firm-level knowledge growth and public knowledge growth. As both are lower, the cut in public R&D lowers economic growth along the several margins discussed in Section 4.

Note that this system admits a solution $\hat{Z} > 0$ for $k = 0$ or $\kappa = 0$. The solution comes from the second equation, which becomes

$$z^* = [\phi\alpha f(s_G) - \rho - \sigma] \frac{\beta\theta(\epsilon - 1)}{\nu\epsilon} - \rho - \sigma.$$

This is the same solution as in our baseline model studied in the previous sections. If in addition we set $s_G = 0$ and note that $f(0) = 1$, we recover the solution for the baseline Schumpeterian model with no government that the literature has studied extensively. We elaborate on this point in some detail in the next subsection, where we discuss some interesting properties of our cross-fertilization model that are not immediately apparent from the setup and the global dynamics.

6.3 Some interesting properties

As we argued in Section 2, the CES specification for the knowledge accumulation processes (8)-(9) allows us to test whether the model with public R&D nests the model with no government that the literature has studied extensively. Thus, we now set $s_G = 0$ and check the model's dynamics starting from initial conditions n_0 and $k_0 = D_0/Z_0$, where D_0 is a constant. The appendix shows that this setup produces the phase diagrams in Figures 2 and 3.

Case 1. For all initial conditions (k_0, n_0) the economy converges to the unique steady state $(0, n^*)$, which is the steady state with endogenous growth driven by private R&D activity of the baseline Schumpeterian model with no government.

Case 2. There is a set of zero growth steady states, the union of the point $(\bar{k}^*, \bar{n}^*)$ and the points $(\tilde{k}^*, \bar{n}^*)$ for $\tilde{k}^* \in (0, \bar{k}^*)$. All initial conditions (k_0, n_0) yield paths that converge to a point in this set. The value $\bar{k}^*$ is determined by the parameters. In contrast, the value $\tilde{k}^*$ depends on the specific path dictated by the initial condition and the law of motion of the system. Therefore, in this configuration the economy exhibits path-dependence for the steady-state value of the productivity of labor in private R&D. There is a watershed trajectory (in blue) such that for all initial conditions to its right the economy converges to $(\bar{k}^*, \bar{n}^*)$, while for all initial conditions to its left the economy converges to a point in the path-dependent set.

One interpretation of this exercise is that it shows the dynamics of a scale-invariant endogenous

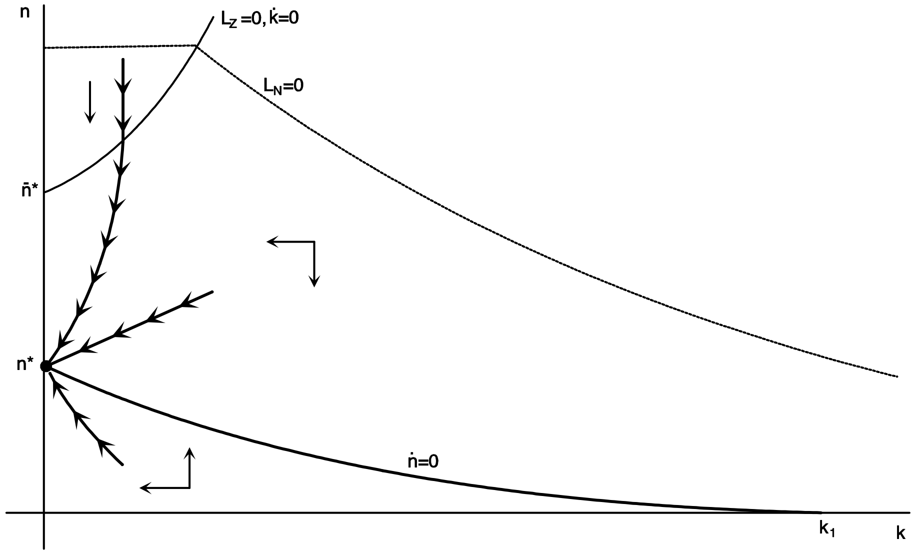

Figure 2: Shutting down public R&D ($s_G = 0$), Case 1

growth model with no government and endogenous productivity of labor in R&D, measured by

$$\frac{\dot{Z}/Z}{L_Z/N} = \alpha \left(\frac{1 + \kappa k^\eta}{1 + \kappa} \right)^{\frac{1}{\eta}} = \alpha \left(\frac{1 + \kappa \left(\frac{D_0}{Z} \right)^\eta}{1 + \kappa} \right)^{\frac{1}{\eta}},$$

that falls throughout the transition to the steady state. The model is thus consistent with the claims of increasing cost of innovation that are often used as arguments against endogenous growth theory (see Jones 2022 for a recent review). Regardless of the empirical correctness of the claims, evidence of a falling ratio of growth to R&D input, whether in firm-level or aggregate data, does not invalidate scale-invariant Schumpeterian growth theory. It only questions the predictions for this particular ratio of models that rely on power functions for the specification of the R&D technology. To see this point more generally, note that the baseline model of innovation-led growth that we take from Peretto and Connolly (2007) specifies firm-level knowledge accumulation as $\dot{Z}_i = \alpha K_i L_{Z_i}$. Replacing this representation with something like $\dot{Z}_i = \alpha (\varkappa + K_i) L_{Z_i}$, where $\varkappa$ is a constant standing for any other source of labor productivity in private R&D, produces qualitatively the dynamics discussed above. This is just another way to write a process that exhibits firm-level diminishing returns to knowledge. The critical difference is that the traditional specification looses such diminishing returns in the symmetric equilibrium of the model since the R&D technology reduces to $\hat{Z} = \alpha L_Z/N$. The new specification, in contrast, does not: the productivity of labor in R&D is always decreasing in the average knowledge stock since the R&D technology reduces to $\hat{Z} = \alpha \left(\frac{\varkappa}{Z} + 1 \right) L_Z/N$.

We can also verify that if public knowledge is essential to private innovation there is a discontinuity at $s_G = 0$ because the system dynamics do not recover the baseline Schumpeterian model with no government. To see this, let $\eta \to 0$. The key equation is the condition for $\hat{Z} = 0$, which

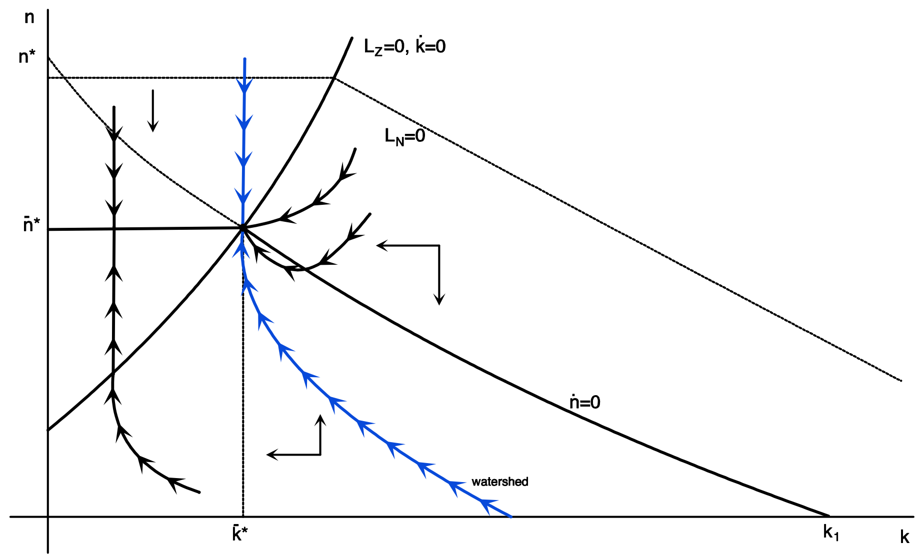

Figure 3: Shutting down public R&D ($s_G = 0$), Case 2

now reads

$$n \leq n_{L_Z=0}(k) = \frac{\frac{\theta(\epsilon-1)}{\epsilon} E^* \alpha k^{\kappa_Z}}{\rho+\sigma}.$$

This function does not have an intercept and therefore precludes access to the vertical axis. Consequently, the only feasible dynamics are similar to those discussed in Case 2 above. We conclude that the cross-fertilization model with essentiality of public knowledge for private innovation is a fundamentally different theory of the growth process. In particular, if it is the correct one, one could argue that all models of endogenous growth with no government are necessarily empirically wrong. Our nesting procedure, in contrast, says that the two specifications coexist peacefully and that abstracting from the role of public R&D is a harmless simplifying assumption.

6.4 Public R&D and growth in the very long run: a catalyst for takeoff

The result that under substitutability ($\eta > 0$) setting $s_G = 0$ recovers the baseline model with no government allows us to explore further the long-run relation between the public production of knowledge and private innovation. Specifically, we can ask the question: can the government, by setting up institutions whose task is to produce knowledge that private agents can freely use, initiate a dynamic process that eventually culminates in the activation of private profit-driven innovation? According to our model the answer is yes. In the language of much of the literature, this property is the ultimate relation of complementarity between public R&D and private R&D: public R&D *causes* the economy's eventual takeoff driven by private R&D.

We illustrate the mechanism in the phase diagram in Figure 4. The economy is initially in the baseline configuration with no public R&D and has fundamentals insufficient to motivate firms to do in-house R&D. Point $(0, n(0))$ in the graph represents such an initial configuration: it is in

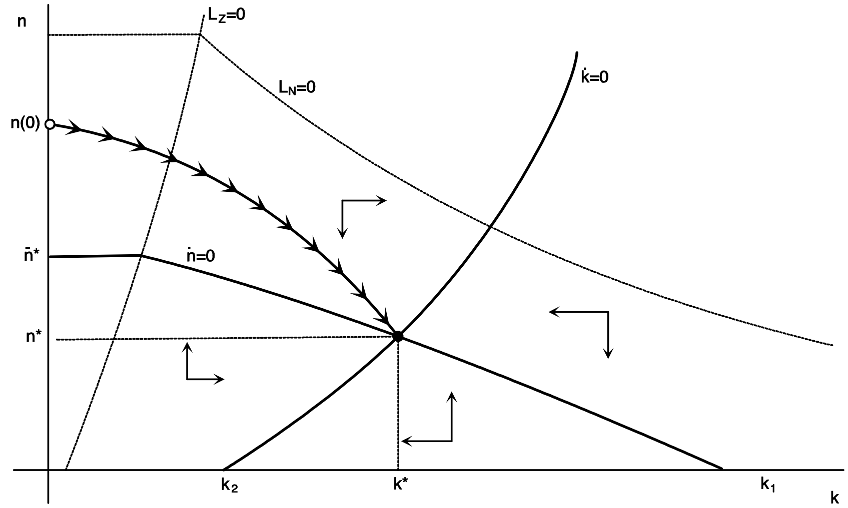

Figure 4: Transition path of economy that institutes public R&D at time $t = 0$ and eventually activates private R&D.

the region where firms do not do in-house R&D and also features zero public knowledge, $k = 0$. When the government begins the production of public knowledge, private agents respond to the crowding-out effect by reducing gross investment in entry. The economy thus starts moving to the right as the mass of firms per capita shrinks and the ratio of public knowledge to private knowledge grows. Two things happen as this process unfolds: firm size rises and the potential productivity of in-house R&D rises. Both forces favor in-house R&D, but firms still do not do it because the return that it would deliver is too low. In this sense, the productivity of in-house R&D is a latent variable: it rises because of the rising contribution of public knowledge but does not yet have an effect on the behavior of firms.

Eventually, however, firm size and the productivity of in-house R&D rise to the point where in-house R&D delivers a rate of return that meets the reservation rate of return of savers. This event occurs when the path in the diagram crosses the boundary $L_Z = 0$. As firms start investing in-house, the economy experiences a growth acceleration that replicates the takeoff observed historically in the advanced economies in the aftermath of the Industrial Revolution. The process continues until the economy converges to the steady state (k^*, n^*) characterized in the previous subsections. As shown there, throughout this growth acceleration, as the ratio of public to private knowledge rises, the productivity of private in-house R&D rises.

We cannot think of a better illustration of the rich dynamic relationship between private and public R&D and of the different effects of public R&D at different time horizons. We showed above that, in the region of the state-space where in-house R&D is positive, the endogenous market structure mechanism causes the ambiguous short-run effect of public R&D on private R&D to turn into an unambiguously positive long-run effect. We showed here that if we adopt a different view of the long run, and we focus not on the model's steady state but on the model's global dynamics and escape from corner solutions, then the model replicates the secular growth acceleration initiated by the Industrial Revolution and suggests that the establishment of public institutions devoted to the production and diffusion of knowledge was one of the catalysts that precipitated that process.[8]

7 Quantitative exercise

In this section, we investigate the quantitative properties of the general model. We calibrate the model to the U.S. economy and then, as we did for the baseline model studied in Section 4, we posit an unanticipated, immediate and permanent cut of the public R&D employment share.

7.1 Calibration

We calibrate the model's parameters to match U.S. data for 1964 to 2021, a period that exhibited a large decline of the ratio of Federal R&D (public R&D) to GDP. We discuss the targeted moments and the calibration procedure in Appendix A; here we focus on the results. The following table reports the calibrated parameter values:

ρ	ϵ	γ	σ	κ	θ	$\eta = \delta$	ϕ	β	α	ξ	ω
0.04	3.333	0.035	0.062	0.429	0.019	0.287	2.454	0.563	0.607	6.147	0.653

7.2 Effects of the public R&D cut

We now feed to the calibrated model an unanticipated, immediate and permanent cut of the public R&D employment share s_G from 0.017 to 0.017/2 to mimic (conservatively, see the appendix for detals) the magnitude of the reduction in public R&D effort that we see in the data. We report the dynamic response of three sets of variables. The first set characterizes the labor reallocation caused by the public R&D cut. Obviously labor flows from public to private employment, but it turns out that the crucial margin of action is within private employment, from entry and in-house R&D to production and viceversa. The second set of variables characterizes the endogenous market structure. We focus on the net entry rate, driving dynamics, and firm size, driving incentives. The third set of variables characterizes the evolution of technology. We focus on the ratio of public knowledge to private knowledge, the growth rates of the two stocks of knowledge, and the growth rate of TFP, which is the ultimate driver of welfare. The ratio of public knowledge to private knowledge is of paramount importance since it drives the *endogenous* productivity of labor in in-house R&D. In our figures, we mark the steady state prior to the policy implementation with an asterisk.

7.2.1 Labor reallocation

Figure 5 shows the dynamic response of four measures of R&D. Firm-level R&D, L_Z/N, jumps

[8] Recently, Lehmann-Hasemeyer, Prettner and Tscheuschner (2020) have argued in a model of the Unified Growth Theory class that the Scientific Revolution was a key driver of the takeoff to sustained modern economic growth. Their perspective is comlementary to ours.

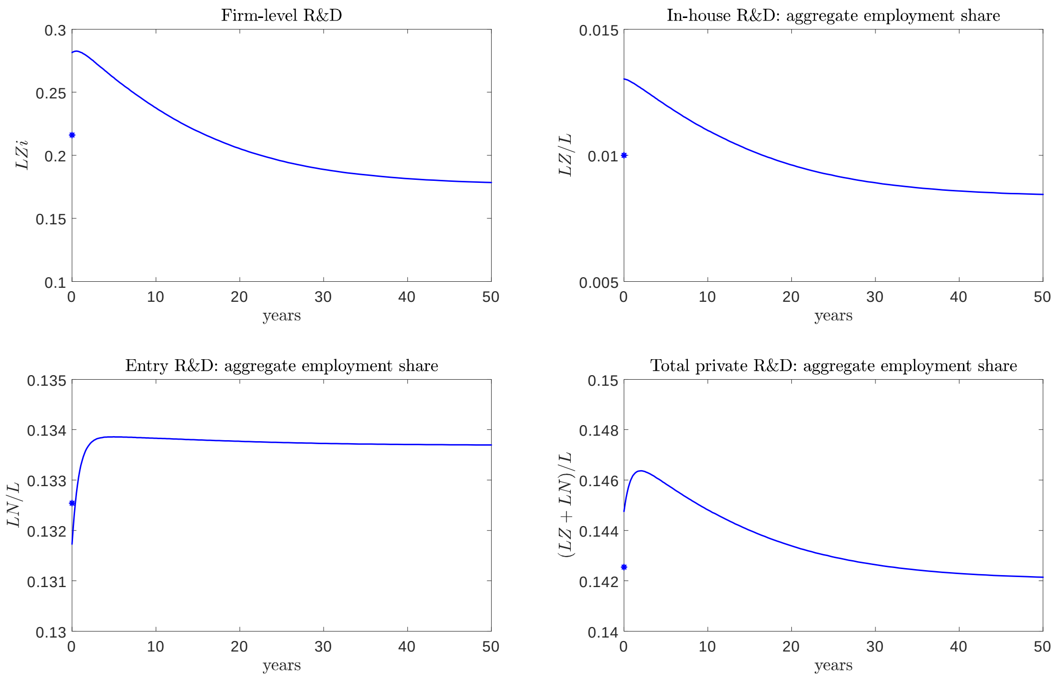

Figure 5: Reallocation of labor in response to the public R&D cut.

up, rises further for a short time, and then gradually converges from above to the new steady-state value, which is lower than the initial one. As we discussed in Section 4, in the flow specification of the model only two competing forces are at play at the time of the change in public R&D: firm size, LE/N, rises because of the weaker crowding-out effect and the personnel-interaction effect, $1+\xi s_G$, weakens. The key mechanism here is that the cut of public R&D slows down the accumulation of public knowledge and thus gradually reduces the productivity of labor in private R&D. This dynamic effect, however, unfolds in time. In our calibration, at $t = 0$ the return to in-house R&D rises more than the return to entry, leading to a reallocation of labor to firm-level R&D. This effect is large; the subsequent decline is likewise large and rapid: within about 15 years firm-level R&D returns to the original value. It then keeps falling very slowly until in the new steady state it registers a small decline from the original steady-state value.

The dynamic response of the employment share of aggregate in-house R&D, L_Z/L, has a similar pattern except for the momentum-like part: it jumps up at $t = 0$ and then starts falling rapidly, returning to the original level in 15 years. It then keeps falling very slowly until it converges to the lower new steady-state value.

The sum of L_Z/L and L_N/L gives us the path of the total private R&D share. It jumps up at $t = 0$ and rises further for a short time due to the very large contribution of firm-level R&D. It then peaks and rapidly declines: in about 15 years, it returns to the initial value and then keeps falling very slowly. Remarkably, in the new steady state it is lower than the original steady state only by

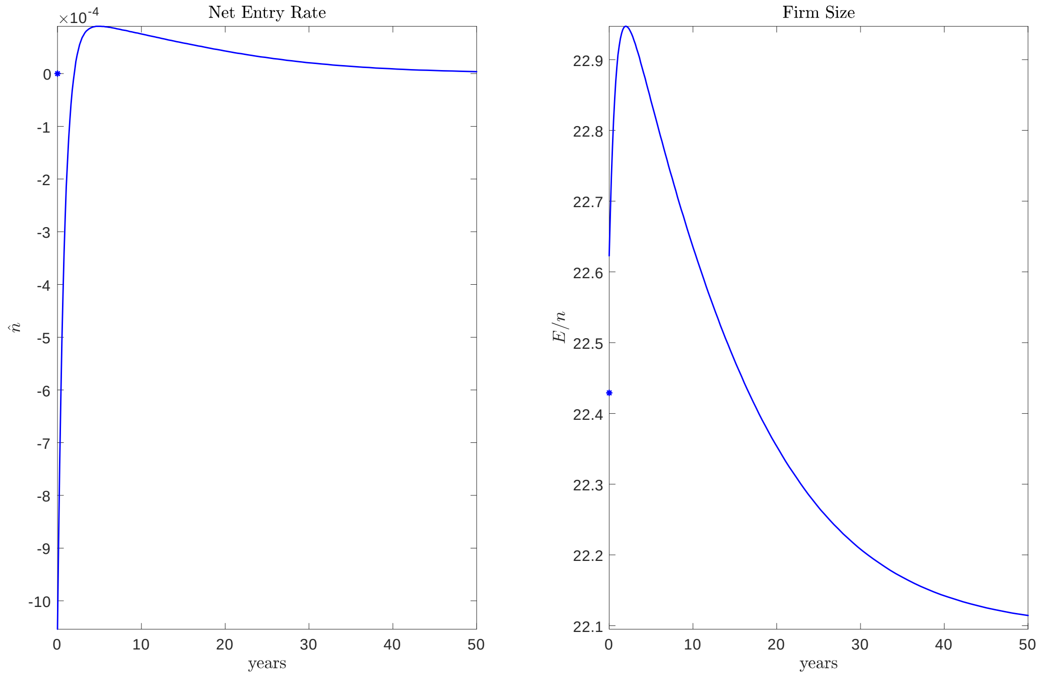

Figure 6: Market structure response to the public R&D cut.

a tiny amount. This result implies that the main margin of adjustment in aggregate employment is from public knowledge production to private goods production.

The comovements that our model produces are consistent with the empirical findings on the substitutability or complementarity of public and private R&D obtained from cross-sectional or short-run panel data. They are also consistent with the findings from long-run panel data when we look at the model's output past the 10-15 years mark. We discussed such evidence at length and do not revisit it here. Instead, we note that one novel pattern that our cross-fertilization model produces is the potential non-monotonic path of key variables, some of which exhibit momentum-like dynamics that to our knowledge is not reported in the literature. In future research it will be interesting to check whether such a pattern is in the data. This will require extending the analysis with dynamic panel methods.

7.2.2 Market structure

Figure 6 shows the effect of the a cut of the public R&D employment share, s_G, on the net entry rate, $\hat{N}$, and firm size, $LE/N = E/n$. Our calibration selects the transition path for Case 2 in the phase diagram in Figure 1.

The left panel shows the response of the net entry rate $\hat{N}$. It initially jumps down, quickly recovers and then converges slowly back to its original value of zero. This differs quite drastically from the baseline model of Section 4.2, where the entry rate jumps up and then reverts to zero monotonically. As we discussed, in that specification of the model only two competing forces are at play: the larger firm size, LE/N, due to the weaker crowding-out effect, and the weaker personnel-interaction effect, $1+\xi s_G$. Here, the cut of public R&D reduces over time the productivity of labor in private R&D. At $t = 0$, however, the new effect is not yet in play and thus in our calibration rises the return to in-house R&D rises more than the return to entry, producing a massive reallocation of labor to firm-level R&D. Afterward, the entry rate rises because of the rapid decline of firm-level R&D driven by the falling public to private knowledge ratio, D/Z, which depresses the productivity if labor in private R&D. As we remarked earlier, these dynamics unfold very rapidly. Indeed, the entry rate overshoots its steady-state value of zero within roughly 2-3 years and then reverts to zero extremely slowly.

As stated, firm size, LE/N, jumps up at the instant of the policy change. Along the transition, its evolution is the inverse of the evolution of the mass of firms per capita, which is dictated by the entry rate. The result is the hump-shaped path in the figure. This path is consistent with the analytical counterpart that we obtained in Section 4 in the flow version of our model, with the additional twist that here the behavior is non-monotonic: after the initial jump, firm size rises further for a very short period of time, roughly 2-3 year, and then falls very rapidly for about 10-15 years, when it slows down until it converges to the new steady state after more than 50 years. Overall, firm size LE/N rises significantly at its peak but decreases only by a tiny percentage in the long run.

7.2.3 Knowledge and TFP growth

Figure 7 shows the response of the ratio of public knowledge to private knowledge, D/Z, the growth rates of two knowledge stocks, $\hat{D}$ and $\hat{Z}$, and TFP growth, $\hat{T}$. When the cut of the public R&D employment share, s_G, takes place, the growth rate of public knowledge, $\hat{D}$, falls while the growth rate of the firms' knowledge, $\hat{Z}$, accelerates because of the massive labor reallocation to in-house R&D discussed earlier.

Afterwards, the sharp rise of the differential between the growth rates of private and public knowledge drives the rapid decrease of the knowledge ratio, D/Z, which in our model drives the productivity of labor in private R&D. As a result, private knowledge growth starts decelerating while public knowledge growth starts accelerating because it benefits from the cross-fertilization coming from the private sector. Eventually, the gap between the two growth rates closes and the economy converges to the new steady state where the knowledge ratio is permanently lower. We remind the reader that in our model the dynamics of the knowledge ratio drives the dynamics of the firm-level measure of the productivity of labor in R&D

$$\frac{\dot{Z}/Z}{L_Z/N} = \alpha \left(\frac{1+\kappa k^{\eta}}{1+\kappa} \right)^{\frac{1}{\eta}}.$$

Our model thus attributes the reported decline in this measure to the cuts in public R&D that characterized the U.S. economy over the last several decades.

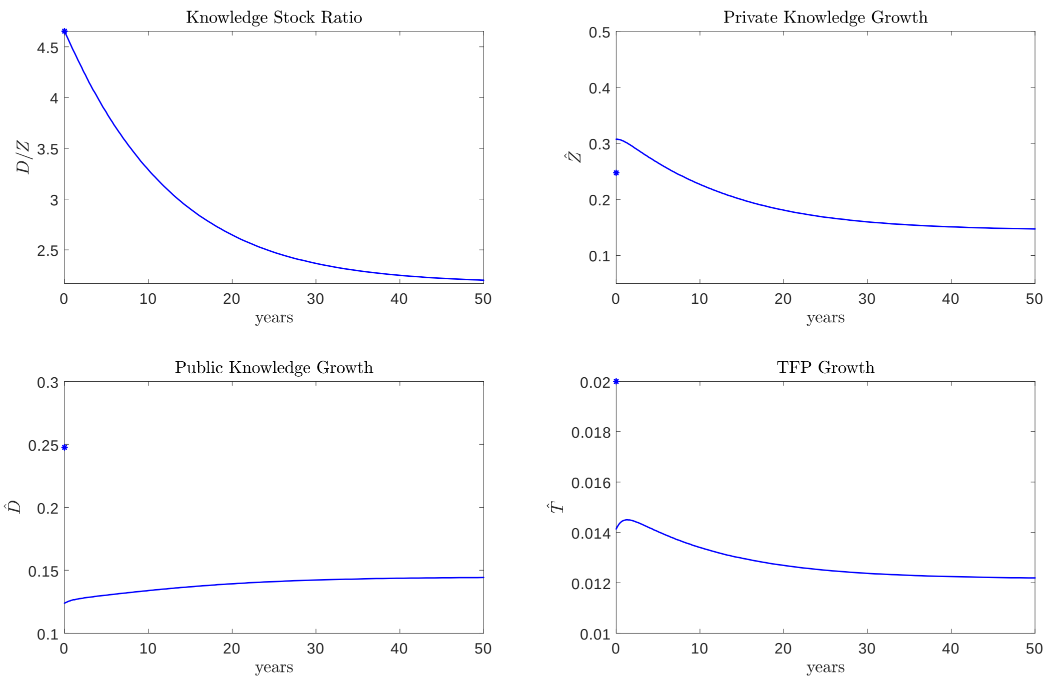

Figure 7: Response of the knowledge ratio, D/Z, the growth rates of the two knowledge stocks, $\hat{D}$ and $\hat{Z}$, and of TFP, $\hat{T}$, to the public R&D cut.

The growth rate of TFP, $\hat{T}$, is a weighted sum of public knowledge growth, $\hat{D}$, firm knowledge growth, $\hat{Z}$, and product variety expansion, $\hat{N}$. The balance of the three forces cause TFP growth to fall at the time of the spending cut, to then rapidly rise due to the recovery of the entry rate and to the explosion of firm-level knowledge growth driven by the labor reallocation discussed earlier. After it peaks, TFP growth continues to decrease over time due to the prolonged decline of firm-level R&D. As this process unfolds, TFP growth declines significantly from its peak to its new steady-state value. The takeaway result is that the halving of public R&D effort causes steady-state growth to decline from 2% to 1.44%. This is a large effect.

7.2.4 Welfare

Differently from Section 5, where we had the luxury of the closed-form solution for the dynamics, here we need to write the household's welfare in less compact form. To minimize this aspect of the exercise, we write the transitional part of the process in terms of TFP, $T = N^{\omega} Z^{\theta} D^{\gamma}$, which is our model's summary statistic for the state of technology. Following the same procedure as in Section 5, we obtain that the public R&D cut yields the following welfare differential with respect to the

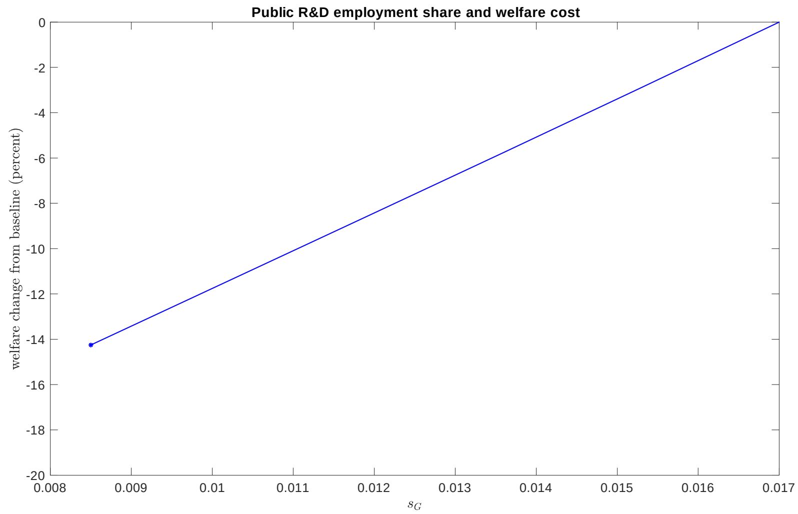

Figure 8: Percentage change in welfare caused by a reduction of s_G.

baseline of no cut (see the Appendix for the derivation):

$$U - U_0 = \frac{\log \frac{E^*}{E_0}}{\rho - \lambda} + \frac{(\theta + \gamma)\left(\hat{Z}^* - \hat{Z}_0\right)}{(\rho - \lambda)^2} + \int_0^\infty e^{-(\rho-\lambda)t}\left[\int_0^t \left(\hat{T}(s) - \hat{T}^*\right) ds\right] dt.$$

The welfare change is the sum of three terms: the change in expenditure per capita from E_0 to E^*; the change in steady-state growth from $(\theta + \gamma)\,\hat{Z}^*$ to $(\theta + \gamma)\,\hat{Z}_0$ (recall that in steady state D/Z is constant); the temporary deviation of TFP from the new steady state. The last term is where the lack of an analytical solution makes things less tidy. Nevertheless, the mechanism is clear.

The first term is positive because the economy experiences the crowding-out effect in reverse due to the cut in public R&D employment. The second term is negative because the public R&D cut yields lower steady-state growth. The sign of the third term is in principle ambiguous because the transition exhibits an initial sharp acceleration of private knowledge growth and above average product variety growth for most of it, dynamics that can compensate for the slowdown of public knowledge growth. Our TFP indicator gives the model-consistent balancing of these competing forces: in our exercise TFP growth falls, overshooting its new steady-state value, and then follows a hump-shaped path, rising above the new steady-state value, to finally turn around and converge from above to the new steady state. The key is that throughout the process TFP growth is below the baseline value. Therefore, the economy experiences a cumulative loss of TFP relative to the baseline path. In Figure 7, the magnitude of this loss is the area below the baseline TFP growth rate and the path of TFP growth caused by the public R&D cut. If this loss dominates the gain from higher expenditure per capita, welfare falls.

Figure 8 shows our quantitative results for a wide range of values of the public R&D employment share, s_G. It plots the percentage change in welfare expressed in consumption equivalent units. The interpretation of the graph is the following. For each value of $s_G \leq 0.017$, the line describes the welfare change generated by feeding to the model an unanticipated, immediate and permanent cut of the public R&D employment share from the baseline value of 0.017 to the value on the horizontal axis. The left endpoint of the line is the effect of the cut of s_G in half that we investigated in our quantitative exercise above. As one can see, the cut causes a percentage reduction in welfare of slightly more than 14%.

8 Conclusion

This paper explored the long-run and short-run transition effects of public R&D on market structure (i.e., number of firms, firm size), innovation by firms (both in-house R&D and entry), and ultimately on economic growth and welfare. The analysis uses the Schumpeterian growth model with endogenous market structure proposed by Peretto and Connolly (2007), whose core mechanism is that variables that in equilibrium depend on firm size (i.e., private R&D per firm, the growth rates of productivity and of consumption per capita) do not respond to the crowding-out effect of public R&D in steady state but respond to it during the transitional dynamics. This means that the model can explain the salient finding of the empirical literature, namely, results differ drastically across studies that use short-run and long-run data.

In the model, in response to a reduction of public R&D employment, the long-run growth rate of income per capita always decreases, while in the short run it can exhibit an initial increase. In addition, public R&D and private R&D always comove positively in steady state, whereas during the transitional dynamics the comovement can be negative. These results agree with the empirical findings of studies that try to disentangle short-run and long-run behavior of variables.

The theoretical model proposes a richer characterization of the interaction of public and private knowledge — CES rather than the popular Cobb-Douglas representation — wherein the productivity of labor in private R&D is decreasing in the ratio of public to private knowledge with a strictly positive lower bound. Consequently, the model exhibits fully endogenous growth while it accommodates a decline of R&D productivity throughout a transition initiated by a cut in public R&D activity. In other words, the model attributes the dynamics of growth and private R&D productivity observed in the data to the dramatic reduction in public R&D (roughly a halving) experienced by the US economy from 1964 to today.

A quantitative exercise shows that in response to a halving of the public R&D employment share, the economy experience a long transition characterized by non-monotonic behavior of key variables like firm size, R&D per firm and economic growth. Throughout, the productivity of labor in private R&D falls gradually by a large amount. The long-run (steady-state) effect is a reduction of the growth rate of income per capita from 2% to 1.44%. Interestingly, in steady state the share of private employment in innovation (both in-house R&D and entry) changes only little. Therefore, the main margin of adjustment is from public production of knowledge to private production of goods. One might be then tempted to conclude that the reduction of the growth rate of income per capita is mostly due to the reduction of the growth rate of public knowledge. Not so. The large reduction of the ratio of public to private knowledge causes a large reduction in the productivity of labor in private R&D. Consequently, even if private R&D per firm remains roughly the same due to the endogenous adjustment in the number of firms, such R&D effort supports a lower growth rate of private knowledge, which falls from 0.248 to 0.145.

A Appendix: Calibration

In this appendix, we discuss the calibration procedure. We first report our standard targets and their relation to our model's variables if such relation is direct. We also perform some robustness checks by comparing values obtained from different sources and the consistency of our measurements with available non-targeted moments.

A.1 Standard targets

Our standard targets are the following.

- Population growth rate: $\lambda = 0.01$.
- Interest rate: $r = \rho = 0.04$.
- Per capita GDP growth rate: $\hat{T} = 0.02$.
- Stock market capitalization to GDP ratio: $A/Y = 1.2$, average for 1995-2019 calculated at https://fred.stlouisfed.org/series/DDDM01USA156NWDB/.
- Consumption expenditure to GDP ratio: $LE/Y = 0.675$.
- Labor share of GDP: $wL/Y = 0.65$.
- Population to firms ratio: $L/N = 21.6$, calculated from the Business Dynamics Statistics (BDS).
- Firms' death rate: $\sigma = 0.0618$, calculated from the BDS. This value gives a profit discount rate $r + \sigma = 0.04 + 0.0618 = 0.1018$, which is reasonable for the rate of return to corporate equity adjusted for exit risk.
- Private R&D employment share: $L_Z/L = 0.01$, calculated from InfoBrief, October 2016, NSF 17-302; see https://www.nsf.gov/statistics/2017/nsf17302/. We can do a robustness check with the industrial R&D to GDP ratio $wL_Z/Y = 0.0148$. We calculate

$$\frac{L_Z}{L} = \frac{wL_Z}{wL} = \frac{wL_Z}{Y}\frac{Y}{wL} = \frac{0.0148}{0.65} = 0.022.$$

  This is twice the NSF value. However, industrial R&D includes non-wage expenditures. If we think of the standard split between labor and other costs of 2/3 and 1/3, we estimate $wL_Z/Y = 0.0148 \times \frac{2}{3} = 0.0098$, which then gives $L_Z/L = \frac{0.009}{0.65} = 0.0138$. This value is now remarkably close to the NSF number.
- The targets $L_Z/L = 0.01$ and $L/N = 21.6$ give $L_Z/N = (L_Z/L) \times (L/N) = 0.01 \times 21.6 = 0.216$.

- Productivity growth rate: $\theta\hat{Z}+\gamma\hat{D}=0.01347$, average annual TFP growth rate in manufacturing from 1995 to 2013 calculated from U.S. Bureau of Labor Statistics.
- Public R&D employment share: $s_G=0.017$. Data on basic and public research employment are usually hard to find. According to the NSF, scientists and engineers make up about 4.2 percent of the total US workforce in 2006. Data from 2003 suggest that about 59 percent of these are employed by industry and the remaining 41 percent are affiliated with government and university.

A.2 Calibration

We begin with the equations that we use in the calibration. Then we discuss the parameters that we identify directly from specific data moments. Finally, we discuss the parameters that we calibrate jointly from sets of data moments.

For simplicity we denote

$$\alpha f\left(s_G\right)\left(\frac{1+\kappa k^{\eta}}{1+\kappa}\right)^{\frac{1}{\eta}}\equiv\Psi.$$

We work with the following steady-state equations:

$$LE=\beta A; \tag{A.1}$$

$$E=\frac{1-s_G}{1-\left(\rho-\lambda\right)A/LE}; \tag{A.2}$$

$$\rho+\sigma+\hat{Z}=\frac{LE}{\epsilon N}\theta\left(\epsilon-1\right)\Psi; \tag{A.3}$$

$$\rho+\sigma=\left(\frac{LE}{\epsilon N}-\phi-\frac{L_Z}{N}\right)\frac{\beta N}{LE}; \tag{A.4}$$

$$\hat{Z}=\Psi\frac{L_Z}{N}; \tag{A.5}$$

$$\hat{D}=\hat{Z}=\left(\frac{1+\kappa k^{-\delta}}{1+\kappa}\right)^{\frac{1}{\delta}}\frac{L}{N}s_G; \tag{A.6}$$

$$\hat{T}=\omega\lambda+\theta\hat{Z}+\gamma\hat{D}. \tag{A.7}$$

These relations identify directly several parameters

Elasticity of substitution, ϵ We take the value $\epsilon=3.3333$ from the literature, which documents that the average markup ratio is $\frac{P}{MC}=\frac{\epsilon}{\epsilon-1}=1.3$.[9]

[9] The macro-oriented literature reports estimates of the average markup between 1.1 and 1.4. Recent micro-oriented studies, such as Hall (2018), report estimates of the average industry markup in the range 1.0 to 1.8 in 2015, while De Loecker, Eeckhout and Unger (2020) report that the weighted average of the markup ratio is 1.61 in 2016. Our target of 1.3 is safely in the range spanned by this literature.

Elasticity of firm TFP with respect to public knowledge capital, γ The estimates in the literature vary widely. Guellec and Van Pottelsberghe (2004) find the value 0.17, whereas Park (1995) and Cole et al. (2009) find that this elasticity is not significantly different from zero. Van Elk et al. (2019) use augmented production function models that include the interactions between R&D variables and additional production factors, such as public capital, the stock of inward and outward foreign direct investments and the shares of high-tech imports and exports. They find a significant value of the elasticity in the 0.027 to 0.04 range. We choose the value $\gamma = 0.035$ in the middle of this range.

Sunk entry cost, $1/\beta$ The free-entry condition (A.1) says that to calibrate β all we need is an estimate of the consumption-wealth ratio, LE/A. To obtain it, we use NIPA data to calculate

$$\beta = \frac{\text{household consumption}}{\text{stock market capitalization}} = \frac{\text{household consumption / GDP}}{\text{stock market capitalization / GDP}} = \frac{0.675}{1.2} = 0.5625.$$

We then use (A.2) to calculate

$$E = \frac{1 - s_G}{1 - (\rho - \lambda)\, A/LE} = \frac{1 - 0.017}{1 - (0.04 - 0.01)\,/0.5625} = 1.0384.$$

This value is extremely close to the value $E = 1.0385$ calculated directly from NIPA data using the procedure

$$\frac{\text{household consumption}}{\text{wage income}} = \frac{\text{household consumption / GDP}}{\text{wage income / GDP}} = \frac{0.675}{0.65} = 1.0385.$$

In other words, recalling that we normalize $w = 1$, we have two different calibrated values of the ratio E/w, one of them model-free, that agree to the third decimal place.

The elasticity of productivity w.r.t. private knowledge, θ Recall our target $\theta\hat{Z} + \gamma\hat{D} = (\theta + \gamma)\,\hat{Z} = 0.01347$. We use (A.5) to write

$$(\theta + \gamma)\,\hat{Z} = (\theta + \gamma)\,\Psi\frac{L_Z}{N} \Rightarrow (\theta + \gamma)\,\Psi = \frac{(\theta + \gamma)\,\hat{Z}}{\frac{L_Z}{N}} = \frac{0.01347}{0.216} = 0.0624.$$

We then solve (A.3) for

$$\theta = \frac{(\theta + \gamma)\,\hat{Z} + \gamma\,(r + \sigma)}{\frac{\epsilon - 1}{\epsilon}\frac{(\theta+\gamma)\hat{Z}}{\frac{L_Z}{L}}E - r - \sigma}.$$

Then, for $\gamma = 0.035$ we have

$$\theta = \frac{(\theta + \gamma)\,\hat{Z} + \gamma\,(r + \sigma)}{\frac{\epsilon - 1}{\epsilon}\frac{(\theta+\gamma)\hat{Z}}{\frac{L_Z}{L}}E - r - \sigma} = \frac{0.01347 + 0.035\,(0.04 + 0.0618)}{\frac{3.333 - 1}{3.333} \times \frac{0.01347}{0.01} \times 1.0384 - 0.04 - 0.0618} = 0.0194.$$

Now that we have θ we calculate:

$$\hat{Z} = \frac{(\theta+\gamma)\,\hat{Z}}{\theta+\gamma} = \frac{0.01347}{0.0194+0.035} = 0.2475;$$

$$\Psi = \frac{0.0624}{0.0194+0.035} = 1.1471.$$

Love of variety, ω We use (A.7) to write

$$\omega = \frac{\hat{T}-(\theta+\gamma)\,\hat{Z}}{\lambda} = \frac{0.02-0.01347}{0.01} = 0.653.$$

Fixed operating cost, ϕ We use (A.4) to write

$$\begin{aligned}\phi &= \frac{LE}{N}\left(\frac{1}{\epsilon}-\frac{\rho+\sigma}{\beta}\right)-\frac{L_Z}{N}\\ &= 1.0384\times 21.6\left(\frac{1}{3.333}-\frac{0.04+0.0618}{0.5625}\right)-0.216 = 2.4543.\end{aligned}$$

The knowledge accumulation quintet, $(\alpha,\xi,\kappa,\eta,\delta)$ We start from the two equations:

$$\hat{Z} = \alpha\,(1+\xi s_G)\left(\frac{1+\kappa k^{\eta}}{1+\kappa}\right)^{\frac{1}{\eta}}\frac{L_Z}{N} = \Psi\frac{L_Z}{N}; \tag{A.8}$$

$$\hat{Z} = \left(\frac{1+\kappa k^{-\delta}}{1+\kappa}\right)^{\frac{1}{\delta}}\frac{L}{N}s_G. \tag{A.9}$$

We differentiate the first to obtain:

$$\frac{d\log\hat{Z}}{d\log Z} = -\frac{\kappa k^{\eta}}{1+\kappa k^{\eta}}; \tag{A.10}$$

$$\frac{d\log\hat{Z}}{d\log s_G} = \frac{(1+\xi)\,s_G}{1+\xi s_G}; \tag{A.11}$$

$$\frac{d\log\hat{Z}}{d\log D} = \frac{\kappa k^{\eta}}{1+\kappa k^{\eta}}. \tag{A.12}$$

The first expression is the elasticity of private innovation with respect to private knowledge capital; the second is the elasticity of private innovation with respect to public R&D employment; the third is the elasticity of private innovation with respect to public knowledge capital. The two elasticities with respect to Z and D provide the same information for the calibration. Thus, at this stage we have 4 equations in 6 unknowns, α, ξ, κ, η, δ and k. We use the identifying assumption $\eta=\delta$. This brings us to 4 equations in 5 unknowns, α, ξ, κ, η and k. Next, we note that Jaffe (1989) measures the exponent of D in a Cobb-Douglas representation of private knowledge growth. We take such claim at face value and map it to our model, obtaining $1-\chi=0.3$, which gives us $\kappa=(1-\chi)/\chi=0.42857$. We now have the 4 equations (A.8)-(A.11) in 4 unknowns, α, ξ, η, k.

We take $d\log\hat{Z}/d\log s_G=0.11$ from Jaffe (1989) and Acs et al. (2001). Equation (A.11) then gives us directly

$$\xi=\frac{1-\frac{s_G}{\frac{d\log\hat{Z}}{d\log s_G}}}{\frac{s_G}{\frac{d\log\hat{Z}}{d\log s_G}}-s_G}=\frac{1-\frac{0.017}{0.11}}{\frac{0.017}{0.11}-0.017}=6.1467.$$

Next we need an estimate of $-\frac{d\log\hat{Z}}{d\log Z}$. We take it from the semi-endogenous growth literature whose scientific contribution is based on the claim that there exist diminishing returns to knowledge in knowledge production. For example, Jones (2022) summarizes the evidence as supporting the range 0.2 to 3 or higher. We use the value 0.4.[10] Note that in our formulation, $-\frac{d\log\hat{Z}}{d\log Z}=\frac{d\log\hat{Z}}{d\log D}$. With this estimate in hand, equation (A.10) yields

$$k^{\eta}=\frac{1}{\kappa}\frac{-\frac{d\log\hat{Z}}{d\log Z}}{1+\frac{d\log\hat{Z}}{d\log Z}}.$$

Noting that $\kappa=0.42857$, we compute

$$k^{\eta}=\frac{1}{\kappa}\frac{-\frac{d\log\hat{Z}}{d\log Z}}{1+\frac{d\log\hat{Z}}{d\log Z}}=\frac{1}{0.42857}\frac{0.4}{1-0.4}=1.5556.$$

We are now down to 2 equations in (α,η). Equation (A.9) gives us directly

$$\eta=\frac{\log\left(\frac{1+\kappa k^{-\eta}}{1+\kappa}\right)}{\log\left(\frac{\hat{Z}}{\frac{L}{N}s_G}\right)}=\frac{\log\left(\frac{1+0.42857\frac{1}{1.5556}}{1+0.42857}\right)}{\log\left(\frac{0.2475}{21.6\times 0.017}\right)}=0.287.$$

This then gives us

$$k=(1.5556)^{\frac{1}{0.287}}=4.6527.$$

Equation (A.8) then gives us

$$\alpha=\frac{\frac{\Psi}{\left(\frac{1+\kappa k^{\eta}}{1+\kappa}\right)^{\frac{1}{\eta}}}}{1+\xi s_G}=\frac{\frac{1.1471}{\left(\frac{1+0.42857\times 1.5556}{1+0.42857}\right)^{\frac{1}{0.287}}}}{1+6.1467\times 0.017}=0.6068.$$

This value is in line with analogous calibrations that abstract from public R&D.

Policy experiment We do not have a time series for the public R&D employment share. Figure A.1, however, reports the available time series for the public R&D expenditure to GDP ratio, which

[10] Note that 1 is the highest admissible value consistent with our CES formulation. Values of the elasticity larger that 1 are admissible only in the semi-endogenous growth model that features exclusively power functions.

displays a halving over the period 1964-2021. To mimic this pattern, we feed to the model a change in s_G of the same order of magnitude as the change in the public R&D expenditure to GDP ratio, i.e., we go from s_G to $s'_G = s_G/2$. In the data, the ratio of the 2021 value to the 1964 value is about 1/3, so our experiment is somewhat conservative. Inspecting the figure, however, suggests that 1/2 is a reasonable approximation to the ratio that one obtains by splitting the sample in two subperiods, 1964 to about 1990 and 1990 to 2021. The reason is that the public R&D to GDP ratio appears to stabilize ovet the last 20 years or so.

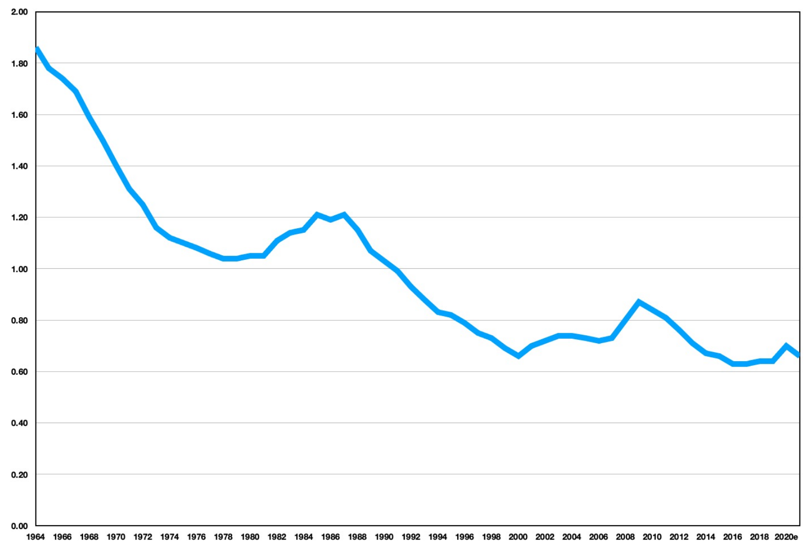

Figure A.1: Federal R&D share of GDP (percent), 1964-2021. Source: National Patterns of R&D resources, National Center for Science and Engineering Statistics, US National Science Foundation.

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