Inflation Persistence and a new Phillips Curve*

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Abstract

Auclert et al. (2024) recently argued that, to first order, menu-costs models deliver the same New Keynesian Phillips Curves as time-dependent models in response to AR(1) shocks. We show here that when considering a broader class of shocks, menu-costs models can generate qualitatively and quantitatively different Philips curves than implied by time-dependent models. Shocks to the growth rate of nominal demand generate inflation persistence in the model, in line with the data, but at odd with the standard time-dependent NKPC. Changes in the extensive margin of price adjustment in the menu-cost model generate history dependence that is captured by the lagged inflation rate. Once we control for lagged nominal demand growth, the explanatory power of lagged inflation drops significantly. The reason is that nominal demand growth is a second determinant of inflation in the Phillips curve in menu-cost models and inflation therefore inherits the persistence of the process for nominal demand.

Keywords: Phillips Curve, Menu cost, State Dependent Pricing, Monetary Economics

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1 Introduction

The New Keynesian model is the dominant paradigm for studying business cycles and stabilization policies in modern macroeconomics. The standard model features time-dependent pricing frictions a la Calvo. The model delivers the so-called New Keynesian Phillips Curve (NKPC), which relates current and future marginal cost (gaps) to the inflation rate. The model, while providing an elegant characterization is not consistent with the empirical evidence on inflation, calling into question its relevance for positive and normative analysis.

A key shortcoming of the NKPC is its lack of inflation persistence (Fuhrer, 2010). Whereas an extensive empirical literature estimating NKPC finds inflation persistence, theoretical models resort to indexation to past inflation rates to link current to past inflation rates (Christiano et al., 2005). Inflation indexation indeed adds inflation persistence to the theoretical model but such adhoc model features render a discussion of monetary policy a challenge. Understanding the dynamics of inflation and the sources of persistence is a prerequisite to avoid the Lucas-critique. It would have also put the debate between "team transitory" and "team permanent" about the recent inflation surge on a solid theoretical foundation.

The main reason for the empirical shortcoming of the NKPC is its purely-forward looking nature. Current inflation is linked to current and expected future real marginal costs. All past variables, including past inflation rates and past labor market variables, have no effect on the current inflation rate once current and future real marginal costs are taken into account. As a result inflation inherits its persistence from the persistence of real marginal costs, implying that the model persistence falls short of the empirical persistence. The NKPC also seems to be quite flat as inflation is largely decoupled from real variables which according to the theory are its only determinants Hazell et al. (2022); Hagedorn (2023). Taking this decoupling into account renders generating persistence in the model even more difficult.

This paper shows that menu-cost models can overcome this empirical shortcoming. The key difference is that the menu cost model features an extensive and an intensive margin whereas only the intensive margin operates in the New Keynesian model since the price adjustment probability is exogenous due to the Calvo pricing assumption. The extensive margin reflects firms' endogenous choice when to adjust their price. It captures both the change in the overall probability of price adjustment and the changes in the probability to increase or decrease the price, respectively (Caballero and Engel, 2007).[1] In terms of the Phillips curve the extensive margin adds a second determinant. The intensive margin of the menu-cost model gives rise to the standard New Keynesian real marginal cost determinant and nominal demand growth captures the extensive margin. Consistent with empirical evidence, the extensive margin accounts for about 70-80% of inflation movements in our model. In terms of Inflation persistence,

[1] This definition differs from the definition in Klenow and Kryvtsov (2008); Midrigan (2011) who restrict the extensive margin to changes in the overall price adjustment probability.

nominal demand growth movements are a significant contributor to inflation dynamics and inflation inherits its persistence from nominal demand growth. Nominal demand growth has the same implications for persistence as a persistent exogenous cost-push shock would have in the New Keynesian model. The difference is that although the demand growth rate movements resemble an exognenous cost-push shock, their effects on inflation are endogenous. The menu cost model thus delivers persistency in inflation without being subject to the Lucas critique.

Concretely, we implement several Phillips curve regressions. We find that the NKPC specification with the expected discounted sum of marginal costs deliver a coefficient on lagged inflation of an magnitude as observed in empirical work. This already shows that our menu-cost model can generate inflation persistence without including adhoc inflation indexation as in New Keynesian models. We then add lagged nominal demand growth to the same regression. We find that lagged nominal demand growth has a sizeable coefficient and that the coefficient on lagged inflation is now close to zero.

The reason for out findings is the second determinant of inflation in the Phillips curve, the growth rate of nominal demand, which accounts for the persistence in the menu-cost model. In contrast, real marginal costs is the only determinant in the New Keynesian model which generates little persistence. The extensive margin is by construction history dependent since a firm current decision whether to adjust its price depends on its price set in the past. Lagged nominal demand growth captures this history dependence if included in the regression and if it is left out then the lagged inflation rate captures the history dependence. Adding the entire state space including the distribution of prices to the regression would eliminate the history dependence. Such a regression is of no practical relevance and including all past prices renders considering persistency a quite meaningless question.

The regressions underlying our main results assume that we can measure all variables without any error. We obtain the same findings when we consider specifications which resemble approaches used in empirical work. Following Hazell et al. (2022) instrument the expected discounted sum of marginal cost since expectations are not included in their dataset. Our instrumental variable regression replicates their approach. It delivers a large coefficient on lagged inflation if only the instrumented marginal cost term is included and adding nominal demand growth reduces the coefficient close to zero.

We also estimate a hybrid Phillips curve as in Gali and Gertler (1999) which describes inflation as a function of three determinants: past inflation, current real marginal costs and expected future inflation. Again, we confirm our main findings. Lagged inflation matters in the regression including only real marginal costs and becomes unimportant when nominal demand growth is included as a regressor.

Our findings are in line with the theoretical results in Auclert et al. (2024) (ARRS). These authors show that the Phillips curve has two determinants, three if trend inflation is taken into account, where the first determinant captures the intensive margin and the other determinants capture the extensive margin. ARRS then ask the quantitative question whether the model can be represented through one determinant, the expected discounted sum of marginal costs. Their model simulations show that the answer is yes. However, the representation through one-determinant is a misspecified NKPC since at least two factors are theoretically necessary. It cannot thus not be interpreted as saying that the menu cost model gives rise to a NKPC. Instead the correct interpretation is that the menu cost models gives rise to a Phillips curve with the same single determinant as in the NKPC but in which the slope coefficient captures both the intensive and the extensive margin and not only the intensive if it was a correctly specified NKPC. This interpretation is consistent with Golosov and Lucas (2007), in which the selection effects/extensive margin in the menu-cost model delivers a steeper Phillips curve. However, a steep NKPC seems inconsistent with the data (Hazell et al., 2022).

We show that the quantitative result and its interpretation in ARRS depends on the properties of the demand growth rate which is the driving force in the model. If we consider a permanent increase in the level of nominal demand as in ARRS or Alvarez and Lippi (2014); Alvarez et al. (2021), the response of inflation in our menu-cost model has the same front-loading property as the New Keynesian model based on Calvo pricing. Firms incentive to front-load price decisions because they can delay and time their pricing decisions Midrigan (2006) is not strong enough to overcome the well-known front-loading finding from the New Keynesian literature. Accordingly, the inflation rate jumps in response to the demand shock and then decays gradually to its steady-state value.

In contrast, the inflation rate response does not feature this extreme front-loading if we allow for autocorrelation in the growth rate of nominal demand as observed in the data (Nakamura and Steinsson, 2010; Midrigan, 2011). Firms take into account that a current increase in nominal demand is followed by further future demand increases. Firms therefore exercise the option of not adjusting the price immediately since they can adjust the price in a future period at a given cost. Not adjusting the price when the opportunity arises in a Calvo setting is potentially very costly since the next opportunity might be far in the future. In a menu-cost model this cost is bounded by the fixed cost of price adjustment at which firms can always adjust their price. As result the front-loading incentive is much weaker in models with state-dependent pricing (Midrigan, 2006). Correspondingly, we find that the inflation response is hump-shaped for a demand increase and U-shaped for a demand decrease. An increase in the inflation rate is then followed a larger inflation increase which is not captured by real marginal costs. In a regression, the deviation form the NKPC front-loading shape shows up as inflation persistence. Our results already hold in a linearized model but become stronger if we allow for non-linear dynamics.

The rest of the paper is structured as follows. Section 2 presents our menu-cost model with idiosyncratic productivity and idiosyncratic fixed adjustment costs. Section 3 presents the calibration and computational strategy. The computational method is laid out in Section

?? and builds on the sequence-space method developed by Boppart et al. (2018) and extended in Auclert et al. (2021). Section 3.1 follows the calibration strategy in Midrigan (2011) and establishes the good model fits including key moments of the price distribution and the observed behavior of the intensive and the extensive margin (Alvarez et al., 2019). Our results are presented in Section 4. Finally, section 5 concludes.

2 Model

We describe a state-dependent pricing model with idiosyncratic productivity shocks and stochastic price adjustment costs. Our main focus is on the firm side to understand how exogenous aggregate nominal demand translates into inflation. The firm model is therefore quite detailed whereas the household model is kept quite simple. The main purpose of including the household sector is to endogenously derive flexible wages which equal marginal costs and to obtain the demand schedule which firms take as given. We first describe the household sector before describing the firm side.

2.1 Households

We assume a representative household with preferences over consumption $\{c_t\}_{t=0}^{\infty}$ and hours $\{h_t\}_{t=0}^{\infty}$,

$$\sum_{t=0}^{\infty} \beta^t u(C_t, h_t) \tag{1}$$

Households consume differentiated goods $c_t(i)$ at a price $p_t(i)$ indexed by $i \in [0, 1]$. The composite consumption C_t is assumed to be a Dixit-Stiglitz aggregator of differentiated goods $c_t(i)$,

$$C_t = \Big[\int_0^1 c_t(i)^{\frac{\epsilon-1}{\epsilon}} \Big]^{\frac{\epsilon}{\epsilon-1}}. \tag{2}$$

Each period the household chooses $c_t(i)$ at a price $p_t(i)$ to maximize utility (1) subject to the budget constraint

$$\int_0^1 p_t(i) c_t(i) di \leq W_t l_t + \Pi_t, \tag{3}$$

where Π_t is distributed profits and W_t is the nominal wage.

This requires that household demand for each good i is

$$c_t(i) = \Big(\frac{p_t(i)}{P_t}\Big)^{-\epsilon} \frac{D_t}{P_t},$$

where

$$P_t = \Big[\int_0^1 p_t(i)^{1-\epsilon} \Big]^{\frac{1}{1-\epsilon}} \tag{4}$$

is the price index and total nominal expenditures satisfies

$$P_t C_t = \int_0^1 p_t(i) c_t(i) di, \tag{5}$$

Households' hours choice h_t satisfies

$$\frac{W_t}{P_t} = \frac{u_h(c_t, h_t)}{u_c(c_t, h_t)}.$$

2.2 Firms

There is a measure one of firms indexed by $i \in [0,1]$ producing differentiated goods. Firm i hires labor $n_t(i,z)$ to produce output with idiosyncratic productivity z_{it} and aggregate productivity Z_t,

$$y_t(i,z) = z_{it} Z_t n_t(i,z).$$

A firm $i \in [0,1]$ with price $p_t(i)$ faces demand

$$y(p_t(i), P_t, D_t) := \Big(\frac{p_t(i)}{P_t}\Big)^{-\epsilon} \frac{D_t}{P_t},$$

taking aggregate nominal demand D_t and the price level P_t as given. The nominal cost of producing $y_t(i)$ units of real output is

$$P_t mc(\frac{D_t}{P_t}) \frac{y_t(i)}{z_{it}},$$

where $mc_t = MC(\frac{D_t}{P_t})$ is real marginal costs, which depends on real aggregate demand D_t/P_t and are thus common to all firms. Since labor is the only input into production, real marginal cost equals the real wage,

$$MC(\frac{D_t}{P_t}, Z_t) = \frac{W_t}{P_t}\frac{1}{Z_t} = \frac{-u_h(C_t, h_t)}{u_c(C_t, h_t)}\frac{1}{Z_t} = \frac{-u_h(\frac{D_t}{P_t}, \frac{D_t}{P_t}\frac{1}{Z_t})}{u_c(\frac{D_t}{P_t}, \frac{D_t}{P_t}\frac{1}{Z_t})}\frac{1}{Z_t},$$

where $\frac{W_t}{P_t}$ is the hourly wage to produce Z_t units of output and taking into account that in equilibrium $C_t = \frac{D_t}{P_t}$ and $h_t = \frac{D_t}{P_t}\frac{1}{Z_t}$. In quantitative analysis we assume that

$$u(c_t, h_t) = \frac{c_t^{1-\sigma}}{1-\sigma} - \frac{h_t^{1+\varphi}}{1+\varphi},$$

and thus obtain for marginal costs,

$$MC(\frac{D_t}{P_t}, Z_t) = \frac{(\frac{D_t}{P_t}\frac{1}{Z_t})^{\varphi}}{\frac{D_t}{P_t}^{-\sigma}}\frac{1}{Z_t} = (\frac{D_t}{P_t})^{\varphi+\sigma}(\frac{1}{Z_t})^{1+\varphi}$$

We set out to rewrite real profits as a function of real variables. A firm's state is its (nominal) price p, its productivity z, aggregate nominal demand D, and the aggregate price level P. Lower-case variables denote firm-specific variables, upper-case denote aggregate variables.

The period t nominal profit of the firm is given by

$$\Pi(p_t, z_t, P_t, D_t, Z_t) = \left(\frac{p_t}{P_t}\right)^{1-\epsilon} D_t - MC\left(\frac{D_t}{P_t Z_t}\right)\left(\frac{p_t}{P_t}\right)^{-\epsilon}\frac{D_t}{z_t Z_t}.$$

and real profits are given by

$$\frac{\Pi(p_t, P_t, D_t, Z_t)}{P_t} = \left(\frac{p_t}{P_t} - MC\left(\frac{D_t}{P_t Z_t}\right)\frac{1}{z_t Z_t}\right)\left(\frac{p_t}{P_t}\right)^{-\epsilon}\frac{D_t}{P_t}.$$

Define the firm-specific markup by $\mu_t = \frac{p_t/P_t}{MC(D_t/(P_t Z_t))/(z_t Z_t)}$. We can the rewrite real profits as

$$\frac{\Pi(\mu_t, z_t, D_t/P_t, Z_t)}{P_t} = \underbrace{(\mu_t - 1)\,\mu_t^{-\epsilon} z_t^{\epsilon-1}}_{\text{idiosyncratic}} \times \underbrace{\left(MC\left(\frac{D_t}{P_t Z_t}\right)\right)^{1-\epsilon}\frac{D_t}{P_t}Z_t^{\epsilon-1}}_{\text{aggregate}}.$$

We postulate that the firm can change its price (prior to production) if paying a fixed cost $z_t^{\epsilon-1}\varepsilon_t$ where ε_t is an idiosyncratic shock drawn each period. We write the firm problem recursively. Productivity is evolving according to a random walk in logs, $z_{t+1} = \eta_{t+1} z_t$.

The recursive formulation of the risk-neutral profit-maximizing firm's problem, under a perfect-foresight path for aggregate variables, is described by

$$\begin{aligned}
V_t^{noadj}(\mu, z) &= (\mu-1)\mu^{-\epsilon}z^{\epsilon-1} \times (MC_t)^{1-\epsilon}\frac{D_t}{P_t}Z_t^{\epsilon-1} + \beta\mathbb{E}V_{t+1}(\mu', z') \\
&\quad \text{s.t. } z' = \eta' z \\
&\quad\quad\ \mu' = \eta'\frac{P_t}{P_{t+1}}\frac{MC_t}{MC_{t+1}}\frac{Z_{t+1}}{Z_t}\mu \\
V_t^{adj}(\mu, z|\varepsilon) &= \max_{\mu^*}(\mu^*-1)(\mu^*)^{-\epsilon}z^{\epsilon-1} \times (MC_t)^{1-\epsilon}\frac{D_t}{P_t}Z_t^{\epsilon-1} - z^{\epsilon-1}\varepsilon + \beta\mathbb{E}V_{t+1}(\mu', z') \\
&\quad \text{s.t. } z' = \eta' z \\
&\quad\quad\ \mu' = \eta'\frac{P_t}{P_{t+1}}\frac{MC_t}{MC_{t+1}}\frac{Z_{t+1}}{Z_t}\mu^* \\
V_t(\mu, z|\varepsilon) &= \max\{V_t^{noadj}(\mu, z), V_t^{adj}(\mu, z|\varepsilon)\} \\
V_t(\mu, z) &= \mathbb{E}_\varepsilon\left[V_t(\mu, z|\varepsilon)\right]
\end{aligned}$$

Since the problem is homothetic in z, we can eliminate z as a state variable. We guess and verify that all value functions satisfy $V(\mu, z) = v(\mu)z^{\epsilon-1}$:

$$\begin{aligned}
v_t^{noadj}(\mu) &= (\mu-1)\mu^{-\epsilon} \times (MC_t)^{1-\epsilon}\frac{D_t}{P_t}Z_t^{\epsilon-1} + \\
&\quad \beta\mathbb{E}\left[(\eta')^{\epsilon-1}v_{t+1}\left(\eta'\frac{P_t}{P_{t+1}}\frac{MC_t}{MC_{t+1}}\frac{Z_{t+1}}{Z_t}\mu\right)\right] \\
v_t^{adj}(\mu|\varepsilon) &= \max_{\mu^*}(\mu^*-1)(\mu^*)^{-\epsilon} \times (MC_t)^{1-\epsilon}\frac{D_t}{P_t}Z_t^{\epsilon-1} - \varepsilon \\
&\quad + \beta\mathbb{E}\left[(\eta')^{\epsilon-1}v_{t+1}\left(\eta'\frac{P_t}{P_{t+1}}\frac{MC_t}{MC_{t+1}}\frac{Z_{t+1}}{Z_t}\mu^*\right)\right] \\
v_t(\mu|\varepsilon) &= \max\{v_t^{noadj}(\mu), v_t^{adj}(\mu|\varepsilon)\} \\
v_t(\mu) &= \mathbb{E}_\varepsilon\left[v_t(\mu|\varepsilon)\right]
\end{aligned}$$

2.3 Equilibrium

As in Midrigan (2011) we assume that nominal spending equals exogenous nominal demand D_t,

$$D_t = P_tC_t = \int_0^1 p_t(i)c_t(i)di$$

The aggregate price level, through the Dixit-Stiglitz aggregator, is given by

$$\begin{aligned}
P_t = \left(\int p_{it}^{1-\epsilon}di\right)^{1/(1-\epsilon)} &= \left(\int \left(\mu_{it}P_tMC_t/(z_tZ_t)\right)^{1-\epsilon}di\right)^{1/(1-\epsilon)} = \\
&\left(\int \mu_{it}^{1-\epsilon}z_{it}^{\epsilon-1}\right)^{1/(1-\epsilon)}\frac{P_tMC_t}{Z_t}
\end{aligned}$$

so we get the equilibrium condition that real marginal cost times the economy-wide markup equals one,

$$1 = \left(\int \mu_{it}^{1-\epsilon}z_{it}^{\epsilon-1}di\right)^{1/(1-\epsilon)}\frac{MC_t}{Z_t}. \tag{6}$$

Equivalently, aggregation of quantities yields the equilibrium condition

$$\left(\int p_t(i)^{1-\epsilon}di\right)^{1/(\epsilon-1)} = P_t$$

since it is equivalent to the equilibrium conditions that supply, Y_t, equals demand, D_t/P_t,

$$D_t/P_t = Y_t = \left(\int y_t(i)^{(\epsilon-1)/\epsilon} di\right)^{\epsilon/(\epsilon-1)} = \left(\int \left(p_t(i)^{-\epsilon} P_t^{\epsilon-1} D_t\right)^{(\epsilon-1)/\epsilon} di\right)^{\epsilon/(\epsilon-1)} =$$
$$= \left(\int p_t(i)^{1-\epsilon} di\right)^{\epsilon/(\epsilon-1)} P_t^{\epsilon-1} D_t.$$

3 Computation and Calibration

3.1 Calibration

The calibration strategy follows Midrigan (2011). The model period is a week. We choose the idiosyncratic firm productivity shock and stochastic (exponential) adjustment cost parameters to match key steady state targets: The frequency of (regular) weekly price changes, 2.9%, and the distribution of the size of (regular) price changes We use the same targets as in Midrigan: 10% of prices changes are less than 3 percent, 25% of prices changes are less than 5 percent, 50% of prices changes are less than 9 percent, 75% of prices changes are less than 13 percent and 90% of prices changes are less than 21 percent. Figure 1 shows these 5 data moments (blue dots) and the distribution of prices changes in our calibrated model, confirming that we are able to match all five data targets. Figure 1 also shows that the distribution of prices has no mass points. We choose $\sigma = 1$ to be balanced-growth path consistent so that we can also consider permanent aggregate technology shocks. We set $\varphi = 1$ consistent with a Frsich elasticity of 0.5.

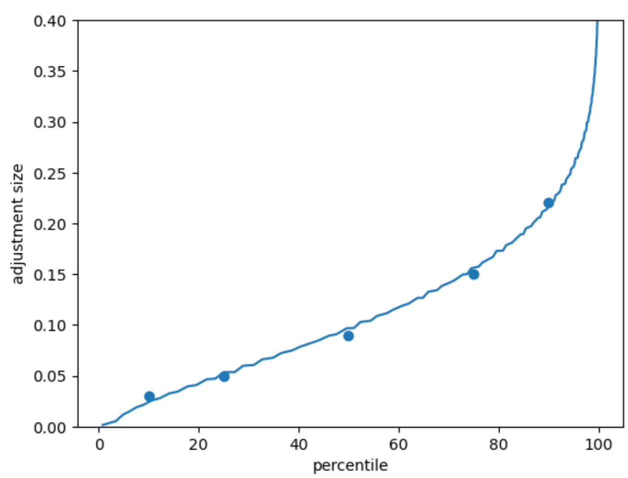

Figure 1: Distribution of prices changes in the model (line) and in the data (5 dots)

Following Alvarez et al. (2019) inflation satisfies the accounting identity

$$1 + \pi = \lambda^+ \Delta^+ - \lambda^- \Delta^-,$$

where λ^+ is the frequency of price increases, λ^- is the frequency of price decreases, Δ^+ is the average size of price increases and Δ^- is the average size of price decreases. Total differentiation inflation with respect to demand D_t delivers a decomposition into an extensive and an intensice margin,

$$\frac{\partial \Delta 1 + \pi_t}{\partial \Delta D_t} = \underbrace{\frac{\partial \lambda^+}{\partial \Delta D_t}\Delta^+ - \frac{\partial \lambda^-}{\partial \Delta D_t}\Delta^-}_{\text{Extensive Margin}} + \underbrace{\lambda^+ \frac{\partial \Delta^+}{\partial \Delta D_t} - \lambda^- \frac{\partial \Delta^-}{\partial \Delta D_t}\Delta^-}_{\text{Intensive Margin}}.$$

The extensive margin is positive since $\frac{\partial \lambda^+}{\partial \Delta D_t} > 0$, $\frac{\partial \lambda^-}{\partial \Delta D_t} < 0$, $\Delta^+ > 0$ and $\Delta^- < 0$. Defining λ as the overall frequency of price changes, the extensive margin can be further decomposed into the selection effect and changes in the total frequency of price changes,

$$\frac{\partial \lambda^+}{\partial \Delta D_t}\Delta^+ - \frac{\partial \lambda^-}{\partial \Delta D_t}\Delta^- = \underbrace{\frac{\partial (\lambda^+ - \lambda)}{\partial \Delta D_t}\Delta^+ - \frac{\partial (\lambda^- - \lambda)}{\partial \Delta D_t}\Delta^-}_{\text{Selection}} + \underbrace{\frac{\partial \lambda}{\partial \Delta D_t}(\Delta^+ - \Delta^-)}_{\text{Total Frequency}}$$

By the same arguments as above, the selection effect is positive. In response to an increase in nominal demand growth, the probability to increase the price, λ^+ increases where the the probability to decrease the price, λ^- increases. The selection effect is thus positive even if the overall frequency of price changes is constant, $\frac{\partial \lambda}{\partial \Delta D_t} = 0$.

In general both components of the extensive margin are positive, although certain assumption imply $\frac{\partial \lambda}{\partial \Delta D_t} = 0$ for small changes in demand (Alvarez et al., 2019). In particular, both components are positive in response to large shocks as non-linear effects kick in.

$$\begin{aligned}
\frac{\partial \Delta 1 + \pi_t}{\partial \Delta D_t} &= \frac{\partial (\lambda^+ - \lambda)}{\partial \Delta D_t}\Delta^+ - \frac{\partial (\lambda^- - \lambda)}{\partial \Delta D_t}\Delta^- + \frac{\partial \lambda}{\partial \Delta D_t}(\Delta^+ - \Delta^-) + \lambda^+ \frac{\partial \Delta^+}{\partial \Delta D_t} - \lambda^- \frac{\partial \Delta^-}{\partial \Delta D_t}\Delta^- \quad \textbf{Linear} \\
&+ \frac{\partial^2 (\lambda^+ - \lambda)}{\partial^2 \Delta D_t}\Delta^+ + \frac{\partial^2 \lambda}{\partial^2 \Delta D_t}\Delta^+ + 2\frac{\partial^2 \lambda^+}{\partial^2 \Delta D_t}\frac{\partial \Delta^+}{\partial \Delta D_t} + \lambda^+ \frac{\partial^2 \Delta^+}{\partial^2 \Delta D_t} \quad \textbf{Second Order (+)} \\
&- \frac{\partial^2 (\lambda^- - \lambda)}{\partial^2 \Delta D_t}\Delta^- - \frac{\partial^2 \lambda}{\partial^2 \Delta D_t}\Delta^- - 2\frac{\partial^2 \lambda^-}{\partial^2 \Delta D_t}\frac{\partial \Delta^-}{\partial \Delta D_t} + \lambda^- \frac{\partial^2 \Delta^-}{\partial^2 \Delta D_t} \quad \textbf{Second Order (-)} \\
&+ \ldots \quad \textbf{ThirdOrder},
\end{aligned}$$

where the second derivative of the total frequency of price changes, $\frac{\partial^2 \lambda}{\partial^2 \Delta D_t}$, is positive.

Comparing the steady-state properties of the intensive and extensive margins to empirical results in Alvarez et al. (2019) shows that our calibrated model captures both margins well. Concretely, we conduct this experiment: Increase steady-state growth rate of nominal demand to increase the steady-state inflation rate while keeping all other parameters unchanged. Figures 2 shows the size of price increases Δ^+ and size of price decreases Δ^- as a function of the annual inflation rate in the data and in the model. Figures 3 shows the monthly frequency of prices increases λ^+, prices decreases λ^- and of all price changes, $\lambda^+ + \lambda^-$ as a function of the annual inflation rate in the data and in the model. Figures 3 shows the extensive margin, the selection effect $\lambda^+ - \lambda^-$ and the total frequency, $\lambda^+ + \lambda^-$. The Figures lead to the conclusion that the model replicates the data well.

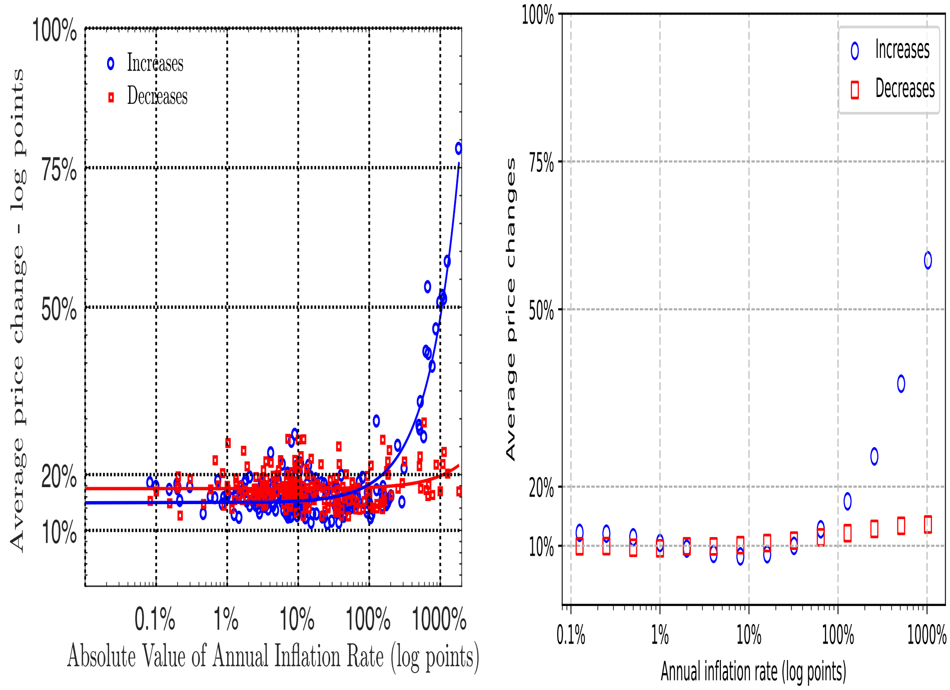

Figure 2: Intensive margin in the data and the model: Size of price increases Δ^+ and size of price decreases Δ^-. Left panel: data. Right panel: model

3.2 Computation Method

We solve the model using standard methods. We solve for the firm price setting problem using dynamic programming. In order to solve for the steady state, we discretize the state space and simulation the idiosyncratic shocks via non-stochastic simulation following Young (2010). To deal with the random walk shocks for productivity, we divide through by idiosyncratic productivity and express the cross-sectional distributions in terms of mark-up gaps (current markup relative to desired markup). To compute aggregate statistics, we then integrate this distribution based on the permanent productivity neutral measure, following the method of Harmenberg (2023).

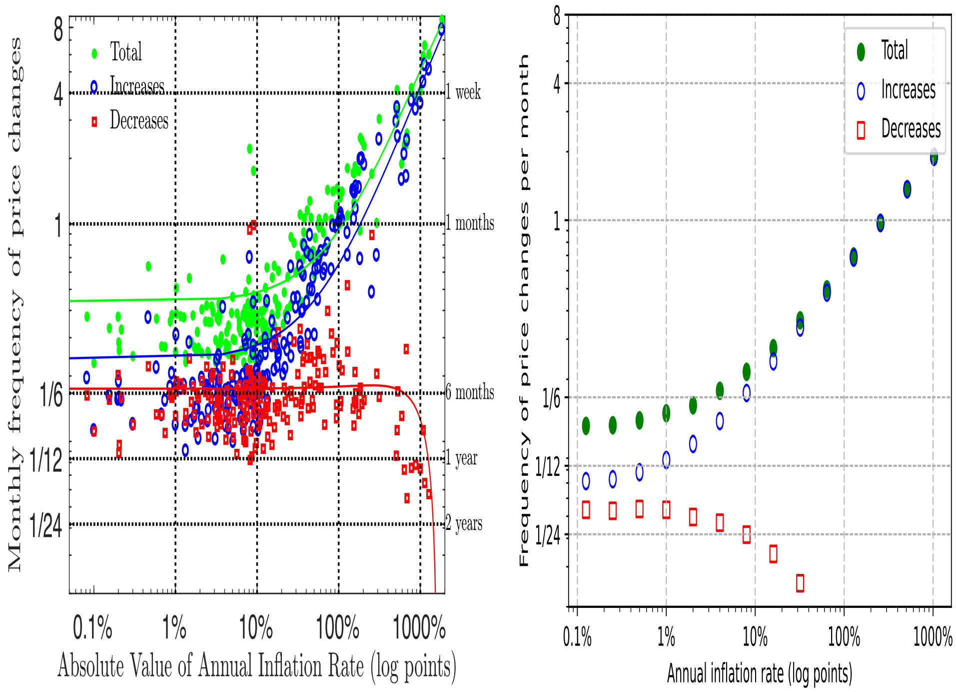

Figure 3: Monthly Frequency of prices changes: Increases λ^+, Decreases λ^- and total $\lambda^+ + \lambda^-$. Left panel: data. Right panel: model

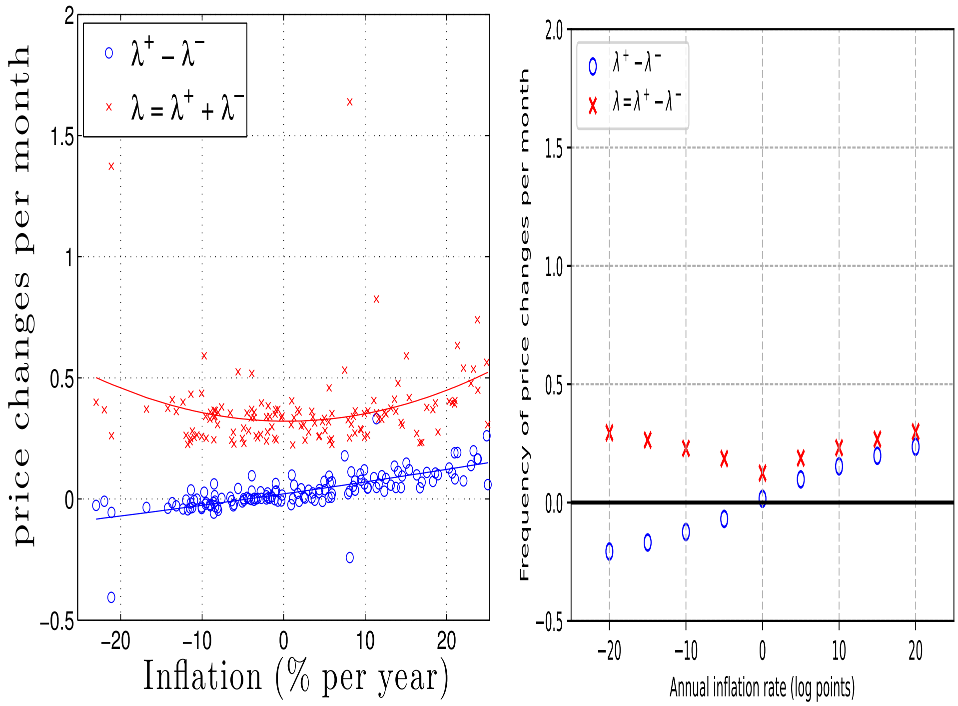

Figure 4: Extensive margin in the data and the model: Selection $\lambda^+ - \lambda^-$ and Total Frequency $\lambda^+ + \lambda^-$

4 Results

This Section presents the our main results on the response of the calibrated model to shocks to nominal demand growth ΔD_t. We linearize model with small MIT-shocks in sequence space (Boppart et al., 2018; Auclert et al., 2021). We assume that the weekly process for nominal demand growth is autocorrelated,

$$\Delta D_t = \rho_D^w \Delta D_{t-1} + \epsilon_t^D,$$

where $\rho_D^w = 0.95$ matches the autocorrelation of nominal demand at the quarterly frequency of $\rho_D = 0.5$ (Nakamura and Steinsson, 2010; Midrigan, 2011) To explain our results and to relate to Auclert et al. (2024) we also consider permanent level shocks ($\rho_D^w = 0$). We simulate the economy to obtain weekly model generated data as in Midrigan (2011) and implement quarterly Phillips curve regressions consistent with frequency typically found in empirical studies. Before showing the Phillip curve regressions, we first present impulse responses of inflation and its driving forces so as to explain the model mechanisms.

4.1 Impulse Responses

We first show the weekly impulse responses to a negative demand shock $\epsilon_t < 0$ of inflation pi_t, marginal costs mc_t and the discounted sum of marginal costs, $\sum_{k=0}^{T} mc_{t+k}$ in Figure 5. The left panel shows the three variables when the first element is normalized to -1 and the right panel shows the best (affine-)linear fit of mc_t and $\sum_{k=0}^{T} mc_{t+k}$ to the inflation rate. It is evident that neither marginal costs nor the discounted sum of marginal costs can fully explain the inflation rate. This is equivalent to an R^2 lower than in the regressions underlying the right panel and assigns a role for the lagged inflation rate. Indeed the regression

$$\pi_t = c_0 + \kappa mc_t + \gamma pi_{t-1} \tag{7}$$

delivers a coefficient $\alpha^\pi = 0.7828$ on lagged inflation. Likewise, the regression

$$\pi_t = c_0 + \kappa E_t \sum_{k=0}^{T} mc_{t+k} + \gamma pi_{t-1} \tag{8}$$

yields coefficient $\alpha^\pi = 0.2907$.[2]

The shape of the impulse responses are consistent with the regression results. The inflation rate response is U-shaped whereas the response of $\sum mc$ shows the front-loading properties known from New Keynesian Phillips Curves. The strongest response is observed on impact and then gradually dies out. Clearly, a front-loaded curve cannot perfectly fit a U-shaped curve.

[2]This regression uses the correct model expectations, rendering $E_t \sum_{k=0}^{T} mc_{t+k}$ a Period t variable which can be included in the regression.

The reason for the U-shape is the muted front-loading in menu-cost models as emphasized in Midrigan (2006). Firms can delay the price adjustment since they know that prices can always be adjusted at a fixed cost. The incentive to delay is strengthened if the growth rates of demand are autocorrelated. Firms are then less inclined to adjust their prices immediately at the time of the initial shock since they know that demand will further decrease in the future. It can then be profitable to wait and adjust the price later. In terms of inflation persistence, this means that an initial decrease in inflation is followed by a larger decrease in the next period, implying autocorrelation in inflation rates not captured by mc or $\sum mc$. Figure 6 replicates the same exercise but for $\rho_D = 0$, showing that this conclusion depends on the autocorrelation in nominal growth rates. The left panel again shows the three variables when the first element is normalized to -1 and the right panel shows the best (affine-)linear fit of mc_t and $\sum_{k=0}^{T} mc_{t+k}$ to the inflation rate. Now, the three curves, π, mc and $\sum mc$ are almost on top of each other. Correspondingly, the regression (7) for mc and regression (8) for $\sum mc$ deliver smaller coefficients on lagged inflation, $\alpha^{\pi} = 0.127$ for mc and $\alpha^{\pi} = 0.065$ for $\sum mc$. The permanent shock in contrast to the autocorrelated growth shock does not induce incentives to delay price adjustments so that the impulse response has the same front-loading shape as the NKPC. Auclert et al. (2024) reach the same conclusion for the same permanent level shock, establishing that the difference in results is due to our autocorrelated growth rate shocks which break the extreme front-loading in the NKPC.

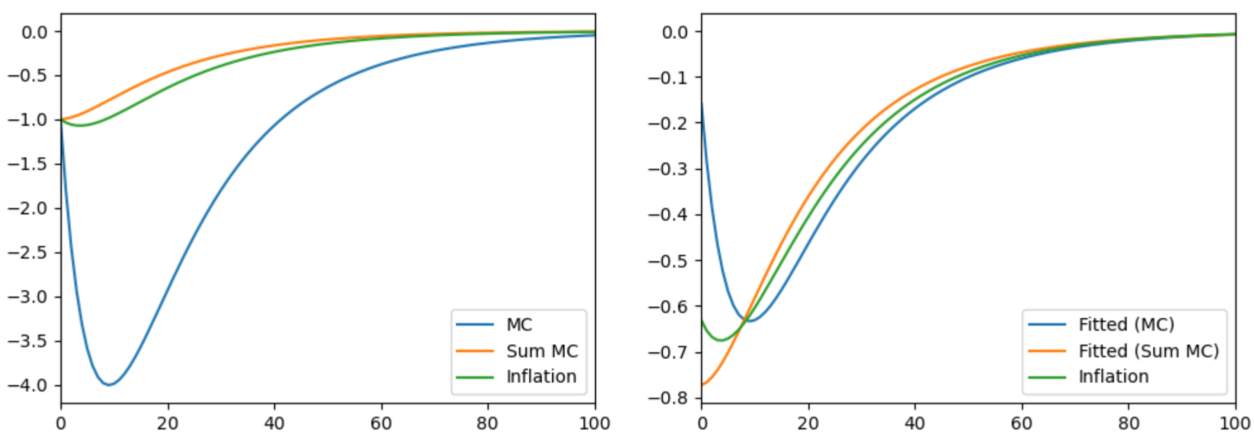

Figure 5: Weekly IRFs $\rho_D = 0.5$

NEW: Understanding the extensive margin

Can we also plot the weekly IRFs for permanent level and autocorrelated growth shock for

1. $\lambda_t^+ - \lambda_{ss}^+$
2. $\lambda_t^- - \lambda_{ss}^-$

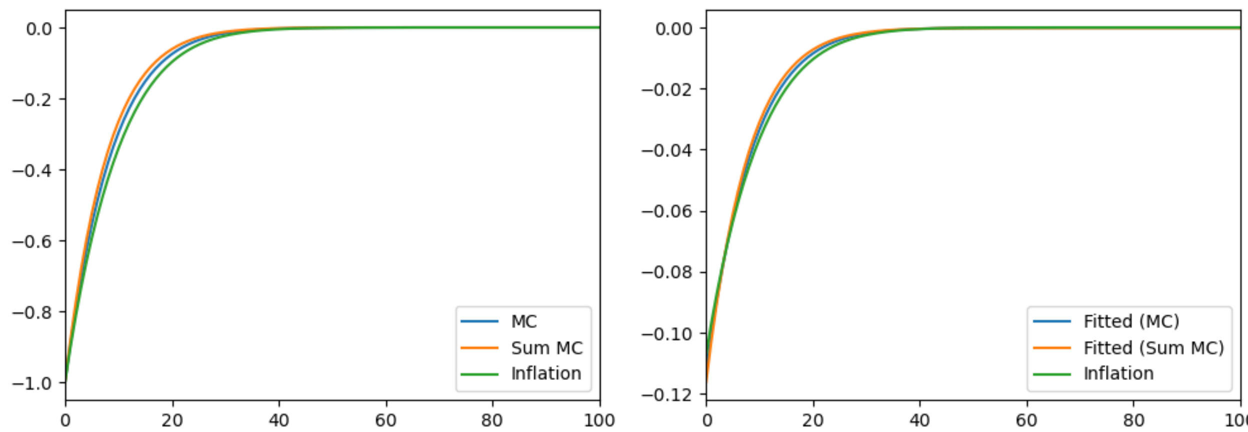

Figure 6: Understanding the results: Weekly IRFs $\rho_D = 0$

3. $(\lambda_t^+ - \lambda_{ss}^+)\Delta_{ss}^+ - (\lambda_t^- - \lambda_{ss}^-)\Delta_{ss}^-$

4. $\Delta_t^+ - \Delta_{ss}^+$

5. $\Delta_t^- - \Delta_{ss}^-$

Recall that λ^+ is the frequency of price increases, λ^- is the frequency of price decreases, Δ^+ is the average size of price increases and Δ^- is the average size of price decreases. Note that that both Δ^+ and Δ^- are positive numbers. [Easiest is just to add these as statistics when computing the IRFs, then you can do it directly] A subscript ss means the steady-state value and a superscript t means time (since shocks in the IRF)

We can then regress these 5 statistics on the lagged inflation rate, π_{t-1}. We need these regressions only for demand shocks with $\rho_D = 0$and $rho_D = 0.5$ (from superimposing shocks)

4.2 Phillips Curve Results

We now first implement the Calvo specification of the Phillips Curve regression,

$$\pi_t = \kappa \sum \mathbb{E}[\beta^k mc_{t+k}] + \gamma\pi_{t-1} + \nu_t, \tag{9}$$

on our simulated data. The estimate coefficient γ is the parameter of interest as it describes the inflation persistence taking into account the NKPC determinant $\sum mc$. We run regression which assume that all variables are measured consistently with the model. In particular the expectation of future marginal costs use the model expectations and are thus a Period t variable which can therefore be included in the regressions. We consider specifications which resemble approaches used in empirical work in Section 4.3 below. The first row of Table I shows that our model delivers a large coefficient of lagged inflation rate, $\gamma = 0.4994$. The inflation is persistence in the model is in the range of empirical estimates although we control for $\sum mc$

Table I: Main Regression Results

	$\sum$ mc	π_{t-1}	ΔD_{t-1}
Calvo Specification	0.0027	0.4994	
	(0.0000)	(0.0069)	
Full Specification	0.0016	0.0529	7.0428
	(0.0000)	(0.0065)	(0.0797)

Standard errors in parentheses.

in the regression. The autocorrelation of inflation is close to 0.8. The coefficient on $\sum mc$ is positive consistent with the theory model and small consistent with empirical evidence.

The inflation persistence captures the history dependence of price setting and is largely muted if we control for lagged nominal demand growth, a driving force in the model. Adding the lagged nominal demand growth rate to the previous regression,

$$\pi_t = \kappa \sum \mathbb{E}[\beta^k mc_{t+k}] + \gamma \pi_{t-1} + \delta \Delta D_{t-1} + \nu_t, \tag{10}$$

confirms this. The second raw of Table I shows that the coefficient of the lagged inflation rate is smaller by an order of magnitude and close to zero, $\gamma = 0.0529$. At the same time, we estimate a lrge and significant coefficient on lagged ΔD_{t-1}. Nominal demand growth as the driving force in the model largely captures the history dependence and as a result reduces the coefficient on lagged inflation, which does not provide substantial information about history not already captured by nominal demand growth. Since nominal demand growth and marginal costs are positively correlated, the coefficient on $\sum mc$ is lower in the second row than in the first row of Table I .

The key conclusions are

- The New Keynesian specification of the Phillips curve delivers a positive coefficient on $\sum mc$ and a sizeable coefficient on lagged inflation
- Adding nominal demand growth yields a positive coefficient and significantly reduces the coefficient on lagged inflation.

4.3 Other Specifications of Phillips Curve regressions

The regressions underlying our main results assume that we can observe all model variables without error. In this Section we consider specifications which resemble approaches used in empirical work. We first follow Hazell et al. (2022) (HHNS)and instrument the expected

Table II: New Keynesian Phillips Curve Results: Hazell et al. (2022) Approach

	$\sum$ mc	π_{t-1}	ΔD_{t-1}
Calvo Specification	0.0040	0.3663	
	(0.0077)	(0.0084)	
Full Specification	0.0025	0.0896	5.3829
	(0.0000)	(0.0076)	(0.0997)

Standard errors in parentheses.

discounted sum of marginal cost since expectations are not included in their dataset. Our instrumental variable regression replicates their approach. We implement the regression:

$$\pi_t = \kappa \sum_{s=t}^{t+20} \beta^{s-t} mc_s + \gamma \pi_{t-1} + \nu_t \qquad \textstyle\sum_{s=t}^{t+20} \beta^{s-t} mc_s \text{ instrumented with } mc_t, \tag{11}$$

where we follow HHNS and truncate the sum after 20 quarters. The first row of Table II shows again that inflation is persistent with a coefficient $\gamma = 0.3663$. Using instruments instead of the correct model variables as in Table I leads to a larger coefficient on $\sum mc$ and a smaller but sizeable coefficient on lagged inflation, which is within the range of empirical estimates. As in the main results, adding lagged nominal growth as an additional regressor,

$$\pi_t = \kappa \sum_{s=t}^{t+20} \beta^{s-t} mc_s + \gamma \pi_{t-1} + \delta \Delta D_{t-1} + \nu_t, \qquad \textstyle\sum_{s=t}^{t+20} \beta^{s-t} mc_s \text{ instrumented with } mc_t, \tag{12}$$

reduces the coefficient of lagged inflation, $\gamma = 0.0896$. Nominal demand growth again has a large and sizeable coefficient. Using the approach in HHNS delivers the same conclusion as the benchmark regression: A large coefficient on lagged inflation if only the instrumented marginal cost term is included and adding nominal demand growth reduces the coefficient close to zero.

We also estimate a hybrid Phillips curve as in Gali and Gertler (1999) which describes inflation as a function of three determinants: past inflation, current real marginal costs and expected future inflation.[3] Again, we confirm our main findings. Lagged inflation matters in the regression including only real marginal costs, $\gamma = 0.3529$ and becomes unimportant when nominal demand growth is included as a regressor, $\gamma = 0.0533$.

[3]The specification follows Auclert et al. (2024) and adds an i.i.d. term to marginal cost to avoid multicollineariy issues.

$$\pi_t = \kappa mc_t + \gamma \pi_{t-1} + \zeta \mathbb{E}\pi_{t+1} + \epsilon_t,$$

Table III: New Keynesian Hybrid Phillips Regression Results

	mc_t	π_{t-1}	$E_t\pi_{t+1}$	ΔD_{t-1}
Calvo Specification	1.0786	0.3529	0.3042	
	(0.0243)	(0.0065)	(0.0051)	
Full Specification	0.4614	0.0533	0.2764	6.0622
	(0.0157)	((0.0054)	(0.0051)	(0.0751)

Standard errors in parentheses.

5 Conclusion

This paper finds that menu-cost models can generate inflation persistence in line with empirical evidence, in contrast to the standard New Keynesian model. The reason is that while the New Keynesian Phillips Curve (NKPC) posits a one-to-one relationship between marginal cost (gaps) and inflation, menu-cost models decouple inflation from real activity. Nominal and marginal cost (gaps) determine the inflation rate so that inflation can inherit its persistence from nominal demand in menu-cost models whereas real marginal costs is the only source of persistence in the NKPC.

Future work will explore whether the inflation persistence in menu-cost models deliver the same implications as the New Keyensian model, for example imply a “disinflationary boom” (Ball, 1994). A related important question is about the optimal policy in models with inflation persistence. Doe they differ from the prescriptions of the New Keyenesian model? How does an optimal disinflationary policy look like? It is conceivable that the optimal policy should address the source of the persistence, that it differs from conventional recommendations and that the answer depends on the source of the persistnce.

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