Pitfalls of Information Spillovers in Persuasion

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Abstract

We study a multiple-receiver Bayesian persuasion model in which the sender wants to achieve an outcome and commits to an experiment which sends correlated messages to homogeneous receivers. Receivers are connected in a network and can perfectly observe their immediate neighbors' messages. After updating their beliefs, receivers choose an action to match the true state of the world. Surprisingly, the sender's gain from persuasion does not change monotonically with network density. We characterize a class of networks in which *increased* communication among the receivers is *strictly better* for the sender and hence *strictly worse* for the receivers.

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1 Introduction

Multiple-receiver Bayesian persuasion models with private communication often assume that receivers do not exchange information with each other between receiving messages from the sender and taking their action. However, in reality people usually deliberate before taking an action and might consult friends and acquaintances in search of additional opinions and information. We model such communication among receivers prior to making a decision with a simple setup that captures *limited information spillovers*: receivers are in a fixed network and neighbors can observe each other's private messages. An application of such communication are social networks like Facebook or Twitter, where senders (e.g. sellers, political parties) can target receivers (e.g. consumers, voters) with adverts. For example, if a person likes or shares an ad or a video on Twitter it is visible to all of their followers. Thus, senders are aware that the information they share will spread through the network of their followers.[1]

Incorporating a communication network with limited information sharing significantly complicates the sender's problem of optimal persuasion as she must also take into account the intricacies of the information flow between receivers when designing a communication protocol. An immediate question that arises is whether the receivers always benefit from communicating more with each other. Alternatively, can the sender actually benefit from greater information sharing between the receivers? Surprisingly, the answer we provide is in the affirmative.

1.1 Illustrative Example

Consider a company that develops a new product and wishes to achieve a critical mass of big corporate clients, which will allow the company to be self-sufficient and would significantly increase the demand for the product in the future. The product's quality is either good (G) or bad (B). Suppose that 5 out of 9 corporate clients constitute a critical mass.[2] The clients initially believe that the quality is good with probability $1/3$ and they would buy if they have a belief of at least $1/2$. The company prepares reports about the product's quality, which are privately distributed among the clients. The communication protocol of the company, which we call an *experiment*, can be formalized by distributions $\pi(\cdot|G)$ and $\pi(\cdot|B)$ on a set of signals. Let $\bar{g} = (g, \dots, g)$ denote the *signal* in which all agents observe *message* g and define $\bar{b}$ analogously. Messages g and b can be interpreted as recommendations to buy and to not buy,

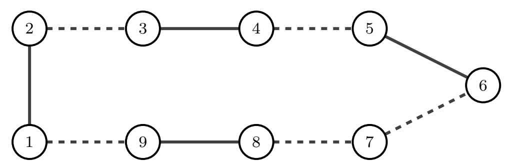
[1]Several papers study the use of social media to spread (fake) news; see Allcott and Gentzkow (2017), Grinberg, Joseph, Friedland, Swire-Thompson, and Lazer (2019), and Zhuravskaya, Petrova, and Enikolopov (2020).

[2]We assume that all clients demand one unit of the product. In a related study (Kerman and Tenev, 2022) we relax this assumption and consider heterogeneous demands without information spillovers.

respectively. While π is known by the clients, under private communication they only observe their own message.

First, assume that the clients do not communicate with each other, i.e. they form an empty network. Let T be the set of signals in which 5 agents observe g and 4 agents observe b, i.e. $|T|=\binom{9}{5}$. An optimal experiment, π, is given in the following table, where every signal in T is targeted with equal probability, $9/10 \cdot 1/|T|$.

π	G	B
$\bar{g}$	1	0
T	0	$\frac{9}{10}$
$\bar{b}$	0	$\frac{1}{10}$



After observing g, a client's belief that the quality is good is $(1/3 \cdot 1)/(1/3 \cdot 1 + 2/3 \cdot 5/9 \cdot 9/10) = 1/2$. Hence, after all realizations except $\bar{b}$ at least five clients buy the product. Thus, the *value* of π (the probability of reaching the critical mass) is 14/15.

Now assume that there are four communicating pairs of clients (1-2, 3-4, 5-6, and 8-9), as shown by the solid edges above, who exchange the information from the reports before making their decisions. Experiment π is no longer optimal: any signal in which agent 1 observes b and agent 2 observes g will result in both agents not buying the product; agent 2 will deduce that the true state is B from agent 1's message. In this case, the communicating pairs always take the same action (regardless of their observed message) and optimal communication leads to reaching the critical mass with lower probability, 8/9. Hence, the additional communication constrains the company and makes it worse off.

It is natural to expect that if the clients communicate even more, then the optimal value will decrease further. In fact, if all clients communicate with each other, optimal communication is public (which gives the lower bound of the value). Now consider an intermediate case given by the dashed and solid edges above (circle network).[3] Surprisingly, not only does the company improve upon the previous case, but it can reach the empty network optimal value 14/15. This can be achieved via a simple modification to π: let T be the set of signals in which two consecutive agents observe b and all others observe g (i.e. $|T|= 9$), e.g. $(g, g, b, b, g, g, g, g, g)$. This ensures that exactly 5 agents buy the product after all realizations in T and allows the company to fully exploit private communication on the circle network. Hence, the company benefits from more communication among the clients, i.e. the optimal value is higher on a denser network.[4] The example highlights the *non-monotonic* relationship between the network density and the sender's gain from persuasion in the presence of information spillovers.

[3]The information of agent 1 spills over to agents 2 and 9 and not to any other, the information of agent 2 spills over to agents 1 and 3 and not to any other, and so on.

[4]Density is the ratio of the number of actual links and the number of potential links. Hence, any network obtained by adding a link to another network is *denser*.

1.2 Contribution and Related Literature

This paper characterizes a class of networks in which increasing communication among receivers benefits the sender and harms the receivers. Two prominent applications of our model which showcase the importance of its insights are marketing and voting. We show that more interconnected groups of agents can be *more susceptible to manipulation*. In the context of marketing, this implies that reaching a critical mass for a subpar product and being stuck in a "bad" equilibrium is more likely. In the presence of network effects (i.e. demand side economies of scale), it could be an uphill battle to escape this bad equilibrium since the deviation of a small number of agents is not sufficient to shift the status quo. The setup can also be interpreted as a voting model with information spillovers, where the critical mass corresponds to the voting quota. In this context, our results imply that the "wrong" outcome is implemented with a higher probability. Hence, more information becomes *harmful* for collective decision-making.

The current model comes closest to Arieli and Babichenko (2019) and Kerman, Herings, and Karos (2023), which build upon the seminal work of Kamenica and Gentzkow (2011). While Arieli and Babichenko (2019) characterize optimal communication for different utility functions of the sender, Kerman et al. (2023) focus on private communication and collective decision making. A crucial difference to the current setup is that in their models a receiver only has access to information revealed to them directly by the sender, whereas in our setup directly connected agents exchange information.

Several working papers consider information spillovers and are closely related to ours. In Galperti and Perego (2023) agents are able to employ mixed strategies and information diffuses through all directed paths in the network.[5] They show that under these conditions the revelation principle can be recovered. In contrast, we show that under limited information spillovers and employing only pure strategies this is no longer the case.[6] In a model with similar spillovers Candogan (2020) analyzes agents who observe the experiments assigned to their neighbors instead of messages.

The findings in Babichenko, Talgam-Cohen, Xu, and Zabarnyi (2021) closely complement ours. The authors define information-dominating pairs (one of two agents observes all information channels that the other one does) and show that an information structure is *weakly* better for the sender than another if and only if every such pair in the former is also information-dominating in the latter. In contrast, we explore when an intervention in the network structure can *strictly* benefit the sender. Taken together, our results provide a blueprint for approaching the problem of persuading agents in an environment with limited information spillovers.

[5] In a sense, the two models can be seen as two possible extremes of information sharing: information in their model acts close to a *global* public good, while in ours it is strictly a *local* public good.

[6] It is natural to assume that buyers/voters employ pure strategies when making binary purchase/voting decisions.

Liporace (2021) considers spillover effects similar to ours, however, the sender only knows the degree distribution of the agents, but not the network structure. Yet, the paper also shows that the sender can benefit from a denser network. In another model of information sharing, Egorov and Sonin (2020) consider a sender who communicates publicly with receivers in a fixed network, where a receiver either relies on his neighbors to learn the provided information or obtains it directly from the sender for a cost. In contrast, the receivers in our model have costless access to information.

Finally, our paper also contributes to the research on private communication and voting games. Some studies in this literature compare public and private communication under different settings (Wang, 2013; Mathevet, Perego, and Taneva, 2020; Titova, 2022; Sun, Schram, and Sloof, 2023), while others investigate voting games that focus on different voting rules (Bardhi and Guo, 2018; Chan, Gupta, Li, and Wang, 2019). We show that private communication is beneficial also with information spillovers.

2 Setup

2.1 Communication

Let $N = \{1, \ldots, n\}$ be the set of receivers and $\Omega = \{X, Y\}$ be the set of states of the world. For any set S denote by $\Delta(S)$ the set of probability distributions over S with finite support. The receivers share a common prior belief $\lambda^0 \in \Delta^\circ(\Omega)$ about the true state of the world, where $\Delta^\circ(\Omega)$ denotes the set of strictly positive probability distributions on Ω.

Let S_i be a finite set of *messages* the sender can send to receiver i, and let $S = \prod_{i \in N} S_i$, where the elements of S are called *signals*. An *experiment* is a function $\pi : \Omega \to \Delta(S)$ which maps each state of the world to a joint probability distribution over signal realizations. Let Π be the set of all experiments.

For each signal $s \in S$, let $s_i \in S_i$ denote the message for receiver i. For each $\pi \in \Pi$, define $S^\pi = \{s \in S | \exists\, \omega \in \Omega : \pi(s|\omega) > 0\}$, i.e. the signals in S which are sent with positive probability by π. Similarly, for each $i \in N$, define $S_i^\pi = \left\{s_i \in S_i | \exists\, \omega \in \Omega : \sum_{t \in S : t_i = s_i} \pi(t|\omega) > 0\right\}$, which is the set of messages receiver i observes with positive probability under π.

2.2 Information Spillovers

An *undirected network* is a map $g : N \times N \to \{0, 1\}$ with $g_{ij} = g(i, j)$ and $g_{ij} = g_{ji}$. Given a set of receivers N, let $G(N)$ be the set of all networks. We assume that receivers are in a fixed network and each receiver in the network observes his direct neighbors' message realizations.[7] Thus, in a non-empty network a receiver gathers more information about the true state than he would from the *same* experiment under the *empty* network. For any network $g \in G(N)$, we denote the *empty* network with the *same number* of receivers by g_0.

Let $N_i(g) = \{j \in N | g_{ij} = 1\} \cup \{i\}$ be the *neighborhood* of receiver i in g and let $\delta_i^g = |N_i(g)| - 1$ be the *degree* of i in g. Let $s_i(g) = (s_j)_{j \in N_i(g)}$ be the *information set* of receiver i in s, which is the vector of messages receiver i observes upon realization s.

For any $g \in G(N)$, we call $g' \in G(N)$ an *extension of* g (denoted $g \subsetneq g'$) if for all $i \in N$ it holds that $N_i(g) \subseteq N_i(g')$ and there exists $j \in N$ such that $N_j(g) \subsetneq N_j(g')$. In words, g' is a network formed by adding one or multiple links to g.

Let $A_i^\pi(g, s) = \{t \in S^\pi | t_i(g) = s_i(g)\}$ be the *association set* of agent i given s, i.e. the set of signals i considers possible upon realization s. For any $g \in G(N)$, $\pi \in \Pi$, and $s \in S^\pi$, the posterior belief vector $\lambda^{s,g} \in \Delta(\Omega)^n$ is defined by

$$\lambda_i^{s,g}(\omega) = \frac{\sum_{t \in A_i^\pi(g,s)} \pi(t|\omega)\lambda^0(\omega)}{\sum_{\omega' \in \Omega} \sum_{t \in A_i^\pi(g,s)} \pi(t|\omega')\lambda^0(\omega')}, \quad i \in N, \omega \in \Omega.$$

That is, $\lambda_i^{s,g}(\omega)$ is receiver i's posterior belief that the state is ω upon observing $s_i(g)$.

2.3 Receivers' Decisions

For each $i \in N$, let $B_i = \{x, y\}$ be the set of *actions* of receiver i. Let $B = \prod_{i \in N} B_i$ denote the space of action profiles and $Z = \{x, y\}$ be the set of *outcomes*, where outcome x corresponds to achieving the critical mass and y to the opposite case.

Let $z^k : B \to Z$ be a map, where $z^k(a)$ is the outcome when the action profile is a and the *critical mass* is k. Formally,

$$z^k(a) = \begin{cases} x & \text{if } |\{i \in N : a_i = x\}| \geq k, \\ y & \text{otherwise.} \end{cases}$$

Throughout the paper we assume that $k = \lfloor \frac{n+1}{2} \rfloor$, i.e. the critical mass is a simple majority. The sender's utility function $v : Z \to \{0, 1\}$ has value 1 if the outcome is x and 0 otherwise. For each $i \in N$, let $u_i : B_i \times \Omega \to \{0, 1\}$ be the utility function of receiver i such that $u_i(x, X) = u_i(y, Y) = 1$ and $u_i(x, Y) = u_i(y, X) = 0$. That is, the receivers want their actions to match the true state of the world.

[7] Limited information spillovers can be observed in different contexts: while Gatewood (1984) shows that there is only limited information sharing between groups among Alaskan salmon seiners, Ali and Miller (2016) show that agents in communities may not transmit one neighbor's information to another to ensure cooperation. Other types of limited information spillovers include cases where agents only observe their direct neighbors' beliefs (Molavi and Jadbabaie, 2011; Anunrojwong and Sothanaphan, 2018) or actions (Corten and Buskens, 2010; Karamched, Stolarczyk, Kilpatrick, and Josic, 2020).

For any $g \in G(N)$, $\pi \in \Pi$, and $i \in N$, let $S_i^\pi(g) = \prod_{j \in N_i(g)} S_j^\pi$ be the space of message vectors that i can observe. Given their utility functions, receivers choose the action that corresponds to the state which they think is more likely. In particular, the *strategy* of agent i is given by $\alpha_i^{\pi,g} : S_i^\pi(g) \to B_i$ such that for any realization $s \in S^\pi$ it holds that

$$\alpha_i^{\pi,g}(s_i(g)) = \begin{cases} x & \text{if } \lambda_i^{s,g}(X) \geq \dfrac{1}{2}, \\ y & \text{otherwise.} \end{cases}$$

Throughout the paper we assume that $\lambda^0(X) < \lambda^0(Y)$, since otherwise receivers already take the sender's preferred action. Define the set of signals which achieve the critical mass on g under π as $Z_x^g(\pi) = \left\{ s \in S^\pi | z^k\left(\alpha^{\pi,g}(s)\right) = x \right\}$.

Let $a \in B$ be an action profile and $z = z^k(a)$ be an outcome. The *value* of an experiment $\pi \in \Pi$ for critical mass k is defined as the sender's expected utility under π on network g. As we fix λ^0 and $\alpha^{\pi,g}$ throughout the paper, we write $V_k^\pi(g) = V_k^\pi(\lambda^0, g, \alpha^{\pi,g})$, where

$$V_k^\pi(g) = \mathbb{E}_{\lambda^0}\left[\mathbb{E}_\pi\left[v(z^k\left(\alpha^{\pi,g}(s)\right))\right]\right] = \lambda^0(X) \sum_{s \in Z_x^g(\pi)} \pi(s|X) + \lambda^0(Y) \sum_{s \in Z_x^g(\pi)} \pi(s|Y).$$

That is, given n, k, and g, the value of an experiment is equal to the probability of reaching the critical mass. An experiment $\pi^* \in \Pi$ is *optimal on* g for critical mass k if $V_k^{\pi^*}(g) = \sup_{\pi \in \Pi} V_k^\pi(g)$.

3 Complexity and Bounds

On the empty network, an optimal experiment is straightforward (à la Kamenica and Gentzkow (2011)), anonymous, and sends x to all receivers with probability 1 if the state is X and to a set of k receivers (selected randomly with equal probability) if the state is Y.[8] However, limited information spillovers create a serious tractability issue for optimality in arbitrary networks. In particular, neither of the three characteristics of optimal experiments on the empty network outlined above hold in general in our setup.[9] Straightforwardness does not hold as the same recommendation is interpreted differently by different agents due to information spillovers. Anonymity trivially does not hold when agents are in a network. Finally, the sender might benefit from garbling information in state X and thus revealing the true state in X is not necessarily optimal.

While the information a receiver gathers on a non-empty network g can always be replicated on g_0, the converse is not necessarily true. This implies that the upper

[8] An experiment is *straightforward* if for all $i \in N$ it holds that (*i*) $S_i^\pi \subseteq B_i$ and (*ii*) for all $g \in G(N)$ and $s \in S^\pi$ with $s_i = a_i$, $\alpha_i^{\pi,g}(s_i(g)) = a_i$.

[9] Cf. Kerman and Tenev (2021) for an illustration.

bound of the sender's gain from persuasion is the optimal value on the empty network which we denote by V_k^n. Since our model boils down to the setup of Kerman et al. (2023) when the network is empty, we conclude that $V_k^n = \min\left\{\frac{n+k}{k}\lambda^0\left(X\right), 1\right\}$.[10] On the other hand, the optimal public experiment (i.e. agents observe the same message within every signal) is *independent* of the network structure and thus guarantees the sender a lower bound, which we denote by V^p. In particular, it always yields the same value $V^p = 2\lambda^0(X)$ for any k, as either all agents are persuaded or none are.[11] Note that $V^p < V_k^n$.

Proposition 1. *The value of an optimal experiment lies within* $[V^p, V_k^n]$.

The proofs and formal statements of the results can be found in the Appendix. Notice that V^p is the optimal value on the complete network. Hence, starting from the empty network and extending it to the complete network implies that the optimal value must strictly decrease after some extension. Moreover, Proposition 1 implies that if the receivers could form links without cost, they would form a complete network. However, surprisingly, we show that forming a limited number of links might not be in their best interest.

4 Detrimental Information

Let us return to our illustrative example in which the sender benefits from the creation of multiple links and therefore the change in the optimal value is non-monotonic. To systematically explain this result, we make two observations. First, if two agents have exactly the same neighborhood, then they can be treated identically. This is a feature we call *Symmetry*. Because of the way we model information spillovers the sender cannot separate the beliefs of agents who receive the same aggregate information.

Lemma 1 (Symmetry)**.** *If two agents have the same neighborhood, then it is optimal to send them the same message within each signal.*

Second, the sender's persuasion capability is not hindered on a circle network provided that it is not complete, i.e. when $n > 3$. While limited information spillovers on a circle network decrease the variety of minimal winning coalitions that can be persuaded in state Y (relative to the empty network), it is still sufficiently rich as to allow the sender to fully exploit private communication.

Lemma 2. *The sender can achieve V_k^n on a circle network whenever $n > 3$.*

[10] Note that the result of Kerman et al. (2023) also follows from Corollary 2 of Arieli and Babichenko (2019).

[11] Since receivers share a common prior, the situation is equivalent to persuading a single receiver as in Kamenica and Gentzkow (2011).

This result can also be explained by the findings in Babichenko et al. (2021). In their terminology, a receiver information-dominates another if he observes at least the same information channels. The authors show that the sender can achieve the upper bound of the value if there are no information-dominating pairs, which is the case for the circle network. However, in their setup the sender employs a continuum of messages and an experiment that achieves the upper bound is not provided. On the other hand, we prove the result by constructing an experiment that employs only two messages and achieves V_k^n.

We can now state our first result on the non-monotonicity of the optimal value.

Proposition 2. *If all agents in a network have degree at most 1, then the sender can either achieve* V_k^n *or there exists an extension that strictly benefits the sender.*[12]

Note that all agents having degree at most one is equivalent to a network consisting of pairs and singletons. Proposition 2 helps explain the observation in our illustrative example. Intuitively, in these types of networks having pairs and at most one singleton presents the greatest limitation for the sender's persuasion capabilities. We prove the result by establishing the existence of networks on which the sender cannot achieve V_k^n so that extending them to a circle is a strict improvement to the value.

The intuition behind Proposition 2 is as follows. On the one hand the existence of multiple connected pairs constrains the sender's possible choice of experiments by Lemma 1, since the pairs will take the same action as each other even if they observe different messages. This implies that the optimal value is likely to be less than V_k^n. On the other hand, extending this network to a circle allows the sender to exploit private communication and induce different actions. Consequently, by Lemma 2, the sender can achieve V_k^n and benefit from a denser network.

Intuitively, the network of pairs can be treated as a weighted voting setup or as a situation where customers have non-unitary demand for a product. Indeed, in reality voter blocs vote for the same alternative and customers within the same segment make the same purchase decisions. Moreover, all voters within a bloc (or all customers within a segment) have the same ex-ante belief, as well as the same ex-post belief. This makes it more difficult for the sender to reach a critical mass optimally. However, in a circle network voter blocs (or customer segments) are not as strictly delineated and thus the sender might induce different ex-post beliefs and achieve a higher probability of persuasion.

In the remainder of the section we will focus on stars (i.e. structures which have a center connected to all agents and all other agents are only connected to the center). There are two main reasons for stars to be a point of interest. First, a star is a commonly observed structure in friendship networks on social media outlets. The center in a star can be interpreted as an information hub or opinion leader in society. Because of their increased access to information, companies and political

[12]The logic of the proposition easily applies to networks which consist of connected triples and at most one singleton.

parties strategically incorporate such agents in marketing and political campaigns. Stars can also represent smaller scale situations, such as a board of directors in which the chairperson is the center node.

Second, the optimal value on any network with a star component of sufficiently large size is less than V_k^n. This is an important observation, since in this case the sender cannot achieve the upper bound of the value. The intuition is as follows. In an optimal experiment on a network with a sufficiently large star component, *the center node always observes the same message in state* X and thus is only persuaded when he cannot change the outcome. This implies that it is in the sender's best interest *not* to attempt to persuade the center node.

Proposition 3. *The optimal value on a network containing a star component with more than* k *agents is strictly less than* V_k^n.

Since the sender does not attempt to persuade the center node, it is as if the sender is persuading k out of $n-1$ agents, in which case V_k^{n-1} is the upper bound of the value. More generally, a similar result could be derived for networks with a star component that has more than one center, say $m > 1$. In this case it follows from applying *Symmetry* to the centers that the sender can achieve V_k^{n-m}. This implies that for a fixed number of agents n, the optimal value *monotonically* decreases as m increases and becomes V^p when m is sufficiently high (e.g. $m = n$).[13]

Next, we show that networks with star components are important for the non-monotonicity of the optimal value when a network is extended. Particularly, whenever there is a large enough star component, the sender can *benefit* from creating new links.

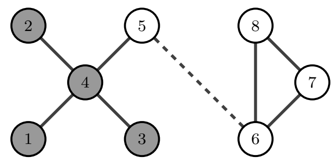
Proposition 4. *For any network containing a star component with more than* k *agents, there exists an extension that strictly benefits the sender.*

Proposition 4 implies that the change in the optimal value as the network becomes denser can be non-monotonic. In particular, it is easy to find a sequence of extensions that starts from g_0 and ends with a network that has a star component such that the value monotonically decreases. However, as we illustrate in Example 5, we can always extend this network in a way that benefits the sender. Therefore, in this new sequence of extensions, the change in value will be non-monotonic.

Example 5. Let $n = 8$, $\lambda^0(X) = 1/3$, and $k = 4$.[14] Consider network g below (without dashed edge). By Proposition 3, for any $\pi \in \Pi$ that is optimal on g it holds that $V_4^\pi(g) \leq V_4^{n-1} = V_4^7 = 11/12$.

[13]Note that this is in the same spirit as in a result in Candogan (2019), where the sender's payoff is decreased as some nodes in the network have a higher degree.

[14]Note that we consider a network with even number of agents only for ease of exposition.



π'	X	Y
$\bar{x}$	1	0
s	0	$\frac{1}{2}$
t	0	$\frac{1}{2}$

Now consider network g' (with dashed edge). Let $\bar{x}_i = x$ for all $i \in N$, $s \in S$ be such that $s_i = y$ for $i = 6$, and $s_j = x$ for $j \in N \setminus \{6\}$. Similarly, let $t \in S$ be such that $t_i = y$ for $i \in \{1, 2, 3\}$ and $t_j = x$ for $j \in N \setminus \{1, 2, 3\}$. Consider π' given above that employs s and t. It is easy to verify that x is implemented after any signal realization and thus the optimal value strictly increases to $V_4^{\pi}(g') = V_4^8 = 1$. △

Proposition 4 and Example 5 highlight the importance of the network structure for the sender's gain from persuasion.[15] We can interpret this result from two different angles. For the sender, nodes with many sources of information (information hubs) are difficult to persuade. To increase the probability of persuasion, the sender can either try to break social ties (i.e. create more singleton nodes) or alternatively, she can try to encourage more communication. While the former strikes as a polarizing approach, the latter is usually perceived as unifying and democratic. However, both can be equally detrimental for the receivers.

From the perspective of the receivers, forming more links seems like a natural improvement as it allows access to more sources of information (e.g. following more people on Twitter). Nevertheless, our result implies that receiving information from multiple sources might harm the receivers when these sources are highly correlated.[16] In Example 5, agent 5 forming a link with agent 6 allows the sender to employ a rougher partition of the network and diminish the influence of the information hub. In this way, the sender can leverage to her advantage the natural properties of real-life social networks, which usually exhibit high degrees of clustering, e.g. around opinion leaders (Jackson and Rogers, 2007).

5 Conclusion

The paper tests the naive intuition that more information provided to the receivers through the network would make them less manipulable. Interestingly, it is possible that the sender achieves a higher value on the denser of two networks. More

[15] Example 5 also illustrates an important difference between our results and Babichenko et al. (2021): while they show that V_k^n can be achieved if there are no information-dominating pairs, the sender can achieve V_k^n on g' even though agent 4 information dominates agents 1, 2, and 3.

[16] Increasing concentration of media ownership (Vizcarrondo, 2013; Noam, 2016) suggests that news from different sources might be highly correlated. Some theoretical studies in different contexts (Colla and Mele, 2010; Currarini, Ursino, and Chand, 2020) show how correlation between information sources can have a negative effect on a decision-maker.

importantly, the value of an optimal experiment *does not always decrease monotonically* when the network is extended. This is due to the fact that in some network structures, additional connections enable the sender to fully exploit all channels of information transmission among agents to her benefit.

Our results imply that simply encouraging more communication among receivers is not necessarily a good solution to collective decision making problems. In fact, increased communication might make it less likely that the receivers choose the "correct" outcome. Thus, a policy intervention that encourages the creation of more social ties requires a specific analysis of the network structure to ensure maximum efficacy, lest it yield counterproductive results.

Appendix A Proofs

Proof of Proposition 1. Let $\pi \in \Pi$. For each $i \in N$, assume that $|S_i^\pi(g)| = c(i)$. Let $R(i) = \left\{m_i^1, \ldots, m_i^{c(i)}\right\} \subseteq S_i$ be a set of distinct messages for i. Moreover for any $j \in N$, $q \in \{1, \ldots, c(i)\}$, and $q' \in \{1, \ldots, c(j)\}$ let $m_i^q \neq m_j^{q'}$.

For each $i \in N$, let $\phi_i : S_i^\pi(g) \to R(i)$ be a bijection, so each *information set* of i is mapped to a *unique message* in $R(i)$. For each $\omega \in \Omega$ and $s' \in S$, define $\pi' \in \Pi$:

$$\pi'(s'|\omega) = \begin{cases} \pi(s|\omega) & \text{if } \phi_i(s_i(g)) = s'_i, \quad \forall i \in N, \\ 0 & \text{otherwise.} \end{cases}$$

Note that the definition of π' implies that there is a bijection $\phi : S^\pi \to S^{\pi'}$ such that for each $i \in N$, $\phi(s) = s'$ if and only if $\phi_i(s_i(g)) = s'_i$. Hence, π' is an experiment.

We want to show that the value of π' under the empty network is equal to the value of π under g, i.e., $V_k^{\pi'}(g_0) = V_k^\pi(g)$. What remains to be shown is that each receiver i has the same posterior belief upon observing $s_i(g)$ under π and upon observing $\phi_i(s_i(g))$ under π'. Let $s' \in S^{\pi'}$ be such that $s'_i \in \left\{m_i^1, \ldots, m_i^{c(i)}\right\}$. For any $\omega \in \Omega$, we have

$$\begin{aligned} \lambda_i^{s'}(\omega) &= \frac{\sum_{s \in S^{\pi'}: s_i = s'_i} \pi'(s|\omega)\lambda^0(\omega)}{\sum_{\omega' \in \Omega} \sum_{s \in S^{\pi'}: s_i = s'_i} \pi'(s|\omega')\lambda^0(\omega')} = \frac{\sum_{s \in S^\pi: s_i(g) = \phi^{-1}(s'_i)} \pi(s|\omega)\lambda^0(\omega)}{\sum_{\omega' \in \Omega} \sum_{s \in S^\pi: s_i(g) = \phi^{-1}(s'_i)} \pi'(s|\omega')\lambda^0(\omega')} \\ &= \frac{\sum_{s \in A_i^\pi(g, \phi^{-1}(s'))} \pi(s|\omega)\lambda^0(\omega)}{\sum_{\omega' \in \Omega} \sum_{s \in A_i^\pi(g, \phi^{-1}(s'))} \pi(s|\omega')\lambda^0(\omega')} = \lambda_i^{\phi^{-1}(s'), g}(\omega). \end{aligned}$$

Thus, for each $s \in S^\pi$ it holds that $\alpha^{\pi, g}(s) = \alpha^{\pi', g_0}(\phi(s))$. Hence, $V_k^{\pi'}(g_0) = V_k^\pi(g)$. Since any $\pi \in \Pi$ on some network g can be replicated on the empty network, $V_k^n \geq V_k^\pi(g)$. Finally, since receivers share a common prior and have homogeneous preferences it follows from Kamenica and Gentzkow (2011) that $V_k^\pi(g) \geq V^p$ for any optimal $\pi \in \Pi$. $\square$

Lemma 1. *Let $\pi \in \Pi$ and let $g \in G(N)$ and $i, j \in N$ be such that $N_i(g) = N_j(g)$. Then there exists $\pi' \in \Pi$ such that for any $s \in S^{\pi'}$ it holds that $s_i = s_j$ and $V_k^{\pi'}(g) = V_k^\pi(g)$.*

Proof. First, note that since $\bar{N}_i(g) = \bar{N}_j(g)$, for any $s \in S^\pi$ we have $A_i^\pi(g,s) = A_j^\pi(g,s)$. Hence, i and j have the same posterior belief, i.e. for any $\omega \in \Omega$ and any $s \in S^\pi$ it holds that $\lambda_i^{s,g}(\omega) = \lambda_j^{s,g}(\omega)$.

Let $|S_i^\pi \times S_j^\pi| = c$. Let $R = \{m^1, \dots, m^c\}$ be a set of distinct messages. Define a bijection $\phi : S_i^\pi \times S_j^\pi \to R$. That is, for any *tuple* $(s_i, s_j), (t_i, t_j) \in S_i^\pi \times S_j^\pi$ it holds that $\phi(s_i, s_j) = \phi(t_i, t_j)$ if and only if $(s_i, s_j) = (t_i, t_j)$, so that each distinct *combination of messages* of i and j (and not every distinct neighborhood) is mapped to a distinct message in R.

Define $S' = \{s' \in S |\, s \in S^\pi, s'_{-ij} = s_{-ij} \text{ and } \phi(s_i, s_j) = s'_i = s'_j \in R\}$. In words, S' consists of signals obtained by replacing the messages of i and j with distinct messages in R (for each distinct message combination) and leaving the other receivers' messages unchanged, in each signal in S^π. Let $\tau : S^\pi \to S'$ be a bijection such that for any $s \in S^\pi$ we have $\tau(s) = s'$ if $\tau(s_i, s_j) = s'_i = s'_j$ and $s'_{-ij} = s_{-ij}$.

For every $s \in S^\pi$ and $\omega \in \Omega$, define $\pi'(\tau(s)|\omega) = \pi(s|\omega)$. It is clear that π' is an experiment. Note that since the probability weights are the same under π and π', receivers i and j still have the same posterior belief under π', i.e. for any $\omega \in \Omega$ and $s \in S^{\pi'}$ it holds that $\lambda_i^{s,g}(\omega) = \lambda_j^{s,g}(\omega)$.

Next, we show that for any $r \in \bar{N}_i(g)$, $\omega \in \Omega$, and $s \in S^\pi$ we have $\lambda_r^{s,g}(\omega) = \lambda_r^{\tau(s),g}(\omega)$. That is,

$$\lambda_r^{s,g}(\omega) = \frac{\sum_{t \in A_r^\pi(g,s)} \pi(t|\omega)\lambda^0(\omega)}{\sum_{\omega' \in \Omega} \sum_{t \in A_r^\pi(g,s)} \pi(t|\omega')\lambda^0(\omega')} = \frac{\sum_{t \in A_r^\pi(g,s)} \pi'(\tau(t)|\omega)\lambda^0(\omega)}{\sum_{\omega' \in \Omega} \sum_{t \in A_r^\pi(g,s)} \pi'(\tau(t)|\omega')\lambda^0(\omega')}$$
$$= \frac{\sum_{t' \in A_r^{\pi'}(g,\tau(s))} \pi'(t'|\omega)\lambda^0(\omega)}{\sum_{\omega' \in \Omega} \sum_{t' \in A_r^{\pi'}(g,\tau(s))} \pi'(t'|\omega')\lambda^0(\omega')} = \lambda_r^{\tau(s),g}(\omega).$$

Finally, any $r \notin \bar{N}_i(g)$ has the same posterior belief under π and π', as it is not affected by the transformation. Hence, $V_k^{\pi'}(g) = V_k^\pi(g)$. $\square$

Lemma 2. *Let $g \in G(N)$ be a circle and $n > 3$. Then there exists $\pi \in \Pi$ such that $V_k^\pi(g) = V_k^n$.*

Proof. Let $w_1 = (\lambda^0(X)/\lambda^0(Y))(n/k)$. For all $i \in N$, define $T_i = \{i \pmod n, (i+1) \pmod n, \dots, (i+n-2-k) \pmod n\}$. For each T_i, define $s^{T_i} \in S$ such that $s_j^{T_i} = x$ for all $j \in T_i$ and $s_\ell^{T_i} = y$ for all $\ell \notin T_i$. Let $\bar{x}_i = x$ and $\bar{y}_i = y$ for all $i \in N$. Let $\pi \in \Pi$ be defined as $\pi(\bar{x}|X) = 1$, $\pi\left(s^{T_i}|Y\right) = w_1/n$ for all $i \in N$, and $\pi(\bar{y}|Y) = 1 - w_1$. That is, π sends x to all agents in state X, and sends x to $k+2$ agents who are indexed consecutively in state Y. It is easy to verify that π is a probability distribution and that $V_k^\pi(g) = \lambda^0(X) \cdot 1 + \lambda^0(Y) w_1 = \min\left\{\frac{n+k}{k}\lambda^0(X), 1\right\} = V_k^n$. $\square$

Proposition 2. *Let $g \in G(N)$ be such that for all $i \in N$ it holds that $\delta_i \leq 1$. Let $\pi \in \Pi$ be optimal on g. Then either (i) $V_k^{\pi}(g) = V_k^n$ or (ii) there exists $g' \supsetneq g$ and $\pi' \in \Pi$ such that $V_k^{\pi'}(g') > V_k^{\pi}(g)$.*

Proof. We first introduce a technical lemma.

Lemma A1. *Let $g \in G(N)$ and $\pi \in \Pi$ be such that $V_k^{\pi}(g) < 1$. If there exists $s \in S^{\pi}$ with $s \in Z_x^g(\pi)$ and $|\{i \in N : \alpha_i^{\pi,g}(s_i(g)) = x\}| > k$, then $V_k^n > V_k^{\pi}(g)$.*

Proof. Suppose that there exists $s \in S^{\pi}$ such that $|\{i \in N : \alpha_i^{\pi,g}(s_i(g))\}| > k$. We can transform π on g into $\hat{\pi} \in \Pi$ on g_0 such that agents have the same action patterns and $V_k^{\hat{\pi}}(g_0) = V_k^{\pi}(g)$. Then, there exist at least k signals that can be obtained via s by removing the excess agents who chose x. Define $\pi' \in \Pi$ such that $\pi'(s|Y) = 0$, where $\pi(s|Y)$ is distributed evenly among $k-1$ such signals and takes some probability from $\bar{y}$ for the kth signal. The probabilities of all other signals are preserved under π'. Thus, $V_k^{\pi'}(g_0) > V_k^{\hat{\pi}}(g_0)$ and hence $V_k^n > V_k^{\hat{\pi}}(g_0) = V_k^{\pi}(g)$. □

We will establish the existence of cases in which the sender cannot achieve V_k^n. First consider the case $|\{i \in N : \delta_i = 0\}| = 0$, that is the network consists of pairs, which implies that $n = 2m$ for $m \in \mathbb{N}$. Consider the case that $k = 2\ell + 1$. By Lemma A1 and *Symmetry*, $V_k^{\pi}(g) < V_k^n$.

Now consider the case $|\{i \in N : \delta_i = 0\}| = 1$, that is the network consists of pairs and one singleton, which implies that $n = 2m + 1$. If $k = 2\ell$, then $j \in N$ with $\delta_j = 0$ is a dummy player and it follows that $V_k^{\pi}(g) < V_k^n$. If $k = 2\ell + 1$, then the sender has two options: (i) the experiment targets minimal winning coalitions, in which case the singleton node is a veto player and thus $V_k^{\pi}(g) < V_k^n$ or (ii) the experiment targets winning coalitions not all of which are minimal, in which case by Lemma A1 $V_k^{\pi}(g) < V_k^n$.

Whenever $V_k^{\pi}(g) < V_k^n$, consider the extension $g' \supsetneq g$ such that g' is a circle. Then, by Lemma 2 there exists $\pi' \in \Pi$ such that $V_k^{\pi'}(g') = V_k^n > V_k^{\pi}(g)$. □

Proposition 3. *Let $g \in G(N)$ have a star component with set of agents C and $|C| > k$. Then for any $\pi \in \Pi$ it holds that $V_k^{\pi}(g) \leq V_k^{n-1}$.*

Proof. We first give the definition of an anchor, which will be useful in the rest of the proof.

Definition A1. Let $\pi \in \Pi$. A signal $s \in S^{\pi}$ is an *anchor* if $\pi(s|X)\lambda^0(X) \geq \pi(s|Y)\lambda^0(Y)$. The set of all anchors is denoted by $An(\pi)$.

Denote the center of the star component by $c \in N$. We first show that transferring the information of c to the periphery nodes leads to no loss of information to c, as he can uniquely reconstruct the information from the combination of messages of the peripheral nodes. This means that c can be sent a single message in all signals, without changing the value.

Lemma A2. *Let $g \in G(N)$ have a star component with set of agents C and $|C| > k$. For any $\hat{\pi} \in \Pi$ and $\hat{s} \in S^{\hat{\pi}}$ such that $\alpha_c^{\hat{\pi},g}(\hat{s}) = x$, there exists $\pi \in S^{\pi}$ such that $|S_c^{\pi}| = 1$ and $V_k^{\hat{\pi}}(g) = V_k^{\pi}(g)$.*

Proof. Note that for two anchors $s, t \in S^{\hat{\pi}}$ with $s_c \neq t_c$, it holds that $A_c^{\hat{\pi}}(g, s) \cap A_c^{\hat{\pi}}(g, t) = \emptyset$. Let $S' \subseteq S$. Define a bijection $\tau : S^{\hat{\pi}} \to S'$ such that $\tau(s) = s'$ if (i) $s'_c = x$, (ii) for $j \in C \setminus \{c\}$ it holds that $s'_j = (s_j, s_c)$, and (iii) for $\ell \in N \setminus C$ it holds that $s'_\ell = s_\ell$. That is, in signals in S' the center always observes x and the messages of nodes $C \setminus \{c\}$ are modified so that they contain the information previously provided by c in signal s. So, the information that c reveals to nodes in $C \setminus \{c\}$ is shifted to them while c observes the same message in every signal.

For any $\omega \in \Omega$ and $s' \in S'$ such that $\tau(s) = s'$, let $\pi \in \Pi$ be defined by $\pi(s'|\omega) = \hat{\pi}(\tau^{-1}(s')|\omega)$. As the probabilities of corresponding signals are the same under π as under $\hat{\pi}$ and c's information under $\hat{\pi}$ is shifted to nodes in $C \setminus \{c\}$ under π (which are observed by c), c's action does not change. Moreover, the actions of nodes in $C \setminus \{c\}$ and in $N \setminus C$ do not change either. To see this, note that for any $i \in N$ and $t' \in A_i^{\pi}(g, s')$ there exists $t \in A_i^{\hat{\pi}}(g, s)$ such that $\tau(t) = t'$. This, together with the definition of τ implies that $\sum_{t' \in A_i^{\pi}(g,s')} \pi(t'|\omega) = \sum_{t \in A_i^{\hat{\pi}}(g,s)} \hat{\pi}(t|\omega)$. Thus, every node has the same posterior belief upon observing $s \in S^{\hat{\pi}}$ and $\tau(s) \in S^{\pi}$. Hence, $V_k^{\pi}(g) = V_k^{\hat{\pi}}(g)$. □

Next, we show that whenever there is an experiment $\hat{\pi}$ with a signal in which c chooses x, there is another experiment which *preserves the value* and in which in all signals where c chooses x, all peripheral agents also choose x.

Lemma A3. *Let $g \in G(N)$ have a star component with set of agents C and $|C| > k$. For any $\hat{\pi} \in \Pi$ and $\hat{s} \in S^{\hat{\pi}}$ such that $\alpha_c^{\hat{\pi},g}(\hat{s}) = x$, there exists $\pi \in S^{\pi}$ where for every $s \in S^{\pi}$ such that $\alpha_c^{\pi,g}(s) = x$ it holds that $\alpha_i^{\pi,g}(s) = x$ for all $i \in C$ and $V_k^{\hat{\pi}}(g) = V_k^{\pi}(g)$.*

Proof. Put simply, by means of unique messages, every peripheral node in C can uniquely identify the signals in which c, which has *strictly* more information than a peripheral node, chooses x. Hence, all peripheral nodes are persuaded whenever c is.

Suppose that there is a signal $t \in S^{\hat{\pi}}$ in which c chooses x. Hence, there is at least one anchor $s \in An(\hat{\pi})$ with $s_i = t_i$ for all $i \in C$ and for every $r \in S^{\hat{\pi}}$ such that $r_i = s_i$ for all $i \in C$, c also chooses x. The action patterns of nodes in C in such signals are: (a) all x; (b) c chooses x and zero or more nodes in $C \setminus \{c\}$ choose x.

Consider case (b). It must be true that if some node $\ell \in C \setminus \{c\}$ chooses y this is because it associates t with *more* signals than c. In other words, in all signals $r \in S^{\hat{\pi}}$ where $(r_\ell, r_c) = (t_\ell, t_c)$ node ℓ chooses y and this *includes* the signals in which c *does not* choose x. As this also includes the associated anchors, $A_c^{\hat{\pi}}(g, t) \subsetneq A_\ell^{\hat{\pi}}(g, t)$.

Notice the trivial fact that for every $s, t \in S^{\hat{\pi}}$ with $s_c \neq t_c$ and $i \in C$, it holds that $A_i^{\hat{\pi}}(g, s) \cap A_i^{\hat{\pi}}(g, t) = \emptyset$. So, whenever c receives different messages in different signals, these signals belong to *disjoint* association sets and the same observation holds for every $i \in C$.

Let T be the set of signals in which receiver ℓ chooses y and c chooses x. That is,

$$T = \left\{ t \in S^{\hat{\pi}} \middle| \, \alpha_\ell^{\hat{\pi},g}(t_\ell(g)) = y \text{ and } \alpha_c^{\hat{\pi},g}(t_c(g)) = x \right\}.$$

Define a bijection such that in signals in $\hat{\pi}$ in which ℓ chooses y and c chooses x, the message of ℓ is changed to a *unique* message that is specific to each distinct information set of c and keep all other messages the same. Formally, let $T' \subsetneq S$ and define $\phi : T \to T'$ such that for any $t \in T$ it holds that $\phi(t) = t'$ if $t'_\ell = t_c(g) \in S'_\ell \setminus S_\ell^{\hat{\pi}}$ and $t'_{-\ell} = t_{-\ell}$. Now for any $\omega \in \Omega$ define a new experiment $\pi \in \Pi$, which transforms the signals in T according to ϕ and keeps all other signals the same while preserving the probability weights:

$$\pi\left(s'|\omega\right) = \begin{cases} \hat{\pi}(s'|\omega) & \text{if } s' \in S^{\hat{\pi}} \setminus T, \\ \hat{\pi}(\phi^{-1}(s')|\omega) & \text{if } s' \in T. \end{cases}$$

Let $s' \in S^\pi$ be such that $\phi(s) = s'$ for some $s \in T$. Then,

$$\lambda_\ell^{s',g}(X) = \frac{\sum_{t' \in A_\ell^\pi(g,s')} \pi(t'|X)\lambda^0(X)}{\sum_{\omega\in\Omega} \sum_{t' \in A_\ell^\pi(g,s')} \pi(t'|\omega)\lambda^0(\omega)} = \frac{\sum_{t' \in A_\ell^\pi(g,s')} \hat{\pi}(\phi^{-1}(t')|X)\lambda^0(X)}{\sum_{\omega\in\Omega} \sum_{t' \in A_\ell^\pi(g,s')} \hat{\pi}(\phi^{-1}(t')|\omega)\lambda^0(\omega)}$$
$$= \frac{\sum_{t \in A_\ell^{\hat{\pi}}(g,s) \cap A_c^{\hat{\pi}}(g,s)} \hat{\pi}(t|X)\lambda^0(X)}{\sum_{\omega\in\Omega} \sum_{t \in A_\ell^{\hat{\pi}}(g,s) \cap A_c^{\hat{\pi}}(g,s)} \hat{\pi}(t|\omega)\lambda^0(\omega)} = \frac{\sum_{t \in A_c^{\hat{\pi}}(g,s) \subseteq T} \hat{\pi}(t|X)\lambda^0(X)}{\sum_{\omega\in\Omega} \sum_{t \in A_c^{\hat{\pi}}(g,s) \subseteq T} \hat{\pi}(t|\omega)\lambda^0(\omega)} \geq \frac{1}{2},$$

where $\phi(t) = t'$ and the third equality follows from the definition of ϕ; $A_\ell^{\hat{\pi}}(g,s) \cap A_c^{\hat{\pi}}(g,s) = A_c^{\hat{\pi}}(g,s) \subseteq T$ follows from $A_c^{\hat{\pi}}(g,t) \subsetneq A_\ell^{\hat{\pi}}(g,t)$ and the inequality follows from the definition of case (b). Similarly, it holds that $\lambda_c^{s',g}(X) \geq 1/2$. This implies that in π node ℓ will choose x whenever c chooses x in π. Additionally, node c will still choose x in the corresponding signals in $\hat{\pi}$ and π. Thus, the transformation does not change the action of c in any signal, it only *increases* the number of x actions. Observe that for $s \in A_\ell^\pi(g,t) \setminus A_c^\pi(g,t)$ such that $t \in T$, it holds that $\alpha_\ell^{\hat{\pi},g}(t_\ell(g)) = y$ and the transformation will not decrease the value, as in such s nodes ℓ and c must already be choosing y. Hence, $V_k^\pi(g) = V_k^{\hat{\pi}}(g)$. □

Lemma A2 implies that the center of a star component can be sent the same message in all signals without affecting the value. Moreover, Lemma A3 implies that whenever c chooses x, *all peripheral nodes* choose x. Hence, c's action will *never be decisive* since $|N \setminus C| < k$. Even persuading all $|N \setminus C|$ will necessitate persuading a member of C. This member can never be c because, that means that there will always be *strictly more* players than the critical mass, who choose x. The center of a star then becomes a *dummy player* (who should never be persuaded) and thus, $V_k^\pi(g) \leq V_k^{n-1}$. □

Proposition 4. *Let $g \in G(N)$ have a star component with set of agents C and $|C| > k$. Let $\pi \in \Pi$ be optimal on g. There exists $g' \supsetneq g$ and $\pi' \in \Pi$ such that $V_k^{\pi'}(g') = V_k^n > V_k^\pi(g)$.*

Proof. Let P be the set of peripheral nodes in C and $M \subseteq P$ with $|M| = k - |N \setminus C|$. Note that $V_k^\pi(g) \leq V_k^{n-1}$ by Proposition 3. First suppose that n is even. Consider $g' \supseteq g$ such that for some $i \in N \setminus C$, $g'_{ij} = 1$ for all $j \in M$. Let $s^1 \in S$ be defined as $s_i^1 = x$ for all $i \in (N \setminus C) \cup M \cup \{c\}$ and $s_j^1 = y$ otherwise. Similarly, define $s^2 \in S$ as $s_i^2 = x$ for all $i \in C$ and $s_j^2 = y$ otherwise. Now, define $\pi' \in \Pi$ as $\pi'(\bar{x}|X) = 1$, $\pi'(s^1|Y) = \lambda^0(X)/\lambda^0(Y)$, $\pi'(s^2|Y) = \lambda^0(X)/\lambda^0(Y)$, and $\pi'(\bar{y}|Y) = 1 - 2\lambda^0(X)/\lambda^0(Y)$. Hence, it follows that $V_k^{\pi'}(g') = 3\lambda^0(X) > V_k^{n-1} = V_k^\pi(g)$.

Now suppose that n is odd. We first provide the adapted definition of information-domination due to Babichenko et al. (2021) and state a corollary to one of their results.

Definition A2. Given a network $g \in G(N)$, an ordered pair of receivers (i, j) is *information-dominating* if $N_j(g) \subseteq N_i(g)$.

Corollary A1. *Let $g \in G(N)$ and suppose that for any $\pi \in \Pi$ it holds that $V_k^\pi(g) < V_k^n$. If $g' \in N$ with $g \subsetneq g'$ has no information-dominating pairs, then there exists $\pi' \in \Pi$ such that $V_k^{\pi'}(g') = V_k^n > V_k^\pi(g)$.*

Thus, we need to show that we can reach a network g' with no information-dominating pairs by extending g. Then, either (i) $|P| = |N \setminus C|$ or (ii) $|P| > |N \setminus C|$. In case (i), consider $g' \supsetneq g$ such that each $i \in N \setminus C$ is connected to a unique node in P. Then, there are no information-dominating pairs in g' and therefore $V_k^{\pi'}(g') > V_k^\pi(g)$. In case (ii), fix $j \in N \setminus C$. Consider $g' \supsetneq g$ such that each $i \in (N \setminus C) \setminus \{j\}$ is connected to a unique node in P and j is connected all the nodes that are not connected to any node in $N \setminus C$. Then, there are no information-dominating pairs in g' and therefore $V_k^{\pi'}(g') = V_k^n > V_k^\pi(g)$. □

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