Trading Relationships in Over-the-Counter Markets

Alex Maciocco*
University of California - Irvine

Preliminary Draft
This version: February 28, 2024

Abstract

This paper formalizes a decentralized asset market where investors form long-term trading relationships with dealers. Relationships impact both the provision and price of liquidity in OTC markets. More stable relationships cause trade sizes and volume to rise while transaction costs tend to fall. I endogenize the formation of relationships and calibrate the model to the inter-dealer municipal bond market. Policies aimed at eliminating trading relationships result in a rise in transaction costs and a decrease in trading volume, yet they can be welfare improving.

JEL Classification: G12, D83

Keywords: Search, OTC markets, relationship trading, liquidity

*Please address any correspondence to Alex Maciocco, Department of Economics, University of California, Irvine, 3151 Social Science Plaza A, Irvine, California 92697-5100, USA. E-mail: amaciocc@uci.edu. Website: www.alexmaciocco.com.

1 Introduction

In over-the-counter (OTC) markets, investors solicit quotes from dealers and have the flexibility to accept, reject, or negotiate the offers they receive. Typically, the trading protocols used do not provide anonymity, meaning both parties involved in the transaction are aware of each other's identities. This characteristic of OTC markets enables the formation of long-term trading relationships, as investors can freely select their preferred counterparties.[1] Empirical evidence supports this notion, with several studies finding that market participants maintain persistent relations with a limited number of trading partners (e.g., Hendershott, Li, et al. 2020). However, this behavior contrasts with a common assumption in search-theoretic models of OTC markets, which posits that interactions with dealers are short-lived. Since stronger relationships correlate with tighter spreads (Di Maggio, Kermani, and Song 2017) and reduced trading delays (Afonso, Kovner, and Schoar 2013), their omission from conventional theories of OTC markets neglects a dimension of trade that is known to impact the objects of study of such theories.

The key contribution of this paper is to introduce trading relationships in a market where opportunities to trade an asset occur infrequently and terms of trade are negotiated bilaterally. The model is closely based on that of Lagos and Rocheteau (2009). Meetings between investors and dealers originate according to a random search technology, but in contrast to Lagos and Rocheteau (2009), the meetings are long-lived. This formalization of relationships is similar in spirit to worker-firm relationships in Mortensen and Pissarides (1994). Importantly, this type of meeting arrangement enables repeated trade between an investor-dealer pair. Effectively, relationships give investors the ability to temporarily bypass search frictions that are associated with finding a counterparty. By allowing investors to contract with dealers over a long-term horizon, they obtain a form of liquidity insurance: a payment in exchange for the right to trade continuously. I use this framework to study the role relationships play in shaping conventional measures of market liquidity.

I find that the stability (longevity) of trading relationships has important implications for market liquidity, as measured by volume and transaction costs. When trading relationships become longer-lived, the volume of trade increases monotonically, and transaction costs decline. These liquidity improvements are twofold. First, on the extensive margin, stable relationships mechanically increase trading volume by providing more investors with the opportunity to trade. Less obviously, a second effect arises from the intensive margin. Because investors can contract with dealers over a long-term horizon, relationships mitigate a 'hold-up' problem.[2] In Lagos and Rocheteau (2009), whenever an investor trades, it is never with the same dealer that she acquired her current asset holdings from. Thus, any portfolio investment made by the investor cannot yield gains from trade with the current dealer with which she is bargaining. Furthermore, since the investor does not appropriate the entire surplus generated from her investment, given the next dealer she meets will

[1]In many platforms, such as MarketAxess' Open Trading product, investors looking to send a Request For Quote (RFQ) to dealers have the option to send non-anonymous RFQ's to those dealers with whom they have existing relationships, or anonymous RFQ's to all other dealer's participating in the market.

[2]For a discussion on how long term contracts can address the hold-up problem described in Goldberg (1976), refer to Klein, Crawford, and Alchian (1978).

have some bargaining power, the investor under-invests by trading in smaller quantities. In my model, since investors make the portfolio investment with the same dealer with whom they bargain and generate surplus with, the hold-up problem is not as severe. This leads to more extreme asset positions, larger trade volume, and lower fees per unit of the asset traded.

To understand how dealers incentives to provide intermediation are influenced by the stability of relationships, I consider the free entry of dealers. I show that more stable relationships can not only eliminate a potential multiplicity of equilibria but also generate a unique steady-state equilibrium with more favorable liquidity properties —such as more participating dealers, higher trade volume, and lower fees. The mechanism at work is that of Lagos and Rocheteau (2007). By making the contact rate a function of the measure of participating dealers, two countervailing forces generate a multiplicity of equilibria. First, as more dealers enter, the expected profits of any one dealer decline as a result of increased competition. The second effect is that as the number of dealers increases, the meeting rate of investors rises, prompting them to trade in larger quantities. These larger trades tend to produce more surplus which increases the expected profits of dealers. For any given level of participation, making relationships more stable increases the expected fees those participating dealers will receive. It means that ceteris paribus, longer-lived relationships provide greater incentives for dealers to enter.

I expand the baseline environment by introducing the coexistence of spot trading into the model and find that spot transactions are more costly for investors as opposed to trading via relationships. Furthermore, I show that under plausible parametric choices, average intermediation fees for spot transactions can decline as relationships are more unstable which is in contrast to the behavior of trading fees in the baseline environment. This feature of my model is able to generate non-monotone patterns for market-wide measures of transaction costs and trade volume which highlights a potential trade-off between improving liquidity for spot transactions at the expense of relationship traders. It suggests that depending on the composition and structure of a given market, certain policies that affect relationship stability can yield differential outcomes for market liquidity.

Lastly, I endogenize the formation and destruction of trading relationships by introducing a heterogeneous flow cost associated with maintaining a relationship. This extended model is solved numerically and calibrated using data moments from the inter-dealer municipal bond market. To quantify the value of trading relationships within this market, I shut down the ability of investors to form relationships. Through this exercise, I find that the absence of trading relationships leads to a 40% increase in spreads, coupled with a 9% decrease in trading volume.

1.1 Empirical Evidence of Trading Relationships

Certain asset classes are inherently unfit for continuous all to all trading. Bonds, derivative instruments, and unsecured loans all have idiosyncratic properties such as credit risk, lot size, and expiration dates that make them particularly difficult to standardize. As a result, these non-fungible assets often trade in decentralized, fragmented markets where each trade is bilaterally negotiated between a buyer-seller pair. Investors are generally limited in the number of dealers they can *feasibly* contact for quotes. Bessembinder and Maxwell (2008) document that quoted prices from dealers in the corporate bond market are firm "as long as the breath is warm", implying that investors only ever get to sample a subset of the market before quotes are rendered obsolete.

Since investors in OTC markets need to search over a potentially large pool of counterparties in order to fulfill their trading needs, associated delays arising from market fragmentation pose a challenge from a risk management standpoint; Investors need the ability to offload or onboard assets quickly to satisfy liquidity and hedging requirements (Hendershott, Li, et al. 2020). In practice, it is found that relationships are often used as a mechanism to fulfill liquidity needs both in normal times (Afonso, Kovner, and Schoar 2013; Riggs et al. 2020; Han, Nikolaou, and Tase 2022) and in times of crisis (Di Maggio, Kermani, and Song 2017). Investors not only find assets more readily with their relationship counterparties, those with whom prior relations already exist, but also do so at better prices.[3]

Finding quality assets at reasonable prices remains a time consuming and costly process in decentralized environments. It comes as no surprise then that empirical studies have found the existence of either a core-periphery network structure or the presence of long-term bilateral relationships, if not both, in virtually all OTC asset markets.[4] In an effort to economize on informational or search frictions, investors build trading relationships to provide a form of liquidity insurance. In this way, relationships act as a tool for investors to limit uncertainty around asset purchases and help them to circumvent costly trading delays.

1.2 Related Literature

This paper contributes to an extensive theoretical literature that seeks to understand the role of search frictions on liquidity in OTC markets. Duffie, Gârleanu, and Pedersen (2005) (DGP hereafter), although not the first to study bid-ask spreads or decentralized asset markets (e.g., Amihud and Mendelson 1980; Rubinstein and Wolinsky 1987), are the first to show that endogenous bid-ask spreads arise naturally as a result of trading frictions and depend critically on investors' outside options. This approach is a departure from earlier literature using dealer inventory considerations or asymmetrically informed investors to explain bid-ask spreads. While stylized, the model of DGP captures two key features of OTC markets which are present in my model as well: bilateral meetings and bargaining over prices. The framework is extended in a number of ways to provide explanations for many relevant empirical features of decentralized asset markets.[5]

Most relevant to this paper, Lagos and Rocheteau (2009) (LR hereafter) expand the economic setting of

[3]There exists ample evidence of this phenomenon in the market for federal funds such as in Ashcraft and Duffie (2007), Cocco, Gomes, and Martins (2009), Afonso, Kovner, and Schoar (2013), Bräuning and Fecht (2017), but also for longer term lending markets as in Li (2021) and even in the corporate bond market as documented by Di Maggio, Kermani, and Song (2017).

[4]See for example Li and Schürhoff (2019) (municipal bonds), Hollifield, Neklyudov, and Spatt (2017) (ABS), Han, Nikolaou, and Tase (2022) (triparty repos), Iercosan and Jiron (2017) (CDS), Di Maggio, Kermani, and Song (2017) (corporate bonds), Afonso, Kovner, and Schoar (2013) (federal funds).

[5]See Weill (2020) for a comprehensive review of the search based literature on OTC markets.

DGP by allowing for unrestricted asset holdings and more general investor preferences. This more general economic setting provides a new channel through which investors can bypass search frictions, namely, their portfolio size. LR show that the resulting asset dispersion and so called *liquidity hedging* behavior by investors is a key determinant of market liquidity. This dimension of portfolio choice is present in my model as well. Whereas LR focus exclusively on one-time, spot transactions, I build on their framework and consider the existence of repeated trade between investors and dealers in long-term matches.

Zhang (2018) considers repeat trades between investor-dealer pairs in an environment with private valuations. They show that dealer's screening behavior can lead to liquidity distortions even if investors do not face severe search frictions given the existence of trading relationships. My model differs in that the degree to which relationships are unstable generates liquidity distortions in its own right without the need for asymmetric information. The liquidity distortion in Zhang (2018) is obtained via the breakdown of trade that occurs as a result of asymmetric information. In contrast, the liquidity distortion in my model is realized directly through the degree of relationship instability which affects transaction costs and trade volume through the endogenous trade sizes.

There exists a complementary strand of papers on OTC markets that are interested in studying the formation of networks.[6] This literature shares a common element with my paper in that it acknowledges that trade is not fully random but instead occurs via repeated interactions with the same counterparties. As an example, Sambalaibat (2019) endogenously generates trading networks by allowing for ex-ante heterogeneous investors to choose which dealers to form relationships with. The relationships in the model lack a long-term component since by assumption, they consist of a bond purchase and a sale back to the same dealer, but the relationship is over after the round trip trade is completed. In contrast, I allow for investors to trade multiple times with dealers which is more consistent with the idea of a trading relationship. Furthermore, while Sambalaibat (2019) can explain the persistent interdealer trading links present in many OTC markets, nothing can be said about the stability of investor-dealer relationships as evidenced in Li and Schürhoff (2019). On the other hand, I study precisely the effects of relationship stability on market liquidity.

Relationships have also been studied extensively in alternative economic environments such as banking, credit arrangements, and labor (e.g., Chiu, Eisenschmidt, and Monnet 2020; Bethune et al. 2022; Mortensen and Pissarides 1994). In the case of labor market models, a key difference with my paper is that a worker's labor, the analogue of the asset in my paper, is not re-traded. In my model, investors' need to trade the asset multiple times drives inefficiencies through a hold-up problem with dealers, which is partially mitigated by relationships. This source of inefficiency is not present in most models of labor markets. Bethune et al. (2022) construct a model of corporate lending where entrepreneurs must form relationships with banks in order to secure external financing from suppliers of capital. The relationships between entrepreneurs and banks serve as a technology to circumvent the breakdown of trade arising from commitment problems. The

[6]See for example Malamud and Rostek (2017), Babus and Hu (2017), Chang and Zhang (2021), Hendershott, Li, et al. (2020), Wang (2017).

relationships in my model are a technology that saves on search costs, whereas the relationships in Bethune et al. (2022) are technologies that improve credibility.

2 Environment

Time is continuous and the horizon infinite. There are two types of infinitely-lived agents: a unit measure of investors and a unit measure of dealers. There is one asset and one perishable good, which I use as a numéraire. The asset is durable, perfectly divisible, and in fixed supply, $A \in \mathbb{R}_+$. The numéraire good is produced and consumed by all agents. The instantaneous utility function of an investor is $u_i(a) + c$, where $a \in \mathbb{R}_+$ represents the investor's asset holdings, $c \in \mathbb{R}$ is the net consumption of the numéraire good ($c < 0$ if the investor produces more than she consumes), and $i \in \{1, ..., I\} \equiv \mathcal{I}$ indexes a preference shock. The utility function $u_i(a)$ is strictly increasing, concave, continuously differentiable and satisfies the Inada condition that $u_i'(0) = \infty$. Investors receive idiosyncratic preference shocks that occur with Poisson arrival rate λ. Conditional on the preference shock, the investor draws preference type i with probability π_i, and $\sum_{i=1}^{I} \pi_i = 1$. These preference shocks capture the notion that investors value the services provided by the asset differently over time, and will generate a need for investors to periodically change their asset holdings. The instantaneous utility of a dealer is simply c. All agents discount at the same rate $r > 0$.

There is a competitive market for the asset. Dealers can continuously buy and sell in this market at price p, while investors can only access through a dealer. I assume that investors and dealers can form lasting relationships. Hence, investors are either matched (connected) or unmatched (unconnected). An unmatched investor forms a relationship with a dealer according to a Poisson process with arrival rate α. Once the investor and the dealer have made contact, they negotiate the terms of a long term contract that specifies the quantity of assets that the dealer will acquire (or sell) in the market on behalf of the investor, conditional on the history of preference shocks of the investor, and the discounted sum of the intermediation fees that the investor will pay the dealer for their services. A relationship is destroyed at rate δ.

3 Equilibrium

I focus on steady-state equilibria where the asset price and the distribution of investors across states are constant through time.

3.1 Bargaining

Long term trading relationships are made explicit by a contract specifying two components. The first is a path of assets the dealer will acquire for the investor during the relationship, conditional on the investor's idiosyncratic history of preference types, for all possible histories. This object is best thought of as an asset rule that assigns for any history of preference types an according asset position. Therefore, it is possible to define a desired asset path as a mapping from the partial histories of preference types whilst in a relationship into a time path of asset positions. Let t_0 denote the time a relationship is formed. Define the partial history of investor preference types from the time a relationship is formed up to time t as the time path $i_t = \{i(t_0), \ldots, i(t)\}$. We can define the mapping $\mathbf{a} : i_t \longmapsto a_t \ \forall t$ where a_t is the optimal steady state asset position of an investor at time t and $\mathbf{a}$ is the resulting asset path. An investor-dealer pair will bargain over the asset path acquired for the investor by the dealer during the course of the relationship. It is assumed the dealer can commit to providing assets to the investor in the future. The second component of the contract is an expected discounted sum of intermediation fees, Φ, paid to the dealer for his services.

Terms of the contract are determined by the generalized Nash bargaining Solution. The problem of an investor-dealer pair is given by

$$\max_{\mathbf{a},\Phi}[V_i(a_0, \mathbf{a}) - W_i(a_0) - \Phi]^{1-\eta}\Phi^{\eta}$$

where $V_i(a_0, \mathbf{a})$ is the utility of a matched investor before the discounted sum of fees are paid to the dealer when the investor has preference type i, initial asset holdings a_0, and holds an asset path $\mathbf{a}$ throughout the relationship. $W_i(a_0)$ is the value of being unmatched with portfolio a_0. Dealers' bargaining power is η. The solution to the bargaining problem is given by the following equations

$$\mathbf{a} = \arg\max_{\mathbf{a}'}\Big\{V_i(a_0, \mathbf{a}') - W_i(a_0)\Big\} \tag{1}$$

$$\Phi_i(a_0) = \eta[V_i(a_0, \mathbf{a}) - W_i(a_0)]. \tag{2}$$

As a result of preferences that are linear in the numeraire, the outcome of Nash bargaining will be such that the asset path maximizes the joint surplus of a long-term relationship and the discounted sum of intermediation fees splits the surplus created by the match according to the dealer's bargaining power.

3.1.1 Bargaining Without Dealer Commitment

I provide strategic foundations to the axiomatic solution discussed in the previous section and consider a bargaining approach that does not require the assumption of dealer commitment. I view the trading relationship as a sequence of alternating offer games with discounting and exogenous risk of breakdown, in the spirit of Rubinstein (1982), between an investor and a dealer. An investor-dealer pair only interact at discrete points in time. A proposal made by either agent consists of an asset position for the investor and an intermediation fee paid to the dealer. The receiver of the offer is free to accept or reject the proposed contract. In the case where an offer is accepted, the players remain matched but the bargaining game ends and both players receive their according payoffs. I assume counteroffers take time. In the case of rejection, the game continues on unless either a preference shock is received, after which I assume a new bargaining game begins, the relationship is destroyed or a new counterparty is found.

I find that taking the limit as counteroffer speeds approach zero yields investor asset demands that are identical to those obtained by Nash bargaining.[7] Thus, dealer commitment is not requisite to sustain asset

[7] The two asset positions are identical up to some transformation. I only require that the ratio of counteroffer speeds in the alternating offer game equals the ratio of bargaining powers used in Nash bargaining.

allocations that maximize the joint surplus of a relationship. Furthermore, the strategic approach yields per-trade intermediation fees that can be used in supplement to the discounted sum of fees in my analysis. For further details on the case of bargaining without commitment, I direct the reader to Appendix A where the bargaining game is fully characterized and solved.

3.2 Bellman Equations

Consider first an investor and a dealer in a relationship.

Proposition 1 *The lifetime utility of a matched investor is linear in wealth such that* $V_i(a) = pa + V_i$

One can think of the matched investor as selling all her assets at unit price p, which generates a wealth pa, before reoptimizing her portfolio. The term V_i solves the following HJB equation:

$$rV_i = \max_{a' \geq 0} \left\{ u_i(a') - rpa' + \delta[W_i(a') - V_i(a')] + \lambda \sum_{j \in \mathcal{I}} \pi_j [V_j(a') - V_i(a')] \right\}. \quad (3)$$

At each point in time, the investor chooses her asset holdings, a', so as to maximize the right side of (3). The first two terms, $u_i(a') - rpa'$, correspond to the instantaneous utility of the investor net of the flow cost of holding the asset. One can think of the investor as renting the asset from the dealer at the rental price rp. The third term, $W_i(a') - V_i(a')$, corresponds to the event where the investor gets disconnected from the dealer at Poisson arrival rate δ. At the time of separation, the investor cannot readjust her asset holdings and is stuck with the asset position she had previously chosen. The last term corresponds to the arrival of preference shocks at rate λ. The new preference type is j with probability π_j. We can make use of the linearity of the value function, $V_i(a)$, to notice that $V_j(a') - V_i(a')$ is independent of a' and simplify the problem further as

$$rV_i = \max_{a' \geq 0} \left\{ u_i(a') - rpa' + \delta[W_i(a') - V_i(a')] \right\} + \lambda \sum_{j \in \mathcal{I}} \pi_j \left[V_j - V_i \right]. \quad (4)$$

So from (4), the investor maximizes her instantaneous utility net of the rental cost of the asset and the cost from losing access to the dealer, which creates a potential illiquidity.

Using that $V_i'(a) = p$, the first-order condition for the optimal asset holdings is

$$u_i'(a_i) + \delta W_i'(a_i) = (r + \delta)\, p. \quad (5)$$

The left side is the marginal instantaneous utility from the asset taking into account the risk of separation while the right side is the holding cost of the asset.

I now turn to the value of an unmatched investor with preference type i and asset holdings, a. It solves the following Bellman equation:

$$rW_i(a) = u_i(a) + \lambda \sum_{j \in \mathcal{I}} \pi_j \left[W_j(a) - W_i(a) \right] + \alpha(1 - \eta) \left[V_i(a) - W_i(a) \right]. \quad (6)$$

The unmatched investor enjoys $u_i(a)$ from holding the asset. At Poisson arrival rate λ, she draws a new preference type. At Poisson rate α, the investor meets a dealer. They negotiate the terms of a long term contract that specifies asset holdings during the relationship and a discounted sum of intermediation fees. The outcome of this negotiation is given by (1) and (2). As a result of the quasi-linear preferences, the discounted sum of intermediation fees is simply a fraction of the joint surplus. Therefore, at rate α an unmatched investor meets a dealer and enjoys her share of the joint surplus. Alternatively, we could think of the unmatched investor as contacting a dealer at a *bargaining-adjusted* rate $\alpha(1-\eta)$ and extracting the full surplus, which corresponds to the last term of (6).

At this point we will make an observation that will allow to compute the value function, $W_i(a)$, in closed form. According to (6), from the view point of the investor, the economy is payoff-equivalent to one where she gains access to the competitive asset market at rate $\alpha(1-\eta)$. Upon access, the duration of participation in the market is exponentially distributed with mean $1/\delta$. Hence, we can rewrite (6) as

$$W_i(a) = U_i(a) + \mathbb{E}_i\left[e^{-rT}V_{s(T)}(a)\right], \tag{7}$$

where T is exponentially distributed with mean $1/\left[\alpha(1-\eta)\right]$ and where

$$U_i(a) \equiv \mathbb{E}_i\left[\int_0^T e^{-rt}u_{s(t)}(a)dt\right].$$

The expectation is with respect to T, the time to gain effective access to the market, and the history of preference shocks, $s(t)$, conditional on the initial preference type, $s(0)=i$. The function $U_i(a)$ represents the discounted sum of utility flows until the next access to the market at rate $\alpha(1-\eta)$. It solves the following Bellman equation:

$$rU_i(a) = u_i(a) + \lambda\sum_{j\in\mathcal{I}}\pi_j\left[U_j(a)-U_i(a)\right] - \alpha(1-\eta)U_i(a). \tag{8}$$

It adds the discounted utility flows until the next effective access to the market occurs are rate $\alpha(1-\eta)$. We take a weighted sum of (8) to compute the expected discounted sum of utility flows across preference types:

$$\sum_i \pi_i U_i(a) = \frac{\sum_i \pi_i u_i(a)}{r+\alpha(1-\eta)}.$$

It is simply the expected instantaneous utility with respect to the preference type discounted at rate $r+\alpha(1-\eta)$. We substitute this expression back into (8) and solve to obtain:

$$U_i(a) = \frac{\left[r+\alpha(1-\eta)\right]u_i(a) + \lambda\sum_j \pi_j u_j(a)}{\left[r+\alpha(1-\eta)\right]\left[r+\lambda+\alpha(1-\eta)\right]}. \tag{9}$$

We see from (9) that $\left[r+\alpha(1-\eta)\right]U_i(a)$ is a weighted average of the current instantaneous utility of the investor, $u_i(a)$, and her expected future utility at the time the next preference shock occurs, $\sum_j \pi_j u_j(a)$. The weight on the current utility increases with the rate of time preference, r, and the rate of effective access to the market, $\alpha(1-\eta)$.

Using that T is distributed according to an exponential distribution with parameter $\alpha(1-\eta)$, we can compute the second term of (7) recursively as the solution to the following Bellman equation:

$$rX_i(a) = \alpha(1-\eta)[V_i(a)-X_i(a)] + \lambda\sum_{j\in\mathcal{I}}\pi_j[X_j(a)-X_i(a)]$$

where

$$X_i(a) \equiv \mathbb{E}_i \left[e^{-rT} V_{s(T)}(a) \right]. \tag{10}$$

Employing the same method used to solve (8), we obtain:

$$X_i(a) = \frac{\alpha(1-\eta)}{r+\alpha(1-\eta)} \left[\frac{[r+\alpha(1-\eta)]V_i(a) + \lambda \sum_j \pi_j V_j(a)}{r+\alpha(1-\eta)+\lambda} \right]. \tag{11}$$

It is a discounted weighted sum of maximum attainable lifetime utilities while matched. When λ, the rate at which a preference shock occurs increases, more weight is put on the average value of being matched. When investors meet dealers more frequently or become more impatient (increases in α and r), more weight is put on the value of being matched *now*.

It follows that the expected discounted utility of the unmatched investor can be re-expressed as below

$$W_i(a) = U_i(a) + X_i(a). \tag{12}$$

The lifetime value of an unmatched investor is the sum of two components: the utilities an investor enjoys while unmatched (first term) and the utility from being matched at a later date (second term).

3.3 Asset Demands

We are now in position to obtain the demand for asset holdings of the matched investors. Differentiate the Bellman equation (12) to obtain the marginal benefit of one unit of asset for an unmatched investor:

$$W_i'(a) = U_i'(a) + \frac{\alpha(1-\eta)}{r+\alpha(1-\eta)} p.$$

It is the discounted sum of the utility flows until the investor has access to a dealer plus the expected discounted resale price. We substitute this expression into the first-order condition for the choice of asset holdings, (5), to obtain:

$$u_i'(a_i) = rp + \delta \left[\frac{rp}{r+\alpha(1-\eta)} - U_i'(a_i) \right]. \tag{13}$$

The optimal asset holdings are such that the instantaneous marginal utility of the asset is equal to the rental price of the asset, net of a term that captures the cost of being temporarily stuck with the asset when the trading relationship is severed at rate δ. This cost is equal to the difference between the expected resale price of the asset and the discounted sum of utility flows of that asset when the investor is unmatched. We substitute $U_i'(a)$ obtained from (9) to rewrite the individual demand for assets as:

$$\frac{(\lambda + r + \alpha(1-\eta) + \delta)\,(r+\alpha(1-\eta))u_i'(a_i) + \delta\lambda \sum_j \pi_j u_j'(a_i)}{(\lambda + r + \alpha(1-\eta))\,(r+\alpha(1-\eta)+\delta)} = rp. \tag{14}$$

The left side of (14) is strictly decreasing in a_i, it goes to $+\infty$ as a_i approaches 0 and to 0 as a_i goes to infinity. Hence, there is a unique $a_i > 0$ solution to (14) and it is decreasing in the asset price, p.

We can use (14) to study the effects of the stability of trading relationships on asset demands. It can be checked that the left side of (14) is a weighted average of the marginal instantaneous utility, $u_i'(a_i)$, and the expected marginal utility, $\sum_j \pi_j u_j'(a_i)$. The weight associated with the current utility is decreasing in δ

while the weight associated with the expected utility increases with δ. Hence, if $u_i'(a_i) > \sum_j \pi_j u_j'(a_i)$, then an increase in δ leads to a decrease in asset demand and vice-versa if $u_i'(a_i) < \sum_j \pi_j u_j'(a_i)$.

We can also check that the model admits as limiting cases both the model of Lagos and Rocheteau (2009) and the model of a frictionless asset market. Suppose first that $\delta \to +\infty$, i.e., matches with dealers are short lived. From (14), the asset demand is given by

$$\frac{(r + \alpha(1-\eta))u_i'(a_i) + \lambda \sum_j \pi_j u_j'(a_i)}{\lambda + r + \alpha(1-\eta)} = rp.$$

This expression corresponds to the asset demand in Lagos and Rocheteau (2009). At the opposite, suppose that, $\delta \to 0$. In that case,

$$u_i'(a_i) = rp.$$

The asset demand is the one of a frictionless market where the marginal utility of the asset is equal to the rental price of the asset.

3.4 Intermediation Fees

I now compute the intermediation fees incurred by investors in order to have access to a trading relationship. Substituting (12), the value of an unmatched investor, into (4), the HJB for a matched investor, and solving for V_i yields that

$$V_i = \frac{(\kappa + \lambda) Z_i(a_i)}{(r+\lambda)(\kappa + \lambda + \delta)} + \frac{[\kappa\lambda(\kappa+\lambda) + \delta\lambda\alpha(1-\eta)] \sum_j \pi_j Z_j(a_j)}{(r+\lambda)(\kappa+\lambda+\delta)(\kappa+\lambda) r}. \tag{15}$$

where $\kappa \equiv r + \alpha(1-\eta)$ and

$$Z_i(a) \equiv u_i(a) + \delta U_i(a) - \frac{r(\kappa+\delta)}{\kappa} pa.$$

Hence, from (12) and (15) we obtain a closed form solution for the surplus of a match, $V_i(a) - W_i(a)$. It is equal to the difference between the value of being matched net of fees and the value of being unmatched. Finally, the discounted sum of fees received by the dealer over a relationship is:

$$\Phi_i(a) = \eta[V_i(a) - W_i(a)]. \tag{16}$$

It is a constant fraction η of the total surplus from a relationship.

3.5 Distribution of Investors Across States

I now turn to the distribution of investors across states. I denote n^m the measure of matched investors and $n^u = 1 - n^m$ the measure of unmatched investors. In a steady state, the flow of new relationships is αn^u while the destruction of existing relationships is δn^m. Hence, the steady-state measure of relationships is

$$n^m = \frac{\alpha}{\alpha + \delta}.$$

I adopt the notation n_{ji}^s to denote the measure of investors with match status $s \in \{u, m\}$ who holds a_j and has preference type i. Note that here I used the observation that in a steady state all investors must hold asset holdings corresponding to some preference type, i.e., the support of the distribution of assets holdings is $\{a_i\}_{i=1}^I$. Because matched investors can adjust their asset holdings instantly,

$$n_{ii}^m = \pi_i n^m = \frac{\alpha \pi_i}{\alpha+\delta} \text{ for all } i \in \mathcal{I} \tag{17}$$

$$n_{ji}^m = 0 \text{ for all } j \neq i. \tag{18}$$

Matched investors always hold assets corresponding to their preference type. Moreover, the distribution of preference types across matched investors corresponds to the invariant distribution, $\{\pi_i\}_{i=1}^I$. Among unmatched investors, the laws of motion of n_{ji}^u are given by:

$$\dot{n}_{ii}^u = \delta n_{ii}^m - \alpha n_{ii}^u + \lambda \pi_i \sum_{k \neq i} n_{ik}^u - \lambda(1-\pi_i) n_{ii}^u \quad \text{for all } i \in \mathcal{I}$$

$$\dot{n}_{ji}^u = \lambda \pi_i \sum_{k \neq i} n_{jk}^u - \left[\lambda(1-\pi_i)+\alpha\right] n_{ji}^u \quad \text{for all } j \neq i.$$

At a steady state, $\dot{n}_{ii}^u = \dot{n}_{ji}^u = 0$. We can use the observation that $\sum_k n_{jk}^u = \pi_j n^u$ to obtain:

$$n_{ji}^u = \frac{\lambda \pi_i \pi_j n^u}{\lambda+\alpha} = \frac{\delta \lambda \pi_i \pi_j}{(\lambda+\alpha)(\alpha+\delta)} \text{ for all } i \neq j \tag{19}$$

$$n_{ii}^u = \frac{\delta \pi_i n^m + \lambda \pi_i^2 n^u}{\alpha+\lambda} = \frac{\delta \alpha \pi_i + \lambda \delta \pi_i^2}{(\lambda+\alpha)(\alpha+\delta)} \text{ for all } i \in \mathcal{I} \tag{20}$$

3.6 Market Clearing and Equilibrium

It is possible to characterize market clearing in terms of flows. The measure of investors who have access to the market at a given point in time is n^m. The average quantity of assets held by matched investors is A^m while the average quantity of assets held by unmatched investors is A^u. In the steady state, $\delta n^m A^m = \alpha n^u A^u$ implies $A^m = A^u = A$. Hence, the flow of matched investors who receive a preference shock bring to the market $\lambda n^m A$ units of asset. By the law of large numbers, among these investors, a fraction π_i are of type i. Hence, they demand $\lambda n^m \sum_i \pi_i a_i$. It implies that market clearing requires

$$\sum_{i \in \mathcal{I}} \pi_i a_i = A. \tag{21}$$

The right side is the fixed asset supply. The left side of (21) is decreasing in p, from $+\infty$ when $p=0$ to 0 when $p=+\infty$. Hence, there is a unique p solution to (21). We are now in a position to define an equilibrium which can be characterized recursively.

Definition 1 *A steady-state equilibrium of the OTC market with trading relationships is the following list of objects,* $\{a_i\}_{i=1}^I$, $\{n_{ji}^s\}_{(j,i)\in\{1,\ldots,I\}^2, s\in\{m,u\}}$, $\{\Phi_{ji}\}_{(j,i)\in\{1,\ldots,I\}^2}$, p, *solution to the following. Given the asset demands in (14), the market clearing condition, (21), gives both* p, *and the support of the distribution of asset holdings,* $\{a_i\}_{i=1}^I$. *The distribution of investors across states is given by (17)-(18) and (19)-(20). Finally, the trading costs,* $\Phi_{ji} = \Phi_i(a_j)$, *are obtained from (16).*

3.7 Some Special Cases

Linear utility Suppose the flow utility from holding an asset is $\varepsilon_i a$ with $\varepsilon_1 < \varepsilon_2 < \ldots < \varepsilon_I$. For this specification, we need to allow for corner solutions for the choice of asset holdings. It is easy to show that only investors with the highest preference type will want to hold assets, i.e., $a_1 = ... = a_{I-1} = 0$ and $a_I > 0$. From (14), the asset price solves

$$p = \frac{[r + \lambda + \alpha(1-\eta) + \delta]\,[r + \alpha(1-\eta)]\,\varepsilon_I + \delta\lambda\bar{\varepsilon}}{r\,[r + \lambda + \alpha(1-\eta)]\,[r + \alpha(1-\eta) + \delta]},$$

where $\bar{\varepsilon} = \sum_j \pi_j \varepsilon_j$. Using that $\varepsilon_I > \bar{\varepsilon}$, it follows that $\partial p/\partial\delta < 0$. So as trading relationships become more stable, the asset price increases. Each matched investor of type I holds A/π_I. Hence, the volume of trade is

$$\mathcal{V} = (n^m \pi_I)\,\lambda(1-\pi_I)\frac{A}{\pi_I} = \frac{\alpha}{\alpha+\delta}\lambda(1-\pi_I)A.$$

The volume of trade increases as more investors are matched.

Linear utility with satiation Suppose the utility takes the form $u_i(a) = \varepsilon_i \min\{1, a\}$. The investor has a constant marginal utility for the asset until asset holdings reach $a = 1$ after which the marginal utility is zero. We now assume that ε has a continuous cumulative distribution, $F(\varepsilon)$. The mean of the distribution is $\bar{\varepsilon}$. There is a threshold, ε_R, above which investors desire to hold one unit of the asset. It satisfies:

$$p = \frac{[r + \lambda + \alpha(1-\eta) + \delta]\,[r + \alpha(1-\eta)]\,\varepsilon_R + \delta\lambda\bar{\varepsilon}}{r\,[r + \lambda + \alpha(1-\eta)]\,[r + \alpha(1-\eta) + \delta]}.$$

Market clearing requires

$$1 - F(\varepsilon_R) = A.$$

The share of investors who desire to hold the asset is equal to the asset supply. We can rewrite this condition as

$$\varepsilon_R = F^{-1}(1-A).$$

It follows that $\partial p/\partial\delta > 0$ if $A > 1 - F(\bar{\varepsilon})$ and $\partial p/\partial\delta < 0$ if $A < 1 - F(\bar{\varepsilon})$.

Logarithmic utility Suppose now that $u_i(a) = \varepsilon_i \ln a$. From (14), the asset demand of a type i investor solves:

$$a_i = \frac{[r + \lambda + \alpha(1-\eta) + \delta]\,[r + \alpha(1-\eta)]\,\varepsilon_i + \delta\lambda\bar{\varepsilon}}{[r + \lambda + \alpha(1-\eta)]\,[r + \alpha(1-\eta) + \delta]\,rp}.$$

It follows that $\partial a_i/\partial\delta < 0$ if $\varepsilon_i > \bar{\varepsilon}$ and positive otherwise. So asset holdings become more dispersed when trading relationships are more stable. From (21), the asset price solves

$$p = \frac{\bar{\varepsilon}}{rA}.$$

The asset price is independent of trading frictions, α, bargaining power of dealers, η, and the stability of relationships, δ. So, trading relationships affect the volume of trade but not asset prices.

3.8 Liquidity Measures

To help gauge market liquidity in the model, I look at multiple dimensions of liquidity such as trade sizes, trade volume, and transaction costs. The measures used are discussed below.

Trade Volume Investors can be classified as either mismatched or not with respect to their portfolios. Mismatched investors have assets that are not in line with their preference type. In contrast, investors who are not mismatched are satisfied with their current asset holdings. In addition to trading contingent on a preference shock, mismatched investors will also engage in *realignment* trades upon forming a relationship. Thus, volume of trade can be expressed as

$$\mathcal{V} = \alpha \sum_{i,j} n_{ji}^{u} |a_i - a_j| + \lambda \sum_{i,j} n_{ii}^{m} \pi_j |a_j - a_i|.$$

It is the sum of two components. The first is the sum of all realignment trades and the second is the trading that takes place due to the arrival of preference shocks while in a relationship.

Proposition 2 *When investor preferences are represented by $u_i(a) = \varepsilon_i ln(a)$, trading volume exhibits the following characteristics: $\partial \mathcal{V}/\partial \delta \leq 0$ and $\partial^2 \mathcal{V}/\partial \delta^2 \geq 0$.*

When relationships are less stable, not only are there fewer investors who are able to trade, but the ones who do transact do so in smaller quantities. These two effects taken together imply that trading volume is a convex function of relationship instability.

Trading Fees per Unit of Time and Asset Traded I use trading costs paid *per unit of asset traded* to resemble more closely a bid-ask spread that one might observe in financial markets. Denote $\mathcal{V}_{ji}$ as the volume of trade attributable to an investor of type i with asset holdings j and note that

$$\frac{\Phi_i(a_j)}{\mathcal{V}_{ji}}$$

represents the average deviation from the interdealer price that an investor pays. To construct my measure of transaction costs, I weight the individual intermediation fees paid per unit of asset traded by the fraction of total volume an investor accounts for then I sum over all investors.[8] Hence, the measure of trading fees I use, commonly referred to as the effective half-spread, is

$$\mathcal{F} = n^m \sum_{i,j} \frac{f_{ij} \Phi_i(a_j)(r+\delta)}{\mathcal{V}_{ji}} \frac{\mathcal{V}_{ji}}{\mathcal{V}} = n^m \frac{\sum_{i,j} f_{ij} \Phi_j(a_i)(r+\delta)}{\mathcal{V}}$$

where $f_{ij} \equiv n_{ij}^u / n^u$ is the fraction of matched investors who negotiated a fee payment of $\Phi_j(a_i)$ for access to a relationship and hence pay $\Phi_j(a_i)(r+\delta)$ per unit of time.[9] One can think of $\mathcal{F}$ as the average fee a dealer receives per unit of intermediation he provides to the market.

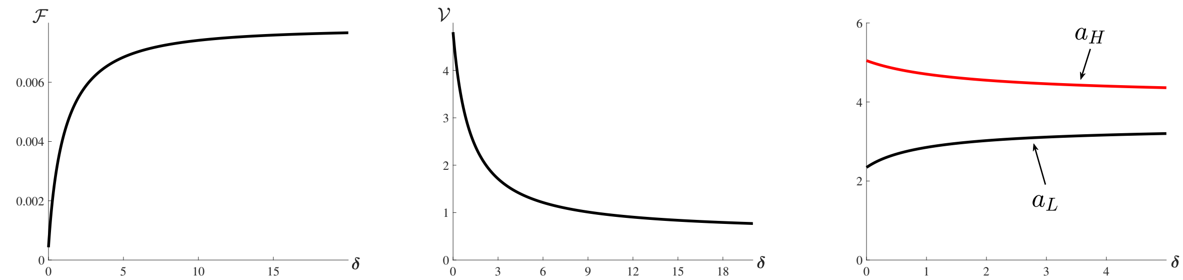
Proposition 3 *Assume $u_i(a) = \varepsilon_i ln(a)$, then* (i) *$\partial \Phi_i(a)/\partial \delta$ approaches 0 as δ approaches ∞* (ii) *$\partial \mathcal{F}/\partial \delta > 0$ for δ sufficiently large.*

As relationships are made fully unstable, the aggregate measure of transaction costs rises. It implies that more unstable relationships make it more costly for investors to trade.

[8] Appendix D considers an alternative measure of transaction costs and shows that they are qualitatively equivalent.

[9] When analyzing steady state measures of trading fees, we may encounter situations where it appears as if no fees are being paid (constituting a highly liquid market), when it is solely an artifact of the equivalence between the discounted sum of fees and the flow payments. To avoid these potential missteps, I re-express the discounted sum of intermediation fees as a payment per unit of time.

Exploiting the fact that the model is solved in closed form, I use the following parametrization for numerical examples.[10] Preferences of investors are given by $u_i(a) = \varepsilon_i a^{1-\sigma}/(1-\sigma)$, $r = 0.05$, $\sigma = 3$, and binary valuation types $\varepsilon_l = 1$ and $\varepsilon_h = 10$ with associated probabilities $\pi_l = 0.667$ and $\pi_h = 0.333$. Investors contact dealers at rate $\alpha = 3$ and receive preference shocks at rate $\lambda = 4$. Dealer bargaining power is set to $\eta = 0.5$. The supply of assets is normalized to $A = 1$.



(a) Market-Wide Trading Costs (b) Trading Volume (c) L-type and H-type Portfolios

Figure 1: Effects of a Change in Relationship Stability

The key parameter in this model is δ, the volatility of relationships. A high value indicates short lived relationships and vice versa. Panel 1(a) plots $\mathcal{F}$ as a function of relationship volatility. Changing δ has three distinct effects on trading costs. The first effect has to do with how relationship volatility affects match surpluses. Shorter lived relationships generate less value for investors, since they cannot enjoy their connection status for as long, thereby leading to a reduction in the discounted sum of fees paid to the dealer. This first effect decreases transaction costs. Second, making relationships shorter-lived decreases the number of matched investors who pay fees in the first place. This effect on the distribution of investors decreases trade volume (panel 1(b)) and total intermediation fees paid. The third distinct effect from changing δ is concerned with the dispersion of asset holdings. As relationships are shorter lived, investors put more weight on their average preference type when choosing their portfolio. Panel 1(c) shows how the distribution of asset holdings approaches an average level as relationships are short-lived. Investors use their portfolios to limit trading needs as relationships are more unstable. This type of liquidity hedging reduces volume and increases trading fees per unit of asset traded.

4 Free Entry of Dealers

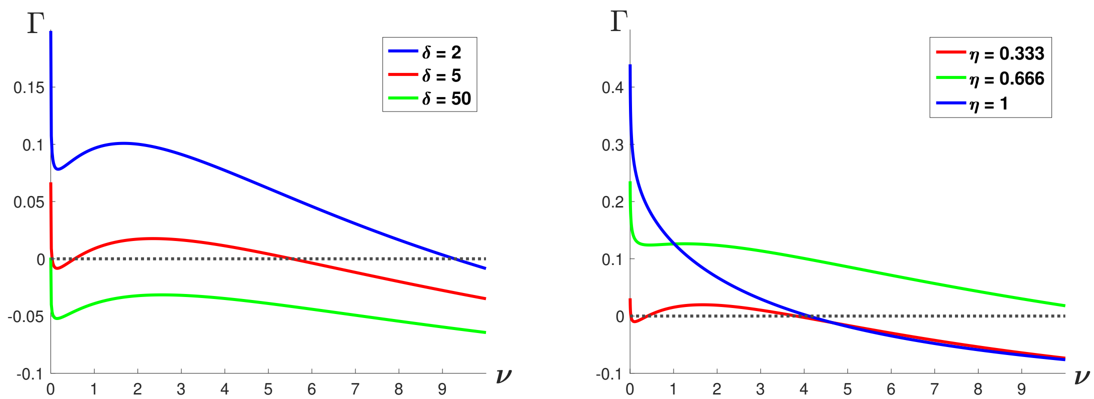
Dealers' decisions to provide their intermediation services cannot be taken as given. In this section, I am concerned with understanding how relationship stability impacts a dealer's decision to make markets. I assume that investors rate of contact varies with the amount of dealers presently active in the market. Specifically, denote $\alpha(\nu)$ as the (endogenous) rate at which investors form relationships which depends

[10]The parameter values are taken from the numerical exercises in Zhang (2018) who calibrate certain parameters themselves and also borrow from existing literature.

critically on the measure ν of active dealers.[11] I assume that $\lim_{\nu \to 0} \alpha(\nu) = 0$ and $\lim_{\nu \to \infty} \alpha(\nu) = \infty$. Furthermore, assume dealers pay a flow cost γ to operate in the market. The remaining characteristics of the environment remain unchanged from Section 3.

Denoting dealer profits as $\Gamma \equiv \alpha(\nu)/\nu \cdot \sum_{i,j} n_{ij}^u \Phi_i(a_j) - \gamma$, the free entry condition implies that $\Gamma = 0$. A particular dealer is contacted at rate $\alpha(\nu)/\nu$ and earns $\sum_{i,j} n_{ij}^u \Phi_i(a_j)$ on average. The difference between dealer revenues and their operating cost, γ, must be zero in equilibrium.

The following graphs were obtained with the subsequent parameter values: $u_i(a) = \varepsilon_i ln(a)$, $r = 0.1$, $\eta = 0.5$, $\delta = 2$, $\lambda = 1$, $A = 1$, $\pi_L = 0.667$, $\pi_H = 0.333$, $\varepsilon_L = 1$, $\varepsilon_H = 4$, $\alpha(\nu) = 5\nu^{0.9}$.



(a) Dealer Profits for Varying Stability of Relationships

(b) Dealer Profits for Varying Bargaining Powers

Figure 2: Multiplicity of Equilibria

Relationship stability affects the number of steady-state equilibria. For example, when relationships are short lived (high δ), there can exist a unique 'low liquidity' equilibrium that is characterized by a small number of dealers present in the market, low asset dispersion, low volume of trade, and high transaction costs. The high degree of relationship instability does not provide dealers with enough incentive for them to provide their services. Instead, few dealers enter and capture the limited amount of fees investors are willing to pay for short-lived relationships thereby creating an illiquid market.

As relationships are rendered more stable (decrease in δ), multiplicity of equilibria appears. A larger measure of dealers has two effects. First, it reduces the probability a dealer will be contacted by an investor which tends to decrease per-dealer profits. Second, an increase in ν causes an increase in the rate at which investors contact dealers. This causes investors to choose larger portfolio sizes tending to increase fees, via larger gains from trade, resulting in higher per-dealer profits. These two opposing effects generate the non-monotone behavior of dealer profits giving rise to multiple equilibria: a low-liquidity, intermediate-liquidity, and high-liquidity state. Moving from low to high liquidity equilibria, we find increases in the measure of

[11] This formalization is adopted from Lagos and Rocheteau (2007). Presumably, if there are many dealers, investors will have larger contact rates while if there are few dealers in the market, investors will make contact with dealers more infrequently.

active dealers, dispersion of portfolios, and trade volume in addition to reductions in trading fees.

In a market where relationships are long-lived, the uniqueness of equilibria is recovered but with more favorable liquidity properties compared to the unique equilibrium under short-lived relationships. It is characterized by high trading volume and asset dispersion, a large number of dealers, and low trading fees. Since trading relationships are long lived, investors' gains from entering a relationship are large. Accordingly, many dealers enter to capture the existing profits. In this way, relationship stability is both a tool that incentivizes dealers to make markets, and a mechanism that can be used to coordinate on higher liquidity equilibria.

5 Spot Trading vs. Relationship Trading

To account for the possibility of non-relationship investor-dealer trades, I assume that an investor may contact a unit measure of two types of dealers: Relationship Dealers (RD) that trade exclusively via relationships and Spot Dealers (SD) who engage only in one off transactions. I allow for RD and SD to have different bargaining powers which we denote η and η_s, respectively. As before, RD are contacted at rate α by investors upon which a relationship is formed. Novel to this section is the arrival of trading opportunities with SD at a Poisson arrival rate, α_s. This model setup captures the idea that investors have more than one way in which to trade.[12] Since many derivations are identical in nature to those of Section 3 (up to the addition of some new parameters to describe spot transactions) they are relegated to Appendix D.

5.1 Bargaining Problem (Spot Transactions)

Spot trades are formalized in a similar fashion to relationships with two important differences. First, the assets acquired by the dealer on behalf of the investor will be a one-time acquisition, not an asset path. Second, the intermediation fee paid to the dealer will be a one-time fee, not a discounted sum of fees. I assume the generalized Nash bargaining Solution is used.

An investors' surplus from spot trading is the capital gain on her lifetime utility net of the price she pays to readjust her portfolio. The capital gain is given by $W_i(a') - W_i(a)$, where a' denotes her new asset holdings, and the price she pays for the readjustment is $p(a' - a) - \phi_i(a)$, where $\phi_i(a)$ is the one-time intermediation fee the dealer receives. The outcome of bargaining is given by

$$a_i^s = \arg\max_{a'} \{W_i(a') - pa'\} \tag{22}$$

$$\phi_i(a) = \eta_s [W_i(a_i^s) - W_i(a) - p(a_i^s - a)]. \tag{23}$$

where a_i^s denotes an asset position acquired from a spot trade. An investor chooses a portfolio that equates

[12] There often exists parallel markets for many assets: an OTC style market where relationships prove to be important and a Limit-Order-Book market where trading occurs all-to-all with anonymity. See relevant evidence in the market for corporate bonds (Hendershott and Madhavan (2015)), treasury securities (Barclay, Hendershott, and Kotz (2006)), and foreign exchange contracts (Holden et al. (2021)). I view the RD in this section as the OTC market and SD as the more centralized, spot trading LOB market. Alternatively, we could view this setting as a single OTC market where a certain number of dealers are willing to form relationships (core) and other dealers prefer to only engage in spot transactions (periphery).

the marginal value of being unmatched to the price of the asset. Fees paid to the dealer are a constant fraction η_s of the surplus that is created from trade.

5.2 Spot Trade Asset Demands

I now turn to the determination of asset demands by investors in spot trades. From (22) the FOC for spot trade asset demands is given by

$$W_i'(a') = p.$$

It equates the marginal benefit of the asset for an unmatched investor to the price of the asset. Differentiating $W_i(a)$, which can be found in Appendix D, and substituting it into the above equation yields the following asset demand equation

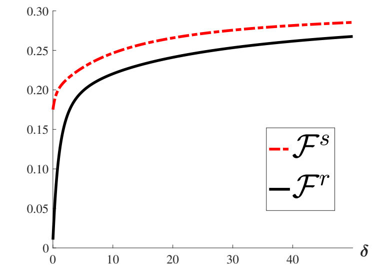
$$\frac{[r + \alpha(1-\eta) + \alpha_s(1-\eta_s)]u_i'(a') + \lambda \sum_j \pi_j u_j'(a')}{\lambda + r + \alpha(1-\eta) + \alpha_s(1-\eta_s)} = rp. \tag{24}$$

Noticeably, δ does not appear in (24). It implies that relationship volatility only affects the spot trading asset decision through potential effects via the interdealer price, p. Whereas increasing δ means putting more weight on future marginal utilities for relationship trades, it has no effect on the marginal benefit for spot traders. The only parameters that affect how much weight is allocated to current versus future marginal utilities are the bargaining-adjusted arrival rates of trading opportunities, $\alpha(1-\eta)$ and $\alpha_s(1-\eta_s)$, the rate at which preference shocks arrive, λ, and the rate of time preference, r. An important determinant of market liquidity will be how different the portfolios chosen by spot traders are from the portfolios chosen for relationship trades.

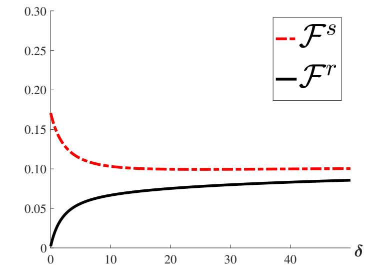
5.3 Trading Volume and Transaction Costs

I distinguish between liquidity measures used for relationships and spot trades to compare both types of trading arrangements. In particular, $\mathcal{V}^r$ and $\mathcal{F}^r$ denote our measures of relationship trade volume and relationship transaction costs, respectively, which remain largely unchanged compared to Section 3 while $\mathcal{V}^s$ and $\mathcal{F}^s$ are the analogous measures for spot transactions and are defined in Appendix D. Functional forms and all parameter values already specified remain unchanged. Investors meet spot dealers at rate $\alpha_s = 6$ and I consider symmetric RD and SD bargaining powers so that $\eta = \eta_s = 0.5$. I consider 3 different 'regimes' which differ in the intensity of preference shocks.[13] Regime 1 is such that investors rarely need to trade with $\lambda = 1.5$, Regime 2 exhibits an intermediate need to trade with $\lambda = 15$, and Regime 3 resembles a market where investors change preference types frequently with $\lambda = 150$.

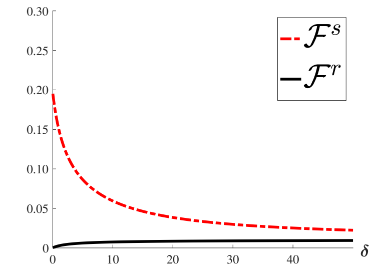
[13]Different asset markets exhibit varying needs to trade. For example, corporate bonds often trade infrequently and are many times held to maturity as opposed to treasury securities which are traded frequently for a variety of purposes.



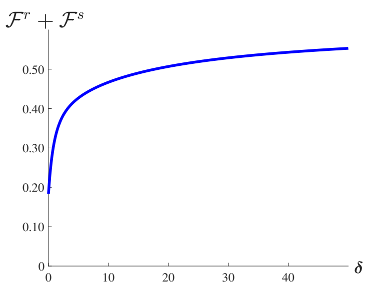
(a) Fees by Trade Type (Regime 1)



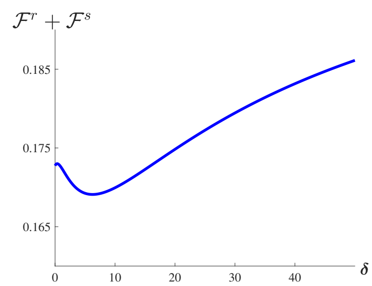
(b) Fees by Trade Type (Regime 2)



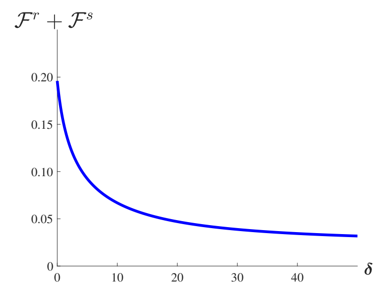
(c) Fees by Trade Type (Regime 3)



(d) Total Market Fees (Regime 1)



(e) Total Market Fees (Regime 2)



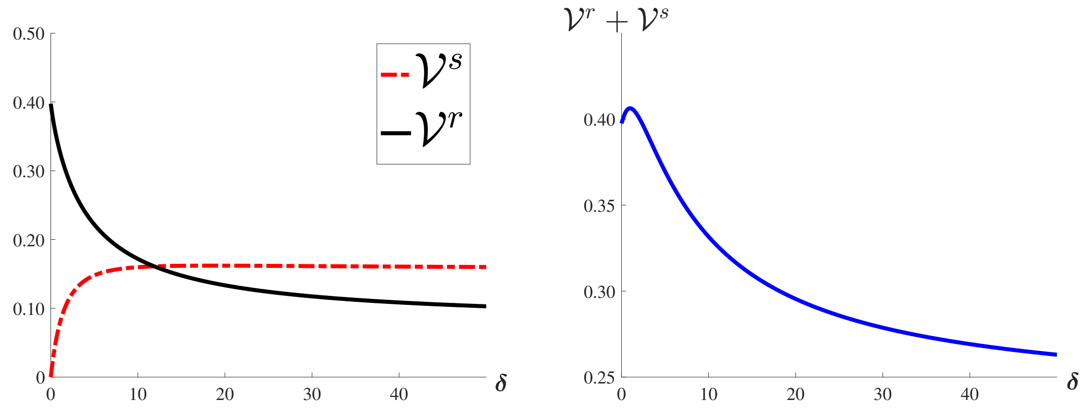
(f) Total Market Fees (Regime 3)

Figure 3: Effects of Relationship Stability on Trading Costs Under Regimes 1-3

As relationships become increasingly volatile, trading costs per unit of asset traded for both spot trades and relationships converge to the same level. The intuition being that in the limit as $\delta \to \infty$, relationships are so short lived that they are not any different from spot trades. In fact, we see from (56) that taking the limit as $\delta \to \infty$ yields the asset demand equation for spot trades, (24).

The next important observation is with respect to the levels of spot fees and relationship fees. We see clearly that per unit of asset traded, relationship fees are bounded above by spot trading costs. On average, trading vis-à-vis a relationship is more liquid than trading via spot transactions along this dimension. The difference in trading costs between both arrangements is exacerbated as investors trade more frequently, with the connection status of matched investors becoming increasingly important.

The third, and perhaps most important, observation is that the slope of spot trading fees per unit of asset traded can be either positive or negative leading to potentially non-monotone behavior of market-wide trading costs. This is driven by the fact that as preference shocks are increasingly frequent, relationship portfolios will approach those levels of spot portfolios. As a result, the gains from trade from a spot transaction will be lower, on average, since the portfolio of a mismatched investor will likely already be close to the desired level for a spot transaction. Increasing the arrival of destruction shocks amplifies this phenomenon.



(a) Spot and Relationship Volume

(b) Total Market Volume

Figure 4: Effects of Relationship Stability on Trading Volume

As before, relationship trading volume is diminished as δ increases for two reasons. The number of agents who are able to trade declines in addition to the quantities they wish to trade. The trade volume for spot transactions on the other hand has the opposite effect. Increasing the arrival rate of destruction shocks means that there are more unmatched investors in the market. From time to time, these unmatched investors will be able to contact a spot dealer and engage in a one-off trade. The amount of spot trades happening in the market is increasing in δ. As a result, as relationships are more unstable, spot traders engage in more trades of the same quantities. These two opposing effects yield non-monotone behavior of total market-wide trade volume, the sum of spot and relationship trade volume.

6 Endogenous Relationships

Trading relationships are costly to form (Hendershott, Li, et al. 2020). As a result, investors maintain finite trading networks keeping only the most beneficial relationships. In practice, counterparties are chosen so as to offset trading needs or risk profiles (Afonso, Kovner, and Schoar 2013). Investors who are more inclined to purchase a certain class of assets will pair themselves with dealers who more often than not hold those assets in their inventories. This implies that not all relationships are created equal.

I formalize this notion by assuming investors incur a match specific flow cost to maintaining a relationship.[14] I denote by $\mathcal{X}$ the flow cost borne by investors in relationships. Once the cost is drawn, it remains constant for the duration of the relationship. If the investor chooses not to form a relationship, they still retain the opportunity to engage in a spot trade with the dealer. This is in contrast to section 5 where the investor's choice of trading arrangement, spot or relationship, was exogenous.

[14]Note that this formalization is identical to describing a flow benefit to being in a relationship so that investors drawing low benefits would be equivalent to incurring high costs. The cost need not be thought of as purely a monetary or opportunity cost but also captures the extent to which a particular relationship is a good match.

6.1 Bellman Equations

The lifetime value of an investor currently in a relationship is again $V_i(a) = pa + V_i$ where V_i solves the following HJB equation instead

$$rV_i = \max_{a'} \left\{ u_i(a') - rpa' - \chi + \delta\big[W_i(a') - V_i(a')\big] + \lambda \sum_{j \in \mathcal{I}} \pi_j \big[\max\left\{V_j,\ \Omega_j\right\} - V_i\big] \right\} \tag{25}$$

where $\Omega_i \equiv \max_{a''}\{W_i(a'') - pa''\}$. Matched investors choose their portfolio optimally at every point in time. They receive flow utility net of the cost of acquiring the asset, incur a flow cost to maintain the relationship, are unmatched at rate δ and receive preference shocks at rate λ. Whenever an investor switches to a new preference type, they re-evaluate the cost of maintaining the relationship against the cost of accessing the asset market infrequently. If accessing the market infrequently is less costly to the investor than staying matched, the investor will engage in a final trade, choosing assets that will be optimal for her spell as an unmatched individual, and terminate the relationship. Importantly, since investors can trade when they receive the preference shock, this decision is independent of asset holdings.

Proposition 4 *There exists a unique reservation cost χ_i^* that makes investors indifferent between forming relationships and spot trading. It is defined implicitly such that $V_i = \Omega_i$.*

By definition, χ_i^* makes investors indifferent between forming relationships and spot trading. Since $\partial V / \partial \chi < 0$, it follows that for $\chi > \chi_i^*$ investors prefer to spot trade while for $\chi \leq \chi_i^*$, investor will choose to form a relationship.

The difference for unmatched investors in this environment is that relationship formation, as well as the choice to trade via spot or relationship transactions, is endogenous. The changes are incorporated below. Using the notion of the reservation cost, χ_i^*, we can write the HJB as

$$rW_i(a) = u_i(a) + \lambda \sum_{j \in \mathcal{I}} \pi_j [W_j(a) - W_i(a)] + \alpha(1-\eta)\big[\max\{V_i, \Omega_i\} + pa - W_i(a)\big]. \tag{26}$$

Unmatched investors enjoy some flow utility, receive preference shocks at rate λ, and contact dealers at rate α after which they obtain a fraction $(1-\eta)$ of the joint surplus from whichever trading arrangement they choose (spot trade or relationship). Investors choose to form a relationship if the match specific cost they draw is less than their reservation threshold. Otherwise, they spot trade. Regardless of the trading arrangement chosen, investors offload their current portfolios generating some wealth pa and incur a capital gain in lifetime value. This is captured by the last two terms in the square brackets.

Asset demands remain identical to those in Section 5. The match-specific cost affects the decision of whether to form or destroy a relationship, but has no direct effect on the asset positions that are chosen.

6.2 Calibration

I calibrate the model to match a variety of moments from the inter-dealer municipal bond market and choose a unit of time to represent one month.[15] The rate of time preference is set to 5% per year, i.e., $r = 0.05/12$. The supply of assets is normalized to $A = 1$. I set the number of preference types to $I = 30$ and assume that the ε_i are equally spaced between the 0^{th} and 90^{th} percentiles of a Generalized Pareto distribution $G(\mu, \zeta, \beta)$ with location parameter μ, shape parameter ζ and scale parameter β.[16] The π_i are chosen so as to create a valid probability distribution (details are left to Appendix C). Furthermore, investors' utility function is $u_i(a) = \varepsilon_i a^{1-\sigma}/1 - \sigma$.

There are 9 parameters that I jointly calibrate to minimize the sum of squared deviations of 13 model moments from their data counterparts. The calibrated parameters are χ, α, δ, σ, λ, η, and the three parameters governing the distribution of valuations: μ, ζ, and β. The model is solved numerically and the procedure is detailed in Appendix C. Expressions for all targeted model moments can also be found there.

Municipal bonds outstanding at the end of 2017 totalled \$3.948 trillion (SIFMA 2024) with inter-dealer trading volume totaling \$530.561 billion (MSRB 2018). These figures imply a monthly inter-dealer turnover (volume as a percentage of supply) of 1.12%. Wu (2018) uses data from 2017 and reports an estimate for the average interdealer effective spread (expressed as a percentage of the midpoint price) of 76.6 basis points.[17] I use this estimate as my target for the average effective spread paid in my model. The MSRB (2018) Fact Book also reports that 2017 saw 3.826 million interdealer transactions. Furthermore, Clowers (2012) reports that there exists approximately 1.138 million unique municipal securities. Taken together, these figures imply that the average bond is traded 0.2801 times per month. Together with the figure for asset turnover, the number of trades per security per month imply that on average, each trade is roughly 4% of the outstanding asset supply.

Li and Schürhoff (2019) calculate a dealer relationship stability transition matrix using data from 1998 to 2012. They find that conditional on trading together in a given month, the probability that two dealers maintain a relationship with each other in the following month is 66%. It implies a monthly relationship separation probability of 34%. This probability is analogous to the model's endogenous separation probability which is computed as the average probability, conditional on being matched, that at least 1 destruction shock arrives or at least one preference shock arrives such that the relationship is terminated. Similarly, the same transition matrix also reports that new relations are formed from one month to the next with a probability

[15] I choose to target the interdealer portion of the municipal bond market since transaction level data is more readily available for inter-dealer markets as opposed to customer to dealer segments. As a result, moments on relationship stability, a key feature of the model, are more readily available for interdealer markets.

[16] The choice of distribution is not crucial for matching moments related to transactions cost or trade volume. However, when the distribution of valuations is not heavily skewed, the model has difficulty in matching the distribution of trade sizes. Intuitively, bond markets have many participants who trade in small quantities, and a few participants who trade in very large quantities.

[17] The data used by Wu (2018) only includes interdealer trades of less than \$100k. However, a majority of interdealer trades ($\approx$ 83%) fall within this category. Thus, while imperfect, the spread represents an sensible measure of transactions costs for that year in the interdealer market.

of 2%. This serves as the counterpart for the endogenous relationship formation probability given by the model.

The remaining moments I choose to target are aimed at capturing features of trade size heterogeneity. The MSRB (2018) reports the number of inter dealer trades that fall within 8 categories of trade sizes which I will call categories 1-8 for simplicity.[18] I then re-express these categories as fractions of the largest trade size to obtain a distribution of normalized trade sizes comparable to the calibration exercise done in Lester, Rocheteau, and Weill (2015). The fraction of interdealer trades that fall within each category is reported in Table 1.

A summary of the calibrated parameter values and model fit is provided in Table 1. The calibrated value for the contact rate implies that unmatched dealers meet other dealers approximately every 1.1 trading days which is in line with estimates commonly found in the literature.[19] Dealers who initiate the transaction appear to receive a small share of the trade surplus, as indicated by the high bargaining power parameter. Intuitively, if dealers contact each other quickly, the model requires a large bargaining power to match the moments on transaction costs. In other words, if the contact rate is very large, to match the data, the model requires a smaller bargaining-adjusted contact rate $\alpha(1-\eta)$. The model does well in matching moments closely, but has some difficulty in matching exactly the distribution of trade sizes.

Distribution of Normalized Trade Sizes Figure 5 reports the empirical distribution of normalized trade sizes and its theoretical counterpart. The model does well in roughly matching the empirical distribution but underestimates slightly the proportion of large sized trades. The average normalized trade size in the model is 1.15% which is smaller but of comparable magnitude compared to the average normalized trade size seen in the data of 2.59%.

Spreads The model also predicts effective spreads conditional on being matched and unmatched. The effective spread for relationship trades is 69.35bps while for spot transactions it is 108.91bps. The difference in levels between the two effective spreads appears of reasonable magnitude and is in line with empirical evidence that trading with relationship dealers is less expensive. For example, Di Maggio, Kermani, and Song (2017) find an inter-dealer relationship discount of approximately 20bps in the market for corporate bonds. The difference in the level thus seems of a reasonable magnitude and would be interesting to confirm empirically.

Who Forms Relationships? The calibrated model also speaks to what types of investors choose to form relationships. Figure 6 plots $V_i - \Omega_i$ for each preference type for the calibrated set of parameters. A positive value indicates that a dealer endogenously chooses to form a relationship while a negative value indicates

[18]The 8 categories of reported trade sizes are $0 - $25k, $25k - $50k, $50k - $75k, $75k - $100k, $100k - $500k, $500k - $1mm, $1mm - $2mm, and $2mm+.

[19]See for a reference the calibration section of Hugonnier, Lester, and Weill (2020).

Parameter	**Notation**	**Calibrated Value**
CRRA coefficient	σ	1.358
Destruction Shock Arrival Rate	δ	0.288
Preference Shock Arrival Rate	λ	0.306
Relationship Flow Cost	χ	0.0428
Contact Rate	α	19.04
Bargaining Power	η	0.9979
Preference Type Distribution Shape Parameter	ζ	2.19
Preference Type Distribution Scale Parameter	β	1.23
Preference Type Distribution Location Parameter	μ	2.625
Endogenous Variable	**Target**	**Model Value**
Effective Spread	76.60bps	76.63bps
Asset Turnover	1.119%	1.118%
Relationship Separation Probability	34.00%	34.30%
Relationship Formation Probability	2.00%	1.89%
Trades per Security	0.2813	0.2818
Percent of Trades in Category 1	50.12%	50.02%
Percent of Trades in Category 2	20.36%	23.03%
Percent of Trades in Category 3	4.83%	6.53%
Percent of Trades in Category 4	7.69%	5.36%
Percent of Trades in Category 5	12.92%	13.66%
Percent of Trades in Category 6	2.19%	1.00%
Percent of Trades in Category 7	1.01%	0.29%
Percent of Trades in Category 8	0.87%	0.11%

Table 1: Calibrated Parameters and Model Fit

that a dealer instead chooses spot transactions.

The Value of Relationships To quantify the value that trading relationships add to the inter-dealer municipal bond market, I shut down the ability of agents to form relationships and study the resulting impact on various endogenous measures of liquidity. This experiment can be thought of as taking δ or χ to infinity. Table 2 reports the results. One interpretation of such an extreme shift in market structure would be the adoption of fully anonymous trading protocols such as Alternative Trading Systems (ATS), which have risen in popularity in recent years in the interdealer portion of the market (Wu 2021).

Endogenous Variable	**Model with Relationships**	**No Relationships**	**Percent Change**
Effective Spread	76.63bps	166.59bps	117.39%
Asset Turnover	1.118%	0.675%	-39.62%
Trades per Security	0.2818	0.2793	-0.89%
Welfare	-8.8711	-8.8744	-0.037%

Table 2: Value of Relationships

When relationships cannot be formed, agents trade less (both in number of trades and quantity traded) and do so at wider spreads. The mechanisms at play remain identical to those described earlier in the

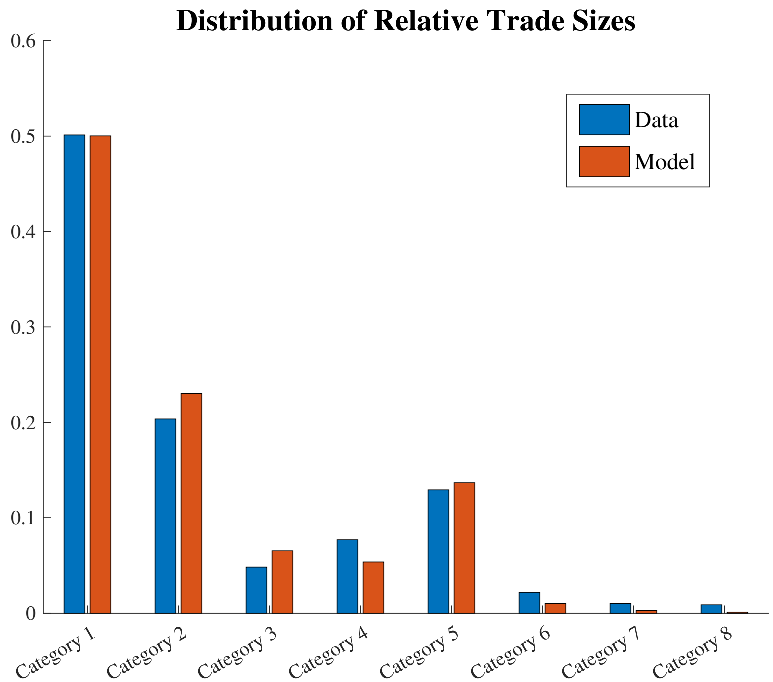


Figure 5: Distribution of Normalized Trade Sizes

paper: relationships partially solve a hold-up problem that increases the size of investor trades. The model predicts that spreads rise by approximately 31 basis points, while asset turnover and the number of trades per security fall by 1.5% and 0.818, respectively. These figures imply that eliminating relationships corresponds to a 40.85% increase in spreads, and decline in turnover and trades per security of 9.38% and 79.18% respectively. Of course, these results are conditional on other parameters remaining fixed at their calibrated values. To address this concern, I look at how much certain parameters would need to change for market liquidity to remain constant if relationships were eliminated. Thus, one takeaway of the following results is to say that any policies that may eliminate trading relationships would need to improve liquidity along other dimensions to prove beneficial.

Policy Counterfactual I examine the effects of scaling down the distribution of relationship specific maintenance costs on the model's endogenous variables. One example of such a parametric shift would be a policy aimed at matching natural counterparties. For any given bond, certain dealers may be frequent buyers while others may be frequent sellers. It would seem natural that these two dealers should form a trading relationship since those demanding liquidity would be matched with those supplying it. However, these potential counterparties may be hard to locate and relationships may be formed with less ideal trading partners instead, or not at all. A trade mechanism known as match auctions that attempts to resolve similar issues and has been proposed for use in treasury markets (Chaboud et al. 2022).[20]

[20] One important difference being that match auctions are currently anonymous trading protocols, which would not allow for the formation of trading relationships.

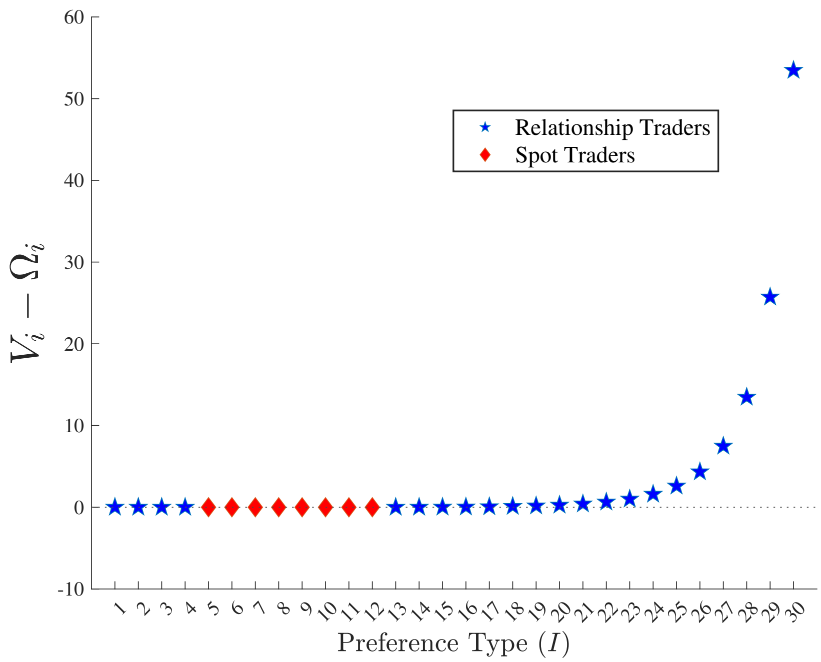


Figure 6: Relationship Surplus

Endogenous Variable	**Model Value**	$\overline{\chi} \downarrow 50\%$
Effective Spread	76.34bps	65.46bps
Asset Turnover	15.98%	16.84%
Relationship Separation Probability	98.48%	98.47%
Relationship Formation Probability	5.61%	9.35%
Fraction of Matched Investors	10.00%	16.64%
Proportion of Retail Sized Trades	79.24%	86.31%
Trades per Security	1.033	1.567

Table 3: Counterfactual Analysis

Table 3 reports the effects of scaling down the distribution of match-specific costs. Liquidity generally improves in that the effective spread is reduced by approximately 11 basis points while turnover and the number of trades per security increase as well.

7 Conclusion

Despite a large body of empirical work documenting the existence of trading relationships in financial markets and its importance for spreads and trading volume, the search based literature on OTC markets spurred by Duffie, Gârleanu, and Pedersen (2005) has largely ignored this empirical finding in theoretical work. I build a model that encompasses the salient features of relationship trading within decentralized markets, namely, repeated interaction between an investor-dealer pair. The tractability of the model admits closed form solutions and can easily adopt a variety of functional forms in numerical exercises and analytical results alike. Investors temporarily circumvent search frictions via relationships which impact portfolio decisions, trading volume, and transaction costs. When there is free entry of dealers, relationship stability plays a role in determining both the number and respective liquidity properties of surviving steady state equilibria. More stable relationships allow for coordination on a unique, higher liquidity outcome (i.e. greater liquidity provision, larger trade volume, lower transaction costs). When spot trading and relationship trading coexist, standard measures of market liquidity may change in a non-monotone fashion as the stability of relationships is altered which highlights a trade-off in liquidity between differing trading arrangements. Finally, relationships are endogenized in the model and calibrated to a real world OTC market. The results suggest that relationships add value to the inter-dealer municipal bond market by positively impacting various measures of market liquidity.

References

Afonso, Gara, Anna Kovner, and Antoinette Schoar (2013). "Trading partners in the interbank lending market". In: *FRB of New York Staff Report*(620).

Amihud, Yakov and Haim Mendelson (1980). "Dealership market: Market-making with inventory". In: *Journal of financial economics* 8(1), pp. 31–53.

Ashcraft, Adam B and Darrell Duffie (2007). "Systemic illiquidity in the federal funds market". In: *American Economic Review* 97(2), pp. 221–225.

Babus, Ana and Tai-Wei Hu (2017). "Endogenous intermediation in over-the-counter markets". In: *Journal of Financial Economics* 125(1), pp. 200–215.

Barclay, Michael J, Terrence Hendershott, and Kenneth Kotz (2006). "Automation versus intermediation: Evidence from Treasuries going off the run". In: *The Journal of Finance* 61(5), pp. 2395–2414.

Bessembinder, Hendrik and William Maxwell (2008). "Markets: Transparency and the corporate bond market". In: *Journal of economic perspectives* 22(2), pp. 217–234.

Bethune, Zachary et al. (2022). "Lending relationships and optimal monetary policy". In: *The Review of Economic Studies* 89(4), pp. 1833–1872.

Bräuning, Falk and Falko Fecht (2017). "Relationship lending in the interbank market and the price of liquidity". In: *Review of Finance* 21(1), pp. 33–75.

Chaboud, Alain et al. (2022). "All-to-all trading in the US Treasury market". In: *FRB of New York Staff Report*(1036).

Chang, Briana and Shengxing Zhang (2021). "Endogenous market making and network formation". In: *Available at SSRN 2600242*.

Chiu, Jonathan, Jens Eisenschmidt, and Cyril Monnet (2020). "Relationships in the interbank market". In: *Review of Economic Dynamics* 35, pp. 170–191.

Clowers, Nicole (2012). "Overview of Market Structure, Pricing, and Regulation". In: *United States Government Accountability Office Report to Congressional Committees*.

Cocco, Joao F, Francisco J Gomes, and Nuno C Martins (2009). "Lending relationships in the interbank market". In: *Journal of Financial Intermediation* 18(1), pp. 24–48.

Di Maggio, Marco, Amir Kermani, and Zhaogang Song (2017). "The value of trading relations in turbulent times". In: *Journal of Financial Economics* 124(2), pp. 266–284.

Duffie, Darrell, Nicolae Gârleanu, and Lasse Heje Pedersen (2005). "Over-the-counter markets". In: *Econometrica* 73(6), pp. 1815–1847.

Edwards, Amy K, Lawrence E Harris, and Michael S Piwowar (2007). "Corporate bond market transaction costs and transparency". In: *The Journal of Finance* 62(3), pp. 1421–1451.

Goldberg, Victor P (1976). "Regulation and administered contracts". In: *the Bell journal of economics*, pp. 426–448.

Han, Song, Kleopatra Nikolaou, and Manjola Tase (2022). "Trading relationships in secured markets: Evidence from triparty repos". In: *Journal of Banking & Finance* 139, p. 106486.

Hendershott, Terrence, Dan Li, et al. (2020). "Relationship trading in over-the-counter markets". In: *The Journal of Finance* 75(2), pp. 683–734.

Hendershott, Terrence and Ananth Madhavan (2015). "Click or call? Auction versus search in the over-the-counter market". In: *The Journal of Finance* 70(1), pp. 419–447.

Holden, Craig W et al. (2021). "What is the impact of introducing a parallel OTC market? theory and evidence from the chinese interbank fx market". In: *Journal of Financial Economics* 140(1), pp. 270–291.

Hollifield, Burton, Artem Neklyudov, and Chester Spatt (2017). "Bid-ask spreads, trading networks, and the pricing of securitizations". In: *The Review of Financial Studies* 30(9), pp. 3048–3085.

Hugonnier, Julien, Benjamin Lester, and Pierre-Olivier Weill (2020). "Frictional intermediation in over-the-counter markets". In: *The Review of Economic Studies* 87(3), pp. 1432–1469.

Iercosan, Diana and Alexander Jiron (2017). "The value of trading relationships and networks in the cds market". In: *Available at SSRN 2901743*.

Klein, Benjamin, Robert G Crawford, and Armen A Alchian (1978). "Vertical integration, appropriable rents, and the competitive contracting process". In: *The journal of Law and Economics* 21(2), pp. 297–326.

Lagos, Ricardo and Guillaume Rocheteau (2007). "Search in asset markets: Market structure, liquidity, and welfare". In: *American Economic Review* 97(2), pp. 198–202.

Lagos, Ricardo and Guillaume Rocheteau (2009). "Liquidity in asset markets with search frictions". In: *Econometrica* 77(2), pp. 403–426.

Lester, Benjamin, Guillaume Rocheteau, and Pierre-Olivier Weill (2015). "Competing for order flow in OTC markets". In: *Journal of Money, Credit and Banking* 47(S2), pp. 77–126.

Li, Dan and Norman Schürhoff (2019). "Dealer networks". In: *The Journal of Finance* 74(1), pp. 91–144.

Li, Yi (2021). "Reciprocal lending relationships in shadow banking". In: *Journal of Financial Economics* 141(2), pp. 600–619.

Malamud, Semyon and Marzena Rostek (2017). "Decentralized exchange". In: *American Economic Review* 107(11), pp. 3320–62.

Mortensen, Dale T and Christopher A Pissarides (1994). "Job creation and job destruction in the theory of unemployment". In: *The review of economic studies* 61(3), pp. 397–415.

MSRB (2018). "2017 Fact Book". In.

Riggs, Lynn et al. (2020). "Swap trading after Dodd-Frank: Evidence from index CDS". In: *Journal of Financial Economics* 137(3), pp. 857–886.

Rubinstein, Ariel (1982). "Perfect equilibrium in a bargaining model". In: *Econometrica: Journal of the Econometric Society*, pp. 97–109.

Rubinstein, Ariel and Asher Wolinsky (1987). "Middlemen". In: *The Quarterly Journal of Economics* 102(3), pp. 581–593.

Sambalaibat, Batchimeg (2019). "Endogenous specialization and dealer networks". In: *Available at SSRN 2676116*.

SIFMA (2024). "US Fixed Income Securities: Issuance, Trading Volume, Outstanding". In.

Wang, Chaojun (2017). "Core-periphery trading networks". In: *Manuscript, Stanford University*.

Weill, Pierre-Olivier (2020). "The search theory of over-the-counter markets". In: *Annual Review of Economics* 12, pp. 747–773.

Wu, Simon Z (2018). "Transaction Costs for Customer Trades in the Municipal Bond Market: What Is Driving the Decline?" In: *Municipal Securities Rulemaking Board* 1, p. 29.

Wu, Simon Z (2021). "Characteristics of Municipal Securities Trading on Alternative Trading Systems and Broker's Broker Platforms". In.

Zhang, Shengxing (2018). "Liquidity misallocation in an over-the-counter market". In: *Journal of Economic Theory* 174, pp. 16–56.

A Limited Commitment of Dealers

Here, I relax the assumption that dealers are able to commit to providing the assets that investors demand throughout the entire length of the relationship. I construct an extensive form game representing the strategic bargaining process over an intermediation fee and asset position of an investor-dealer pair.

A.1 Game Design

The bargaining problem is represented as an alternating offer game with discounting and exogenous risk of breakdown. An investor-dealer pair only interact at discrete points in time. A proposal made by either agent consists of an asset position for the investor and an intermediation fee paid to the dealer. The receiver of the offer is free to accept or reject the proposed contract. If an investor (dealer) rejects a proposal, I assume they must wait Δ_I (Δ_d) units of time before formulating their own offer. In the case where an offer is accepted, the players remain matched but the bargaining game ends and both players receive their according payoffs. In the case of rejection, the game continues on unless either the relationship is destroyed, a new counterparty is found, or a new preference shock is received after which I assume a new bargaining game begins.

A.2 Equilibrium

I restrict my attention to equilibria where investors and dealers use stationary strategies so that proposals and acceptance rules will be the same in all periods for a given agent.

A.2.1 Maximization Problems

Let $V_i(a)$ ($W_i(a)$) denote the expected lifetime utility of a matched (unmatched) investor with preference type i and a units of the asset. The function $\Pi_i(a)$ is the expected lifetime utility of a dealer who is in a relationship with an investor having characteristics (i, a). If the investor makes an offer, she chooses her terms of trade to maximize her expected utility net of the fees incurred to rebalance her portfolio, subject to a dealer indifference condition, as follows

$$\max_{a',\phi'} \left\{-\phi' + U_i(a, a') + \Upsilon_i^I(a') : \phi' + \Upsilon^d(a') \geq R^d(a)\right\} \tag{27}$$

where

$$U_i(a, a') \equiv \mathbb{E}_i \left[\int_0^\tau e^{-rt} u_i(a') dt\right] - p(a' - a)$$

is the expected utility of an investor from now until the next shock occurs (preference or destruction) at time $\tau = min(\tau_\lambda, \tau_\delta)$ where τ_λ and τ_δ are exponentially distributed with means $1/\lambda$ and $1/\delta$, respectively. The functions $\Upsilon_i^I(a')$ and $\Upsilon^d(a')$ are defined as

$$\Upsilon_i^I(a') \equiv \mathbb{E}\left[e^{-r\tau}\mathbb{1}_{\{\tau=\tau_\lambda\}} V_{s(\tau_\lambda)}(a')\right] + \mathbb{E}\left[e^{-r\tau}\mathbb{1}_{\{\tau=\tau_\delta\}} W_i(a')\right] = \frac{\lambda}{r+\lambda+\delta}\sum_{j\in\mathcal{I}} \pi_j V_j(a') + \frac{\delta}{r+\lambda+\delta} W_i(a')$$

$$\Upsilon^d(a') \equiv \mathbb{E}\left[e^{-r\tau}\mathbb{1}_{\{\tau=\tau_\lambda\}} \Pi_{s(\tau_\lambda)}(a')\right] = \frac{\lambda}{r+\lambda+\delta}\sum_{j\in\mathcal{I}} \pi_j \Pi_j(a')$$

and represent the expected continuation values of an investor and dealer, respectively. $R^d(a)$ denotes the reservation value of a dealer and is taken as given by the investor, it is defined precisely shortly. The investor will always propose an intermediation fee so as to make the dealer indifferent between accepting and rejecting the proposal so that the inequality constraint will always bind in equilibrium. Substituting the binding constraint into the objective function reduces the investors problem into a choice of asset holdings that solves the following maximization problem

$$\max_{a'} \left\{ U_i(a,a') + \Upsilon_i^I(a') + \Upsilon^d(a') - R^d(a) \right\}. \tag{28}$$

The functions $\Pi(\cdot)$, $V(\cdot)$, and $W(\cdot)$ are taken as given by the investor when formulating her offer. The portfolio choice will pin down the intermediation fees so that an investor's proposed contract will be entirely summarized by the following:

$$a_I(i) = \arg\max_{a'} \{U_i(a,a') + \Upsilon_i^I(a') + \Upsilon^d(a') - R^d(a)\} \tag{29}$$

$$\phi_I(i,a) = R^d(a) - \Upsilon^d(a_I(i)). \tag{30}$$

The proposal consists of an asset position that maximizes the joint value of the investor-dealer pair while the intermediation fee makes the dealer indifferent between accepting the proposal or rejecting it.

If the dealer gets the chance to make an offer he will maximize his expected payoff, equal to the current per-trade fees plus his expected discounted continuation value, such that the investor is indifferent between accepting and rejecting his proposal. His problem is written in a similar fashion to the investors problem as follows:

$$\max_{a',\phi'} \left\{ \phi' + \Upsilon^d(a') : U_i(a,a') + \Upsilon_i^I(a') - \phi' \geq R^I(a) \right\}. \tag{31}$$

The dealer's proposed contract is summarized by the following equations

$$a_d(i) = \arg\max_{a'} \{U_i(a,a') + \Upsilon_i^I(a') + \Upsilon^d(a') - R^I(a)\} \tag{32}$$

$$\phi_d(i,a) = U_i(a,a_d(i)) + \Upsilon_i^I(a_d(i)) - R^I(a). \tag{33}$$

It is an asset position that maximizes the joint value of a relationship and an intermediation fee that equals the largest payment an investor would be willing to make.

Inspecting (29) and (32), it follows that the assets proposed in a contract will maximize the joint value of a relationship irrespective of who is making the offer. Thus, in my notation, I write a_i as the asset holdings proposed in a contract when the investor is of type i.

A.2.2 Bellman Equations

The expected lifetime utility of a matched investor can be written as below.[21]

$$V_i(a) = \mathbb{E}\left[\int_0^\tau e^{-rt} u_i(a_i) dt\right] - p(a_i - a) - \phi_d(i,a) + \frac{\delta}{r+\delta+\lambda} W_i(a_i) + \frac{\lambda}{r+\delta+\lambda} \sum_{j \in \mathcal{I}} \pi_j V_j(a_i) \tag{34}$$

[21] In writing the Bellman equations here I assume that dealers make the first offer. However, the problem can be written identically when investors make the first offer. Since I am interested in the limit as the time between counteroffers goes to zero, any first-mover advantage will be eliminated implying that this exposition is without loss in generality.

It equals the discounted utility of holding the asset until the next shock arrives net of the cost of acquiring it, plus the expected continuation values when that shock is realized. The lifetime utility of an unmatched investor with characteristics (i, a) solves

$$rW_i(a) = u_i(a) + \lambda \sum_{j \in \mathcal{I}} \pi_j \big[W_j(a) - W_i(a)\big] + \alpha\big[V_i(a) - W_i(a)\big] \tag{35}$$

with the obvious difference from equation (6) that the exogenous dealer bargaining power does not enter the above equation. The expected lifetime utility of a dealer solves

$$\Pi_i(a) = \phi_d(i, a) + \frac{\lambda}{r + \delta + \lambda} \sum_{j \in \mathcal{I}} \pi_j \Pi_j(a_i). \tag{36}$$

A dealer enjoys the current per-trade fees paid to him by the investor, plus an expected continuation value of remaining in the relationship. Crucially, the dealer's continuation value depends on the assets he trades with the investor today.

A.2.3 Reservation Utilities

Here I write the expressions for the reservation utilities of investors and dealers, respectively. In my notation I make use of the fact that on the equilibrium path, counteroffers will always be accepted.

An investors reservation utility can be broken down into two components as below.

$$\begin{aligned} R^I(a) = \Delta_I u_i(a) + e^{-r\Delta_I}\Bigg(&\delta\Delta_I W_i(a) + \lambda\Delta_I \sum_{j \in \mathcal{I}} \pi_j \left[U_j(a, a_j) - \phi_d(a, j) + \Upsilon^I_j(a_j)\right] \\ &+ \alpha\Delta_I \left[U_i(a, a_i) - \phi_d(a, i) + \Upsilon^I_i(a_i)\right] \\ &+ (1 - \delta\Delta_I - \lambda\Delta_I - \alpha\Delta_I) \left[U_i(a, a_i) - \phi_I(a, i) + \Upsilon^I_i(a_i)\right]\Bigg). \end{aligned} \tag{37}$$

The first component represents the utility flow enjoyed by an investor during an interval of length Δ_I, should an agreement not be reached. The second component represents the discounted expected utility an investor will receive after this interval of length Δ_I has passed. The terms inside the brackets correspond to the following events. With probability $\delta\Delta_I$ a relationship is destroyed and the investor becomes unmatched. With probability $\lambda\Delta_I$ an investor changes type after which a new bargaining game begins and the dealer makes an offer. With probability $\alpha\Delta_I$ an investor meets a new dealer who makes an offer that the investor accepts. Lastly, with complement probability $(1 - \delta\Delta_I - \lambda\Delta_I - \alpha\Delta_I)$ the investor is neither unmatched nor changes type nor meets a new dealer and is thus able to make a counteroffer.

A dealers reservation utility is simply his expected discounted utility from rejecting an offer. So,

$$R^d(a) = e^{-r\Delta_d}\Bigg(\lambda\Delta_d \sum_{j \in \mathcal{I}} \pi_j \big[\phi_d(a, j) + \Upsilon^d(a_j)\big] + (1 - \delta\Delta_d - \lambda\Delta_d - \alpha\Delta_d)\big[\phi_d(a, i) + \Upsilon^d(a_i)\big]\Bigg). \tag{38}$$

Over an interval of length Δ_d after a dealer's rejection, four events are possible. First, the game ends with probability $\delta\Delta_d$ after which the dealer receives a payoff of zero. Second, with probability $\lambda\Delta_d$, an investor changes type and a new game begins, with the dealer making an offer. Third, with probability $\alpha\Delta_d$ an investor meets a new counterparty and the dealer receives a payoff of zero. Lastly, with complement probability $(1-\delta\Delta_d-\lambda\Delta_d-\alpha\Delta_d)$, the investor's characteristics remain unchanged and the dealer makes a counteroffer.

Definition 2 *An equilibrium of the alternating offer game for an investor-dealer pair must satisfy 3 conditions. First, a set of proposals $\{a_I,\phi_I\}$ and $\{a_d,\phi_d\}$ that satisfy equations (29),(30) and (32),(33) taking as given the Bellman equations V, W, and Π as well as the reservation values R^d and R^I. Second, equations (34), (35), and (36) that define the values V, W, and Π. Lastly, equations (37) and (38) that define the reservation values for investors and dealers, respectively.*

A.3 Immediate Counteroffers

I assume that the ratio Δ_I/Δ_d is constant and equal to θ so that an investor takes θ times longer to formulate a counteroffer compared to a dealer. Equivalently, I can write $\Delta_d \equiv \Delta$ and $\Delta_I \equiv \theta\Delta$. I am interested in finding a solution to the bargaining game described above when $\Delta \to 0$. When counteroffers are immediate, I remove any first-mover advantage in the bargaining procedure. The intermediation fees reflect comparative advantages in terms of proposal speed and outside options instead of who is the first to make an offer.

A.3.1 Per-Trade Intermediation Fees

It can be checked from (30) and (33) that as the time between counteroffers goes to zero, $\phi_d=\phi_I$. Thus, in the limit as counteroffers are immediate, contracts proposed by investors will exactly match those contracts offered by dealers. Combining (30) and (33) while using the expressions for the reservation values (37) and (38) it follows that $\phi_d(i,a)$ can be expressed as

$$\phi_d(i,a)=\Gamma_1[U_i(a,a_i)+\Upsilon_i^I(a_i)]+\Gamma_2\sum_{j\in\mathcal{I}}\pi_j[U_j(a,a_j)+\Upsilon_j^I(a_j)]+\Gamma_3\Upsilon^d(a_i)+\Gamma_4\sum_{j\in\mathcal{I}}\pi_j\Upsilon^d(a_j)$$
$$+\Gamma_5u_i(a)+\Gamma_6\sum_{j\in\mathcal{I}}\pi_ju_j(a)+\Gamma_7W_i(a)+\Gamma_8\sum_{j\in\mathcal{I}}\pi_jW_j(a) \quad (39)$$

where the Γ coefficients are provided in Section A.6. There exists a direct mapping from the ratio of counteroffer speeds, θ, into the ratio of bargaining powers used in Nash Bargaining, $\frac{\eta}{1-\eta}$, given by $\theta \to \frac{\eta}{1-\eta}$.[22] Taking the limit as $\Delta \longrightarrow 0$ yields per-trade fees given by

$$\phi_i(a)=\eta\left[\frac{(r+\delta+\lambda)[U_i(a,a_i)+\Upsilon_i^I(a_i)+\Upsilon^d(a_i)]}{\kappa+\delta+\lambda}\right]-\eta\left[\frac{\lambda\alpha\sum_j\pi_j[U_j(a,a_j)+\Upsilon_j^I(a_j)+\Upsilon^d(a_j)]}{(\kappa+\delta+\lambda)(\kappa+\delta)}\right]$$
$$-\eta\left[\frac{u_i(a)+\delta W_i(a)}{\kappa+\delta+\lambda}\right]-\frac{\eta}{1-\eta}\left[\frac{\lambda\sum_j\pi_ju_j(a)+\lambda\delta\sum_j\pi_jW_j(a)}{(\kappa+\delta+\lambda)(\kappa+\delta)}\right]-\Upsilon^d(a_i). \quad (40)$$

The fees paid by an investor for trade to occur depends on how fast she can formulate an offer relative to the dealer, how much she stands to gain if trade occurs, and her outside options from holding the asset without trade. I illustrate three special cases below.

[22] In anticipation of this result, I replace $\frac{\theta}{1+\theta}$ with η and $\frac{1}{1+\theta}$ with $1-\eta$. It follows that $\theta=0$ is equivalent to $\eta=0$ and $\theta\to\infty$ is equivalent to $\eta=1$.

Fast Dealers When dealers respond substantially faster than investors ($\theta \longrightarrow \infty$), (40) reduces to

$$\phi_i(a) = U_i(a, a_i) + \Upsilon_i^I(a_i) - \left[\frac{\delta(r+\delta)W_i(a) + \delta\lambda\sum_j \pi_j W_j(a)}{(r+\delta)(r+\lambda+\delta)} + \frac{(r+\delta)u_i(a) + \lambda\sum_j \pi_j u_j(a)}{(r+\delta)(r+\lambda+\delta)}\right]$$

so that the fees paid by an investor equal exactly the expected utility from accepting a proposal (first two terms) net of the investors utility should an agreement not be reached (third term). This corresponds to the outcome of the Nash Bargaining solution when dealers have all the bargaining power.

Fast Investors When investors have maximum advantage with respect to counteroffer speeds ($\theta = 0$), combining equation (40) with (36) implies that a dealer's lifetime utility is zero so that (40) reduces to

$$\phi_i(a) = 0$$

which corresponds to the outcome of the Nash Bargaining solution when dealers have no bargaining power. It means that investors need not pay any fees when they are significantly faster than dealers and instead enjoy the full joint surplus.

Equal Counteroffer Times When investors and dealers are symmetric in their ability to generate counteroffers ($\theta = 1$), we obtain the following expression for the intermediation fees:

$$\phi_i(a) = \frac{1}{2}\left[\frac{(r+\delta+\lambda)[U_i(a,a_i) + \Upsilon_i^I(a_i) + \Upsilon^d(a_i)]}{r + \frac{\alpha}{2} + \delta + \lambda}\right] - \frac{1}{2}\left[\frac{\lambda\alpha\sum_j \pi_j[U_j(a,a_j) + \Upsilon_j^I(a_j) + \Upsilon^d(a_j)]}{(r+\frac{\alpha}{2}+\delta+\lambda)(r+\frac{\alpha}{2}+\delta)}\right]$$
$$- \frac{1}{2}\left[\frac{u_i(a) + \delta W_i(a)}{r+\frac{\alpha}{2}+\delta+\lambda}\right] - \left[\frac{\lambda\sum_j \pi_j u_j(a) + \lambda\delta\sum_j \pi_j W_j(a)}{(r+\frac{\alpha}{2}+\delta+\lambda)(r+\frac{\alpha}{2}+\delta)}\right] - \Upsilon^d(a_i).$$

Investors and dealers split the surplus created by a trade equally amongst themselves.

A.3.2 Bellman Equations Revisited

It follows from the results above that the maximum lifetime utility attainable by an investor and dealer, respectively, can be written as

$$V_i(a) = (1-\eta)\left[\frac{(r+\alpha+\delta+\lambda)[U_i(a_i,a) + \Upsilon_i^I(a_i) + \Upsilon^d(a_i)]}{\kappa+\delta+\lambda}\right] + \eta\left[\frac{\lambda\alpha\sum_j \pi_j[U_j(a_j,a) + \Upsilon_j^I(a_j) + \Upsilon^d(a_j)]}{(\kappa+\delta+\lambda)(\kappa+\delta)}\right]$$
$$+ \eta\left[\frac{u_i(a) + \delta W_i(a)}{\kappa+\delta+\lambda}\right] + \frac{\eta}{1-\eta}\left[\frac{\lambda\sum_j \pi_j u_j(a) + \lambda\delta\sum_j \pi_j W_j(a)}{(\kappa+\delta+\lambda)(\kappa+\delta)}\right] \quad (41)$$

and

$$\Pi_i(a) = \eta\left[\frac{(r+\delta+\lambda)[U_i(a,a_i) + \Upsilon_i^I(a_i) + \Upsilon^d(a_i)]}{\kappa+\delta+\lambda}\right] - \eta\left[\frac{\lambda\alpha\sum_j \pi_j[U_j(a,a_j) + \Upsilon_j^I(a_j) + \Upsilon^d(a_j)]}{(\kappa+\delta+\lambda)(\kappa+\delta)}\right]$$
$$- \eta\left[\frac{u_i(a) + \delta W_i(a)}{\kappa+\delta+\lambda}\right] - \frac{\eta}{1-\eta}\left[\frac{\lambda\sum_j \pi_j u_j(a) + \lambda\delta\sum_j \pi_j W_j(a)}{(\kappa+\delta+\lambda)(\kappa+\delta)}\right]. \quad (42)$$

The joint surplus of a relationship then, $V_i(a) + \Pi_i(a)$, is linear in wealth since

$$S_i(a) \equiv V_i(a) + \Pi_i(a) = \frac{(r+\delta+\lambda)[U_i(a,a_i) + \Upsilon_i^I(a_i) + \Upsilon^d(a_i)]}{\kappa+\delta+\lambda}$$

so that $S_i'(a) = p$. Using this and the choice of asset holdings given by both (29) and (32), it follows that the FOC for the portfolio decision can be written as

$$u_i'(a_i) + \delta W_i'(a_i) = (r+\delta)p. \tag{43}$$

It is identical to equation (5) with the only exception being that $W_i(a)$ solves (35) instead of (6).

A.4 Asset Demands

Equation (35) can be differentiated with respect to an investor's current portfolio to obtain

$$rW_i'(a) = u_i'(a) + \lambda \sum_{j\in\mathcal{I}} \pi_j \left[W_j'(a) - W_i'(a)\right] + \alpha\left[V_i'(a) - W_i'(a)\right].$$

Differentiating (40) and substituting into into the above equation after using the fact that $V_i'(a) = p - \phi_i'(a)$ yields an expression for $W_i'(a)$ in terms of current and future marginal utilities, $u_i'(a)$ and $\sum_j \pi_j u_j'(a)$, and the interdealer price p. After some algebra, one obtains that

$$\frac{(\alpha+(1+\theta)(r+\delta+\lambda))\,(\alpha+r(1+\theta))u_i'(a_i) + \delta\lambda(1+\theta)^2\sum_j \pi_j u_j'(a_i)}{(\alpha+(1+\theta)(r+\lambda))\,(\alpha+(1+\theta)(r+\delta))} = rp.$$

Using the fact that a mapping exists between θ, the ratio of counteroffer speeds, and η, dealers' bargaining power, the above equation can be re-expressed as

$$\frac{(\lambda+r+\alpha(1-\eta)+\delta)\,(r+\alpha(1-\eta))u_i'(a_i) + \delta\lambda\sum_j \pi_j u_j'(a_i)}{(\lambda+r+\alpha(1-\eta))\,(r+\alpha(1-\eta)+\delta)} = rp$$

which exactly coincides with the asset demands under the generalized Nash solution.

A.5 Intermediation Fees and Trade Sizes

Differentiating (40) with respect to an investor's current portfolio yields that

$$\phi_i'(a) = \left(\frac{\eta}{\kappa+\lambda+\delta}\right)\left[p(r+\delta) - u_i'(a) - \delta W_i'(a)\right] - \left(\frac{\eta}{\kappa+\lambda+\delta}\right)\frac{\lambda}{\kappa}\left[p(\alpha\eta - r) + \frac{\sum_j \pi_j u_j'(a)}{1-\eta}\right]. \tag{44}$$

It is the sum of two terms.[23] From the FOC given by (43), the first term equals zero when $a = a_i$. Since both $u_i(a)$ and $W_i(a)$ are strictly concave, it follows that the first term has the same sign as $a - a_i$. The following proposition provides sufficient conditions to determine the sign of $\phi_i'(a)$.

Proposition 5 *Suppose $a > a_i$ then*

i) $\partial\phi_i(a)/\partial a \geq 0$ *when* $p(\alpha\eta - r) + \sum_j \pi_j u_j'(a)/(1-\eta) \leq 0$

ii) $\partial\phi_i(a)/\partial a < 0$ *when* $p(\alpha\eta - r) + \sum_j \pi_j u_j'(a)/(1-\eta) > \kappa/\lambda[p(r+\delta) - u_i'(a) - \delta W_i'(a)]$

and opposite otherwise.

Proof of Proposition 5.

■

[23] Note that the function $W_i(a)$ referenced in equation (44) is not the same at the function that solves (6) but instead is given by (35).

As opposed to Lagos and Rocheteau (2009) where intermediation fees increase as a function of trade sizes, per-trade intermediation fees in my model need not be increasing functions of trade sizes. This result is more in line with empirical evidence of asset markets that find larger trade sizes generally receive more favorable prices (e.g. Edwards, Harris, and Piwowar 2007). Given the nearly identical structure of my model and that of Lagos and Rocheteau (2009), I can attribute this result to the existence of long term relationships between investors and dealers.

A.6 Coefficients for Section A.3.1

We have that

$$\Gamma_1 \equiv \frac{1 - e^{-r\theta\Delta}(1 - \delta\theta\Delta - \lambda\theta\Delta)}{\gamma_1}$$
$$\Gamma_2 \equiv \gamma_2[1 - (1 - \delta\theta\Delta)e^{-r\theta\Delta}] - \frac{\lambda\theta\Delta e^{-r\theta\Delta}}{\gamma_1}$$
$$\Gamma_3 \equiv \frac{e^{-r\theta\Delta}(1 - \delta\theta\Delta - \lambda\theta\Delta - \alpha\theta\Delta)[e^{-r\Delta}(1 - \delta\Delta - \lambda\Delta - \alpha\Delta) - 1]}{\gamma_1}$$
$$\Gamma_4 \equiv \frac{e^{-r(1+\theta)\Delta}(1 - \delta\theta\Delta - \lambda\theta\Delta - \alpha\theta\Delta)\lambda\Delta}{\gamma_1} - \gamma_2 e^{-r\theta\Delta}(1 - \delta\theta\Delta - \lambda\theta\Delta - \alpha\theta\Delta)[1 - e^{-r\Delta}(1 - \delta\Delta - \alpha\Delta)]$$
$$\Gamma_5 \equiv -\frac{\theta\Delta}{\gamma_1}$$
$$\Gamma_6 \equiv -\theta\Delta\gamma_2$$
$$\Gamma_7 \equiv -\frac{e^{-r\theta\Delta}\delta\theta\Delta}{\gamma_1}$$
$$\Gamma_8 \equiv -\gamma_2 e^{-r\theta\Delta}\delta\theta\Delta$$

where γ_1 and γ_2 are given by

$$\gamma_1 \equiv 1 - \alpha\theta\Delta e^{-r\theta\Delta} - e^{-r\Delta(1+\theta)}(1 - \delta\theta\Delta - \lambda\theta\Delta - \alpha\theta\Delta)(1 - \delta\Delta - \lambda\Delta - \alpha\Delta)$$
$$\gamma_2 \equiv \frac{e^{-r\theta\Delta}[\lambda\theta\Delta + e^{-r\Delta}(1 - \delta\theta\Delta - \lambda\theta\Delta - \alpha\theta\Delta)\lambda\Delta]}{\gamma_1\big(\gamma_1 - e^{-r\theta\Delta}[\lambda\theta\Delta + e^{-r\Delta}(1 - \delta\theta\Delta - \lambda\theta\Delta - \alpha\theta\Delta)\lambda\Delta]\big)}.$$

Taking the limit as $\Delta \to 0$ and applying l'Hôpitals rule where necessary yields that

$$\lim_{\Delta\to 0}\Gamma_1 = \frac{\theta(r+\delta+\lambda)}{\alpha+(1+\theta)(r+\delta+\lambda)}$$
$$\lim_{\Delta\to 0}\Gamma_2 = -\frac{\theta\lambda\alpha}{(\alpha+(1+\theta)(r+\delta+\lambda))(\alpha+(1+\theta)(r+\delta))}$$
$$\lim_{\Delta\to 0}\Gamma_3 = -\frac{r+\delta+\lambda+\alpha}{\alpha+(1+\theta)(r+\delta+\lambda)}$$
$$\lim_{\Delta\to 0}\Gamma_4 = -\frac{\theta\lambda\alpha}{(\alpha+(1+\theta)(r+\delta+\lambda))(\alpha+(1+\theta)(r+\delta))}$$
$$\lim_{\Delta\to 0}\Gamma_5 = -\frac{\theta}{\alpha+(1+\theta)(r+\delta+\lambda)}$$
$$\lim_{\Delta\to 0}\Gamma_6 = -\frac{\lambda\theta(1+\theta)}{(\alpha+(1+\theta)(r+\delta+\lambda))(\alpha+(1+\theta)(r+\delta))}$$
$$\lim_{\Delta\to 0}\Gamma_7 = -\frac{\delta\theta}{\alpha+(1+\theta)(r+\delta+\lambda)}$$
$$\lim_{\Delta\to 0}\Gamma_8 = -\frac{\delta\lambda\theta(1+\theta)}{(\alpha+(1+\theta)(r+\delta+\lambda))(\alpha+(1+\theta)(r+\delta))}.$$

B Proofs

Proof of Proposition 1. Since a matched investor is connected to the interdealer market, we can think of her lifetime utility from the moment she chooses to reoptimize her portfolio onward. Thus,

$$V_i(a) = \max_{a' \geq 0} \left\{ \int_0^{\tau} e^{-rt} u_i(a') dt - p(a' - a) + \mathbb{E}\big[e^{-r\tau} \mathbb{1}_{\{\tau = \tau_\delta\}} \sum_{j \in \mathcal{I}} \pi_j V_j(a')\big] + \mathbb{E}\big[e^{-r\tau} \mathbb{1}_{\{\tau = \tau_\delta\}} W_i(a')\big] \right\}$$

where $\tau \equiv \min(\tau_\delta, \tau_\lambda)$ and τ_δ and τ_λ are exponentially distributed times with respective means of $1/\delta$ and $1/\lambda$. Expanding the above equation further we obtain that

$$V_i(a) = pa + \max_{a' \geq 0} \left\{ \frac{u_i(a')}{r + \lambda + \delta} - pa' + \frac{\lambda}{r + \lambda + \delta} \sum_{j \in \mathcal{I}} \pi_j V_j(a') + \frac{\delta}{r + \lambda + \delta} W_i(a') \right\}.$$

The first term is an investor's wealth, pa, and the second term can be fully summarized by an investor's current preference type. It follows that the lifetime utility of a matched investor can be written as

$$V_i(a) = pa + V_i.$$

Hence, her value function is linear in her wealth. ■

Proof of Proposition 2. Volume of trade is given by the following equation

$$\mathcal{V} = \alpha \sum_{i,j} n^u_{ji} |a_i - a_j| + \lambda \sum_{i,j} n^m_{ii} \pi_j |a_j - a_i|.$$

Using both the expressions for the distribution of investors (Section 3.5) and a_i (Section 3.7), $\mathcal{V}$ is easily reexpressed, after a few lines of algebra, as follows:

$$\mathcal{V} = \frac{(\delta + \lambda + \alpha)(\delta + \lambda + r + \alpha(1 - \eta))}{(\delta + \alpha)(\delta + r + \alpha(1 - \eta))} \left[\frac{\alpha\lambda(r + \alpha(1 - \eta)) \sum_{i,j} \pi_i \pi_j |\varepsilon_i - \varepsilon_j|}{(\alpha + \lambda)(r + \alpha(1 - \eta) + \lambda) rp} \right].$$

After taking the first and second derivatives of $\mathcal{V}$ with respect to δ where we use the fact that p is independent of the relationship stability parameter under log-utility, we easily obtain the desired results that $\frac{\partial \mathcal{V}}{\partial \delta} \leq 0$ and $\frac{\partial^2 \mathcal{V}}{\partial \delta^2} \geq 0$. ■

Proof of Proposition 3. The discounted sum of intermediation fees, $\Phi_i(a)$, after combining like terms can be expressed as follows

$$\Phi_i(a) = \frac{rpa}{\kappa} - \frac{\kappa u_i(a) + \lambda \sum_j \pi_j u_j(a)}{\kappa(\kappa + \lambda)} + \frac{\lambda[\alpha(1 - \eta)(2(\kappa + \delta + \lambda) + r) + r(r + \lambda)] \sum_j \pi_j p a_j}{\kappa(\kappa + \lambda)(\kappa + \lambda + \delta)}$$
$$- \frac{r(\kappa + \delta) p a_i}{\kappa(\kappa + \lambda + \delta)} + \frac{u_i(a_i)}{(\kappa + \lambda)} + \frac{\delta\lambda \sum_j \pi_j u_j(a_i)}{\kappa(\kappa + \lambda)(\kappa + \lambda + \delta)} + \frac{\lambda \sum_j \pi_j u_j(a_j)}{(\kappa + \delta)(\kappa + \lambda)} + \frac{\delta\lambda^2 \sum_j \pi_j \sum_k \pi_k u_k(a_j)}{\kappa(\kappa + \delta)(\kappa + \lambda)(\kappa + \delta + \lambda)}.$$

To establish the first part of the proposition, differentiate the above expression with respect to δ to obtain

$$\Phi_i'(a) = \frac{rp}{\kappa}\frac{\partial a}{\partial \delta} - \frac{\varepsilon_i}{a(\kappa+\lambda)}\frac{\partial a}{\partial \delta} - \frac{\bar{\varepsilon}\lambda}{a\kappa(\kappa+\lambda)}\frac{\partial a}{\partial \delta} - \frac{\lambda[\alpha(1-\eta)(2(\kappa+\delta+\lambda)+r)+r(r+\lambda)]}{\kappa(\kappa+\lambda)(\kappa+\lambda+\delta)}p\sum_j \pi_j \frac{\partial a_j}{\partial \delta}$$
$$+\frac{\lambda r\sum_j \pi_j p a_j}{\kappa(\kappa+\lambda+\delta)^2} - \frac{\lambda r p a_i}{\kappa(\kappa+\lambda+\delta)^2} - \frac{r(\kappa+\delta)}{\kappa(\kappa+\lambda+\delta)}p\frac{\partial a_i}{\partial \delta} + \frac{\varepsilon_i}{a(\kappa+\lambda)}\frac{\partial a_i}{\partial \delta}$$
$$+\frac{\lambda\bar{\varepsilon} ln(a_i)}{\kappa(\kappa+\lambda+\delta)^2} + \frac{\delta\lambda}{\kappa(\kappa+\lambda)(\kappa+\lambda+\delta)}\frac{\bar{\varepsilon}}{a_i}\frac{\partial a_i}{\partial \delta} + \frac{\lambda}{(\kappa+\delta)(\kappa+\lambda)}\sum_j \pi_j \frac{\varepsilon_j}{a_j}\frac{\partial a_j}{\partial \delta} - \frac{\lambda\sum_j \pi_j \varepsilon_j ln(a_j)}{(\kappa+\delta)^2(\kappa+\lambda)}$$
$$+\frac{\lambda^2(\kappa(\kappa+\lambda)-\delta^2)\sum_j \pi_j \bar{\varepsilon} ln(a_j)}{\kappa(\kappa+\delta)^2(\kappa+\lambda)(\kappa+\delta+\lambda)^2} + \frac{\delta\lambda^2}{\kappa(\kappa+\delta)(\kappa+\lambda)(\kappa+\delta+\lambda)}\sum_j \pi_j \frac{\bar{\varepsilon}}{a_j}\frac{\partial a_j}{\partial \delta}.$$

after using the fact that $u_i(a) = \varepsilon_i ln(a)$ and the interdealer price is not affected by market frictions under log-utility. One can notice that taking $\delta \to \infty$ means the above expression reduces to $\Phi_i'(a) = 0$ after using the fact that $\partial a/\partial \delta$ approaches 0 and that a_i approaches a constant when δ becomes large. This establishes the first part of the proposition. To show that the second claim is true, notice that $\mathcal{F}$ can be expressed as follows

$$\mathcal{F} = \frac{n^m \sum_{i,j} f_{ij}\Phi_j(a_i)(r+\delta)}{\mathcal{V}} = \frac{\frac{r+\delta}{\alpha+\delta}\sum_i \frac{\alpha(\alpha\pi_i+\lambda\pi_i^2)}{\lambda+\alpha}\Phi_i(a_i)}{\mathcal{V}} + \frac{\frac{r+\delta}{\alpha+\delta}\sum_j\sum_{i\neq j}\frac{\alpha\lambda\pi_i\pi_j}{\lambda+\alpha}\Phi_j(a_i)}{\mathcal{V}}.$$

Using the quotient rule, we obtain that

$$\partial\mathcal{F}/\partial\delta = \frac{\sum_i \overbrace{\Psi_1[\frac{r+\delta}{\alpha+\delta}\frac{\partial\Phi_i(a_i)}{\partial\delta} + \frac{\alpha-r}{(\alpha+\delta)^2}\Phi_i(a_i)]\mathcal{V}}^{\text{approaches 0 as }\delta\to\infty} - \overbrace{\Psi_1\frac{\partial\mathcal{V}}{\partial\delta}\frac{r+\delta}{\alpha+\delta}\Phi_i(a_i)}^{<0}}{\mathcal{V}^2}$$
$$+\frac{\sum_j\sum_{i\neq j}\Psi_2[\frac{r+\delta}{\alpha+\delta}\frac{\partial\Phi_j(a_i)}{\partial\delta} + \frac{\alpha-r}{(\alpha+\delta)^2}\Phi_j(a_i)]\mathcal{V} - \Psi_2\frac{\partial\mathcal{V}}{\partial\delta}\frac{r+\delta}{\alpha+\delta}\Phi_j(a_i)}{\mathcal{V}^2}.$$

where Ψ_1 and Ψ_2 denote some coefficients independent of δ. When δ becomes large, the first term in both summations approaches zero where we use the fact that $\partial\Phi_i(a)/\partial\delta$ approaches 0 as $\delta \to \infty$, a claim which has already been established in the first part of this proposition. The second term of either summation is negative since by proposition 2, $\partial\mathcal{V}/\partial\delta \leq 0$. Thus, since these negative terms are subtracted, it implies that $\partial\mathcal{F}/\partial\delta > 0$. ■

Proof of Proposition 4. A reservation cost renders an investor indifferent between forming a relationship and spot trading so that by definition

$$V_i(a, \mathcal{X}^*) = \max_{a''}\Big\{W_i(a'') - p(a''-a)\Big\}.$$

As a result of the linear preferences, the above equation can be simplified to the following equation

$$V_i(\mathcal{X}^*) = \max_{a''}\Big\{W_i(a'') - pa''\Big\}.$$

It is an implicit equation that determines the reservation cost, $\mathcal{X}^*$, for each investor of type i. One can check by inspection that an investor's preference type fully determines the reservation cost that makes her indifferent between forming or maintaining a relationship and spot trading. Thus, the reservation cost for an investor of type i depends only on her current preference type and is written as $\mathcal{X}_i^*$. Taking the derivative of the LHS with respect to χ where we use equation (25) yields that $\partial V/\partial \chi$ is bounded above and below so that

$$\partial V/\partial \chi \in \left[\frac{-1}{r+\delta}, \frac{-1}{r+\delta+\lambda}\right]$$

is always negative. The first bound corresponds to when an investor always forms a relationship while the second represents the case when the investor never forms relationships. The derivative of the RHS with respect to χ equals zero. Thus it implies that there is a unique reservation cost such that the investor is indifferent between forming or maintaining a relationship and spot trading. ■

C Endogenous Relationships Numerical Procedure

Here I detail the numerical procedure used to compute the solutions to equations (25) and (**??**). I also numerically compute the endogenous distribution of investors and calculate measures of market liquidity.

C.1 Preparations

To begin, rewrite (25) and (26) in such a way so as to resemble a standard Bellman equation. We have that

$$V_i(\chi) = \beta_m \left[\max_{a'} \left\{ u_i(a') - (r+\delta)pa' - \chi + \delta W_i(a') + \lambda \sum_{j=1}^{I} \pi_j \max \left\{ V_j(\chi), \max_{a''} \{ W_j(a'') - pa'' \} \right\} \right\} \right] \quad (45)$$

where

$$\beta_m \equiv \frac{1}{r+\lambda+\delta}$$

and

$$W_i(a) = \beta_u \left[u_i(a) + \lambda \sum_{j=1}^{I} \pi_j W_j(a) + \alpha(1-\eta) \left(pa + \int_{\underline{\chi}}^{\overline{\chi}} \max \left\{ V_i(\chi'), \max_{a''} \{ W_i(a'') - pa'' \} \right\} dF(\chi') \right) \right] \quad (46)$$

where

$$\beta_u \equiv \frac{1}{r+\lambda+\alpha(1-\eta)}.$$

For what follows, we will make use of the steady state distribution of asset holdings to discretize the continuous state space of asset positions. More precisely, recognize that in a steady state, an investors portfolio choice will be the solution to one of two equations. Either the investor will spot trade, and update her portfolio with choice of asset holdings a_i^s, solution to (22), or the investor will trade via relationships choosing a_i, solution to (5).

I also discretize the distribution of match specific costs, $F(\chi)$. With this in mind, (46) is rewritten as

$$W_i(a) = \beta_u \left[u_i(a) + \lambda \sum_{j=1}^{I} \pi_j W_j(a) + \alpha(1-\eta) \left(pa + \sum_{k=1}^{M} \rho_k \max \left\{ V_i(\chi^k), \max_{a''} \{ W_i(a'') - pa'' \} \right\} \right) \right] \quad (47)$$

where ρ_k denotes the probability of drawing a match-specific cost, χ^k, from the PMF resulting from the discretization of its continuous analogue, $f(\chi)$, with M partitions.

C.2 Distribution of Investors

The laws of motion for the distribution of investors can be written as below.

$$\dot{n}^{m}_{ii} = \alpha(\pi_i - n^{m}_{ii}) - \delta n^{m}_{ii} - \lambda(1-\pi_i)n^{m}_{ii} + \lambda\pi_i \sum_{j\neq i} n^{m}_{jj} \quad \text{for all } i \in \{1,...,I\} \text{ such that } V_i - \Omega_i \geq 0 \tag{48}$$

$$\dot{n}^{m}_{ii} = 0 \quad \text{for all } i \in \{1,...,I\} \text{ such that } V_i - \Omega_i < 0 \tag{49}$$

$$\dot{n}^{ur}_{ii} = \delta n^{m}_{ii} - \alpha n^{ur}_{ii} + \lambda\pi_i \sum_{k\neq i} n^{ur}_{ik} - \lambda(1-\pi_i)n^{ur}_{ii} \quad \text{for all } i \in \{1,...,I\} \tag{50}$$

$$\dot{n}^{ur}_{ji} = \lambda\pi_i \sum_{k\neq i} n^{ur}_{jk} - \lambda(1-\pi_i)n^{ur}_{ji} - \alpha n^{ur}_{ji} \quad \text{for all } j \neq i \tag{51}$$

$$\dot{n}^{us}_{ii} = \alpha \sum_{k=k^*_i}^{M} \sum_{j} \rho_k n^{ur}_{ji} + \alpha \sum_{k=k^*_i}^{M} \sum_{j\neq i} \rho_k n^{us}_{ji} - \alpha \sum_{k=1}^{k^*_i} \rho_k n^{us}_{ii} + \lambda\pi_i \sum_{k\neq i} n^{us}_{ik} - \lambda(1-\pi_i)n^{us}_{ii} + \pi_i\lambda \sum_{k=k^*_i}^{M} \sum_{j\neq i} n^{m}_{jjk} \tag{52}$$

$$\dot{n}^{us}_{ji} = \lambda\pi_i \sum_{k\neq i} n^{us}_{jk} - \lambda(1-\pi_i)n^{us}_{ji} - \alpha n^{us}_{ji} \quad \text{for all } j \neq i \tag{53}$$

C.3 Algorithm Philosophy

To begin, I arbitrarily choose an inter-dealer market price. Given the price, we then compute the value functions through value function iteration and obtain the relationship formation thresholds. Once the thresholds are known, we can compute the endogenous distribution of investors. Next, given the distribution of investors, we calculate the total assets held by investors. If the asset demand deviates from the supply of assets, we change the inter-dealer price accordingly and repeat the process until the asset market clears. Once we have an inter-dealer price that clears the market, we can calculate our measures of liquidity.

C.4 Algorithm

Denote I the number of preference types, K the number of steady state asset positions, and M the number of match-specific costs an investor can draw.[24] We let $\mathbf{Z}_{i,j}$ denote the element from the i^{th} row and j^{th} column of a given matrix, $\mathbf{X}$, where the **boldface** denotes a matrix or vector. The algorithm proceeds as follows:

1. Depending on the choice of distribution for the match specific costs, it may need to be discretized in different ways. If the Uniform distribution or any other discrete distribution is used, nothing needs to be done. In the calibration of the model, the distribution is chosen to be Normal and the discretization process is standard. Should another distribution be preferred, more care will need to be taken to generate an appropriate discrete approximation of the continuous PDF.

2. Fix an inter-dealer market price, p (a natural initial guess is the price resulting from section 5).

[24] Note that $K = 2I$ in a steady state.

3. Construct a $K \times 1$ column vector of steady state asset holdings, denoted by $\mathbf{A}$ where the first I rows correspond to relationship portfolios and rows $I+1$ to K correspond to spot trade portfolios.

4. Construct a $1 \times M$ row vector of equally spaced match-specific costs, χ, denoted by $\mathbf{X}$ where the element in the first column position is $\underline{\chi}$ and the element in the M^{th} column position is $\overline{\chi}$.

5. Construct an $I \times 1$ column vector of preference type probabilities, π, denoted by $\mathbf{\Pi}$ where the element in the first row position is π_1 and the element in the I^{th} row position is π_I. The sum of all rows must equal 1.

6. Construct an $M \times 1$ column vector of match-specific cost probabilities, ρ, denoted by $\mathbf{P}$ where the element in the first row position is ρ_1, the probability of drawing $\underline{\chi}$, and the element in the M^{th} row position is ρ_M, the probability of drawing $\overline{\chi}$. The sum of all rows must equal 1.

7. Denote $\mathbf{V}^0$ the initial guess for (45). It is an $M \times I$ matrix where the i^{th} row corresponds to match specific cost χ^i with $i \in \{1, \ldots, M\}$, and the j^{th} column corresponds to preference type $j \in \{1, \ldots, I\}$. Guess any arbitrary $\mathbf{V}^0$.

8. Denote $\mathbf{W}^0$ the initial guess for (47). It is a $K \times I$ matrix where the i^{th} row corresponds to the investors current portfolio, and the j^{th} column corresponds to preference type $j \in \{1, \ldots, I\}$. Guess any arbitrary $\mathbf{W}^0$.

9. Set $t = 0$. Choose a maximum number of iterations, T, and a convergence criterion, ϵ.

10. Calculate $\mathbf{V}^{t+1}$ as the solution to:
$$\mathbf{V}^{t+1}_{i,j} = \beta_m \left[u_j(\mathbf{A}_{j,1}) - (r+\delta)p\mathbf{A}_{j,1} - \mathbf{X}_{1,i} + \delta \mathbf{W}^t_{j,j} + \lambda \sum_{s=1}^{I} \mathbf{\Pi}_{s,1} \max\left\{ \mathbf{V}^t_{i,s}, \mathbf{W}^t_{s+I,s} - p\mathbf{A}_{s+I,1} \right\} \right]$$
$\forall i \in \{1, \ldots, M\}$ and $j \in \{1, \ldots, I\}$

11. Calculate $\mathbf{W}^{t+1}$ as the solution to:
$$\mathbf{W}^{t+1}_{i,j} = \beta_u \left[u_j(\mathbf{A}_{i,1}) + \lambda \sum_{s=1}^{I} \mathbf{\Pi}_{s,1} \mathbf{W}^t_{i,s} + \alpha(1-\eta) \left(p\mathbf{A}_{i,1} + \sum_{s=1}^{M} \mathbf{P}_{s,1} \max\left\{ \mathbf{V}^t_{s,j}, \mathbf{W}^t_{j,j+I} - p\mathbf{A}_{j+I,1} \right\} \right) \right]$$
$\forall i \in \{1, \ldots, K\}$ and $j \in \{1, \ldots, I\}$

12. *Statement*: $|\mathbf{V}^{t+1}_{i,j} - \mathbf{V}^t_{i,j}| < \epsilon$ and $|\mathbf{W}^{t+1}_{q,j} - \mathbf{W}^t_{q,j}| < \epsilon$ $\forall i \in \{1, \ldots, M\}$, $j \in \{1, \ldots, I\}$, and $q \in \{1, \ldots, K\}$. If the previous statement is true, proceed to the next step. If the previous statement is false, and $t < T$, set $t = t+1$ and go to step 9. Else, terminate code.[25]

13. Construct an $M \times I$ matrix, denoted by $\mathbf{\Gamma}$, where $\mathbf{\Gamma}_{i,j} \equiv \mathbf{V}_{i,j} - \mathbf{W}_{j+I,j} + p\mathbf{A}_{j+I,1}$

[25] You have reached the maximum number of iterations. Choose better initial guesses $\mathbf{V}^0$ and $\mathbf{W}^0$ or increase the maximum number of iterations.

14. For each column of $\mathbf{\Gamma}$, identify the row that contains smallest element in absolute value. Call the identified row number of column j as r_j^*.

15. Construct an $I \times 1$ column vector, denoted by $\mathbf{X}^*$ where $\mathbf{X}^*_{i,1} \equiv \mathbf{X}_{1,r_i^*}$. It is a row vector of reservations costs where the i^{th} row corresponds to χ_i^*.

16. Numerically solve equations (48)-(53) using a solver.

17. Compute the total asset demand as the sum of all asset holdings given the distribution of investors.

18. *Statement*: $|AssetDemand - AssetSupply| < \epsilon_p$. If the previous statement is true, proceed to next step. If the statement is false and Asset Demand > Asset Supply, set $p = p + \Delta$ and go to step 2. If the statement is false and Asset Demand < Asset Supply, set $p = p - \Delta$ and go to step 2.

19. You are done with the main loop and can now calculate the equilibrium objects of your choosing.

C.5 Calibration Details

The calibration of the model proceeds by minimizing the sum of squared percentage deviations of model moments from their targeted data counterparts.

C.5.1 Expressions for Model Equivalents of Targeted Moments

Asset turnover in the model is given by

$$\frac{\mathcal{V}^r + \mathcal{V}^s}{A}.$$

I use this estimate as my target for the average effective spread paid in my model which is given by

$$\frac{2\alpha}{p(\mathcal{V}^r + \mathcal{V}^s)} \sum_{i,j} \left(\sum_{k=1}^{k_i^*} \rho_k n_{ji}^{ur} \Phi_i(a_j) \; + \; \sum_{k=1}^{k_i^*} \rho_k n_{ji}^{us} \Phi_i(a_j^s) \; + \sum_{k=k_i^*+1}^{\overline{k}} \rho_k n_{ji}^{ur} \phi_i(a_j) \; + \sum_{k=k_i^*+1}^{\overline{k}} \rho_k n_{ji}^{us} \phi_i(a_j^s) \right).$$

This probability is analogous to the model's endogenous, average separation probability for relationships which can be computed as

$$\sum_{i,k} \frac{n_{iik}^m}{n^m} \left(1 - e^{-(\delta + \lambda \sum_j \pi_j \mathbb{I}_{\chi_k > \chi_j^*})}\right).$$

It is the average probability, conditional on being matched, that at least 1 destruction shock arrives or at least one preference shock arrives such that the relationship is terminated. Similarly, the same transition matrix can also be used to calculate the annualized rate at which new relations are formed, 5.53%, which serves as a proxy for the endogenous, average relationship formation probability in the model given by

$$\sum_{i,j} \frac{n_{ji}^{ur} + n_{ji}^{us}}{n^u} \left(1 - e^{-\alpha \sum_k \rho_k \mathbb{I}_{\chi_k \le \chi_i^*}}\right)$$

which gives the probability that an unmatched investor will form a relationship within the next year.

D Supplementary Material

D.1 Alternative Liquidity Measures

I consider here an alternative measure of transaction costs and show that qualitatively, similar results are obtained. Instead of using a volume weighted measure of transaction costs, as in section 3, I consider here a non-weighted sum. Consider the following expression

$$\tilde{\mathcal{F}} = n^m (r + \delta) \sum_{i,j} \frac{f_{ij} \Phi_j(a_i)}{\mathcal{V}_{ij}}.$$

where $\mathcal{V}_{ij}$ is the amount of assets that an investor of type i with asset holdings corresponding to type j trades over the entire course of the relationship.

Binary Preference Types For the case of two preference types, volume of trade attributed to a specific individual can be calculated in closed form by calculating the probabilities that a preference shock arrives before a destruction shock and noticing the geometric series after using the fact that all trades are of the same size. It takes the following simple expression:

$$\mathcal{V}_{ij} = |a_i - a_j| \left(1 + \frac{\lambda \delta \pi_j + 2\pi_j \pi_i \lambda^2}{\delta(\lambda + \delta)} \right) \quad \text{for all } j \neq i$$
$$\mathcal{V}_{ii} = |a_i - a_j| \left(\frac{\lambda \delta \pi_j + 2\pi_j \pi_i \lambda^2}{\delta(\lambda + \delta)} \right) \quad \text{for all } i \in \{1, ..., I\}$$

The following graph plots $\tilde{\mathcal{F}}$ for two preference types and the parameter values used in the baseline calibration of Section 3.

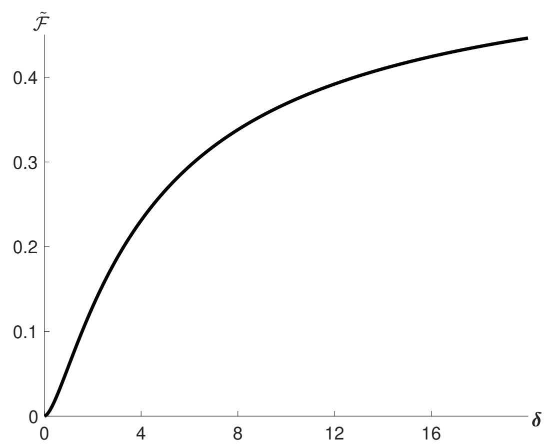


Figure 7: Alternative Measure of Trading Costs

The same qualitative features hold as opposed to the measure of transaction costs that is used in the main body of the text. Transaction costs per unit of asset traded rise as relationships are more unstable. Furthermore, since this measure of transaction costs is not weighted by the volume share of an individual investor, it leaves open the possibility of interpreting an illiquid market as a result of a few investors who incur a large bid-ask spread but trade a very little amount of the asset. Nevertheless, it shows that the results described in section 3 are also robust to alternative measures of trading costs.

D.2 Section 5 Supplementary Material

Bellman Equations Even though matched investors are able to contact spot trade dealers, given their match status, their portfolios are always optimal implying they will never need to trade with a spot dealer. As a result, the value function of a matched investor remains as in Section 3. Making use of the solution to the bargaining problem for spot transactions, the lifetime utility of an unmatched investor solves

$$rW_i(a) = u_i(a) + \lambda \sum_{j \in \mathcal{I}} \pi_j [W_j(a) - W_i(a)] + \alpha(1-\eta)[V_i(a) - W_i(a)] + \alpha_s(1-\eta_s)[V_i^s(a) - W_i(a)] \quad (54)$$

where

$$V_i^s(a) \equiv pa + \Omega_i$$

denotes the value of being matched for a spot trade and $\Omega_i \equiv \max_{a'}\{W_i(a') - pa'\}$. An unmatched investor enjoys some flow utility, changes type with intensity λ, and meets a RD or SD, respectively, at effective rates $\alpha(1-\eta)$ and $\alpha_s(1-\eta_s)$.

Solving for $W_i(a)$ yields that

$$W_i(a) = \mathbb{E}[e^{-r\tau} pa] + \mathbb{E}_i \left[\int_0^\tau e^{-rt} u_{s(t)}(a) dt \right] + \Delta_i \quad (55)$$

We break down the value of being unmatched into three different components. The first is the expected wealth of the investor the next time she is able to trade, (i.e. when she next meets either type of dealer). This event occurs at some future time τ that is exponentially distributed with parameter $\alpha(1-\eta) + \alpha_s(1-\eta_s)$. The second component is the expected utility the investor enjoys until time τ. Finally, the third term Δ_i stands in for the expected value of being able to engage in a spot trade or trade vis-à-vis a relationship at time τ, respectively.

Using equation (54), multiplying each side by π_i, summing over all i and collecting like terms yields that

$$\sum_i \pi_i W_i(a) = \frac{\sum_i \pi_i u_i(a)}{r + \alpha(1-\eta) + \alpha_s(1-\eta_s)} + pa \frac{\alpha(1-\eta) + \alpha_s(1-\eta_s)}{r + \alpha(1-\eta) + \alpha_s(1-\eta_s)} + \frac{\sum_i \pi_i [\alpha(1-\eta)V_i + \alpha_s(1-\eta_s)\Omega_i]}{r + \alpha(1-\eta) + \alpha_s(1-\eta_s)}.$$

Substituting the above equation back into (54) and solving for $W_i(a)$ yields

$$W_i(a) = pa \frac{\alpha(1-\eta) + \alpha_s(1-\eta_s)}{\kappa_s} + \frac{\kappa_s u_i(a) + \lambda \sum_j \pi_j u_j(a)}{\kappa_s(\kappa_s + \lambda)} + \frac{\kappa_s[\alpha(1-\eta)V_i + \alpha_s(1-\eta_s)\Omega_i + \lambda \sum_j \pi_j [\alpha(1-\eta)V_j + \alpha_s(1-\eta_s)\Omega_j]]}{\kappa_s(\kappa_s + \lambda)}$$

where $\kappa_s \equiv r + \alpha(1-\eta) + \alpha_s(1-\eta_s)$ is defined for notational convenience. The above equation expresses the lifetime value of an unmatched investor as the sum of three terms: the expected wealth of an investor at a future time τ that is exponentially distributed with parameter $\alpha(1-\eta) + \alpha_s(1-\eta_s)$, the utility the investor enjoys up until that time τ, and the continuation value of the investor at time τ. It follows then that

$$W_i(a) = \mathbb{E}[e^{-r\tau} pa] + \mathbb{E}_i \left[\int_0^\tau e^{-rt} u_{s(t)}(a) dt \right] + \Delta_i$$

which coincides exactly with equation (55).

Relationship Trade Asset Demands The first-order condition for the optimal asset holdings of a matched investor is given by (5). After differentiating (55) and substituting the expression into (5), an investors' choice of asset holdings while in a relationship can be reduced to the following equation:

$$\frac{[\lambda+\delta+r+\alpha(1-\eta)+\alpha_s(1-\eta_s)][r+\alpha(1-\eta)+\alpha_s(1-\eta_s)]u_i'(a_i)+\delta\lambda\sum_j \pi_j u_j'(a_i)}{[\lambda+r+\alpha(1-\eta)+\alpha_s(1-\eta_s)][\delta+r+\alpha(1-\eta)+\alpha_s(1-\eta_s)]}=rp. \tag{56}$$

It equates the expected marginal utility of holding the asset (left side) to the marginal cost (right side). As before, the expected marginal utility of holding the asset is a weighted sum of current utility and future expected utility. The weight on current utility is increasing in r, $\alpha(1-\eta)$, and $\alpha_s(1-\eta_s)$ while the weight on future utility is increasing in λ and δ. As investors are more impatient or as they expect to be able to contact dealers more quickly, their current valuation of the asset dominates. Conversely, when investors expect to change types more quickly or as relationships are shorter lived, they place more weight on the expected marginal utility.

Distribution of Investors Our measure of matched investors remains as given by (17) and (18). We distinguish between investors whose last trade was in a relationship, with measure n^{ur}, and those whose last trade was a spot transaction, with measure n^{us}. We have then that $n^u = n^{us} + n^{ur}$. This distinction will prove important since the quantity of assets acquired via spot trades can be different than that acquired through relationships. The laws of motion for unmatched investors are given by the following four equations.

$$\begin{aligned}
\dot{n}_{ii}^{ur} &= \delta n_{ii}^{m} - \alpha n_{ii}^{ur} - \beta n_{ii}^{ur} + \lambda\pi_i \sum_{k\neq i} n_{ik}^{ur} - \lambda(1-\pi_i)n_{ii}^{ur} \quad \text{for all } i\in\{1,...,I\}\\
\dot{n}_{ji}^{ur} &= \lambda\pi_i \sum_{k\neq i} n_{jk}^{ur} - \lambda(1-\pi_i)n_{ji}^{ur} - \alpha n_{ji}^{ur} - \beta n_{ji}^{ur} \quad \text{for all } j\neq i\\
\dot{n}_{ii}^{us} &= \beta \sum_{k} n_{ki}^{ur} + \beta \sum_{k\neq i} n_{ki}^{us} - \alpha n_{ii}^{us} + \lambda\pi_i \sum_{k\neq i} n_{ik}^{us} - \lambda(1-\pi_i)n_{ii}^{us} \quad \text{for all } i\in\{1,...,I\}\\
\dot{n}_{ji}^{us} &= \lambda\pi_i \sum_{k\neq i} n_{jk}^{us} - \lambda(1-\pi_i)n_{ji}^{us} - \alpha n_{ji}^{us} - \beta n_{ji}^{us} \quad \text{for all } j\neq i
\end{aligned}$$

The flow of investors whose last trade was a spot transaction is $\alpha_s n^{ur}$ while the flow of investors whose last trade was in a relationship is αn^{us}. Thus, we have that $n^{us} = \frac{\alpha_s}{\alpha+\alpha_s}n^u$ and $n^{ur} = \frac{\alpha}{\alpha+\alpha_s}n^u$. In a steady state, solving the above equations yields the following equations.

$$n_{ii}^{ur} = \frac{(\alpha+\alpha_s)\alpha\delta\pi_i + \lambda\alpha\delta\pi_i^2}{(\lambda+\alpha+\alpha_s)(\alpha+\delta)(\alpha+\alpha_s)} \quad \text{for all } i\in\mathcal{I} \tag{57}$$

$$n_{ji}^{ur} = \frac{\delta\lambda\alpha\pi_i\pi_j}{(\lambda+\alpha+\alpha_s)(\alpha+\delta)(\alpha+\alpha_s)} \quad \text{for all } i\neq j \tag{58}$$

$$n_{ii}^{us} = \frac{\alpha_s\delta(\alpha+\alpha_s)\pi_i + \alpha_s\delta\lambda\pi_i^2}{(\lambda+\alpha+\alpha_s)(\alpha+\delta)(\alpha+\alpha_s)} \quad \text{for all } i\in\mathcal{I} \tag{59}$$

$$n_{ji}^{us} = \frac{\alpha_s\delta\lambda\pi_i\pi_j}{(\lambda+\alpha+\alpha_s)(\alpha+\delta)(\alpha+\alpha_s)} \quad \text{for all } i\neq j \tag{60}$$

Market Clearing In a steady state, the average amount of assets held by matched investors, A^m, unmatched investors whose last trade was spot, A^{us}, and unmatched investors whose last trade was via relationships, A^{ur} must be constant so that $\dot{A^m} = \dot{A^{us}} = \dot{A^{ur}} = 0$ where

$$\dot{A^{us}} = \alpha_s n^{ur} A^{ur} - \alpha n^{us} A^{us} \tag{61}$$

$$\dot{A^{ur}} = \delta n^m A^m - \alpha n^{ur} A^{ur} - \alpha_s n^{ur} A^{ur} \tag{62}$$

$$\dot{A^m} = \alpha n^{us} A^{us} + \alpha n^{ur} A^{ur} - \delta n^m A^m. \tag{63}$$

It follows then that $A^m = A^{us} = A^{ur} = A$ and the flow supply of assets is given by

$$\lambda n^m A + \alpha_s n^u A.$$

A fraction n^m of investors receive a trading shock and bring to the market A units of the asset. Similarly, a fraction n^u of investors contact a spot dealer and bring to the market A units of the asset as well.

The flow demand is written as

$$\lambda n^m \sum_{i \in \mathcal{I}} \pi_i a_i + \alpha_s n^u \sum_{i \in \mathcal{I}} \pi_i a_i^s.$$

The flow of matched investors who receive a trading shock is λn^m. Using the law of large numbers, a fraction π_i will be of type i, and hence will demand a_i. Also, at rate α_s, unmatched investors will receive the opportunity to spot trade. With probability π_i, they will be of type i and demand a_i^s.

Equating supply and demand yields the market clearing condition:

$$\lambda n^m A + \alpha_s n^u A = \lambda n^m \sum_{i \in \mathcal{I}} \pi_i a_i + \alpha_s n^u \sum_{i \in \mathcal{I}} \pi_i a_i^s. \tag{64}$$

The left side of (64) is a constant, whereas the right side is decreasing in p. Thus, there is a unique interdealer market price, p, that is a solution to (64).

Trading Volume Volume of trade is calculated as the sum of all realignment trades executed at the onset of a relationship and subsequent trades made during the relationship after receiving a preference shock. The novelty with the addition of spot trades is that we must keep track of what *type* of assets investors hold. More precisely, asset positions that investors demand from spot trades will be different from those demanded by investors in relationships, so the magnitude of the realignment trades will vary depending on what was the last trade an investor engaged in. Relationship trading volume can be expressed as

$$\mathcal{V}^r = \sum_{i,j} \Big(\alpha n_{ji}^{us} |a_i - a_j^s| + \alpha n_{ji}^{ur} |a_i - a_j| + \lambda n_{ii}^m \pi_j |a_j - a_i| \Big). \tag{65}$$

The first two terms in the summation operator capture the volume of trade driven by realignment trades of investors who are newly matched and hold a portfolio that is aligned for spot transactions and relationships, respectively. The last term represents volume of trade by investors currently in a relationship who trade after receiving a preference shock.

One way to view volume of trade for spot transactions is as consisting only of realignment trades by investors. Spot volume, which we denote as $\mathcal{V}^s$, is given by the following expression:

$$\mathcal{V}^s = \alpha_s \sum_{i,j} \left(n_{ji}^{us} |a_i^s - a_j^s| + n_{ji}^{ur} |a_i^s - a_j| \right). \tag{66}$$

There will be those investors who hold a spot portfolio but are mismatched with respect to their preference type (first term), and those investors who hold relationship portfolios (potentially mismatched) and wish to transition to spot portfolios (second term).

Trading Costs Our measure of expected fees per unit of asset traded for relationships remains largely unchanged with the exception that we pay closer attention to the distribution of asset holdings to correctly calculate fees paid. We have then

$$\mathcal{F}^r = \frac{n^m (r+\delta) \sum_{i,j} \left(f_{ij}^s \Phi_j(a_i^s) + f_{ij} \Phi_j(a_i) \right)}{\mathcal{V}^r} \tag{67}$$

where $f_{ij}^s \equiv n_{ij}^{us}/n^u$. Expected spot trade fees can be obtained by multiplying the distribution of investors and the according fees that they pay. Hence, the measure of spot trade fees per unit of asset traded is

$$\mathcal{F}^s = \frac{\alpha_s \sum_{i,j} \left(n_{ij}^{us} \phi_j(a_i^s) + n_{ij}^{ur} \phi_j(a_i) \right)}{\mathcal{V}^s}. \tag{68}$$