Optimal Merger Remedies*

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Abstract

This paper studies optimal merger remedies when an antitrust authority has a consumer surplus standard. Remedies are modeled as asset divestitures which reduce the marginal cost of the firm receiving the assets, at the expense of the merged firm. If a merger affects only a single market, asset divestitures on their own are not sufficient for the merger to be implemented—synergies are also required. As the market becomes less competitive, it is less likely that any given merger with remedies is implemented. If instead a merger affects several different markets, and the authority cares about consumer surplus aggregated over all markets, then it is optimal to divest as many assets as feasible in some markets and no assets in all remaining markets. In contrast to the single-market analysis, we also show that when a market is less competitive, it is optimal to divest assets in more competitive markets.

Keywords: Antitrust, merger policy, structural remedies, divestitures, data.

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1 Introduction

Antitrust authorities regularly clear mergers subject to the implementation of remedies. For example in the U.S., more than 60% of the mergers that were challenged by the authorities between 2003 and 2012 were later approved subject to remedies (Kwoka, 2014). Similarly, between 1990 and 2014, the European Commission was around 15 times less likely to prohibit a merger, than it was to clear it subject to remedies (Affeldt, Duso, and Szücs, 2018). Typically remedies are "structural", in that they involve the merging parties divesting assets; these assets may include manufacturing facilities, personnel, intellectual property and data.[1] The aim of these asset divestitures is to strengthen existing competitors, or to facilitate entry of new firms, and thereby prevent a merger from harming consumers.

Surprisingly, despite their importance, the existing literature on merger policy largely ignores the possibility of remedies. Allowing for remedies raises several natural questions. For instance, to what extent can remedies substitute for efficiencies—which, absent remedies, would be required for a merger to not harm consumers? And how do market characteristics affect whether remedies are required and, if so, which assets the merger partners should optimally divest, and to whom? Moreover, a merger will often affect several different markets; for example, a supermarket merger impacts consumers in different geographic markets, while a pharmaceutical merger impacts consumers in different product (drug) markets. To the extent that an authority is willing to balance gains and losses across different markets, how does this affect the remedies optimally proposed by the merger partners? For example, is it better to divest few assets in many markets, or many assets in few markets?

In this paper, we offer a tractable framework to study optimal merger remedies. In our model, a merger between two firms may generate synergies, as well as a set of assets that could be feasibly divested to another firm. (This asset-receiving outsider may or may not already be selling in the market.) Divesting assets may raise the marginal cost of the merged firm, while lowering the marginal cost of the firm that receives the assets. In our baseline analysis divesting assets does not generate any revenue for the merged firm, but we relax this later on. We consider the following game. First, the merger partners decide which assets (if any) to divest and to whom, and then propose this to an antitrust authority. Second, using a consumer surplus standard, the authority accepts or blocks this proposal. Finally, firms compete in a Cournot fashion.

We begin by analyzing a merger which affects a single market. First, we characterize the set of mergers and associated divestitures that would be blocked by the antitrust authority. We demonstrate that as the market becomes less competitive, this set increases. Second, we then examine the optimal merger proposal decision by the merger partners. We show that if a

[1] This is contrast to "behavioral" remedies, which involve the merging parties making commitments about future conduct. Structural remedies are usually preferred because they are easier to implement and do not require monitoring (see, e.g., CMA, 2018).

merger without remedies would not be blocked, then this is what the merger partners would propose. If instead such a merger would be blocked, then if the merger partners choose to propose a merger, it must leave consumer surplus unchanged. We show that for the merger partners to optimally propose such a merger, it must generate substantial synergies (in a way that we make precise in the text). That is, remedies cannot fully substitute for merger-induced efficiencies. Moreover, we show that if there is no merger (and associated divestitures) which is both profitable and acceptable to the authority, then the same is true as the market becomes less competitive.

We then turn to a merger which affects multiple markets. If the antitrust authority blocks a merger that harms consumers in any one market, then our single-market analysis applies. However, such a "market-by-market" approach may often be inappropriate. For instance, if the same consumers are present in many of the affected markets (Crane, 2015), or "if a merger with massive competitive benefits would be made unlawful by unfixable anticompetitive effects in a single tiny market" (Werden, 2017), it may be more appropriate to balance gains in some markets against losses in others. Indeed, this is consistent with antitrust policy in some countries.[2] We therefore focus on the benchmark case where the antitrust authority clears a merger (and associated remedies) provided that consumer surplus aggregated across all markets is not reduced.

Within this multimarket setting, we show that, conceptually, one can view the merged firm as choosing (through its proposed remedies) a consumer surplus level and associated profit in each market. In particular, as consumer surplus in any given market is increased, the merged firm's profit in that market decreases. We introduce the concept of a *remedies exchange rate*, which measures how many dollars the merged firm must give up in a market when consumer surplus is increased by one dollar through divestitures. We demonstrate that, under certain conditions, this exchange rate improves as the induced level of consumer surplus increases. As a result, the optimal merger and remedies proposal is 'bang-bang': it involves no asset divestitures in some markets, and maximal asset divestitures in all other markets. We then show—in contrast to our single-market analysis—that as a market is made less competitive, it is more likely that the merged firm proposes no asset divestitures for that market.

We then extend our model in two directions. First, we allow for negotiations between the merger partners and the antitrust authority, concerning which remedies should be implemented. Second, we allow for the merged firm to receive revenues from its divested as-

[2]For example, in the UK, the CMA can clear a merger at Phase 1 if a significant lessening of competition (SLC) and its adverse effects in one market are outweighed by consumer gains "in any market in the United Kingdom (whether or not in the market(s) in which the SLC has occurred or may occur)." (CMA, 2018) See also OECD (2016) for discussion of the so-called "balancing clause" in German merger law. Werden (2017) points out that even the 2010 U.S. merger guidelines allow the market-by-market rule to be broken in certain circumstances.

sets. These revenues are determined endogenously through a bargaining process between the merger partners and the asset-receiving outsider.

Related Literature Much of the literature on mergers focuses on a single market, and assumes that the authority must either accept or reject the merger, without allowing for remedies. At the heart of many of these papers is the Williamson (1968) trade-off, whereby a merger can raise prices due to a market power effect, or lower them due to efficiencies. Farrell and Shapiro (1990) formalize this trade-off in a homogeneous goods Cournot model, and show that for a merger not to harm consumers, it must generate efficiencies in the form of lower marginal costs. Moreover, Nocke and Whinston (2010) show that any merger that does not harm consumers is profitable for the merger partners.

There is a small literature on structural merger remedies. In a homogeneous goods setting, Vergé (2010) and Vasconcelos (2010) assume that firms have the same cost functions, and impose strong functional form assumptions on those cost functions. Vergé (2010) shows, for example, that with three firms and no merger synergies, a merger must harm consumers even accounting for asset divestitures. In a similar vein, focusing on four symmetric firms, Vasconcelos (2010) shows that in equilibrium remedies are never used. Meanwhile Cosnita-Langlais and Tropeano (2012) examine whether merger partners that are privately informed about the magnitude of any synergies, can credibly signal that information through the remedies that they propose to the antitrust authority.

While in our paper a merger may affect different product or geographic markets, Johnson and Rhodes (2021) and Nocke and Schutz (2023) consider mergers between differentiated multiproduct firms within a single market. The former studies mergers between vertically differentiated firms in a Cournot setting, and finds for example that mergers without synergies can raise consumer surplus, but only when certain necessary observable conditions on the pre-merger industry structure are satisfied. The latter studies mergers between multi-product firms in a differentiated Bertrand setting, and among other things shows that absent synergies the merger's harm to consumers is proportional to the naively-computed change in the Herfindahl index.[3] However none of these papers consider merger remedies.

The rest of the paper proceeds as follows. Section 2 studies optimal remedies when a merger affects a single market, while Section 3 considers the case where a merger affects multiple markets. Section 4 extends our baseline analysis to allow for bargaining between the antitrust authority and the merger partners, and for the merger partners to earn revenue from divesting assets. Section 5 concludes with a discussion of future avenues for research. All omitted proofs are available in the Appendix.

[3]Along these lines, Nocke and Whinston (2022) show that the synergies required for a merger not to harm consumers are increasing in the naively-computed Herfindahl index.

2 Single-Market Analysis

2.1 The Setting

There is a set $\mathcal{N}$ of firms producing a homogeneous good with constant returns and competing in a Cournot fashion. The marginal cost of firm i is denoted $c_i \geq 0$. Inverse demand is given by $P(Q)$, where Q denotes aggregate output. We impose standard assumptions on demand ensuring that there exists a unique Nash equilibrium in quantities for any vector of marginal costs: $P(0) > 0$, $\lim_{Q\to\infty} P(Q) = 0$, and for all Q such that $P(Q) > 0$, $P'(Q) < 0$ and $\sigma(Q) < 1$, where $\sigma(Q) \equiv -QP''(Q)/P'(Q)$ is the curvature of inverse demand.

We consider a potential merger among two *active* firms, i.e., firms that produce a strictly positive output pre merger. The set of (potential) merger partners is exogenous and denoted $\mathcal{M} \subset \mathcal{N}$. The set of non-merging outsiders is denoted $\mathcal{O} \equiv \mathcal{N} \setminus \mathcal{M}$; not all of these outsider need be active pre-merger, and the set of active outsiders may be affected by the merger.

The merger partners have a t-dimensional vector of assets, $K \in \mathbb{R}^t_+$, that could feasibly be divested to a single rival firm. (For simplicity, we treat the amount of each asset as a continuous variable.) A remedy is denoted (k, i), where $k \leq K$ is the vector of assets being divested and $i \in \mathcal{O}$ is the asset-receiving outsider (which may or may not be active pre merger). Let $\mathcal{P} \subseteq ([0, K], \mathcal{O})$ denote the set of feasible mergers and associated divestitures. Following a divestiture $(k, i) \in \mathcal{P}$, the merged firm's marginal cost is $\bar{c}_M(-k)$ and the receiving outsider's marginal cost is $\bar{c}_i(k)$. The merger has no effect on any outsider's marginal cost, except through divestitures; that is, $\bar{c}_i(0) = c_i$. We assume that post-merger marginal costs $\bar{c}_M(-k)$ and $\bar{c}_i(k)$ are weakly decreasing (in each argument) and $\mathcal{C}^2$. For now, we assume that the merger partners do not receive any revenue from divesting assets (but we relax this assumption later on).

We assume that the antitrust authority applies a consumer surplus standard. It therefore blocks the merger and associated remedy (k, i) if and only if the resulting post-merger consumer surplus is strictly lower than the pre-merger level.[4] The game proceeds as follows: The merger partners decide whether or not to propose their merger and any remedy (k, i); if so, the authority then accepts or blocks the proposal; given the resulting market structure, firms compete in a Cournot fashion. Throughout, we focus on equilibria in which a merger is proposed if and only if it would not be blocked and is strictly profitable.[5]

2.2 Preliminary Analysis

We begin by briefly recapping standard analysis of Cournot oligopoly. To ease notation we drop the firm subscript.

[4] A merger without remedy is denoted $(0, i)$, $i \in \mathcal{O}$.

[5] Equivalently, we could assume that there is an arbitrarily small merger proposal cost.

A firm with marginal cost c chooses output q to maximize its profit $(P(Q) - c)\, q$. The firm's *output fitting-in function* $r(Q; c)$ is the output level that solves its first-order condition:

$$r(Q; c) = \frac{\max\{P(Q) - c, 0\}}{-P'(Q)}. \tag{1}$$

We can then use the output fitting-in function to rewrite the expression for firm profit, and thereby obtain the firm's *profit fitting-in function*:

$$\pi(Q; c) = -\left(r(Q; c)\right)^2 P'(Q). \tag{2}$$

Notice that given our assumptions on market demand $P(Q)$, both $r(Q; c)$ and $\pi(Q; c)$ are weakly decreasing in Q and c, and strictly so whenever $c < P(Q)$.

The pre-merger aggregate equilibrium output Q^* is then the unique Q which solves

$$Q = \sum_{i \in \mathcal{N}} r(Q; c_i). \tag{3}$$

(Note that an arbitrary subset of firms may be inactive in the pre-merger equilibrium, in which case $r(Q^*; c_i) = 0$.) Following a merger with remedy (k, i), the aggregate equilibrium output $\overline{Q}^*(k, i)$ is the unique solution in Q to

$$Q = r\left(Q; \overline{c}_M(-k)\right) + \sum_{j \in \mathcal{O} \setminus \{i\}} r(Q; c_j) + r(Q; \overline{c}_i(k)). \tag{4}$$

(Note that the merger may induce some firms to become active or inactive.) Consumer surplus as a function of aggregate output Q is given by

$$v(Q) = \int_0^Q [P(z) - P(Q)] dz, \tag{5}$$

and is thus strictly increasing in Q.

Comparative statics are well behaved. The following result is standard and the proof is therefore omitted.

Lemma 1. *A decrease in the marginal cost of any active firm i leads to a strict increase in aggregate equilibrium output and in firm i's equilibrium profit, and a strict decrease in the equilibrium profit of any other active firm $j \neq i$.*

2.3 Equilibrium Analysis

We now turn to the equilibrium analysis of our two-stage game. We proceed by backward induction: we first characterize the set of merger proposals that would be approved by the antitrust authority and then derive the merger partner's optimal proposal.

Let $\mathcal{B} \subseteq \mathcal{P}$ denote the antitrust authority's *blocking set*, i.e., the set of mergers (k, i) that strictly reduce consumer surplus relative to the pre-merger outcome, and $\mathcal{U} \subset \mathcal{P}$ the set that leaves consumer surplus unchanged.

We begin by examining when a merger without divestitures, $(0, i)$, lies in the blocking set.

Lemma 2. *There exists a $\widehat{c}_M < \min_{i \in \mathcal{M}} c_i$ such that a merger without divestitures lies in the blocking set if and only if $\overline{c}_M(0) > \widehat{c}_M$. (Note that $\widehat{c}_M$ could be negative, in which case a merger without divestitures is always blocked.)*

Proof. It follows from Lemma 1 that this cutoff $\widehat{c}_M$ exists. Moreover, it solves

$$r(Q^*; \widehat{c}_M) = \sum_{i \in \mathcal{M}} r(Q^*; c_i),$$

which immediately implies that $\widehat{c}_M < \min_{i \in \mathcal{M}} c_i$. □

Hence, as was pointed out by Farrell and Shapiro (1990), for a merger among active firms not to harm consumers, it must involve synergies in that $\widehat{c}_M < \min_{i \in \mathcal{M}} c_i$.

As shown by Nocke and Whinston (2010), the cutoff $\widehat{c}_M$ decreases with the pre-merger market price $P(Q^*)$. That is, as the market becomes less competitive, the set of mergers without divestitures that would be blocked by the antitrust authority increases. The following proposition shows that the same still holds in the more general case with arbitrary divestitures.

Proposition 1. *Suppose that merger (k, i) is in the blocking set $\mathcal{B}$. Then, that same merger remains in the blocking set after a decrease in the competitiveness of the market, i.e., an increase in the pre-merger price $P(Q^*)$—either because of a change in demand or because of an increase in the marginal cost of some firm $h \notin \mathcal{M} \cup \{i\}$. That is, the blocking set increases (in the set order) as the market becomes less competitive.*

Proof. From equations (3) and (4), merger (k, i) strictly reduces aggregate output (and is therefore in the blocking set) if and only if

$$\sum_{j \in \mathcal{N}} r(Q^*; c_j) > r(Q^*; \overline{c}_M(-k)) + r(Q^*; \overline{c}_i(k)) + \sum_{j \in \mathcal{N} \setminus (\mathcal{M} \cup \{i\})} r(Q^*; c_j).$$

Using equation (1) and the assumption that both merger partners are active pre-merger, this condition can be rewritten as

$$2P(Q^*) - \sum_{j \in \mathcal{M}} c_j - \max\{P(Q^*) - \overline{c}_M(-k), 0\} - \max\{P(Q^*) - \overline{c}_i(k), 0\} + \max\{P(Q^*) - c_i, 0\} > 0.$$

As $P(Q^*)$ increases, the merger partners remain active, so this inequality remains valid. To complete the proof, note that the lefthand side is weakly increasing in $P(Q^*)$. □

Whether or not a merger is in the blocking set depends on the relative strengths of the merger's market power and efficiency effects. One novel element of our analysis is that the efficiency effect now consists of two parts: the (standard) impact on the insiders' marginal cost and the (new) impact on the marginal cost of the asset-receiving outsider. In the simplest case in which both the merged firm is active post merger and the asset-receiving outsider is active pre merger, both components of the efficiency effect, $\min_{j \in \mathcal{M}} c_j - \overline{c}_M(-k)$ and $\overline{c}_i(k) - c_i$, are independent of how competitive the market is. Since the market power effect of the merger, $P(Q^*) - \max_{j \in \mathcal{M}} c_j$, is stronger when the market is less competitive, the result follows immediately. In the more interesting case in which the asset-receiving outsider becomes active only after receiving the assets, $\overline{c}_i(k) < P(Q^*) < c_i$, the new component of the efficiency effect becomes $P(Q^*) - \overline{c}_i(k)$ and is therefore also stronger when the market is less competitive. As the proof demonstrates, however, that effect does not outweigh the change in the market power effect.

Folding backwards, we now analyze the merger partners' proposal decision. Clearly, if all feasible mergers are in the blocking set, then no merger is proposed. We now focus on the more interesting case in which not all feasible mergers would be blocked.

We start with the simplest case in which synergies are sufficiently strong so that a merger without remedies would not be blocked when proposed.

Proposition 2. *Suppose that $\overline{c}_M(0) \leq \widehat{c}_M$. Then, it is an equilibrium for the merger partners to propose a merger without divestitures, $(0, i)$, and the antitrust authority approves it.*

Proof. It follows from Lemma 2 that a merger without remedies is not in $\mathcal{B}$ and so will not be blocked.

First, we show that such a merger is strictly profitable. To see this, note that if $\overline{c}_M(0) = \widehat{c}_M$ the merger is strictly profitable because:

$$\begin{aligned} \pi(Q^*; \widehat{c}_M) &= [P(Q^*) - \widehat{c}_M] r(Q^*; \widehat{c}_M) \\ &= [P(Q^*) - \widehat{c}_M] \sum_{i \in \mathcal{M}} r(Q^*; c_i) \\ &> \sum_{i \in \mathcal{M}} [P(Q^*) - c_i] r(Q^*; c_i), \end{aligned}$$

where the second equality follows from the fact that a merger with post-merger marginal cost $\widehat{c}_M$ does not affect aggregate output, and the inequality from Lemma 2, according to which $\overline{c}_M(0) < \min_{i \in \mathcal{M}} c_i$. Next, note from Lemma 1 that the merged firm's equilibrium profit is decreasing in marginal cost, implying that the merger is also strictly profitable for any $\overline{c}_M(0) \leq \widehat{c}_M$.

Finally, note that a merger with divestitures must lead to weakly lower profit. This follows from Lemma 1 and the fact that divestitures would weakly increase the merged firm's marginal cost and weakly lower the marginal cost of one asset-receiving outsider. □

We now turn to the more interesting case where $\overline{c}_M(0) > \widehat{c}_M$ such that a merger without remedies would be blocked. We first investigate whether remedies may serve as a substitute for synergies. The following proposition shows that even if remedies can prevent consumer harm, for the merger to be profitable, synergies are still necessary.

Proposition 3. *Suppose that $\overline{c}_M(0) > \widehat{c}_M$. Suppose also that the merger (k, i) weakly increases consumer surplus. Then, the merger is strictly profitable only if $\overline{c}_M(-k) < \min_{j \in \mathcal{M}} c_j$. That is, for a merger not to harm consumers requires synergies even after divestitures.*

Proof. Suppose towards a contradiction that the merger is strictly profitable and yet $\overline{c}_M(-k) \geq \min_{j \in \mathcal{M}} c_j$. Then,

$$\begin{aligned} \pi(\overline{Q}^*; \overline{c}_M(-k)) &\leq \pi(Q^*; \overline{c}_M(-k)) \\ &\leq \pi(Q^*; \min_{j \in \mathcal{M}} c_j) \\ &< \sum_{j \in \mathcal{M}} \pi(Q^*; c_j), \end{aligned}$$

where the first line follows because the merger weakly increases aggregate output, the second line follows by the assumption that $\overline{c}_M(-k) \geq \min_{j \in \mathcal{M}} c_j$, and the third because both merger partners are active pre merger. But this means that the merger is strictly unprofitable, a contradiction. □

Hence, divestitures are not a perfect substitute for synergies: for a merger to be profitable and not harm consumers requires not only that $\overline{c}_M(0) < \min_{j \in \mathcal{M}} c_j$ but actually that $\overline{c}_M(-k) < \min_{j \in \mathcal{M}} c_j$. Although this result may seem surprising, the intuition is straightforward. To fix ideas, suppose that the merger does not involve synergies after divestitures so that $\overline{c}_M(-k) = \min_{j \in \mathcal{M}} c_j$. Now, if the merger does not affect consumer surplus, the market price is unchanged as well, implying that the merged firm's profit is equal to the pre-merger profit of the more efficient merger partner, which is strictly less than the sum of the pre-merger profits of the merger partners.

In contrast to the case of mergers without divestitures, it no longer holds that a merger that does not harm consumers is necessarily profitable. The tension arises because divestitures that benefit consumers harm the merged firm, both by increasing aggregate output and by raising the merged firm's marginal cost. This suggests that the merger partners will never propose a merger that strictly benefits consumers, as the next proposition will indeed demonstrate. Before stating the result, we derive the level of post-merger marginal cost, $\tilde{c}$, that would leave the merger partners' joint profit unchanged, conditional on the merger not changing consumer surplus. That is, $\tilde{c}$ solves $\pi(Q^*;\tilde{c}) = \sum_{j\in\mathcal{M}} \pi(Q^*;c_j)$. Using equation (2), this yields

$$\tilde{c}_M = P(Q^*) - \sqrt{\sum_{j\in\mathcal{M}} [P(Q^*) - c_j]^2}.$$

Proposition 4. *Suppose that $\overline{c}_M(0) > \widehat{c}_M$. Let $\underline{c}_M \equiv \min_{(k,i)\in\mathcal{U}} \overline{c}_M(-k)$ denote the lowest post-merger marginal cost associated with any merger that leaves consumer surplus unchanged. If $\underline{c}_M < \tilde{c}_M$, then one such merger will be proposed and approved in equilibrium; otherwise, no merger will be proposed.*

Proof. To see why a eventually implemented merger must leave consumer surplus unchanged, suppose by contraction that merger (k,i) would strictly increase consumer surplus. As merger $(0,i)$ would strictly reduce consumer surplus (since $\overline{c}_M(0) > \widehat{c}_M$), there must exist by continuity another merger (k',i), with $k' \leq k$, that leaves consumer surplus unchanged (and is therefore not in the blocking set). This alternative merger (k',i) results in weakly lower marginal cost of the merged firm and a strictly lower aggregate output than (k,i). As a result, the merged firm's profit is strictly larger under (k',i). Next, note that conditional on leaving consumer surplus unchanged, the merged firm's profit is maximized when its marginal cost is minimized. Hence, any merger that is eventually implemented must lead to post-merger marginal cost $\underline{c}_M$. It follows from Proposition 3 that such a merger is strictly profitable if and only if $\underline{c}_M < \tilde{c}_M$. □

Our earlier Proposition 1 demonstrated that, as the industry becomes less competitive, the merger partners may have to offer larger remedies for the merger not to be blocked. The next proposition reveals that this is in conflict with the profitability of the merger.

Proposition 5. *Suppose that $\overline{c}_M(0) > \widehat{c}_M$. If no merger is proposed and approved in equilibrium, then the same is true after a decrease in the competitiveness of the market, i.e., after an increase in the pre-merger price $P^*(Q^*)$—either because of a change in demand or because of an increase in the marginal cost of some firm $h \notin \mathcal{M} \cup \{i\}$.*

Proof. This follows because $\tilde{c}_M$ is strictly decreasing in $P(Q^*)$. □

Proposition 5 implies that for a merger to be proposed merger-induced synergies will need to be larger (in the sense that $\overline{c}_M(\cdot)$ needs to be lower) in less competitive industries.

3 Multimarket Analysis

In this section we consider the empirically relevant case in which the merger partners are active in more than one market. If the antitrust authority adopts a "market-by-market" approach, whereby it blocks a merger that harms consumers in any single market, then our previous analysis applies. However, as discussed in the Introduction, there are good reasons why an antitrust authority may deviate from this approach. We therefore now analyze the case where the authority is willing to trade off gains and losses across markets.

3.1 The Setting

There is a continuum of independent markets, indexed by $h \in [0,1]$, and a set $\mathcal{N} = \{1, ..., N\}$ of firms, each of which is present in a subset of markets. Within each market, competition is as in our earlier single-market setting, with all variables now having a superscript denoting the market.

As before, we consider a merger among the two firms in set $\mathcal{M} \subset \mathcal{N}$. For simplicity, we assume that both merger partners are active in each market pre merger.[6]

In this section, we assume that the antitrust authority cares about consumer surplus aggregated over all markets. In particular, the authority's blocking set $\mathcal{B} \subset ([0, K^h]^h, \mathcal{O}^h)^{h \in [0,1]}$ is the set of mergers that satisfy

$$\overline{V}^* \equiv \int_{[0,1]} v^h(\overline{Q}^{h*}(k^h, i^h))dh < \int_{[0,1]} v^h(Q^{h*})dh \equiv V^*. \tag{6}$$

Similarly, the merger partners propose a merger if and only if that merger will not be blocked and would strictly increase their profit aggregated over all markets, i.e.,

$$\overline{\Pi}^*_M \equiv \int_{[0,1]} \pi^h(\overline{Q}^{h*}(k^h, i^h); \overline{c}^h_M(-k^h))dh > \sum_{i \in \mathcal{M}} \int_{[0,1]} \pi^h(Q^{h*}; c^h_i)dh \equiv \Pi^*_M. \tag{7}$$

We consider again a two-stage game in which the merger partners first decide whether or not to propose their merger and a set of divestitures $((k^h, i^h))^{h \in [0,1]}$; if so, the authority then accepts or blocks the proposal. To rule out uninteresting cases, we assume that the complement of the blocking set is non-empty and that a merger without divestitures in any market is strictly profitable; if either condition were violated, no merger would be proposed in equilibrium.

3.2 Equilibrium Analysis

Mirroring our analysis of the single-market case, we first consider the antitrust authority's optimal blocking set.

We saw in the single-market analysis that, as a market becomes less competitive, the

[6]If only one (or no) merger partner were active in a given market, then we implicitly assume that there are no synergies and no divestible assets, so that we can ignore such a market without loss.

blocking set increases, as any given merger (k, i) is more "likely" to harm consumers. We now show that this is not necessarily the case in a multimarket setting: when markets are heterogeneous, a merger may harm consumers in some markets and benefit them in others. Suppose now that we make a positive measure of markets less competitive, for instance, by increasing the marginal costs of firms not involved directly in the merger. On the one hand, from Proposition 1, this implies that the merger may now harm consumers in some markets where it previously benefitted them. This, in turn, implies that a merger is now optimally blocked that was not previously. On the other hand, it is easy to construct examples where the magnitude of the harm to consumers induced by the merger is reduced (while remaining positive).[7] As a result, it may no longer be optimal to block the merger if the gains in some markets now outweigh the (smaller) losses in others.

We now turn to the first stage of the game and examine the merger partners' optimal proposal. The simplest case arises when even a merger with no divestiture in any market would not be blocked by the antitrust authority. Given our assumption that such merger would be strictly profitable, it follows from our single-market analysis, that it would be proposed (and approved) in equilibrium.

The interesting (and more relevant) case arises when a merger without divestitures in any market would be in the authority's blocking set. Following the logic of our single-market analysis, the merger partners would never find it optimal to propose a merger that strictly increases aggregate consumer surplus. Amongst those mergers that would leave aggregate consumer surplus unchanged, the merger partners chooses one that maximizes their profit. A profit-maximizing set of divestitures need not leave consumer surplus in each market unchanged; indeed, such a set may not even exist. Instead, in general, the merger partners will optimally propose divestitures that will increase consumer surplus in some markets and decrease it in others. We now show that we can recast the merger partners' problem as one of choosing a (feasible) consumer surplus level in each market, subject to the constraint that aggregate consumer surplus remains unchanged.

To this end, it is instructive to introduce the concepts of the *synergies* and *divestitures curves*.

3.2.1 The Synergies Curve

The synergies curve in a given market is the locus of all feasible combinations of market-level consumer surplus $v(\overline{Q}^*)$ and the merged firm's profit $\pi(\overline{Q}^*; \overline{c}_M)$ arising from different values of the post-merger marginal cost $\overline{c}_M(0)$. As illustrated by Figure 1, since both v and π are strictly decreasing in $\overline{c}_M(0)$, the synergies curve is upward sloping.

[7] See the Online Appendix for such an example.

3.2.2 The Divestitures Curve

The divestitures curve in a given market is the locus of all feasible combinations of market-level consumer surplus $v(\overline{Q}^*)$ and the merged firm's profit $\pi(\overline{Q}^*; \overline{c}_M))$ arising from different asset divestitures.

It is useful to distinguish between two cases. We begin with the case in which divestible assets are redundant in the following sense.

Definition 1. *Assets are said to be* ***redundant*** *if* $\overline{c}_M(-k) = \overline{c}_M(0)$ *for all* $k \leq K$.

The redundant assets cases arises, for instance, when the divestible assets are data or intellectual property or are held in duplicate by the merged firm, such that divesting them has no effect on the merged firm's cost.

Redundant assets case. Consider a single market h and start by fixing the identity of the asset-receiving outsider, firm i. Dropping the market superscript for notational simplicity, let $\overline{v}_{ND} \equiv v(\overline{Q}^*(0, i))$ denote the consumer surplus resulting from no divestitures. As more assets are divested to firm i, the merged firm's marginal cost does not change (by definition of redundant assets) whereas firm i's marginal cost decreases; hence, market-level output and consumer surplus both increase and the merged firm's market-level profit decreases. If all K assets are divested to firm i, the resulting consumer surplus is $v(\overline{Q}^*(K, i))$. Let $\overline{v}_{\max} \equiv \max_{i \in \mathcal{O}} v(\overline{Q}^*(K, i))$ be the highest achievable level of consumer surplus in that market. By continuity, any consumer surplus level between $\overline{v}_{ND}$ and $\overline{v}_{\max}$ can be achieved through some merger (k, i). Note also that there may be more than one merger that induces a given v in that interval but they all result in the same profit for the merged firm: the merged firm's profit depends only on its own marginal cost and market-level output, and divestitures affect only the latter when assets are redundant. For a given market, denote by $\mathcal{P}(v)$ the set of divestitures that would result in consumer surplus $v \in [\overline{v}_{ND}, \overline{v}_{\max}]$. That is,

$$d_M(v) = \pi_M(\overline{Q}^*(k, i); \overline{c}_M(-k)) \text{ for } (k, i) \in \mathcal{P}(v).$$

.

Figure 2 combines the synergies curve with the divestitures curve in the redundant assets case. The two curves meet at the outcome associated with no divestitures.

Consider a change in the set of divested assets that induces an increase in aggregate output $\overline{Q}^*$. The effect on the merged firm's profit is given by

$$\frac{\partial \pi(Q; \overline{c}_M)}{\partial Q} = QP'(Q)s_M(Q)\left[2 - s_M(Q)\sigma(Q)\right] \leq 0, \tag{8}$$

where $s_M(Q) \equiv r(Q; \overline{c}_M)/Q$ is the merged firm's market share. The fact that this is negative

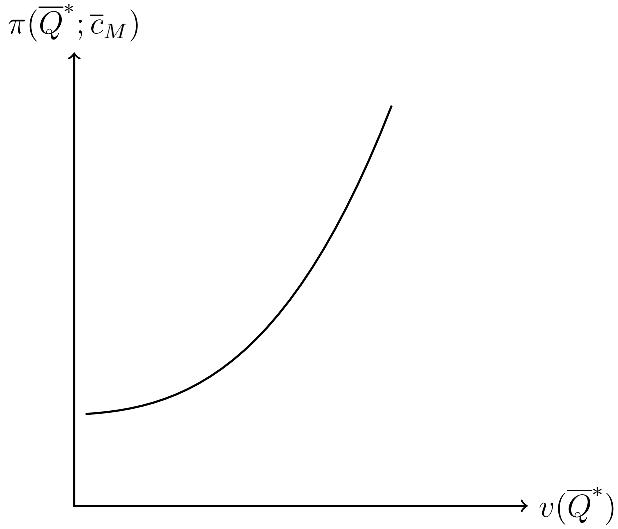

Figure 1: The synergies curve

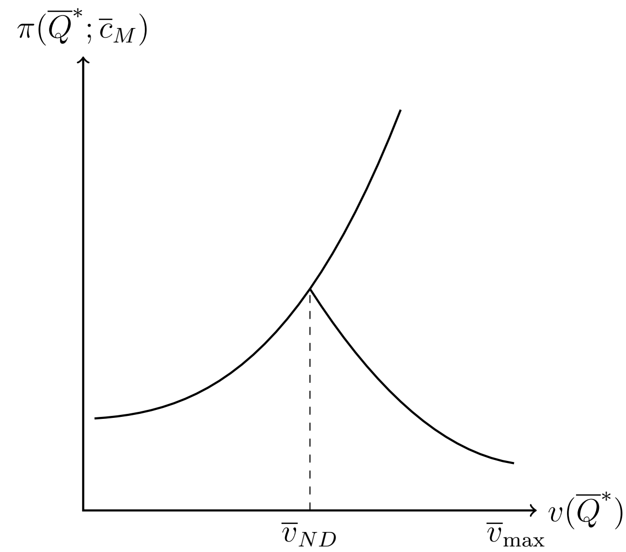

Figure 2: The synergies and divestitures curves with redundant assets

(and strictly so as long as the merged firm is active) follows from Lemma 1. Meanwhile, the effect on market-level consumer surplus is $dv(Q)/dQ = -QP'(Q) > 0$. Hence, the divestitures curve slopes down. Indeed, the slope of the curve gives how much profit the merged firm has to give up to increase consumer surplus; we call this slope the *remedies exchange rate*.

The remedies exchange rate is given by

$$-d'(v) = s_M(Q(v))\,[2 - s_M(Q(v))\sigma(Q(v))] \geq 0, \tag{9}$$

where $Q(v)$ is the output such that consumer surplus in that market equals v.

We henceforth assume the following regularity condition on market demand:

Assumption 1. *For all $Q > 0$ such that $P(Q) > 0$, market demand satisfies*

$$\min\{3 - \sigma(Q), 2[1 - \sigma(Q)][2 - \sigma(Q)]\} + Q\sigma'(Q) \geq 0.$$

Assumption 1 holds provided that $\sigma'(Q)$ is not too negative, and therefore ensures that inverse demand does not become "too concave" as market-level output increases. The condition is trivially satisfied by any demand function with constant curvature, such as linear demand. It is also satisfied by demands that are derived from many common distributions. For example, if demand is proportional to $1 - F(p)$, Condition 1 is satisfied for various common distributions F; see the Online Appendix for further details.

The following lemma characterizes some useful properties of the divestitures curve in the redundant assets case.

Lemma 3. *Suppose that assets are redundant. The divestitures curve $d_M(v)$, defined on $[\overline{v}_{ND}, \overline{v}_{\max}]$, is weakly decreasing and convex (and strictly so if $v > \overline{v}_{ND}$ and $d_M(v) > 0$).*

We already explained why the divestitures curve is decreasing: this is because consumer surplus is strictly increasing whereas the profit fitting-in function is decreasing in market-level output. We now explain why the divestitures curve is convex. First, note that consumer surplus is convex in output. As such, as v increases, an additional dollar increase in v can be achieved through successively smaller increases in output Q. Secondly, in the special case where market demand is linear, a unit increase in Q reduces market price by the same amount. However when Q is higher, the merged firm's market share is lower, and so it is hurt less by any given reduction in the market price. This explains why the divestitures curves is convex when demand is linear. Thirdly, when demand is non-linear, a given increase in output Q does not reduce market price by the same amount. Assumption 1 ensures that as Q increases demand does not become too concave, and so price does not fall too quickly and hence the divestitures curve is still convex.

Complementary assets case. We now turn to the case where assets are no longer redundant (in that $\overline{c}_M(-k)$ is not constant for all $k \leq K$) but instead are *complementary* as defined below. We then show that, under complementarity, the divestitures curve has the same properties as when assets are redundant.

Let $c^*_M(c;i)$ denote the minimized marginal cost of the merged firm when assets are divested to outsider i in such a way that the outsider's marginal cost equals c:

$$c^*_M(c;i) \equiv \min_{k \in [0,K]} \overline{c}_M(-k) \quad \text{s.t. } \overline{c}_i(k) = c.$$

For simplicity, we assume that $c^*_M(c;i)$ is twice differentiable. As an example, in the case of one-dimensional assets, this would hold if both $\overline{c}_M(-k)$ and $\overline{c}_i(k)$ are twice differentiable, with $\overline{c}_i'(k) < 0$ for all $k \in [0, K]$.

Definition 2. *Assets are said to be* ***complementary*** *if $c^*_M(c;i)$ is strictly decreasing and concave for all $(c;i) \in ([\overline{c}_i(K), c_i]; \mathcal{O})$.*

A simple example of when assets are complementary in the sense of Definition 2 is when assets are one-dimensional and each firm has increasing returns from these assets. As the merged firm divests more and more of these assets, its marginal cost increases by less and less, whilst the cost of the asset-receiving outsider decreases by more and more.

We now define a *conditional* divestitures curve $d_M(v;i)$ on $[\overline{v}_{ND}, \overline{v}_{\max}(i)]$ as the highest achievable profit for the merged firm, conditional on divesting assets to outsider i and inducing

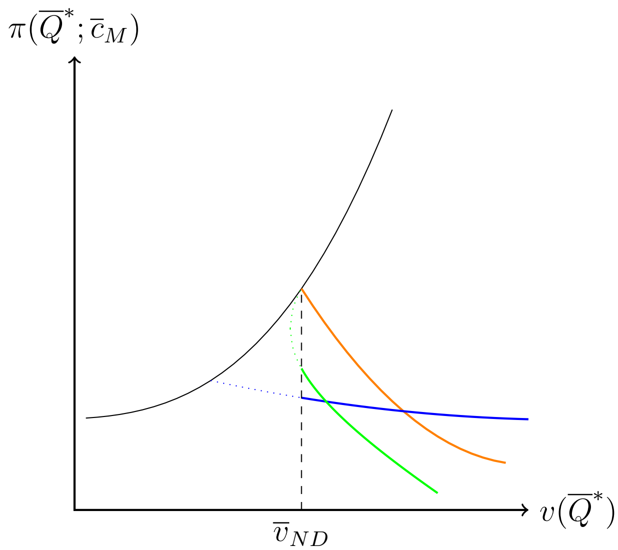

Figure 3: Examples of conditional divestitures curves with complementary assets

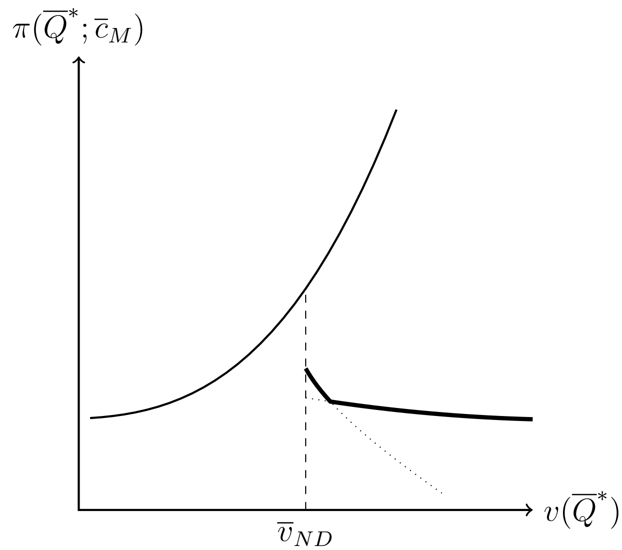

Figure 4: Example of the divestitures curve with complementary assets

consumer surplus level v, where $\overline{v}_{\max}(i) \equiv \max_{k\in[0,K]} v(\overline{Q}^*(k,i))$ is the highest achievable level of v. Figure 3 depicts three such conditional divestitures curves, one for each outsider (as well as the synergies curve). The orange conditional divestitures curves meets with the synergies curve at $v(\overline{Q}^*) = \overline{v}_{ND}$. This represents a situation where even small divestitures reduce the asset-receiving outsider's marginal cost more than they increase the merged firm's marginal cost. Hence, even small divestitures raise consumer surplus above $\overline{v}_{ND}$, and thus the conditional divestitures curve is continuous in v around $\overline{v}_{ND}$. In the figure, the green and blue conditional divestitures curves are discontinuous at $\overline{v}_{ND}$. The green curve depicts the case where small divestitures reduce the asset-receiving outsider's marginal cost less than they increase the merged firm's marginal cost. Hence small divestitures actually *reduce* aggregate output and consumer surplus, as shown by the backward-bending dotted green curve in the figure. Only when asset divestitures are large enough, can consumer surplus exceed $\overline{v}_{ND}$. The blue curve in the figure represents a situation where the asset-receiving outsider is initially inactive. Conceptually, one can think of proceeding in two steps. First, assets are divested until this firm's marginal cost has been reduced to a level where it is just active; this is the same as moving down the synergies curve from the point $v = \overline{v}_{ND}$. Second, divesting more assets to this firm then leads to increased output and consumer surplus. The dotted blue curve represents divestitures where consumer surplus is still below $\overline{v}_{ND}$, while the thick blue line represents the conditional divestitures curve (i.e., outcomes where consumer surplus weakly exceeds $\overline{v}_{ND}$).

Lemma 4. *The slope of the conditional divestitures curve is*

$$-d'_M(v;i) = s_M(Q(v)) \left[2 - s_M(Q(v))\sigma(Q(v)) - 2\frac{(\overline{n}(Q(v);i) + 1 - \sigma(Q(v)))c^{*\prime}_M(c(v);i)}{1 + c^{*\prime}_M(c(v);i)} \right], \tag{10}$$

where $\overline{n}(Q(v);i)$ is the number of active firms when induced consumer surplus is v and the asset-receiving outsider is i, and $c(v)$ is that outsider's marginal cost at consumer surplus v.

Proof. Omitting the dependence of $Q(v)$ on v, we begin by differentiating the profit fitting-in function:

$$\frac{d\pi(Q; c^*_M(c(Q);i))}{dQ} = QP'(Q)s_M(Q) \left[2 - s_M(Q)\sigma(Q) - \frac{2c^{*\prime}_M(c(Q);i)}{P'(Q)} \frac{dc}{dQ} \right].$$

Note that, from the price fitting-in functions, when $1 + c^{*\prime}_M(c(Q);i) > 0$ we have that

$$\frac{dc}{dQ} = P'(Q)\frac{\overline{n}(Q;i) + 1 - \sigma(Q)}{1 + c^{*\prime}_M(c(Q);i)}.$$

Combining the above two equations, and using the fact that $v'(Q) = -P'(Q)Q$, we obtain the expression for $d'_M(v;i)$ in the statement of the lemma.

□

Comparing with the slope of the divestitures curve in the redundant assets case (see equation (9)), there is a new term in square brackets—which reflects that now the merged firm's marginal cost changes as it moves down a conditional divestitures curve. To see that this term is positive note that, for v to increase beyond $\overline{v}_{ND}$, the asset-receiving outsider has to be active and its marginal cost to decrease. By assumption, this increases the merged firm's marginal cost but by less than the decrease in the asset-receiving outsider's marginal cost, as otherwise v would not increase; that is $0 > c^{*\prime}_M > -1$. The fact that this new, third term is positive implies that $d'_M(v;i) \leq 0$, and strictly so when $s_M(Q(v)) > 0$.

The bold curve in Figure 4 is the upper envelope of the conditional divestitures curves; this is the (efficient) divestitures curve $d_M(v)$, defined on $[\overline{v}_{ND}, \overline{v}_{\max}]$, where $\overline{v}_{\max} \equiv \max_i \overline{v}_{\max}(i)$. Note that even if all conditional divestitures curves were backward bending, it would never be optimal to offer divestitures on the backward-bending part; this would make it harder to satisfy the antitrust authority and is clearly worse for the merged firm's profit compared to not offering any divestitures at all.

The following result is the analog of Lemma 3.

Lemma 5. *Suppose that assets are complementary. The divestitures curve $d_M(v)$, defined on $[\overline{v}_{ND}, \overline{v}_{\max}]$, is weakly decreasing and convex (and strictly so if $v > \overline{v}_{ND}$ and $d_M(v) > 0$).*

The convexity of the divestitures curve follows from the fact that each conditional divestitures curve is convex. The intuition is similar to the redundant assets case, except now as we move down a conditional divestitures curve, the merged firm's marginal cost is increasing. Asset complementarity, however, implies that the induced increase in the merged firm's marginal cost becomes smaller and smaller, therefore reinforcing the convexity of the conditional divestitures curve when the merged firm's marginal cost is held fixed. Intuitively, because the divestitures curve is the upper envelope of the convex conditional divestitures curves, it is convex as well. The proof makes this intuition precise, taking into account that the upper bounds of the supports of the conditional divestitures curves may be different for different curves.

As the divestitures curve is decreasing and convex, it is necessarily continuous, except possibly at $v = \overline{v}_{ND}$. A discontinuity may arise at $v = \overline{v}_{ND}$ when the asset-receiving outsider is either initially inactive or the merged firm's marginal cost increases faster than the outsider's marginal cost decreases (i.e., when $c_M^{*\prime}(c; i) < -1$). The continuity of the divestitures curve above $\overline{v}_{ND}$ implies that the merged firm's marginal cost increases continuously as we move down the divestitures curve. In the case of one-dimensional assets, this in turn implies that the amount of divested assets also increase continuously as v is increased. However, this may not hold when assets are multi-dimensional.

3.2.3 Optimal Divestitures

As shown above, the merger partners' problem can be cast as one of choosing a feasible consumer surplus level in each market, subject to the constraint that consumer surplus aggregated over all markets remains unchanged. Recalling that aggregate consumer surplus pre merger is denoted V^*, the merger partners' optimization problem can be written as

$$\mathcal{L} = \max_{(v^h \in [\overline{v}^h_{ND}, \overline{v}^h_{\max}])^{h \in [0,1]}} \int_{[0,1]} \left[d_M^h(v^h) + \lambda(v^h - V^*) \right] dh, \tag{11}$$

where $\lambda > 0$ is the Lagrange multiplier on the aggregate consumer surplus constraint. Market h's contribution to the Langrangian is $d_M^h(v^h) + \lambda(v^h - V^*)$. Its derivative with respect to v^h is positive if and only if $\lambda > -d_M^{h\prime}(v^h)$, i.e., the (absolute value of the) slope of the divestitures curve—the remedies exchange rate—is less than the Lagrange multiplier λ. (Recall that the remedies exchange rate -$d_M^{h\prime}(v^h)$ indicates how many dollars the merger partners have to give up in a market h in order to increase consumer surplus in that market by one dollar.) From Lemmas 3 and 5, the remedies exchange rate improves as consumer surplus increases. If it is optimal to do some divestitures in market h, which holds if $-d_M^{h\prime}(v^h) < \lambda$, then it is optimal to increase v^h all the way to $v^h_{\max}$, as $-d_M^{h\prime}(v^h)$ is decreasing in v^h.

Let

$$a^h \equiv \frac{d_M^h(\overline{v}_{ND}^h) - d_M^h(\overline{v}_{\max}^h)}{\overline{v}_{\max}^h - \overline{v}_{ND}^h} \tag{12}$$

denote the average remedies exchange rate along the divestitures curve in market h. In the following, we assume for simplicity that markets are heterogeneous in the following sense: the distribution of average remedies exchange rates across markets has a continuous and strictly increasing cumulative distribution function.

The following result is an immediate implication:

Proposition 6. *Suppose that assets in each market are either redundant or complementary. Then, the solution to the merger partner's maximization problem is "bang bang": the merger partners optimally propose in each market h consumer surplus $v_{\max}^h$ if $a^h < \lambda$ and consumer surplus v_{ND}^h if the inequality is reversed. The Lagrange multiplier λ is the unique solution to*

$$\int_{[0,1]} \{\overline{v}_{ND}^h \mathbf{1}_{\{a^h > \lambda\}} + \overline{v}_{\max}^h \mathbf{1}_{\{a^h < \lambda\}}\} dh = V^*. \tag{13}$$

Proof. To prove the "bang bang" result, note that the contribution of market h to the Lagrangian in equation (11) is $d_M^h(v^h) + \lambda(v^h - V^*)$. This contribution is convex in v^h, by Lemmas 3 and 5. Hence it is optimal to choose $v^h \in \{\overline{v}_{ND}^h, \overline{v}_{\max}^h\}$. It is optimal to choose $v^h = \overline{v}_{\max}^h$ if and only if

$$d_M^h(\overline{v}_{\max}^h) + \lambda(\overline{v}_{\max}^h - V^*) > d_M^h(\overline{v}_{ND}^h) + \lambda(\overline{v}_{ND}^h - V^*),$$

which after simplifications yields equation (12). Finally, the constraint $\int_{[0,1]} v^h(\overline{Q}^{h*}(k^h, i^h))dh = V^*$ must be satisfied; given the 'bang bang' solution, this simplifies to equation (13). The left-hand side of this equation is (by assumption) strictly less than V^* when $\lambda < \min_h a^h$, and strictly larger than V^* when $\lambda > \max_h a^h$; since the left-hand side is also continuous and strictly increasing in λ, there is a unique λ which solves equation (13). □

Lemmas 3 and 5 already established the convexity of the divestitures curves; that is, the remedies exchange rate improve as we move down the divestitures curve in any given market. This immediately implies the optimal solution has a "bang-bang" property: either no assets are divested or all assets are divested to the outsider to whom divesting leads to the largest increase in consumer surplus. It then follows that it is optimal to divest no assets in markets where the average exchange rate is highest, i.e., where $a^h > \lambda$. The value of that multiplier λ is pinned down by equation (13), which says that the induced aggregate level of consumer surplus is equal to its pre-merger level.

Having established that the merger partners optimally propose either no or maximal divestitures in each market, the question arises how the market characteristics affect the choice between these two extreme forms of divestitures.

Proposition 7. *Consider market h and suppose that assets in this market are redundant. Suppose that it is optimal to propose no divestitures in that market. Then, if the market is made (weakly) less competitive pre-merger by (strictly) increasing the marginal cost of a non-asset-receiving outsider, it remains optimal to propose no divestitures.*

Proof. We begin by noting that, by Lemma 1, the increase in the outsider's marginal cost weakly reduces both $\overline{v}^h_{ND}$ and $\overline{v}^h_{\max}$. Dropping the market superscript, we now show that the average remedies exchange rate a is decreasing in both $\overline{v}_{ND}$ and $\overline{v}_{\max}$. Taking the derivative of a in equation (12) with respect to $\overline{v}_{ND}$ is proportional to

$$\frac{\partial \pi(Q(\overline{v}_{ND}); \overline{c}_M(0))}{\partial \overline{v}_{ND}} + a = d'_M(\overline{v}_{ND}) + a, \tag{14}$$

which is negative by convexity of $d_M(v)$ on $[\overline{v}_{\max} - \overline{v}_{ND}]$, as shown in Lemma 3. Similar steps can be used to prove that a is also decreasing in $\overline{v}_{\max}$. This establishes that a weakly increases in the outsider's marginal cost, which means that a remains above λ, implying that no divestitures is still optimal. □

In our single-market analysis, Proposition 1 showed that the merger partners will have to propose more divestitures in less competitive markets, as otherwise the merger may not be approved. Proposition 7 reveals that the opposite prediction obtains in a multimarket setting with redundant assets: the merger partners will optimally offer *fewer* divestitures in less competitive markets, as the average remedy exchange rate in these markets is worse. Intuitively, as the market becomes less competitive, the merged firm's market share increases, which—from equations (9) and (10)—makes the remedies exchange rate even less favorable, because the merged firm is hurt more by a given increase in consumer surplus. Hence, it is more expensive for the merger partners to offer divestitures in such markets.

In the complementary assets case, stronger assumptions are needed to guarantee that the same prediction holds. The proof of Proposition 7 involves showing that the average exchange rate is decreasing in $\overline{v}_{ND}$ and $\overline{v}_{\max}$. While in the complementary assets case, it still holds that the derivative of the average exchange rate with respect to $\overline{v}_{ND}$ is proportional to the left-hand side in equation (14). However, it no longer holds that

$$\frac{\partial \pi(Q(\overline{v}_{ND}); \overline{c}_M(0))}{\partial \overline{v}_{ND}} = d'_M(\overline{v}_{ND}),$$

as in the case of complementary assets, the slope of the divestitures at $v = \overline{v}_{ND}$ involves changing the merged firm's profit through two channels: the increase in consumer surplus (or, equivalently, aggregate output) and the increase in marginal cost from divesting assets.[8]

[8]In the complementary assets case, the derivative $d'_M(\overline{v}_{ND})$ is not defined when the divestitures has a downward jump at that point; but in this case, we can think of $d'_M(\overline{v}_{ND}) = -\infty$.

As a result, convexity of the divestitures curve may not be sufficient to imply that proposing no divestitures is more likely to be optimal as the market becomes less competitive.

Proposition 8. *Consider market h and suppose that assets in this market are complementary. Suppose that it is optimal to propose no divestitures in that market. Then, if the market is made less competitive by slightly increasing the marginal cost of a non-asset-receiving outsider, it remains optimal to propose no divestitures, provided (i) the firm whose marginal cost has been increased remains active at $\overline{v}_{\max}$ and (ii) the number of active firms is weakly smaller at $\overline{v}_{\max}$ than at $\overline{v}_{ND}$.*

Note that the assumption that the number of active firms is weakly smaller at $\overline{v}_{\max}$ than at $\overline{v}_{ND}$ would hold, for example, if the firm receiving all of the assets at $\overline{v}_{\max}$ is already active absent divestitures.

4 Extensions

We now show that our main insights are robust to allowing for bargaining between the merger partners and the antitrust authority, and to allowing the merger partners to earn revenues from asset divestitures.

4.1 Bargaining

To be completed.

4.2 Asset Revenues

To be completed.

5 Conclusion

Merger remedies are widely used in practice, and mergers often affect multiple different markets, but these issues have been largely ignored by the existing literature. In this paper we provide a framework to study optimal merger remedies when the merger partners may be active in more than one market.

If the merger affects only a single market, or if the antitrust authority follows a "market-by-market" approach, the existing literature has shown that any merger that is profitable and does not harm consumers must involve synergies. We demonstrate that this remains true even when the merger partners can offer remedies. In addition, we show that as a market becomes less competitive, there is a sense in which larger divestitures are required for the merger not to harm consumers. As a result, it is less likely that any merger will be proposed and accepted in less competitive markets.

In the merger instead affects multiple markets, and the antitrust authority is willing to balance gains and losses across these markets, then the notion of a remedies exchange rate becomes key. We show that in any given market, the remedies exchange rate improves as assets are divested in such a way that the level of consumer surplus increases. Hence optimal merger remedies are "bang-bang": no divestitures in some markets, and the largest feasible divestitures in other markets. Perhaps surprisingly, our analysis reveals that it is optimal for the merger partners to propose divestitures in the more competitive markets, and no divestitures in the less competitive markets.

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A Omitted Proofs

Proof of Lemma 3. It is straightforward to see that $d_M(v)$ is twice differentiable, except at a point v' at which $s_M(Q(v')) = 0$ and $s_M(Q(v)) > 0$ for $v < v'$.

Consider first the case in which $s_M(Q(v')) = 0$ for some $v' \in [\overline{v}_{ND}, \overline{v}_{\max}]$, implying that $s_M(Q(v)) = 0$ for all $v > v'$. It is then immediate from equation (9) that $d_M(v) = 0$ for all $v \geq v'$, and so $d_M(v)$ is weakly decreasing and convex.

Consider second the case in which $s_M(Q(v)) > 0$. To simplify the exposition we henceforth omit the dependence of $Q(v)$ on v. Note that because $s_M(Q) = r(Q; \overline{c}_M)/Q$ and $\overline{c}_M$ is a constant in the redundant assets case, we have that

$$\frac{ds_M(Q)}{dQ} = -\frac{1 + s_M(Q)[1 - \sigma(Q)]}{Q},$$

and

$$d''_M(v) = 2\left[\frac{1}{s_M(Q)} - \sigma(Q)\right]\left[\frac{1}{s_M(Q)} + 1 - \sigma(Q)\right] + Q\sigma'(Q).$$

By Assumption 1, $d''_M(v)$ is weakly positive, and strictly so if $s_M(Q) < 1$ (which holds if $v > \overline{v}_{ND}$). □

Proof of Lemma 5. We first show that any conditional divestitures curve $d_M(v; i)$ is weakly decreasing and convex, and strictly so when $v > \overline{v}_{ND}$ and $d_M(v; i) > 0$.

From the expression in Lemma 4, and using the fact that $0 > c^{*\prime}_M(c; i) > -1$, it is straightforward to see that $d_M(v; i)$ is weakly decreasing, and strictly so when $d_M(v; i) > 0$ (or, equivalently, when $s_M(Q(v)) > 0$).

We now show that any conditional divestitures curve $d_M(v; i)$ is convex on $[\overline{v}_{ND}, \overline{v}_{\max}(i)]$, and strictly so when both $v > \overline{v}_{ND}$ and $d_M(v; i) > 0$.

Let $v'(i) \equiv \sup\{v \in [\overline{v}_{ND}, \overline{v}_{\max}(i)] | s_M(Q(v)) > 0\}$. As $s_M(Q(v)) = 0$ for all $v \in (v'(i), \overline{v}_{\max}(i)]$, it is immediate from the equation in the statement of Lemma 4 that $d_M(v; i)$ is equal to zero (and thus weakly convex) on $[v'(i), \overline{v}_{\max}(i)]$.

We now show that $d_M(v; i)$ is weakly convex on $[\overline{v}_{ND}, v'(i))$, and strictly so for $v > \overline{v}_{ND}$. It is straightforward to see from Lemma 4 that $d_M(v; i)$ is twice differentiable almost everywhere on $[\overline{v}_{ND}, v'(i))$. Consider first any v in that interval at which $d''_M(v; i)$ exists. As $s_M(Q) = r(Q; c^*_M(c(Q); i))/Q$, we have:

$$\frac{ds_M(Q)}{dQ} = -\frac{1}{Q}\left[1 + s_M(Q)[1 - \sigma(Q)] - \frac{c^{*\prime}_M(c(Q); i)}{P'(Q)}\frac{dc}{dQ}\right].$$

First, one can then check that the derivative with respect to Q of $-s_M(Q)$ multiplied by the first term in square brackets in equation (10) is proportional to

$$2\left[\frac{1}{s_M(Q)}-\sigma(Q)\right]\left[\frac{1}{s_M(Q)}+1-\sigma(Q)-\frac{[\overline{n}(Q;i)+1-\sigma(Q)]c_M^{*\prime}(c(Q);i)}{(1+c_M^{*\prime}(c(Q);i))s_M(Q)}\right]+Q\sigma'(Q).$$

Since $-1 < c_M^{*\prime}(c(Q);i) < 0$, Assumption 1 implies that this expression is weakly positive, and strictly so for any $v > \overline{v}_{ND}$ (which implies that $s_M(Q) < 1$).

Second, one can also check that when it exists, the derivative with respect to Q of $-s_M(Q)$ multiplied by the second term in square brackets in equation (10) is weakly greater than a term that is proportional to

$$[\overline{n}(Q;i)+1-\sigma(Q)]\left[\frac{1}{s_M(Q)}+1-\sigma(Q)-\frac{[\overline{n}(Q;i)+1-\sigma(Q)]c_M^{*\prime}(c(Q);i)}{(1+c_M^{*\prime}(c(Q);i))s_M(Q)}\right]+Q\sigma'(Q).$$

Note that for any $v > \overline{v}_{ND}$ such that $d_M(v;i) > 0$, both the merged firm and the asset-receiving outsider are active, implying that $\overline{n}(Q;i) \geq 2$. Since the second-bracketed term in the above equation exceeds 1, it follows from Assumption 1 that the whole expression is weakly positive, and strictly so for any $v > \overline{v}_{ND}$ (which implies that $s_M(Q) < 1$). It then follows that for any v where $d_M(v;i)$ is twice-differentiable, we have $d''_M(v;i) > 0$.

As $s_M(Q(v))$, $\sigma(Q(v))$ and $c_M^{*\prime}(c;i)$ are all differentiable everywhere on that interval, $d_M(v;i)$ is twice differentiable, except possibly at $v = \overline{v}_{ND}$ (where $d_M(v;i)$ may jump downwards) and at any v at which $\overline{n}(Q(v);i)$ jumps down (inducing an upward jump in the slope of $d_M(v;i)$). Note that any such non-differentiability preserves the convexity of $d_M(v;i)$.

We have thus shown that $d_M(v;i)$ is weakly convex on $[\overline{v}_{ND}, v'(i))$, and strictly so for $v > \overline{v}_{ND}$. Before, we had already shown that $d_M(v;i) = 0$ for all $(v'(i), \overline{v}_{\max}(i)]$. As $d_M(v;i)$ is continuous everywhere, it follows that it is weakly convex everywhere, and strictly so in $(\overline{v}_{ND}, v'(i))$.

It remains to show that divestitures curve $d_M(v)$, which is the upper envelope of the conditional divestitures curve, is weakly convex, and strictly so on $(\overline{v}_{ND}, \max_i v'(i))$. If all the conditional divestitures curves had the same support, then the result would follow trivially from the fact that the upper envelope of convex functions is convex. If, however, there are two asset-receiving outsiders i_1 and i_2 with $\overline{v}_{\max}(i_1) < \overline{v}_{\max}(i_2)$, this raises the possibility that the upper envelope has a downward jump at $\overline{v}_{\max}(i_1)$, namely if $d_M(\overline{v}_{\max}(i_1)) = d_M(\overline{v}_{\max}(i_1); i_1)$, implying that $d_M(v)$ is not convex. We now show that this possibility cannot arise. To see this, note that the merged firm's marginal cost is $\overline{c}_M(-K)$ when all K assets are divested to outsider i, inducing consumer surplus $\overline{v}_{\max}(i)$, for $i = i_1, i_2$. Note that by continuity, we can divest strictly fewer than K assets to outsider i_2 and induce consumer surplus level $v = \overline{v}_{\max}(i_1)$; this results in a post-merger marginal cost for outsider i_2 strictly exceeding $\overline{c}_{i_2}(K)$, which by the definition of complementarity implies that the merged firm's marginal cost is strictly lower than $\overline{c}_M(-K)$. But this implies that $d_M(\overline{v}_{\max}(i_1); i_2) > d_M(\overline{v}_{\max}(i_1); i_1)$, a contradiction. □

Proof of Proposition 8. We begin by proving that the average remedies exchange rate a^h increases when market h is made less competitive in the way described in the proposition. To simplify the exposition, we henceforth drop the market-level superscript. Note that if we let $Q(v)$ denote the market-level output associated with consumer surplus level v, we can rewrite a from equation (12) as

$$\frac{\pi(Q(\overline{v}_{ND});\overline{c}_M(-K)) - \pi(Q(\overline{v}_{\max});\overline{c}_M(-K))}{\overline{v}_{\max} - \overline{v}_{ND}} + \frac{\pi(Q(\overline{v}_{ND});\overline{c}_M(0)) - \pi(Q(\overline{v}_{ND});\overline{c}_M(-K))}{\overline{v}_{\max} - \overline{v}_{ND}}. \tag{15}$$

We need to show that the average exchange rate is increasing in the marginal cost of the non-asset-receiving outsider. The first term can be shown to be increasing, using exactly the same steps as in the proof of Proposition 7. Next, we show that the numerator of the second term is increasing. To this end, we rewrite the numerator as

$$-\int_{\overline{c}_M(0)}^{\overline{c}_M(-K)} \frac{\partial \pi(Q(\overline{v}_{ND});c)}{\partial c} dc = 2\int_{\overline{c}_M(0)}^{\overline{c}_M(-K)} r(Q(\overline{v}_{ND});c)dc.$$

This expression increases as the market is made less competitive, because $r(Q;c)$ is decreasing in Q while $\overline{v}_{ND}$ also decreases as the market becomes less competitive.

Finally, we show that the denominator of the second term, $\overline{v}_{\max} - \overline{v}_{ND}$, decreases as the market becomes less competitive. Using our assumption that the firm whose marginal cost is being increased is still active at $\overline{v}_{\max}$, the derivative of $\overline{v}_{\max} - \overline{v}_{ND}$ with respect to the marginal cost of that outsider is

$$-\frac{Q(\overline{v}_{\max})}{\overline{n}(Q(\overline{v}_{\max})) + 1 - \sigma(Q(\overline{v}_{\max}))} + \frac{Q(\overline{v}_{ND})}{\overline{n}(Q(\overline{v}_{ND})) + 1 - \sigma(Q(\overline{v}_{ND}))},$$

where $\overline{n}(Q(v))$ denotes the number of active firms when consumer surplus is equal to v.[9] This expression is negative because $2 \leq \overline{n}(Q(\overline{v}_{\max}) \leq \overline{n}(Q(\overline{v}_{ND})$ by assumption, and also because Assumption 1 ensures that $Q/(n+1-\sigma(Q))$ is weakly increasing in Q. (Note that $2 \leq \overline{n}(Q(\overline{v}_{\max})$ because at $\overline{v}_{\max}$ both the asset-receiving outsider and the firm whose cost has been increased are active.) □

[9] If, initially, there exists a firm with marginal cost equal to $P(Q(v))$, that firm will become active following a small increase in the marginal cost of the non-asset-receiving outsider. In this case, $\overline{n}(Q(v))$ includes the firm that is just inactive before the change in competitiveness.

B Online Appendix

B.1 Example where a decrease in market competitiveness reduces the harm to consumers of a merger

Consider a market with $n = 4$ firms and market demand curve $P(Q) = 1 - Q$. Suppose that pre-merger marginal costs are $c_1 = c_2 = c_3 = 1/2$ and $c_4 = 1$. We can then compute the following pre-merger outcomes: $Q^* = 3/8$, $P(Q^*) = 5/8$, and $v(Q^*) = 9/128$. (Note that before the merger, only firms $1, 2$, and 3 are active.) Now consider a merger between firms 1 and 2 and a divestiture $(k, 4)$ such that $\overline{c}_M(-k) = 1/2$ and $\overline{c}_4(k) = 1/2 + \epsilon$ where $\epsilon \in (0, 1/6)$. We can then compute the following post-merger outcomes: $\overline{Q}^* = (3 - 2\epsilon)/8$, $P(\overline{Q}^*) = (5 + 2\epsilon)/8$, and $v(\overline{Q}^*) = (3 - 2\epsilon)^2/128$. (Note that after the merger, the merged firm as well as firms 3 and 4 are active.) Hence the merger and divestitures reduce aggregate consumer surplus by

$$v(Q^*) - v(\overline{Q}^*) = \frac{\epsilon(3 - \epsilon)}{32}. \tag{16}$$

Now suppose we make the market less competitive, by raising firm 3's pre-merger marginal cost such that $c_3 = 1/2 + \epsilon$. We can then compute the following pre-merger outcomes: $Q^* = (3 - 2\epsilon)/8$, $P(Q^*) = (5 + 2\epsilon)/8$, and $v(Q^*) = (3 - 2\epsilon)^2/128$. Again consider a merger between firms 1 and 2 and a divestiture $(k, 4)$ such that $\overline{c}_M(-k) = 1/2$ and $\overline{c}_4(k) = 1/2 + \epsilon$. We can then compute the following post-merger outcomes: $\overline{Q}^* = (3 - 4\epsilon)/8$, $P(\overline{Q}^*) = (5 + 4\epsilon)/8$, and $v(\overline{Q}^*) = (3 - 4\epsilon)^2/128$. (Note that again, before the merger firms $1, 2$, and 3 are active, whereas after the merger the merged firm and also firms 3 and 4 are active.) Hence the merger and divestitures reduce aggregate consumer surplus by

$$v(Q^*) - v(\overline{Q}^*) = \frac{3\epsilon(1 - \epsilon)}{32}. \tag{17}$$

Notice that (16) is strictly greater than (17), i.e., although the merger reduces consumer surplus in both cases, the magnitude is lower in the less competitive market.

Now consider a multimarket context with two types of market. Type A markets are the same as those described above, while type B markets are such that a merger strictly raises consumer surplus. Note that there exist weights on these two markets such that, when the type A markets are competitive the merger would be blocked by the antitrust authority, but when the type A markets are made less competitive the merger would not be blocked.

B.2 Examples of distributions satisiying Assumption 1

Suppose that $Q = \alpha[1 - F(P)]$ for some $\alpha > 0$, where for simplicity we will henceforth omit the dependence of P on Q. Differentiating this equation, we find that

$$P'(Q) = -\frac{1}{\alpha f(P)} \quad \text{and} \quad P''(Q) = -\frac{f'(P)}{\alpha^2 f(P)^3},$$

which then implies that

$$\sigma(Q) = -\frac{QP''(Q)}{P'(Q)} = -\frac{[1 - F(P)]f'(P)}{f(P)^2}.$$

We now use this to check Assumption 1 for different distributions.

Generalized Pareto We have $F(P) = 1 - (1 + \eta P)^{-1/\eta}$ and $f(P) = (1 + \eta P)^{-(1+\eta)/\eta}$ for $\eta < 0$. Hence $\sigma(Q) = 1 + \eta$ and Assumption 1 holds.

Logistic We have $F(P) = 1/(1 + e^{-P})$ and $f(P) = e^{-P}/(1 + e^{-P})^2$. Hence

$$\sigma(Q) = 1 - e^{-P} \quad \text{and} \quad \sigma'(Q) = -\frac{(1 + e^{-P})^2}{\alpha}.$$

One can check that Assumption 1 holds if and only if

$$\min\{2 + e^{-P}, 2e^{-P}(1 + e^{-P})\} \geq e^{-P}(1 + e^{-P})$$

which is clearly true for any $P > 0$.

Generalized Extreme Value Distribution We have $F(P) = 1 - e^{-e^P}$ and $f(P) = e^{-e^P}e^P$. Hence

$$\sigma(Q) = 1 - e^{-P} \quad \text{and} \quad \sigma'(Q) = -\frac{e^{-P}}{\alpha e^{-e^P}e^P}.$$

One can check that Assumption 1 holds if and only if

$$\min\{1 + 2e^P, 2(1 + e^{-P})\} \geq e^{-P},$$

which is clearly true for any $P > 0$.