The Macroeconomics of Narratives

Joel P. Flynn*
Yale

Karthik A. Sastry†
Princeton

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Abstract

We study the macroeconomic implications of *narratives*, defined as beliefs about the economy that spread contagiously. In an otherwise standard business-cycle model, narratives generate persistent and belief-driven fluctuations. Sufficiently contagious narratives can "go viral," generating hysteresis in the model's unique equilibrium. Empirically, we use natural-language-processing methods to measure firms' narratives. Consistent with the theory, narratives spread contagiously and firms expand after adopting optimistic narratives, even though these narratives have no predictive power for future firm fundamentals. Quantitatively, narratives explain 32% and 18% of the output reductions over the early 2000s recession and Great Recession, respectively, and 19% of output variance.

*Yale University Department of Economics, 30 Hillhouse Avenue, New Haven, CT, 06511. Email: joel.flynn@yale.edu.

†Princeton University Department of Economics, 192A Julis Romo Rabinowitz Building, Princeton, NJ, 08544, Email: ksastry@princeton.edu.

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1 Introduction

At least since Keynes (1936), economists have hypothesized that waves of "spontaneous optimism" generate business cycles. But what drives these fluctuations in beliefs and how much do they matter? The *Narrative Economics* of Shiller (2017, 2020) postulates that contagious narratives induce aggregate shifts in beliefs and drive macroeconomic events. Our aim is to make progress toward understanding the business-cycle relevance of economic narratives.

We introduce a framework within which the macroeconomic importance of narratives can be studied theoretically, empirically, and quantitatively. Motivated by the literature on narratives and firm organization (see *e.g.*, Vaara, Sonenshein, and Boje, 2016), we define narratives as subjective and potentially incorrect models of the world. By altering beliefs, narratives influence economic actions such as hiring. Moreover, motivated by Shiller (2017), we allow narratives to gain or lose prevalence over time for two distinct reasons: direct feedback from their prevalence, as in epidemiological models, and indirect feedback from the economic activity that narratives induce, as in models of learning. We refer to these forces as *contagiousness* and *associativeness*, respectively.

Theoretically, we embed narratives in a real business cycle model. We characterize the unique equilibrium of this model and find that narratives generate belief-driven fluctuations in economic activity. Our model formalizes the notion that narratives can "go viral" out of nowhere: small and transitory fundamental shocks can generate large and persistent movements in the economy as a new narrative takes over. Finally, we show that the dynamic, general-equilibrium effects of narratives can be identified given firm-level panel data on narratives and decisions.

Empirically, we develop a method to measure narratives and their influence. Specifically, we apply natural-language-processing methods to measure narratives in the universe of earnings calls and 10-K regulatory filings, in which all US public firms discuss "perspectives on [their] business results and what is driving them" (SEC, 2011). We find that optimistic narratives predict greater hiring by firms. Moreover, optimism does *not* predict positive future fundamentals. This is consistent with our interpretation that optimism captures a non-fundamental narrative and inconsistent with the alternative interpretation that optimism encodes firm-level news about fundamentals. Moreover, narrative optimism spreads contagiously and associatively among firms. Together, these estimates empirically discipline how narratives affect decisions and how they spread.

Quantitatively, we adopt a "micro-to-macro" approach to understand the effect of narratives on the macroeconomy. We find that fluctuations in narrative optimism account for 32% and 18% of the output reductions over the early 2000s recession and Great Recession, respectively. We further find that stable aggregate fluctuations in narrative optimism can arise from the interaction among many specific and individually viral narratives. Taken together, our analysis suggests that narratives are a significant cause of macroeconomic fluctuations.

Model. In our framework, narratives are the common building blocks of agents' heterogeneous beliefs. We formally define an economy's *narratives* as a set of prior distributions that agents can combine with outside data to form beliefs. This is motivated by the *realist* and *interpretative* schools of narrative analysis in organizational economics (Vaara, Sonenshein, and Boje, 2016), which postulate that narratives provide a lens through which individuals understand the world. At an individual level, we model narrative adoption as a Markov process that depends on one's own past narrative, the relative prevalence of narratives in the population, and endogenous economic states. The second feature incorporates the possibility of *contagion* emphasized by Shiller (2017). The third feature allows agents to associate certain macroeconomic states with certain more plausible models, as would be natural in any model of learning.

We embed this notion of narratives in a real business cycle model. Specifically, the agents are heterogeneous firms, the fundamental is aggregate productivity, and the two available narratives are *optimism* and *pessimism* about productivity. The remaining microfoundations are intentionally standard, following the literature on business cycles with dispersed information (Lorenzoni, 2009; Angeletos and La'O, 2010): the consumption, production, and labor supply side of the model is a real variant of the standard model of Woodford (2003b) and Galí (2008).

Theoretical Results. Our main results analytically characterize how narratives can generate non-fundamental fluctuations in aggregate output, hysteresis, and boom-bust cycles.

We first establish that there is a unique equilibrium in which aggregate output is log-linear in aggregate productivity and a non-linear function of the fraction of optimists in the population. We refer to the latter effect as the *non-fundamental component* of aggregate output, because it is driven by the state of narratives rather than any fundamental change in productivity. We decompose this component into a partial-equilibrium effect of optimism on firm hiring as well as a general-equilibrium *narrative multiplier* driven by strategic complementarities: even if a firm is pessimistic, the presence of other, more optimistic firms causes them to produce more. The equilibrium effect of narratives is smaller if firms have access to more precise information, as this leads them to rely less on narratives to form their beliefs. However, because of the narrative multiplier, the power of the truth to stop false narratives is weakest exactly when narratives are strongest. Importantly, as equilibrium is unique, fluctuations in our model are not driven by sunspots or movements between multiple equilibria.

We next describe the dynamics of the fraction of optimists in the population ("optimism"), the key new state variable in our economy. For a fixed level of aggregate productivity, we show that there always exists a steady-state level of optimism, but there may be multiple. We provide a necessary and sufficient condition for a particularly extreme type of steady-state multiplicity: if narratives are sufficiently contagious and/or the narrative multiplier is sufficiently large, then unanimous optimism and unanimous pessimism are both stable steady states. That is, in the economy's *unique* equilibrium, contagious optimism or contagious pessimism can "go viral" or die out entirely, depending on the initial prevalence of optimism.

Our characterization of narrative dynamics implies three key properties of narrative business cycles. First, under the aforementioned conditions of high contagiousness and a high narrative multiplier, the economy can feature *narrative hysteresis*: history determines whether the economy is optimistic and high-output or pessimistic and low-output. Second, productivity shocks have *endogenously persistent* effects on output because of narrative evolution. This can generate hump-shaped impulse responses to perfectly transitory shocks. In cases consistent with narrative hysteresis, transitory shocks can even have permanent effects because a new narrative takes hold. This represents an important difference relative to models of dispersed information (see *e.g.,* Woodford, 2003a; Lorenzoni, 2009; Angeletos and La'O, 2010), in which one-time shocks can have persistent effects only if fundamentals are themselves persistent (otherwise there is nothing to learn about over time). Third, when hit by repeated shocks, the economy can experience *boom-bust cycles* because of fluctuations between high and low optimism. In the hysteresis case, these can take the form of slow oscillations between periods of stable extreme pessimism (reminiscent of "lost decades") and stable extreme optimism (reminiscent of "roaring decades").

We finally show how the model can be identified using information on relative choices of optimistic versus pessimistic firms, the updating process of firms' narratives (in particular, the extent of social contagion), and otherwise standard macroeconomic parameters that control general-equilibrium effects. This motivates us to take a "micro-to-macro" approach in the remainder of the paper, the final result of which will be a quantitative appraisal of the properties identified above.

Measurement. To empirically evaluate our framework and quantify its predictions, we construct a dataset comprising US public firms' narratives and decisions. To measure narratives, we apply natural-language-processing methods to the text of firms' end-of-year 10-K reports and earnings conference calls. We argue that these text datasets are ideal for measuring narratives because they include unstructured information on how managers rationalize and interpret economic outcomes and they can be readily linked to firms' choices. Specifically, we construct a textual proxy for narrative optimism by computing the intensity of positive and negative sentiment using the financial dictionary introduced by Loughran and McDonald (2011). To complement our study of narrative optimism, we apply two other techniques to measure more granular narratives. Our first such technique computes the similarity between firms' language and the language that best characterizes the *Perennial Economic Narratives* introduced by Shiller (2020) using a method that has also been applied in Hassan, Hollander, Van Lent, and Tahoun (2019) and Flynn and Sastry (2024). These "narratively identified narratives" are motivated by the historical evidence of relevance and contagiousness provided by Shiller (2020). Our second granular technique estimates a Latent Dirichlet Allocation (LDA) model (Blei, Ng, and Jordan, 2003), which extracts an underlying set of topics based on the frequency with which certain words co-occur within documents. This unsupervised method allows the data to speak flexibly about firms' narratives. We combine these measures of firms' narratives with Compustat data on their decisions and financial performance.

Empirical Results. First, we estimate how narrative optimism affects firm decisions. Our model implies that the partial-equilibrium effect of optimism on hiring is identified in a firm-level panel regression that controls for firm and time fixed effects, which sweep out narratives' correlation with aggregate fundamentals and their equilibrium effects. Implementing this strategy, we estimate a partial-equilibrium effect of optimism on hiring of 3.6 percentage points. We moreover show that the effect of optimism on hiring is quantitatively robust to controlling for canonical measures of firm-level fundamentals: productivity, leverage, stock returns, and Tobin's Q.

The key threat to our interpretation that narratives drive the positive effect of optimism on hiring is that optimism may reflect news about future firm-level fundamentals that is not captured by our controls. That is, optimism could capture information about firms' future prospects that is available inside the firm but not to an outside econometrician. Two simple tests distinguish between the narrative interpretation and the news interpretation. First, under the news interpretation, optimism must be predictive of strong future performance of the firm, such as greater profitability or positive stock returns. Second, under the news interpretation, firms must have more optimistic forecasts for future performance, but these should not be predictably more optimistic than realized future performance.

Implementing these two tests, we find that measured optimism is non-fundamental and predicts over-optimistic beliefs. Specifically, in firm-level local projection regressions, we find that optimism *negatively* predicts future stock returns and profitability. That is, firms that are currently optimistic and accelerating hiring do *worse*, not better, in the near future. Second, using managerial guidance data from IBES, we show that optimism in language predicts that managers' sales forecasts exceed realizations. That is, managers predictably overestimate firm performance after writing optimistic reports or giving optimistic earnings calls. Notwithstanding our rejection of the "news" explanation, to further isolate a plausibly exogenous shifter of narratives that is necessarily independent of news, we also study changes in optimism driven by plausibly exogenous changes in CEOs (*i.e.,* those caused by death, illness, personal issues, or voluntary retirement, as coded by Gentry, Harrison, Quigley, and Boivie, 2021). This strategy yields quantitatively similar effects of optimism on hiring.

We next estimate how narrative optimism spreads across firms. Our model implies that we can recover the contagiousness and associativeness of optimism by estimating a firm-level linear probability model for the likelihood that a firm remains or becomes optimistic. We find that greater aggregate optimism and higher aggregate real GDP growth are associated with a greater probability that a firm is optimistic in the following year—that is, optimism is contagious and associative. We also find evidence of contagiousness and associativeness at the industry level when we non-parametrically control for aggregate conditions with time fixed-effects. Moreover, both these aggregate and industry-level results are robust to controlling for future economic conditions. This finding is inconsistent with the key threat to our interpretation: that aggregate optimism drives future optimism through its correlation with omitted positive news about economic conditions. To further test the validity of our interpretation, we construct a granular instrumental variable (Gabaix and Koijen, 2020) for aggregate optimism based on idiosyncratic shocks to the optimism of large firms. We find similar results using this approach.

Quantification. We finally calibrate our model to quantify the extent to which fluctuations in narratives explain historical business cycle fluctuations and understand the extent to which narratives generate hysteresis. We leverage the fact that our empirical estimates identify both the partial equilibrium effects of narratives on hiring and the nature of narrative diffusion.

We first study the extent to which narratives generate non-fundamental fluctuations in output.

Decomposing aggregate output into the components attributable to optimism versus fundamentals, we find that measured aggregate movements in optimism account for 32% of output loss during the early 2000s recession and 18% during the Great Recession. More systematically, fluctuations in optimism account for 19% of output variance as well as 33% of the short-run (one-year) and 79% of the medium-run (two-year) autocovariance in output. Thus, narrative dynamics lead to strong endogenous persistence (internal propagation): the model generates persistent business cycles even with close to i.i.d. shocks.[1] This represents an important difference between our model of narratives and those of noise shocks or dispersed information (see *e.g.,* Woodford, 2003a; Lorenzoni, 2009; Angeletos and La'O, 2010), which require persistent exogenous shocks to explain the time series.

We next study the potential for narrative hysteresis. For optimism, we quantitatively reject the theoretical condition required for hysteresis in both optimism and output dynamics. But we do not reject this condition for many granular (narratively identified and topic) narratives, implying that it is possible for these narratives either to die out or "go viral" depending on initial conditions.

Finally, we study an enriched model that allows multiple latent narratives to form a basis for overall optimism, to evaluate Shiller's (2020) hypothesis that many narratives may be mutually reinforcing. Surprisingly, we find that the interaction of many jointly evolving and highly contagious narratives that can individually feature hysteresis nevertheless underlie stable fluctuations in emergent aggregate optimism and output.

Related Literature. Our work relates to a large literature on belief-driven business cycles. Some studies postulate that shocks directly to beliefs affect aggregate supply and/or demand and cause fluctuations (*e.g.*, Lorenzoni, 2009; Angeletos and La'O, 2010, 2013; Angeletos, Collard, and Dellas, 2018; Benhima, 2019; Christiano, Ilut, Motto, and Rostagno, 2008; Benhabib, Wang, and Wen, 2015; Nimark, 2014; Chahrour, Nimark, and Pitschner, 2021). Others focus on the interaction of other shocks with fixed belief differences (*e.g.*, Caballero and Simsek, 2020; Guerreiro, 2022), the tendency of agents to over-extrapolate via diagnostic expectations (*e.g.*, Maxted, 2020; Bordalo, Gennaioli, Shleifer, and Terry, 2021; Bianchi, Ilut, and Saijo, 2024), or the effects of slow and/or misspecified learning (*e.g.*, Marcet and Sargent, 1989a,b; Eusepi and Preston, 2011; Adam, Marcet, and Beutel, 2017; Kozlowski, Veldkamp, and Venkateswaran, 2020). Our work can be understood as micro-founding both belief dynamics and belief disagreement through the social transmission of narratives, as well as providing novel evidence for the importance of narratives.

The emphasis on social dynamics is shared with a literature on macroeconomic models of epidemiological "contagion" in beliefs (Carroll, 2001; Burnside, Eichenbaum, and Rebelo, 2016; Carroll and Wang, 2022; Jamilov, Kohlhas, Talavera, and Zhang, 2024). This relates to a theoretical literature studying the survival dynamics of competing heterogeneous models (*e.g.*, Brock and Hommes, 1997; Molavi, Tahbaz-Salehi, and Vedolin, 2021; Bohren and Hauser, 2021). To this literature, we add direct measurement of narrative dynamics and the connection to qualitative and quantitative macroeconomic predictions. In particular, we show why joint modeling of macro and

[1] Normatively, we show that contagious optimism can be welfare-improving even if it is unfounded. Quantitatively, we find that optimism is welfare-improving and welfare-equivalent to a 1.3% production subsidy.

belief dynamics is necessary to assess whether macroeconomic narratives can "go viral."

We relate to an empirical literature that proposes techniques to measure narratives following Shiller (2017): Andre, Haaland, Roth, and Wohlfart (2022) implement open-ended surveys to understand narratives underlying inflation, Goetzmann, Kim, and Shiller (2022) measure narratives about financial crashes in news media, and Macaulay and Song (2022) measure how news coverage of narratives affects sentiment on social media. Our empirical approach differs in its use of text data from firms to extract narratives, uncover their effects on decision-making, and study their spread. Our analysis therefore relates to a literature studying the relationship between firm-level outcomes and their language (*e.g.*, Loughran and McDonald, 2011; Hassan, Hollander, Van Lent, and Tahoun, 2019) and measured beliefs (*e.g.,* Gennaioli, Ma, and Shleifer, 2016; Coibion, Gorodnichenko, and Ropele, 2020). In contrast to these papers, we calibrate a model to match our firm-level findings and study the general equilibrium consequences of narratives for the business cycle.[2]

Outline. The paper proceeds as follows. In Section 2, we introduce our macroeconomic model with contagious narratives. In Section 3, we provide theoretical results on macroeconomic dynamics. In Section 4, we describe our data and measurement. In Section 5, we describe our empirical results. In Section 6, we quantify the role of narratives. In Section 7, we study an enriched model that features multiple narratives. Section 8 concludes.

2 A Narrative Business-Cycle Model

We first describe our framework, incorporating contagious narratives into an otherwise standard real business cycle model.

2.1 Narratives and Beliefs

What are Narratives? In the literature on narratives and firm organization, narratives are characterized as a *common set* of stories that allow people to make sense of the world (see, *e.g.*, Isabella, 1990; Maitlis, 2005; Loewenstein, Ocasio, and Jones, 2012). To be specific, Vaara, Sonenshein, and Boje (2016) review the organizational literature on narratives and describe two perspectives on what narratives are and how they impact decisionmakers. The first is the *realist approach*, in which "narratives [act] as data to be used to explain or understand other phenomena." The second is the *interprative approach*, in which narratives "provide a means for individual, social and organizational sensemaking and sensegiving." Under both approaches, narratives provide models that agents combine with experience to understand the world. Motivated by this, we model narratives as models that agents combine with information to arrive at their beliefs. Formally, this corresponds to associating narratives with prior beliefs.

[2] Bachmann and Elstner (2015), Barrero (2022), and Ma, Ropele, Sraer, and Thesmar (2020) share this approach of combining information on firms' beliefs with a macroeconomic model. They study the different but complementary question of whether firms' biased beliefs induce quantitatively significant misallocation.

Our approach is different from how some in the economics literature have modeled narratives as encoding causal reasoning (Spiegler, 2016; Eliaz and Spiegler, 2020; Andre, Haaland, Roth, and Wohlfart, 2022; Macaulay and Song, 2022), as formalized by Pearl (2009) in directed acyclic graphs (DAGs). In our analysis, we prefer to take the broader view of narratives as priors without the specific structure of DAGs for three reasons: narratives are not necessarily causal, there is empirical evidence that people are not good at engaging in causal reasoning, and, to study equilibrium, it is both inessential and ill-posed to consider causation. In our framework, narratives *do* provide explanations or rationalizations for why people think in a certain way, but those explanations need not be causal. We now make our three arguments more precise.

First, narratives expressed in communication and writing are not necessarily *causal*. In particular, as Bruner (1991) writes in his foundational work on the interpretative approach to narratives:

> The loose link between intentional states and subsequent action is the reason why narrative accounts cannot provide causal explanations. What they supply instead is the basis for interpreting why a character acted as he or she did.

Given the inherent complexity of emergent macroeconomic phenomena, we argue that Bruner's argument may be especially applicable. Consistent with this, the nine "Perennial Economic Narratives" identified by Shiller (2020) as prevalent and important all resist a simple causal structure (*e.g.*, "Panic versus Confidence" or "Labor-Saving Machines Replace Many Jobs").[3] Thus, we argue that it is natural to model narratives as providing models, which naturally provide interpretations that explain actions without reliance on one specific causal structure.

Second, a large literature in psychology documents that the majority of people are incapable of performing simple causal ("if ..., then ...") reasoning. For example, in the Wason (1966; 1968) Card Test, the experimenter places four cards showing "3," "8," "Blue," and "Red" on a table and asks the subject to flip two cards to test the claim that "if a card shows an even number on one face, then its opposite face is blue." The finding, which has since been replicated many times (Manktelow, 2021), is that the vast majority of respondents get the answer wrong.[4] "If, then" reasoning underlies the "do" operator at the heart of Pearl's (2009) DAG-based formalization of causality. The inability to perform a single chain of "if, then" reasoning makes it dubious that people are able to engage in the long chains of reasoning that may be necessary to provide a fully causal account of macroeconomic phenomena.

Third, to study how people make decisions, it is necessary only to understand their preferences and their beliefs. Concretely, under any consequentialist objective (which nests expected utility theory), it does not matter to a firm *why* its productivity and demand are high, it matters only that

[3] The full list is: Panic versus Confidence; Frugality versus Conspicuous Consumption; The Gold Standard versus Bimetallism; Labor-Saving Machines Replace Many Jobs; Automation and Artificial Intelligence Replace Almost All Jobs; Real Estate Booms and Busts; Stock Market Bubbles; Boycotts, Profiteers, and Evil Businesses; and The Wage-Price Spiral and Evil Labor Unions.

[4] In a famous follow-up experiment, Cosmides and Tooby (1992) show that respondents are more likely to be correct (but still often incorrect) when the experiment is re-contextualized to the more familiar task of identifying underage drinking.

its productivity and demand *are* high. If we are willing to assume, as we do, that narratives do not affect preferences, then it is essential only to understand how narratives affect agents' beliefs. This renders the DAG formalism redundant. A more fundamental issue is that equilibrium reasoning cannot be expressed in the form of a DAG, since interactions are not acyclic. To give one stylized example: in the Keynesian cross, consumption affects income and income affects consumption. This is a cycle and cannot be represented in the DAG formalism. The same applies in the macroeconomic model that we study (for instance, because firms' hiring affects wages which affect firms' hiring).

Narratives in Our Model. Motivated by the reasons described above, we postulate that narratives are described by a small set of prior beliefs about a time-varying exogenous state. Specifically, time is discrete and indexed by $t \in \mathbb{N}$, and the exogenous macroeconomic state is $\theta_t \in \Theta \subseteq \mathbb{R}$. In equilibrium, as we will see shortly, agents' beliefs about θ_t will also govern their prior beliefs about endogenous objects (*e.g.*, GDP, wages, and what narratives will catch on in the future).

We describe narratives, indexed by $k \in \mathcal{K}$, as probability distributions $N_{k,t} \in \Delta(\Theta)$ within the set of narratives $k \in \mathcal{K}$. In our model, the fundamental θ_t describes the strength of productivity. Thus, a pessimistic narrative $N_{P,t}$ corresponds to the view that "productivity in the economy is low on average," while an optimistic narrative $N_{O,t}$ corresponds to the view that "productivity in the economy is high on average."

Agents combine narratives to form priors about the fundamental by placing a vector of weights $\lambda_t = \{\lambda_{k,t}\}_{k \in \mathcal{K}} \in \Lambda \subseteq \Delta(\mathcal{K})$ on each narrative. An agent with narrative weights λ_t has an induced prior distribution over fundamentals given by the following linear combination of distributions:

$$\pi_{\lambda_t}(\theta_t) = \sum_{k \in \mathcal{K}} \lambda_{k,t} N_{k,t}(\theta_t) \tag{1}$$

for all $\theta_t \in \Theta$. Continuing the example, an agent who is fully pessimistic might place weight $\lambda_{P,t} = 1$ on the pessimistic narrative and complementary weight $\lambda_{O,t} = 0$ on the optimistic narrative, so their subjective probabilities for each state θ_t are $\pi(\theta_t) = N_{P,t}(\theta_t)$.

In our analysis, we will assume that $\log \theta_t$ evolves according to an AR(1) process:

$$\log \theta_t = (1-\rho)\mu + \rho \log \theta_{t-1} + \sigma \nu_t \tag{2}$$

where $\nu_t \sim N(0,1)$ is i.i.d.. For simplicity, for our main theoretical analysis, we will suppose that there are two competing narratives about the macroeconomic state: an optimistic narrative under which $\mu = \mu_O$ and a pessimistic narrative under which $\mu = \mu_P$, where $\mu_O > \mu_P$. The *true* distribution of the fundamental need not coincide with either narrative. Firms either believe the optimistic narrative or the pessimistic narrative. Hence, each agent $i \in [0,1]$ has a prior belief regarding the fundamental that can be described as:

$$N\left(\left[\lambda_{it}\mu_O + (1-\lambda_{it})\mu_P\right](1-\rho) + \rho \log \theta_{t-1}, \sigma^2\right) \tag{3}$$

where $\lambda_{it} = 1$ corresponds to an agent believing in the optimistic narrative, and $\lambda_{it} = 0$ corresponds to an agent believing in the pessimistic narrative. In Appendix B.5, we show that our results are unchanged if narratives correspond to idiosyncratic (as opposed to aggregate) fundamentals. In Appendix B.6, we extend our analysis to allow for multiple narratives that differ in the mean, persistence, and volatility of productivity. In Section 7, we will enrich the model to allow optimism to be driven by a large set of underlying and more specific narratives.

2.2 Narrative Dynamics

How do Narratives Spread? In a static setting, narratives correspond to heterogeneous beliefs. Therefore, the core distinguishing feature of a theory of narratives is how they evolve. We suppose that narratives can spread in two distinct and potentially complementary ways.

The first channel is *contagion*: narratives spread between people as a function of their prevalence. Shiller (2020) writes that "contagion is the heart of narrative economics." He argues that failure to model narrative contagion is an important shortcoming of existing macroeconomic models:

> We need to incorporate the contagion of narratives into economic theory. Otherwise, we remain blind to a very real, very palpable, very important mechanism for economic change, as well as a crucial element for economic forecasting. If we do not understand the epidemics of popular narratives, we do not fully understand changes in the economy and in economic behavior.

In our analysis, we will define *contagiousness* as the propensity of a narrative to spread as a function of its prevalence, holding all else fixed. We will not take a stand on the deep origins of contagiousness. These might correspond, as Shiller (2020) hypothesizes, to features such as "human interest, identity, and patriotism." We will take contagiousness as a primitive property of a narrative and study macroeconomic dynamics conditional on contagiousness.

The second channel is *association*: narratives are more likely to spread when they better describe the world. This is the standard view in economic theories in which belief dynamics are governed by learning. In the context of "Narrative Economics," associativeness implies (for example) that a narrative that is obviously contradicted by data may fail to catch on even if it is contagious. Moreover, failing to account for association may lead an observer to spuriously attribute the spread of a narrative to contagion even when it does not exist: for example, the observation that people become more optimistic when the economy is doing well does not imply the existence of contagion. Thus, contagion alone is not sufficient for an adequate theory of narrative dynamics.

Contagion and Association in Our Model. We now formalize these notions. We summarize the prevalence of narratives by the cross-sectional distribution of narratives in the population, $Q_t \in \Delta(\Lambda)$. This represents the distribution of agents' distributions of narrative weights at some time period. For example, in an economy populated by only optimists $\lambda^O = (0, 1)$ and pessimists $\lambda^P = (1, 0)$, with some abuse of notation we can collapse the distribution of narratives into the scalar sufficient statistic $Q_t = \int_{[0,1]} \lambda_{it} \, \mathrm{d}i \in [0, 1]$, which corresponds to the fraction of the population that is optimistic. We also define $Y_t \in \mathcal{Y}$ as a relevant endogenous macroeconomic outcome. Specifically, this will correspond to aggregate output.

In full generality, we describe the dynamics of narratives via an updating rule $P_t : \Lambda \times \mathcal{Y} \times \Delta(\Lambda) \to \Delta(\Lambda)$, which returns the probabilities $\{P_{t,\lambda'}(\lambda, Y, Q)\}_{\lambda' \in \Lambda}$ that an agent with narrative weights λ changes their weights to λ' when the endogenous state is Y and the distribution of narratives in the population is Q. Hence, conditional on realized endogenous outcomes given by Y_t and distribution of narratives given by Q_t, the next period's distribution of narratives is:

$$Q_{t+1,\lambda'} = \sum_{\lambda \in \Lambda} Q_{t,\lambda} P_{t,\lambda'}(\lambda, Y_t, Q_t) \tag{4}$$

The dependence on Y_t allows us to capture association, where the endogenous state of the economy leads agents to adopt narratives that best describe that state (*e.g.,* "aggregate output is high, therefore productivity is high"). The dependence on Q_t allows us to capture contagion, as the distribution of narratives itself affects the likelihood that people retain or switch to a narrative.

In our main analysis with two narratives (optimism and pessimism), we need to describe the probability that optimists remain optimistic, P_O, and the probability that pessimists become optimistic, P_P. We specify that both probabilities depend on aggregate output Y_t, the fraction of optimists in the population Q_t, and an aggregate *narrative* shock to how agents update ε_t, which has distribution G. This shock captures shifts in economic narratives that are wholly unrelated to economic conditions. Hence, the fraction of optimists evolves according to:

$$Q_{t+1} = Q_t P_O(\log Y_t, Q_t, \varepsilon_t) + (1 - Q_t) P_P(\log Y_t, Q_t, \varepsilon_t) \tag{5}$$

We assume that P_O and P_P are continuous and almost everywhere differentiable. In Section 7, we extend this model to allow for many jointly evolving narratives.[5]

2.3 Technology and Preferences

The consumption, production, and labor supply side of the model is intentionally standard, following Angeletos and La'O (2010), and is a real variant of the models described in Woodford (2003b) and Galí (2008). There is a continuum of monopolistically competitive intermediate goods firms of unit measure, indexed by i, and uniformly distributed on the interval $[0, 1]$. They hire labor L_{it} monopsonistically at wage w_{it} to produce a differentiated variety in quantity x_{it} that they sell at price p_{it} according to the production function:

$$x_{it} = \theta_{it} L_{it}^{\alpha} \tag{6}$$

where $\alpha \in (0, 1]$ is the return-to-scale in production and θ_{it} is the firm's Hicks-neutral productivity.

Narratives correspond to different beliefs about the distribution of the common component

[5] See Appendix B.3 for a discussion of how our approach differs from imposing Bayesian learning dynamics.

of firms' productivity. Concretely, firm productivity θ_{it} is comprised of a common, aggregate component θ_t, an idiosyncratic time-invariant component γ_i, and an idiosyncratic time-varying component $\tilde{\theta}_{it}$:

$$\theta_{it} = \tilde{\theta}_{it}\gamma_i\theta_t \tag{7}$$

Firms know that $\log \gamma_i \sim N(\mu_\gamma, \sigma_\gamma^2)$, know their own value of γ_i, and believe that $\log \tilde{\theta}_{it} \sim N(0, \sigma_{\tilde{\theta}}^2)$ and independently and identically distributed (i.i.d.) across firms and time. We assume that firms can observe all previous macroeconomic outcomes. Firms receive idiosyncratic Gaussian signals about $\log \theta_t$ with noise $e_{it} \sim N(0, \sigma_e^2)$ that is i.i.d. across firms and time: $s_{it} = \log \theta_t + e_{it}$. We define the signal-to-noise ratio as:

$$\kappa = \frac{1}{1 + \frac{\sigma_e^2}{\sigma_\theta^2}} \tag{8}$$

which indexes how much firms update their beliefs about aggregate productivity upon receiving the signal s_{it}. By allowing for these signals, our model nests the case in which beliefs are fully driven by narratives ($\kappa = 0$) as well as the case in which narratives have no bearing on the posterior beliefs that are relevant for decisions ($\kappa = 1$).

A final goods firm competitively produces aggregate output Y_t by using a constant elasticity of substitution (CES) production function:

$$Y_t = \left(\int_{[0,1]} x_{it}^{\frac{\epsilon-1}{\epsilon}} \, \mathrm{d}i \right)^{\frac{\epsilon}{\epsilon-1}} \tag{9}$$

where $\epsilon > 1$ is the elasticity of substitution between varieties.

A representative household consumes final goods C_t and supplies labor $\{L_{it}\}_{i\in[0,1]}$ to the intermediate goods firms with isoelastic, separable, expected discounted utility preferences:

$$\mathcal{U}\left(\{C_t, \{L_{it}\}_{i\in[0,1]}\}_{t\in\mathbb{N}}\right) = \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\gamma}}{1-\gamma} - \int_{[0,1]} \frac{L_{it}^{1+\psi}}{1+\psi} \, \mathrm{d}i \right) \right] \tag{10}$$

where household expectations are arbitrary (and potentially correct), $\gamma \in \mathbb{R}_+$ indexes the size of income effects in the household's supply of labor, and $\psi \in \mathbb{R}_+$ is the inverse Frisch labor supply elasticity to each firm.

Finally, we define the composite parameter:

$$\omega = \frac{\frac{1}{\epsilon} - \gamma}{\frac{1+\psi-\alpha}{\alpha} + \frac{1}{\epsilon}} \tag{11}$$

which indexes the strength of strategic complementarity. So that complementarity is positive but not so extreme that the model features multiple equilibria, we assume that $\omega \in [0, 1)$. This requires that income effects in labor supply do not overwhelm aggregate demand externalities (in the sense of Blanchard and Kiyotaki, 1987) and that these externalities are not too large.

2.4 Equilibrium

We study a standard rational expectations equilibrium. All agents optimize, firms form their expectations by combining all available information with their narratives, narratives spread dynamically in accordance with the law governing association and contagion, and all markets clear. Formally:

Definition 1 (Narrative Rational Expectations Equilibrium)**.** *An equilibrium is a path for all variables:*

$$\mathcal{E} = \left\{ Y_t, C_t, Q_t, \theta_t, \varepsilon_t, \{L_{it}, x_{it}, p_{it}, w_{it}, \lambda_{it}, s_{it}, \tilde{\theta}_{it}\}_{i \in [0,1]} \right\}_{t \in \mathbb{N}} \tag{12}$$

1. *Narrative weights λ_{it} and the fraction of optimists Q_t follow a Markov process consistent with Equation 5.*
2. *Firms' production x_{it} maximizes expected profits under the household's stochastic discount factor given their narrative weights λ_{it}, signals s_{it}, and knowledge of $\mathcal{E}$.*
3. *Consumption C_t and labor supply $\{L_{it}\}$ are consistent with household expected utility maximization.*
4. *All markets clear.*

3 Macroeconomic Dynamics with Narratives

We now study the equilibrium dynamics of narratives and output in our model. We find that narratives induce non-fundamental fluctuations in the economy and have the potential to generate endogenous persistence and hysteresis. Moreover, we show how to use firm-level panel data to identify the model's parameters and test its predictions.

3.1 Characterizing Equilibrium Dynamics

To solve for equilibrium production, it suffices to solve for intermediate goods production. These firms maximize expected profits, as priced by the representative household:

$$\Pi_{it} = \mathbb{E}_{it}[C_t^{-\gamma} (p_{it} x_{it} - w_{it} L_{it})] \tag{13}$$

These firms act as monopolists in the product market and monopsonists in the labor market.

To characterize intermediate goods firms' optimal production, we solve for the equilibrium conditions implied by household optimization, final goods optimization, and market clearing. First, the final goods firm maximizes profits taking as given the prices set by intermediate goods firms. This implies the constant-price-elasticity demand curve, $p_{it} = Y_t^{\frac{1}{\epsilon}} x_{it}^{-\frac{1}{\epsilon}}$. Increases in aggregate output shift out this demand curve via aggregate demand externalities. Second, the intratemporal Euler equation of the representative household implies that labor supply is given by $L_{it}^{\psi} = w_{it} C_t^{-\gamma}$. Third, given the production technology of the firm, when it commits to producing x_{it}, its implied labor input is given by $L_{it} = \theta_{it}^{-\frac{1}{\alpha}} x_{it}^{\frac{1}{\alpha}}$. Finally, by imposing goods market clearing $C_t = Y_t$, we obtain that each intermediate goods firm solves the following profit maximization problem:

$$\max_{x_{it}} \mathbb{E}_{it} \left[Y_t^{-\gamma} \left(Y_t^{\frac{1}{\epsilon}} x_{it}^{1-\frac{1}{\epsilon}} - Y_t^{\gamma} \theta_{it}^{-\frac{1+\psi}{\alpha}} x_{it}^{\frac{1+\psi}{\alpha}} \right) \right] \tag{14}$$

Taking the first-order condition of this program, we have that optimal production solves:

$$\left(1 - \frac{1}{\epsilon}\right) \mathbb{E}_{it} \left[Y_t^{\frac{1}{\epsilon}-\gamma} \right] x_{it}^{-\frac{1}{\epsilon}} = \frac{1+\psi}{\alpha} \mathbb{E}_{it} \left[\theta_{it}^{-\frac{1+\psi}{\alpha}} \right] x_{it}^{\frac{1+\psi-\alpha}{\alpha}} \tag{15}$$

where the left-hand side is the marginal expected revenue of the firm from expanding production and the right-hand side is the marginal expected cost of this expansion. A given firm's narrative affects their expected marginal costs of production, via the expectation of idiosyncratic productivity, and their expected marginal benefits of production, via the expectation of aggregate output (which encompasses aggregate demand externalities, asset pricing forces, and wage pressure). Moreover, these beliefs about output depend on the narratives held by other firms.

Substituting this best reply into the final goods production function, any equilibrium *conditional on any process of narrative evolution* solves the following functional fixed-point equation:

$$\begin{aligned} \log Y_t = \frac{\epsilon}{\epsilon - 1} \log \mathbb{E}_t \Bigg[\exp \Bigg\{ \frac{\frac{\epsilon-1}{\epsilon}}{\frac{1+\psi-\alpha}{\alpha} + \frac{1}{\epsilon}} \Bigg(\log \left(\frac{1-\frac{1}{\epsilon}}{\frac{1+\psi}{\alpha}} \right) \\ - \log \mathbb{E}_{it} \left[\exp \left\{ -\frac{1+\psi}{\alpha} \log \theta_{it} \right\} \right] + \log \mathbb{E}_{it} \left[\exp \left\{ \left(\frac{1}{\epsilon} - \gamma \right) \log Y_t \right\} \right] \Bigg) \Bigg\} \Bigg] \end{aligned} \tag{16}$$

where the outer expectation operator integrates over productivity shocks $(\tilde{\theta}_{it}, \gamma_i)$, narrative loadings λ_{it}, and signals s_{it}.

By employing a functional guess-and-verify argument, we obtain that the model has a unique *quasi-loglinear* equilibrium in which log output depends linearly on log aggregate productivity and non-linearly, but separably, on the fraction of optimists in the population:

Theorem 1 (Equilibrium Characterization)**.** *There exists a unique quasi-loglinear equilibrium:*

$$\log Y(\log \theta_t, \log \theta_{t-1}, Q_t) = a_0 + a_1 \log \theta_t + a_2 \log \theta_{t-1} + f(Q_t) \tag{17}$$

Moreover, in the unique quasi-loglinear equilibrium, we have that:

$$f(Q_t) = \frac{1}{1-\omega} \frac{\epsilon}{\epsilon - 1} \log \left(1 + Q_t \left[\exp \left\{ \frac{\epsilon - 1}{\epsilon} \alpha \delta^{OP} \right\} - 1 \right] \right) \tag{18}$$

where δ^{OP} is given by:

$$\delta^{OP} = \frac{1}{\alpha} \frac{\frac{1+\psi}{\alpha}}{\frac{1+\psi-\alpha}{\alpha} + \frac{1}{\epsilon}} \left(1 + \frac{\kappa\omega}{1-\kappa\omega} \right) (1-\kappa)(1-\rho)(\mu_O - \mu_P) \tag{19}$$

Proof. See Appendix A.1, which also provides the formulas for $a_0, a_1 > 0$, and $a_2 > 0$. □

Before discussing the implications for and intuition behind this result, we first discuss the important issue of whether there exist other equilibria that are not the unique quasi-loglinear equilibrium. We show that the unique quasi-loglinear equilibrium is the only equilibrium that survives a natural refinement that requires an equilibrium to obtain as a limit of economies with bounded fundamentals as that bound is taken to be large. We believe that this refinement is of independent interest and could be used in dispersed information economies with unbounded fundamentals in which the uniqueness of equilibrium is an open question outside of the log-linear class, such as Angeletos and La'O (2013) and Benhabib, Wang, and Wen (2015).

Remark 1 (The Unique Quasi-Loglinear Equilibrium is the Unique Equilibrium Under a "Boundedness" Refinement)**.** Theorem 1 establishes uniqueness within the quasi-loglinear class. As best replies and aggregation are non-linear and the space of fundamentals is not compact, one cannot use classical arguments based on Blackwell's sufficient conditions to ensure that the fixed point operator implicit in Equation 16 is a contraction. In reality, of course, productivity is bounded (potentially for some extremely large bound), and using unbounded fundamentals is a simplifying approximation of reality. Hence, we might regard an equilibrium that does not survive this refinement as a product of the artifice of unbounded fundamentals. In Appendix A.1, we show that there is a unique equilibrium when fundamentals are restricted to lie in a compact set (Lemma 1). Moreover, the claimed quasi-loglinear equilibrium is an ε-equilibrium for any $\varepsilon > 0$ for some sufficiently large support for fundamentals (Lemma 2). Hence, the quasi-loglinear equilibrium is the limit of the unique equilibrium with bounded fundamentals as the bound becomes large.

3.2 Narratives Drive Non-Fundamental Fluctuations in Output

We now unpack the economics of Theorem 1 to study how narratives drive non-fundamental fluctuations in output and examine the ability of information to prevent narratively driven fluctuations.

The Effect of Optimism on Output. First, we observe that optimism affects output in a way that is separable from fundamentals via the function f. This function is non-linear because firms' heterogeneous priors induce heterogeneity in production conditional on productivity and hence also misallocation. Notwithstanding this non-linearity, it turns out that it is useful to summarize the role of optimism by computing the difference in aggregate output when everyone in the economy is optimistic versus when everyone in the economy is pessimistic:

$$\Delta_t \equiv \log Y(\log\theta_t, \log\theta_{t-1}, 1) - \log Y(\log\theta_t, \log\theta_{t-1}, 0) \tag{20}$$

By Theorem 1, we observe that $\Delta_t = \Delta = f(1) - f(0)$, which has an intuitive structure:

Corollary 1 (The Effect of Optimism on Output)**.** *The effect on aggregate output of moving from a fully pessimistic economy to a fully optimistic economy, Δ_t, is invariant to time and the state of the economy and is given by:*

$$\Delta = \frac{1}{1-\omega} \times \alpha \times \delta^{OP} \tag{21}$$

In this expression, as we will later justify formally, δ^{OP} is the partial equilibrium effect of a firm's optimism on the amount of labor the firm hires when we hold fixed the behavior of all other firms and fundamentals. To find the general equilibrium effect of this on aggregate output, Theorem 1 implies that we can first convert the effect of hiring into the output effect via the returns-to-scale parameter α and then apply a *narrative multiplier* $\frac{1}{1-\omega}$. This multiplier is large when strategic complementarities in production arising from aggregate demand externalities are much larger than strategic substitutability that arises from income effects in household labor supply. This multiplier captures the intuitive idea that even a pessimistic firm will produce more if a large fraction of *other* firms is optimistic, as this optimism increases aggregate demand.

The Power of the Truth. Second, this result formalizes and provides nuance for Shiller's (2020) argument that "the truth is not enough to stop false narratives." Specifically, let us define the *power of the truth* as $|\frac{\partial \Delta}{\partial \kappa}|$. This measures how the effect of narratives on firms' hiring and aggregate output changes as they receive more precise information about productivity.

Corollary 2 (The Power of the Truth)**.** *The power of the truth $|\frac{\partial \Delta}{\partial \kappa}|$ is positive and increasing in the precision of private information κ. The power of truth is strictly increasing in the precision of private information if and only if strategic complementarity is strictly positive ($\omega > 0$).*

The key implication of this result is that providing information to agents is least effective at stopping narratives exactly when narratives are at their most powerful. Conversely, when private information is precise and narratives are weak, the marginal effects of better private information are strong. In this sense, the ability of information to prevent narratives faces an *adverse selection* problem: narratives are the hardest to stop exactly when they are the strongest.

This result depends critically on the presence of general equilibrium interactions and strategic complementarity. That is, when there is no strategic complementarity and $\omega = 0$, the power of the truth is constant. To see this, observe that:

$$\Delta \propto \left(1 + \frac{\kappa\omega}{1-\kappa\omega}\right)(1-\kappa)(\mu_O - \mu_P) \tag{22}$$

where the constant of proportionality does not depend on κ. Thus, when $\omega = 0$, we have that the power of the truth is given by $|\frac{\partial \Delta}{\partial \kappa}| \propto (\mu_O - \mu_P)$, which depends on the differences in beliefs across narratives but not the precision of agents' information. Intuitively, providing more information simply scales down how much agents rely on their narratives. The aforementioned adverse selection problem arises when there is strategic complementarity and $\omega > 0$ because increasing the precision of private information now has a second effect: agents know that other agents will be responding more to their signals and relying less on their priors. Because agents want to produce more when others produce more, agents' narratives about fundamentals become paradoxically *more* important as they now use narratives more aggressively in forecasting the actions that others will take. This effect dampens the ability of information to stop narratives and it does so by more exactly when information is weakest and narratives are strongest.

3.3 The Dynamics of Narratives and Output

We now use the characterization of output in Theorem 1 to describe the economy's dynamics via a first-order nonlinear stochastic difference equation for aggregate optimism.

Corollary 3 (Narrative Dynamics)**.** *In the unique quasi-loglinear equilibrium, the fraction of optimists Q_t evolves according to $Q_{t+1} = T(Q_t, \log\theta_t, \log\theta_{t-1}, \varepsilon_t)$, where*

$$\begin{aligned} T(Q_t, \log\theta_t, \log\theta_{t-1}, \varepsilon_t) = Q_t P_O(a_0 + a_1\log\theta_t + a_2\log\theta_{t-1} + f(Q_t), Q_t, \varepsilon_t) + \\ (1-Q_t)P_P(a_0 + a_1\log\theta_t + a_2\log\theta_{t-1} + f(Q_t), Q_t, \varepsilon_t) \end{aligned} \tag{23}$$

This result has two important implications. First, narratives can be self-propagating. Formally, holding fixed the fundamental and narrative shocks $(\log\theta_t, \log\theta_{t-1}, \epsilon_t)$, optimism evolves non-linearly: individuals' proclivity to hold onto their current narrative, social contagiousness, and associativeness shape the spread of narratives via P_O and P_P. Second, narratives provide a propagation channel through which fundamental shocks can have endogenously persistent effects. For example, a one-time productivity shock today can increase aggregate output and thereby increase future optimism through the associativeness mechanism. Moreover, this increased optimism can then self-propagate even in the absence of subsequent shocks. We now unpack and formalize these ideas by characterizing the properties of the dynamical system for narratives and output.

Steady-States and Narrative Hysteresis. We begin by isolating the propagation of narratives without shocks. Formally, let $T_\theta(Q) = T(Q, \theta, \theta, 0)$ denote the transition map for aggregate optimism when aggregate productivity is fixed and there is no narrative shock. We say that a level of optimism Q^*_θ is a deterministic steady state for the level of productivity θ if it is a fixed point of the corresponding map, $T_\theta(Q^*_\theta) = Q^*_\theta$. The following result establishes that a deterministic steady state always exists and provides necessary and sufficient conditions for extreme optimism and pessimism to be (stable) steady states.

Theorem 2 (Steady State Multiplicity and Stability)**.** *The following statements are true:*

1. *There exists a deterministic steady-state level of optimism for every $\theta \in \Theta$.*
2. *There exist thresholds θ_P and θ_O such that: $Q = 0$ is a deterministic steady state for θ if and only if $\theta \leq \theta_P$ and $Q = 1$ is a deterministic steady state for θ if and only if $\theta \geq \theta_O$. Moreover, these thresholds are given by:*

$$\theta_P = \exp\left\{\frac{P_P^{-1}(0;0) - a_0}{a_1 + a_2}\right\} \quad \text{and} \quad \theta_O = \exp\left\{\frac{P_O^{-1}(1;1) - a_0 - \Delta}{a_1 + a_2}\right\} \tag{24}$$

   where $P_P^{-1}(x;Q) = \sup\{Y : P_P(Y,Q,0) = x\}$ and $P_O^{-1}(x;Q) = \inf\{Y : P_O(Y,Q,0) = x\}$.
3. *Extreme pessimism is stable if $\theta < \theta_P$ and $P_O(P_P^{-1}(0;0), 0, 0) < 1$ and extreme optimism is stable if $\theta > \theta_O$ and $P_P(P_O^{-1}(1;1), 1, 0) > 0$.*

Proof. See Appendix A.3. □

If extreme optimism or extreme pessimism is a stable steady state, then the optimistic (or pessimistic) narrative has a tendency to "go viral" and fully infect the entire population. The conditions under which this occurs can be checked with only a few parameters, which we will later be able to discipline empirically: the responsiveness of output to productivity (a_1, a_2), the impact of all agents being optimistic on output Δ, the highest level of output such that all pessimists remain pessimistic when everyone is a pessimist $P_P^{-1}(0; 0)$, and the lowest level of output such that all optimists remain optimistic when all other agents are optimists $P_O^{-1}(1; 1)$.

Of particular interest is the case in which, for fixed values of other parameters and fundamentals, *either* extreme optimism or extreme pessimism could go viral depending on initial conditions. This can induce fully history-dependent, long-run changes in output, a property which we refer to as *narrative hysteresis*. The following corollary characterizes exactly when this can happen:

Corollary 4 (Narrative Hysteresis)**.** *Extreme optimism and pessimism are simultaneously deterministic steady states for θ if and only if $\theta \in [\theta_O, \theta_P]$, which is non-empty if and only if*

$$P_O^{-1}(1; 1) - P_P^{-1}(0; 0) \leq \Delta \tag{25}$$

Intuitively, this condition is more likely to hold if the optimistic narrative has a large effect on output (high Δ), if a relatively low output can be consistent with self-fulfilling optimism (low $P_O^{-1}(1; 1)$), or if a relatively high output can be consistent with self-fulfilling pessimism (high $P_P^{-1}(0; 0)$).

3.4 Hysteresis, Endogenous Persistence, and Boom-Bust Cycles

So far, we have analyzed dynamics at an abstract level. We now introduce a parametric family of narrative updating rules that embody the key forces of narrative transmission. This will allow us to understand the key features of narrative business cycles. Later, we will take this model to the data and quantify its implications.

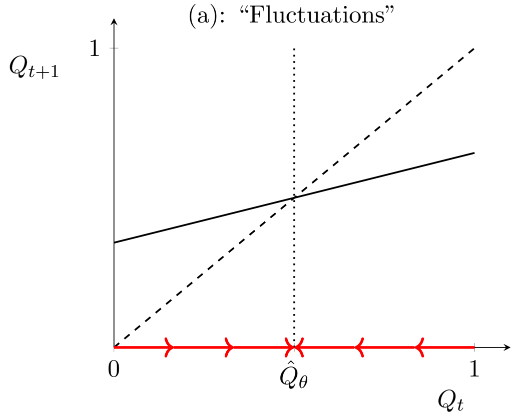
The *linear-associative-contagious* (LAC) model for updating rules is:

$$\begin{aligned} P_O(\log Y, Q, \varepsilon) &= \left[\frac{u}{2} + r \log Y + sQ + \varepsilon\right]_0^1 \\ P_P(\log Y, Q, \varepsilon) &= \left[-\frac{u}{2} + r \log Y + sQ + \varepsilon\right]_0^1 \end{aligned} \tag{26}$$

where $[z]_0^1 = \max\{\min\{z, 1\}, 0\}$ and ε is i.i.d. $N(0, \sigma_\varepsilon^2)$. The parameter $u \geq 0$ captures *stubbornness*, or all agents' proclivity not to change narratives. The parameter $r \geq 0$ captures *associativeness*, or the extent to which agents associate high output with the optimistic narrative. The parameter $s \geq 0$ captures contagiousness, or the direct effect of peers' narrative weights on one's own. Finally, the aggregate shock allows narratives to fluctuate autonomously. This last feature has no impact on the theoretical properties described below, but it will help us later match data on observed narrative dynamics.

We now illustrate three qualitative properties of narrative business cycles:

Figure 1: Fluctuations *vs.* Hysteresis in the Linear-Associative-Contagious Model



Notes: In each subfigure, the solid line is an example transition map T_θ, the dashed line is the 45-degree line, the dotted vertical line indicates the interior steady state $\hat{Q}_\theta$, and the red arrows indicate the dynamics. Both correspond to the linear-associative-contagious model with different calibrations for the underlying parameters. In panel (a) ("fluctuations"), the condition for extremal multiplicity (Equation 27) does not hold. In panel (b) ("hysteresis"), the condition does hold, and $Q = 0$ and $Q = 1$ are stable steady states.

1. *Hysteresis and the criticality threshold*: despite equilibrium uniqueness, there can be multiple steady states and a critical level of narrative adoption away from which the economy diverges.
2. *Endogenous persistence of output*: stubbornness, associativeness, and contagiousness generate state-dependent and size-dependent persistence of one-time shocks.
3. *Boom-bust cycles*: even when hit by i.i.d. stochastic shocks, the economy features a tendency toward boom-bust cycles.

In Appendix B.1, we formalize these properties of shock responses in a larger class of non-parametric updating rules. Below, we describe them using examples from the LAC class.

Hysteresis and the Criticality Threshold. Figure 1 visualizes the transition map for two example calibrations of the updating rule, fixing the state θ and the calibration of other parameters. In panel (a), stubbornness, contagiousness, and associativeness are relatively low. The transition map T_θ crosses the 45-degree line once, from above. Therefore, the interior steady state denoted by $\hat{Q}_\theta$ is stable, and optimism tends to converge to this level if perturbed away from it. For this reason, we refer to this as a case that admits "fluctuations" if hit by shocks. In panel (b), stubbornness, contagiousness, and associativeness are relatively high. The transition map T_θ intersects the 45-degree line three times: twice at the extremes of $Q = 0$ and $Q = 1$ and once from below at an interior level $\hat{Q}_\theta$. Paths for Q that start slightly to the left or right of $\hat{Q}_\theta$ converge, respectively, to the stable points of $Q = 0$ or $Q = 1$. In this sense, dynamics of optimism display hysteresis: holding fixed fundamentals, the long-run behavior of the economy depends on initial conditions. Lemma 3 in the Appendix formalizes these ideas by showing exactly when $\hat{Q}_\theta$ is on the boundary of two basins of attraction for, respectively, extreme pessimism and extreme optimism.

In the LAC case, we can analytically compute the condition in Corollary 4 which determines when extreme optimism and pessimism are both stable steady states. In particular, $P_O^{-1}(1;1)$ solves $1 = \frac{u}{2} + rP_O^{-1}(1;1) + s$, so $P_O^{-1}(1;1) = \frac{1}{r} - \frac{u}{2r} - \frac{s}{r}$; $P_P^{-1}(0;0)$ solves $0 = -\frac{u}{2} + rP_P^{-1}(0;0)$, so $P_P^{-1}(0;0) = \frac{u}{2r}$. Thus, extreme optimism and pessimism can coexist as steady states if and only if:

$$M = u + s + r\Delta - 1 \geq 0 \tag{27}$$

which is to say that stubbornness, associativeness, contagiousness, and the equilibrium impact of optimism on output are sufficiently large. This expression clarifies that strong *static complementarities*, which would manifest in high Δ, are sufficient but not necessary for extremal multiplicity. In particular, stubbornness and contagiousness contribute *dynamic complementarity* that can also induce extremal multiplicity. Thus the parameter M, which incorporates both static and dynamic complementarity, is the correct gauge for the "strength" of narratives.

Moreover, as suggested by panel (b) of Figure 1, the model with stable extremal steady states has an unstable, intermediate steady state $\hat{Q}_\theta \in (0,1)$ that solves $\hat{Q}_\theta = T_\theta(\hat{Q}_\theta)$, or

$$\hat{Q}_\theta = \frac{u}{2}(2\hat{Q}_\theta - 1) + s\hat{Q}_\theta + r(a_0 + (a_1 + a_2)\log\theta + f(\hat{Q}_\theta)) \tag{28}$$

We refer to this value of optimism as the *criticality threshold* because it separates regions of the state space that are attracted to extreme optimism versus extreme pessimism. Under the approximation that $f(Q) \approx \Delta Q$, which we later find to be quantitatively accurate, we have that:

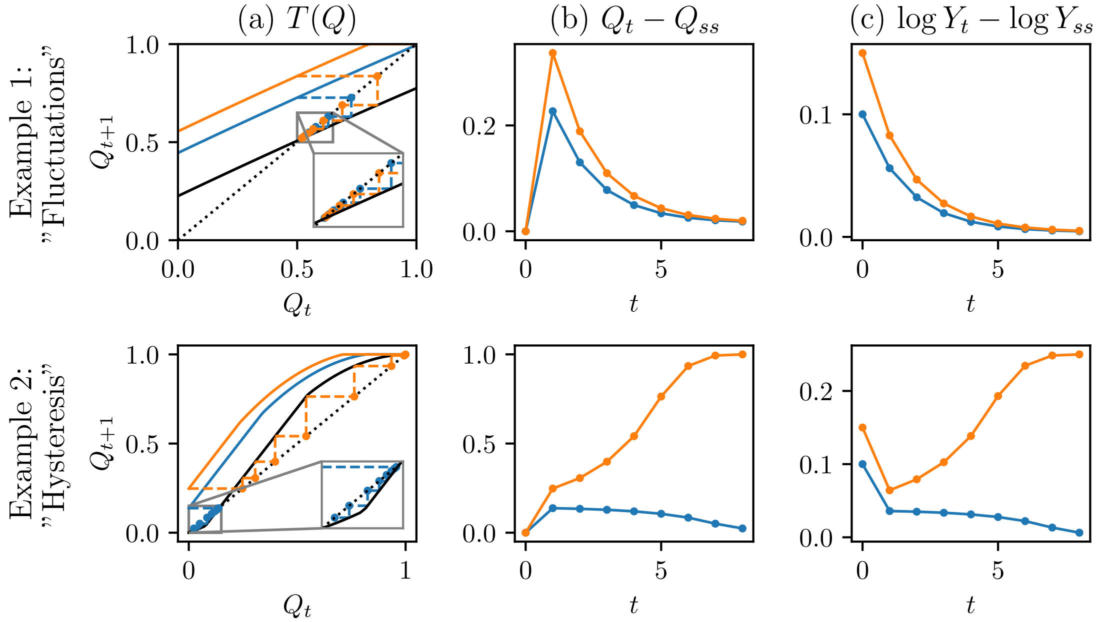
$$\hat{Q}_\theta \approx \frac{\frac{u}{2} - r(a_0 + (a_1 + a_2)\log\theta)}{M} \tag{29}$$

Hence, greater contagiousness (s), associativeness (r), and static economic impact of narratives (Δ) reduce $\hat{Q}_\theta$, or equivalently decrease the lower bound of initial optimism that is consistent with the optimistic narrative eventually going viral.

Endogenous Persistence. We now study how the economy with narratives responds to shocks. Figure 2 illustrates how the economy responds to shocks under a "fluctuations" versus "hysteresis" calibration of the model. In both calibrations, productivity shocks are perfectly transitory ($\rho = 0$). The blue and orange lines of each plot respectively illustrate smaller and larger one-time productivity shocks at $t = 0$. In both cases, because of positive associativeness, these correspond to one-time upward shifts in the transition maps $T(Q)$ (panel (a)). The dots and dashed lines in panel (a) trace out the dynamic response of optimism to each shock using the transition map. Panels (b) and (c) illustrate the impulse response functions of optimism and log output.

In the fluctuations case (row 1), narratives create endogenous persistence in the economic boom. Because of positive associativeness ($r > 0$), stubbornness ($u > 0$), and contagiousness ($s > 0$), optimism remains elevated for several periods before smoothly converging back to the steady state. At $t = 0$, output is elevated above its steady-state value *only* because of the productivity shock; for $t \geq 1$, output is elevated because of the persistent increase in optimism, even though productivity has returned to its steady-state value. The large shock (orange) leads to a larger and more persistent

Figure 2: Endogenous Persistence in Shock Responses



Notes: This figure illustrates the response of the economy to transitory productivity shocks under two different model calibrations. The top row corresponds to a "fluctuations" calibration and the bottom row corresponds to a "hysteresis" calibration, as defined in the main text. The orange lines correspond to a larger productivity shock and the blue lines correspond to a smaller productivity shock. Column (a) shows the transition map for aggregate optimism (black), its perturbations under each shock (colors), and the paths of optimism (dots and dashed lines). The inset graphs zoom in near the interior steady state. Column (b) shows the impulse response of optimism relative to the interior steady-state value (top row) and relative to extreme pessimism (bottom row). Column (c) shows the impulse response of log output relative to the respective steady-state values.

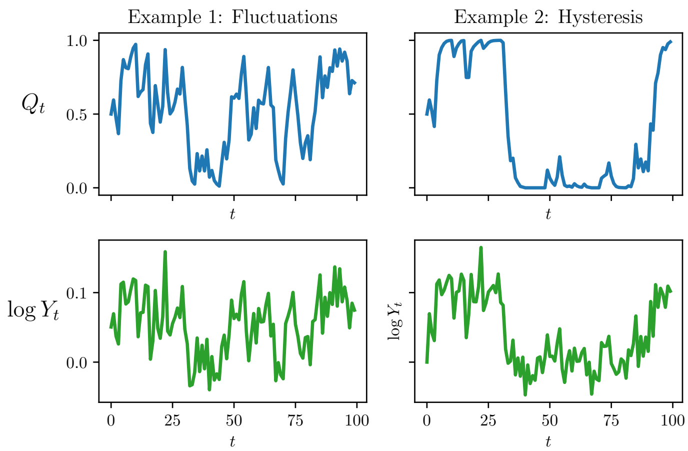
boom than the small shock (blue).

In the hysteresis case (row 2), the small shock leads to a highly persistent boom, whereas the large shock leads to a *regime shift*. This discontinuity of shock responses as a function of shock size emerges because large shocks can push the economy above the unstable interior steady state (panel (a)). Intuitively, the large shock seeds enough optimism for the optimistic narrative to "go viral." This also induces a non-monotone response of output (panel (c)): while the direct effect of productivity disappears after one period, the effect of viral optimism grows over time.

In sum, both regimes feature *endogenous persistence* of output even in response to one-time shocks. In the hysteresis regime, there is the possibility also of *permanent* economic effects of temporary shocks. Propositions 1 and 2 in the Appendix formalize these properties, and moreover characterize when endogenous persistence is large enough to generate "hump-shaped" impulse response functions of output in response to perfectly transitory shocks.

Boom-Bust Cycles. We finally study how these different cases map to time-series properties of the macroeconomy. To visualize this, we simulate from "fluctuations" and "hysteresis" calibrations

Figure 3: Simulated Paths of the Economy under "Fluctuations" *vs.* "Hysteresis"



Notes: Each panel corresponds to a simulated time series of the model, with identical time paths of i.i.d. productivity shocks but differing calibrations of narrative evolution across the two examples.

of the model for 100 periods in Figure 3. Productivity shocks are *common* across the two simulations and there are no direct shocks to narrative evolution ($\sigma_\varepsilon = 0$). In both simulations, output is persistent despite the lack of persistent driving shocks. This arises because of persistent variation in optimism. In the fluctuations case, the narrative state variable tends to revert to a steady state of $Q = 0.5$. In the hysteresis case, the narrative state variable is attracted toward extreme optimism or extreme pessimism. For example, in the middle of the sample path, a sequence of large negative shocks pushes the economy toward the pessimistic steady state. Thus, in both cases, narrative evolution leads to persistent "business-cycle" variations. In the hysteresis case, narrative dynamics can lead to more dramatic, medium-frequency "boom and bust" patterns. Proposition 3 in the Appendix formalizes this observation and provides analytical bounds on the period of boom-bust cycles in the model.

3.5 Additional Theoretical Results and Extensions

Here, we briefly summarize additional results and extensions contained within the Appendix.

Welfare Implications. In Appendix B.2, we study the normative implications of optimism and provide conditions under which its presence is welfare improving, despite its being misspecified. Intuitively, optimism acts as if it were an *ad valorem* price subsidy for firms, which induces firms to hire more and can undo distortions caused by market power.

Multi-dimensional Narratives. In Appendix B.4, we generalize the model to feature a continuum of models regarding the mean of productivity. In Appendix B.6, we generalize the model to allow for arbitrarily many narratives regarding the mean, persistence, and volatility of fundamentals, which is essentially exhaustive within the Gaussian class. In both cases, we characterize equilibrium output and narrative evolution.

Bayesian Updating. An important model that is ruled out by our conditions on the updating rule is one in which firms observe aggregate variables $\log Y_t$ and Q_t and use Bayes' rule to update their beliefs over models. As we formalize in Appendix B.3, this "Bayesian benchmark" contradicts the dependence of firms' updating on Q_t and ε_t conditional on $\log Y_t$ (respectively, contagiousness and shocks). Moreover, this "Bayesian benchmark" predicts that agents converge to holding the better-fitting empirical model exponentially quickly. Later, we will show that such a prediction is at odds with our finding of cyclical dynamics for aggregate optimism (Figure A1). However, in principle, richer Bayesian models that are consistent with our empirical results might be nested by our reduced-form updating probabilities.

Persistent Idiosyncratic Shocks and Narrative Updating. In Appendix B.5, we show that a model in which narratives concern the probability distribution of *idiosyncratic* fundamentals yields the same predictions as our main model. Thus, as long as there are aggregate narrative dynamics, narratives need not describe the macroeconomy *per se*. In Appendix B.7, we extend the model to allow for narrative updating that depends on the realization of a persistent idiosyncratic productivity state. When idiosyncratic shocks are fully transitory, this is of no consequence and our equilibrium characterization is identical. However, when idiosyncratic shocks are persistent, the fact that narrative updating depends on idiosyncratic shock realizations induces dependence between an agent's narrative and their idiosyncratic productivity state. This matters for equilibrium output only insofar as it induces a time-varying covariance between optimism and productivity. We find no empirical evidence for the cyclicality of this covariance. We therefore abstract from this channel.

Contrarianism, Endogenous Cycles, and Chaos. While this model generates narratively driven fluctuations, it cannot generate fully endogenous cycles and chaotic dynamics. In Appendix B.8, we extend this model to allow for contrarianism and the possibility that pessimists may be more likely to become optimists than optimists are to remain optimists. Allowing for these features generates the possibility of endogenous cycles of arbitrary period and topological chaos (sensitivity to arbitrarily small changes in initial conditions). This model also admits a structural test for the presence of cycles and chaos that we bring directly to the data and reject at the 95% confidence level that either cycles or chaos obtain.

Narratives in Games and the Role of Higher-Order Beliefs. We have studied narratively driven fluctuations in a business-cycle model, but our insights apply to co-ordination games much more generally. Taking a more abstract perspective, in Appendix B.9, we study contagious narratives in beauty contests (in the sense of Morris and Shin, 2002), in which agents' best replies are a linear function of their expectations of fundamentals and the average actions of others. Many models of aggregative games in macroeconomics and finance can be recast as such games when (log-)linearized (for a review, see Angeletos and Lian, 2016). We characterize equilibrium in this context and show how optimism percolates through the hierarchy of higher-order beliefs about fundamentals. This allows us to show that the narrative multiplier in our model can be understood as arising from the effect of narratives on agents' higher-order beliefs about the state of the economy.

3.6 Toward Measurement: Identification of Model Parameters

We have shown that the theoretical properties of narrative business cycles hinge on four critical objects: (1) the effect of optimism on hiring, (2) agents' updating rules, (3) the persistence and volatility of exogenous shocks, and (4) the extent of private information. We now show how to identify these objects conditional on calibrating four standard macroeconomic production and preference parameters. This will form the basis for our measurement, empirical strategies, and quantification.

Step I: Identification of The Effect of Optimism. Formally, Theorem 1 implies that f, the effect of narrative optimism on output in the unique quasi-loglinear equilibrium, is identified given knowledge of both δ^{OP} and the standard macroeconomic parameters $(\alpha, \epsilon, \gamma, \psi)$. Moreover, the model implies that δ^{OP} can be recovered via a simple regression of firms' hiring on their optimism with firm fixed-effects, time fixed-effects, and controls for firm-level productivity and lagged labor:

Corollary 5 (Firm Hiring Regression)**.** *In the unique quasi-loglinear equilibrium, firms' hiring decisions obey the following equation:*

$$\Delta \log L_{it} = \gamma_i + \chi_t + \tau_1 \log \theta_{it} + \tau_2 \log L_{i,t-1} + \delta^{OP} \lambda_{it} + \zeta_{it} \tag{30}$$

where the time fixed-effect is given by $\chi_t = c_1 \log \theta_t + c_2 \log \theta_{t-1} + c_3 f(Q_t)$ *for some constants* $c_1, c_2,$ *and* c_3*, and* ζ_{it} *is an i.i.d. normal random variable with zero mean. Thus, conditional on* $(\alpha, \epsilon, \gamma, \psi)$*,* δ^{OP} *uniquely identifies* f*, the equilibrium effect of optimism on aggregate output.*

Proof. See Appendix A.4. □

The "time fixed-effect" of this regression absorbs two aggregate equilibrium forces: the general-equilibrium effect of optimism on hiring, $c_3 f(Q_t)$, and the effects of aggregate productivity on aggregate output, $c_1 \log \theta_t + c_2 \log \theta_{t-1}$. Without the time fixed effect, the regression would produce a biased estimate for δ^{OP} because of the correlation of optimism with aggregate economic fundamentals. These facts highlight formally the necessity of combining cross-sectional variation and a structural model for general-equilibrium interaction to identify the effect of narratives on economic outcomes.[6]

Step II: Identification of Updating Rules. In the linear-associative-contagious (LAC) model for updating rules, we can identify u, r, and s by estimating a linear probability model for the evolution of optimism at the firm level. The residual term in this regression corresponds to idiosyncratic shocks to updating (since the model is probabilistic) plus the aggregate shock ε.

[6] We note that our argument for identifying the partial-equilibrium effect of optimism on hiring and aggregating these effects via the model also applies if narratives concern firms' idiosyncratic productivity (Appendix B.5). Thus, our empirical and quantitative analysis is not sensitive to this modeling choice. As we explain formally in the Appendix, only the exact identification of the underlying parameters κ and $\mu_O - \mu_P$ would change, while the outcome-relevant objects δ^{OP} and f remain the same.

Step III: Identification of Private Information and the Shock Processes. To obtain the law of motion of aggregate output, we require the four parameters that govern the persistence of productivity ρ, the volatility of productivity innovations σ, the signal-to-noise ratio for productivity κ, and the volatility of optimism shocks σ_ε. The key to our identification strategy for the first three parameters is the following observation:

Corollary 6 (Fundamental Output is an ARMA(1,1))**.** *In the unique quasi-loglinear equilibrium, the fundamental component of output,* $\log Y_t^f = \log Y_t - f(Q_t) - a_0$*, follows an ARMA(1,1) process:*

$$\log Y_t^f = \rho \log Y_t^f + a_1 \sigma \nu_t + a_2 \sigma \nu_{t-1} \tag{31}$$

where (a_0, a_1, a_2) *are the coefficients chacterized in Theorem 1 and* ν_t *is i.i.d.* $N(0,1)$*.*

The coefficients of this ARMA(1,1) process impose three restrictions on the three parameters (ρ, σ, κ). We finally observe that, conditional on all other parameters, the scaling of the narrative shock σ_ε is identified by the observed time-series variance of aggregate optimism $\text{Var}[Q_t]$.

Roadmap: From Identification to Measurement, Estimation, and Quantification. In the next three sections, motivated by these identification arguments, we measure the required variables, estimate the required relationships, and use the estimated parameters to quantify the role of narratives in driving the business cycle.

4 Data and Measurement

We now develop a panel dataset on firms' narratives and decisions. In particular, we measure textual proxies for narratives by applying natural-language-processing techniques to two corpora of language: the universe of public firms' SEC Forms 10-K and a large sample of earnings calls.

4.1 Data

Text. Our main source of firm-level textual data is SEC Form 10-K. Each publicly traded firm in the US submits an annual 10-K to the SEC. These forms provide "a detailed picture of a company's business, the risks it faces, and the operating and financial results of the fiscal year." Moreover, "company management also discusses its perspective on the business results and what is driving them" (SEC, 2011). This description is consistent with our premise that narratives constitute a view of the world and its rationalization via some model.

We download all SEC Forms 10-K from the SEC Edgar database from 1995 to 2019. This yields a corpus of 182,259 `html` files comprising the underlying text of the 10-K. We describe our exact method for processing the text data in Appendix C.1. The three key steps are pre-processing the raw text data to isolate English-language words, associating words with their common roots via lemmatization, and fitting a bigram model that groups together co-occurring two-word phrases. We then count the occurrences of all words, including bigrams, in all documents to obtain the bag-of-words representation (*i.e.,* a vector of word counts) for each document. Our final sample consists of 100,936 firm-by-year observations from 1995 to 2018.

As an alternative source of text data, we use public firms' sales and earnings conference calls. Our initial sample consists of 158,810 documents from 2002 to 2014. We apply the same natural-language-processing techniques that we employ for the 10-Ks to this corpus. We average variables over the periods between successive 10-Ks to obtain a firm-by-fiscal-year dataset. Our final sample consists of 25,589 firm-by-year observations. We describe more details in Appendix C.2.

Firm Fundamentals and Choices. We compile our dataset of firm fundamentals and choices using Compustat Annual Fundamentals from 1995 to 2018. This dataset includes information from firms' financial statements on employment, sales, input expenses, capital, and other financial variables. We apply standard selection criteria to screen out firms that are very small, report incomplete information, or were likely involved in an acquisition. As is standard, we also drop firms in the financial and utilities sectors due to their markedly different production and/or market structure. More details about our sample selection are in Appendix D.1. We organize firms into 44 industries, which are defined at the NAICS 2-digit level, but for Manufacturing (31-33) and Information (51), which we split into the 3-digit level.

Manager and Analyst Beliefs. We collect data from the International Brokers' Estimate System (IBES) on quantitative sales forecasts by companies. Specifically, we use the IBES Guidance dataset which records, for specific variables, a numerical management expectation recorded from press releases or transcripts of corporate events. We restrict to the first recorded forecast per fiscal year of that year's sales.

4.2 Recovering Narrative Optimism from Language

As our primary measure of the narratives adopted by firms, we categorize firms as "optimistic" or "pessimistic" in a given fiscal year based on their use of language. Toward this goal, we first categorize individual words as either positive or negative using the dictionaries constructed by Loughran and McDonald (2011). These dictionaries adjust standard tools for sentiment analysis to more precisely score financial communications, in which certain words (*e.g.*, the leading example "liability") have specific definitions.[7] We first define $\mathcal{W}_P$ as the set of positive words and $\mathcal{W}_N$ as the set of negative words. For reference, we print the 20 most common words in each set in Table A1. We calculate positive and negative sentiment as:

$$\text{pos}_{it} = \sum_{w \in \mathcal{W}_P} \text{tf}(w)_{it} \qquad \text{neg}_{it} = \sum_{w \in \mathcal{W}_N} \text{tf}(w)_{it} \tag{32}$$

where $\text{tf}(w)_{it}$ is the term frequency of all bigrams including word w in the time-t 10-K of firm i. We then construct a one-dimensional measure of net sentiment, sentiment_{it}, by computing the

[7] Loughran and McDonald (2011) generate the dictionaries based on human inspection of the most common words in the 10-Ks and their usage in context. We describe more details of our methodology in Appendix C.3.

across-sample z-scores of both positive and negative sentiment and taking their difference. Finally, we define a firm i as being optimistic at time t if its sentiment is above the entire-sample median:

$$\text{opt}_{it} = \mathbb{I}\left[\text{sentiment}_{it} \geq \text{med}\left(\text{sentiment}_{it}\right)\right] \tag{33}$$

Aggregating this optimism measure across firms, we find that aggregate optimism is persistent, with an autocorrelation of 0.75, and cyclical, with a correlation of -0.37 with the contemporaneous level of unemployment (see Figure A1). Both features are to be expected in our model. The former is a result of stubbornness, contagiousness, associativeness, and autocorrelation of fundamentals. The latter could reflect either direction of causality: narratives might reflect current conditions, or narratives might have an economically significant effect on real outcomes. Because the time series evidence cannot distinguish among these different explanations, it is insufficient to calibrate the model and identify its key mechanisms. Thus, in the next section, we will use cross-sectional variation in narratives to isolate their effects and characteristics.

Measuring Other Narratives. Measuring narrative optimism is sufficient for describing firms' prior beliefs in our model, as there is only one dimension of fundamental uncertainty. But this strategy abstracts from the details of what firms discuss. This could mask richer dynamics that underlie the evolution of sentiment. In Section 7, we describe and implement methods for measuring more specific firm narratives.

5 Empirical Results

We now use our firm-level dataset to estimate how narrative optimism affects decisions and spreads. In the process, we test our narrative interpretation of the data against a key alternative interpretation that optimism is driven by news. We find strong evidence against the news interpretation and in favor of our narrative interpretation.

5.1 Narrative Optimism Drives Hiring

We first estimate the relationship between narrative optimism and hiring. The estimating equation is derived in Corollary 5. Specifically, we estimate the following firm-by-fiscal-year model:

$$\Delta \log L_{it} = \delta^{OP} \text{opt}_{it} + \gamma_i + \chi_{j(i),t} + \tau' X_{it} + \varepsilon_{it} \tag{34}$$

The outcome variable is the log difference of the firm's employment across fiscal years ("hiring") and the main regressor, opt_{it}, is the binary indicator for optimism whose construction is described in Section 4. We control for firm and industry-by-time fixed effects to sweep out fixed differences across firms and non-parametric trends and cycles within industries. We include a suite of firm-level time-varying controls X_{it} including current and past TFP, lagged labor, and financial variables.[8]

[8] To measure total factor productivity, we estimate a constant-returns-to-scale, Cobb-Douglas, two-factor production function in materials and capital, for each industry. More details are provided in Appendix D.2.

Viewed through the lens of the model, the estimated effect δ^{OP} combines two margins: the effect of optimism on beliefs and the effect of beliefs on input choices. We could obtain a null result of $\delta^{OP} = 0$ if optimism in language has no influence on firms' choices over and above other measured fundamentals.

We find that optimistic firms hire more than pessimistic firms holding fixed other observed fundamentals (Table 1). We first estimate the model with no additional controls other than fixed effects and estimate $\hat{\delta}^{OP} = 0.0355$ with a standard error of 0.0030 (column 1). In column 2, we add controls for current and lagged TFP, and lagged labor ($\log \hat{\theta}_{it}, \log \hat{\theta}_{i,t-1}, \log L_{i,t-1}$). These controls proxy both for time-varying firm fundamentals and, to first order, adjustment costs in labor.[9] Our point estimate $\hat{\delta}^{OP} = 0.0305$ (SE: 0.0030) is quantitatively comparable to our uncontrolled estimate. To formalize this, in Appendix E.1 we report the robustness of our estimate to selection on unobservables using the method of Oster (2019). We find that our finding of a positive effect of optimism on hiring is robust by the benchmark suggested by Oster (2019) (see Table A2).

In column 3, we add measures of firms' financial characteristics, the (log) book-to-market ratio, last fiscal year's log stock return (inclusive of dividends), and leverage (total debt over total assets). These controls proxy for Tobin's q and firm-level financial frictions, features absent from our model but potentially relevant in practice. These controls are conservative in that they may absorb variation in both omitted firm fundamentals and optimism itself. The point estimate remains positive and quantitatively similar. In column 4, we estimate a specification with the controls from column 2 but no firm fixed effects to guard against small-sample bias from strict exogeneity violations (Nickell, 1981). We find similar results.[10]

To test if optimism predicts (and does not merely describe) hiring, we finally estimate a specification in which the outcome and control variables are time-shifted one year in advance:

$$\Delta \log L_{i,t+1} = \delta_{-1}^{OP} \text{opt}_{it} + \tau' X_{i,t+1} + \gamma_i + \chi_{j(i),t+1} + \varepsilon_{i,t+1} \tag{35}$$

where δ_{-1}^{OP} is the effect of lagged optimism on hiring and the (time-shifted) control variables $X_{i,t+1}$ are those studied in column 2. In this specification, hiring takes place in fiscal year $t+1$ after the filing of the 10-K at the end of fiscal year t. Our point estimate in column 5 is similar in magnitude to our comparable baseline estimate (column 2). In Table A5, we report results from our baseline regression Equation 34, using $\text{opt}_{i,t-1}$ as an instrument for opt_{it}. This is robust to any identification concern arising from the simultaneous determination of opt_{it} and $\Delta \log L_{it}$, but estimates the original parameter δ^{OP} rather than δ_{-1}^{OP}. Our estimates are positive, statistically significant, and larger than our baseline estimates.

Robustness and Alternative Strategies. To further isolate plausibly exogenous variation in the narratives considered by firms, we study the effects on hiring of changes in narratives induced by plausibly exogenous CEO turnover. We provide the details in Appendix E.2. Specifically, we

[9] To evaluate robustness to richer adjustment dynamics, in Table A3, we control for up to three lags of productivity and labor and our financial controls and continue to find a significant impact of optimism on hiring.

[10] In Table A4, we report standard errors for the estimates of Table 1 under alternative clustering approaches.

Table 1: Narrative Optimism Predicts Hiring

	(1)	(2)	(3)	(4)	(5)
	Outcome is				
	$\Delta \log L_{it}$				$\Delta \log L_{i,t+1}$
opt_{it}	0.0355	0.0305	0.0250	0.0322	0.0216
	(0.0030)	(0.0030)	(0.0032)	(0.0028)	(0.0037)
Firm FE	✓	✓	✓		✓
Industry-by-time FE	✓	✓	✓	✓	✓
Lag labor		✓	✓	✓	✓
Current and lag TFP		✓	✓	✓	✓
Log Book to Market			✓		
Stock Return			✓		
Leverage			✓		
N	71,161	39,298	33,589	40,580	38,402
R^2	0.259	0.401	0.419	0.142	0.398

Notes: For columns 1-4, the regression model is Equation 34 and the outcome is the log change in firms' employment from year $t-1$ to t. The main regressor is a binary indicator for the optimistic narrative, defined in Section 4.2. In column 5, the regression model is Equation 35, the outcome is the log change in firms' employment from year t to $t+1$, and control variables are dated $t+1$. In all specifications, we trim the 1% and 99% tails of the outcome variable. Standard errors are two-way clustered by firm ID and industry-year.

estimate a variant of Equation 34 over firm-year observations corresponding to the death, illness, or voluntary retirement of a CEO, as measured by Gentry, Harrison, Quigley, and Boivie (2021). We find quantitatively similar effects of narrative optimism on hiring as those reported in Table 1.

In the Appendix, we also report several additional results that probe the robustness of our main specification. We summarize them briefly here. First, Table A6 repeats the analysis of Table 1 with our conference-call-based optimism measure, and finds similar results. Second, Table A7 repeats our main analysis for different measured inputs—employment (the baseline), total variable input expenditure, and investment—and demonstrates a positive and comparably sized effect of optimism on all three. Third, in Figure A2 we re-create the regression models of the first three columns of Table 1 with indicators for each decile of our continuous sentiment measure. We find monotonically increasing associations of hiring with sentiment, implying that our binary construction is not masking non-monotone effects of the continuous measure.

5.2 Testing the Narrative Interpretation *vs.* the News Interpretation

Viewed through the lens of the model, our estimate of the effect of optimism on hiring is sufficient to quantify the static, partial equilibrium effect of narrative optimism on aggregates (Corollary 5). When combined with an external calibration of general-equilibrium forces, which allows us to measure also the un-estimated effects in the "intercept" of Equation 34, this estimate is sufficient to measure the static, general-equilibrium effect of optimism on aggregate output. But this calibration strategy, in isolation, does not directly validate our hypothesis for what textual optimism represents.

Indeed, an alternative interpretation of our findings above is that narrative optimism measures news about future firm-level fundamentals. This would be inconsistent with our model, in which optimism conveys no news about those fundamentals. Moreover, it would create an identification threat that is not handled by controlling for measured *past* fundamentals of the firm. To test the "narratives" interpretation against the "news" interpretation, we observe the following: the defining property of "news" is that it is predictive of future fundamentals.

Thus, we can perform two tests that distinguish between the news and narrative interpretations. First, under the news interpretation, a firm's optimism should positively predict its subsequent performance. By contrast, under the narrative interpretation, optimism should be unpredictive of future firm fundamentals and negatively predictive of future firm performance. Second, under the news interpretation, if a firm processes the news in a Bayesian fashion, then their forecasts should be more optimistic but they should not be predictably more likely to exceed realized performance. By contrast, under the narrative interpretation, firms' forecasts should be predictably over-optimistic.

Test I: Narrative Optimism Predicts Poor Future Performance. Thus motivated, to test the "news" hypothesis, we estimate projection regressions of firm fundamentals and performance Z_{it}, either TFP growth $\Delta \log \hat{\theta}_{it}$, log stock returns R_{it}, or changes in profitability $\Delta \pi_{it}$, on optimism at leads and lags k:[11]

$$Z_{it} = \beta_k \, \text{opt}_{i,t-k} + \gamma_i + \chi_{j(i),t} + \varepsilon_{it} \tag{36}$$

Under the "news" hypothesis, we would expect $\beta_k > 0$ for $k > 0$: that is, optimistic firms are both more productive and more successful than their pessimistic counterparts in the future. Under the "narrative" hypothesis, we should expect to see that $\beta_k = 0$ for $k > 0$ for firm productivity (as a measure of fundamentals) and that $\beta_k < 0$ for $k > 0$ for firm performance.

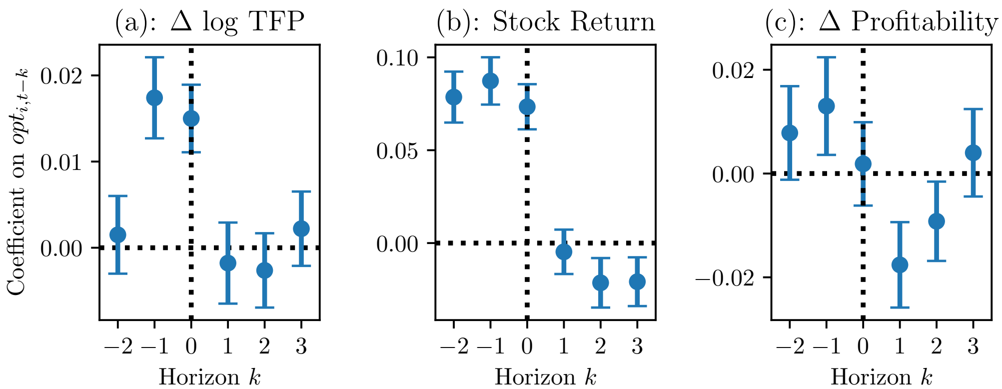
Our findings, reported in Figure 4, strongly contradict this "news" hypothesis and are, instead, consistent with our "narrative" hypothesis. For $k < 0$ and all three outcome variables, we find evidence of $\beta_k > 0$. That is, a firm doing well today in terms of TFP growth, stock-market returns, and/or profitability is likely to become optimistic in the future. However, for $k > 0$, and all three outcome variables, we find no positive association. That is, a firm that is optimistic this year does not on average do better next year.[12] We find, in sharp contrast, that optimistic firms have *negative* stock returns and decreasing profitability in the future. This is consistent with our finding that optimistic firms persistently increase input expenditure (column 5 of Table 1), but see no increase in productivity (panel (a) of Figure 4). Figure A3 replicates this analysis with conference-call-based optimism and finds similar results.[13] Finally, in Figure A4 we replicate this

[11] We measure profitability as earnings before interest and taxes (EBIT) divided by the previous fiscal year's total variable costs (cost of goods sold (COGS) plus selling, general, and administrative expense (SGA), minus depreciation).

[12] To further investigate the effects on stock prices, we also estimate the correlation of optimism with stock returns near the 10-K filing date (Table A8). We find essentially no evidence of stock response on or before the filing day, and weak evidence of positive returns (about 15-25 basis points) in the four days after. The latter finding is consistent with those in Loughran and McDonald (2011).

[13] Jiang, Lee, Martin, and Zhou (2019) relatedly find that positive textual sentiment in firm disclosures, by their own measure, predicts negative excess returns over the subsequent year.

Figure 4: Dynamic Relationship between Optimism and Firm Performance



Notes: The regression model is Equation 36, and each coefficient estimate is from a different regression. The outcomes are (a) the log change in TFP, calculated as described in Appendix D.2, (b) the log stock return inclusive of dividends over the fiscal year, and (c) changes in profitability, defined as earnings before interest and taxes (EBIT) as a fraction of the previous fiscal year's variable costs. In all specifications, we trim the 1% and 99% tails of the outcome variables. Error bars are 95% confidence intervals, based on standard errors clustered at the firm and industry-year level.

analysis with other financial fundamentals (leverage, capital structure, payout policy, and stock return volatility): consistent with the findings above, optimistic firms face relatively *tighter* future financial conditions, not looser ones.

Test II: Narrative Optimism Predicts Over-Optimistic Beliefs. We next directly test whether optimistic firms hold over-optimistic beliefs. We do this by linking a subset of our data on narrative optimism with data on managerial guidance forecasts. We construct the variable $\text{GuidanceOverOpt}_{it}$ as an indicator of managers' guidance minus the realization exceeding the sample median.[14] We estimate the following regression model:

$$\text{GuidanceOverOpt}_{i,t+1} = \beta \, \text{opt}_{it} + \tau' X_{it} + \chi_{j(i),t} + \varepsilon_{it} \tag{37}$$

The control variables X_{it} are current and lagged TFP and lagged labor, all in log units. As we have guidance data for only a small subset of firms, we do not include firm fixed effects. Our findings are reported in Table 2. We find a positive correlation that increases when we add the aforementioned control variables (columns 1 and 2). That is, textual optimism corresponds to forecasts that are predictably more likely to exceed subsequent performance. This is exactly what we would predict under the narrative hypothesis.[15]

[14] When managers' guidance is reported as a range, we code a point-estimate forecast as the range's midpoint. The method of comparing to the median corrects for the fact that, in more than half of our observations, guidance is lower than the realized value, presumably due to asymmetric incentives.

[15] In an analogous regression in which the outcome measures managerial optimism relative to contemporaneous *analyst* forecasts, we find an imprecise positive effect in an uncontrolled model and a zero effect in the controlled model (Table A10). These findings are consistent with a story in which narratives are shared between management and investors, potentially due to persuasion in communications. Loughran and McDonald (2011) relatedly find

Table 2: Narrative Optimism Predicts Over-Optimistic Forecasts

	(1)	(2)
	Outcome is $\text{GuidanceOverOpt}_{i,t+1}$	
opt_{it}	0.0354	0.0561
	(0.0184)	(0.0257)
Ind.-by-time FE	✓	✓
Lag labor		✓
Current and lag TFP		✓
N	3,817	2,159
R^2	0.173	0.193

Notes: The regression model is Equation 37. The outcome is a binary indicator for whether sales guidance was high relative to realized sales. Standard errors are two-way clustered by firm ID and industry-year.

Given that we have found that guidance correlates with narrative optimism, it is also natural to ask if narrative optimism affects firm decisions conditional on guidance and *vice versa*. In Table A9, we show that textual optimism predicts hiring over and above measured beliefs.[16] This finding is consistent with the idea that managers' non-quantitative *soft information* (Liberti and Petersen, 2019) is important for decisions. Our language analysis may be able to pick up nuances in managers' perspectives that are not reflected in the guidance data or other standard measures.

Summary. Based on these tests, we argue that there is strong evidence in favor of the narrative interpretation. To be concrete, to argue against the narrative interpretation of the data, one would have to argue that firms that use positive language, subsequently expand hiring and investment, have predictably over-optimistic forecasts, and perform worse in the future were somehow correct in their optimism.

5.3 Narrative Optimism is Contagious and Associative

We next estimate how optimism spreads across firms. Specifically, we estimate a version of the linear-associative-contagious updating rule (Equation 26) in our panel data:

$$\text{opt}_{it} = u\ \text{opt}_{i,t-1} + s\ \overline{\text{opt}}_{t-1} + r\ \Delta \log Y_{t-1} + \gamma_i + \varepsilon_{it} \tag{38}$$

where $\overline{\text{opt}}_{t-1}$ is average optimism in the previous period, $\Delta \log Y_{t-1}$ is US real GDP growth, and γ_i is an individual fixed effect. In our model interpretation, u measures stubbornness, s measures contagiousness, and r measures associativeness.

We find strong evidence of all three forces (Table 3). That is, firms are significantly more likely to be optimistic in year t if, in the previous year, they were optimistic, if other firms were optimistic,

that, in Fama-MacBeth predictive regressions of standardized unexpected earnings, 10-K negativity predicts higher earnings surprises in the subsequent quarter.

[16] Our main model controls for expected capital expenditures. In robustness checks, we study analogous models that control for predicted sales and earnings per share growth (Tables A11 and A12).

Table 3: Narrative Optimism is Contagious and Associative

	(1)	(2)
	Outcome is opt_{it}	
Own lag, $\text{opt}_{i,t-1}$	0.209	0.214
	(0.0071)	(0.0080)
Aggregate lag, $\overline{\text{opt}}_{t-1}$	0.290	
	(0.0578)	
Real GDP growth, $\Delta \log Y_{t-1}$	0.804	
	(0.2204)	
Industry lag, $\overline{\text{opt}}_{j(i),t-1}$		0.276
		(0.0396)
Industry output growth, $\Delta \log Y_{j(i),t-1}$		0.0560
		(0.0309)
Firm FE	✓	✓
Time FE		✓
N	64,948	52,258
R^2	0.481	0.501

Notes: The regression model is Equation 38 for column 1 and Equation 39 for column 2. Aggregate and industry optimism are averages of the narrative optimism variable over the respective sets of firms. Industry output growth is the log difference in sectoral value-added calculated from BEA data, linked to two-digit NAICS industries. Standard errors are two-way clustered by firm ID and industry-year.

and if the economy grew. Our finding of $s > 0$, in particular, is consistent with Shiller's (2020) hypothesis that decision-relevant narratives are contagious.

Our estimation of Equation 38 levers only time-series variation. While this is the level of variation that is relevant for calibrating the model, one may worry that the small sample size leaves open the door to spurious correlation. We therefore also study a model that allows for contagiousness and associativeness at finer levels. Specifically, we estimate the following variation of the original regression at the industry level:

$$\text{opt}_{it} = u_{\text{ind}}\ \text{opt}_{i,t-1} + s_{\text{ind}}\ \overline{\text{opt}}_{j(i),t-1} + r_{\text{ind}}\ \Delta \log Y_{j(i),t-1} + \gamma_i + \chi_t + \varepsilon_{it} \tag{39}$$

where $\overline{\text{opt}}_{j(i),t-1}$ is the leave-one-out mean of optimism within industry $j(i)$ and $\Delta \log Y_{j(i),t-1}$ is the growth of sectoral value-added, measured by linking BEA sector-level data to our NAICS-based classification.[17] The time fixed effect χ_t absorbs aggregate contagiousness and associativeness. We find strong evidence for contagiousness and weaker evidence for associativeness within industries (column 2 of Table 3).[18]

As a robustness check, we also measure contagiousness at a finer level by defining narrow sets of peers that share equity analysts for firms listed on the New York Stock Exchange, following Kaustia and Rantala (2021). We find both a quantitatively similar industry-level effect and an independent

[17] These data are available only from 1997.

[18] In Table A13, we report standard errors for Table 3 under alternative clustering.

Table 4: Narrative Optimism is Contagious, Controlling for Past and Future Outcomes

	(1)	(2)	(3)	(4)	(5)
	Outcome is opt_{it}				
Aggregate lag, $\overline{\text{opt}}_{t-1}$	0.290	0.339	0.235		
	(0.0578)	(0.0763)	(0.1278)		
Ind. lag, $\overline{\text{opt}}_{j(i),t-1}$				0.276	0.241
				(0.0396)	(0.0434)
Firm FE	✓	✓	✓	✓	✓
Time FE				✓	✓
Own lag, $\text{opt}_{i,t-1}$	✓	✓	✓	✓	✓
$(\Delta \log Y_{t+k})^2_{k=-2}$		✓	✓		
$(\Delta \log Y_{j(i),t+k})^2_{k=-2}$			✓		✓
N	64,948	49,631	38,132	52,258	38,132
R^2	0.481	0.484	0.497	0.501	0.498

Notes: The regression model is Equation 40 for columns 1-3, and an analogous industry-level specification for columns 4 and 5 (*i.e.,* Equation 39 with past and future controls). Columns 1 and 4 correspond, respectively, with columns 1 and 3 of Table 3. The added control variables are two leads, two lags, and the contemporaneous value of: real GDP growth (columns 2-3) and industry-level output growth (columns 3 and 6). Standard errors are two-way clustered by firm ID and industry-year.

peer-set effect (Table A14). Finally, we find consistent evidence of stubbornness, contagiousness, and associativeness for the continuous measure of sentiment (Table A15).

Inspecting the Mechanism: Spillovers are Not Driven by Common Shocks. The coefficients of interest (u, r, and s) identify stubbornness, associativeness, and contagiousness, when idiosyncratic optimism, aggregate optimism, and GDP are unrelated to other factors that affect changes in optimistic sentiment at the firm level. Since the key regressor is *lagged* aggregate optimism, our estimates are not threatened by the reflection problem of Manski (1993). Nevertheless, our estimates may be contaminated by omitted variables bias because aggregate optimism is correlated with common shocks to the economy that are in the error term.

To test for this possibility, we augment our previous regressions to include controls for past and future fundamentals in the form of two leads and lags of real GDP or value-added growth at the aggregate and industry levels. Specifically, we estimate

$$\text{opt}_{it} = u \ \text{opt}_{i,t-1} + s \ \overline{\text{opt}}_{t-1} + \gamma_i + \sum_{k=-2}^{2} \left(\eta_k^{\text{agg}} \Delta \log Y_{t+k} + \eta_k^{\text{ind}} \Delta \log Y_{j(i),t+k} \right) + \varepsilon_{it} \tag{40}$$

We estimate an analogous specification at the industry level, but with the aggregate leads and lags absorbed. If common positive shocks to the economy and sectors were driving some or all of the estimated spillovers, we would expect to find a severely attenuated estimate of the contagiousness coefficient s. Even under our interpretation, future output growth could be a "bad control" that is caused by optimism and absorbs some of its effect.

We report our estimates of the contagiousness coefficients in Table 4, adding the "bad controls" one at a time (columns 2 and 3) and find similar results to our baseline (column 1). Similarly, for our industry-level estimates, we find no statistically significant evidence of coefficient attenuation as additional controls are added (columns 4 and 5). In Table A16, we report analogous estimates with the continuous sentiment variable and find similar results. Taken together, these estimates build confidence that our baseline contagiousness results are not driven by omitted aggregate shocks.

Alternative Identification Strategies. To further test whether our measure of contagiousness captures spillovers, we pursue two additional instrumental variables strategies. First, in Appendix E.3, we use size-weighted idiosyncratic shocks to firm-level optimism as an instrument for aggregate size-weighted optimism (a granular IV à la Gabaix and Koijen, 2020). While not comparable to our main estimates as the measure of spillovers is different, we recover a statistically significant contagiousness effect. Second, in Appendix E.2, we use spillovers from the same plausibly exogenous CEO changes to construct instruments for industry and peer-set optimism. We find similar (albeit noisier) point estimates.

6 Quantifying the Impact of Narratives

We now combine our model and empirical results to gauge the quantitative effects of narratives on business cycles.

6.1 Estimating the Model

In Section 3, we showed that we could estimate the model in three steps. We now combine our empirical estimates from Section 5 with this three-step approach to estimate the model. We provide the point estimates of model parameters in Table 5 and provide additional details in Appendix F.

Step I: Estimation of the Effect of Optimism. To estimate the static relationship between output and optimism, we need to estimate f. In turn, f requires knowledge of: δ^{OP}, the partial-equilibrium effect of optimism on hiring; α, the returns-to-scale parameter; ϵ, the elasticity of substitution between varieties; and ω, the extent of complementarity (which itself depends on γ, indexing income effects in labor supply, and ψ, the inverse Frisch elasticity of labor supply). In our main analysis, we combine our baseline regression estimate of $\hat{\delta}^{OP} = 0.0355$ (see Table 1) with an external calibration of α, ϵ, γ, and ψ, which together also pin down ω.

For the external calibration, we impose that intermediate goods firms have constant returns-to-scale or $\alpha = 1$, which has been argued by Basu and Fernald (1997) and Foster, Haltiwanger, and Syverson (2008) to be a reasonable assumption for large US firms. Second, as noted by Angeletos and La'O (2010), γ indexes wealth effects in labor supply, which are empirically very small (Cesarini, Lindqvist, Notowidigdo, and Östling, 2017). Hence, we set $\gamma = 0$. Third, we calibrate the inverse Frisch elasticity of labor supply at $\psi = 0.4$ based on standard macroeconomic estimates (Peterman, 2016). Finally, we calibrate the elasticity of substitution to match estimated markups from De Loecker, Eeckhout, and Unger (2020) of 60%, which implies that $\epsilon = 2.6$. Hence,

Table 5: Model Calibration

Fixed	ϵ	Elasticity of substitution	2.6
	γ	Income effects in labor supply	0
	ψ	Inverse Frisch elasticity	0.4
	α	Returns-to-scale	1
Calibrated	$\mu_O - \mu_P$	Belief effect of optimism	0.028
	κ	Signal-to-noise ratio	0.344
	ρ	Persistence of productivity	0.086
	σ	Std. dev. of the productivity innovation	0.011
	u	Stubbornness	0.208
	r	Associativeness	0.804
	s	Contagiousness	0.290
	σ_ε	Std. dev. of the optimism shock	0.044

Notes: "Fixed" parameters are externally set. "Calibrated" parameters are chosen to hit various moments. Our specific calibration methods are described in Section 6.1.

altogether, this calibration implies an aggregate degree of strategic complementarity of $\omega = 0.49$. In Section 6.2, we study the sensitivity of our results to this external calibration, and we introduce two other estimation strategies for complementarity: using estimates of demand multipliers from the literature and inferring a demand multiplier for optimism using our own firm-level regressions.

Step II: Estimation of Updating Rules. To estimate the parameters of the LAC updating rules, we use the linear probability model estimated in Table 3.[19] This yields values of $u = 0.208$ for stubbornness, $r = 0.804$ for associativeness, and $s = 0.290$ for contagiousness.

Step III: Estimation of Private Information and the Shock Processes. To estimate the extent of private information and the persistence and volatility of productivity shocks, we showed that we need to estimate the model-implied ARMA(1,1) process for fundamental output. Now that we have estimated f, we can compute fundamental output as:

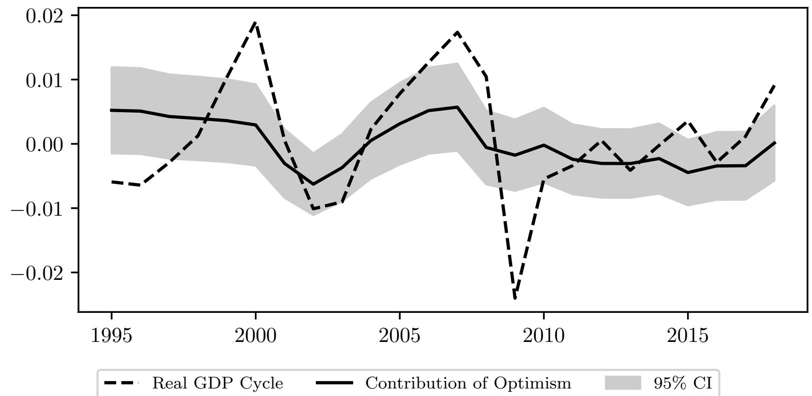
$$\log Y_t^f = \log Y_t - f(Q_t) \tag{41}$$

To calculate $\log Y_t^f$ in the data, we take $\log Y_t$ as band-pass filtered US real GDP (Baxter and King, 1999), Q_t as our measured time series of aggregate optimism (see Figure A1), and f as our calibrated function.[20] We estimate by maximum-likelihood the ARMA(1,1) process for Y_t^f and then set (ρ, σ, κ) to exactly match the three estimated ARMA parameters. Upon obtaining κ, the restriction on κ and $\mu_O - \mu_P$ imposed by δ^{OP} yields the value of $\mu_O - \mu_P$. Finally, we estimate the variance of optimism shocks, σ_ε^2, to match the time-series variance of optimism.

[19] While the linear probability model does not necessarily yield probabilities between zero and one, our estimates of u, r and s imply updating probabilities that are always between zero and one so long as output does not deviate by more than 30% (holding fixed ε_t), *i.e.,* there is a five-standard-deviation optimism shock.

[20] We apply the Baxter and King (1999) band-pass filter to post-war quarterly US real GDP data (Q1 1947 to Q1 2022). We use a lead-lag length of 12 quarters, a low period of 6 quarters, and a high period of 32 quarters. We then average these data to the annual level.

Figure 5: The Effect of Optimism on Historical US GDP



Notes: The "Real GDP Cycle" is calculated from a Baxter and King (1999) band-pass filter capturing periods between 6 and 32 quarters. The "Contribution of Optimism" is the model-implied effect of optimism on log output. The 95% confidence interval incorporates uncertainty from the estimation of δ^{OP} using the delta method.

The Estimated Model Features Almost i.i.d. Shocks. Before proceeding to the quantitative results, we observe an important property of the estimated model: our point estimate for the persistence of exogenous productivity shocks is $\rho = 0.086$. As we have only allowed for i.i.d. optimism shocks, this means that our model only requires *almost* i.i.d. exogenous shocks to match the time-series properties of output. Thus, our estimates imply that narratives generate strong internal propagation. This represents an important difference between our theory of narrative dynamics and theories based on learning and dispersed information (see *e.g.*, Woodford, 2003a; Lorenzoni, 2009; Angeletos and La'O, 2010), all of which require exogenously persistent fundamentals about which agents slowly learn.

6.2 How Does Optimism Shape the Business Cycle?

Using the calibrated model, we now study the effects of optimism on the business cycle via two complementary approaches: (i) gauging the historical effect of swings in business optimism on US GDP and (ii) exploring the full dynamic implications of contagious and associative optimism.

The Effects of Optimism on US GDP. In our empirical exercise, which leveraged cross-sectional data on US firms' optimism, the general-equilibrium effect of optimism on total production was the unidentified "missing intercept." Now, equipped with the model calibration of general-equilibrium forces, we can return to the question of how changes in optimism have historically affected the US business cycle. Concretely, we calculate the time series of $f(Q_t)$, where f is the calibrated function mapping aggregate optimism to aggregate output, which depends on the partial-equilibrium effect of optimism on hiring, returns-to-scale, and the demand multiplier, and Q_t observed annual time series for business optimism. We take the observed time path of aggregate optimism as given, and therefore use the estimated dynamics of optimism only to determine the shocks that rationalize this observed path.

Figure 5 illustrates our findings by plotting the cyclical component of real GDP (dashed line) and the contribution of measured optimism toward output according to our model (solid line with grey 95% confidence interval). Cyclical optimism explains a meaningful portion of fluctuations, particularly the booms of the mid-1990s and the mid-2000s and the busts of 2000-2002 and 2007-2009. The decline in the optimism component of GDP explains 31.65% (SE: 2.68%) of the output loss between 2000 and 2002 and 18.06% (SE: 1.53%) of the output loss between 2007 and 2009.

To unpack the model-implied causes of the historical business cycle, we plot the sequence of fundamental output and optimism shocks that our model requires to match the realized optimism and output time series in Figure A5. Our model accounts for the early 2000s recession with a large negative optimism shock ($\varepsilon_{2001} = -0.08$, or -1.8 standard deviations in our calibration) and a moderate-sized shock to fundamental output. For the Great Recession, our model implies a larger shock to fundamentals along with a smaller optimism shock ($\varepsilon_{2008} = -0.06$ or -1.4 standard deviations). The larger contribution of, and shock to, optimism at the outset of the early 2000s recession is consistent with a story that a break in confidence, associated with the "dot com" crash in the stock market, spurred a recession despite sound economic fundamentals. This is further consistent with independent textual evidence that "crash narratives" in financial news were especially rampant in this period (Goetzmann, Kim, and Shiller, 2022).

Contagious Narratives and Economic Fluctuations. We now fully describe the role of narrative dynamics in shaping the business cycle via the estimated process for how optimism spreads. To produce a summary statistic for the contribution of optimism toward the covariance structure of output, we observe that the covariance of output at lag $\ell \geq 0$ can be decomposed into four terms:

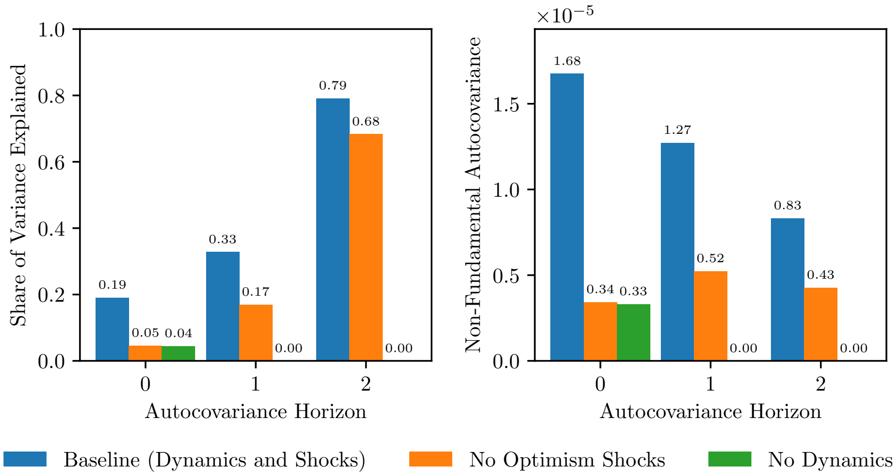
$$\begin{aligned} \text{Cov}[\log Y_t, \log Y_{t-\ell}] = {} & \text{Cov}[\log Y_t^f, \log Y_{t-\ell}^f] + \text{Cov}[f(Q_t), f(Q_{t-\ell})] \\ & + \text{Cov}[f(Q_t), Y_{t-\ell}] + \text{Cov}[f(Q_{t-\ell}), Y_t] \end{aligned} \tag{42}$$

The first term captures the volatility and persistence of exogenous fundamentals (*i.e.,* the driving productivity shocks). The second term captures the volatility and persistence of the non-fundamental component of output. The last two terms capture the relationship of optimism with past and future fundamentals, which arises from the co-evolution of narratives with economic outcomes. We therefore define non-fundamental variance as the total autocovariance arising from endogenous optimism as the sum of the last three terms, as well as its fraction of total variance, at each lag ℓ:

$$\begin{aligned} \text{Non-Fundamental Autocovariance}_\ell &= \text{Cov}[\log Y_t, \log Y_{t-\ell}] - \text{Cov}[\log Y_t^f, \log Y_{t-\ell}^f] \\ \text{Share of Variance Explained}_\ell &= \frac{\text{Non-Fundamental Autocovariance}_\ell}{\text{Cov}[\log Y_t, \log Y_{t-\ell}]} \end{aligned} \tag{43}$$

We calculate these statistics at horizons $\ell \in \{0, 1, 2\}$ and under three model variants: the baseline model with optimism shocks, a variant model which turns off the shocks (or sets $\sigma_\varepsilon^2 = 0$), and a

Figure 6: The Contribution of Optimism to Output Variance



Notes: The left panel plots the fraction of variance, one-year autocovariance, and two-year autocovariance explained by endogenous optimism in model simulations. The right panel plots the total non-fundamental autocovariance. Both quantities are defined in Equation 43. In each figure, we plot results under three model scenarios: the baseline model with optimism shocks and optimism dynamics (blue), a variant model with no optimism shocks, or $\sigma_\varepsilon^2 = 0$ (orange), and a variant model with shocks but no dynamics for narrative spread, or $u = r = s = 0$ (green).

variant model that keeps optimism shocks but shuts down the endogenous evolution of narratives (by setting $u = r = s = 0$).[21]

Optimism explains 19% of contemporary variance ($\ell = 0$), and this fraction increases with the lag (Figure 6). At one-year and two-year lags, optimism explains 33% and 79% of output autocovariance, respectively. Thus, most medium-frequency (two-year) dynamics are produced by contagious optimism instead of fundamentals. The model without endogenous dynamics of optimism explains only 4% of output variance and, as optimism shocks are i.i.d., 0% of output auto-covariance. Moreover, while the model without optimism shocks matches only 5% of output variance, it accounts for 17% and 69% of one-year and two-year output autocovariance. Interestingly, the separate contributions to output variance of shocks and endogenous dynamics sum to less than one-half of their joint explanatory power. This result establishes that the contagiousness and associativeness of narratives are amplifying propagation mechanisms for exogenous sentiment shocks.

Sensitivity Analysis. In Table A19, we report a sensitivity analysis of the conclusions above to different calibrations for the macroeconomic parameters. We first focus on the calibration of macroeconomic complementarity and, by extension, the demand multiplier. Recall that $f(Q) \approx \frac{\alpha\delta^{OP}}{1-\omega}Q$, where $\frac{1}{1-\omega}$ is the general equilibrium demand multiplier in our economy, α indexes the returns-to-scale, and δ^{OP} is the partial equilibrium effect of optimism on hiring. Our baseline

[21] As discussed in Appendix F, we always add a constant to LAC updating so 0.5 is the interior steady-state when output is at its steady state. Thus, the "no dynamics" variant sets $Q_{t+1} = 0.5 + \epsilon_t$.

calibration implies a multiplier of $\frac{1}{1-\omega} = 1.96$. In rows 1, 2, 3, and 4 we vary the multiplier by: (i) adjusting the inverse-Frisch elasticity to 2.5 to match microeconomic estimates (Peterman, 2016), (ii) allowing for greater income effects in labor supply $\gamma = 1$, (iii) matching the empirical estimates of the demand multiplier of 1.33 from Becko, Flynn, and Patterson (2024), and (iv) estimating the general equilibrium multiplier semi-structurally by using the extent of omitted variables bias from omitting a time fixed effect in the regression of hiring on optimism (see Appendix F.3 for the details). Our numerical results from adjusting the multiplier, holding fixed $(\delta^{OP}, \alpha, \epsilon)$, convey that the contribution of optimism is increasing in this number. We finally consider sensitivity to the calibrations of the elasticity of substitution ϵ (row 5 of Table A19) and the returns-to-scale α (row 6 of Table A19) holding fixed the multiplier (via adjustment in ψ). Changing ϵ has close to no effect on our results, due to the aforementioned near-linearity of f. Reducing α, or assuming decreasing returns to scale, dampens the effect of optimism on output because it implies a smaller production effect of our estimated effect of optimism on hiring.

6.3 Can Contagious Optimism Generate Hysteresis?

We have shown that the dynamics of optimism generate quantitatively significant business cycles. However, we have not yet explored the implications of narratives for hysteresis and long-run movements in output. Our theoretical analysis delimited two qualitatively different regimes for macroeconomic dynamics with contagious optimism: one with stochastic fluctuations around a stable steady state, and one with hysteresis and (almost) global convergence to extreme steady states. Are economic narratives strong enough to generate hysteresis?

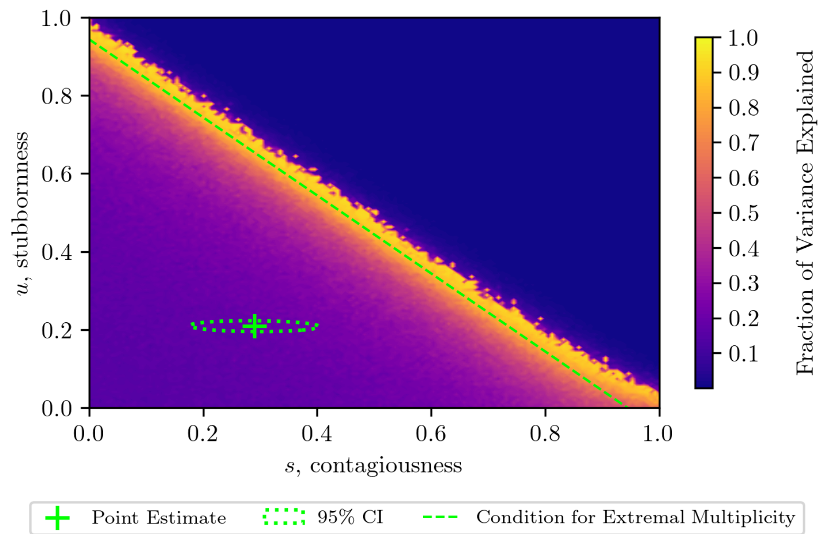
For the LAC case which we have taken to the data, the necessary and sufficient condition for extremal multiplicity is given by Equation 27. We compute the empirical analog of this condition:

$$\hat{M} = \hat{u} + \hat{s} + \hat{r}\frac{\alpha}{1-\omega}\hat{\delta}^{OP} - 1 \tag{44}$$

If $\hat{M} > 0$, the calibrated model features hysteresis in the dynamics of optimism and output; if $\hat{M} < 0$, the model features oscillations around a stable steady state. We find $\hat{M} = -0.44 < 0$ with a standard error of 0.052, implying stable oscillations and ruling out hysteresis dynamics. This reflects the fact that decision-relevance, stubbornness, contagiousness, and associativeness are sufficiently small for narrative optimism.

We explore the sensitivity of this conclusion to our calibration of the two parameters to which it is most sensitive: stubbornness and contagiousness. In Figure 7, we plot our point estimate of contagiousness and stubbornness as a plus and its 95% confidence interval as a dotted ellipse. We also plot, as a dashed line, the condition for $M = 0$; to the left of this line, $M < 0$, and to the right of this line, $M > 0$. In the Figure, we shade the fraction of variance explained by non-fundamental optimism. Given the statistical precision in the estimates of stubbornness and contagiousness, we are confident that narratives contribute stable fluctuations to the economy and explain about 20% of the variance in output.

Figure 7: Variance Decomposition for Different Values of Stubbornness and Contagiousness



Notes: Calculations vary u and s, holding fixed all other parameters at their calibrated values. The shading corresponds to the fraction of variance explained by optimism, or Share of Variance Explained$_0$ defined in Equation 43. The plus is our calibrated value from Table 5, and the dotted line is the boundary of a 95% confidence set. The dashed line is the condition of extremal multiplicity from Corollary 4 and Equation 27.

How does extremal multiplicity interact with our model's predictions for non-fundamental volatility? To isolate the role of endogenous propagation, our theoretical discussion of extremal multiplicity considered paths of the economy without shocks. In the quantitative model, the economy is constantly buffeted with shocks that move optimism away from its steady state(s). Near the condition for extremal multiplicity, non-fundamental variance reaches essentially 100% of total variance. This is because even small shocks have the potential to "go viral," and the force pulling the economy toward an interior steady state (*i.e.,* balanced optimism and pessimism) is weak.[22]

Finally, far to the right of the extremal multiplicity condition, contagious optimism explains little output variance. This is because the economy quickly settles into an extreme steady state, fully optimistic or fully pessimistic, and moves quickly back to this steady state in response to shocks. Figure A7 shows this quantitatively by plotting, against the same (s, u) grid, the fraction of time that the optimistic fraction Q_t lies outside of $[0.25, 0.75]$—this is 0% at the baseline calibration, and essentially 100% in the calibrations featuring extremal multiplicity. In this region, while sentiment does not greatly affect output dynamics, it does affect the *static* level of output. Moreover, path dependence in the early history of our simulation determines whether output is permanently high (optimism goes viral) or permanently low (pessimism goes viral). Thus, the "M test" provides an accurate diagnostic for whether economic narratives can "go viral" even in the presence of shocks.

[22]Due to the presence of shocks to optimism, this prediction is symmetric around the extremal multiplicity threshold; in the variant model which turns off optimism shocks, the extremal multiplicity condition sharply delineates the regime in which optimism fluctuations contribute to output variance from the regime in which there is complete hysteresis (Figure A6).

7 Unpacking Optimism Via Narrative Constellations

In our main analysis, we developed, empirically tested, and quantified a model in which a narrative shaped business cycle dynamics. But this aggregate approach might mask richer interactions between more specific, granular narratives. In particular, Shiller (2020) argues that *constellations* of many smaller and semantically related narratives may reinforce one another to create strong economic and social effects, and that the *confluence* of seemingly unrelated narratives may explain business-cycle fluctuations. In this final section, we extend our theoretical model and measurement technique to study how the confluence of granular narratives can generate emergent fluctuations in overall sentiment and economic activity. We find that we can explain the data with a model in which granular narratives are *individually* prone to violent fluctuations and hysteresis, but emergent optimism nevertheless admits stable fluctuations.

7.1 A Model of Narrative Confluence and Constellations

To understand how a constellation structure for optimism may affect our macroeconomic predictions, we first describe an enriched model in which multiple narratives interact to determine emergent optimism. There is a latent space of K granular narratives. Agents either do or do not believe in each narrative, and we denote individuals' narrative weights by $\lambda_{it} = (\lambda_{1,it}, \ldots, \lambda_{K,it}) \in \{0,1\}^K$. We let $Q_t^k = \int_0^1 \lambda_{k,it} \, \mathrm{d}i \in [0,1]$ denote the share of population that adopts each narrative.

Optimism emerges from the confluence of many narratives. To model this tractably, we assume that the aggregate fraction of optimists, Q_t, depends linearly on the fraction of agents adopting each narrative:

$$Q_t = \left[\sum_{k=1}^{K} \zeta^k Q_t^k \right]_0^1 \tag{45}$$

where $(\zeta^k)_{k=1}^K$ are *constellation weights* controlling the marginal effect of each narrative on emergent optimism and the $[\cdot]_0^1$ restricts the fraction to $[0,1]$. A particular narrative may be more or less influential depending on its value of $|\zeta^k|$.

Each granular narrative k evolves via a linear-associative-contagious process. That is, we let (P_1^k, P_0^k) respectively denote functions returning the probability that an agent who currently does or does not hold narrative k at time t holds the narrative at time $t+1$, and we define:

$$\begin{aligned} P_1^k(\log Y, Q, \varepsilon) &= \left[\frac{u^k}{2} + r^k \log Y + s^k Q_t^k + \varepsilon^k \right]_0^1 \\ P_0^k(\log Y, Q, \varepsilon) &= \left[-\frac{u^k}{2} + r^k \log Y + s^k Q_t^k + \varepsilon^k \right]_0^1 \end{aligned} \tag{46}$$

We allow for k-specific stubbornness, associativeness, and contagiousness, as well as independent shocks $\epsilon^k \sim N(0, \sigma_{\varepsilon,k}^2)$. The dynamics of different granular narratives interact in this model through associativeness. For example, narratives may contribute to optimism, boosting the economy, and thereby indirectly promoting other narratives associated with a good economy.

The rest of the model is the same as the baseline. Thus, while dynamics are the same *conditional* on the process for optimism, the process for emergent optimism through the latent evolution of narratives may differ.

7.2 Measurement and Calibration

We introduce two strategies to measure granular narratives. The first is a partially supervised method that detects firms' discussion of the nine *Perennial Economic Narratives* described by Shiller (2020). The second is an unsupervised Latent Dirichlet Allocation model (Blei, Ng, and Jordan, 2003), which flexibly identifies clusters of topics discussed by firms. We then describe how we combine this measurement with LASSO regressions to discipline the key parameters of the constellation model.

Narrative Identification of Narratives. In his book *Narrative Economics*, Robert Shiller identifies a set of nine *Perennial Economic Narratives* that recur throughout American history. These are: Panic versus Confidence; Frugality versus Conspicuous Consumption; The Gold Standard versus Bimetallism; Labor-Saving Machines Replace Many Jobs; Automation and Artificial Intelligence Replace Almost All Jobs; Real Estate Booms and Busts; Stock Market Bubbles; Boycotts, Profiteers, and Evil Businesses; and The Wage-Price Spiral and Evil Labor Unions. We quantify US firms' adoption of these narratives by measuring the similarity of the firms' language with the language Shiller uses to describe each narrative. This method "narratively identifies narratives" because it uses prior knowledge from Shiller's historical study to inform our approach.

Formally, we use a method related to prior work by Hassan, Hollander, Van Lent, and Tahoun (2019) and Flynn and Sastry (2024). For each narrative k, we first compute the term-frequency-inverse-document-frequency (tf-idf) score to obtain a set of words most indicative of that narrative:

$$\text{tf-idf}(w)_k = \text{tf}(w)_k \times \log\left(\frac{1}{\text{df}(w)}\right) \tag{47}$$

where $\text{tf}(w)_k$ is the number of times that word w appears in the chapter corresponding to narrative k in *Narrative Economics* and $\text{df}(w)$ is the fraction of 10-K documents containing the word. Intuitively, if a word has a higher tf-idf score, it is common in Shiller's description of a narrative but relatively uncommon in 10-K filings. We define the set of 100 words with the highest tf-idf score for narrative k as $\mathcal{W}_k$. We print the twenty most common words in each $\mathcal{W}_k$ in Table A17.

We initially score document (i, t) for narrative k by the total frequency of narrative words:

$$\widehat{\text{Shiller}}_{it}^{k} = \sum_{w \in \mathcal{W}_k} \text{tf}(w)_{it} \tag{48}$$

We then compute a binary measure of narrative adoption by comparing to the in-sample median: $\text{Shiller}_{it}^{k} = \mathbb{I}[\widehat{\text{Shiller}}_{it}^{k} > \text{med}(\widehat{\text{Shiller}}_{it}^{k})]$. In Figure A8, we plot the raw time series for the aggregate variable corresponding to each chapter's narrative.

Unsupervised Recovery of Narratives via LDA. While "narrative identification" may help us focus on an *ex ante* reasonable set of narratives, this method will invariably miss other topics—for example, those that pertain more heavily to our sample period than to the broader sweep of US economic history studied by Shiller. To identify narratives without relying on external references, we apply Latent Dirichlet Allocation (LDA), a hierarchical Bayesian model in which documents are constructed by combining a latent set of topic narratives (Blei, Ng, and Jordan, 2003).

More specifically, given our corpus of 10-Ks with M documents, we postulate that there are $K = 100$ topics. First, the number of words in each document is drawn from a Poisson distribution with parameter ξ. Second, the distribution of topics in each document is given by $\vartheta = (\vartheta_1, \dots, \vartheta_M)$, over which we impose a Dirichlet prior with parameter $\alpha = \{\alpha_k\}_{k \in \mathcal{K}}$, where α_k represents the prior weight that topic k is in any document. Third, the distribution of words across topics is given by $\phi = (\phi_1, \dots, \phi_K)$, over which we impose a Dirichlet prior with parameter $\beta = \{\beta_{jk}\}_{k \in \mathcal{K}}$, where β_{jk} is the prior weight that word j is in topic k. Finally, we assume that individual words in each document d are generated by first drawing a topic z from a multinomial distribution with parameter ϑ, and then selecting a word from that topic by drawing a word from a multinomial distribution with parameter ϕ_z. Intuitively, in an LDA, the set of documents is formed of a low-dimensional space of narratives of co-occurring words. To estimate the LDA, we use the Gensim implementation of the variational Bayes algorithm of Hoffman, Bach, and Blei (2010), which makes estimation of LDA on our large dataset feasible when standard Markov Chain Monte Carlo methods would be slow.[23] In Table A18, we print the top ten terms associated with each of our estimated topics. Given the estimated LDA, we construct the document-level narrative score as the posterior probability of that topic in the estimated document-specific topic distribution $\hat{p}$:

$$\widehat{\text{topic}}_{it}^{k} = \hat{p}(k|d_{it}) \tag{49}$$

We then compute a binary measure of topic discussion by comparing to the in-sample median: $\text{topic}_{it}^{k} = \mathbb{I}[\widehat{\text{topic}}_{it}^{k} > \text{med}(\widehat{\text{topic}}_{it}^{k})]$.

Calibration. To calibrate the new parameters of the model, we proceed in four steps. We summarize the steps below and provide more details in Appendix F.2.

First, we estimate the constellation weights in the following firm-level regression:

$$\text{opt}_{it} = \sum_{k=1}^{9} \zeta_{\text{Shiller}}^{k} \cdot \text{Shiller}_{it}^{k} + \sum_{k=1}^{100} \zeta_{\text{topic}}^{k} \cdot \text{topic}_{it}^{k} + \gamma_i + \chi_{j(i),t} + \varepsilon_{it} \tag{50}$$

This model estimates the marginal effects of each granular narrative on the propensity toward optimism. As throughout our analysis, we control for firm fixed effects and non-parametric sector-by-time trends.

An obvious challenge in estimating Equation 50 is the high dimensionality of the regressors. In

[23] For computational reasons, we estimate the model using all available documents from a randomly sampled 10,000 of our 37,684 unique possible firms. We score all documents with this estimated model.

more practical terms: while our narrative identification methods are designed to pick up recurring language, we may suspect that only a strict subset of them affect firms' economic beliefs and decisions. To address this challenge, we estimate Equation 50 with the Rigorous Square-Root post-LASSO method of Belloni, Chen, Chernozhukov, and Hansen (2012) and Belloni, Chernozhukov, Hansen, and Kozbur (2016). Specifically, this method uses a LASSO variable selection technique to identify the subset of regressors that are relevant for predicting optimism, and then obtains a consistent estimate of the ζ^k coefficients via OLS. Applying this method yields a relevant subset of 30 topic narratives and 8 Shiller narratives. Table A25 in the Appendix prints each of the selected narratives and their respective ζ^k.

Second, we estimate stubbornness, associativeness, and contagiousness for each narrative just as in the main analysis, by estimating variants of Equation 38. This step fixes the parameters (u^k, r^k, s^k) for each selected narrative. These estimates are also reported in Table A25.

Third, we calibrate the variance of narrative shocks, $\sigma^2_{\varepsilon,k}$, to match the time-series variance of each granular narrative. Specifically, we minimize the sum of square deviations of model-generated variances from measured time-series variances.

Finally, to calibrate the rest of the model, we proceed exactly as described in Section 6.1. Since all of these strategies are conditioned on the realizations of (emergent) optimism, they also remain valid in this enriched model.

7.3 Results: Macro Dynamics When Narratives "Go Viral"

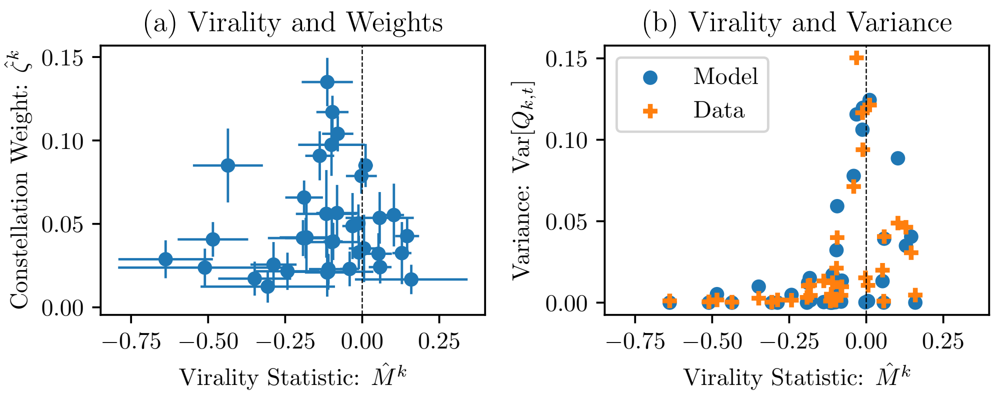
Comparing the model with constellations to the baseline quantification, we find that emergent optimism explains a comparable amount of the variance and autocorrelation of output. For example, optimism explains 16% of output variance and 31% of the first-lag autocovariance, compared to 19% and 33% in our baseline calibration. We report these results in Figure A9. This result implies that the constellation model implies very similar time-series properties for emergent optimism compared to the baseline model, even though the time-series variance of emergent optimism is an untargeted moment.

However, this similarity belies significant heterogeneity in how the granular narratives spread, which is in turn related to each narrative's tendency to "go viral." The unconditional variance of emergent optimism in the model can be decomposed as

$$\mathrm{Var}[Q_t] = \sum_{k=1}^{K} (\zeta^k)^2 \mathrm{Var}[Q_t^k] + R \tag{51}$$

where R is a remainder that accounts for (i) the flooring and capping of Q_t and (ii) the covariance between the narrative evolutions. In our simulations, we find that R is only 3% of the total variance in Q_t. This implies that the variance of emergent optimism can be approximately described as a weighted sum of the variances of the granular narratives, with weights given by the square of their

Figure 8: The Viral Components of Emergent Optimism



Notes: Panel (a) plots our estimates of the virality statistic $\hat{M}^k$, defined in Equation 52, against our estimates of the constellation weights $\hat{\zeta}^k$, from Equation 50. The solid lines are 95% confidence intervals. Panel (b) plots our estimated virality statistics against their simulated variances (blue circles) and their empirical time-series variances (orange crosses).

importance in driving optimism. Next, we observe that the narrative-specific M statistics,

$$\hat{M}^k = \hat{u}^k + \hat{s}^k + \hat{r}^k \frac{\alpha \hat{\zeta}^k \hat{\delta}^{OP}}{1-\omega} - 1 \tag{52}$$

correspond to the correct hysteresis test statistic if narrative k were the only component of emergent optimism. Intuitively, $\hat{M}^k$ captures each granular narrative's "tendency toward virality."

We find that many narratives have M statistics that exceed (or nearly exceed) the criticality threshold of zero (see Panel (a) of Figure 8). These narratives, unlike aggregate optimism, can therefore go viral. Moreover, emergent optimism places large weights on many of these viral narratives (see Panel (a) of Figure 8). Thus, aggregate optimism is significantly driven by viral granular narratives. Finally, the narratives that our model predicts as being close to the virality threshold are precisely the highest variance narratives in the data (see Panel (b) of Figure 8). This provides empirical validation of M as a diagnostic for virality.

Taken together, we find that emergent optimism is largely driven by viral and volatile narratives. Moreover, despite the virality of its underlying components, emergent optimism is stable and its effect on the business cycle is almost unchanged relative to our baseline model with a single non-viral narrative. Thus, while modeling underlying narrative constellations is descriptively interesting for understanding narrative dynamics, it appears inessential for studying the contribution of narratives to the aggregate business cycle.

8 Conclusion

This paper studies the macroeconomic implications of contagious narratives. We develop a conceptual framework in which narratives form building blocks of agents' beliefs, affect agents' decisions, and spread contagiously and associatively between agents. We develop a narrative business-cycle model and find that narratives can generate non-fundamentally driven boom-bust cycles and hysteresis. To take this model to the data, we measure narratives among US firms. We find that contagious and associative *narrative optimism* affects firms' decisions and beliefs without representing news about fundamentals. When we calibrate the model to match the data, we find that the business-cycle implications of narratives are quantitatively significant: measured declines in optimism account for approximately 32% of the peak-to-trough decline in output over the early 2000s recession and 18% over the Great Recession. Finally, we show that the interaction of many simultaneously evolving and highly contagious narratives, some of which are individually prone to hysteresis, can nevertheless underlie stable fluctuations in emergent optimism and output. Taken together, our analysis shows that narratives may be a significant cause of the business cycle.

Our analysis leaves open at least two important areas for future study. First, we have analyzed how firms' narratives matter and abstracted away from studying households' narratives. It seems reasonable that similar mechanisms could operate on the household side of the economy, where contagious narratives might influence spending and investing decisions. Moreover, co-evolving narratives on both the "supply side" and the "demand side" of the economy might have mutually reinforcing effects. From this perspective, narratives have the potential to explain even more of the business cycle than our analysis suggests. Second, there remains much more to study about what "makes a narrative a narrative"—that is, in the language of our model, what microfounds the set of narratives and their contagiousness? A richer study of these issues would cast further light on policy issues, including both the interaction of standard macroeconomic policies with narratives and the potential effects of directly "managing narratives" via communication. Moreover, probing these deeper origins of narratives could further enrich the study of narrative constellations beyond our analysis, to account for the full economic, semantic, and psychological interactions between narratives in a complex world.

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Appendices

Contents

A Omitted Derivations and Proofs

A.1 Proof of Theorem 1

Proof. We guess and verify that there exists a unique quasi-loglinear equilibrium. That is, there exists a unique equilibrium of the following form:

$$\log Y(\theta_t, \theta_{t-1}, Q_t) = a_0 + a_1 \log \theta_t + a_2 \log \theta_{t-1} + f(Q_t) \tag{53}$$

for some parameters $a_0, a_1, a_2 \in \mathbb{R}$ and function $f : [0, 1] \to \mathbb{R}$. To verify this conjecture, we need to compute best replies under this conjecture and show that when we aggregate these best replies that the conjecture is consistent and, moreover, that it is consistent for a unique tuple (a_0, a_1, a_2, f).

From the arguments in the main text, we have Equation 16 holds. Thus, we need to compute two objects: $\log \mathbb{E}_{it}\left[\theta_{it}^{-\frac{1+\psi}{\alpha}}\right]$ and $\log \mathbb{E}_{it}\left[Y_t^{\frac{1}{\epsilon}-\gamma}\right]$. We can compute the first object directly. Conditional on a signal s_{it} and a narrative weight λ_{it}, we have that the distribution of the aggregate component of productivity is:

$$\log \theta_t | s_{it}, \lambda_{it} \sim N\left(\kappa s_{it} + (1-\kappa)\mu(\lambda_{it}, \theta_{t-1}), \sigma^2_{\theta|s}\right) \tag{54}$$

by the standard formula for the conditional distribution of jointly normal random variables, where:

$$\mu(\lambda_{it}, \theta_{t-1}) = (1-\rho)(\mu_O \lambda_{it} + \mu_P(1-\lambda_{it})) + \rho \log \theta_{t-1}\,,\ \kappa = \frac{1}{1+\frac{\sigma_e^2}{\sigma_\theta^2}}\,,\ \sigma^2_{\theta|s} = \frac{1}{\frac{1}{\sigma_\theta^2}+\frac{1}{\sigma_e^2}} \tag{55}$$

with κ being the signal-to-noise ratio and $\sigma^2_{\theta|s}$ the variance of fundamentals conditional on the signal. Thus, the conditional distribution of idiosyncratic productivity is given by:

$$\log \theta_{it} | s_{it}, \lambda_{it} \sim N\left(\log \gamma_i + \kappa s_{it} + (1-\kappa)\mu(\lambda_{it}, \theta_{t-1}), \sigma^2_{\theta|s} + \sigma^2_{\tilde{\theta}}\right) \tag{56}$$

where we will denote the above mean by μ_{it} and variance by η^2. Hence, rewriting and using the moment generating function of a normal random variable, we have that:

$$\begin{aligned} \log \mathbb{E}_{it}\left[\theta_{it}^{-\frac{1+\psi}{\alpha}}\right] &= \log \mathbb{E}_{it}\left[\exp\left\{-\frac{1+\psi}{\alpha}\log\theta_{it}\right\}\right] \\ &= -\frac{1+\psi}{\alpha}\mu_{it} + \frac{1}{2}\left(\frac{1+\psi}{\alpha}\right)^2\eta^2 \end{aligned} \tag{57}$$

Under our conjecture (Equation 53), we can moreover compute:

$$\begin{aligned} \log \mathbb{E}_{it}\left[Y_t^{\frac{1}{\epsilon}-\gamma}\right] &= \log \mathbb{E}_{it}\left[\exp\left\{\left(\frac{1}{\epsilon}-\gamma\right)(a_0 + a_1\log\theta_t + a_2\log\theta_{t-1} + f(Q_t))\right\}\right] \\ &= \left(\frac{1}{\epsilon}-\gamma\right)[a_0 + a_1(\mu_{it} - \log\gamma_i) + a_2\log\theta_{t-1} + f(Q_t)] \\ &\quad + \frac{1}{2}a_1^2\left(\frac{1}{\epsilon}-\gamma\right)^2\left[\eta^2 - \sigma^2_{\tilde{\theta}}\right] \end{aligned} \tag{58}$$

Thus, we have that best replies under our conjecture are given by:

$$\log x_{it} = \frac{1}{\frac{1+\psi-\alpha}{\alpha} + \frac{1}{\epsilon}} \left[\log \left(\frac{1-\frac{1}{\epsilon}}{\frac{1+\psi}{\alpha}} \right) + \frac{1+\psi}{\alpha} \mu_{it} - \frac{1}{2} \left(\frac{1+\psi}{\alpha} \right)^2 \eta^2 \right.$$
$$\left. + \left(\frac{1}{\epsilon} - \gamma \right) [a_0 + a_1(\mu_{it} - \log \gamma_i) + a_2 \log \theta_{t-1} + f(Q_t)] + \frac{1}{2} a_1^2 \left(\frac{1}{\epsilon} - \gamma \right)^2 \left[\eta^2 - \sigma^2_{\tilde{\theta}} \right] \right] \tag{59}$$

To confirm the conjecture, we must now aggregate these levels of production and show that they are consistent with the conjecture. Performing this aggregation we have that:

$$\begin{aligned} \log Y_t &= \log \left[\left(\int_{[0,1]} x_{it}^{\frac{\epsilon-1}{\epsilon}} \right)^{\frac{\epsilon}{\epsilon-1}} \right] \\ &= \frac{\epsilon}{\epsilon-1} \log \mathbb{E}_t \left[\exp \left\{ \frac{\epsilon-1}{\epsilon} \log x_{it} \right\} \right] \\ &= \frac{\epsilon}{\epsilon-1} \log \mathbb{E}_t \left[\mathbb{E}_t \left[\exp \left\{ \frac{\epsilon-1}{\epsilon} \log x_{it} \right\} | \lambda_{it} \right] \right] \end{aligned} \tag{60}$$

Moreover, expanding the terms in Equation 59, we have that:

$$\begin{aligned} \log x_{it} = & \frac{1}{\frac{1+\psi-\alpha}{\alpha} + \frac{1}{\epsilon}} \left[\log \left(\frac{1-\frac{1}{\epsilon}}{\frac{1+\psi}{\alpha}} \right) \right. \\ & + \frac{1+\psi}{\alpha} [\log \gamma_i + \kappa s_{it} + (1-\kappa)\mu(\lambda_{it}, \theta_{t-1})] \\ & - \frac{1}{2} \left(\frac{1+\psi}{\alpha} \right)^2 \left(\sigma^2_{\theta|s} + \sigma^2_{\tilde{\theta}} \right) \\ & + \left(\frac{1}{\epsilon} - \gamma \right) [a_0 + a_1 \left(\kappa s_{it} + (1-\kappa)\mu(\lambda_{it}, \theta_{t-1}) \right) + a_2 \log \theta_{t-1} + f(Q_t)] \\ & \left. + \frac{1}{2} a_1^2 \left(\frac{1}{\epsilon} - \gamma \right)^2 \sigma^2_{\theta|s} \right] \end{aligned} \tag{61}$$

which is, conditional on λ_{it}, normally distributed as both $\log \gamma_i$ and s_{it} are both normal. Hence, we write $\log x_{it} | \lambda_{it} \sim N(\delta_t(\lambda_{it}), \hat{\sigma}^2)$, where:

$$\begin{aligned} \delta_t(\lambda_{it}) = & \frac{1}{\frac{1+\psi-\alpha}{\alpha} + \frac{1}{\epsilon}} \left[\log \left(\frac{1-\frac{1}{\epsilon}}{\frac{1+\psi}{\alpha}} \right) \right. \\ & + \frac{1+\psi}{\alpha} [\mu_\gamma + \kappa \log \theta_t + (1-\kappa)\mu(\lambda_{it}, \theta_{t-1})] - \frac{1}{2} \left(\frac{1+\psi}{\alpha} \right)^2 \left(\sigma^2_{\theta|s} + \sigma^2_{\tilde{\theta}} \right) \\ & + \left(\frac{1}{\epsilon} - \gamma \right) [a_0 + a_1 \left(\kappa \log \theta_t + (1-\kappa)\mu(\lambda_{it}, \theta_{t-1}) \right) + a_2 \log \theta_{t-1} + f(Q_t)] \\ & \left. + \frac{1}{2} a_1^2 \left(\frac{1}{\epsilon} - \gamma \right)^2 \sigma^2_{\theta|s} \right] \end{aligned} \tag{62}$$

and:

$$\hat{\sigma}^2 = \left(\frac{1}{\frac{1+\psi-\alpha}{\alpha}+\frac{1}{\epsilon}}\right)^2 \left[\left(\frac{1+\psi}{\alpha}\right)^2 \sigma_\gamma^2 + \kappa^2 \left[\frac{1+\psi}{\alpha} + a_1\left(\frac{1}{\epsilon}-\gamma\right)\right]^2 \sigma_e^2\right] \tag{63}$$

Thus, we have that:

$$\mathbb{E}_t\left[\exp\left\{\frac{\epsilon-1}{\epsilon}\log x_{it}\right\}|\lambda_{it}\right] = \exp\left\{\frac{\epsilon-1}{\varepsilon}\delta_t(\lambda_{it}) + \frac{1}{2}\left(\frac{\epsilon-1}{\varepsilon}\right)^2\hat{\sigma}^2\right\} \tag{64}$$

and so:

$$\begin{aligned}\mathbb{E}_t\left[\mathbb{E}_t\left[\exp\left\{\frac{\epsilon-1}{\epsilon}\log x_{it}\right\}|\lambda_{it}\right]\right] &= Q_t \exp\left\{\frac{\epsilon-1}{\varepsilon}\delta_t(1) + \frac{1}{2}\left(\frac{\epsilon-1}{\varepsilon}\right)^2\hat{\sigma}^2\right\} \\ &+ (1-Q_t)\exp\left\{\frac{\epsilon-1}{\varepsilon}\delta_t(0) + \frac{1}{2}\left(\frac{\epsilon-1}{\varepsilon}\right)^2\hat{\sigma}^2\right\} \\ &= \left[Q_t\exp\left\{\frac{\epsilon-1}{\epsilon}(\delta_t(1)-\delta_t(0))\right\} + (1-Q_t)\right]\exp\left\{\frac{\epsilon-1}{\epsilon}\delta_t(0) + \frac{1}{2}\left(\frac{\epsilon-1}{\varepsilon}\right)^2\hat{\sigma}^2\right\}\end{aligned} \tag{65}$$

Yielding:

$$\log Y_t = \delta_t(0) + \frac{1}{2}\frac{\epsilon-1}{\epsilon}\hat{\sigma}^2 + \frac{\epsilon}{\epsilon-1}\log\left(Q_t\exp\left\{\frac{\epsilon-1}{\epsilon}(\delta_t(1)-\delta_t(0))\right\} + (1-Q_t)\right) \tag{66}$$

where we define $\alpha\delta^{OP} = \delta_t(1) - \delta_t(0)$ and compute:

$$\delta_t(1) - \delta_t(0) = \frac{1}{\frac{1+\psi-\alpha}{\alpha}+\frac{1}{\epsilon}}\left(\frac{1+\psi}{\alpha} + a_1\left(\frac{1}{\epsilon}-\gamma\right)\right)(1-\kappa)(1-\rho)(\mu_O-\mu_P) = \alpha\delta^{OP} \tag{67}$$

and note that this is a constant. Finally, we see that $\delta_t(0)$ is given by:

$$\begin{aligned}\delta_t(0) = \frac{1}{\frac{1+\psi-\alpha}{\alpha}+\frac{1}{\epsilon}}&\left[\log\left(\frac{1-\frac{1}{\epsilon}}{\frac{1+\psi}{\alpha}}\right) + \frac{1+\psi}{\alpha}\left(\mu_\gamma + (1-\kappa)((1-\rho)\mu_P + \rho\log\theta_{t-1}\right) - \frac{1}{2}\left(\frac{1+\psi}{\alpha}\right)^2\left(\sigma_{\theta|s}^2 + \sigma_{\tilde{\theta}}^2\right)\right. \\ &+ \left(\frac{1}{\epsilon}-\gamma\right)(a_0 + a_1(1-\kappa)((1-\rho)\mu_P + \rho\log\theta_{t-1})) + \frac{1}{2}a_1^2\left(\frac{1}{\epsilon}-\gamma\right)^2\sigma_{\theta|s}^2 \\ &\left. + \left[\frac{1+\psi}{\alpha} + a_1\left(\frac{1}{\epsilon}-\gamma\right)\right]\kappa\log\theta_t + \left(\frac{1}{\epsilon}-\gamma\right)(a_2\log\theta_{t-1} + f(Q_t))\right]\end{aligned} \tag{68}$$

By matching coefficients between Equations 66 and Equation 53, we obtain a_0, a_1, a_2, and f.

We first match coefficients on $\log\theta_t$ to obtain an equation for a_1:

$$a_1 = \frac{\left[\frac{1+\psi}{\alpha} + a_1\left(\frac{1}{\epsilon}-\gamma\right)\right]\kappa}{\frac{1+\psi-\alpha}{\alpha}+\frac{1}{\epsilon}} \tag{69}$$

Under our maintained assumption that $\frac{\frac{1}{\epsilon}-\gamma}{\frac{1+\psi-\alpha}{\alpha}+\frac{1}{\epsilon}} \in [0,1)$, as $\kappa \in [0,1]$, we have that this has a unique solution:

$$a_1 = \frac{\frac{\frac{1+\psi}{\alpha}\kappa}{\frac{1+\psi-\alpha}{\alpha}+\frac{1}{\epsilon}}}{1-\frac{\left(\frac{1}{\epsilon}-\gamma\right)\kappa}{\frac{1+\psi-\alpha}{\alpha}+\frac{1}{\epsilon}}} = \frac{1}{1-\kappa\omega}\frac{\frac{1+\psi}{\alpha}\kappa}{\frac{1+\psi-\alpha}{\alpha}+\frac{1}{\epsilon}} \tag{70}$$

which is in terms of primitive parameters and is moreover positive.

Second, we match coefficients on $\log\theta_{t-1}$ to obtain an equation for a_2:

$$a_2 = \frac{1}{\frac{1+\psi-\alpha}{\alpha}+\frac{1}{\epsilon}}\left[\left(\frac{1+\psi}{\alpha}+\left(\frac{1}{\epsilon}-\gamma\right)a_1\right)(1-\kappa)\rho+\left(\frac{1}{\epsilon}-\gamma\right)a_2\right] \tag{71}$$

This implies that:

$$\begin{aligned} a_2 &= \frac{1}{1-\omega}\frac{1}{\frac{1+\psi-\alpha}{\alpha}+\frac{1}{\epsilon}}\left[\frac{1+\psi}{\alpha}+\left(\frac{1}{\epsilon}-\gamma\right)a_1\right](1-\kappa)\rho \\ &= \frac{1}{1-\omega}\frac{1}{\frac{1+\psi-\alpha}{\alpha}+\frac{1}{\epsilon}}\left[\frac{1+\psi}{\alpha}+\left(\frac{1}{\epsilon}-\gamma\right)\frac{1}{1-\kappa\omega}\frac{\frac{1+\psi}{\alpha}\kappa}{\frac{1+\psi-\alpha}{\alpha}+\frac{1}{\epsilon}}\right](1-\kappa)\rho \end{aligned} \tag{72}$$

which is in terms of primitive parameters.

Third, by collecting terms with Q_t we obtain an equation for f:

$$f(Q) = \frac{\frac{1}{\epsilon}-\gamma}{\frac{1+\psi-\alpha}{\alpha}+\frac{1}{\epsilon}}f(Q)+\frac{\epsilon}{\epsilon-1}\log\left(1+Q\left[\exp\left\{\frac{\epsilon-1}{\epsilon}\alpha\delta^{OP}\right\}-1\right]\right) \tag{73}$$

which has a unique solution as $\frac{\frac{1}{\epsilon}-\gamma}{\frac{1+\psi-\alpha}{\alpha}+\frac{1}{\epsilon}} \in [0,1)$ and can be solved to yield:

$$f(Q) = \frac{\frac{\epsilon}{\epsilon-1}}{1-\frac{\frac{1}{\epsilon}-\gamma}{\frac{1+\psi-\alpha}{\alpha}+\frac{1}{\epsilon}}}\log\left(1+Q\left[\exp\left\{\frac{\epsilon-1}{\epsilon}\alpha\delta^{OP}\right\}-1\right]\right) \tag{74}$$

where we observe that δ^{OP} depends only on primitive parameters and a_1, for which we have already solved.

Finally, by collecting constants, we obtain an equation for a_0:

$$\begin{aligned} a_0 = \frac{1}{\frac{1+\psi-\alpha}{\alpha}+\frac{1}{\epsilon}}\Bigg[&\log\left(\frac{1-\frac{1}{\epsilon}}{\frac{1+\psi}{\alpha}}\right)+\frac{1+\psi}{\alpha}\left(\mu_\gamma+(1-\kappa)(1-\rho)\mu_P\right)-\frac{1}{2}\left(\frac{1+\psi}{\alpha}\right)^2\left(\sigma^2_{\theta|s}+\sigma^2_{\tilde{\theta}}\right) \\ &+\left(\frac{1}{\epsilon}-\gamma\right)\left(a_0+a_1(1-\kappa)(1-\rho)\mu_P\right)+\frac{1}{2}a_1^2\left(\frac{1}{\epsilon}-\gamma\right)^2\sigma^2_{\theta|s}\Bigg]+\frac{1}{2}\frac{\epsilon-1}{\epsilon}\hat{\sigma}^2 \end{aligned} \tag{75}$$

Solving this equation yields:

$$a_0 = \frac{1}{1 - \frac{\frac{1}{\epsilon}-\gamma}{\frac{1+\psi-\alpha}{\alpha}+\frac{1}{\epsilon}}} \left[\frac{1}{\frac{1+\psi-\alpha}{\alpha}+\frac{1}{\epsilon}} \left[\log\left(\frac{1-\frac{1}{\epsilon}}{\frac{1+\psi}{\alpha}}\right) + \frac{1+\psi}{\alpha}\left(\mu_\gamma + (1-\kappa)(1-\rho)\mu_P\right) - \frac{1}{2}\left(\frac{1+\psi}{\alpha}\right)^2 \left(\sigma^2_{\theta|s} + \sigma^2_{\tilde{\theta}}\right) \right.\right.$$
$$\left.\left. + \left(\frac{1}{\epsilon}-\gamma\right) a_1 (1-\kappa)(1-\rho)\mu_P + \frac{1}{2} a_1^2 \left(\frac{1}{\epsilon}-\gamma\right)^2 \sigma^2_{\theta|s} \right] + \frac{1}{2}\frac{\epsilon-1}{\epsilon}\hat{\sigma}^2 \right] \tag{76}$$

which we observe depends only on parameters, a_1, and $\hat{\sigma}^2$. Moreover, $\hat{\sigma}^2$ depends only on parameters and a_1. Thus, given that we have solved for a_1, we have now recovered a_0, a_1, a_2 and f uniquely and verified that there exists a unique quasi-loglinear equilibrium. Finally, to obtain the formula for the best reply of agents, simply substitute a_0, a_1, a_2 and f into Equation 61 and label the coefficients as in the claim. □

A.2 Proof of the Claims in Remark 1

We now prove the claims made in Remark 1. We have already shown that there exists a unique quasi-loglinear equilibrium. More generally, we seek to rule out an equilibrium of any other form. To do so, we show that there is a unique equilibrium when fundamentals are bounded by some $M \in \mathbb{R}$, $\log \theta_t \in [-M, M]$, $\log \gamma_i \in [-M, M]$, $\log \tilde{\theta}_{it} \in [-M, M]$, and $e_{it} \in [-M, M]$.

Lemma 1. *When fundamentals are bounded, there exists a unique equilibrium*

Proof. To this end, we can recast any equilibrium function $\log Y(\theta, \theta_{-1}, Q)$ as one that solves the fixed point in Equation 16. In the case where fundamentals are bounded, this can be accomplished by demonstrating that the implied fixed-point operator is a contraction by verifying Blackwell's sufficient conditions. More formally, consider the space of bounded, real-valued functions $\mathcal{C}$ under the L^∞-norm and consider the operator $V_M : \mathcal{C} \to \mathcal{C}$ given by:

$$V_M(g)(\theta, \theta_{-1}, Q) = \frac{\epsilon}{\epsilon-1} \log \mathbb{E}_{(\theta,\theta_{-1},Q)} \left[\exp\left\{ \frac{\frac{\epsilon-1}{\epsilon}}{\frac{1+\psi-\alpha}{\alpha}+\frac{1}{\epsilon}} \left(\log\left(\frac{1-\frac{1}{\epsilon}}{\frac{1+\psi}{\alpha}}\right) \right.\right.\right.$$
$$\left.\left.\left. - \log \mathbb{E}_{(s,Q)}\left[\exp\left\{-\frac{1+\psi}{\alpha}\log\theta_{it}\right\}\right] + \log \mathbb{E}_{(s,Q)}\left[\exp\left\{\left(\frac{1}{\epsilon}-\gamma\right) g\right\}\right] \right) \right\} \right] \tag{77}$$

The following two conditions are sufficient for this operator to be a contraction: (i) monotonicity: for all $g, h \in \mathcal{C}$ such that $g \geq h$, we have that $V_M(g) \geq V_M(h)$ (ii) discounting: there exists a parameter $c \in [0, 1)$ such that for all $g \in \mathcal{C}$ and $a \in \mathbb{R}_+$ and $V_M(g + a) \leq V_M(g) + ca$. Thus, as the space of bounded functions under the L^∞-norm is a complete metric space, if Blackwell's conditions hold, then by the Banach fixed-point theorem, there exists a unique fixed point of the operator V_M.

To complete this argument, we now verify (i) and (ii). To show monotonicity, observe that

$\frac{1}{\epsilon} - \gamma \geq 0$ as $\omega \geq 0$ and recall that $\epsilon > 1$. Thus, we have that:

$$\log \mathbb{E}_{(s,Q)} \left[\exp \left\{ \left(\frac{1}{\epsilon} - \gamma \right) g \right\} \right] \geq \log \mathbb{E}_{(s,Q)} \left[\exp \left\{ \left(\frac{1}{\epsilon} - \gamma \right) h \right\} \right] \tag{78}$$

for all (s, Q). And so $V_M(g)(\theta, Q) \geq V_M(h)(\theta, Q)$ for all (θ, Q). To show discounting, observe that:

$$\log \mathbb{E}_{(s,Q)} \left[\exp \left\{ \left(\frac{1}{\epsilon} - \gamma \right) (g + a) \right\} \right] = \log \mathbb{E}_{(s,Q)} \left[\exp \left\{ \left(\frac{1}{\epsilon} - \gamma \right) g \right\} \right] + \left(\frac{1}{\epsilon} - \gamma \right) a \tag{79}$$

And so:

$$\begin{aligned} V_M(g + a)(\theta, \theta_{-1}, Q) &= \frac{\epsilon}{\epsilon - 1} \log \mathbb{E}_{(\theta,\theta_{-1},Q)} \Bigg[\exp \Bigg\{ \frac{\frac{\epsilon-1}{\epsilon}}{\frac{1+\psi-\alpha}{\alpha} + \frac{1}{\epsilon}} \Bigg(\log \left(\frac{1 - \frac{1}{\epsilon}}{\frac{1+\psi}{\alpha}} \right) \\ &- \log \mathbb{E}_{(s,Q)} \left[\exp \left\{ -\frac{1+\psi}{\alpha} \log \theta_{it} \right\} \right] + \log \mathbb{E}_{(s,Q)} \left[\exp \left\{ \left(\frac{1}{\epsilon} - \gamma \right) g \right\} \right] + \left(\frac{1}{\epsilon} - \gamma \right) a \Bigg) \Bigg\} \Bigg] \\ &= V_M(g)(\theta, \theta_{-1}, Q) + \omega a \end{aligned} \tag{80}$$

where $\omega \in [0, 1)$ by assumption. Note that the modulus of contraction ω is precisely the claimed strategic complementarity parameter in Equation 11. This verifies equilibrium uniqueness. □

Away from the case with bounded fundamentals, the above strategy cannot be used to demonstrate uniqueness. Even though the fixed-point operator still satisfies Blackwell's conditions, the relevant function space now becomes any L^p-space for $p \in (1, \infty)$ and the sup-norm over such spaces can be infinite, making Blackwell's conditions insufficient for V to be a contraction. In this case, we show that the unique quasi-loglinear equilibrium in the unbounded fundamentals case is an appropriately-defined ε-equilibrium for any $\varepsilon > 0$. Let the unique quasi-loglinear equilibrium we have guessed and verified be $\log Y^*$. We say that g is a ε-equilibrium if

$$||g - V_M(g)||_p < \varepsilon \tag{81}$$

where $|| \cdot ||_p$ is the L^p-norm. In words, g is a ε-equilibrium if its distance from being a fixed point is at most ε. The following Lemma establishes that Y^* is a ε-equilibrium for bounded fundamentals for any $\varepsilon > 0$ for some bound M:

Lemma 2. *For every $\varepsilon > 0$, there exists an $M \in \mathbb{N}$ such that $\log Y^*$ is a ε-equilibrium.*

Proof. Now extend from $\mathcal{C}$, $V_M : L^p(\mathbb{R}) \to L^p(\mathbb{R})$ as in Equation 77. We observe that V_M is continuous in the limit in M in the sense that $V_M(g) \to V(g)$ as $M \to \infty$ for all $g \in L^p(\mathbb{R})$. This observation follows from noting that both $\log \mathbb{E}_{(s,Q)} \left[\exp \left\{ -\frac{1+\psi}{\alpha} \log \theta_{it} \right\} \right]$ and $\log \mathbb{E}_{(s,Q)} \left[\exp \left\{ \left(\frac{1}{\epsilon} - \gamma \right) g \right\} \right]$ are convergent pointwise for $M \to \infty$ for all (s, Q). In Proposition 1, we showed that $V(\log Y^*) = \log Y^*$. Thus, we have that: $V_M(\log Y^*) \to V(\log Y^*) = \log Y^*$, which implies that:

$$\lim_{M \to \infty} || \log Y^* - V_M(\log Y^*)||_p = 0 \tag{82}$$

which implies that for every $\varepsilon > 0$, there exists a $\bar{M} \in \mathbb{N}$ such that:

$$|| \log Y^* - V_M(\log Y^*)||_p < \varepsilon \quad \forall M \in \mathbb{N} : M > \bar{M} \tag{83}$$

Completing the proof. □

A.3 Proof of Theorem 2

Proof. We prove the three claims in sequence.

(1) The map $T_\theta : [0,1] \to [0,1]$ is continuous for all $\theta \in \Theta$ as f, P_O and P_P are continuous functions. Moreover, it maps a convex and compact set to itself. Thus, by Brouwer's fixed point theorem, there exists a Q^*_θ such that $Q^*_\theta = T_\theta(Q^*_\theta)$ for all $\theta \in \Theta$.

(2) To characterize the existence of extremal steady states, observe that $Q = 1$ is a steady state for θ if and only if $T_\theta(1) = P_O(a_o + (a_1 + a_2)\log\theta + f(1), 1, 0) = 1$ and $Q = 0$ is a steady state for θ if and only if $T_\theta(0) = P_P(a_0 + (a_1 + a_2)\log\theta, 0, 0) = 0$. Thus, $Q = 1$ is a steady state if and only if $P_O^{-1}(1; 1) \leq a_0 + (a_1 + a_2)\log\theta + f(1)$ and $Q = 0$ is a steady state if and only if $P_P^{-1}(0; 0) \geq a_0 + (a_1 + a_2)\log\theta$. To obtain the result as stated, we re-arrange these inequalities in terms of $\log\theta$ and exponentiate.

(3) To analyze the stability of the extremal steady states, observe that if $T'_\theta(Q^*) < 1$ at a steady state Q^*, then Q^* is stable. When it exists (which it does almost everywhere), we have that:

$$\begin{aligned} T'_\theta(Q) = {} & P_O(a_0 + (a_1 + a_2)\log\theta + f(Q), Q, 0) - P_P(a_0 + (a_1 + a_2)\log\theta + f(Q), Q, 0) \\ & + Q\frac{\mathrm{d}}{\mathrm{d}Q}P_O(a_0 + (a_1 + a_2)\log\theta + f(Q), Q, 0) \\ & + (1 - Q)\frac{\mathrm{d}}{\mathrm{d}Q}P_P(a_0 + (a_1 + a_2)\log\theta + f(Q), Q, 0) \end{aligned} \tag{84}$$

Thus, for $\theta < \theta_P$ and $Q = 0$:

$$\begin{aligned} T'_\theta(0) &= P_O(a_0 + (a_1 + a_2)\log\theta, 0, 0) - P_P(a_0 + (a_1 + a_2)\log\theta, 0, 0) \\ &\quad + \frac{\mathrm{d}}{\mathrm{d}Q}P_P(a_0 + (a_1 + a_2)\log\theta + f(Q), Q, 0) \mid_{Q=0} \\ &= P_O(a_0 + (a_1 + a_2)\log\theta, 0, 0) \end{aligned} \tag{85}$$

where the second equality follows by observing that all of P_P, $\frac{\partial P_P}{\partial \log Y}$, and $\frac{\partial P_P}{\partial Q}$ are zero for $\theta < \theta_P$. Thus, we have that $T'_\theta(0) < 1$ when $P_O(a_0 + (a_1 + a_2)\log\theta, 0, 0) < 1$. Moreover, for $\theta < \theta_P$, we have that: $P_O(a_0 + (a_1 + a_2)\log\theta, 0, 0) \leq P_O(a_0 + (a_1 + a_2)\log\theta_P, 0, 0) = P_O(P_P^{-1}(0; 0), 0, 0)$. Thus, a sufficient condition for $T'_\theta(0) < 1$ for $\theta < \theta_P$ is that $P_O(P_P^{-1}(0; 0), 0, 0) < 1$.

For $\theta > \theta_O$ and $Q = 1$, we have that:

$$\begin{aligned}T'_\theta(1) &= P_O(a_0 + (a_1 + a_2)\log\theta + f(1), 1, 0) - P_P(a_0 + (a_1 + a_2)\log\theta + f(1), 1, 0) \\ &\quad + \frac{\mathrm{d}}{\mathrm{d}Q} P_O(a_0 + (a_1 + a_2)\log\theta + f(1), 1, 0) \mid_{Q=1} \\ &= 1 - P_P(a_0 + (a_1 + a_2)\log\theta + f(1), 1, 0)\end{aligned} \tag{86}$$

where the second equality follows by observing that $P_O = 1$ and both $\frac{\partial P_O}{\partial \log Y}$ and $\frac{\partial P_O}{\partial Q}$ are zero for $\theta > \theta_O$. Hence, we have that $T'_\theta(1) < 1$ when $P_P(a_0 + (a_1 + a_2)\log\theta + f(1), 1, 0) > 0$. For $\theta > \theta_O$ we have that $P_P(a_0+(a_1+a_2)\log\theta+f(1), 1, 0) \geq P_P(a_0+(a_1+a_2)\log\theta_O+f(1), 1) = P_P(P_O^{-1}(1, 1), 1, 0)$. Thus, a sufficient condition for $T'_\theta(1) < 1$ for $\theta > \theta_O$ is that $P_P(P_O^{-1}(1, 1), 1, 0) > 0$. □

A.4 Proof of Corollary 5

Proof. From Equation 61 in the proof of Proposition 1, we have that the log production of firm i at time t is described in the unique quasi-log-linear equilibrium by:

$$\begin{aligned}\log x_{it} = \frac{1}{\frac{1+\psi-\alpha}{\alpha} + \frac{1}{\epsilon}} \Bigg[&\log\left(\frac{1 - \frac{1}{\epsilon}}{\frac{1+\psi}{\alpha}}\right) + \frac{1+\psi}{\alpha}\left[\log\gamma_i + \kappa s_{it} + (1-\kappa)\mu(\lambda_{it}, \theta_{t-1})\right] \\ &- \frac{1}{2}\left(\frac{1+\psi}{\alpha}\right)^2 \left(\sigma^2_{\theta|s} + \sigma^2_{\tilde{\theta}}\right) + + \frac{1}{2} a_1^2 \left(\frac{1}{\epsilon} - \gamma\right)^2 \sigma^2_{\theta|s} \\ &+ \left(\frac{1}{\epsilon} - \gamma\right)\left[a_0 + a_1\left(\kappa s_{it} + (1-\kappa)\mu(\lambda_{it}, \theta_{t-1})\right) + a_2 \log\theta_{t-1} + f(Q_t)\right]\Bigg]\end{aligned} \tag{87}$$

We substitute this expression into the production function to obtain an equation for hiring $\log L_{it} = \frac{1}{\alpha}(\log x_{it} - \log\theta_{it})$. Subtracting lagged labor from both sides yields Equation 30. □

B Additional Theoretical Results and Extensions

This appendix covers several additional results and model extensions. First, we provide formal results on the model's impulse response functions and its propensity to undergo boom-bust cycles (B.1). Second, we theoretically characterize and quantify the normative implications of narrative fluctuations (B.2). Third, we study equilibrium dynamics under a benchmark model of Bayesian model updating and contrast these predictions with those obtained in our main analysis (B.3). Fourth, fifth, sixth, and seventh we extend the baseline model to respectively incorporate a continuum of different levels of optimism (B.4), narratives about idiosyncratic fundamentals (B.5), multi-dimensional narratives (B.6), and narrative updating that depends on idiosyncratic fundamentals (B.7). In each case, we characterize equilibrium dynamics and show how our main theoretical insights extend. Eighth, we show how endogenous cycles and chaotic dynamics can obtain when agents are contrarian and implement an empirical test for their presence (B.8). Ninth and finally, we highlight the role of higher-order beliefs and show how our analysis could generalize to other settings by deriving a similar law of motion for optimism in abstract, linear beauty contest games à la Morris and Shin (2002) (B.9).

B.1 Impulse Responses and Stochastic Fluctuations

This Appendix generalizes and formalizes the observations about narrative business cycle dynamics from Section 3.3.

First, we define two important types of updating rules that satisfy a natural single-crossing condition. We say that T is *strictly single-crossing from above* (SSC-A) if for all $\theta \in \Theta$ there exists $\hat{Q}_\theta \in [0,1]$ such that $T_\theta(Q) > Q$ for all $Q \in (0, \hat{Q}_\theta)$ and $T_\theta(Q) < Q$ for all $Q \in (\hat{Q}_\theta, 1)$. We say that T is *strictly single-crossing from below* (SSC-B) if for all $\theta \in \Theta$ there exists $\hat{Q}_\theta \in [0,1]$ such that $T_\theta(Q) > Q$ for all $Q \in (\hat{Q}_\theta, 1)$ and $T_\theta(Q) < Q$ for all $Q \in (0, \hat{Q}_\theta)$. If T is either SSC-A or SSC-B, we say that it is SSC. The left and right panels of Figure 1 respectively illustrate examples of SSC-A and SSC-B transition maps.

Lemma 3 (Steady States under the SSC Property)**.** *If T_θ is SSC, then there exist at most three deterministic steady states. These correspond to extreme pessimism $Q = 0$, extreme optimism $Q = 1$, and intermediate optimism $Q = \hat{Q}_\theta$. Moreover, when T_θ is SSC-A: intermediate optimism is stable with a basin of attraction that includes $(0,1)$; and whenever extreme optimism or extreme pessimism are steady states that do not coincide with $\hat{Q}_\theta$, they are unstable with respective basins of attraction $\{0\}$ and $\{1\}$. When T_θ is SSC-B: whenever extreme optimism is a steady state, it is stable with basin of attraction $(\hat{Q}_\theta, 1]$; whenever extreme pessimism is a steady state it is stable with basin of attraction $[0, \hat{Q}_\theta)$; and intermediate optimism is always unstable with basin of attraction $\{\hat{Q}_\theta\}$.*

Proof. Fix $\theta \in \Theta$. We first study the SSC-A case. By SSC-A of T we have that there exists $\hat{Q}_\theta \in [0,1]$ such that $T_\theta(Q) > Q$ for all $Q \in (0, \hat{Q}_\theta)$ and $T_\theta(Q) < Q$ for all $Q \in (\hat{Q}_\theta, 1)$. As T_θ is continuous we have that $T_\theta(\hat{Q}_\theta) = \hat{Q}_\theta$. Consider now some $Q_0 \in (0,1)$ such that $Q_0 \neq \hat{Q}_\theta$. We have that $T_\theta(Q_0) > \hat{Q}_\theta$ if $Q_0 < \hat{Q}_\theta$ and $T_\theta(Q_0) < \hat{Q}_\theta$ if $Q_0 > \hat{Q}_\theta$. Hence, there exists at most one $Q^* \in (0,1)$ such that $T_\theta(Q^*) = Q^*$. Thus, there exist at most three steady states $Q^* = 0$, $Q^* = \hat{Q}_\theta$, and $Q^* = 1$.

To find the basins of attraction of these steady states, fix $Q_0 \in (0,1)$ and consider the sequence $\{T_\theta^n(Q_0)\}_{n\in\mathbb{N}}$. For a steady state Q^*, its basin of attraction is:

$$\mathcal{B}_\theta(Q^*) = \left\{Q_0 \in [0,1] : \lim_{n\to\infty} T_\theta^n(Q_0) = Q^*\right\} \tag{88}$$

First, consider $Q_0 \in (0, \hat{Q}_\theta)$. We now show by induction that $T_\theta^n(Q_0) \geq T_\theta^{n-1}(Q_0)$ for all $n \in \mathbb{N}$. Consider $n = 1$. We have that $T_\theta(Q_0) > Q_0$ as T is SSC-A and $Q_0 < \hat{Q}_\theta$. Suppose now that $T_\theta^n(Q_0) \geq T_\theta^{n-1}(Q_0)$. We have that:

$$T_\theta^{n+1}(Q_0) = T_\theta \circ T_\theta^n(Q_0) \geq T_\theta \circ T_\theta^{n-1}(Q_0) = T_\theta^n(Q_0) \tag{89}$$

by monotonicity of T_θ, which proves the inductive hypothesis. Observe moreover that the sequence $\{T_\theta^n(Q_0)\}_{n\in\mathbb{N}}$ is bounded as $T_\theta^n(Q_0) \in [0,1]$ for all $n \in \mathbb{N}$. Hence, by the monotone convergence theorem, $\lim_{n\to\infty} T_\theta^n(Q_0)$ exists. Toward a contradiction, suppose that $Q_0^\infty = \lim_{n\to\infty} T_\theta^n(Q_0) > \hat{Q}_\theta$. By SSC-A of T we have that $T_\theta(Q_0^\infty) > Q_0^\infty$, but this contradicts that $Q_0^\infty = \lim_{n\to\infty} T_\theta^n(Q_0)$. Thus, we have that $Q_0^\infty = \hat{Q}_\theta$. Hence, $(0, \hat{Q}_\theta) \subseteq \mathcal{B}_\theta(\hat{Q}_\theta)$. Second, consider $Q_0 = \hat{Q}_\theta$. We have that $T_\theta(\hat{Q}_\theta) = \hat{Q}_\theta$. Thus, $Q_0^\infty = \hat{Q}_\theta$. Hence, $\hat{Q}_\theta \in \mathcal{B}_\theta(\hat{Q}_\theta)$. Third, consider $Q_0 \in (\hat{Q}_\theta, 1)$. Following the arguments of the first part, we have that $(\hat{Q}_\theta, 1) \subseteq \mathcal{B}_\theta(\hat{Q}_\theta)$. Thus, $(0,1) \subseteq \mathcal{B}_\theta(\hat{Q}_\theta)$. Moreover, if $Q = 0$ or $Q = 1$ are steady states, they can only have basins of attraction in $[0,1] \setminus \mathcal{B}_\theta(\hat{Q}_\theta)$, which implies that they are unstable and can only have basins of attraction $\{0\}$ and $\{1\}$.

The analysis of the SSC-B case follows similarly. By SSC-B of T we have that there exists $\hat{Q}_\theta \in [0,1]$ such that $T_\theta(Q) > Q$ for all $Q \in (\hat{Q}_\theta, 1)$ and $T_\theta(Q) < Q$ for all $Q \in (0, \hat{Q}_\theta)$. As T_θ is continuous, we have that $T_\theta(\hat{Q}_\theta) = \hat{Q}_\theta$. Consider now some $Q_0 \in (0,1)$ such that $Q_0 \neq \hat{Q}_\theta$. Observe that $T_\theta(Q_0) < \hat{Q}_\theta$ if $Q_0 < \hat{Q}_\theta$ and $T_\theta(Q_0) > \hat{Q}_\theta$ if $Q_0 > \hat{Q}_\theta$. Hence, there exists at most one $Q^* \in (0,1)$ such that $T_\theta(Q^*) = Q^*$. Thus, there exist at most three steady states $Q^* = 0$, $Q^* = \hat{Q}_\theta$, and $Q^* = 1$.

To find the basins of attraction of these steady states, first consider $Q_0 \in (0, \hat{Q}_\theta)$. We now show by induction that $T_\theta^n(Q_0) \leq T_\theta^{n-1}(Q_0)$ for all $n \in \mathbb{N}$. Consider $n = 1$. We have that $T_\theta(Q_0) < Q_0$ as T is SSC-B and $Q_0 < \hat{Q}_\theta$. Suppose now that $T_\theta^n(Q_0) \leq T_\theta^{n-1}(Q_0)$. We have that:

$$T_\theta^{n+1}(Q_0) = T_\theta \circ T_\theta^n(Q_0) \leq T_\theta \circ T_\theta^{n-1}(Q_0) = T_\theta^n(Q_0) \tag{90}$$

by monotonicity of T_θ, which proves the inductive hypothesis. Observe moreover that the sequence $\{T_\theta^n(Q_0)\}_{n\in\mathbb{N}}$ is bounded as $T_\theta^n(Q_0) \in [0,1]$ for all $n \in \mathbb{N}$. Hence, by the monotone convergence theorem, $\lim_{n\to\infty} T_\theta^n(Q_0)$ exists. Finally, toward a contradiction, suppose that $Q_0^\infty = \lim_{n\to\infty} T_\theta^n(Q_0) > 0$. By SSC-B of T we have that $T_\theta(Q_0^\infty) < Q_0^\infty$, but this contradicts that $Q_0^\infty = \lim_{n\to\infty} T_\theta^n(Q_0)$.

Thus, we have that $Q_0^\infty = 0$. Hence, $[0, \hat{Q}_\theta) \subseteq \mathcal{B}_\theta(0)$. Second, consider $Q_0 = \hat{Q}_\theta$. We have that $T_\theta(\hat{Q}_\theta) = \hat{Q}_\theta$. Thus, $Q_0^\infty = \hat{Q}_\theta$. Hence $\hat{Q}_\theta \in \mathcal{B}_\theta(\hat{Q}_\theta)$. Third, consider $Q_0 \in (\hat{Q}_\theta, 1]$. By the exact arguments of the first part, we have that $(\hat{Q}_\theta, 1] \subseteq \mathcal{B}_\theta(1)$. Observing $\mathcal{B}_\theta(0)$, $\mathcal{B}_\theta(\hat{Q}_\theta)$, and $\mathcal{B}_\theta(1)$ are disjoint completes the proof. □

In the SSC-A case there is a unique, (almost) globally stable steady state (left panel of Figure 1). In the SSC-B class, there exists a state-dependent criticality threshold $\hat{Q}_\theta \in [0, 1]$, below which the economy converges to extreme, self-fulfilling pessimism and above which the economy converges to extreme, self-fulfilling optimism (right panel of Figure 1). These two classes delineate two qualitatively different regimes for narrative dynamics: one with stable narrative convergence around a long-run steady state (SSC-A) and one with a strong role for initial conditions and hysteresis (SSC-B).

We now study how the economy responds to deterministic and stochastic fundamental and narrative shocks. For this analysis, we restrict attention to the SSC class, noting that this is an assumption solely on primitives.[24]

Hump-Shaped and Discontinuous Impulse Responses. We consider the responses of aggregate output and optimism in the economy to a one-time positive shock to fundamentals from a steady state corresponding to $\theta = 1$:

$$\theta_t = \begin{cases} 1, & t = 0, \\ \hat{\theta}, & t = 1, \\ 1, & t \geq 2. \end{cases} \tag{91}$$

where $\hat{\theta} > 1$. We would like to understand when the impulse response to a one-time shock is *hump-shaped*, meaning that there exists a $\hat{t} \geq 2$ such that Y_t is increasing for $t \leq \hat{t}$ and decreasing thereafter. Moreover, we would like to understand how big a shock needs to be to send the economy from one steady state to another, as manifested as a discontinuity in the IRFs in the shock size $\hat{\theta}$. For simplicity, we focus on the case with i.i.d. productivity shocks in which $\rho = 0$.

In the SSC-A case, IRFs are continuous in the shock but can nevertheless display hump-shaped dynamics as a result of the endogenous evolution of optimism.

Proposition 1 (SSC-A Impulse Response Functions)**.** *In the SSC-A case, suppose that* $Q_0 = \hat{Q}_1 \in$

[24] This is without a substantive loss of generality as we can always represent any non-SSC T_θ as the concatenation of a set of restricted functions that are SSC on their respective domains. Concretely, whenever T_θ is not SSC, we can represent its domain $[0, 1]$ as a collection of intervals $\{I_j\}_{j \in \mathcal{J}}$ such that $\cup_{j \in \mathcal{J}} I_j = [0, 1]$ and the restricted functions $T_{\theta,j} : I_j \to [0, 1]$ defined by the property that $T_{\theta,j}(Q) = T_\theta(Q)$ for all $Q \in I_j$ are either SSC-A or SSC-B for all $j \in \mathcal{J}$. Thus, applying our results to these restricted functions, we have a complete description of the global dynamics.

$(0,1)$. *The impulse response of the economy is given by:*

$$\log Y_t = \begin{cases} a_0 + f(\hat{Q}_1), & t = 0, \\ a_0 + a_1 \log \hat{\theta} + f(\hat{Q}_1), & t = 1, \\ a_0 + f(Q_t), & t \geq 2 \end{cases} \qquad Q_t = \begin{cases} \hat{Q}_1, & t \leq 1, \\ Q_2, & t = 2, \\ T_1(Q_{t-1}), & t \geq 3. \end{cases} \tag{92}$$

Moreover, $Q_2 = \hat{Q}_1 P_O(a_0 + a_1 \log \hat{\theta} + f(\hat{Q}_1), \hat{Q}_1, 0) + (1 - \hat{Q}_1) P_P(a_0 + a_1 \log \hat{\theta} + f(\hat{Q}_1), \hat{Q}_1, 0) > \hat{Q}_1$, Q_t *is monotonically declining for all* $t \geq 2$, *and* $Q_t \to \hat{Q}_1$. *The IRF is hump-shaped if and only if* $\hat{\theta} < \exp\{(f(Q_2) - f(\hat{Q}_1))/a_1\}$.

Proof. By Proposition 1 and substituting the form of the shock process from Equation 91, we obtain the formula for the output IRF. For the fraction of optimists, we see that:

$$\begin{aligned} Q_2 &= \hat{Q}_1 P_O(a_0 + a_1 \log \hat{\theta} + f(\hat{Q}_1), \hat{Q}_1, 0) + (1 - \hat{Q}_1) P_P(a_0 + a_1 \log \hat{\theta} + f(\hat{Q}_1), \hat{Q}_1, 0) \\ &> \hat{Q}_1 P_O(a_0 + f(\hat{Q}_1), \hat{Q}_1, 0) + (1 - \hat{Q}_1) P_P(a_0 + f(\hat{Q}_1), \hat{Q}_1, 0) = \hat{Q}_1 \end{aligned} \tag{93}$$

and $Q_t = T_1(\log Y_{t-1}, Q_{t-1})$ for $t \geq 3$ by iterating forward. That Q_t monotonically declines to $\hat{Q}_1$ follows from Lemma 3 as we are in the SSC-A case. The hump shape is obtained if $\log Y_1 \leq \log Y_2$. This corresponds to

$$\log Y_1 = a_0 + a_1 \log \hat{\theta} + f(\hat{Q}_1) \leq a_0 + f(Q_2) = \log Y_2 \tag{94}$$

which rearranges to the desired expression. □

All persistence in the IRF of output derives from persistence in the IRF of optimism. There is a hump in the IRF for output if the boom induced by optimism exceeds the direct effect of the shock. This contrasts with the SSC-B case, wherein impulse responses can be discontinuous in the shock size. The following proposition characterizes the IRFs from the pessimistic steady state; those from the optimistic steady state are analogous.

Proposition 2 (SSC-B Impulse Response Functions)**.** *In the SSC-B case, suppose that* $\theta_O < 1 < \theta_P$ *and that* $Q_0 = 0$. *The impulse response of the economy is given by:*

$$\log Y_t = \begin{cases} a_0, & t = 0, \\ a_0 + a_1 \log \hat{\theta}, & t = 1, \\ a_0 + f(Q_t), & t \geq 2 \end{cases} \qquad Q_t = \begin{cases} 0, & t \leq 1, \\ P_P(a_0 + a_1 \log \hat{\theta}, 0, 0), & t = 2, \\ T_1(Q_{t-1}), & t \geq 3. \end{cases} \tag{95}$$

These impulse responses fall into the following four exhaustive cases:

1. $\hat{\theta} \leq \theta_P$, *No Lift-Off:* $Q_t = 0$ *for all* $t \in \mathbb{N}$.
2. $\hat{\theta} \in (\theta_P, \theta^*)$, *Transitory Impact:* Q_t *is monotonically declining for all* $t \geq 2$ *and* $Q_t \to 0$.
3. $\hat{\theta} = \theta^*$, *Permanent (Knife-edge) Impact:* $Q_t = \hat{Q}_1$ *for all* $t \geq 1$

4. $\hat{\theta} > \theta^*$*, Permanent Impact:* Q_t *is monotonically increasing for all* $t \geq 2$ *and* $Q_t \to 1$

where the critical shock threshold is $\theta^* = \exp\{(P_P^{-1}(\hat{Q}_1; 0) - a_0)/a_1\} > \theta_P$*. In the transitory case, the output IRF is hump-shaped if and only if* $\hat{\theta} < \exp\{f(P_P(a_0 + a_1 \log \hat{\theta}, 0, 0))/a_1\}$*.*

Proof. We first derive the IRF functions. The formula for the output IRF follows Proposition 1. For the IRF for the fraction of optimists, we simply observe that $Q_0 = Q_1 = 0$ and $Q_2 = P_P(a_0 + a_1 \log \hat{\theta}, 0, 0)$, and that $Q_t = T_1(Q_{t-1})$ for $t \geq 3$ by iterating forward.

We now describe the properties of the IRFs as a function of the size of the initial shock $\hat{\theta}$. First, observe that $Q_2 = P_P(a_0 + a_1 \log \hat{\theta}, 0, 0)$. Thus, we have that $Q_2 = 0$ if and only if $P_P^{-1}(0; 0) \geq a_0 + a_1 \log \hat{\theta}$ which holds if and only if $\hat{\theta} \leq \theta_P$. For any $\hat{\theta} > \theta_P$ it follows that $Q_2 > 0$. As we lie in the SSC class, by Lemma 3, we have that the steady states $Q = 0$, $Q = 1$, and $Q = \hat{Q}_1$ have basins of attraction given by $[0, \hat{Q}_1)$, $(\hat{Q}_1, 1]$, $\{\hat{Q}_1\}$. Thus, if $Q_2 < \hat{Q}_1$, we have monotone convergence of Q_t to 0. If $Q_2 = \hat{Q}_1$, then $Q_t = \hat{Q}_t$ for all $t \in \mathbb{N}$. If $Q_2 > \hat{Q}_1$, we have monotone convergence of Q_t to 1. Moreover, the threshold for $\hat{\theta}$ such that $Q_2 = \hat{Q}^*$ is $\exp\left\{\frac{P_P^{-1}(\hat{Q}_1;0)-a_0}{a_1}\right\}$.

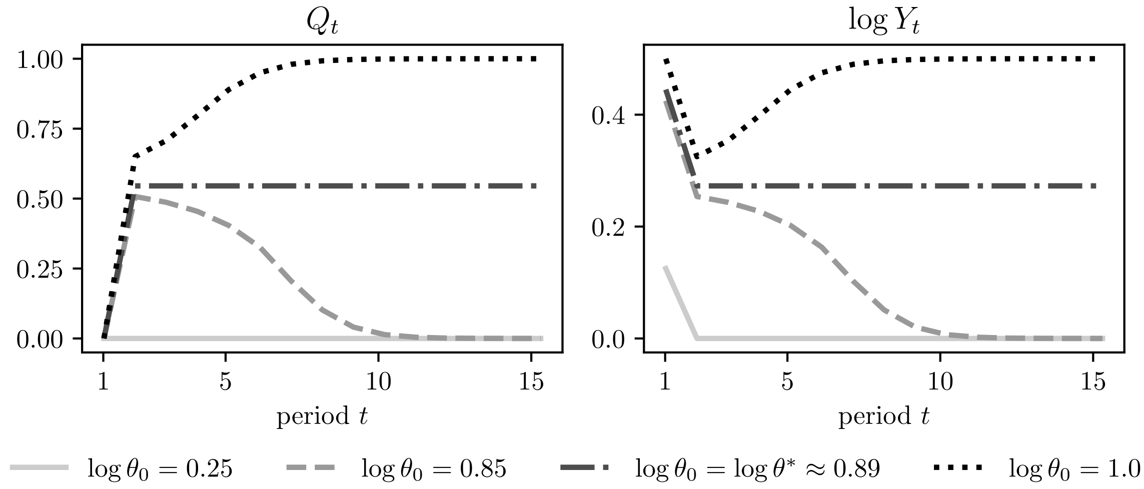
Finally, to find the condition such that the IRF is hump-shaped, we observe that this occurs if and only if $f(Q_2) > a_1 \log \hat{\theta}$ as Q_t is monotonically decreasing for $t \geq 2$, which is precisely the claimed condition. □

To understand this result, we first inspect the IRFs. At time $t = 0$, the economy lies at a steady state of extreme pessimism with $\log \theta_0 = 0$ and so $\log Y_0 = a_0$. At time $t = 1$, the one-time productivity shock takes place and output jumps up to $\log Y_1 = a_0 + a_1 \log \hat{\theta}$ as everyone remains pessimistic. At time $t = 2$, agents observe that output rose in the previous period. As a result, a fraction $P_P(\log Y_1, 0)$ of the population becomes optimistic. For output, the one-time productivity shock has dissipated, so output is now given by its unshocked baseline a_0 plus the equilibrium output effect of optimism $f(Q_2)$. From this point, the IRF evolves deterministically and its long-run behavior depends solely on whether the fraction that became initially optimistic exceeds the criticality threshold $\hat{Q}_1$ that delineates the basins of attraction of the steady states of extreme optimism and extreme pessimism.

As a result, productivity shocks have the potential for the following four qualitatively distinct effects, described in Proposition 2 and illustrated numerically in Figure 9. First, if a shock is small and no agent is moved toward optimism, the shock has a one-period impact on aggregate output. Second, if some agents are moved to optimism by the transitory boost to output but this fraction lies below the criticality threshold, then output steadily declines back to its pessimistic steady-state level as optimism was not sufficiently great to be self-fulfilling. Third, in the knife-edge case, optimism moves to a new (unstable) steady state and permanently increases output. Fourth, when enough agents are moved to optimism by the initial boost to output, then the economy converges to the fully optimistic steady state and optimism is completely self-fulfilling.

The impulse responses to narrative shocks are identical to those described above. One can take the formulas in Propositions 1 and 2 from $t \geq 2$ and set Q_2 equal to the value of Q that obtains

Figure 9: Illustration of IRFs in an SSC-B Case



Notes: The plots show the deterministic impulse responses of Q_t and $\log Y_t$ in a model calibration with LAC updating. The four initial conditions correspond to the four cases of Proposition 2.

following the narrative shock ε. It follows that the qualitative nature of the impulse response to a narrative shock is identical to that of a fundamental shock.

Stochastic Boom-Bust Cycles. Having characterized the deterministic impulse propagation mechanisms at work in the economy, we now turn to understand the stochastic properties of the path of the economy as it is hit by fundamental and narrative shocks. For simplicity, we once again restrict to the case of i.i.d. fundamentals, in which $\rho = 0$.

To this end, we analytically study the period of boom and bust cycles: the expected time that it takes for the economy to move from a state of extreme pessimism to a state of extreme optimism, and *vice versa*. Formally, define these expected stopping times as:

$$T_{PO} = \mathbb{E}\left[\min\{\tau \in \mathbb{N} : Q_\tau = 1\} | Q_0 = 0\right], \; T_{OP} = \mathbb{E}\left[\min\{\tau \in \mathbb{N} : Q_\tau = 0\} | Q_0 = 1\right] \tag{96}$$

where the expectation is taken under the true data generating process for the aggregate component of productivity H, which may or may not coincide with one of the narratives under consideration, and that of the narrative shocks G.

The following result provides sharp upper bounds, in the sense that they are attained for some (H, G), on these stopping times as a function of deep structural parameters:

Proposition 3 (Period of Boom-Bust Cycles)**.** *The expected regime-switching times satisfy the* *following inequalities:*

$$\begin{aligned} T_{PO} &\leq \frac{1}{1-\mathbb{E}_G\left[H\left(\exp\left\{\frac{P_P^{\dagger}(1;0,\varepsilon)-a_0}{a_1}\right\}\right)\right]} \\ T_{OP} &\leq \frac{1}{\mathbb{E}_G\left[H\left(\exp\left\{\frac{P_O^{\dagger}(0;1,\varepsilon)-a_0-f(1)}{a_1}\right\}\right)\right]} \end{aligned} \tag{97}$$

where $P_P^{\dagger}(x;Q,\varepsilon) = \inf\{Y : P_P(Y,Q,\varepsilon) = x\}$ *and* $P_O^{\dagger}(x;Q,\varepsilon) = \sup\{Y : P_O(Y,Q,\varepsilon) = x\}$. *Moreover, when* $P_O^{\dagger}(0;1,0) - P_P^{\dagger}(1;0,0) \leq f(1)$*, these bounds are tight in the sense that they are attained for some processes for fundamentals and narrative shocks* (H,G)*.*

Proof. We prove this result by first constructing fictitious processes for optimism that bound above and below the true optimism process for all realizations of $\{\theta_t\}_{t\in\mathbb{N}}$ before the stopping time. We can then use this to bound the stopping times' distributions in the sense of first-order stochastic dominance and use this fact to bound the expectations.

First, consider the case where we seek to bound $\tau_{PO} = \min\{t \in \mathbb{N} : Q_t = 1, Q_0 = 0\}$. In the model, we have that $Q_{t+1} = T(Q_t,\nu_t)$. Fix a path of fundamentals and narrative shocks $\{\nu_t\}_{t\in\mathbb{N}} = \{\theta_t,\varepsilon_t\}_{t\in\mathbb{N}}$ and define the fictitious $\overline{Q}$ process as:

$$\overline{Q}_{t+1} = \mathbb{I}[T(\overline{Q}_t,\nu_t) = 1] \tag{98}$$

with $\overline{Q}_0 = 0$. We prove by induction that $\overline{Q}_t \leq Q_t$ for all $t \in \mathbb{N}$. Consider first the base case that $t = 1$:

$$\overline{Q}_1 = \mathbb{I}[T(0,\nu_0) = 1] \leq T(0,\nu_0) = Q_1 \tag{99}$$

Toward the inductive hypothesis, suppose that $\overline{Q}_{t-1} \leq Q_{t-1}$. Then we have that:

$$\overline{Q}_t = \mathbb{I}[T(\overline{Q}_{t-1},\nu_{t-1}) = 1] \leq \mathbb{I}[T(Q_{t-1},\nu_{t-1}) = 1] \leq T(Q_{t-1},\nu_{t-1}) = Q_t \tag{100}$$

where the first inequality follows by the property that $T(\cdot,\nu)$ is a monotone increasing function.

As $\overline{Q}_t \leq Q_t$ for all $t \in \mathbb{N}$, we have that:

$$\overline{\tau}_{PO} = \min\{t \in \mathbb{N} : \overline{Q}_t = 1, \overline{Q}_0 = 0\} \geq \min\{t \in \mathbb{N} : Q_t = 1, Q_0 = 0\} = \tau_{PO} \tag{101}$$

Else, we would have at $\overline{\tau}_{PO}$ that $Q_{\overline{\tau}_{PO}} < \overline{Q}_{\overline{\tau}_{PO}}$, which is a contradiction.

We now have a pathwise upper bound on τ_{PO}. We now characterize the distribution of the bound. Observe that the possible sample paths for $\{\overline{Q}_t\}_{t\in\mathbb{N}}$ until stopping are given by the set:

$$\mathcal{G}_{PO} = \{(0^{(n-1)},1)\} : n \geq 1\} \tag{102}$$

Moreover, conditional on $\overline{Q}_{t-1} = 0$, the distribution of $\overline{Q}_t$ is independent of $\{\nu_s\}_{s\leq t-1}$. Thus, the fictitious stopping time $\overline{\tau}_{PO}$ has a geometric distribution with parameter given by $\mathbb{P}[Q_{t+1} = 1|Q_t =$

0]. This parameter is given by:

$$\begin{aligned}\mathbb{P}[Q_{t+1}=1|Q_t=0] &= \mathbb{P}\left[P_P(a_0+a_1\log\theta_t,0,\varepsilon_t)=1\right] \\ &= \mathbb{P}\left[\theta_t \geq \exp\left\{\frac{P_P^{\dagger}(1;0,\varepsilon_t)-a_0}{a_1}\right\}\right] \\ &= 1-\mathbb{E}_G\left[H\left(\exp\left\{\frac{P_P^{\dagger}(1;0,\varepsilon)-a_0}{a_1}\right\}\right)\right]\end{aligned} \tag{103}$$

Thus, we have established a stronger result and provided a distributional bound on the stopping time:

$$\tau_{PO} \prec_{FOSD} \overline{\tau}_{PO} \sim \text{Geo}\left(1-\mathbb{E}_G\left[H\left(\exp\left\{\frac{P_P^{\dagger}(1;0,\varepsilon)-a_0}{a_1}\right\}\right)\right]\right) \tag{104}$$

An immediate corollary is that:

$$T_{PO}=\mathbb{E}[\tau_{PO}] \leq \mathbb{E}[\overline{\tau}_{PO}] = \frac{1}{1-\mathbb{E}_G\left[H\left(\exp\left\{\frac{P_P^{\dagger}(1;0,\varepsilon)-a_0}{a_1}\right\}\right)\right]} \tag{105}$$

We can apply appropriately adapted arguments for the other case, where we now define:

$$\underline{Q}_{t+1}=\mathbb{I}[T(\underline{Q}_t,\nu_t)\neq 0] \tag{106}$$

with $\underline{Q}_0=1$. In this case, by an analogous induction have that $\underline{Q}_t \geq Q_t$ for all $t\in\mathbb{N}$ for all sequences $\{\nu_t\}_{t\in\mathbb{N}}$. And so, we have that if $\underline{Q}_t$ has reached 0 then so too has Q_t. The possible sample paths in this case are:

$$\mathcal{G}_{OP}=\{(1^{(n-1)},0)\}:n\geq 1\} \tag{107}$$

So again the stopping time has a geometric distribution, this time with parameter:

$$\begin{aligned}\mathbb{P}[Q_{t+1}=0|Q_t=1] &= \mathbb{P}\left[\theta_t \leq \exp\left\{\frac{P_O^{\dagger}(0;1,\varepsilon_t)-a_0-f(1)}{a_1}\right\}\right] \\ &= \mathbb{E}_G\left[H\left(\exp\left\{\frac{P_O^{\dagger}(0;1,\varepsilon)-a_0-f(1)}{a_1}\right\}\right)\right]\end{aligned} \tag{108}$$

And so we have:

$$T_{OP} \leq \frac{1}{\mathbb{E}_G\left[H\left(\exp\left\{\frac{P_O^{\dagger}(0;1,\varepsilon)-a_0-f(1)}{a_1}\right\}\right)\right]} \tag{109}$$

It remains to show that these bounds are tight. To do so, we derive a law H such that $Q_t=\overline{Q}_t=\underline{Q}_t$ for all $t\in\mathbb{N}$. Concretely, define the set:

$$\Theta^*=\left(-\infty,\exp\left\{\frac{P_O^{\dagger}(0;1,0)-a_0-f(1)}{a_1}\right\}\right]\cup\left[\exp\left\{\frac{P_P^{\dagger}(1;0,0)-a_0}{a_1}\right\},\infty\right) \tag{110}$$

and suppose that θ takes values only in this set, where the two sub-intervals are disjoint as $P_O^\dagger(0;1,0) - P_P^\dagger(1;0,0) \leq f(1)$. Moreover, suppose that narrative shocks equal zero with probability one. In this case, starting from $Q_t = 1$, the only possible values for Q_{t+1} are zero and one. Moreover, starting from $Q_t = 0$, the only possible values for Q_{t+1} are zero and one. Thus, in either case, $Q_t = \overline{Q}_t = \underline{Q}_t$ pathwise and $T_{OP} = T_{OP}^*$ and $T_{PO} = T_{PO}^*$. It is worth noting that such a distribution can be obtained by considering a limit of normal-mixture distributions. Concretely, suppose that H is derived as a mixture of two normal distributions $N(\mu_A, \sigma^2)$ and $N(\mu_B, \sigma^2)$ for $\mu_A < \exp\left\{\frac{P_O^\dagger(0;1,0)-a_0-f(1)}{a_1}\right\}$ and $\mu_B > \exp\left\{\frac{P_P^\dagger(1;0,0)-a_0}{a_1}\right\}$. Taking the limit as $\sigma \to 0$, the support of H converges to being contained within Θ^*. □

This result establishes that the economy regularly oscillates between times of booms and busts. We establish this result by postulating fictitious processes for optimism and showing that they bound, path-by-path, the true optimism process. This enables us to construct stopping times that dominate the true stopping times in the sense of first-order stochastic dominance and have expectations that can be computed analytically, thus providing the claimed bounds. We establish that these bounds are tight by constructing a family of distributions (H, G) such that the fictitious processes coincide always with the true processes.[25]

We can provide insights into the determinants of the period of boom-bust cycles from these analytical bounds. Concretely, consider the bound on the expected time to reach a bust from a boom. This bound is small when the quantity $\mathbb{E}_G\left[H\left(\exp\left\{\frac{P_P^\dagger(1;0,\varepsilon)-a_0}{a_1}\right\}\right)\right]$ is large, which happens when there is a fat left tail of fundamentals, when it is relatively easier for optimists to switch to pessimism as measured by $P_O^\dagger(0;1,\varepsilon^P)$, and when co-ordination motives are weak as measured by $f(1)$.

B.2 Welfare Implications

In this appendix, we derive the normative implications of narratives for the economy.

Theory. The following result characterizes welfare along any path for the fraction of optimists in the population and the conditions under which a steady state of extreme optimism is preferred to one of extreme pessimism:

Proposition 4 (Narratives and Welfare)**.** *For any path of aggregate optimism* $\mathbf{Q} = \{Q_t\}_{t=0}^{\infty}$,

[25] We moreover show that elements of this family can be attained by taking the limit of normal mixtures with sufficiently dispersed means. Thus, for sufficiently dispersed μ_O and μ_P, we can therefore construct (H, G) for which the bound is attained by taking weighted averages of the optimistic and pessimistic narratives, making the uncertainty under each sufficiently small, and eliminating narrative shocks.

aggregate welfare is given by

$$\begin{aligned}\mathcal{U}(\mathbf{Q}) = U_C^* \sum_{t=0}^{\infty} \beta^t \exp\{(1-\gamma) f(Q_t)\} \\ - U_L^* \sum_{t=0}^{\infty} \beta^t \left(Q_t \exp\{(1+\psi)d_2\} + (1-Q_t)\right) \exp\{(1+\psi) d_3 f(Q_t)\}\end{aligned} \tag{111}$$

for some positive constants U_C^*, U_L^*, d_2 and d_3 that are provided in the proof of the result. Thus, there is higher welfare in an optimistic steady state than in a pessimistic steady state if and only if

$$\frac{U_C^*}{U_L^*} \times \frac{\exp\{(1-\gamma)f(1)\} - 1}{\exp\{(1+\psi)(d_2 + d_3 f(1))\} - 1} > 1 \tag{112}$$

Moreover, when the pessimistic narrative is correctly specified, extreme optimism is welfare-equivalent to an ad valorem price subsidy for intermediate goods producers of:

$$\tau^* = \exp\left\{(1-\omega)\left(\frac{1+\psi-\alpha}{\alpha} + \frac{1}{\epsilon}\right) f(1)\right\} - 1 \tag{113}$$

Proof. We have that welfare for any path of optimism $\mathbf{Q} = \{Q_t\}_{t\in\mathbb{N}}$ is given by:

$$\mathcal{U}(\mathbf{Q}) = \sum_{t=0}^{\infty} \beta^t \left(\mathbb{E}_H \left[\frac{C_t(Q_t, \theta_t)^{1-\gamma}}{1-\gamma} \right] - \mathbb{E}_H \left[\int_{[0,1]} \frac{L_{it}(\gamma_i, s_{it}, Q_t)^{1+\psi}}{1+\psi} \mathrm{d}i \right] \right) \tag{114}$$

By market clearing, we have that $C_t = Y_t$ for all t. Thus, using the formula for equilibrium aggregate output from Proposition 1 and our assumption that $\log \theta_t$ is Gaussian under H, we have that the consumption component of welfare is given by:

$$\begin{aligned}\mathbb{E}_H \left[\frac{C_t^{1-\gamma}(Q_t, \theta_t)}{1-\gamma} \right] &= \mathbb{E}_H \left[\frac{1}{1-\gamma} \exp\{(1-\gamma)\log Y(Q_t, \theta)\} \right] \\ &= \mathbb{E}_H \left[\frac{1}{1-\gamma} \exp\{(1-\gamma)(a_0 + a_1 \log\theta + f(Q_t))\} \right] \\ &= \frac{1}{1-\gamma} \exp\left\{(1-\gamma)(a_0 + a_1\mu_H + f(Q_t)) + \frac{1}{2} a_1^2 \sigma_H^2 \right\} \\ &= \frac{1}{1-\gamma} \exp\left\{(1-\gamma)(a_0 + a_1\mu_H) + \frac{1}{2} a_1^2 \sigma_H^2 \right\} \exp\{(1-\gamma) f(Q_t)\} \\ &= U_C^* \exp\{(1-\gamma) f(Q_t)\}\end{aligned} \tag{115}$$

From Proposition 1, we moreover have that labor employed by each firm can be written as:

$$L_{it} = d_1 \log\theta_t + d_2 \lambda_{it} + d_3 f(Q_t) + v_{it} \tag{116}$$

where v_{it} is Gaussian and i.i.d. over i. Hence given θ and Q_t:

$$\begin{aligned} &\int_{[0,1]} \frac{L_{it}(\gamma_i, s_{it}, Q_t)^{1+\psi}}{1+\psi} \mathrm{d}i \\ &= \frac{1}{1+\psi} \left(Q_t \exp\{(1+\psi)d_2\} + (1-Q_t)\right) \\ &\quad \times \exp\left\{(1+\psi)(d_1 \log\theta + \mu_v + d_3 f(Q_t)) + \frac{1}{2}(1+\psi)^2\sigma_v^2\right\} \end{aligned} \tag{117}$$

Hence, the expectation over θ is given by:

$$\begin{aligned} &\mathbb{E}_H\left[\int_{[0,1]} \frac{L_{it}(\gamma_i, s_{it}, Q_t)^{1+\psi}}{1+\psi} \mathrm{d}i\right] \\ &= \frac{1}{1+\psi} \left(Q_t \exp\{(1+\psi)d_2\} + (1-Q_t)\right) \\ &\quad \times \exp\left\{(1+\psi)d_3 f(Q_t)\right\} \exp\left\{(1+\psi)(d_1\mu_H + \mu_v) + \frac{1}{2}(1+\psi)^2(\sigma_v^2 + d_1^2\sigma_H^2)\right\} \\ &= U_L^* \left(Q_t \exp\{(1+\psi)d_2\} + (1-Q_t)\right) \exp\left\{(1+\psi)d_3 f(Q_t)\right\} \end{aligned} \tag{118}$$

And so total welfare under narrative path $\mathbf{Q}$ is given by:

$$\begin{aligned} \mathcal{U}(\mathbf{Q}) = U_C^* &\sum_{t=0}^{\infty} \beta^t \exp\left\{(1-\gamma) f(Q_t)\right\} \\ &- U_L^* \sum_{t=0}^{\infty} \beta^t \left(Q_t \exp\{(1+\psi)d_2\} + (1-Q_t)\right) \exp\left\{(1+\psi)d_3 f(Q_t)\right\} \end{aligned} \tag{119}$$

The final inequality follows by noting that $f(0) = 0$ and rearranging this expression.

Now consider the benchmark model but where, without loss of generality, all agents are pessimistic $Q_t = 0$ and a planner levies an *ad valorem* subsidy. That is, when the consumer price is $p_{it}^C = Y_t^{\frac{1}{\epsilon}} x_{it}^{-\frac{1}{\epsilon}}$, the price received by the producer is $p_{it}^P = (1+\tau)p_{it}^C$. Under this subsidy, each producer's first-order condition is:

$$\begin{aligned} \log x_{it} = &\frac{1}{\frac{1+\psi-\alpha}{\alpha} + \frac{1}{\epsilon}} \left(\log\left(\frac{1-\frac{1}{\epsilon}}{\frac{1+\psi}{\alpha}}\right) - \log \mathbb{E}_{it}\left[\exp\left\{-\frac{1+\psi}{\alpha}\log\theta_{it}\right\}\right] \right. \\ &\left. + \log \mathbb{E}_{it}\left[\exp\left\{\left(\frac{1}{\epsilon} - \gamma\right)\log Y_t\right\}\right] \right) + \Xi(\tau) \end{aligned} \tag{120}$$

where $\Xi(\tau) = \frac{1}{\frac{1+\psi-\alpha}{\alpha} + \frac{1}{\epsilon}} \log(1+\tau)$. By identical arguments to Proposition 1, we have that there is a unique quasi-loglinear equilibrium, where:

$$\log Y(\theta, \tau) = a_0 + a_1 \log\theta + \frac{1}{1-\omega}\Xi(\tau) \tag{121}$$

and a_0 and a_1 are as in Proposition 1. Hence, in this equilibrium we have that:

$$\log x_{it}(\tau) = \log x_{it}(0) + \frac{1}{1-\omega}\Xi(\tau) \tag{122}$$

Which implies that:

$$\log L_{it}(\tau) = \log L_{it}(0) + \frac{1}{\alpha}\frac{1}{1-\omega}\Xi(\tau) \tag{123}$$

And so, welfare under the subsidy τ is given by:

$$\begin{aligned}\mathcal{U}(\tau) = U_C^* \sum_{t=0}^{\infty} \beta^t \exp\left\{(1-\gamma)\frac{1}{1-\omega}\Xi(\tau)\right\} \\ - U_L^* \sum_{t=0}^{\infty} \beta^t \exp\left\{(1+\psi)d_3\frac{1}{1-\omega}\Xi(\tau)\right\}\end{aligned} \tag{124}$$

as $d_3 = \frac{1}{\alpha}$. Hence:

$$\mathcal{U}(1) = \mathcal{U}(\tau^*) \tag{125}$$

where τ^* is such that $\frac{1}{1-\omega}\Xi(\tau^*) = f(1)$. Hence:

$$\tau^* = \exp\left\{(1-\omega)\left(\frac{1+\psi-\alpha}{\alpha} + \frac{1}{\epsilon}\right) f(1)\right\} - 1 \tag{126}$$

Completing the proof. □

This result sheds light on the potential for non-fundamental optimism to increase aggregate welfare. In the presence of the product market monopoly and labor market monopsony distortions, intermediate goods firms under-hire labor and under-produce goods. As a result, if irrational optimism causes them to produce more output, but not so much that the household over-supplies labor, then it has the potential to be welfare improving. The final part of the proposition then reduces this question to assessing if the implied optimism-equivalent subsidy is less than the welfare-optimal subsidy. Thus, optimism in the economy can serve the role of undoing monopoly frictions and thereby has the potential to be welfare-improving, even when misspecified.

Quantification. Proposition 4 can be directly applied in our numerical calibration from Section 6 to calculate the welfare effects of narrative optimism without approximation. We calculate the average payoff of the representative household under three scenarios. The first corresponds to the calibrated narrative dynamics in simulation, under the assumption that the pessimistic model is correctly specified.[26] The second is a counterfactual scenario with permanent extreme optimism, or $Q_t \equiv 1$ for all t. The third is a counterfactual scenario with permanent extreme pessimism, or $Q_t \equiv 0$ for all t, and an *ad valorem* subsidy of τ to all producers. We use the third scenario to translate the first and second into payoff-equivalent subsidies. We find that both contagious and

[26] Relative to the positive analysis, the normative analysis requires two additional model parameters. We set the idiosyncratic component of productivity to have unit mean and zero variance.

extreme optimism are welfare-increasing relative to extreme pessimism in autarky (*i.e,* $\tau = 0$). In payoff units, they correspond respectively to equivalent subsidies of 1.33% and 2.59%. Our finding of an overall positive welfare effect for contagious optimism suggests that, in our macroeconomic calibration, losses from inducing misallocation are more than compensated by level increases in output.

B.3 Comparison to the Bayesian Benchmark

Consider an alternative model in which each agent i initially believes the optimistic model is correct with probability $\lambda_{i0} \in (0,1)$, and subsequently updates this probability by observing aggregate output and aggregate optimism and applying Bayes' rule under rational expectations. For simplicity, we focus on the case of i.i.d. shocks ($\rho = 0$). Formally, this corresponds to the following law of motion for Q_t:

$$Q_{t+1} = \int_{[0,1]} \mathbb{P}_i[\mu = \mu_O | \{\log Y_j, Q_j\}_{j=0}^t] \, \mathrm{d}i \tag{127}$$

where $\mathbb{P}_i[\mu = \mu_0 | \emptyset] = \lambda_{i0}$ for some $\lambda_{i0} \in (0,1)$ for all $i \in [0,1]$, and conditional probabilities are computed under rational expectations with knowledge of $\{\lambda_{i0}\}_{i \in [0,1]}$. We define the log-odds ratio of an agent's belief as $\Omega_{it} = \log \frac{\lambda_{it}}{1-\lambda_{it}}$. The following Proposition characterizes the dynamics of agents' subjective models under the Bayesian benchmark:

Proposition 5 (Dynamics under the Bayesian Benchmark)**.** *Each agent's log-odds ratio follows a random walk with drift, or* $\Omega_{i,t+1} = \Omega_{it} + a + \xi_t$*, where* $a = \mathbb{E}_H\left[\frac{(\log\theta_t - \mu_P)^2 - (\log\theta_t - \mu_O)^2}{\sigma^2}\right]$ *and* ξ_t *is an i.i.d., mean-zero random variable. The economy converges almost surely to either extreme optimism* ($a > 0$) *or extreme pessimism* ($a < 0$)*. The dynamics of the economy are asymptotically described by:*

$$\log Y_t = \begin{cases} a_0 + a_1 \log\theta_t & \text{if } a < 0, \\ a_0 + a_1 \log\theta_t + f(1) & \text{if } a > 0. \end{cases} \tag{128}$$

Thus, the economy does not feature steady state multiplicity, hump-shaped or discontinuous IRFs, or the possibility for boom-bust cycles.

Proof. The equilibrium Characterization of Proposition 1 still holds. Moreover, Q_0 is known to all agents. Thus, they can identify θ_0 as:

$$\theta_0 = \frac{\log Y_0 - a_0 - f(Q_0)}{a_1} \tag{129}$$

Thus, we have that $\lambda_{i1} = \mathbb{P}[\mu = \mu_O | \theta_0, \lambda_{i0}]$. Moreover, all agents know that $Q_1 = \int_{[0,1]} \lambda_{i1} \, \mathrm{d}i$. Thus, agents can sequentially identify θ_t by observing only $\{Y_j\}_{j \le t}$ (and not $\{Q_j\}_{j \le t}$) by computing:

$$\theta_t = \frac{\log Y_t - a_0 - f(Q_t)}{a_1} \tag{130}$$

Thus, we can describe the evolution of agents' beliefs by computing:

$$\lambda_{i,t+1} = \mathbb{P}_i[\mu = \mu_O | \{\theta_j\}_{j=1}^t] = \lambda_{i,t+1} = \mathbb{P}_i[\mu = \mu_O | \{Y_j\}_{j=1}^t] \tag{131}$$

By application of Bayes rule, we obtain:

$$\lambda_{i,t+1} = \mathbb{P}[\mu = \mu_O | \theta_t, \lambda_{i,t}] = \frac{f_O(\theta_t)\lambda_{i,t}}{f_O(\theta_t)\lambda_{i,t} + f_P(\theta_t)(1-\lambda_{i,t})} \tag{132}$$

which implies that:

$$\begin{aligned} \frac{\lambda_{i,t+1}}{1-\lambda_{i,t+1}} &= \frac{f(\log\theta_t | \mu = \mu_O)}{f(\log\theta_t | \mu = \mu_P)} \frac{\lambda_{i,t}}{1-\lambda_{i,t}} \\ &= \exp\left\{\frac{(\log\theta_t - \mu_P)^2 - (\log\theta_t - \mu_O)^2}{\sigma^2}\right\} \frac{\lambda_{i,t}}{1-\lambda_{i,t}} \end{aligned} \tag{133}$$

Defining $\Omega_{it} = \log\frac{\lambda_{i,t}}{1-\lambda_{i,t}}$ and $a = \mathbb{E}_H\left[\frac{(\log\theta_t - \mu_P)^2 - (\log\theta_t - \mu_O)^2}{\sigma^2}\right]$ and $\xi_t = \frac{(\log\theta_t - \mu_P)^2 - (\log\theta_t - \mu_O)^2}{\sigma^2} - a$, we then have that:

$$\begin{aligned} \Omega_{i,t+1} &= \Omega_{i,t} + \frac{(\log\theta_t - \mu_P)^2 - (\log\theta_t - \mu_O)^2}{\sigma^2} \\ &= \Omega_{it} + a + \xi_t \end{aligned} \tag{134}$$

which is a random walk with drift, with the drift and stochastic increment claimed in the statement. Iterating, dividing by t, and applying the law of large numbers, we obtain:

$$\frac{\Omega_{i,t}}{t} = \frac{1}{t}\Omega_{i,0} + \frac{t-1}{t}a + \frac{1}{t}\sum_{i=1}^t \xi_i \to^{a.s.} a \tag{135}$$

Hence, almost surely, we have that $Q_t \to 1$ if $a > 0$ and $Q_t \to 0$ if $a < 0$.

Hence, the dynamics are asymptotically described by Proposition 1 with $Q_t = 1$ if $a > 0$ and $Q_t = 0$ if $a < 0$. The resulting properties for output follow immediately from combining this characterization for Q_t with the characterization in our main analysis of equilibrium output conditional on optimism and fundamentals (Proposition 1), which continues to hold in the model of this appendix. □

The optimist fraction Q converges to either 0 or 1 in the long run because one model is unambiguously better-fitting, and this will be revealed with infinite data. Moreover, the log-odds ratio converges linearly and so the odds ratio in favor of the better fitting model converges exponentially quickly. Thus the Bayesian benchmark model makes a prediction that is at odds with our finding of cyclical dynamics for aggregate optimism (Figure A1), and moreover, in the long run, rules out the features of macroeconomic dynamics that we derive in Section 3 as consequences of the endogenous evolution of narrative optimism.

B.4 Continuous Narratives

Our main analysis featured two levels of optimism. However, much of our analysis generalizes to a setting with a continuum of levels of optimism. For expositional simplicity, in this section, we abstract from optimism shocks and assume that productivity is i.i.d ($\rho = 0$). The model is as in Section 2, but now $\mu \in [\mu_P, \mu_O]$ and the distribution of narratives is given by $Q_t \in \Delta([\mu_P, \mu_O])$. The probabilistic transition between models is now given by a Markov kernel $P : [\mu_P, \mu_O] \times \mathcal{Y} \times \Delta^2([\mu_P, \mu_O]) \to \Delta([\mu_P, \mu_O])$ where $P_{\mu'}(\mu, \log Y, Q)$ is the density of agents who have model μ who switch to μ' when aggregate output is Y and the distribution of narratives is Q.

Characterizing Equilibrium Output. By modifying the guess-and-verify arguments that underlie Proposition 1, we can obtain an almost identical representation of equilibrium aggregate output:

Proposition 6 (Equilibrium Characterization with Continuous Narratives)**.** *There exists a quasi-loglinear equilibrium:*

$$\log Y(\log \theta_t, Q_t) = a_0 + a_1 \log \theta_t + f(Q_t) \tag{136}$$

Moreover, the density of narratives evolves according to the following difference equation:

$$\mathrm{d}Q_{t+1}(\mu') = \int_{\mu_P}^{\mu_O} P_{\mu'}(\mu, a_0 + a_1 \log \theta_t + f(Q_t), Q_t)\mathrm{d}Q_t(\mu) \tag{137}$$

Proof. By appropriately modifying the steps of the proof of Proposition 1, the result follows. Throughout, simply replace $\lambda_{it}\mu_O + (1-\lambda_{it})\mu_P$ with $\tilde{\mu}_{it} \sim Q_t$ and λ_{it} with $\tilde{\mu}_{it}$ as appropriate. The proof follows as written until the aggregation step. At this point, we instead obtain:

$$\log Y_t = \delta_t(\mu_P) + \frac{1}{2}\frac{\epsilon-1}{\epsilon}\hat{\sigma}^2 + \frac{\epsilon}{\epsilon-1}\log\left(\int_{\mu_P}^{\mu_O} \exp\left\{\frac{\epsilon-1}{\epsilon}(\delta_t(\tilde{\mu}) - \delta_t(\mu_P))\right\}\mathrm{d}Q_t(\tilde{\mu})\right) \tag{138}$$

where $\delta_t(\mu_P) = \delta_t(0)$ and $\delta_t(\tilde{\mu}) - \delta_t(\mu_P) = \alpha\delta^{OP}\frac{\tilde{\mu}-\mu_P}{\mu_O-\mu_P}$. Hence, we have that a_0 and a_1 are as in Proposition 1 and f is instead given by:

$$f(Q) = \frac{\frac{\epsilon}{\epsilon-1}}{1 - \frac{\frac{1}{\epsilon}-\gamma}{\frac{1+\psi-\alpha}{\alpha}+\frac{1}{\epsilon}}}\log\left(\int_{\mu_P}^{\mu_O} \exp\left\{\frac{\epsilon-1}{\epsilon}\alpha\delta^{OP}\frac{\tilde{\mu}-\mu_P}{\mu_O-\mu_P}\right\}\mathrm{d}Q(\tilde{\mu})\right) \tag{139}$$

Completing the proof. □

Importantly, observe that we still obtain a marginal representation in terms of the partial equilibrium effect of going from full pessimism to full optimism on hiring δ^{OP}, as we have empirically estimated.

Equilibrium Dynamics. We have seen that a continuum of models poses no difficulty for the static analysis. The challenge for the dynamic analysis is that the state variable, the evolution of which is fully characterized by Proposition 6, is now infinite-dimensional. This notwithstanding, by use of approximation arguments, we can reduce the dynamics to an essentially identical form to that which we have studied in the main text.

To this end, define the cumulant generating function (CGF) of the cross-sectional distribution of narratives as:

$$K_Q(\tau) = \log\left(\mathbb{E}_Q[\exp\{\tau\tilde{\mu}\}]\right) \tag{140}$$

We therefore have that $\log\left(\mathbb{E}_Q[\exp\{\tau(\tilde{\mu} - z)\}]\right) = K_Q(\tau) - \tau z$. It follows by Equation 139 that:

$$f(Q) = \frac{\frac{\epsilon}{\epsilon-1}}{1-\omega}\left[K_Q\left(\frac{\epsilon-1}{\epsilon}\alpha\delta^{OP}\frac{1}{\mu_O - \mu_P}\right) - \frac{\epsilon-1}{\epsilon}\alpha\delta^{OP}\frac{\mu_P}{\mu_O - \mu_P}\right] \tag{141}$$

By Maclaurin series expansion, we can express the CGF to first-order as:

$$K_Q(\tau) = \mu_Q\tau + O(\tau^2) \tag{142}$$

We therefore have that:

$$f(Q) = \frac{1}{1-\omega}\alpha\delta^{OP}\frac{\mu_Q - \mu_P}{\mu_O - \mu_P} + O\left(\left(\frac{\epsilon-1}{\epsilon}\alpha\delta^{OP}\frac{1}{\mu_O - \mu_P}\right)^2\right) \tag{143}$$

We now can express the static, general equilibrium effects in terms of mean of the narrative distribution. With some abuse of notation, we now write $f(\mu_Q) = f(Q)$. Of course, this CGF-based approach would allow one to consider higher-order effects through the variance, skewness, kurtosis, and higher cumulants as desired.

In the next steps, we provide conditions on updating that allow us to express the dynamics solely in terms of the mean of the narrative distribution. To do this, we assume that $P_{\mu'}(\mu, \log Y, Q) = P_{\mu'}(\mu'', \log Y, \mu_Q)$ for all $Q \in \Delta^2([\mu_P, \mu_O])$ and all $\mu, \mu', \mu'' \in [\mu_P, \mu_O]$. This is tantamount to assuming no stubbornness (all agents update the same regardless of the model they start with) and that contagiousness only matters via the mean. Under this assumption, we can write $P_{\mu'}(\log Y(\log\theta, \mu_Q), \mu_Q)$ and express the difference equation as:

$$\begin{aligned} \mathrm{d}Q_{t+1}(\mu') &= \int_{\mu_P}^{\mu_O} P_{\mu'}(a_0 + a_1\log\theta_t + f(\mu_{Q,t}), \mu_{Q,t})\mathrm{d}Q_t(\mu) \\ &= P_{\mu'}(a_0 + a_1\log\theta_t + f(\mu_{Q,t}), \mu_{Q,t}) \end{aligned} \tag{144}$$

It then suffices to take the mean of Q_{t+1} to express the system in terms of the one-dimensional state variable $\mu_{Q,t}$:

$$\mu_{Q,t+1} = T(\mu_{Q,t}, \theta_t) = \int_{\mu_P}^{\mu_O} \mu' P_{\mu'}(a_0 + a_1\log\theta_t + f(\mu_{Q,t}), \mu_{Q,t})\mathrm{d}\mu' \tag{145}$$

Which is simply a continuous state analog of the difference equation expressed in Corollary 3 expressed in terms of average beliefs.

Steady State Multiplicity. We now obtain the analogous characterization of extremal steady state multiplicity in this setting, *i.e.,* when it is possible that all agents being maximally pessimistic and all agents being maximally optimistic are simultaneously deterministic steady states. To this end, define the following two inverses:

$$\begin{aligned}\hat{P}^{-1}(x;\mu_Q) &= \sup\{Y : P(Y,Q) = \delta_x\} \\ \check{P}^{-1}(x;\mu_Q) &= \inf\{Y : P(Y,Q) = \delta_x\}\end{aligned} \tag{146}$$

where δ_x denotes the Dirac delta function on x. We define analogous objects to the previous θ_O and θ_P:

$$\theta_O = \exp\left\{\frac{\check{P}^{-1}(\mu_O;\mu_O) - a_0 - f(1)}{a_1}\right\},\ \theta_P = \exp\left\{\frac{\hat{P}^{-1}(\mu_P;\mu_P) - a_0 - f(1)}{a_1}\right\} \tag{147}$$

The following result establishes that these thresholds characterize extremal multiplicity:

Proposition 7 (Steady State Multiplicity with Continuous States)**.** *Extreme optimism and pessimism are simultaneously deterministic steady states for θ if and only if $\theta \in [\theta_O, \theta_P]$, which is non-empty if and only if*

$$\check{P}^{-1}(\mu_O;\mu_O) - \hat{P}^{-1}(\mu_P;\mu_P) \leq f(1) \tag{148}$$

Proof. This follows exactly the same steps as the proofs of Proposition 2 and Corollary 4, replacing the appropriate inverses defined above. □

Thus, the same conditions that give rise to multiplicity with binary narratives obtain with a continuum of levels of optimism. Indeed, observe that restricting to first-order approximations above was unnecessary. We could have considered an arbitrary order, say k, of approximation of the CGF and obtained a system of difference equations for the first k cumulants. Proposition 7 would still hold as written, as under the extremal steady states, all higher cumulants are identically zero and remain so under the provided condition. Naturally, however, the general dynamics only reduce to those resembling the simple model under the first-order approximation. Nevertheless, we observe that this is a first-order approximation to the exact equilibrium dynamics and not simply an approximation of the dynamics of an approximate equilibrium.

B.5 Narratives About Idiosyncratic Fundamentals

In the main analysis, we assumed that narratives described properties of aggregate fundamentals. In this section, we characterize equilibrium dynamics when narratives describe properties of idiosyncratic fundamentals. For expositional simplicity, we suppose that productivity shocks are i.i.d. (or $\rho = 0$). Concretely, we now instead suppose that all agents believe that $\log\theta_t \sim N(0,\sigma^2)$, or agree about the distribution of aggregate productivity. Moreover, as in the baseline, all agents believe that others' idiosyncratic productivity follows $\log\tilde{\theta}_{jt} \sim N(0,\sigma^2_{\tilde{\theta}})$ for all $j \neq i$. However, agents disagree about the mean of their own idiosyncratic productivity: optimistic agents believe that $\log\tilde{\theta}_{it} \sim N(\mu_O, \sigma^2_{\tilde{\theta}})$ while pessimistic agents believe that $\log\tilde{\theta}_{it} \sim N(\mu_P, \sigma^2_{\tilde{\theta}})$. The rest of the model is identical.

In this context, dynamics are identical conditional on the static relationship between output and narratives. Moreover, the static relationship between output and narratives is now identical (up to a constant) conditional on estimating the partial equilibrium effect of optimism on hiring. This is formalized by the following result:

Proposition 8 (Equilibrium Characterization with Narratives About Idiosyncratic Fundamentals)**.** *There exists a unique equilibrium such that:*

$$\log Y(\log\theta_t, Q_t) = \tilde{a}_0 + a_1 \log\theta_t + \tilde{f}(Q_t) \tag{149}$$

for coefficients $\tilde{a}_0$ and $a_1 > 0$, and a strictly increasing function f, where a_1 is identical to that from Proposition 1 and

$$\tilde{f}(Q) = \frac{\frac{\epsilon}{\epsilon-1}}{1 - \frac{\frac{1}{\epsilon}-\gamma}{\frac{1+\psi-\alpha}{\alpha}+\frac{1}{\epsilon}}} \log\left(1 + Q\left[\exp\left\{\frac{\epsilon-1}{\epsilon}\alpha\tilde{\delta}^{OP}\right\} - 1\right]\right) \tag{150}$$

where $\tilde{\delta}^{OP}$ is defined in Equation 151.

Proof. The proof follows exactly the steps of the proof of Proposition 1 where the aggregate narrative is replaced with an idiosyncratic one. To be concrete, the computation of $\log\mathbb{E}_{it}\left[\theta_{it}^{-\frac{1+\psi}{\alpha}}\right]$ and the method of aggregation are identical to those in the proof of Proposition 1. The only difference is in the computation of $\log\mathbb{E}_{it}\left[Y_t^{\frac{1}{\epsilon}-\gamma}\right]$. Now, Equation 58 differs in that $\mu_{it} = \log\gamma_i + \kappa s_{it}$. Tracking this through to Equation 62, lines 1, 2, 3, and 5 are identical and line 4 differs only in that the term $(1-\kappa)[\lambda_{it}\mu_O + (1-\lambda_{it})\mu_P]$ is now set equal to zero. The analysis then follows up to Equation 67, at which point we have that the exact formula for δ^{OP} changes and is now given by:

$$\alpha\tilde{\delta}^{OP} = \frac{\frac{1+\psi}{\alpha}}{\frac{1+\psi-\alpha}{\alpha}+\frac{1}{\epsilon}}(1-\kappa)(\mu_O - \mu_P) \tag{151}$$

The formula for $\delta_t(0)$ is identical except for in the second line where the term $a_1(1-\kappa)\mu_P$ is now equal to zero. The formula for a_1 remains the same. Conditional on $\tilde{\delta}^{OP}$, the formula for f remains the same. The formula for a_0 is identical except for the second line where the term $(1/\epsilon - \gamma)a_1(1-\kappa)\mu_P$ is now equal to zero. □

This Proposition makes clear that output differs in this case only up to an intercept and in changing the mapping from structural parameters to the partial-equilibrium effect of optimism on hiring. Nonetheless, interpreted via the model above, our empirical exercise directly identifies the now-relevant parameter $\tilde{\delta}^{OP}$. As a result, neither our theoretical nor quantitative analysis is sensitive to making narratives be about idiosyncratic conditions. The only difference is that the point calibrations for κ and $(\mu_O - \mu_P)$ would change, while the aggregate dynamics would remain identical.

B.6 Multi-Dimensional Narratives

Our baseline model featured two narratives regarding the mean of fundamentals, but we live in a world of many competing narratives regarding many aspects of reality. In this extension, we broaden our analysis to study a class of three-dimensional narratives, which is essentially exhaustive within the Gaussian class. For simplicity, we abstract from narrative shocks in this analysis. Concretely, suppose that agents believe that the aggregate component of fundamentals follows:

$$\log \theta_t = (1 - \rho)\mu + \rho \log \theta_{t-1} + \sigma \nu_t \tag{152}$$

with $\nu_t \sim N(0, 1)$ and i.i.d.. Narratives now correspond to a vector of (μ, ρ, σ), indexing the mean, persistence and variance of the process for fundamentals. The set of narratives can therefore be represented by $\{(\mu_k, \rho_k, \sigma_k)\}_{k \in \mathcal{K}}$. We restrict that agents place Dirac weights on this set, so that they only ever believe one narrative at a time, and let $Q_{t,k}$ be the fraction of agents who believe narrative $(\mu_k, \rho_k, \sigma_k)$ at time t. Finally, we assume that agents face the same signal-to-noise ratio κ, regardless of the narrative that they hold.[27] Together, these assumptions ensure that agents' posteriors are normal and place a common weight on narratives when agents form their expectations of fundamentals.

By modifying the functional guess-and-verify arguments from Proposition 1, we characterize equilibrium output in this setting in the following result:

Proposition 9 (Equilibrium Characterization with Multi-Dimensional Narratives and Persistence)**.** *There exists a quasi-loglinear equilibrium:*

$$\log Y(\log \theta_t, \log \theta_{t-1}, Q_t) = a_0 + a_1 \log \theta_t + a_2 \log \theta_{t-1} + f(Q_t, \theta_{t-1}) \tag{153}$$

for some $a_1 > 0$, $a_2 \geq 0$, and f. In this equilibrium, the distribution of narratives in the population evolves according to:

$$Q_{t+1,k} = \sum_{k' \in \mathcal{K}} Q_{t,k'} P_{k'}(k, a_0 + a_1 \log \theta_t + a_2 \log \theta_{t-1} + f(Q_t, \theta_{t-1}), Q_t) \tag{154}$$

Proof. We follow the same steps as in the proof of Proposition 1, appropriately adapted to this richer setting. First, we guess an equilibrium of the form:

$$\log Y(\log \theta_t, \log \theta_{t-1}, Q_t) = a_0 + a_1 \log \theta_t + a_2 \log \theta_{t-1} + f(Q_t, \theta_{t-1}) \tag{155}$$

To verify that this is an equilibrium, we need to compute agents' best replies under this conjecture, aggregate them, and show that they are consistent with this guess once aggregated.

[27] Formally, this means that the variance of the noise in agents' signals satisfies $\sigma^2_{\varepsilon,k} \propto \sigma^2_k$ across narratives.

We first find agents' posterior beliefs given narrative weights. Let E denote the standard basis for $\mathbb{R}^K$ with k-th basis vector denoted by

$$e_k = \{\underbrace{0,\ldots,0}_{k-1},1,\underbrace{0,\ldots,0}_{K-k}\} \tag{156}$$

We have that $\lambda_{it} = e_k$ for some $k \leq K$. Under this narrative loading, we have that agent's posteriors are given by:

$$\log\theta_{it}|\lambda_{it}, s_{it} \sim N\left(\log\gamma_i + \kappa s_{it} + (1-\kappa)\mu(\lambda_{it},\theta_{t-1}), \sigma^2_{\theta|s}(\lambda_{it}) + \sigma^2_{\tilde{\theta}}\right) \tag{157}$$

with:

$$\begin{aligned}\mu(e_k,\theta_{t-1}) &= (1-\rho_k)\mu_k + \rho_k\log\theta_{t-1}\\ \sigma^2_{\theta|s}(e_k) &= \frac{1}{\frac{1}{\sigma_k^2}+\frac{1}{\sigma^2_{\epsilon,k}}} \quad \kappa = \frac{1}{1+\frac{\sigma^2_{\varepsilon,k}}{\sigma_k^2}}\end{aligned} \tag{158}$$

for all $k \leq K$,where κ does not depend on k as $\sigma^2_{\varepsilon,k} \propto \sigma^2_k$. Hence, we can compute agents' best replies by evaluating:

$$\log\mathbb{E}_{it}\left[\theta_{it}^{-\frac{1+\psi}{\alpha}}\right] = -\frac{1+\psi}{\alpha}\left(\log\gamma_i + \kappa s_{it} + (1-\kappa)\mu(\lambda_{it},\theta_{t-1})\right) + \frac{1}{2}\left(\frac{1+\psi}{\alpha}\right)^2\left(\sigma^2_{\theta|s}(\lambda_{it}) + \sigma^2_{\tilde{\theta}}\right) \tag{159}$$

$$\begin{aligned}\log\mathbb{E}_{it}\left[Y_t^{\frac{1}{\epsilon}-\gamma}\right] &= \left(\frac{1}{\epsilon}-\gamma\right)\left(a_0 + a_1\left(\kappa s_{it} + (1-\kappa)\mu(\lambda_{it},\theta_{t-1})\right) + a_2\log\theta_{t-1} + f(Q_t,\theta_{t-1})\right)\\ &+ \frac{1}{2}\left(\frac{1}{\epsilon}-\gamma\right)^2 a_1^2\sigma^2_{\theta|s}(\lambda_{it})\end{aligned} \tag{160}$$

By substituting this into agents' best replies, we obtain:

$$\begin{aligned}\log x_{it} = \frac{1}{\frac{1+\psi-\alpha}{\alpha}+\frac{1}{\epsilon}}\Bigg[&\log\left(\frac{1-\frac{1}{\epsilon}}{\frac{1+\psi}{\alpha}}\right) + \frac{1+\psi}{\alpha}\left[\log\gamma_i + \kappa s_{it} + (1-\kappa)\mu(\lambda_{it},\theta_{t-1})\right]\\ &-\frac{1}{2}\left(\frac{1+\psi}{\alpha}\right)^2\left(\sigma^2_{\theta|s}(\lambda_{it}) + \sigma^2_{\tilde{\theta}}\right)\\ &+\left(\frac{1}{\epsilon}-\gamma\right)\left[a_0 + a_1\left(\kappa s_{it} + (1-\kappa)\mu(\lambda_{it},\theta_{t-1})\right) + a_2\log\theta_{t-1} + f(Q_t,\theta_{t-1})\right]\\ &+\frac{1}{2}a_1^2\left(\frac{1}{\epsilon}-\gamma\right)^2\sigma^2_{\theta|s}(\lambda_{it})\Bigg]\end{aligned} \tag{161}$$

which we observe is conditional normally distributed as $\log x_{it}|\lambda_{it} \sim N(\delta_t(\lambda_{it}), \hat{\sigma}^2)$ with $\hat{\sigma}^2$ as in

Equation 63 and:

$$
\begin{aligned}
\delta_t(e_k) = \frac{1}{\frac{1+\psi-\alpha}{\alpha}+\frac{1}{\epsilon}} \Bigg[&\log\left(\frac{1-\frac{1}{\epsilon}}{\frac{1+\psi}{\alpha}}\right) \\
&+ \frac{1+\psi}{\alpha}\left[\log\gamma_i + \kappa\log\theta_t + (1-\kappa)\mu(e_k,\theta_{t-1})\right] \\
&- \frac{1}{2}\left(\frac{1+\psi}{\alpha}\right)^2\left(\sigma^2_{\theta|s}(e_k)+\sigma^2_{\tilde{\theta}}\right) \\
&+ \left(\frac{1}{\epsilon}-\gamma\right)\left[a_0 + a_1\left(\kappa\log\theta_t + (1-\kappa)\mu(e_k,\theta_{t-1})\right) + a_2\log\theta_{t-1} + f(Q_t,\theta_{t-1})\right] \\
&+ \frac{1}{2}a_1^2\left(\frac{1}{\epsilon}-\gamma\right)^2\sigma^2_{\theta|s}(e_k)\Bigg]
\end{aligned} \tag{162}
$$

for all $k \leq K$. Aggregating these best replies, using Equation 64, we obtain that:

$$
\begin{aligned}
\log Y_t &= \frac{\epsilon}{\epsilon-1}\log\mathbb{E}_t\left[\mathbb{E}_t\left[\exp\left\{\frac{\epsilon-1}{\epsilon}\log x_{it}\right\}|\lambda_{it}\right]\right] \\
&= \frac{\epsilon}{\epsilon-1}\log\left(\sum_k Q_{t,k}\exp\left\{\frac{\epsilon-1}{\varepsilon}\delta_t(e_k) + \frac{1}{2}\left(\frac{\epsilon-1}{\varepsilon}\right)^2\hat{\sigma}^2\right\}\right) \\
&= \delta_t(e_1) + \frac{1}{2}\frac{\epsilon-1}{\epsilon}\hat{\sigma}^2 + \frac{\epsilon}{\epsilon-1}\log\left(\sum_k Q_{t,k}\exp\left\{\frac{\epsilon-1}{\epsilon}\left(\delta_t(e_k)-\delta_t(e_1)\right)\right\}\right)
\end{aligned} \tag{163}
$$

where $\hat{\sigma}^2$ is a constant, $\delta_t(e_1)$ depends linearly on $\log\theta_t$ and $\log\theta_{t-1}$ and $\delta_t(e_k)-\delta_t(e_1)$ does not depend on $\log\theta_t$ for all $k \leq K$ and can therefore be written as $\delta_{k1}(\theta_{t-1})$. Moreover, by matching coefficients, we obtain that a_1 is the same as in the proof of Proposition 1. And we find that f must satisfy:

$$
f(Q,\theta_{t-1}) = \frac{\frac{1}{\epsilon}-\gamma}{\frac{1+\psi-\alpha}{\alpha}+\frac{1}{\epsilon}}f(Q,\theta_{t-1}) + \frac{\epsilon}{\epsilon-1}\log\left(\sum_k Q_{t,k}\exp\left\{\frac{\epsilon-1}{\epsilon}\delta_{k1}(\theta_{t-1})\right\}\right) \tag{164}
$$

and so:

$$
f(Q,\theta_{t-1}) = \frac{\frac{\epsilon}{\epsilon-1}}{1-\frac{\frac{1}{\epsilon}-\gamma}{\frac{1+\psi-\alpha}{\alpha}+\frac{1}{\epsilon}}}\log\left(\sum_k Q_{t,k}\exp\left\{\frac{\epsilon-1}{\epsilon}\delta_{k1}(\theta_{t-1})\right\}\right) \tag{165}
$$

Completing the proof. □

In the multidimensional narrative case with persistence, the past value of fundamentals interacts non-linearly with the cross-sectional narrative distribution in affecting aggregate output. However, without more structure, the properties of the dynamics generated by this multi-dimensional system are essentially unrestricted.

B.7 Persistent Idiosyncratic Shocks and Belief Updating

We now extend the analysis from Section B.6 to the case where agents' idiosyncratic states drive narrative updating and are persistent. Concretely, in that setting, we let $P_{k'}$ depend on $(Y_t, Q_t, \tilde{\theta}_{it})$ and idiosyncratic productivity shocks evolve according to an AR(1) process:

$$\log \tilde{\theta}_{it} = \rho_{\tilde{\theta}} \log \tilde{\theta}_{i,t-1} + \zeta_{it} \tag{166}$$

where $0 < \rho_{\tilde{\theta}} < 1$ and $\zeta_{it} \sim N(0, \sigma_\zeta^2)$. We let $F_{\tilde{\theta}}$ denote the stationary distribution of $\tilde{\theta}_{it}$, which coincides with the cross-sectional marginal distribution of $\tilde{\theta}_{it}$ for all $t \in \mathbb{N}$.

The additional theoretical complication these two changes induce is that the marginal distribution of narratives Q_t is now insufficient for describing aggregate output. This is because narratives λ_{it} and idiosyncratic fundamentals $\tilde{\theta}_{it}$ are no longer independent as λ_{it} and $\tilde{\theta}_{it}$ both depend on $\tilde{\theta}_{it-1}$. The relevant state variable is now the joint distribution of narratives and idiosyncratic productivity $\check{Q}_t \in \Delta(\Lambda \times \mathbb{R})$. We denote the marginals as Q_t and $F_{\tilde{\theta}}$, and the conditional distribution of narratives given $\tilde{\theta}$ as $\check{Q}_{t,k|\tilde{\theta}} = \frac{\check{Q}_{t,k}(\tilde{\theta})}{f_{\tilde{\theta}}(\tilde{\theta})}$.

Proposition 10 (Equilibrium Characterization with Multi-Dimensional Narratives, Aggregate and Idiosyncratic Persistence, and Idiosyncratic Narrative Updating)**.** *There exists a quasi-loglinear equilibrium:*

$$\log Y(\log \theta_t, \log \theta_{t-1}, \check{Q}_t) = a_0 + a_1 \log \theta_t + a_2 \log \theta_{t-1} + f(\check{Q}_t, \theta_{t-1}) \tag{167}$$

for some $a_1 > 0$, $a_2 \geq 0$, and f.

Proof. This proof follows closely that of Proposition 9. Under narrative loading λ_{it}, we have that the agent's posterior regarding $\log \theta_{it}$ is given by:

$$\log \theta_{it} | \tilde{\theta}_{it-1}, \lambda_{it}, s_{it} \sim N\left(\log \gamma_i + \rho_{\tilde{\theta}} \log \tilde{\theta}_{it-1} + \kappa s_{it} + (1-\kappa)\mu(\lambda_{it}, \theta_{t-1}), \sigma^2_{\theta|s}(\lambda_{it}) + \sigma^2_\xi\right) \tag{168}$$

where $\mu(\lambda_{it}, \theta_{t-1})$, κ, and $\sigma^2_{\theta|s}(\lambda_{it})$ are as in Proposition 9. Then substitute $\log \gamma_i + \rho_{\tilde{\theta}} \tilde{\theta}_{it-1}$ for $\log \gamma_i$ and follow the Proof of Proposition 9 until the aggregation step (Equation 163). We now instead have that:

$$\begin{aligned}
\log Y_t &= \frac{\epsilon}{\epsilon-1}\log \mathbb{E}_t\left[\mathbb{E}_t\left[\exp\left\{\frac{\epsilon-1}{\epsilon}\log x_{it}\right\}|\tilde{\theta}_{it-1},\lambda_{it}\right]\right] \\
&= \frac{\epsilon}{\epsilon-1}\log \mathbb{E}_t\left[\exp\left\{\frac{\epsilon-1}{\epsilon}\delta_t(e_k,\tilde{\theta}_{it-1})+\frac{1}{2}\left(\frac{\epsilon-1}{\epsilon}\right)^2\hat{\sigma}^2\right\}\right] \\
&= \frac{\epsilon}{\epsilon-1}\log\left(\int\sum_k \check{Q}_{t,k|\tilde{\theta}}\exp\left\{\frac{\epsilon-1}{\varepsilon}\delta_t(e_k,\tilde{\theta})+\frac{1}{2}\left(\frac{\epsilon-1}{\varepsilon}\right)^2\hat{\sigma}^2\right\}\mathrm{d}F_{\tilde{\theta}}(\tilde{\theta})\right) \\
&= \delta_t(e_1,1)+\frac{1}{2}\frac{\epsilon-1}{\epsilon}\hat{\sigma}^2 \\
&\quad +\frac{\epsilon}{\epsilon-1}\log\left(\int\sum_k \check{Q}_{t,k|\tilde{\theta}}\exp\left\{\frac{\epsilon-1}{\epsilon}\left(\delta_t(e_k,\tilde{\theta})-\delta_t(e_1,1)\right)\right\}\mathrm{d}F_{\tilde{\theta}}(\tilde{\theta})\right)
\end{aligned} \tag{169}$$

Again, $\hat{\sigma}^2$ is a constant and $\delta_t(e_1,0)$ depends linearly on $\log\theta_t$ and $\log\theta_{t-1}$ and $\delta_t(e_k,\tilde{\theta})-\delta_t(e_1,1)$ does not depend on $\log\theta_t$ for all $k\leq K$. Thus, we may write it as $\delta_{k1}(\theta_{t-1},\tilde{\theta})$. Again, a_1 is the same as in Proposition 1. By the same steps as in Proposition 9, we then have that:

$$f(\check{Q},\theta_{t-1})=\frac{\frac{\epsilon}{\epsilon-1}}{1-\frac{\frac{1}{\epsilon}-\gamma}{\frac{1+\psi-\alpha}{\alpha}+\frac{1}{\epsilon}}}\log\left(\int\sum_k \check{Q}_{t,k|\tilde{\theta}}\exp\left\{\frac{\epsilon-1}{\epsilon}\delta_{k1}(\theta_{t-1},\tilde{\theta})\right\}\mathrm{d}F_{\tilde{\theta}}(\tilde{\theta})\right) \tag{170}$$

Completing the proof. □

We can use this result to study the additional effects induced by persistent idiosyncratic fundamentals. To do this, we restrict to the case of our main analysis with optimism and pessimism. In this context, we have that:

$$f(\check{Q})=\frac{\frac{\epsilon}{\epsilon-1}}{1-\omega}\log\left(\mathbb{E}_{\tilde{\theta}}\left[\check{Q}_{t|\tilde{\theta}}\exp\left\{\frac{\epsilon-1}{\epsilon}\delta_{OP}(\tilde{\theta})\right\}+(1-\check{Q}_{t|\tilde{\theta}})\exp\left\{\frac{\epsilon-1}{\epsilon}\delta_{PP}(\tilde{\theta})\right\}\right]\right) \tag{171}$$

where:

$$\begin{aligned}
\delta_{OP}(\tilde{\theta}) &= \alpha\delta_{OP}+\frac{\frac{1+\psi}{\alpha}}{\frac{1+\psi-\alpha}{\alpha}+\frac{1}{\epsilon}}\rho_{\tilde{\theta}}\log\tilde{\theta} \\
\delta_{PP}(\tilde{\theta}) &= \frac{\frac{1+\psi}{\alpha}}{\frac{1+\psi-\alpha}{\alpha}+\frac{1}{\epsilon}}\rho_{\tilde{\theta}}\log\tilde{\theta}
\end{aligned} \tag{172}$$

We define $\xi = \frac{\frac{1+\psi}{\alpha}}{\frac{1+\psi-\alpha}{\alpha}+\frac{1}{\epsilon}}\rho_{\tilde{\theta}}$ and observe that we can write:

$$\begin{aligned}
&\check{Q}_{t|\tilde{\theta}} \exp\left\{\frac{\epsilon-1}{\epsilon}\delta_{OP}(\tilde{\theta})\right\} + (1-\check{Q}_{t|\tilde{\theta}}) \exp\left\{\frac{\epsilon-1}{\epsilon}\delta_{PP}(\tilde{\theta})\right\} \\
&= Q_{t|\tilde{\theta}} \exp\left\{\frac{\epsilon-1}{\epsilon}\left(\alpha\delta_{OP} + \xi \log\tilde{\theta}\right)\right\} + (1-Q_{t|\tilde{\theta}}) \exp\left\{\frac{\epsilon-1}{\epsilon}\xi\log\tilde{\theta}\right\} \\
&= Q_{t|\tilde{\theta}} \exp\left\{\frac{\epsilon-1}{\epsilon}\xi\log\tilde{\theta}\right\}\left[\exp\left\{\frac{\epsilon-1}{\epsilon}\alpha\delta_{OP}\right\}-1\right] + \exp\left\{\frac{\epsilon-1}{\epsilon}\xi\log\tilde{\theta}\right\}
\end{aligned} \tag{173}$$

Taking the expectation of the relevant terms, we obtain:

$$\begin{aligned}
&\mathbb{E}_{\tilde{\theta}}\left[\check{Q}_{t|\tilde{\theta}} \exp\left\{\frac{\epsilon-1}{\epsilon}\delta_{OP}(\tilde{\theta})\right\} + (1-\check{Q}_{t|\tilde{\theta}}) \exp\left\{\frac{\epsilon-1}{\epsilon}\delta_{PP}(\tilde{\theta})\right\}\right] \\
&= \left[\exp\left\{\frac{\epsilon-1}{\epsilon}\alpha\delta_{OP}\right\}-1\right]\exp\left\{\frac{1}{2}\left(\frac{\epsilon-1}{\epsilon}\xi\right)^2\frac{\sigma_\zeta^2}{1-\rho_{\tilde{\theta}}^2}\right\}Q_t \\
&\quad + \mathrm{Cov}_t\left(Q_{t|\tilde{\theta}}, \tilde{\theta}^{\frac{\epsilon-1}{\epsilon}\xi}\right) + \exp\left\{\frac{1}{2}\left(\frac{\epsilon-1}{\epsilon}\xi\right)^2\frac{\sigma_\zeta^2}{1-\rho_{\tilde{\theta}}^2}\right\}
\end{aligned} \tag{174}$$

Thus, we have that the contribution of optimism to output is given by:

$$\begin{aligned}
f(\check{Q}_t) = \frac{\frac{\epsilon}{\epsilon-1}}{1-\omega}\log\Bigg(&\left[\exp\left\{\frac{\epsilon-1}{\epsilon}\alpha\delta_{OP}\right\}-1\right]\exp\left\{\frac{1}{2}\left(\frac{\epsilon-1}{\epsilon}\xi\right)^2\frac{\sigma_\zeta^2}{1-\rho_{\tilde{\theta}}^2}\right\}Q_t \\
&+ \mathrm{Cov}_t\left(Q_{t|\tilde{\theta}}, \tilde{\theta}^{\frac{\epsilon-1}{\epsilon}\xi}\right) + \exp\left\{\frac{1}{2}\left(\frac{\epsilon-1}{\epsilon}\xi\right)^2\frac{\sigma_\zeta^2}{1-\rho_{\tilde{\theta}}^2}\right\}\Bigg)
\end{aligned} \tag{175}$$

We observe that the first term is almost identical to that in our main analysis. This term is now intermediated by the effect of heterogeneity in previous productivity (to see this, observe that this vanishes when $\rho_{\tilde{\theta}} = 0$). Second, there is a new effect stemming from the covariance of optimism and productivity. Intuitively, when more optimistic firms are also more productive, they increase their production by more and this increases output. Finally, there is a level effect of heterogeneous productivity.

Thus, the sole new qualitative force is the covariance effect. To the extent that this does not vary with time, it can have no effect on dynamics. We investigate this in the data by estimating the regression model

$$\log\hat{\theta}_{it} = \sum_{\tau=1995}^{2019}\beta_\tau\cdot(\mathrm{opt}_{i\tau}\cdot\mathbb{I}[\tau=t]) + \chi_{j(i),t} + \gamma_i + \varepsilon_{it} \tag{176}$$

where $(\chi_{j(i),t}, \gamma_i)$ are industry-by-time and firm fixed effects, and β_s measures the (within-industry, within-firm) difference in mean log TFP for optimistic and pessimistic firms in each year. If the β_s vary systematically with the business cycle, then the shifting productivity composition of optimists over the business cycle is an important component of business-cycle dynamics.

We plot our coefficient estimates β_τ in Figure A10. The estimates are generally positive, but economically small relative to the large observed variation in TFP, $\log \theta_{it}$, which has an in-sample standard deviation of 0.84. Outside of the first two years and last year of the sample, we find limited evidence of time variation. Moreover, the variation that exists is not obviously correlated with the business cycle. This suggests that the compositional effect for optimists driven by narrative updating in response to idiosyncratic conditions is not, at least in our data, quantitatively significant.

B.8 Contrarianism, Endogenous Cycles, and Chaos

The baseline model can generate neither endogenous cycles nor chaotic dynamics without extrinsic shocks to fundamentals (as made formal by Lemma 3). This is because the probability that agents become optimistic is always increasing in the fraction of optimists in equilibrium.

In this appendix, we relax this assumption and delineate precise, testable conditions under which cyclical and chaotic dynamics occur in the absence of fundamental and aggregate shocks. We do so in a model with "contrarian" agents whose updating contradicts recent data and/or consensus. Our analysis of endogenous narratives with contrarianism therefore complements the literature on endogenous cycles in macroeconomic models (see, *e.g.*, Boldrin and Woodford, 1990; Beaudry, Galizia, and Portier, 2020) by providing a further potential micro-foundation for the existence of endogenous cycles.

We begin by defining cycles and chaos. There exists a cycle of period $k \in \mathbb{N}$ if $Q = T^k(Q)$ and all elements of $\{Q, T(Q), \dots, T^{k-1}(Q)\}$ are non-equal. We will say that there are *chaotic dynamics* if there exists an uncountable set of points $S \subset [0,1]$ such that (i) for every $Q, Q' \in S$ such that $Q \neq Q'$, we have that $\limsup_{t\to\infty} |T^t(Q) - T^t(Q')| > 0$ and $\liminf_{t\to\infty} |T^t(Q) - T^t(Q')| = 0$ and (ii) for every $Q \in S$ and periodic point $Q' \in [0,1]$, $\limsup_{t\to\infty} |T^t(Q) - T^t(Q')| > 0$. This definition of chaos is due to Li and Yorke (1975) and can be understood as saying that there is a large set of points such that the iterated dynamics starting from any two points in this set get both far apart and vanishingly close.

A Variant Model with the Potential for Cycles and Chaos. We will study the issue of cycles and chaos under the simplifying assumption that,[28] in equilibrium, the induced probabilities that optimists and pessimists respectively become optimists are quadratic and given by:[29]

$$\tilde{P}_O(Q) = a_O + b_O Q - cQ^2 \, , \; \tilde{P}_P(Q) = a_P + b_P Q - cQ^2 \tag{177}$$

[28] This simplifying assumption is without any qualitative loss as this model can demonstrate the full range of potential cyclical and chaotic dynamics.

[29] This can be microfounded in a generalization our earlier LAC model by taking $P_i(\log Y, Q) = u_i + r_i \log Y + s_i Q - cQ^2$ for $i \in \{O, P\}$ and approximating $f(Q) \approx \frac{\alpha\delta^{OP}}{1-\omega} Q$. In this case:

$$\tilde{P}_i(Q) = (u_i + r_i a_0 + r_i a_1 \log \theta) + \left(r_i \frac{\alpha\delta^{OP}}{1-\omega} + s_i \right) Q - cQ^2$$

with parameters $(a_O, a_P, b_O, b_P, c) \in \mathbb{R}^5$ such that $P_O([0,1]), P_P([0,1]) \subseteq [0,1]$. The parameters a_O and a_P index stubbornness, b_O and b_P capture both contagiousness and associativeness (through the subsumed equilibrium map), and c captures any non-linearity.

The following result describes the potential dynamics:

Proposition 11. *The following statements are true:*

1. *When $\tilde{P}_O \geq \tilde{P}_P$ and both are monotone, there are neither cycles of any period nor chaotic dynamics.*
2. *When $\tilde{P}_O$ and $\tilde{P}_P$ are linear, cycles of period 2 are possible, cycles of any period $k > 2$ are not possible, and chaotic dynamics are not possible.*
3. *Without further restrictions on $\tilde{P}_O$ and $\tilde{P}_P$, cycles of any period $k \in \mathbb{N}$ and chaotic dynamics are possible.*

Proof. The dynamics of optimism are characterized by the transition map

$$\begin{aligned} T(Q) &= Q(a_O + b_O Q - cQ^2) + (1-Q)(a_P + b_P Q - cQ^2) \\ &= a_P + (a_O - a_P + b_P)Q - (c + b_P - b_O)Q^2 \end{aligned} \tag{178}$$

where we define $\omega_0 = a_P$, $\omega_1 = (a_O - a_P + b_P)$, $\omega_2 = (c + b_P - b_O)$ for simplicity. We first show that the dynamics described by T are topologically conjugate to those of the logistic map $\check{T}(x) = \eta x(1-x)$ with

$$\eta = 1 + \sqrt{(a_O - a_P + b_P - 1)^2 + 4a_P(c + b_P - b_O)} \tag{179}$$

Two maps $T : [0,1] \to [0,1]$ and $T' : [0,1] \to [0,1]$ are topologically conjugate if there exists a continuous, invertible function $h : [0,1] \to [0,1]$ such that $T' \circ h = h \circ T$. If T is topologically conjugate to T' and we know the orbit of T', we can compute the orbit of T via the formula:

$$T^k(Q) = \left(h^{-1} \circ T'^k \circ h\right)(Q) \tag{180}$$

Hence, we can prove the properties of interest using known properties of the map $\check{T}$ as well as the mapping from the deeper parameters of T to the parameters of $\check{T}$.

To show the topological conjugacy of T and $\check{T}$, we proceed in three steps:

1. *T is topically topologically conjugate to the quadratic map $\hat{T}(Q) = Q^2 + k$ for appropriate choice of k.* We guess the following homeomorphism $\hat{h}(Q) = \hat{\alpha} + \hat{\beta}Q$. Plugging $\hat{h}$ in $\hat{T}$, we have that:

$$\hat{T}(\hat{h}(Q)) = (k + \hat{\alpha}^2) + 2\hat{\alpha}\hat{\beta}Q + \hat{\beta}^2 Q^2 \tag{181}$$

Inverting $\hat{h}$ and applying it to this expression yields:

$$\hat{h}^{-1}(\hat{T}(\hat{h}(Q))) = \frac{k + \hat{\alpha}(\hat{\alpha} - 1)}{\hat{\beta}} + 2\hat{\alpha}Q + \hat{\beta}Q^2 \tag{182}$$

To verify topological conjugacy, we need to show that $T(Q) = \hat{h}^{-1}(\hat{T}(\hat{h}(Q)))$. Matching coefficients, this is the case if and only if:

$$\omega_0 = \frac{k + \hat{\alpha}(\hat{\alpha} - 1)}{\hat{\beta}},\ \omega_1 = 2\hat{\alpha},\ \omega_2 = -\hat{\beta} \tag{183}$$

We therefore have that:

$$k = \hat{\beta}\omega_0 + \hat{\alpha}(1 - \hat{\alpha}) = -\omega_2\omega_0 + \frac{\omega_1}{2}\left(1 - \frac{\omega_1}{2}\right) \tag{184}$$

with $\hat{h}(Q) = \frac{\omega_1}{2} - \omega_2 Q$.

2. *$\hat{T}$ is topologically conjugate to $\check{T}$ for appropriate choice of η.* We guess the following homeomorphism $\check{h}(Q) = \check{\alpha} + \check{\beta}Q$. Plugging $\check{h}$ in $\check{T}$, we obtain:

$$\check{T}(\check{h}(Q)) = \eta\left(\check{\alpha}(1 - \check{\alpha}) + \check{\beta}(1 - 2\check{\alpha})Q - \check{\beta}^2Q^2\right) \tag{185}$$

Inverting $\check{h}$ and applying it, we obtain:

$$\check{h}^{-1}(\check{T}(\check{h}(Q))) = \frac{\eta\check{\alpha}(1 - \check{\alpha}) - \check{\alpha}}{\check{\beta}} + \eta(1 - 2\check{\alpha})Q - \eta\check{\beta}Q^2 \tag{186}$$

Matching coefficients, we find:

$$k = \frac{\eta\check{\alpha}(1 - \check{\alpha}) - \check{\alpha}}{\check{\beta}},\ 0 = \eta(1 - 2\check{\alpha}),\ 1 = -\eta\check{\beta} \tag{187}$$

We therefore obtain that:

$$k = \eta(\check{\alpha} - \eta(1 - \check{\alpha})) = \frac{\eta}{2}\left(1 - \frac{\eta}{2}\right) \tag{188}$$

which implies that $\eta = 1 + \sqrt{1 - 4k}$ with $\check{h}(Q) = \frac{1}{2} - \frac{1}{1+\sqrt{1-4k}}Q$.

3. *T is topologically conjugate to $\check{T}$ for appropriate choice of η.* We now compose the mappings proved in steps 1 and 2 to show

$$T = \hat{h}^{-1} \circ \check{h}^{-1} \circ \check{T} \circ \check{h} \circ \hat{h} \tag{189}$$

with

$$\begin{aligned}\eta &= 1 + \sqrt{1 - 4\left(-\omega_2\omega_0 + \frac{\omega_1}{2}\left(1 - \frac{\omega_1}{2}\right)\right)} = 1 + \sqrt{(\omega_1 - 1)^2 + 4\omega_2\omega_0} \\ &= 1 + \sqrt{(a_O - a_P + b_P - 1)^2 + 4a_P(c + b_P - b_O)}\end{aligned} \tag{190}$$

and therefore that T is topologically conjugate to $\check{T}$.

Having shown the conjugacy of T to $\check{T}$, we now find bounds on η implied by each case and use this conjugacy to derive the implications for possible dynamics. The following points prove each claim 1-3 in the original Proposition.

1. $\tilde{P}_O \geq \tilde{P}_P$ and both are monotone. Thus, T is increasing and there cannot be cycles or chaos. This implies that $\eta < 3$ (see Weisstein, 2001, for reference).
2. $\tilde{P}_O$ and $\tilde{P}_P$ are linear. It suffices to show that we can attain $\eta > 3$ but that η must be less than $1 + \sqrt{6}$ (see Weisstein, 2001, for reference). In this case, $c = 0$. This is in addition to the requirements that $\max_{Q\in[0,1]} \tilde{P}_i(Q) \leq 1$ and $\min_{Q\in[0,1]} \tilde{P}_i(Q) \geq 0$ for $i \in \{O, P\}$, which can be expressed as:

$$\begin{aligned} \max_{Q\in[0,1]} \tilde{P}_i(Q) &= \max\left\{a_i, a_i + b_i - c, \left(a_i + \frac{b_i^2}{4c}\right)\mathbb{I}[0 \leq b_i \leq 2c]\right\} \leq 1 \\ \min_{Q\in[0,1]} \tilde{P}_i(Q) &= \min\{a_i, a_i + b_i - c\} \geq 0 \end{aligned} \tag{191}$$

   The maximal value of η consistent with these restrictions can therefore be obtained by solving the following program:

$$\begin{aligned} &\max_{(a_O, a_P, b_O, b_P)\in\mathbb{R}^4} (a_O - a_P + b_P - 1)^2 + 4a_P(b_P - b_O) \\ &\text{s.t.} \max\{a_O, a_O + b_O\} \leq 1, \max\{a_P, a_P + b_P\} \leq 1 \\ &\min\{a_O, a_O + b_O\} \geq 0, \min\{a_P, a_P + b_P\} \geq 0 \end{aligned} \tag{192}$$

   Exact solution of this program via Mathematica yields that the maximum value is 5. This implies that the maximum value of η is $1 + \sqrt{5} \approx 3.23$, which is greater than 3 but less than $1 + \sqrt{6}$. Moreover, this maximum is attained at $a_O = 0, a_P = 1, b_O = 0, b_P = -1$.
3. No further restrictions on $\tilde{P}_O$ and $\tilde{P}_P$. We can attain $\eta = 4$ by setting $a_0 = a_P = 0$, $b_O = b_P = 4$, $c = 4$. Thus, cycles of any period $k \in \mathbb{N}$ and chaotic dynamics can occur (see Weisstein, 2001, for reference).

□

The proof of this result follows a classic approach of recasting a quadratic difference equation as a logistic difference equation via topological conjugacy (see, *e.g.,* Battaglini, 2021; Deng, Khan, and Mitra, 2022). The restrictions on structural parameters implied by the hypotheses of the proposition then yield upper bounds on the possible logistic maps and allow us to characterize the possible dynamics using known results.

To understand this result, observe in our baseline case in which T is monotone that cycles and chaos are not possible. This is because there is no potential for optimism to sufficiently overshoot its steady state. By contrast, when $\tilde{P}_O$ and $\tilde{P}_P$ are either non-monotone or non-ranked, two-period cycles can take place where the economy undergoes endogenous boom-bust cycles with periods of high optimism and high output ushering in periods of low optimism and low output (and *vice versa*) as contrarians switch positions and consistently overshoot the (unstable) steady state. Finally, when

$\tilde{P}_O$ and $\tilde{P}_P$ are non-linear and non-monotone, essentially any richness of dynamics can be achieved via erratic movements in optimism that are extremely sensitive to initial conditions.

An Empirical Test for Cycles and Chaos. Proposition 11 shows how to translate an updating rule of the form of Equation 177 into predictions about the potential for cycles and chaos. We now estimate this updating rule in the data to test these predictions empirically. Concretely, in our panel dataset of firms, we estimate the regression model

$$\begin{aligned} \text{opt}_{it} = \alpha_1 \, \text{opt}_{i,t-1} + \beta_1 \text{opt}_{i,t-1} \cdot \overline{\text{opt}}_{i,t-1} + \\ \beta_2 (1 - \text{opt}_{i,t-1}) \cdot \overline{\text{opt}}_{i,t-1} + \tau \left(\overline{\text{opt}}_{i,t-1}\right)^2 + \gamma_i + \varepsilon_{it} \end{aligned} \tag{193}$$

where γ_i is a firm fixed effect. This model allows the effects of contagiousness to depend on agents' previous state. In the mapping to Equation 177, $\alpha = a_P$, $\alpha_1 = a_O - a_P$, $\beta_1 = b_O$, $\beta_2 = b_P$, and $\tau = c$. With estimates of each regression parameter, denoted by a hat, we also obtain an estimate of the logistic map parameter η defined in Equation 179:

$$\hat{\eta} = 1 + \sqrt{(\hat{\alpha}_1 + \hat{\beta}_2 - 1)^2 + 4\hat{\alpha}_1(\hat{\tau} + \hat{\beta}_2 - \hat{\beta}_1)} \tag{194}$$

Since $\hat{\eta}$ is a nonlinear function of estimated parameters in the regression, we can conduct inference on $\hat{\eta}$ using the delta method. Moreover, this constitutes a test for the possibility of cycles and chaos in the model by the logic of Proposition 11. Specifically, as described in the proof of that result, there are two main cases. First, if $\eta < 3$, then case 1 of the result obtains: there are neither cycles of any period nor chaotic dynamics. Second, if $\eta \geq 3$, there can be cycles of period 2 or more and/or chaos. Moreover, if $\eta > 3.57$, chaotic dynamic obtain.

Our estimates are presented in Table A20. Our point estimate of η is 1.443 and the 95% confidence interval is $(0.076, 2.810)$. This rules out, at the 5% level, the presence of cycles and/or chaos. The 99% confidence interval is $(-0.354, 3.240)$, which does not rule out cycles. The p-value for the chaotic dynamics threshold is 0.001. Thus, our results provide strong evidence against the possibility of chaos due to contagious optimism, and marginally weaker evidence against the possibility of cycles. This test complements the literature on endogenous cycles in macroeconomic models (see, *e.g.*, Boldrin and Woodford, 1990; Beaudry, Galizia, and Portier, 2020) by providing a micro-founded test within a structural economic model, which may ameliorate challenges associated with interpreting pure time-series evidence (see, *e.g.,* Werning, 2017).

B.9 Narratives in Games and the Role of Higher-Order Beliefs

We have studied a micro-founded business-cycle model, but the basic insights extend much more generally to abstract, linear beauty contest games. Importantly, these settings provide us with an ability to disentangle the dual roles of narratives in affecting both agents' first-order and higher-order beliefs about fundamentals.

Concretely, suppose that agents' best replies are given by the following beauty contest form (see, *e.g.*, Morris and Shin, 2002):

$$x_{it} = \alpha \mathbb{E}_{it}[\theta_t] + \beta \mathbb{E}_{it}[Y_t] \tag{195}$$

where $\alpha > 0$ and $\beta \in [0, 1)$. This linear form for best replies is commonly justified by (log-)linearization of some underlying best response function (see, *e.g.*, Angeletos and Pavan, 2007). For example, log-linearization of the agents' best replies in the baseline model of this section yields such an equation with $\beta = \omega$ and all variables above standing in for their log-counterparts. Moreover, suppose that aggregation is linear so that $Y_t = \int_{[0,1]} x_{it} \mathrm{d}i$. This can similarly be justified via an appropriate first-order expansion of non-linear aggregators. Finally, we let the structure of narratives be as before.

Toward characterizing equilibrium, we define the average expectations operator:

$$\overline{\mathbb{E}}_t[\theta_t] = \int_{[0,1]} \mathbb{E}_{it} [\theta_t] \, \mathrm{d}i \tag{196}$$

and the higher-order average expectations operator for $k \in \mathbb{N}$ as:

$$\overline{\mathbb{E}}_t^k[\theta_t] = \int_{[0,1]} \mathbb{E}_{it} \left[\overline{\mathbb{E}}_t^{k-1}[\theta_t] \right] \mathrm{d}i \tag{197}$$

Moreover, we observe by recursive substitution that equilibrium aggregate output is given by:

$$Y_t = \alpha \sum_{k=1}^{\infty} \beta^{k-1} \overline{\mathbb{E}}_t^k[\theta_t] \tag{198}$$

We can therefore solve for the unique equilibrium by computing the hierarchy of higher-order expectations. We can do this in closed-form by observing that agents' idiosyncratic first-order beliefs are given by:

$$\mathbb{E}_t[\theta_t | s_{it}, \lambda_{it}] = \kappa s_{it} + (1 - \kappa) \left(\lambda_{it} \mu_O + (1 - \lambda_{it}) \mu_P \right) \tag{199}$$

which allows us to compute average first-order expectations of fundamentals as:

$$\overline{\mathbb{E}}_t[\theta_t] = \kappa \theta_t + (1 - \kappa)(Q_t \mu_O + (1 - Q_t) \mu_P) \tag{200}$$

which is a weighted average between true fundamentals and the average impact of narratives on agents' priors. By taking agents' expectations over this object and averaging, we compute higher-order average expectations as:

$$\overline{\mathbb{E}}_t^k[\theta_t] = \kappa^k \theta_t + (1 - \kappa^k)(Q_t \mu_O + (1 - Q_t) \mu_P) \tag{201}$$

which is again a weighted average between the state and agents' priors, but now with a geometrically increasing weight on narratives as we consider higher-order average beliefs.

The following result characterizes aggregate output and agents' best replies in the unique equilibrium:

Proposition 12 (Narratives and Higher-Order Beliefs)**.** *There exists a unique equilibrium. In this unique equilibrium, aggregate output is given by:*

$$Y_t = \frac{\alpha}{1-\beta}\left(\frac{(1-\beta)\kappa}{1-\beta\kappa}\theta_t + \frac{1-\kappa}{1-\beta\kappa}\left(Q_t\mu_O + (1-Q_t)\mu_P\right)\right) \tag{202}$$

Moreover, agents' actions follow:

$$\begin{aligned} x_{it} = \alpha\frac{1}{1-\beta\kappa}\left[\kappa\theta_t + \kappa e_{it} + (1-\kappa)\left(\lambda_{it}\mu_O + (1-\lambda_{it})\mu_P\right)\right] \\ + \beta\frac{\alpha}{1-\beta}\frac{1-\kappa}{1-\beta\kappa}\left(Q_t\mu_O + (1-Q_t)\mu_P\right) \end{aligned} \tag{203}$$

Proof. To substantiate the arguments in the main text, by aggregating Equation 195, we obtain that:

$$Y_t = \alpha\overline{E}_t[\theta_t] + \beta\overline{E}_t[Y_t] \tag{204}$$

Thus, by recursive substitution k times we obtain that:

$$Y_t = \alpha\sum_{j=1}^{k}\beta^{j-1}\overline{E}_t^j[\theta_t] + \beta^k\overline{E}_t^k[Y_t] \tag{205}$$

Moreover, we have that:

$$\overline{\mathbb{E}}_t^j[\theta_t] = \kappa^j\theta_t + (1-\kappa^j)(Q_t\mu_O + (1-Q_t)\mu_P) \tag{206}$$

and thus that:

$$\begin{aligned} \alpha\sum_{j=1}^{k}\beta^{j-1}\overline{E}_t^j[\theta_t] &= \alpha\sum_{j=1}^{k}\beta^{j-1}\left(\kappa^j\theta_t + (1-\kappa^j)(Q_t\mu_O + (1-Q_t)\mu_P)\right) \\ &= \alpha\sum_{j=1}^{k}\beta^{j-1}(Q_t\mu_O + (1-Q_t)\mu_P) + \alpha\beta^{-1}\sum_{j=1}^{k}(\beta\kappa)^j\left[\theta_t - (Q_t\mu_O + (1-Q_t)\mu_P)\right] \end{aligned} \tag{207}$$

Hence:

$$\begin{aligned} \lim_{k\to\infty}\alpha\sum_{j=1}^{k}\beta^{j-1}\overline{E}_t^j[\theta_t] = \frac{\alpha}{1-\beta}(Q_t\mu_O + (1-Q_t)\mu_P) + \\ \frac{\alpha\kappa}{1-\beta\kappa}\left[\theta_t - (Q_t\mu_O + (1-Q_t)\mu_P)\right] \end{aligned} \tag{208}$$

We therefore have that there is a unique equilibrium if $\lim_{k\to\infty}\beta^k\overline{E}_t^k[Y_t] = 0$. Hellwig and Veldkamp (2009) show in Proposition 1 of their supplementary material that all equilibria differ on a most a measure zero set of fundamentals. In this setting, this implies that $\lim_{k\to\infty} \beta^k \overline{E}_t^k[Y_t] = c$ for some $c \in \mathbb{R}$ for almost all $\theta \in \Theta$. Hence, the equilibrium is given by:

$$\begin{aligned}Y_t &= \frac{\alpha}{1-\beta}(Q_t\mu_O + (1-Q_t)\mu_P) + \frac{\alpha\kappa}{1-\beta\kappa}\left[\theta_t - (Q_t\mu_O + (1-Q_t)\mu_P)\right] + c \\ &= \frac{\alpha}{1-\beta}\left(\frac{(1-\beta)\kappa}{1-\beta\kappa}\theta_t + \frac{1-\kappa}{1-\beta\kappa}\left(Q_t\mu_O + (1-Q_t)\mu_P\right)\right) + c\end{aligned} \tag{209}$$

But then we have that $c = 0$ by computing $\lim_{k\to\infty} \beta^k \overline{E}_t^k[Y_t] = 0$ under this equilibrium.

Finally, to solve for individual actions under this equilibrium, we compute:

$$\begin{aligned}x_{it} &= \alpha\mathbb{E}_{it}[\theta_t] + \beta\mathbb{E}_{it}[Y_t] \\ &= \alpha\mathbb{E}_{it}[\theta_t] + \beta\mathbb{E}_{it}\left[\frac{\alpha}{1-\beta}\left(\frac{(1-\beta)\kappa}{1-\beta\kappa}\theta_t + \frac{1-\kappa}{1-\beta\kappa}\left(Q_t\mu_O + (1-Q_t)\mu_P\right)\right)\right] \\ &= \left(\alpha + \beta\frac{\alpha}{1-\beta}\frac{(1-\beta)\kappa}{1-\beta\kappa}\right)\mathbb{E}_{it}[\theta_t] + \beta\frac{\alpha}{1-\beta}\frac{1-\kappa}{1-\beta\kappa}\left(Q_t\mu_O + (1-Q_t)\mu_P\right) \\ &= \alpha\frac{1}{1-\beta\kappa}\left(\kappa s_{it} + (1-\kappa)\left(\lambda_{it}\mu_O + (1-\lambda_{it})\mu_P\right)\right) \\ &\quad + \beta\frac{\alpha}{1-\beta}\frac{1-\kappa}{1-\beta\kappa}\left(Q_t\mu_O + (1-Q_t)\mu_P\right)\end{aligned} \tag{210}$$

Completing the proof. □

This result allows us to see how narratives affect output by propagating up through the hierarchy of higher-order beliefs. Concretely, we have that the static impulse response of output to a contemporaneous shock to the fraction of optimists in the population is given by:

$$\frac{\partial Y_t}{\partial Q_t} = \frac{\alpha}{1-\beta}\frac{1-\kappa}{1-\beta\kappa}(\mu_O - \mu_P) = \alpha\sum_{j=1}^{\infty}\beta^{j-1}(1-\kappa^j)(\mu_O - \mu_P) \tag{211}$$

The first expression is composed of the relative importance of fundamentals $\frac{\alpha}{1-\beta}$, the impact of prior beliefs on the entire hierarchy of higher-order beliefs about exogenous and endogenous outcomes $\frac{1-\kappa}{1-\beta\kappa}$ and the difference between the two narratives $\mu_O - \mu_P$. The second expression re-expands the heirarchy of beliefs, to highlight how fraction

$$\frac{\beta^{j-1}(1-\kappa^j)}{\frac{1}{1-\beta}\frac{1-\kappa}{1-\beta\kappa}} \tag{212}$$

of the total effect is driven by beliefs of order j. These weights decline more slowly if complementarity β or prior weights $1-\kappa$ are high.

Finally, our result shows how the regression equation relating individual actions with narrative weights, estimated in our main analysis, holds in equilibrium in the linearized beauty contest. Thus, our empirical strategy is compatible with the interpretation that the macroeconomy is best described by a linear beauty contest, and moreover can be ported to other settings where this modeling assumption may be appropriate, such as that of financial speculation (see *e.g.,* Allen, Morris, and Shin, 2006).

C Additional Details on Textual Data

C.1 Obtaining and Processing 10-Ks

Here, we describe our methodology for obtaining and processing raw data on 10-K filings. We start with raw html files downloaded directly from the SEC's EDGAR (Electronic Data Gathering, Analysis, and Retrieval) system. Each of these files corresponds to a single 10-K filing. Each file is identified by its unique accession number. In its heading, each file also contains the end-date for the period the report concerns (*e.g.*, 12/31/2018 for a FY 2018 ending in December), and a CIK (Central Index Key) firm identifier from the SEC. We use standard linking software provided by Wharton Research Data Services (WRDS) to link CIK numbers and fiscal years to the alternative firm identifiers used in data on firm fundamentals and stock prices. We have, in our original dataset, 182,259 files.

We follow the following steps to turn each document, now identified by firm and year, into a bag-of-words representation:

1. *Cleaning raw text.* We first translate the document into unformatted text. Specifically, we follow the following steps in order:
   (a) Removing hyperlinks and other web addresses
   (b) Removing html formatting tags encased in the brackets <>
   (c) Making all text lowercase
   (d) Removing extra spaces, tabs, and new lines.
   (e) Removing punctuation
   (f) Removing non-alphabetical characters
2. *Removing stop words.* Following standard practice, we remove "stop words" which are common in English but do not convey specific meaning in our analysis. We use the default English stop word list in the `nltk` Python package. Example stopwords include articles ("a","the"), pronouns ("I","my"), prepositions ("in","on"), and conjunctions ("and","while").
3. *Lemmatizing documents.* Again following standard practice, we use lemmatization software to reduce words to their common roots. We use the default English-language lemmatizer of the `spacy` Python package. The lemmatizer uses both the word's identity and its content to transform sentences. For instance, when each is used as a verb, "meet," "met," and "meeting" are commonly lemmatized to "meet." But if the software predicts that "meeting" is used as a noun, it will be lemmatized as the noun "meeting."
4. *Estimating a bigram model.* We estimate a bigram model to group together commonly co-occurring words as single two-word phrases. We use the `phrases` function of the `gensim` package. The bigram modeler groups together words that are almost always used together. For instance, if our original text data set were the 10-Ks of public firms Nestlé and General Mills, the model may determine that "ice" and "cream," which almost always appear together, are part of a bigram "ice_cream."

5. *Computing the bag of words representation.* Having now expressed each document as a vector of clean words (*i.e.,* single words and bigrams), we simply collapse these data to frequencies.

Finally, note that our procedure uses *all* of the non-formatting text in the 10K. This includes all sections of the documents, and does not limit to the Management Discussion and Analysis (MD&A) section. This is motivated by the fact that management's discussion is not limited to one section SEC (2011). Moreover, prior literature has found that textual analysis of the entire 10-K versus the MD&A section tends to closely agree, and that limiting scope to the MD&A section has limited practical benefits due to the trade-off of limiting the amount of text per document (Loughran and McDonald, 2011).

C.2 Obtaining and Processing Conference Call Text

We obtain the full text of sales and earnings conference calls from 2002 to 2014 from the Fair Disclosure (FD) Wire service. The original sample includes 261,034 documents, formatted as raw text. We next subset to documents that have reported firm names and stock tickers, which are automatically associated with documents by Lexis Nexis. When matches are probabilistic, we use the first (highest probability) match.[30] We finally restrict to firms that are listed on one of three US stock exchanges: the NYSE, the NASDAQ, or the NYSE-MKT (Small Cap). We finally connect tickers to the firm identifiers in our fundamentals data using the master cross-walk available on Wharton Research Data Services (WRDS). These operations together reduce the sample size to 158,810 calls. We clean these data by conducting steps 1-3 described above in Appendix C.1. We then calculate positive word counts, negative word counts, and optimism exactly as described in the main text for the 10-K data.

C.3 Measuring Positive and Negative Words

To calculate sets of positive and negative 10K words, we use the updated dictionary available online at McDonald (2021) as of June 2020. This dictionary includes substantial updates relative to the dictionaries associated with the original Loughran and McDonald (2011) publication. These changes are reviewed in the *Documentation* available at McDonald (2021).

The Loughran-McDonald dictionary includes 2345 negative words and 347 positive words. The dictionary is constructed to include multiple forms of each relevant word. For instance, the first negative root "abandon" is listed as: "abandon," "abandoned," "abandoning," "abandonment," "abandonments," and "abandons." To ensure consistency with our own lemmatization procedure, we first map each unique word to all of its possible lemmas using the `getAllLemmas` function of the `lemminflect` Python package, which is an extension to the `spacy` package we use for lemmatization. We then construct a new list of negative words by combining the original list of negative words with all new, unique lemmas to which a negative word mapped (and similarly for positive words). This procedure results in new lists of 2411 negative words and 366 positive words, which map exactly

[30] In the essentially zero-measure cases in which there is a tie, we take the alphabetically first ticker.

to the words that appear in our cleaned bag of words representation. We list the top ten most common positive and negative words from this cleaned set in Table A1. In particular, to make the table most legible, we first associate words with their lemmas, then count the sum of document frequencies for each associated word (which may exceed one), and then print the most common word associated with the lemma.

D Additional Details on Firm Fundamentals Data

D.1 Compustat: Data Selection

Our dataset is Compustat Annual Fundamentals. Our main variables of interest are defined in Table A21. We restrict the sample to firms based in the United States, reporting statistics in US Dollars, and present in the "Industrial" dataset. We exclude firms whose 2-digit NAICS is 52 (Finance and Insurance) or 22 (Utilities). This filter eliminates firms in two industries that, respectively, may have highly non-standard production technology and non-standard market structure.

We summarize our definitions of major "input and output" variables in Table A21. For labor choice, we measure the number of employees. For materials expenditure, we measure the sum of reported variable costs (`cogs`) and sales and administrative expense (`xsga`) net of depreciation (`dp`).[31] As in Ottonello and Winberry (2020) and Flynn and Sastry (2024), we use a perpetual inventory method to calculate the value of the capital stock. We start with the first reported observation of gross value of plant, property, and equipment and add net investment or the differences in net value of plant, property, and equipment. Note that, because all subsequent analysis is conditional on industry-by-time fixed effects, it is redundant at this stage to deflate materials and capital expenditures by industry-specific deflators.

We categorize the data into 44 sectors. These are defined at the 2-digit NAICS level, but for the Manufacturing (31-33) and Information (51) sectors, which we classify at the 3-digit level to achieve a better balance of sector size. More summary information about these industries is provided in Appendix F of Flynn and Sastry (2024).

D.2 Compustat: Calculation of TFP

When calculating firms' Total Factor Productivity, we restrict attention to a subset of our sample that fulfils the following inclusion criteria:

1. Sales, material expenditures, and capital stock are strictly positive;
2. Employees exceed 10;
3. Acquisitions as a proportion of assets (`aqc` over `at`) does not exceed 0.05.

The first ensures that all companies meaningfully report all variables of interest for our production function estimation; the second applies a stricter cut-off to eliminate firms that are very small, and lead to outlier estimates of productivity and choices. The third is a simple screening device for large acquisitions which may spuriously show up as large innovations in firm choices and/or productivity.

Our method for recovering total factor productivity is based on cost shares. In brief, we use cost shares for materials to back out production elasticities, and treat the elasticity of capital as the implied "residual" given an assumed mark-up $\mu > 1$ (in our baseline, $\mu = 4/3$) and constant physical returns-to-scale. The exact procedure is the following:

[31] A small difference from Flynn and Sastry (2024) is that, in assessing the firms' costs and later calculating TFP, we do not "unbundle" materials expenditures on labor and non-labor inputs using supplemental data on annual wages.

1. For all firms in industry j, calculate the estimated materials share:

$$\text{Share}_{M,j'} = \frac{\sum_{i:j(i)=j'} \sum_t \text{MaterialExpenditure}_{it}}{\sum_{i:j(i)=j'} \sum_t \text{Sales}_{it}} \tag{213}$$

2. If $\text{Share}_{M,j'} \leq \mu^{-1}$, then set

$$\begin{aligned} \alpha_{M,j'} &= \mu \cdot \text{Share}_{M,j'} \\ \alpha_{K,j'} &= 1 - \alpha_{M,j'} \end{aligned} \tag{214}$$

3. Otherwise, adjust shares to match the assumed returns-to-scale, or set

$$\begin{aligned} \alpha_{M,j'} &= 1 \\ \alpha_{K,j'} &= 0 \end{aligned} \tag{215}$$

To translate our production function estimates into productivity, we calculate a "Sales Solow Residual" $\tilde{\theta}_{it}$ of the following form:

$$\log \tilde{\theta}_{it} = \log \text{Sales}_{it} - \frac{1}{\mu} \left(\alpha_{M,j(i)} \cdot \log \text{MatExp}_{it} + \alpha_{K,j(i)} \cdot \log \text{CapStock}_{it} \right) \tag{216}$$

We finally define our estimate $\log \hat{\theta}$ as the previous net of industry-by-time fixed effects

$$\log \hat{\theta}_{it} = \log \tilde{\theta}_{it} - \chi_{j(i),t} \tag{217}$$

Theoretical Interpretation. The aforementioned method recovers physical productivity ("TFPQ") under the assumptions, consistent with our quantitative model, that firms operate constant returns-to-scale technology and face an isoleastic, downward-sloping demand curve of *known* elasticity (equivalently, they charge a known markup). The idea is that, given the known markup, we can impute firms' (model-consistent) costs as a fixed fraction of sales and then calculate the theoretically desired cost shares. Here, we describe the simple mathematics.

There is a single firm i operating in industry j with technology

$$Y_i = \theta_i M_i^{\alpha_j} K_i^{1-\alpha_j} \tag{218}$$

They act as a monopolist facing the demand curve

$$p_i = Y_i^{-\frac{1}{\epsilon}} \tag{219}$$

for some inverse elasticity $\epsilon > 1$. Observe that this is, up to scale, the demand function faced by monopolistically competitive intermediate goods producers in our model. The firm's revenue is therefore $p_i Y_i = Y_i^{1-\frac{1}{\epsilon}}$. Finally, the firm can buy materials at industry-specific price q_j and rent capital at rate r_j. The firm's program for profit maximization is therefore

$$\max_{M_i,K_i} \left\{(\theta_i M_i^{\alpha_j} K_i^{1-\alpha_j})^{1-\frac{1}{\epsilon}} - q_j M_i - r_j K_i\right\} \tag{220}$$

We first justify our formulas for the input shares (Equation 214). To do this, we solve for the firm's optimal input choices. This is a concave problem, in which first-order conditions are necessary and sufficient. These conditions are

$$\begin{aligned} q_j &= M_i^{-1}\alpha_j \left(1 - \frac{1}{\epsilon}\right)(\theta_i M_i^{\alpha_j} K_i^{1-\alpha_j})^{1-\frac{1}{\epsilon}} \\ r_j &= K_i^{-1}(1-\alpha_j)\left(1 - \frac{1}{\epsilon}\right)(\theta_i M_i^{\alpha_j} K_i^{1-\alpha_j})^{1-\frac{1}{\epsilon}} \end{aligned} \tag{221}$$

Re-arranging, and substituting in $p_i = Y_i^{-\frac{1}{\epsilon}}$, we derive

$$\begin{aligned} \alpha_j &= \frac{\epsilon}{\epsilon - 1}\frac{q_j M_i}{p_i Y_i} \\ 1 - \alpha_j &= \frac{\epsilon}{\epsilon - 1}\frac{r_j K_i}{p_i Y_i} \end{aligned} \tag{222}$$

Or, in words, that the materials elasticity is $\frac{\epsilon}{\epsilon-1}$ times the ratio of materials input expenditures to sales. Observe also that, by re-arranging the two first-order conditions, we can write expressions for production and the price

$$Y = \left(\left(\frac{\epsilon-1}{\epsilon}\right)\theta_i\left(\frac{\alpha_j}{q_j}\right)^{\alpha}\left(\frac{1-\alpha_j}{r_j}\right)^{1-\alpha_j}\right)^{\epsilon} \Rightarrow p = \left(\frac{\epsilon}{\epsilon-1}\right)\theta_i^{-1}\left(\frac{q_j}{\alpha_j}\right)^{\alpha_j}\left(\frac{r_j}{1-\alpha_j}\right)^{1-\alpha_j} \tag{223}$$

and observe that $\theta_i^{-1}\left(\frac{q_j}{\alpha_j}\right)^{\alpha_j}\left(\frac{r_j}{1-\alpha_j}\right)^{1-\alpha_j}$ is the firm's marginal cost. Hence, we can define $\mu = \frac{\epsilon}{\epsilon-1} > 1$ as the firm's markup and write the shares as required:

$$\alpha = \mu\frac{q_j M_i}{p_i Y_i} \tag{224}$$

Finally, we now apply Equations 216 and 217 to calculate productivity. Assume that we observe materials expenditure $q_j M_i$ and capital value $p_{K,j} K_i$, where $p_{K,j}$ is an (unobserved) price of capital. We find

$$\log\tilde{\theta}_i = \left(1 - \frac{1}{\epsilon}\right)(\log\theta_i - \alpha\log q_j - (1-\alpha)\log p_{K,j}) \tag{225}$$

We finally observe that the industry-level means are

$$\chi_j = \left(1 - \frac{1}{\epsilon}\right)\left(\log\bar{\theta}_j - \alpha\log q_j - (1-\alpha)\log p_{K,j}\right) \tag{226}$$

where $\log \bar{\theta}_j$ is the mean of $\log \theta_i$ over the industry. Hence,

$$\log \hat{\theta}_i = \left(1 - \frac{1}{\epsilon}\right)(\log \theta_i) \tag{227}$$

or our measurement captures physical TFP, up to scale.

E Additional Empirical Results

E.1 A Test for Coefficient Stability

Here, we study the bias that may arise from omitted variables in our estimation of the effect of narrative optimism on hiring, or δ^{OP} in Section 5.1, Equation 34, and Table 1. In particular, we apply the method of Oster (2019) to bound bias in the estimate of δ^{OP} under external assumptions about selection on unobservable variables and to calculate an extent of unobservable selection that could be consistent with a point estimate $\delta^{OP} = 0$ that corresponds to our null hypothesis (*i.e.,* "narrative optimism is irrelevant for hiring"). We find that our results are highly robust by this criterion.

Set-up and Review of Methods. To review, our estimating equation is

$$\Delta \log L_{it} = \delta^{OP} \text{opt}_{it} + \gamma_i + \chi_{j(i),t} + \tau' X_{it} + \varepsilon_{it} \tag{228}$$

Hiring and optimism are constructed as described in Section 4, at the level of firms and fiscal years. We treat firm and industry-by-time fixed effects as baseline controls that are necessary for interpreting the regression.[32] As our main "discretionary" controls, we consider current and past TFP and lagged labor—that is, $X_{it} = \{\log \hat{\theta}_{it}, \log \hat{\theta}_{i,t-1}, \log L_{i,t-1}\}$. Under our baseline model, these controls help increase precision, as they are in principle observable variables that explain hiring (Corollary 5). Thus, in this Appendix, we will study the regression model in which the fixed effects are partialed out of both the outcome, main regression, and controls, as indicated below with the $\perp$ superscript:

$$\Delta \log L_{it}^{\perp} = \delta^{OP} \text{opt}_{it}^{\perp} + \tau' X_{it}^{\perp} + \varepsilon_{it}^{\perp} \tag{229}$$

The essence of the method proposed by Oster (2019), who builds on the approach of Altonji, Elder, and Taber (2005), is to extrapolate the change in the coefficient in interest upon the addition of control variables, taking into account the better fit (*i.e.,* additional R^2) from adding the new regressors. To exemplify the logic, consider a case in which we first estimated Equation 229 without controls, obtaining a coefficient estimate of $\hat{\delta}_{NC}^{OP}$ and an R^2 of $\hat{R}_{NC}^2$, and then estimated the same equation with controls, obtaining a coefficient estimate of $\hat{\delta}_{C}^{OP}$ and an R^2 of $\hat{R}_{C}^2$. Both estimates are restricted to a common sample, for comparability. If $\hat{R}_{C}^2 = 1$, then (up to estimation error) we might presume that $\hat{\delta}_{C}^{OP} - \hat{\delta}_{NC}^{OP}$ estimates the entirety of the theoretically possible omitted variables bias, as there is no remaining unmodeled variation in hiring. If $\hat{R}_{C}^2 < 1$ and $\hat{R}_{C}^2 - \hat{R}_{NC}^2$ is small (*i.e.,* the controls did not greatly improve fit), then we might presume that the residual still contains unobserved variables that could contribute toward more bias—in other words, the observed omitted variables bias $\hat{\delta}_{C}^{OP} - \hat{\delta}_{NC}^{OP}$ is only a small fraction of what is possible.

To formalize this idea, Oster (2019) introduces two auxiliary parameters: λ (the *proportional degree of selection*, called δ in the original paper), which controls the relative effect of observed and

[32]The latter, in particular, controls for the effect of fundamentals on hiring in our macroeconomic model. We leverage this interpretation of the *biased* estimate of δ^{OP} from a regression lacking this fixed effect in Appendix F.3.

unobserved controls on the outcome, and $\bar{R}^2$, which is the maximum achievable fit of the regression with all (possibly bias-inducing) controls, presumed in the example above to be 1. Conditional on $\bar{R}^2$, Oster (2019) proposes an intuitively reasonable (and, in special cases and under specific asymptotic arguments, consistent) estimator for the degree of selection required to induce a zero coefficient, $\hat{\lambda}^*$. Conditional on both $\bar{R}^2$ and λ, Oster (2019) also proposes a bias-corrected coefficient estimator, which is $\hat{\delta}^*_{OP}$ in our language.

The key parameter that the researcher has to specify for the first calculation is $\bar{R}^2$: the proportion of variance in the outcome variable (hiring, net of firm and sector-by-time fixed effects) that can be explained by factors that correlate with the variable of interest (optimism) and explain the outcome variable. As the main source of omitted variation that could influence optimism and hiring is news about fundamentals, we benchmark $\hat{\bar{R}}^2$ by estimating a regression in which we include our base control set $X_{it} = \{\log \hat{\theta}_{it}, \log \hat{\theta}_{i,t-1}, \log L_{i,t-1}\}$ and control for two years of future fundamentals and labor choice, or

$$Z_{it} = \{\log \hat{\theta}_{i,t+1}, \log \hat{\theta}_{i,t+2}, \log L_{i,t+1}, \log L_{i,t+2}\}$$

This yields $\hat{\bar{R}}^2 = 0.459$. Oster (2019) also suggests as a benchmark that $\bar{R}^2$ could be taken as three times the R^2 in the controlled regression. We also report robustness to $\bar{R}^2_{\Pi} = 0.387$, three times the value of $R^2 = 0.129$ that we find in the controlled regression. Thus, our baseline value of $\hat{\bar{R}}^2 = 0.459$ is more demanding than that suggested by Oster (2019). We finally construct the bias-corrected coefficients assuming $\lambda = 1$, or equal selection on unobservables and observables, for both values of $\bar{R}^2$.

Results. We report the results of this exercise in Table A2. Under our baseline value of $\hat{\bar{R}}^2 = 0.459$, we find that the degree of selection required to induced a zero coefficient is $\hat{\lambda}^* = 1.69$. This is well above the value of $\hat{\lambda}^* = 1$ that Oster (2019) suggests is likely to be conservative. Under the "three times R^2" benchmark, we obtain that $\hat{\lambda}^* = 2.15$. In both cases, we are robust to there being more selection on unobservables than on observables. According to Oster (2019), approximately 50% of the published top-journal articles in their sample are not robust to this extent of selection.

E.2 Alternative Empirical Strategy: CEO Change Event Studies

To further isolate variation in the narratives held by firms that is unrelated to fundamentals, we study the effects on hiring of changes in narratives induced by plausibly exogenous managerial turnover.

Data. To obtain plausibly exogenous variation in narratives held at the firm level, we will examine the year-to-year change in firm-level narratives stemming from plausibly exogenous CEO changes. To do this, we use the dataset of categorized CEO exits compiled by Gentry, Harrison, Quigley, and Boivie (2021). These data comprise 9,390 CEO turnover events categorized by the reason for the CEO exit. The categorization was performed using primary sources (*e.g.,* press releases, newspaper articles, and regulatory filings) by undergraduate students in a computer lab, supervised by graduate students, with the final dataset checked by both a data outsourcing company and an additional student. We restrict attention to CEO exists caused by death, illness, personal issues, and voluntary retirements. Importantly, we exclude all CEO exits caused by inadequate job performance, quits, and forced retirement.

The Effect of Optimism on Hiring. We first revisit our empirical strategy for measuring the effect of optimism on firms' hiring, using the CEO change event studies. For all firms i and years t such that i's CEO leaves because of death, illness, personal issues or voluntary retirements, we estimate the regression equation

$$\Delta \log L_{it} = \delta^{CEO} \text{opt}_{it} + \psi \, \text{opt}_{i,t-1} + \tau' X_{it} + \chi_{j(i),t} + \varepsilon_{it} \tag{230}$$

This differs from our baseline Equation 34 by including parametric controls for lagged values of the narrative loadings, but removing a persistent firm fixed effect.[33] If the studied CEO changes are truly exogenous, as we have suggested, then the narrative loadings of the new CEO are, conditional on the narrative loadings of the previous CEO, solely due to the differences in worldview across these two senior executives. Of course, CEO exits may be disruptive and reduce firm activity. Any time- and industry-varying effects of CEO exits via disruption are controlled for by the intercept of the regression $\chi_{j(i),t}$, since the equation is estimated only on the exit events. Moreover, any within-industry, time-varying, and idiosyncratic disruption is captured through our maintained productivity control. Under this interpretation, the coefficient of interest δ^{CEO} isolates the effect of optimism on hiring purely via the channel of changing managements' narratives.

We present our results in Table A22. We obtain estimates of δ^{CEO} that are quantitatively similar to our estimates of δ^{OP} in Table 1 (columns 1, 2, and 3). In column 4, we estimate a regression equation on the full sample that measures the direct effect of CEO changes and its interaction with the new management's optimism. Specifically, we estimate

$$\begin{aligned} \Delta \log L_{it} = {} & \delta^{\text{NoChange}} \text{opt}_{it} + \delta^{\text{Change}} (\text{opt}_{it} \times \text{ChangeCEO}_{it}) + \alpha^{\text{Change}} \text{ChangeCEO}_{it} \\ & + \psi \, \text{opt}_{i,t-1} + \tau' X_{it} + \chi_{j(i),t} + \varepsilon_{it} \end{aligned} \tag{231}$$

where ChangeCEO_{it} is an indicator for our plausibly exogenous CEO change events. We find that CEO changes in isolation reduce hiring ($\alpha^{\text{Change}} < 0$) but also that the effect of optimism is magnified when it accompanies a CEO change ($\delta^{\text{Change}} > 0$). This is further inconsistent with a story under which omitted fundamentals lead us to overestimate the effect of optimism on hiring.

Contagiousness from CEO Change Spillovers. We next leverage changes in within-sector and peer-set optimism induced by plausibly exogenous CEO changes as instruments for the level of optimism within these groups. Concretely, we construct an instrument equal to the contribution

[33] With a firm fixed effect, the regression coefficients of interest would be identified only from firms with multiple plausibly exogenous CEO exits.

toward optimism from firms whose CEOs changed for a plausibly exogenous reason, or

$$\overline{\text{opt}}^{\text{ceo}}_{j(i),t-1} = \frac{1}{|M_{j(i),t}|} \sum_{k \in M^c_{j(i),t}} \text{opt}_{k,t-1} \tag{232}$$

where $M_{j(i),t}$ is the set of firms in industry $j(i)$ at time t, and $M^c_{j(i),t}$ is the subset that had plausibly exogenous CEO changes. We construct the peer-set instrument $\overline{\text{opt}}^{\text{ceo}}_{p(i),t-1}$ analogously. We use $(\overline{\text{opt}}^{\text{ceo}}_{j(i),t-1}, \overline{\text{opt}}^{\text{ceo}}_{p(i),t-1})$ as instruments for $(\overline{\text{opt}}_{j(i),t-1}, \overline{\text{opt}}_{p(i),t-1})$ in the estimation of Equation 39. We present the corresponding estimates in Table A23. We find similar point estimates under IV and OLS, although the IV estimates are significantly noisier.

E.3 Measuring Contagiousness via Granular Instrumental Variables

As an alternative strategy to estimate contagiousness, we apply the methods of Gabaix and Koijen (2020) to construct "granular variables" that aggregate idiosyncratic variation in large firms' narrative loadings. We find evidence that the idiosyncratic optimistic updating of large firms induces optimistic updating, a form of contagiousness.

Constructing the Granular Measures. We construct our granular instruments via the following algorithm. We first estimate a firm-level updating regression that controls non-parametrically for aggregate trends and parametrically for firm-level conditions. Specifically, we estimate

$$\text{opt}_{it} = \tau' X_{it} + \chi_{j(i),t} + \gamma_i + u_{it} \tag{233}$$

where $\chi_{j(i),t}$ is an industry-by-time fixed effect (sweeping out industry-specific aggregate shocks), γ_i is a firm fixed effect (sweeping out compositional effects), and X_{it} is the largest vector of controls used in the analysis of Section 5.1, consisting of: lagged log employment, current and lagged log TFP, log stock returns, the log book to market ratio, and leverage. We construct the empirical residuals $\hat{u}_{it}$. To construct the aggregate granular variable, $\overline{\text{opt}}^{g,sw}_t$, we take a sales-weighted average of these residuals:

$$\overline{\text{opt}}^{g,sw}_t = \sum_i \frac{\text{sales}_{it}}{\sum_i \text{sales}_{it}} \hat{u}_{it} \tag{234}$$

To construct an industry-level granular variable, $\overline{\text{opt}}^{g,sw}_{j(i),t}$, we take the leave-one-out sales-weighted average of the $\hat{u}_{it}$:

$$\overline{\text{opt}}^{g,sw}_t = \sum_{i': j(i)=j(i'), i' \neq i} \frac{\text{sales}_{i't}}{\sum_i \text{sales}_{i't}} \hat{u}_{i't} \tag{235}$$

We also construct agggregate and industry (leave-one-out) averages of opt_{it} for comparison. We denote these variables as $\overline{\text{opt}}^{sw}_t$ and $\overline{\text{opt}}^{sw}_{j(i),t}$, respectively.

Empirical Strategy. At the aggregate level, we first consider a variant of our main model Equation 38, but with one of the sales-weighted variables $Z_t \in \{\overline{\text{opt}}_t^{sw}, \overline{\text{opt}}_t^{g,sw}\}$:

$$\text{opt}_{it} = u \ \text{opt}_{i,t-1} + s \ Z_{t-1} + r \ \Delta \log Y_{t-1} + \gamma_i + \varepsilon_{it} \tag{236}$$

The coefficient s measures contagiousness with respect to the sales-weighted measures of optimism. We estimate Equation 236 by OLS, and also estimate a version in which the granular variable $\overline{\text{opt}}_t^{g,sw}$ is an instrumental variable for the raw sales-weighted average $\overline{\text{opt}}_t^{sw}$.

Similarly, at the industry level, we estimate the model

$$\text{opt}_{it} = u_{\text{ind}} \ \text{opt}_{i,t-1} + s_{\text{ind}} \ Z_{j(i),t-1} + r_{\text{ind}} \ \Delta \log Y_{j(i),t-1} + \gamma_i + \chi_t + \varepsilon_{it} \tag{237}$$

for $Z_{j(i),t} \in \{\overline{\text{opt}}_{j(i),t}^{sw}, \overline{\text{opt}}_{j(i),t}^{g,sw}\}$. As above, we estimate this first via OLS for each outcome variable, and then via IV where the granular variable $\overline{\text{opt}}_{j(i),t}^{g,sw}$ is an instrument for the raw sales-weighted average $\overline{\text{opt}}_{j(i),t}^{sw}$.

Results. We present our results in Table A24. First, studying aggregate contagiousness, we find strong evidence that $s > 0$ when measured with the raw sales-weighted average or its granular component (columns 1 and 2). We moreover find significant evidence of $s > 0$ in the IV estimation (column 3). Our IV point estimate of $\hat{s} = 0.308$ greatly exceeds the OLS estimate of $\hat{s} = 0.0847$.

At the industry level, we find strong evidence of contagiousness via the sales-weighted measure (column 4). We find imprecise estimates, centered around 0, for contagiousness measured with the granular variable (column 5) or via the granular IV (column 6). However, the granular IV estimate is noisily estimated and is not significantly different from the point estimate of column 4.

F Additional Details on Model Estimation

In this appendix, we provide complete details on the estimation of the model.

F.1 Normalizations

We begin by making two economically irrelevant normalizations to ease the interpretation of the results. First, we set $a_0 = 0$. As we are not concerned with the level of output in the model, this is a harmless normalization. Second, we normalize the updating rules so that an economy with no productivity shocks and no narrative shocks has an equal fraction of optimists and pessimists. As we have estimated optimism in the data as being above or below the time-series average level of optimism, this is also harmless normalization. More specifically, we update the LAC transition probabilities by introducing a parameter C_P:

$$\begin{aligned} P_O^H(\log Y, Q, \varepsilon) &= [u + r\log Y + sQ + C_P + \varepsilon]_0^1 \\ P_P^H(\log Y, Q, \varepsilon) &= [-u + r\log Y + sQ + C_P + \varepsilon]_0^1 \end{aligned} \tag{238}$$

And we set C_P such that an economy with neutral fundamentals ($\log\theta_t = \log\theta_{t-1} = 0$), equal optimists and pessimists ($Q = 1/2$), and no narrative shocks ($\varepsilon = 0$) continues to have equal optimists and pessimists. Specifically, this implies $C_P = \frac{1-s}{2}$.

F.2 Estimation Methodology

To calibrate the model, we proceed in four steps.

1. *Setting macro parameters.* We first set $(\epsilon, \gamma, \psi, \alpha)$. In Section 6.1 and Table 5, we describe our baseline method based on matching estimates of the deep parameters from the literature. We also consider two other strategies as robustness checks. First, to target estimated fiscal multipliers in the literature, we use the same external calibration of α (returns to scale) and ϵ (elasticity of substitution), and set (γ, ψ) to match the desired multiplier. Since the exact choice of these parameters is arbitrary subject to obtain the correct multiplier, we normalize $\gamma = 0$ and vary only ψ. Second, we match an estimate of the multiplier implied by our own data and an exact formula for the omitted variable bias incurred in estimating the effect of optimism on hiring without controlling for general-equilibrium effects via a time fixed effect. We outline that strategy for estimating the multiplier in Section F.3 below, and we map this to deep parameters exactly as described in our method for matching the literature's estimated multiplier.
2. *Calibrating the effect of optimism on output.* We observe that, conditional on $(\epsilon, \gamma, \psi, \alpha)$ and an estimate of δ^{OP}, we have identified $f(Q_t)$. We take our estimate of δ^{OP} from column 1 in Table 1. This regression identifies δ^{OP} for the reasons described in Corollary 5.
3. *Calibrating the statistical properties of fundamentals (κ, ρ, σ).*

(a) *Computing fundamental output.* We construct a cyclical component of output, $\log \hat{Y}_t$, as band-pass filtered US real GDP (Baxter and King, 1999).[34] We apply our estimated function f to our measured time series of optimism to get an estimated optimism component of output. we then calculate

$$\log \hat{Y}_t^f = \log \hat{Y}_t - \hat{f}(\hat{Q}_t) \tag{239}$$

(b) *Estimating the ARMA representation.* Using our 24 annual observations of $\log \hat{Y}_t^f$, we estimate a Gaussian-errors ARMA(1,1) model via maximum likelihood. Our point estimates are

$$\log \hat{Y}_t^f - 0.086 \log \hat{Y}_t^f = .0078(\zeta_t + .32\, \nu_{t-1}) \tag{240}$$

This implies $\rho = 0.086$, $a_1\sigma = .0078$, and $a_2\sigma = .32$. ρ is therefore identified immediately.

(c) *Calibrating (κ, σ).* We search non-linearly for values of (κ, σ) that satisfy $a_1\sigma = 0.0078$ and $a_2\sigma = 0.32$. There is a unique such pair, reported in Table 5, which also is therefore the maximum likelihood estimate of (κ, σ).

4. *Calibrating the updating rule $(u, r, s, \sigma_\varepsilon^2)$.* The coefficients of the LAC updating model are estimated in column 1 of Table 3. Conditional on the previous calibration, we set σ_ε^2 so that within model Q_t has the same standard deviation as the aggregate optimism time series, which is 0.0533.

F.3 Estimating a Demand Multiplier in Our Empirical Setting

Here, we describe a method for estimating a demand multiplier in our data on optimism and firm hiring. This circumvents the step of external calibration for the multiplier, but relies on correct specification of the time-series correlates of aggregate optimism. Reassuringly, this method yields a general-equilibrium demand multiplier that is comparable to our baseline calibration and our literature-derived calibration.

Mapping the Model to Data. By Corollary 5, we first recall that firms' hiring can be written in equilibrium as

$$\Delta \log L_{it} = \tilde{c}_{0,i} + \tilde{c}_{10} \log \theta_t + \tilde{c}_{11} \log \theta_{t-1} + \tilde{c}_2 f(Q_t) + \tilde{c}_3 \log \theta_{it} + \tilde{c}_4 \log L_{i,t-1} + \delta^{OP} \lambda_{it} + \zeta_{it} \tag{241}$$

where ζ_{it} is an i.i.d. normal random variable with zero mean and λ_{it} is the indicator for having adopted the optimistic narrative.

In the data, our estimating equation without control variables had the following form

$$\Delta \log L_{it} = \gamma_i + \chi_{j(i),t} + \delta^{OP} \text{opt}_{it} + z_{it} \tag{242}$$

[34] Specifically, we filter to post-war quarterly US real GDP data (Q1 1947 to Q1 2022). We use a lead-lag length of 12 quarters, a low period of 6 quarters, and a high period of 32 quarters. We then average these data to the annual level.

This maps to the structural model with $\gamma_i = \tilde{c}_{0,i}$, $\chi_{j(i),t} = \tilde{c}_{10} \log \theta_t + \tilde{c}_{11} \log \theta_{t-1} + \tilde{c}_2 f(Q_t)$, and $z_{it} = \zeta_{it} + \tilde{c}_3 \log \theta_{it} + \tilde{c}_4 \log L_{i,t-1}$. Under the model-implied hypothesis that $\mathbb{E}[z_{it}\text{opt}_{it}] = 0$, then the OLS regression of $\Delta \log L_{it}$ on opt_{it}, conditional on the indicated fixed effects, identifies δ^{OP}.

We consider now an alternative regression equation which is a variant of the above specification without the time fixed effect and with parametric controls for aggregate TFP:

$$\Delta \log L_{it} = \gamma_i + \delta^{OP}\text{opt}_{it} + \tilde{c}_{10} \log \theta_t + \tilde{c}_{11} \log \theta_{t-1} + \tilde{z}_{it} \tag{243}$$

Observe that the new residual, relative to the old residual, is contaminated by the equilibrium effect of optimism. That is, $\tilde{z}_{it} = z_{it} + \tilde{c}_2 f(Q_t)$. To refine this further, we apply the linear approximation $f(Q_t) \approx \frac{\alpha\delta^{OP}}{1-\omega} Q_t$ and the observation that $\tilde{c}_2 = \omega$, so we can write $\tilde{z}_{it} = z_{it} + \frac{\alpha\omega}{1-\omega}\delta^{OP} Q_t$.

We now derive a formula for omitted variables bias in the estimate of δ^{OP} from an OLS estimation of Equation 243. Let X denote a finite-dimensional matrix of data on opt_{it}, firm-level indicators (*i.e.,* the regressors corresponding to the firm fixed effects), and current and lagged aggregate TFP. Similarly, let Y be a finite-dimensional matrix of data on $\Delta \log L_{it}$. The OLS regression coefficient in this finite sample is $\hat{\delta} = ((X'X)^{-1}X'Y)_1$. Using the standard formula for omitted variables bias:

$$\begin{aligned} \mathbb{E}[\hat{\delta}|X] &= \delta^{OP} + \left((X'X)^{-1}\mathbb{E}[X'Q|X] \frac{\alpha\omega}{1-\omega} \delta^{OP} \right)_1 \\ &= \delta^{OP} \left(1 + \frac{\alpha\omega}{1-\omega} \left((X'X)^{-1}\mathbb{E}[X'Q|X] \right)_1 \right) \end{aligned} \tag{244}$$

where Q is the vector of observations of Q_t. We can then observe that:

$$(X'X)^{-1}\mathbb{E}[X'Q|X] = \mathbb{E}\left[(X'X)^{-1}X'Q|X \right] \tag{245}$$

Which is the (expected) OLS estimate of β in the following regression:

$$Q_t = \gamma_i + \beta_O^Q \text{opt}_{it} + \beta_\theta^Q \log \theta_t + \beta_{\theta_{-1}}^Q \log \theta_{t-1} + \varepsilon_t \tag{246}$$

But we observe that, averaging both sides, that $\gamma_i = \beta_\theta^Q = \beta_{\theta_{-1}}^Q = 0$ and $\beta_O^Q = 1$. Thus, $((X'X)^{-1}\mathbb{E}[X'Q|X])_1 = 1$. We therefore obtain that:

$$\mathbb{E}[\hat{\delta} \mid X] = \delta^{OP} \left(1 + \frac{\alpha\omega}{1-\omega} \right) \tag{247}$$

Hence, given a population estimate of the *biased* OLS estimate and an external calibration of α, we can pin down the complementarity ω and the multiplier $\frac{1}{1-\omega}$. Naturally this strategy relies on correctly measuring aggregate TFP as measurement error in that variable would contaminate this estimation. Moreover, it requires us to assume that all variation in aggregate output that is not due to TFP is due to optimism or forces entirely orthogonal to optimism; in view of our running assumption that the spread of optimism is associative, these other forces therefore also have to be completely transitory, lest they be incorporated into current optimism via associative updating in a previous period. These assumptions are strong and are why we do not adopt this strategy for our main quantitative analysis. Nevertheless, we will find similar results, as we now describe.

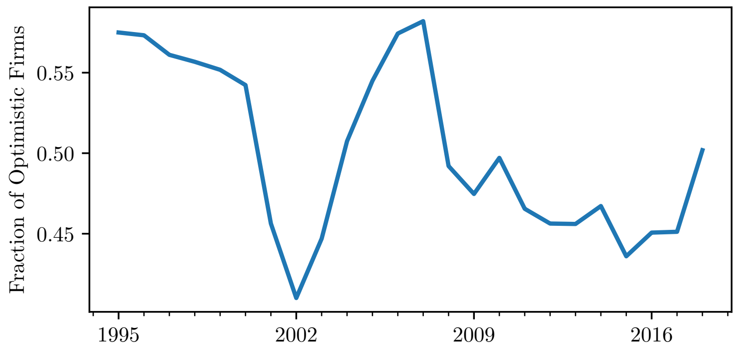
Empirical Application and Results. To operationalize this in practice, we compare estimates of Equation 242 and 243. For the latter, we proxy TFP using the cyclical component of both capacity adjusted and capacity un-adjusted TFP using the data of Fernald (2014).[35] We moreover maintain the assumption of $\alpha = 1$, or constant returns to scale, to map our estimates back to implied multipliers.

Our results are reported in Table A26, along with the associated values of complementarity ω and the multiplier $\frac{1}{1-\omega}$. Using capacity-adjusted and unadjusted TFP, we respectively obtain estimates of 1.46 and 1.37 for the multiplier. These are lower than our baseline estimate, but comparable to our estimates based on structural modeling in the literature. Both estimates are below our baseline calibration of 1.96 but above our multiplier-literature calibration of 1.33. In Table A19, we report our quantitative results under the assumed multiplier of 1.46. We find that, as expected, these estimates imply an role for optimism that is an intermediate between the baseline and multiplier-literature calibrations.

[35] Mirroring our filtering of US real GDP, we apply the Baxter and King (1999) band-pass filter to post-war quarterly data using a lead-lag length of 12 quarters, a low period of 6 quarters, and a high period of 32 quarters. We then average these data to the annual level.

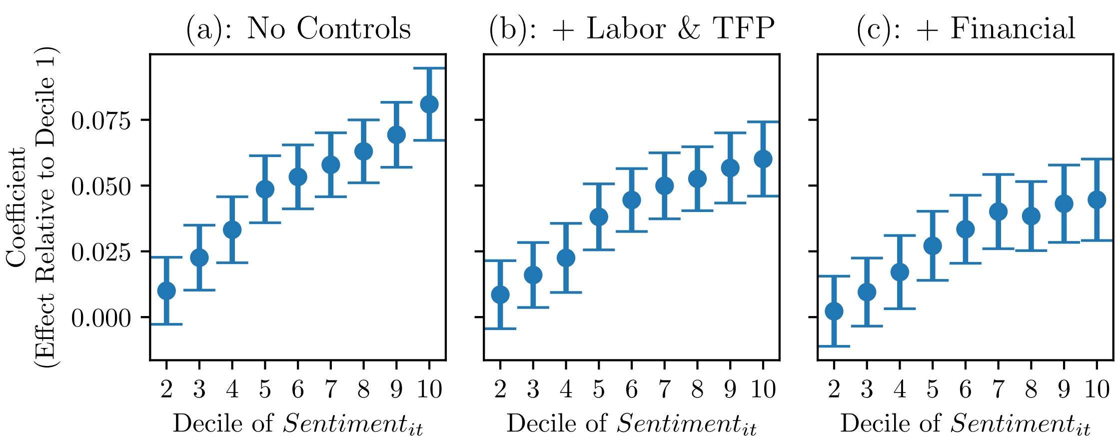
G Additional Figures and Tables

Figure A1: The Time Series of Narrative Optimism



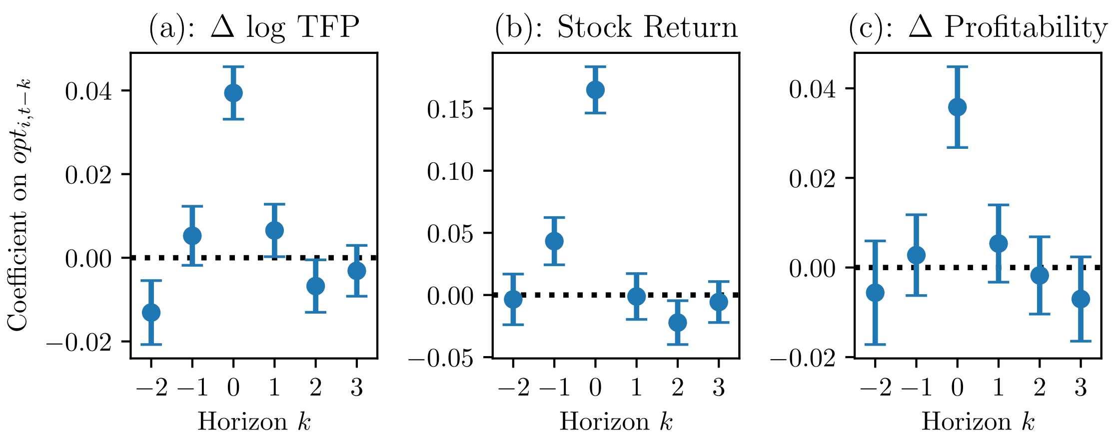
Notes: The plotted variable is the fraction of optimistic firms in each fiscal year. By construction, half of the firm-year observations in our sample are coded as optimistic. Section 4.2 describes our measurement strategy in full detail.

Figure A2: Net Sentiment and Hiring



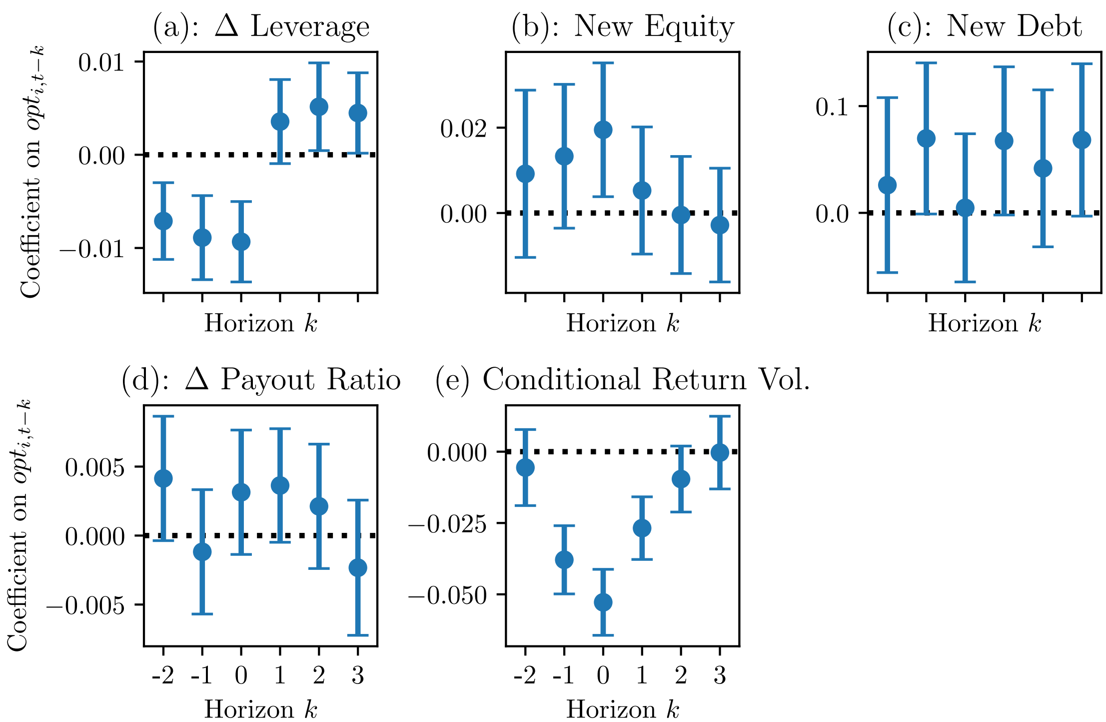
Notes: In each panel, we show estimates from the regression $\Delta \log L_{it} = \sum_{q=1}^{10} \beta_q \cdot (\text{sentiment}_{iqt}) + \tau' X_{it} + \gamma_i + \chi_{j(i),t} + \epsilon_{it}$, where sentiment_{iqt} indicates decile q of the continuous sentiment variable. Panel (a) estimates this equation without controls (like column 1 of Table 1); panel (b) adds controls for lagged labor and current and lagged log TFP (like column 2 of Table 1); and panel (c) adds controls for the log book to market ratio, log stock return, and leverage (like column 3 of Table 1). The excluded category in each regression is the first decile of sentiment_{it}. In all specifications, we trim the 1% and 99% tails of the outcome variable. Error bars are 95% confidence intervals. Standard errors are double-clustered by firm ID and industry-year.

Figure A3: Dynamic Relationship between Optimism and Firm Fundamentals, Conference-Call Measurement



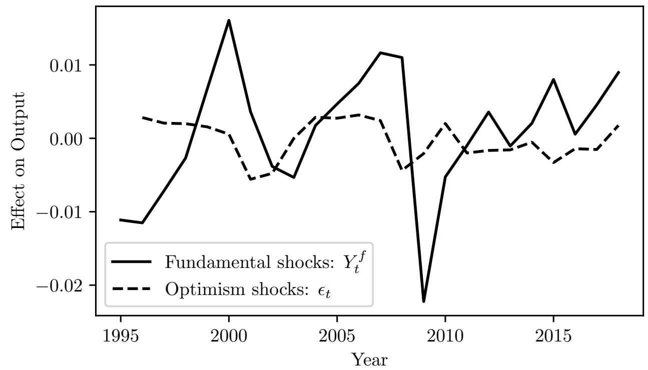
Notes: The regression model is Equation 36 (as in Figure 4), but measuring optimism from sales and earnings conference calls. Each coefficient is estimated from a separate projection regression. The outcomes are (a) the log change in TFP, calculated as described in Appendix D.2, (b) the log stock return inclusive of dividends over the fiscal year, and (c) changes in profitability, defined as earnings before interest and taxes (EBIT) as a fraction of the previous fiscal year's variable costs. In all specifications, we trim the 1% and 99% tails of the outcome variable. Each coefficient is estimated from a separate projection regression. Error bars are 95% confidence intervals.

Figure A4: Dynamic Relationship Between Optimism and Financial Variables



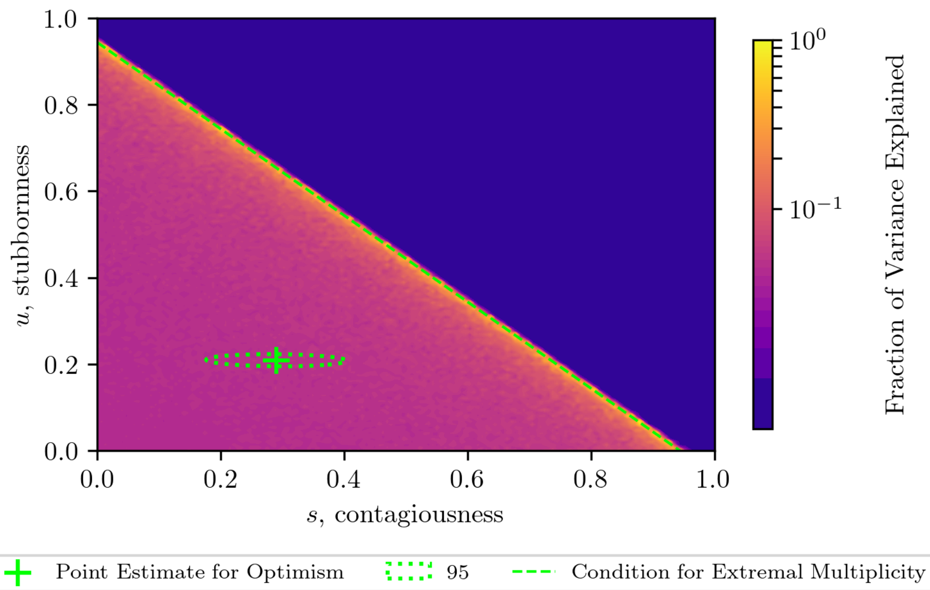
Notes: The regression model is Equation 36 (as in Figure 4), but with financial fundamentals as outcomes. Each coefficient is estimated from a separate projection regression. The outcome variables are: (a) the fiscal-year-to-fiscal-year difference in leverage, which is total debt (short-term debt plus long-term debt); (b) sale of common and preferred stock minus buybacks, normalized by the total equity outstanding in the previous fiscal year; (c) short-term debt plus long-term debt issuance, normalized by the total debt in the previous fiscal year; (d) total dividends divided by earnings before interest and taxes (EBIT); and (e) squared stock returns (volatility). In all specifications, we trim the 1% and 99% tails of the outcome variable. Error bars are 95% confidence intervals. Standard errors are two-way clustered by firm ID and industry-year.

Figure A5: Fundamental and Optimism Shocks That Explain US GDP



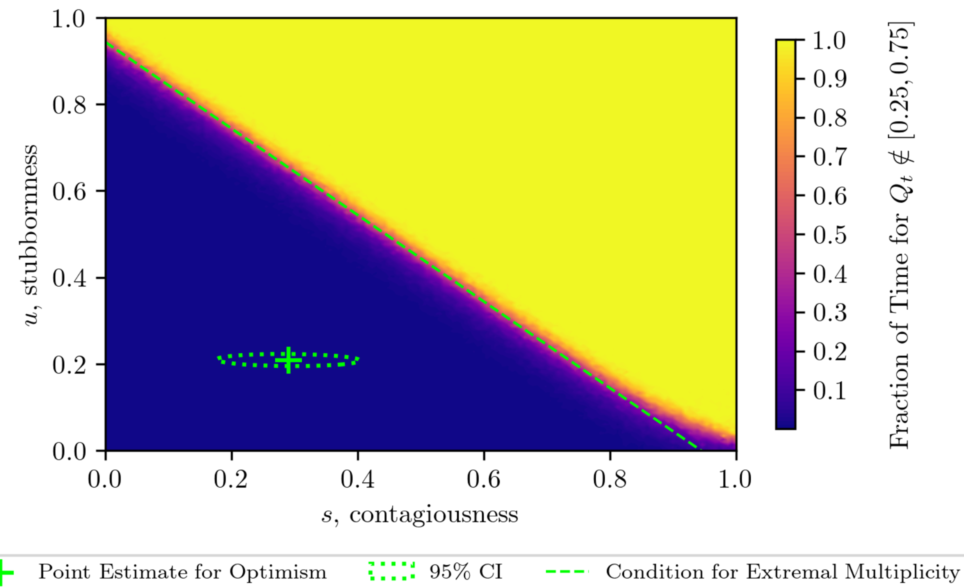
Notes: This figure shows the shocks that rationalize movements in optimism and detrended real GDP in recent US history, as analyzed in Section 6.2. The solid line is the exogenous process for fundamental output and the dashed line is the sequence of shocks in narrative evolution. The dashed line is rescaled by $\delta^{OP}(1-\omega)^{-1}$ to be, up to linear approximation of f, in units of output.

Figure A6: Variance Decomposition for Different Values of Stubbornness and Contagiousness, No Optimism Shocks



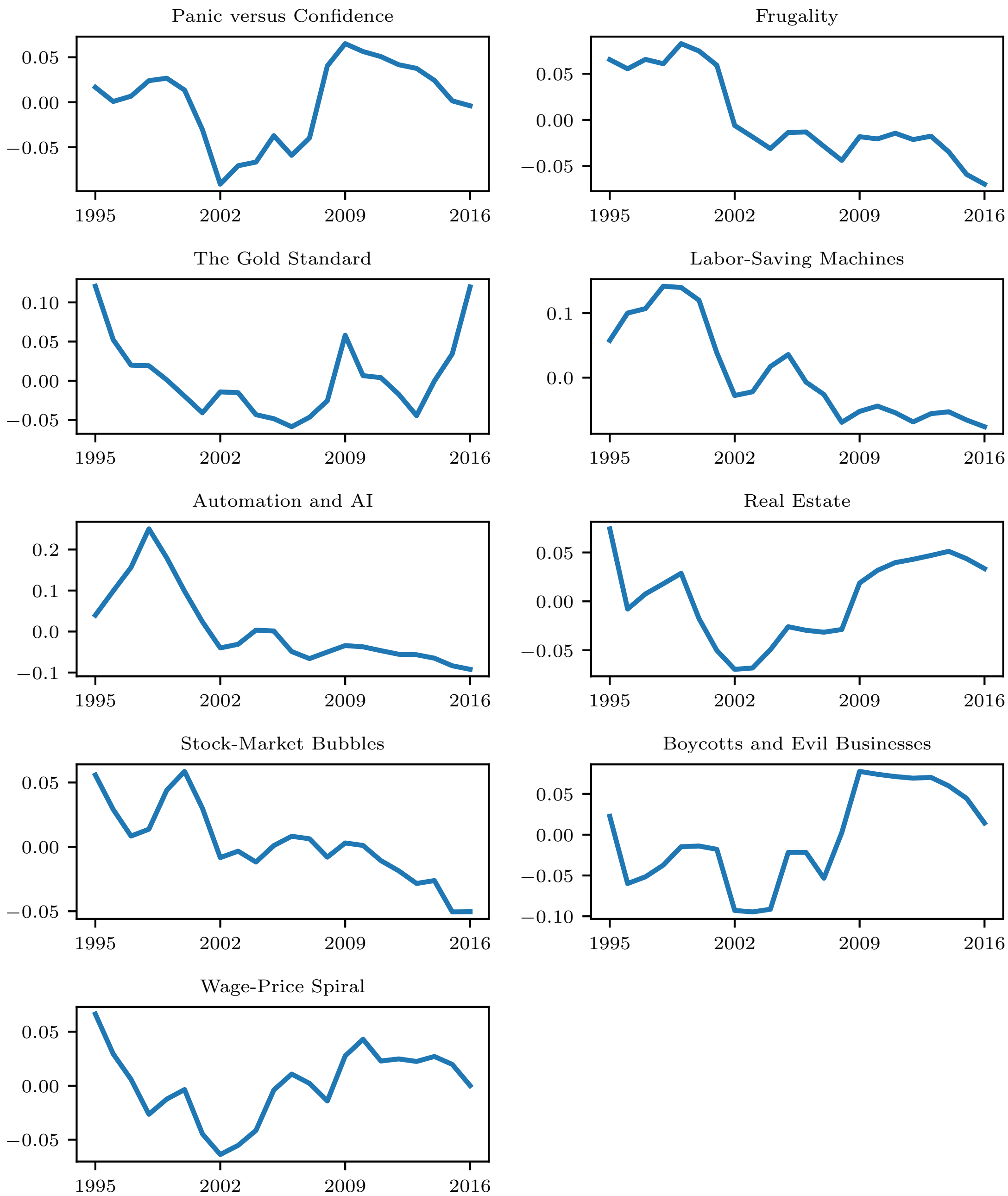
Notes: This Figure replicates Figure A6, with a different color bar scale, in the variant model with no exogenous shocks to optimism. Calculations vary u and s, holding fixed all other parameters at their calibrated values. The shading corresponds to the fraction of variance explained by optimism, or Share of Variance Explained$_0$ defined in Equation 43. The plus is our calibrated value of (u, s), corresponding to a variance share 4.7%, and the dotted line is the boundary of a 95% confidence set. The dashed line is the condition of extremal multiplicity from Corollary 4 and Equation 27.

Figure A7: Tendency Toward Extremal Optimism



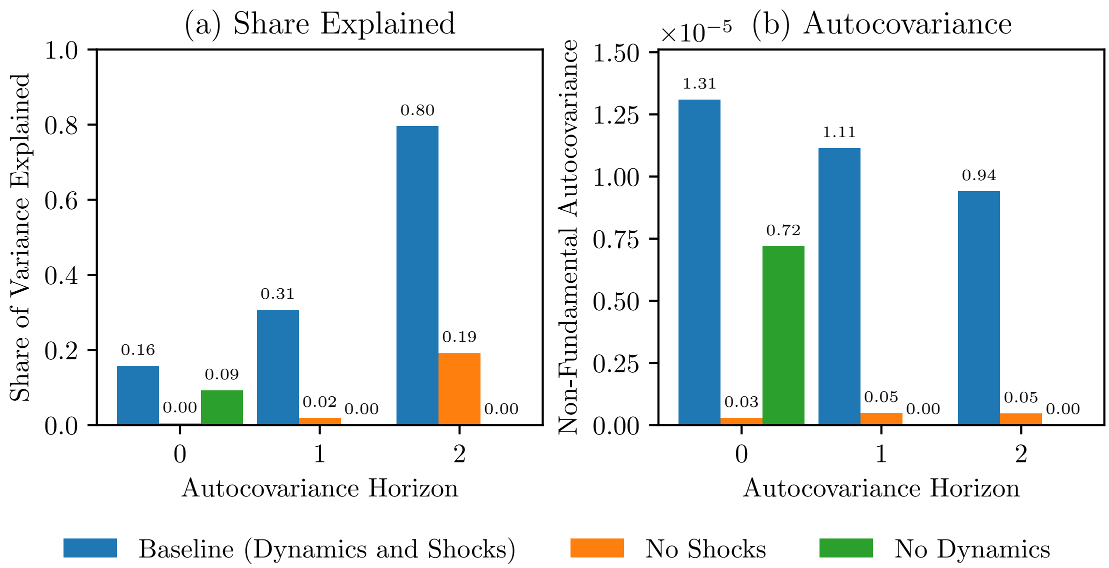
Notes: This Figure plots, in color, the fraction of time that optimism Q_t lies outside of the range $[0.25, 0.75]$ and therefore concentrates at extreme values. Calculations vary u and s, holding fixed all other parameters at their calibrated values. The plus is our calibrated value of (u, s), corresponding to an extremal share of 0%, and the dotted line is the boundary of a 95% confidence set. The dashed line is the condition of extremal multiplicity from Corollary 4 and Equation 27.

Figure A8: Time Series for Shiller's Perennial Economic Narratives



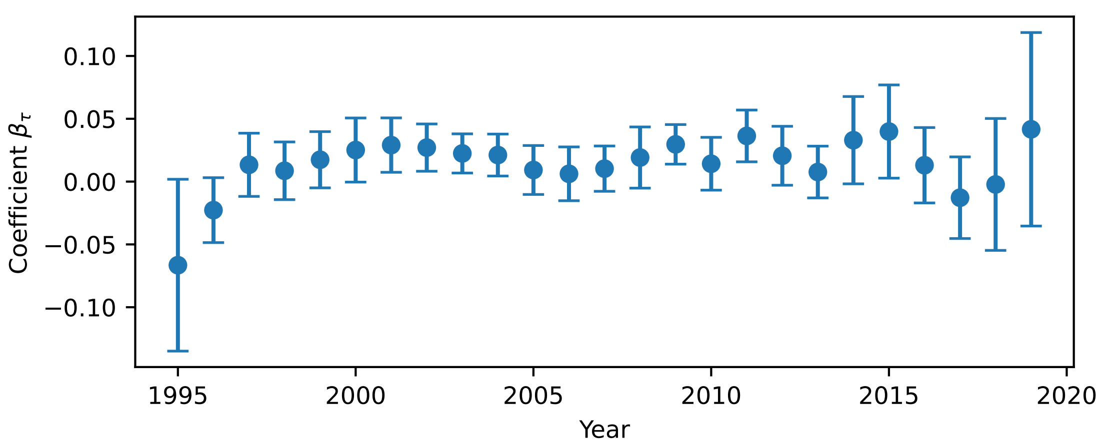
Notes: Each panel plots the time-series average of the narrative variable defined for the corresponding chapter of Shiller (2020)'s *Narrative Economics*. The units are cross-sectional averages of z-score transformed variables.

Figure A9: Optimism and Output Variance in the Constellations Model



Notes: This figure recreates Figure 6 in the model with constellations. The left panel plots the fraction of variance, one-year autocovariance, and two-year autocovariance explained by endogenous optimism in model simulations. The right panel plots the total non-fundamental autocovariance. In each figure, we plot results under three model scenarios: the baseline model with optimism shocks and optimism dynamics (blue), a variant model with no shocks, or $\sigma^2_{\varepsilon,k} = 0$ for all k (orange), and a variant model with shocks but no dynamics for narrative spread, or $u^k = r^k = s^k = 0$ for all k (green).

Figure A10: Time-Varying Relationship Between Optimism and TFP



Notes: Each dot is a coefficient β_τ estimated from Equation 176, corresponding to a year-specific effect of binary optimism (opt_{it}) on log TFP ($\log \hat{\theta}_{it}$). The outcome variable is firm-level log TFP, $\log \theta_{it}$, and the regressors are indicators for binary optimism interacted with year dummies. In the regression, we trim the 1% and 99% tails of the outcome variable. Error bars are 95% confidence intervals, based on standard errors clustered by firm and industry-time.

Table A1: The Twenty Most Common Positive and Negative Words

Positive	Negative
well	loss
good	decline
benefit	disclose
high	subject
gain	terminate
advance	omit
achieve	defer
improve	claim
improvement	concern
opportunity	default
satisfy	limitation
lead	delay
enhance	deficiency
enable	fail
able	losses
best	damage
gains	weakness
improvements	adversely
opportunities	against
resolve	impairment

Notes: The twenty most common lemmatized words among the 230 positive words and 1354 negative words. They are listed in the order of their document frequency. The words are taken from the Loughran and McDonald (2011) dictionary, as described in Section 4.2.

Table A2: Robustness to Assumptions About Unobserved Selection When Estimating the Effect of Narrative Optimism on Hiring

Panel A: Regression Estimates

	(1)	(2)
	Outcome is $\Delta L_{it}^{\perp}$	
$\text{opt}_{it}^{\perp}$	0.0373	0.0305
Controls		✓
N	39,298	39,298
R^2	0.005	0.129

Panel B: Oster (2019) Statistics

	(1)	(2)
	$\bar{R}^2$ is	
	$\hat{\bar{R}}^2 = 0.459$	$\bar{R}^2_{\Pi} = 0.387$
λ^* $(\delta^{OP} = 0)$	1.691	2.151
δ^*_{OP} $(\lambda = 1)$	0.0126	0.0165

Notes: This table summarizes the coefficient stability test described in Appendix E.1. Panel A shows estimates of Equation 229, with and without controls for current and lagged log TFP and lagged log labor. The estimate in column 1 differs from that in column 1 of Table 1 due to restricting to a common sample in columns 1 and 2. The R^2 values are for the model after partialing out fixed effects, and hence correspond with unreported "within-R^2" values in Table 1. Panel B prints the two statistics of Oster (2019). In column 1, we set $\bar{R}^2$ equal to our estimated value of 0.459, calculated as described in the text from an "over-controlled" regression of current hiring on lagged controls and future hiring and productivity. In column 2, we use $\bar{R}^2$ given by three times the R^2 in the controlled hiring regression. The first row (λ^* $(\delta^{OP} = 0)$ reports the degree of proportional selection that would generate a null coefficient. The second row ($\delta^*_{OP}(\lambda = 1)$) is the bias corrected effect assuming that unobservable controls have the same proportional effect as observable controls.

Table A3: Narrative Optimism Predicts Hiring, With More Adjustment-Cost Controls

	(1)	(2)	(3)	(4)
	Outcome is $\Delta \log L_{it}$			
opt_{it}	0.0305	0.0257	0.0235	0.0184
	(0.0030)	(0.0034)	(0.0037)	(0.0039)
Firm FE	✓	✓	✓	✓
Industry-by-time FE	✓	✓	✓	✓
$\log L_{i,t-1}$	✓	✓	✓	✓
$(\log \hat{\theta}_{it}, \log \hat{\theta}_{i,t-1})$	✓	✓	✓	✓
$(\log L_{i,t-2}, \log \hat{\theta}_{i,t-2})$		✓	✓	✓
$(\log L_{i,t-3}, \log \hat{\theta}_{i,t-3})$			✓	✓
Log Book to Market				✓
Stock Return				✓
Leverage				✓
N	39,298	31,236	25,156	21,913
R^2	0.401	0.395	0.396	0.415

Notes: The regression model is Equation 34. Column 1 replicates column 2 of Table 1. Columns 2 and 3 add more lags of firm-level log employment and firm-level log TFP, and column 4 introduces the baseline financial controls (*i.e.,* those in column 3 of Table 1). In all specifications, we trim the 1% and 99% tails of the outcome variable. Standard errors are two-way clustered by firm ID and industry-year.

Table A4: Narrative Optimism Predicts Hiring, Alternative Standard Errors

	(1)	(2)	(3)	(4)	(5)
	Outcome is				
	$\Delta \log L_{it}$				$\Delta \log L_{i,t+1}$
opt_{it}	0.0355	0.0305	0.0250	0.0322	0.0216
	(0.0030)	(0.0030)	(0.0032)	(0.0028)	(0.0037)
	[0.0031]	[0.0026]	[0.0031]	[0.0040]	[0.0034]
	{0.0035}	{0.0026}	{0.0025}	{0.0043}	{0.0036}
Firm FE	✓	✓	✓		✓
Industry-by-time FE	✓	✓	✓	✓	✓
Lag labor		✓	✓	✓	✓
Current and lag TFP		✓	✓	✓	✓
Log Book to Market			✓		
Stock Return			✓		
Leverage			✓		
N	71,161	39,298	33,589	40,580	38,402
R^2	0.259	0.401	0.419	0.142	0.398

Notes: This Table replicates the analysis of Table 1 with alternative standard error constructions. Standard errors in parentheses are two-way clustered by firm ID and industry-year; those in square brackets are two-way clustered by firm ID and year; and those in braces are two-way clustered by industry and year. For columns 1-4, the regression model is Equation 34 and the outcome is the log change in firms' employment from year $t-1$ to t. The main regressor is a binary indicator for the optimistic narrative, defined in Section 4.2. In all specifications, we trim the 1% and 99% tails of the outcome variable. In column 5, the regression model is Equation 35, the outcome is the log change in firms' employment from year t to $t+1$, and control variables are dated $t+1$.

Table A5: Narrative Optimism Predicts Hiring, Instrumenting With Lag

	(1)	(2)	(3)	(4)
	Outcome is $\Delta \log L_{it}$			
opt_{it}	0.0925	0.106	0.102	0.0470
	(0.0130)	(0.0160)	(0.0168)	(0.0053)
Firm FE	✓	✓	✓	
Industry-by-time FE	✓	✓	✓	✓
Lag labor		✓	✓	✓
Current and lag TFP		✓	✓	✓
Log Book to Market			✓	
Stock Return			✓	
Leverage			✓	
N	63,302	35,768	31,071	36,953
First-stage F	773	478	366	3,597

Notes: All columns come from a two-stage-least-squares (2SLS) estimate of Equation 34, using $\text{opt}_{i,t-1}$ as an instrument for opt_{it}. Specifically, the structural equation is

$$\Delta \log L_{it} = \delta^{OP} \cdot \text{opt}_{it} + \gamma_i + \chi_{j(i),t} + \tau' X_{it} + \varepsilon_{it}$$

the endogenous variable is opt_{it} and the excluded instrument is $\text{opt}_{i,t-1}$. In the last row, we report the first-stage F statistic associated with this equation. In all specifications, we trim the 1% and 99% tails of the outcome variable. Standard errors are two-way clustered by firm ID and industry-year.

Table A6: Narrative Optimism Predicts Hiring, Conference-Call Measurement

	(1)	(2)	(3)	(4)	(5)
	Outcome is				
	$\Delta \log L_{it}$				$\Delta \log L_{i,t+1}$
optCC_{it}	0.0277	0.0173	0.0121	0.0237	0.0123
	(0.0038)	(0.0040)	(0.0038)	(0.0038)	(0.0044)
Industry-by-time FE	✓	✓	✓	✓	✓
Firm FE	✓	✓	✓		✓
Lag labor		✓	✓	✓	✓
Current and lag TFP		✓	✓	✓	✓
Log Book to Market			✓		
Stock Return			✓		
Leverage			✓		
N	19,625	11,565	10,851	11,919	11,416
R^2	0.300	0.461	0.467	0.172	0.429

Notes: The regression models are identical to those reported in Table 1, but using the measurement of optimism from sales and earnings conference calls. In all specifications, we trim the 1% and 99% tails of the outcome variable. Standard errors are two-way clustered by firm ID and industry-year. In column 5, control variables are dated $t + 1$.

Table A7: The Effect of Narrative Optimism on All Inputs

	(1)	(2)	(3)	(4)	(5)	(6)
	Outcome is					
	$\Delta \log L_{it}$		$\Delta \log M_{it}$		$\Delta \log K_{it}$	
opt_{it}	0.0355	0.0305	0.0397	0.0193	0.0370	0.0273
	(0.0030)	(0.0030)	(0.0034)	(0.0033)	(0.0034)	(0.0036)
Industry-by-time FE	✓	✓	✓	✓	✓	✓
Firm FE	✓	✓	✓	✓	✓	✓
Lag input		✓		✓		✓
Current and lag TFP		✓		✓		✓
N	71,161	39,298	66,574	39,366	68,864	36,005
R^2	0.259	0.401	0.298	0.418	0.276	0.383

Notes: $\Delta \log M_t$ is the log difference of all variable cost expenditures ("materials"), the sum of cost of goods sold (COGS) and sales, general, and administrative expenses (SGA). $\Delta \log K_t$ is the value of the capital stock is the log difference level of net plant, property, and equipment (PPE) between balance-sheet years $t-1$ and t. In all specifications, we trim the 1% and 99% tails of the outcome variable. Standard errors are two-way clustered by firm ID and industry-year.

Table A8: The Effect of Narrative Optimism on Stock Prices, High-Frequency Analysis

	(1)	(2)	(3)	(4)	(5)	(6)
	Outcome is stock return on					
	Filing Day		Prior Four Days		Next Four Days	
opt_{it}	0.000145	-0.000142	0.00106	0.000963	0.00173	0.00249
	(0.0007)	(0.0007)	(0.0011)	(0.0014)	(0.0012)	(0.0016)
Firm FE	✓	✓	✓	✓	✓	✓
Industry-by-FY FE	✓	✓	✓	✓	✓	✓
Industry-FF3 inter.		✓		✓		✓
N	39,457	39,457	39,396	17,710	39,346	19,708
R^2	0.189	0.246	0.190	0.345	0.206	0.317

Notes: The regression equation for columns (1), (3), and (5) is $R_{i,w(t)} = \beta \text{opt}_{it} + \gamma_i + \chi_{j(i),y(i,t)} + \varepsilon_{it}$ where i indexes firms, t is the 10K filing day, $w(t)$ is a window around the day (the same day, the prior four days, or the next four days), and $y(i,t)$ is the fiscal year associated with the specific 10-K. In columns (2), (4), and (6), we add interactions of industry codes with the filing day's (i) the market minus risk-free rate, (ii) high-minus-low return, and (iii) small-minus-big return. Standard errors are two-way clustered by firm ID and industry-year.

Table A9: Narrative Optimism and Measured Beliefs Have Independent Effects

	(1)	(2)	(3)	(4)
	Outcome is			
	Hiring, $\Delta \log L_{it}$		Investment, $\Delta \log K_{it}$	
opt_{it}	0.0355	0.0311	0.0370	0.0251
	(0.0030)	(0.0068)	(0.0034)	(0.0072)
$\text{ForecastGrowthCapx}_{it}$		0.0564		0.0943
		(0.0062)		(0.0079)
Ind.-by-time FE	✓	✓	✓	✓
Firm FE	✓	✓	✓	✓
N	71,161	7,312	68,864	7,048
R^2	0.259	0.425	0.276	0.472

Notes: We estimate the regression model

$$\Delta \log L_{it} = \delta^{OP} \text{opt}_{it} + \delta^{Z} \text{ForecastGrowthCapx}_{it} + \gamma_i + \chi_{j(i),t} + \varepsilon_{it} \tag{248}$$

where opt_{it} is textual optimism from the 10-K for fiscal year t and $\text{ForecastGrowthCapx}_{it}$ is defined in the text as the log difference between manager guidance about CAPX, for fiscal year t, with last fiscal year's realized value. In all specifications, we trim the 1% and 99% tails of the outcome variable. Standard errors are two-way clustered by firm ID and industry-year.

Table A10: Narrative Optimism and Managerial Optimism Relative to Analysts

	(1)	(2)
	Outcome is $\text{GuidanceOptExAnte}_{i,t+1}$	
opt_{it}	0.0267	-0.000272
	(0.0231)	(0.0353)
Ind.-by-time FE	✓	✓
Lag labor		✓
Current and lag TFP		✓
N	3,044	1,718
R^2	0.161	0.192

Notes: The regression model is a variant of Equation 37 with a different outcome variable. The outcome, GuidanceOptExAnte, is a binary indicators for whether is an indicator of whether managers' sales guidance exceeds the analyst consensus. Standard errors are two-way clustered by firm ID and industry-year.

Table A11: Narrative Optimism Predicts Hiring, Conditional on Measured Beliefs

	(1)	(2)	(3)	(4)
	Outcome is $\Delta \log L_{it}$			
opt_{it}	0.0355	0.0232	0.0311	0.0203
	(0.0030)	(0.0129)	(0.0068)	(0.0164)
$\text{ForecastGrowthSales}_{it}$		0.157		
		(0.0329)		
$\text{ForecastGrowthCapx}_{it}$			0.0564	
			(0.0062)	
$\text{ForecastGrowthEps}_{it}$				0.000961
				(0.0104)
Ind.-by-time FE	✓	✓	✓	✓
Firm FE	✓	✓	✓	✓
N	71,161	2,908	7,312	1,290
R^2	0.259	0.506	0.425	0.638

Notes: opt_{it} is textual optimism from the 10-K for fiscal year t. $\text{ForecastGrowthZ}_{it}$ is defined in the text as the log difference between manager guidance about statistic Z, for fiscal year t, with last fiscal year's realized value. In all specifications, we trim the 1% and 99% tails of the outcome variable. Standard errors are two-way clustered by firm ID and industry-year.

Table A12: Narrative Optimism Predicts Investment, Conditional on Measured Beliefs

	(1)	(2)	(3)	(4)
	Outcome is $\Delta \log K_{it}$			
opt_{it}	0.0370	0.0238	0.0251	0.00503
	(0.0034)	(0.0177)	(0.0072)	(0.0193)
$\text{ForecastGrowthSales}_{it}$		0.172		
		(0.0423)		
$\text{ForecastGrowthCapx}_{it}$			0.0943	
			(0.0079)	
$\text{ForecastGrowthEps}_{it}$				-0.0147
				(0.0102)
Ind.-by-time FE	✓	✓	✓	✓
Firm FE	✓	✓	✓	✓
N	68,864	2,748	7,048	1,245
R^2	0.276	0.496	0.472	0.661

Notes: This table is identical to Table A11, but has net capital investment ΔK_{it} as the outcome. opt_{it} is textual optimism from the 10-K for fiscal year t. $\text{ForecastGrowthZ}_{it}$ is defined in the text as the log difference between manager guidance about statistic Z, for fiscal year t, with last fiscal year's realized value. In all specifications, we trim the 1% and 99% tails of the outcome variable. Standard errors are two-way clustered by firm ID and industry-year.

Table A13: Narrative Optimism is Contagious and Associative, Alternative Standard Errors

	(1)	(2)	(3)
	Outcome is opt_{it}		
Own lag, $\text{opt}_{i,t-1}$	0.209	0.214	0.135
	(0.0071)	(0.0080)	(0.0166)
	[0.0214]	[0.0220]	[0.0281]
	{0.0218}	{0.0221}	{0.0273}
Aggregate lag, $\overline{\text{opt}}_{t-1}$	0.290		
	(0.0578)		
	[0.180]		
	{0.179}		
Real GDP growth, $\Delta \log Y_{t-1}$	0.804		
	(0.2204)		
	[0.635]		
	{0.627}		
Industry lag, $\overline{\text{opt}}_{j(i),t-1}$		0.276	0.207
		(0.0396)	(0.0733)
		[0.0434]	[0.0563]
		{0.0496}	{0.0656}
Industry output growth, $\Delta \log Y_{j(i),t-1}$		0.0560	0.0549
		(0.0309)	(0.0632)
		[0.0328]	[0.0668]
		{0.0428}	{0.0772}
Peer lag, $\overline{\text{opt}}_{p(i),t-1}$			0.0356
			(0.0225)
			[0.0259]
			{0.0329}
Firm FE	✓	✓	✓
Time FE		✓	✓
N	64,948	52,258	8,514
R^2	0.481	0.501	0.501

Notes: This Table replicates the analysis of Table 3 with alternative standard error constructions. Standard errors in parentheses are two-way clustered by firm ID and industry-year; those in square brackets are two-way clustered by firm ID and year; and those in braces are two-way clustered by industry and year. Aggregate, industry, and peer average optimism are averages of the narrative optimism variable over the respective sets of firms. Industry output growth is the log difference in sectoral value-added calculated from BEA data, linked to two-digit NAICS industries.

Table A14: Narrative Optimism is Contagious and Associative, NYSE Peer Set Model

	(1)	(2)
	Outcome is opt_{it}	
Own lag, $\text{opt}_{i,t-1}$	0.214	0.135
	(0.0080)	(0.0166)
Industry lag, $\overline{\text{opt}}_{j(i),t-1}$	0.276	0.207
	(0.0396)	(0.0733)
Industry output growth, $\Delta \log Y_{j(i),t-1}$	0.0560	0.0549
	(0.0309)	(0.0632)
Peer lag, $\overline{\text{opt}}_{p(i),t-1}$		0.0356
		(0.0225)
Firm FE	✓	✓
Time FE	✓	✓
N	52,258	8,514
R^2	0.501	0.501

Notes: The regression model is Equation 39. Industry and peer average optimism are leave-one-out averages of the narrative optimism variable over the respective sets of firms. We define peer sets for the subset of firms traded on the New York Stock Exchange using the method of Kaustia and Rantala (2021). These authors exploit common equity analyst coverage to define peers for each firm. Firm j is a peer of firm i at time t if they have more than C common analysts, where C is chosen so that the probability of having C or more common analysts by chance is less than 1% when analysts following firm i randomly choose the firms they follow among all firms with analysts in period t. Industry output growth is the log difference in sectoral value-added calculated from BEA data, linked to two-digit NAICS industries. Standard errors are two-way clustered by firm ID and industry-year. The sum of coefficients $s_{\text{ind}} + s_{\text{peer}}$, the marginal effect of optimism in both the industry and peer set, is positive and statistically significant (estimate 0.243, standard error 0.075).

Table A15: Narrative Sentiment is Contagious and Associative

	(1)	(2)	(3)
	Outcome is sentiment$_{it}$		
Own lag, sentiment$_{i,t-1}$	0.259	0.279	0.226
	(0.0091)	(0.0106)	(0.0166)
Aggregate lag, $\overline{\text{sentiment}}_{t-1}$	0.253		
	(0.0519)		
Real GDP growth, $\Delta \log Y_{t-1}$	2.632		
	(0.5305)		
Industry lag, $\overline{\text{sentiment}}_{j(i),t-1}$		0.175	0.108
		(0.0360)	(0.0763)
Industry output growth, $\Delta \log Y_{j(i),t-1}$		0.108	0.142
		(0.0522)	(0.1312)
Peer lag, $\overline{\text{sentiment}}_{p(i),t-1}$			0.0234
			(0.0188)
Firm FE	✓	✓	✓
Time FE		✓	✓
N	63,881	51,555	8,338
R^2	0.568	0.599	0.602

Notes: The regression model is a variant of Equation 38 for column 1, and a variant of Equation 39 for columns 2 and 3, with the continuous variable sentiment$_{it}$ (and averages thereof) substituted for binary optimism. Aggregate, industry, and peer average sentiment are averages of the narrative sentiment variable over the respective sets of firms. Industry output growth is the log difference in sectoral value-added calculated from BEA data, linked to two-digit NAICS industries. In all specifications, we trim the 1% and 99% tails of sentiment$_{it}$. Standard errors are two-way clustered by firm ID and industry-year.

Table A16: Narrative Sentiment is Contagious and Associative, Controlling for Past and Future Outcomes

	(1)	(2)	(3)	(4)	(5)
	Outcome is sentiment$_{it}$				
Aggregate lag, $\overline{\text{sentiment}}_{t-1}$	0.253	0.385	0.410		
	(0.0519)	(0.0651)	(0.1103)		
Ind. lag, $\overline{\text{sentiment}}_{j(i),t-1}$				0.175	0.151
				(0.0360)	(0.0409)
Time FE				✓	✓
Firm FE	✓	✓	✓	✓	✓
Own lag, opt$_{i,t-1}$	✓	✓	✓	✓	✓
$(\Delta \log Y_{t+k})_{k=-2}^{2}$		✓	✓		
$(\Delta \log Y_{j(i),t+k})_{k=-2}^{2}$			✓		✓
N	63,881	48,889	37,643	51,555	37,643
R^2	0.568	0.578	0.599	0.599	0.601

Notes: The regression model is a variant of Equation 40 for column 1-3, and an analogous variant of industry-level specification for columns 4 and 5 (*i.e.*, Equation 39 with past and future controls), with the continuous variable sentiment$_{it}$ (and averages thereof) substituted for binary optimism. Columns 1 and 4 correspond, respectively, with columns 1 and 3 of Table A15. The added control variables are two leads, two lags, and the contemporaneous value of: real GDP growth (columns 2-3), and industry-level output growth (columns 3 and 5). In all specifications, we trim the 1% and 99% tails of sentiment$_{it}$. Standard errors are two-way clustered by firm ID and industry-year.

Table A17: The Twenty Most Common Words for Each Shiller Chapter

Panic	Frugality	Gold Standard	Labor-Saving Machines	Automation and AI	Real Estate	Stock Market	Boycotts	Wage-Price Spiral
bank	help	standard	replac	replac	price	chapter	price	countri
consum	hous	book	produc	appear	appear	peopl	profit	labor
appear	buy	money	technolog	show	real	specul	good	union
show	home	run	appear	question	find	drop	consum	ask
forecast	famili	paper	book	suggest	hous	play	start	wage
economi	lost	peopl	power	labor	estat	depress	fall	inflat
suggest	display	metal	save	ask	buy	warn	buy	strong
run	job	depress	problem	run	home	peak	wage	world
concept	peopl	eastern	labor	worker	citi	great	inflat	mile
peopl	explain	almost	innov	vacat	land	today	world	peopl
grew	phrase	depositor	run	autom	movement	get	cut	happen
around	depress	young	wage	human	world	decad	shop	depress
weather	postpon	today	worker	univers	tend	reaction	peopl	war
figur	car	want	electr	world	peopl	newspap	explain	tri
confid	justifi	went	mechan	machin	never	news	campaign	wrote
wall	cultur	decad	human	job	search	storm	play	peak
happen	fashion	idea	world	peopl	specul	saw	depress	great
depress	unemploy	man	machin	answer	explain	memori	behavior	recess
tri	great	newspap	job	around	popul	interview	postpon	went
unemploy	fault	popular	invent	figur	phrase	watch	war	get

Notes: The twenty most common lemmatized words among the 100 words that typify each Shiller (2020) narrative. Our selection procedure is described in Section 4.2.

Table A18: The Ten Most Common Words for Each Selected Topic

Topic 1		Topic 2		Topic 3		Topic 4		Topic 5		Topic 6		Topic 7		Topic 8	
lease	0.047	solid	0.791	foreign	0.097	borrower	0.034	plan	0.066	advertising	0.029	insurance	0.082	derivative	0.078
tenant	0.042	scheme	0.02	currency	0.067	agent	0.029	participant	0.031	retail	0.028	loss	0.031	value	0.05
landlord	0.03	line	0.009	income	0.045	lender	0.022	employee	0.02	brand	0.018	income	0.018	fair	0.048
lessee	0.017	asset	0.008	tax	0.038	agreement	0.02	committee	0.015	credit	0.018	investment	0.017	rate	0.039
rent	0.016	income	0.008	exchange	0.035	loan	0.02	employer	0.014	consumer	0.017	fix	0.016	interest	0.038
lessor	0.014	debt	0.007	comprehensive	0.023	credit	0.018	make	0.013	distribution	0.016	policy	0.015	asset	0.038
property	0.012	tax	0.007	translation	0.023	bank	0.013	account	0.013	card	0.015	business	0.015	hedge	0.025
term	0.011	cash	0.006	loss	0.021	administrative	0.012	provide	0.011	marketing	0.015	life	0.014	gain	0.022
day	0.009	credit	0.006	gain	0.018	interest	0.012	payment	0.01	food	0.013	premium	0.013	credit	0.019
provide	0.008	loss	0.005	financial	0.017	make	0.011	amount	0.01	store	0.013	write	0.012	financial	0.019
Topic 9		Topic 10		Topic 11		Topic 12		Topic 13		Topic 14		Topic 15		Topic 16	
benefit	0.089	stock	0.036	international	0.068	fund	0.059	financial	0.041	corporation	0.119	million	0.036	trustee	0.02
plan	0.08	common	0.033	united	0.065	investment	0.046	income	0.039	board	0.032	debt	0.031	seller	0.016
asset	0.06	financial	0.033	group	0.052	asset	0.032	cash	0.024	meeting	0.02	due	0.023	respect	0.014
pension	0.04	cash	0.022	global	0.031	trading	0.03	consolidated	0.02	stock	0.02	earning	0.022	indenture	0.013
define	0.033	asset	0.019	canada	0.022	value	0.026	approximately	0.018	director	0.016	percent	0.022	holder	0.011
cost	0.031	accounting	0.014	limited	0.022	management	0.022	asset	0.015	president	0.015	segment	0.018	notice	0.011
value	0.023	business	0.013	reference	0.021	market	0.02	statement	0.012	financial	0.013	interest	0.018	provide	0.011
tax	0.022	item	0.012	incorporate	0.017	capital	0.019	share	0.012	officer	0.012	include	0.017	interest	0.011
obligation	0.018	equity	0.011	us	0.013	income	0.017	accounting	0.012	business	0.011	relate	0.015	person	0.01
income	0.018	loss	0.011	sa	0.013	fee	0.015	tax	0.012	vote	0.01	information	0.015	purchaser	0.01
Topic 17		Topic 18		Topic 19		Topic 20		Topic 21		Topic 22		Topic 23		Topic 24	
agreement	0.071	type	0.058	stock	0.152	stock	0.049	gaming	0.035	double	0.405	exhibit	0.042	member	0.499
party	0.018	accounting	0.042	common	0.086	compensation	0.039	service	0.029	solid	0.214	incorporate	0.03	scheme	0.125
provide	0.014	lease	0.039	price	0.037	tax	0.039	network	0.022	income	0.022	reference	0.03	line	0.036
termination	0.011	topic	0.038	exercise	0.036	share	0.028	wireless	0.021	scheme	0.018	item	0.026	amount	0.027
write	0.01	asset	0.037	option	0.036	income	0.023	local	0.019	cash	0.016	registrant	0.023	abstract	0.026
employee	0.009	codification	0.034	purchase	0.034	average	0.019	cable	0.015	loss	0.014	exchange	0.023	asset	0.017
set	0.009	publisher	0.034	agreement	0.03	expense	0.018	provide	0.014	tax	0.014	pursuant	0.019	balance	0.015
notice	0.008	equipment	0.031	share	0.027	asset	0.016	equipment	0.013	balance	0.009	annual	0.018	datum	0.014
information	0.008	balance	0.026	value	0.019	outstanding	0.016	access	0.013	asset	0.007	bank	0.017	type	0.014
day	0.008	definition	0.022	warrant	0.017	weight	0.015	video	0.012	receivable	0.007	financial	0.017	value	0.013
Topic 25		Topic 26		Topic 27		Topic 28		Topic 29		Topic 30					
medical	0.176	june	0.136	executive	0.072	reorganization	0.048	court	0.038	technology	0.018				
health	0.142	march	0.123	compensation	0.03	bankruptcy	0.047	settlement	0.027	revenue	0.017				
care	0.123	note	0.089	employment	0.025	plan	0.044	district	0.021	development	0.015				
provide	0.028	agreement	0.057	officer	0.024	predecessor	0.036	certain	0.019	business	0.013				
management	0.027	august	0.05	board	0.024	successor	0.027	litigation	0.016	customer	0.012				
system	0.027	financial	0.026	committee	0.02	chapter	0.021	action	0.016	stock	0.012				
federal	0.024	interest	0.024	director	0.019	asset	0.019	complaint	0.012	product	0.012				
program	0.023	item	0.016	chief	0.017	court	0.018	damage	0.011	support	0.009				
insurance	0.022	payable	0.015	president	0.017	cash	0.016	approximately	0.011	market	0.009				
service	0.02	due	0.014	annual	0.015	certain	0.014	case	0.01	service	0.008				

Notes: The ten most common words (lemmatized bigrams) in example topics estimated by LDA and selected by our LASSO procedure as relevant for hiring (see Section 5.1). Weights correspond to relative importance for scoring the document. The LDA model and our estimation procedure are described in Section 4.2.

Table A19: Sensitivity Analysis for the Quantitative Analysis

	Parameters						Results			
	α	γ	ψ	ϵ	ω	$\frac{1}{1-\omega}$	$\hat{c}_Q(0)$	$\hat{c}_Q(1)$	2000-02	2007-09
Baseline	1.0	0.0	0.4	2.6	0.490	1.962	0.192	0.335	0.316	0.181
High ψ	1.0	0.0	2.5	2.6	0.133	1.154	0.175	0.359	0.186	0.106
High γ	1.0	1.0	0.4	2.6	-0.784	0.560	0.041	0.184	0.090	0.052
Empirical Multiplier	1.0	0.0	1.15	2.6	0.250	1.333	0.167	0.329	0.215	0.123
Calibrated Multiplier	1.0	0.0	0.845	2.6	0.313	1.455	0.168	0.324	0.235	0.134
High ϵ	1.0	0.0	0.21	5.0	0.490	1.962	0.109	0.240	0.317	0.181
Decreasing RtS	0.75	0.0	0.05	2.6	0.490	1.962	0.125	0.238	0.237	0.135

Notes: This table summarizes the quantitative results under alternative calibrations of the macroeconomic parameters, which we report along side their implied complementarity ω and demand multiplier $\frac{1}{1-\omega}$. We report four statistics as the "results" in the last four columns. The first two are the fraction of output variance explained statically, $\hat{c}_Q(0)$, and at a one-year horizon, $\hat{c}_Q(1)$, by optimism. The second two are the fraction of output losses in the 2000-02 downturn and 2007-09 downturn explained by fluctuations in narrative optimism. Baseline corresponds to our main calibration. High ψ increases the inverse Frisch elasticity to 2.5, or decreases the Frisch elasticity to 0.4. High γ increases the curvature of consumption utility (indexing income effects in labor supply) from 0.0 to 1.0. Empirical Multiplier adjusts ψ to match an output multiplier in line with estimates from Becko, Flynn, and Patterson (2024). Calibrated multiplier adjusts ψ to match our own calculation of the multiplier in our setting in Appendix F.3. High ϵ increases the elasticity of substitution from 2.6 to 5.0, with ψ adjusting to hold fixed the multiplier. Decreasing RtS reduces the returns-to-scale parameter α from 1.0 to 0.75, with ψ adjusting to hold fixed the multiplier.

Table A20: An Empirical Test for Cycles and Chaos

	(1) Outcome is opt_{it}
α: Constant	-0.051
	(0.244)
α_1: $\text{opt}_{i,t-1}$	0.655
	(0.062)
β_1: $\text{opt}_{i,t-1} \cdot \overline{\text{opt}}_{i,t-1}$	0.052
	(1.021)
β_2: $(1 - \text{opt}_{i,t-1}) \cdot \overline{\text{opt}}_{i,t-1}$	0.952
	(1.006)
τ: $(\overline{\text{opt}}_{i,t-1})^2$	-0.062
	(1.034)
η: Logistic parameter	1.443
	(0.698)
Firm FE	✓
N	67,648
R^2	0.480

Notes: The regression model is Equation 193. η is a function of the regression coefficients defined in Equation 194, and interpretable in the model of cycles and chaos in Appendix B.8. Standard errors are two-way clustered by firm ID and industry-year. The standard error for η is calculated using the delta method.

Table A21: Data Definitions in Compustat

	Quantity	Expenditure
Production, x_{it}	—	`sale`
Employment, L_{it}	`emp`	`emp` $\times$ industry wage
Materials, M_{it}	—	`cogs` + `xsga` − `dp`
Capital, K_{it}	`ppegt` plus net investment	—

Table A22: The Effect of Optimism on Hiring, CEO Change Strategy

	(1)	(2)	(3)	(4)
		Outcome is $\Delta \log L_{it}$		
opt_{it}	0.0253	0.0404	0.0362	0.0253
	(0.0131)	(0.0131)	(0.0132)	(0.0029)
$\text{opt}_{it} \times \text{ChangeCEO}_{it}$				0.0220
				(0.0099)
ChangeCEO_{it}				-0.0232
				(0.0088)
Industry-by-time FE	✓	✓	✓	✓
Lag optimism	✓	✓	✓	✓
Lag labor		✓	✓	✓
Current and lag TFP		✓	✓	✓
Log Book to Market			✓	
Stock Return			✓	
Leverage			✓	
N	1,725	982	905	36,953
R^2	0.243	0.375	0.375	0.134

Notes: The regression model is Equation 230 for columns 1-3, and Equation 231 for column 4. The outcome is the log change in firms' employment. opt_{it} is a binary indicator for the optimistic narrative, defined in Section 4.2. ChangeCEO_{it} is a binary indicator for whether firm i changed CEO in fiscal year t due to death, illness, personal issues or voluntary retirement. In all specifications, we trim the 1% and 99% tails of the outcome variable. Standard errors are two-way clustered by firm ID and industry-year.

Table A23: The Contagiousness of Optimism, CEO Change Strategy

	(1)	(2)	(3)	(4)
		Outcome is opt_{it}		
	OLS	IV	OLS	IV
Industry lag, $\overline{\text{opt}}_{j(i),t-1}$	0.275	0.260	0.195	0.272
	(0.0407)	(0.2035)	(0.0760)	(0.5270)
Peer lag, $\overline{\text{opt}}_{p(i),t-1}$			0.0437	0.129
			(0.0236)	(0.1677)
Firm FE	✓	✓	✓	✓
Time FE	✓	✓	✓	✓
Industry output growth, $\Delta \log Y_{j(i),t-1}$	✓	✓	✓	✓
N	50,604	50,604	7,873	7,873
R^2	0.503	0.051	0.508	0.020
First-stage F	—	29.7	—	36.8

Notes: The IV strategies instrument the industry and/or peer lag with the CEO-change variation in those averages. Standard errors are two-way clustered by firm ID and industry-year.

Table A24: Optimism is Contagious and Associative, Granular IV Strategy

	(1)	(2)	(3)	(4)	(5)	(6)
			Outcome is opt_{it}			
	OLS	OLS	IV	OLS	OLS	IV
Own lag, $\text{opt}_{i,t-1}$	0.212	0.213	0.210	0.219	0.220	0.219
	(0.0071)	(0.0071)	(0.0073)	(0.0080)	(0.0081)	(0.0081)
Agg. sales-wt. lag, $\overline{\text{opt}}^{sw}_{t-1}$	0.0847		0.308			
	(0.0421)		(0.1044)			
Real GDP growth, $\Delta \log Y_{t-1}$	1.058	1.104	0.768			
	(0.2205)	(0.2110)	(0.2607)			
Agg. sales-wt. granular lag, $\overline{\text{opt}}^{g,sw}_{t-1}$		0.150				
		(0.0506)				
Ind. sales-wt. lag, $\overline{\text{opt}}^{sw}_{j(i),t-1}$				0.0728		0.0195
				(0.0209)		(0.0459)
Ind. output growth, $\Delta \log Y_{j(i),t-1}$				0.0851	0.0903	0.0886
				(0.0325)	(0.0336)	(0.0333)
Ind. sales-wt. granular lag, $\overline{\text{opt}}^{g,sw}_{j(i),t-1}$					0.00913	
					(0.0216)	
Firm FE	✓	✓	✓	✓	✓	✓
Time FE				✓	✓	✓
N	64,948	64,948	64,948	52,258	50,842	50,842
R^2	0.481	0.481	0.049	0.500	0.503	0.051
First-stage F	—	—	99.1	—	—	262.3

Notes: This table estimates Equations 38 and 39, respectively modeling the spread of optimism at the aggregate and industry level, using granular identification of spillovers (contagiousness). $\overline{\text{opt}}^{sw}_{t-1}$ and $\overline{\text{opt}}^{sw}_{j(i),t-1}$ are sales-weighted averages of aggregate and industry optimism, respectively. $\overline{\text{opt}}^{g,sw}_{t-1}$ and $\overline{\text{opt}}^{g,sw}_{j(i),t-1}$ are (lagged) sales-weighted averages of the *non-fundamentally-predictable* components of firm-level optimism in the aggregate and in the industry, respectively, as explained in Appendix E.3. In columns 3 and 6, we use the granular variables as instruments for the raw sales-weighted averages. Standard errors are two-way clustered by firm ID and industry-year.

Table A25: Calibration Parameters for Narrative Constellation Model

Name	ζ	u	r	s	M	Variance
Lease, Tenant, Landlord...	-0.135	0.063	-0.342	0.820	-0.113	0.003
Solid, Scheme, Line...	0.055	0.434	-2.304	0.678	0.103	0.049
Foreign, Currency, Income...	0.021	0.373	-0.180	0.514	-0.114	0.005
Borrower, Agent, Lender...	-0.066	0.064	0.019	0.747	-0.189	0.010
Plan, Participant, Employee...	-0.023	0.020	0.450	0.871	-0.109	0.012
Advertising, Retail, Brand...	0.056	0.324	0.078	0.594	-0.082	0.005
Insurance, Loss, Income...	-0.039	0.250	-0.132	0.651	-0.099	0.002
Derivative, Value, Fair...	0.042	0.410	0.099	0.407	-0.183	0.011
Benefit, Plan, Asset...	0.097	0.335	-0.500	0.568	-0.100	0.001
Stock, Common, Financial...	-0.041	0.230	-0.212	0.285	-0.484	0.002
International, United, Group...	0.032	0.321	1.369	0.729	0.053	0.020
Fund, Investment, Asset...	0.054	0.219	0.365	0.837	0.057	0.001
Financial, Income, Cash...	0.085	0.084	1.018	0.921	0.011	0.121
Corporation, Board, Meeting...	0.050	0.201	0.812	0.783	-0.012	0.117
Million, Debt, Due...	0.026	0.307	0.138	0.405	-0.288	0.002
Trustee, Seller, Respect...	-0.079	-0.006	-0.165	1.002	-0.003	0.015
Agreement, Party, Provide...	-0.117	0.039	-0.067	0.864	-0.097	0.021
Type, Accounting, Lease...	-0.049	0.371	0.592	0.600	-0.031	0.150
Stock, Common, Price...	0.043	0.198	1.020	0.945	0.146	0.031
Stock, Compensation, Tax...	0.023	0.274	-0.671	0.686	-0.041	0.071
Gaming, Service, Network...	0.042	0.375	0.137	0.444	-0.181	0.004
Double, Solid, Income...	0.032	0.450	-2.022	0.684	0.129	0.046
Exhibit, Incorporate, Reference...	0.033	0.187	0.139	0.802	-0.011	0.094
Member, Scheme, Line...	0.035	0.470	-0.655	0.537	0.006	0.011
Medical, Health, Care...	0.056	0.361	0.026	0.522	-0.116	0.001
June, March, Note...	0.040	0.242	0.312	0.663	-0.094	0.040
Executive, Compensation, Employee...	0.024	0.163	0.880	0.894	0.058	0.041
Reorganization, Bankruptcy, Plan...	-0.085	0.357	-0.119	0.206	-0.436	0.000
Court, Settlement, District...	-0.104	0.363	0.363	0.560	-0.079	0.010
Technology, Revenue, Development...	0.091	0.299	0.674	0.559	-0.138	0.013
Panic versus Confidence	0.017	0.223	-0.143	0.428	-0.349	0.003
The Gold Standard	0.017	0.204	0.724	0.955	0.159	0.005
Labor-Saving Machines	0.024	0.212	0.239	0.278	-0.511	0.001
Automation and AI	0.029	0.214	0.196	0.148	-0.638	0.001
Real Estate	0.022	0.206	-0.130	0.552	-0.243	0.002
Stock-Market Bubbles	0.012	0.221	0.126	0.472	-0.306	0.000
Boycotts and Evil Businesses	0.042	0.168	0.043	0.640	-0.192	0.003
Wage-Price Sprials	0.021	0.221	-0.113	0.669	-0.110	0.002

Notes: This table reports the selected narratives used in the calibration of Section 7. The first set of rows are LDA topic narratives, identified by their three highest-scoring terms, and the second set of rows are chapters of Shiller (2020), identified by shortened forms of their titles. The narratives are selected via post-LASSO estimation of Equation 50, and the first column reports the coefficients. The remaining columns report estimates of stubbornness, associativeness, and contagiousness; the composite statistic M; and the unconditional time-series variance of each narrative. In the estimation, we re-normalize each narrative to have a positive ζ.

Table A26: Multiplier Calibrations via Under-Controlled Regressions of Hiring on Optimism

	(1)	(2)	(3)
	Outcome is ΔL_{it}		
opt_{it}	0.0355	0.0516	0.0486
	(0.0030)	(0.0034)	(0.0033)
Complementarity ω	—	0.313	0.270
Multiplier $\frac{1}{1-\omega}$	—	1.455	1.370
Industry-by-time FE	✓		
Firm FE	✓	✓	✓
Current and lagged adjusted TFP		✓	
Current and lagged unadjusted TFP			✓
N	71,161	65,508	65,508
R^2	0.259	0.207	0.216

Notes: The regression models are introduced in Appendix F.3. The first column replicates Column 1 of Table 1. The second two columns remove the industry-by-time FE and control for the contemporaneous and lagged value of seasonally adjusted log TFP, respectively with and without capacity utilization adjustment, as reported by the updated data series of Fernald (2014). The sample size is lower in columns 2 and 3 due to the band-pass filtering being impossible for the last part of the sample. The remaining rows give the implied complementarity ω and demand multiplier $\frac{1}{1-\omega}$, by comparing the coefficients with that of column 1 and applying the formula in Equation 247. Standard errors are double-clustered by industry-year and firm ID.