A Theory of Bank Liquidity Requirements *

Charles W. Calomiris† Madalen Castells Jauregui ‡ Florian Heider§

Marie Hoerova ¶

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Abstract

We develop an asset-side theory of bank liquidity requirements, which focuses on the risk-management gains from requiring cash reserves. The key role of cash in the bank is to attenuate the banker's moral hazard. Because cash is observable and riskless, its value does not depend on the banker's risk management effort. Greater cash holding improves bank incentives to manage risk in the remaining, non-cash, portfolio of risky assets. Because cash is ring-fenced from moral hazard, it serves as a form of collateral to bank creditors. We allow cash to be generated either initially from the funding of the bank or subsequently by selling risky assets in the market. But buyers of risky assets have limited aggregate resources, which generates a fire-sale discount on risky assets sold by the bank to generate cash. The fire-sale externality leads to a wedge between privately-optimal and socially-optimal liquidity holdings, requiring regulation of bank cash holdings to address the externality. More equity capital ex ante also improves risk management incentives but cash can be complementary to equity because it can be generated in the bad state, at the time when it is hard or impossible for a bank to raise more equity. Our theory has several implications for the design of liquidity regulation that are absent from existing regulation.

Keywords: Bank regulation, liquidity requirement, fire-sale externality, risk management

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†Columbia GSB; NBER

‡European Central Bank (ECB)

§Leibniz Institute for Financial Research SAFE; Goethe University Frankfurt; CEPR

¶European Central Bank (ECB); CEPR

1. Introduction

Liquidity risk is one of the key risks banks must manage to remain solvent. Recognizing the recurrent issues in liquidity risk management of banks, the regulatory overhaul that occurred in the aftermath of the Global Financial Crisis imposed a new set of liquidity requirements on banks: the Liquidity Coverage Ratio (LCR) and the Net Stable Funding Ratio (NFSR), which were intended to complement the revised framework for capital requirements. The stated rationale for these new liquidity requirements was to limit liquidity risk stemming from short-term bank liabilities, which might not be rolled over. In particular, LCR aims to ensure banks have enough liquid assets to meet stressed liquidity outflows for a 30-day period. NSFR is designed to deal with longer-term liquidity mismatches, ensuring that longer-term assets, such as loans, are adequately supported by longer-term, or otherwise stable, liabilities.

Although the implementation of these requirements is under way in many jurisdictions, their conceptual underpinning has not been clearly established. An important economic function banks perform in the economy is liquidity transformation, which goes hand in hand with liquidity risk, which includes the risk of runs, as first modelled by Diamond and Dybvig (1983). Requiring banks to hold liquidity buffers high enough to eliminate liquidity risk implies severely curtailing their liquidity transformation function. An alternative approach to dealing with liquidity risk would rely on a lender of last resort (LOLR) to assist banks when liability holders withdraw funds. Reliance on a LOLR to address liquidity risk would allow banks to continue to perform liquidity transformation. A central motivation of any theory of liquidity requirements should be to explain why such ex ante requirements are preferable to ex post reliance on LOLR support. There is a general recognition that excessive reliance on a LOLR, rather than liquidity requirements, can create increased credit risk due to moral hazard. That recognition posits an important potential connection between liquidity risk and credit risk management. Formalizing that concern is a central motivation of this paper.

We provide a theory of bank liquidity (cash reserve) requirements that identifies the effect of cash on bankers' ex ante incentives to reduce credit risk, which in turn enables banks to better access markets for risk-intolerant, short-term debt. Cash has several key features in our model: 1) it is observable and riskless and therefore does not require risk-management effort; 2) it is verifiable; 3) it is ring-fenced from banker risk-shifting; and 4) its value is invariant to the state of the world, which we show allows cash holdings to improve incentives to manage the risk of risky assets in bad states of the world. These features of cash affect banker risk management incentives have not been recognized in the existing literature on bank liquidity requirements.

First, because cash is observable and riskless, preserving its value does not require any risk-management effort. Second, because cash is verifiable, depositors can credibly threaten to withdraw their deposits if the banker does not maintain adequate liquidity holdings; by the same token, the banker can use cash holdings to credibly signal to depositors that bank risk is adequately managed. This improves banker's access to deposit funding. Third, because cash is ring-fenced from banker moral hazard, it serves as collateral to bank creditors. Holding more cash "collateral" in the bank reduces the potential moral hazard related to bankers prospective gains from increasing the riskiness of risky assets in bad states of the world. Fourth, greater cash can be generated either by holding more assets in cash initially (the value of which is invariant to the evolving state of the world) or by selling risky assets in the market to boost cash after the revelation of the state of the world. However, the strategy of waiting to raise cash from the sale of risky assets has a risk: prospective buyers of banks' risky assets have limited aggregate resources, implying a fire-sale discount on the sale of risky assets. The externality arising from this fire sale discount leads to a wedge between privately-optimal and socially-optimal liquidity holdings, which we show makes it beneficial to require minimum ex ante cash holdings.

Our framework provides a resolution to the "Goodhart's Paradox" related to cash reserve requirements. The original design of LCR stipulated that the cash buffer is to be maintained at all times, raising the issue of why this liquidity cannot be used in a crisis to satisfy withdrawals. Goodhart et al. (2008) provided an analogy of "the weary traveller who arrives at the railway station late at night, and, to his delight, sees a taxi there who could take him to his distant destination. He hails the taxi, but the taxi driver replies that he cannot take him, since local bylaws require that there must always be one taxi standing ready at the station." As LCR was conceived to make sure that banks can better withstand a surge in withdrawals should one occur, mandating that the last cab cannot depart the station seems counterproductive. In our framework, the paradox does not arise. Even if reserves are never actually drawn down to support deposit outflows, they still serve the important objective of incentivizing optimal risk management effort. The presence of the taxi is enough.[1]

Our framework can also explain why banks may see advantages to maintaining cash even when not required to do so by regulators. It has been observed that banks are reluctant to let their LCR drop below the regulatory requirement even when the regulators remove the regulatory minimum in a crisis, as was done between March 2020 and December 2021 by the European Central Bank to deal with the pandemic-induced crisis. From the point of view of

[1] Diamond and Kashyap (2016) set up a model that also highlights the potential incentive properties of regulation which can potentially explain why mandating the presence of some unused liquidity could be beneficial.

our model, banks may want to keep that "last taxi" at the station, to signal that they do good risk management effort. Indeed, it will reduce liquidity risk banks face, by stemming depositors' incentives to withdraw.

We develop a general equilibrium model of cash requirements in which both default and liquidity risk arise. The model has three dates and models the behaviour of three types of agents: depositors, bankers and arbitrageurs. At date 0, bankers raise deposits, by leveraging their inside equity. The banker chooses a portfolio of riskless (cash) and risky assets (loans) whose return depends on banker's risk management effort. At date 1, the aggregate macro state is revealed as either good – that is, associated with lower risk-management effort cost to the banker – or as bad, which is associated with higher risk-management effort cost. After the state is revealed, the banker can choose to top up his cash holdings by selling some of his loans to arbitrageurs (at a fire sale discount that depends on the ratio of the aggregate assets sold to the aggregate resources of the arbitrageurs). At date 2, assets' payoff are realized and depositors consume.

There are two key frictions in the model. First, the banker's risk management effort is not observable, leading to a moral hazard problem in the bad state of the economy when risk management is particularly costly (as in Holmstrom and Tirole (1997)). We show that banker's incentives can be improved by holding more cash because the cash is riskless – its value does does not depend on risk-management effort. However, holding more cash at date 0 is costly for the banker because it means making fewer profitable loans. The banker can decide to invest in cash ex ante (at date 0) or he can generate cash ex post (at date 1) in the bad state as this is when risk-management incentives are jeopardized by the high cost of risk-management. The contract with depositors takes the form of a demandable claim with a fixed return because depositors demand safety. We show that the demandable feature of the contract is crucial: the ability of depositors to withdraw their claims plays an important role in encouraging banks to maintain adequate amounts of cash, especially when "bad" states of the world necessitate a topping up of cash balances. Without giving depositors the right to withdraw, the bank has no incentive to increase cash holdings in bad states of the world. Instead, it would promise to do so but renege on that promise after outsiders had deposited their funds with the bank. That is, the bank must be exposed to potential liquidity risk in order for cash to be able to improve risk management through the threat of depositor withdrawals.

The second key friction relates to the fire-sale discount on risky assets at date 1. Arbitrageurs have limited endowments (held in cash). This leads to cash-in-the-market pricing at date 1 (Allen and Gale (1994)). Cash at date 1 can be generated by the bank through the selling of risky assets, but prospective buyers have limited aggregate resources, which generates an externality from the fire-sale discount. The fire-sale externality leads to a wedge between privately-optimal and socially-optimal liquidity holdings, requiring regulation of bank liquidity cash holdings (as in Stein (2012), and Kara and Ozsoy (2020)). In other words, due to a fire-sale externality, banks' reliance on asset sales to top-up cash holdings is excessive from a social welfare perspective. Consequently, banks' initial net debt (debt minus cash) and their investment in risky assets are inefficiently high.

We show that liquidity (net debt) requirements and risk-adjusted capital requirements are both effective, and are substitutes, in mitigating the inefficiency. While more equity capital ex ante improves risk management incentives, cash has the advantage that it can be generated in the bad state, at the time when it is hard or impossible for a bank to raise more equity. In addition, equity capital inherits the risk property of assets as it is the difference between assets and liabilities. If assets lose value, capital evaporates quickly. The leader in the Economists following the Sillicon Valley Bank collapse illustrates this point (March 17, 2023): "In a crisis once-loyal depositors cold flee, forcing banks to cover deposit outflows by selling assets. If so, the bank's losses would crystallize. Its capital cushion might look comforting today, but most of its stuffing would suddenly become an accounting fiction." By contrast, safe and liquid assets remain safe and liquid in bad states of the world. This underscores the properties "required cash" assets should have from our model's perspective: 1) stable value, incl. in crisis times (business-cycle invariant), 2) no risk-management costs; and 3) can be sold quickly without a discount. Liquidity regulation should focus on safe assets only (or impose high haircuts on less safe assets, to discourage their weight in the liquidity ratio).

Unlike the Basel III approach to liquidity regulation – in which capital standards are a response to default risk and liquidity standards are conceived as protecting against additional exogenous liquidity shocks – our approach recognizes that illiquidity in markets is almost always a direct consequence of severe increases in default risk. Cash requirements play a key role in limiting default risk. Several practical regulatory consequences follow from this difference between our framework and the motivation of Basel III. In particular, the Basel LCR approach views greater reliance on insured deposits as resulting in less need for cash assets because there is less withdrawal risk. In sharp contrast, in our model, insured deposits can undermine market discipline; required holdings of cash are therefore needed to compensate for the absence of market discipline.

Empirically, we believe the evidence is consistent with our view that it is a mistake to see liquidity risk as unrelated to credit risk. For example, during the recent U.S. banking crisis of 2007-2009, the ratios of the market value of equity to assets fell gradually over a period of more than two years, implying a gradual increase in risk. The Global Financial Crisis was not unique in the respect. The history of banking crises is a history of endogenous collapses of market liquidity that are caused by asymmetric information about bank default risk. Because bank liabilities have short duration and mainly take the form of money market instruments, they respond to even small increases in default risk with severe credit rationing (Calomiris and Gorton (1991)). Indeed, it is noteworthy that the history of prudential regulation prior to the 1980s focused on cash ratio requirements, not capital ratio requirements, as the primary regulatory tool. In that sense, our model points toward the need to restore cash requirements to their rightful place in the prudential regulatory toolkit, as a key means of limiting credit risk.

Another empirical observation that motivates our modelling approach is the importance of risk management for explaining cross-sectional differences in the extent to which banks suffer losses during financial crises. Ellul and Yerramilli (2013) show that the centrality of the chief risk officer (CRO) within the bank (as measured by the ratio of CRO compensation relative to CEO compensation) predicts the extent of bank risk taking ex ante and losses ex post. Fahlenbrach et al. (2012) similarly show that there is a bank fixed effect in risk management: banks that had the relatively greatest losses in 2008 tended to be those that also suffered the most severe losses in 1998.

Related literature In our framework, the optimal contract between the banker and her funding sources is a claim senior to the banker's claim and can therefore be though of as debt. Debt contracts economize on the cost of ex post veriffcation (Townsend (1979); Diamond (1984); Gale and Hellwig (1985); Calomiris and Kahn (1991)), reduce the negative signaling of bank type (Myers and Majluf (1984)), allow the trading of the outside claim (Gorton and Pennacchi (1990)) and they can limit the hold-up problem between bankers and their borrowers (Diamond and Dybvig (1983)). Importantly, it will be optimal to make the debt demandable and hence, the banker issues deposits. The seniority of the claim tackles the moral-hazard problem in risk management between the banker and outsiders. The conflict of interest creates a wedge between physical cash-flows and those that can be credibly pledged to outside investors (see Holmstrom and Tirole (1997); Holmström and Tirole (1998)). The ability to withdraw the claim early is necessary to have the banker increase cash holding in the bad macro-economic state when it is more difficult to incentivize risk management. Should she renege on the promise to do so, which is necessary to attract outside investors in the first place, they can withdraw and possibly force the banker into bankruptcy.

The role of cash in our banking environment resembles at first glance the role of collateral in environments with adverse selection (Besanko and Thakor (1987)) or moral hazard (Boot and Thakor (1994)). There are, however, important differences. First, the use of cash can easily be made contingent on the state of the economic environment. While this in principle is also possible for traditional collateral, say a banker's house, it is unlikely to be feasible in practice. The value of the house will also depend on the economic environment and, moreover, the value will probably not be enough relative to the size of the incentive problem of a typical bank. This relates to the second difference. While the amount of collateral is usually taken as given, the amount of cash is endogenous. It depends on the portfolio choice of the banker. Moreover, and again related, the value of collateral is usually assumed to be lower in the hands of lenders than in the hands of borrowers. In contrast, the cost of cash is an opportunity cost. It arises from not having invested in high-risk/high-return assets such as loans.

The role of cash in our framework bears resemblance to the use of variation margins in trading. Biais et al. (2016) show that in the context of derivative trading with moral-hazard in risk-management, optimal hedging contracts may benefit from the use of variation margins. As we show in the model below, cash holdings can be an especially useful means of reducing risk through the effects of cash holdings on the incentives of bankers to expend effort on risk management, especially in economic downturns. Because cash holdings limit the extent to which debtholders lose from high-risk strategies and the extent to which bankers can pursue high-risk strategies, more cash helps to better align managers' incentives with the interest of debtholders. As in Biais et al. (2016), setting aside cash on a separate account has the benefit of curbing the moral-hazard in risk-management but carries the (opportunity) cost of having less invested in a high-return asset. In equilibrium, a withdrawable senior claim (e.g, deposits) for outsiders and a sufficient amount of cash reserves provide an optimal contractual solution to the banking problem that combines all those elements: That solution minimizes the overall costs associated with early liquidation, shirking in risk management and foregone opportunities for profits (from cash holdings).

One of the key insight of the model is that the investment in cash is inefficiently low, which provides a rational for regulation. In the same vein, some studies also claim that banks' liquidity choices are inefficient under market incompleteness, information frictions or externalities (e.g., Farhi et al. (2009); Walther (2016); Carletti et al. (2020); Kashyap et al. (2020); Kara and Ozsoy (2020)). In this paper, banks' inefficient liquidity positions arises due to a fire-sale externality. Fire sales in our model are similar to those in Lorenzoni (2008) and Stein (2012), and fall under the category of "collateral externality", described by Dávila and Korinek (2018), where prices interact with binding constraints.

Finally, there is the long tradition in banking – despite the absence of formal modeling – that has focused on cash requirements as a prudential device. Indeed, with few exceptions, historical prudential regulation prior to the 1980s has concentrated on requirements for cash rather than requirements for capital. For example, the New York Clearing House maintained a 25% cash reserve requirement against deposits for its members.[2] In the famous 1873 Coe Report, authored by George Coe, the President of the New York Clearing House, cash was seen as the essential tool for managing systemic risk (Wicker 2000). In contrast, neither regulators nor bank coalitions set minimum equity capital-to-asset ratios for banks, with few exceptions, until the 1980s.[3]

The remainder of the paper is organized as follows. Section 2 describes the model. Section 3 presents the benchmark case in which effort is observable and there is no moral hazard problem. Section 4 derives the equilibrium when there is moral hazard in bank risk management. Section 5 shows that the market equilibrium is constrained inefficient and that a liquidity requirement implements the social optimum. Section 6 concludes. All proofs are in the Appendix.

2. The model

There are three dates ($t \in \{0, 1, 2\}$), one perishable good that can either be used for consumption or investment and two aggregate states that realize at $t = 1$ and can either be good ($s = g$) with probability q or bad ($s = b$) with probability $1 - q$. The aggregate state is observable but not contractible. The economy is populated by a continuum of bankers, depositors and arbitrageurs each with a unit mass.

Bankers are risk neutral, do not discount the future, and are endowed with $E_0 > 0$ units of the perishable good at $t = 0$. They have access to two investment technologies: a productive technology and a storage technology. The productive technology, transforms $t = 0$ units of the perishable good into $t = 2$ goods, where the unit return depends on bankers' non-observable risk-management effort. Risk-management effort is a binary choice (either they exert or not exert effort) taken at $t = 1$ after the realization of the aggregate state.

[2] An additional motivation for New York City banks to maintain high required reserves was that city's position at the peak of the "pyramid" that connected banks throughout the United States even though it generally did not employ its reserves to protect banks in other regions or non-clearing house members during financial crises.

[3] Some of those exceptions were the state experiments with deposit insurance, for example, in the early 20th century (see Calomiris (1990)).

If effort is exerted the return per unit of loan is Y with probability 1 (riskless). If effort is not exerted the return is Y with probability ε and zero otherwise. In the latter case, bankers earn a private benefit per loan of B_s which depends on the aggregate state $s \in \{g, b\}$. The private benefit in the bad state is higher than in the good state, i.e., $B_g < B_b$. [4] The productive technology represents a portfolio of loans to firms. [5] The storage technology, instead, transforms one unit of the good at time t into one unit of the good at $t+1$ (a unit return of one), thus effectively storing the good ("cash").

Depositors are risk neutral, do not discount the future, and they are endowed with an arbitrarily large amount of the perishable good at $t = 0$. [6] Depositors only have access to the storage technology. Therefore, lending to bankers at $t = 0$ could be an attractive investment choice, as long as the corresponding expected return is at least as high as the return of their outside option, i.e., cash.

Arbitrageurs are born at $t = 1$, are risk neutral and are endowed with m units of the perishable good at $t = 1$. They have access to an investment technology that can transform $K(s)$ units of the perishable good at $t = 1$ into $G(K(s))$ units of the perishable good at $t = 2$. [7]

The following assumptions about the available technologies are crucial in what follows.

Assumption 2.1. *The productive technology has a positive NPV if and only if bankers exert effort in both states:*

$$Y > 1 > qY + (1-q)(\varepsilon Y + B_b).$$

The first inequality implies that when a banker exerts risk-management effort, loan making has a positive NPV, implying that the productive technology dominates the storage technology. The second inequality implies that unless a banker does risk-management effort in both aggregate states, loan making is socially wasteful: even after accounting for the private

[4]This assumption can be viewed as capturing, in a reduced-form, a set-up with costly loan monitoring (whereby the frequency of monitoring should be higher in bad times to achieve the same probability of payments). Or a set up with searching for prospective loan applicants and/or screening out bad borrowers (which is more difficult in bad economic times).

[5]We abstract from modelling the relationship between banks and firms. Instead, we assume that bankers directly invest in risky projects, as the contracting frictions that may arise in the credit market is not central to this paper.

[6]It is assume that depositors' endowment is sufficiently large and that it is not a binding constraint in equilibrium.

[7]Similar to Stein (2012), what is crucial is that when time 1 rolls around and the state of the world is realized, m is fixed. Thus, although it is fine to think of firms as having full access to financial markets at time 0, they cannot go back and raise more at time 1 once they know the state. In order words, m is an unconditional war chest, with the same amount available to firms in the good and bad states. Downward sloping demand.

benefit of a banker, it is more profitable to store funds. Efficiency requires that bankers do risk-management effort in both states since loan making is only productive when a banker exerts effort. The condition also implies that effort is productive in both aggregate states, $Y > \varepsilon Y + B_s$ for $s \in \{g, b\}$.

Assumption 2.2. *The productive technology available to arbitrageurs at $t = 1$ is such that $G'() > 0, G''() < 0$, and $G'(m) > 1$. In addition, $G''''()G'() - 2G''^2() \geq 0$.*

Arbitrageurs' technology is concave, implying that its marginal return is decreasing in the size of the investment. In the bad state, even when all their endowment is invested in this technology, the marginal return is still higher than one – i.e., such investment has a positive net present value. Hence, arbitrageurs will fully invest their scarce resources in the bad state, either in their technology or in an alternative investment opportunity that yields at least the same return (which is higher than one). The last condition is useful to guarantee that the objective functions of the planner is concave.

2.1. The timing of bankers' choices

At $t = 0$ bankers raise D units of funds from depositors (if any), the maximum amount that they can raise being $\bar{D}$. [8] In exchange, bankers promise at $t = 2$ payments contingent on their asset output (i.e., loans and cash): if the output of the loans is Y, the payment to depositors is R^h, and if the output of the loans is zero, the payment to depositors is R^l. Note that when bankers cash holdings are positive, R^l could be positive. Since bankers have the exclusive ability to make loans, they leave depositors no rents, i.e., depositors participation constraint is binding. [9] This implies that depositors' expected payments equals D, consistent with the unit-return on cash. In addition, depositors have the option to liquidate their positions early at $t = 1$. Both, bankers and depositors are subject to limited-liability.

At $t = 0$, bankers can invest their endowment (equity), along with the funds raised from depositors, $E_0 + D$, in loans (the amount invested in loans is denoted L_0, which is also the number of loans since the loan size is normalized to 1) or in cash (the amount invested in cash is denoted C_0). Therefore, the balance sheet constraint (or the budget constraint) is [10]

$$C_0 + L_0 = E_0 + D \tag{1}$$

[8] This is a reduced form to capture an increasing cost of attracting funds due to, e.g., geographical distance. Alternatively, we could assume a limited amount of loan-making opportunities.

[9] It is assumed that bankers represents financial intermediaries with a local monopsony in the deposit market so that depositors earn zero net expected interest.

[10] The storage technology ensures that all agents in the economy are (weakly) better off by investing all their endowment, thus, bankers' consumption at $t = 0$ equals zero.

At $t = 1$, bankers have no new endowment nor technology, but have access to financial markets. Hence, at this date, and after the aggregate state realizes, bankers can liquidate a fraction of their loan portfolio, increasing their cash holdings. In particular, bankers can sell $\Delta L(s)$ units of loans at a price p_s, which leads to $\Delta C(s) = p_s \Delta L(s)$ units of additional cash at $t = 1$:

$$C(s) = C_0 + \Delta C(s) \quad \text{and} \quad L(s) = L_0 - \frac{1}{p_s}\Delta C(s) \tag{2}$$

The maximum increase in cash at $t = 1$ is attained when selling all the loans, this is, the feasibility constraint for cash increase at $t = 1$ is $\Delta C(s) \leq p_s L_0$. As we will later discuss in detail, arbitrageurs sit in the demand side of bankers loans supply. In the bad state, their demand will be positive only when doing so yields a positive net return. The latter requires $p_b < Y$, thus, liquidating loans in the bad state is costly for bankers - *fire-sales*.

After observing the aggregate state realization, the amount of loans their bank has liquidated early and the corresponding increase in cash holding, depositors choose whether to liquidate their position early. If they liquidate, their payoffs are $min\{D, v_s L(s) + C(s)\}$, where the second argument captures the bankruptcy state where bankers fail to meet their obligation and depositors receive the liquidation value of the loans plus the cash, i.e., $v_s L(s) + C(s)$, where $v_s \leq p_s$. If the latter condition holds with strict inequality, depositors face a higher liquidation cost than banker, partially explained by the bankruptcy cost inherent in these processes.

If depositors do not liquidate their position early, they receive their payment at $t = 2$, after bankers take their risk-management effort decision. At $t = 2$ if the high payoff is realized, then depositors receive R^h and the banker receives the residual $YL(s) + C(s) - R^h \geq 0$. If the low payoff is realized, depositors receive R^l and the banker receives the residual $C(s) - R^l \geq 0$. Note that the payoff to the banker is bounded by zero due to limited liability.

The timing of bankers' decision is illustrated in Figure (1):

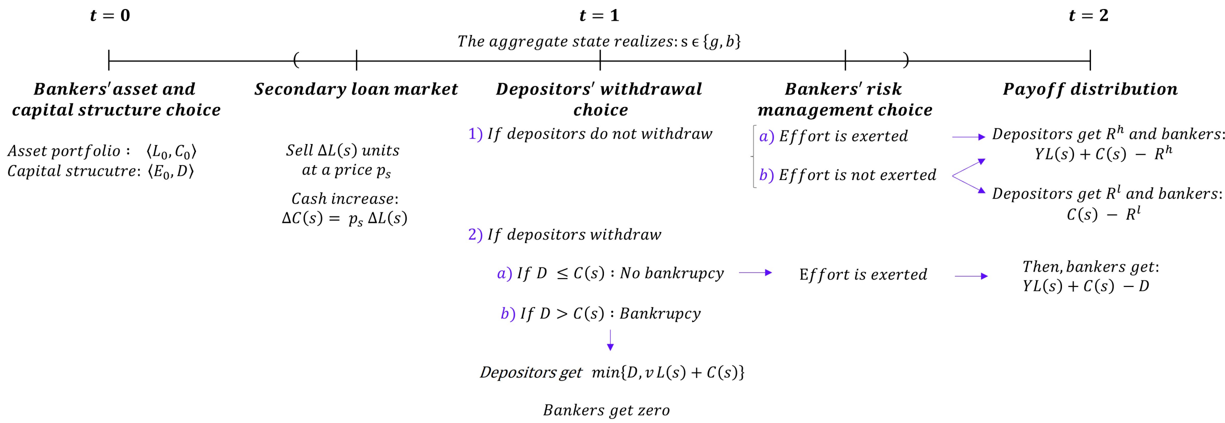


Figure 1: Time line

In the next sections, we will characterize the equilibrium of the model. In section (3) we will assume that effort is not observable, and in section (4) we will relax this assumption. This exercises is useful to assess the inefficiencies associated to the asymmetric information.

3. First-best Equilibrium

By backward induction, bankers' risk-management effort places first in the analysis. Based on assumption (2.1), bankers' effort is essential in ensuring the positive NPV of loans (granted at $t = 0$); otherwise, cash dominates loans in terms of risk-return. Such efficiency gain makes it optimal for bankers to exert risk-management effort, as they would bear the cost otherwise. Therefore, investing in loans yields a riskless return of Y, as it is accompanied by risk-management effort, consequently, the promised payment to depositors at $t = 2$ is $R^h = D$.

It is not optimal for bankers to invest in cash at $t = 0$ nor to accumulate cash at $t = 1$ by liquidating loans early. First, investing in cash at $t = 0$ is costly because it forgoes a more profitable investment opportunity. Second, accumulating cash at $t = 1$ by liquidating loans is costly as those loans might be sold at a discount. Despite being costly, cash does not play any role nor yields any benefit; thus, bankers' optimally set their cash balances to zero at all dates.

Then, bankers choose D and L_0 to maximize their expected payoffs, which after taking equation (1) into account ($L_0 = E_0 + D$), boils down to:

$$\underbrace{YE_0}_{\substack{\text{expected payoffs from}\\ \textbf{direct investment}}} + \underbrace{(Y-1)\,D}_{\substack{\text{expected payoffs from}\\ \textbf{selling claims}}} \tag{3}$$

subject to the feasibility constraint, $D \leq \bar{D}$, and bankers' participation constraint, $D \geq 0$. Assumption (2.1) claims that $Y > 1$ which implies that selling claims to depositors is profitable. To exploit this profit opportunity, they set D at its maximum level, i.e., $D = \bar{D}$, and by doing so, they set the balance sheet at its maximum scale maximizing loans granted. The following proposition summarizes the equilibrium when information is symmetric.

Proposition 1. *When risk-management effort is observable, bankers exerts effort, they operate at the maximum scale and cash is never used.*

4. Equilibrium under Moral Hazard

When effort is not observable, bankers must be induced to exert risk management effort since his effort is no longer contractible by depositors; risk-management effort is optimal for bankers *ex-ante* (at $t = 0$), but it may no longer be the case *ex-post* (at $t = 1$ when the effort decision is taken). To ensure that incentives are aligned in both dates, the incentive-compatibility constraint in both states is essential:

$$YL(s) + C(s) - R^h \geq \varepsilon[YL(s) + C(s) - R^h] + (1-\varepsilon)[C(s) - R^l] + B_s L(s)$$

After some algebra the constraint boils down to $(1-\varepsilon)YL(s) \geq (1-\varepsilon)(R^h - R^l) + B_s L(s)$. In fact, since the risk management effort is taken after the financial contract has been signed with depositors, bankers have an incentive to reduce the risk management effort eliciting a lower payment to depositors than the one promised. The right-hand side of the latter equation, captures this cost saving, that together with the private benefit constitute the total benefit of the risk-shifting. The left hand side of the equation, instead, captures the cost to bankers of their strategic misconduct, this is, the loss in the expected return of the loan portfolio.

The incentive compatibility constraint essentially imposes that the burden of not exerting risk-management effort is not disproportionately beard by depositors, for which a high R^l helps. Therefore, banker set $R^l = C(s)$ at its maximum level. In addition, investing in loans yields a riskless return of Y, as it is accompanied by risk-management effort, and the promised payment to depositors is $R^h = D$. Hence, bankers offer depositors a senior claim on their payoffs, i.e., a debt contract, to ameliorate the moral hazard problem. Substituting these values and simplifying the incentive compatibility constraint we get:

$$D \leq \mathcal{P}_s L(s) + C(s)$$

where $\mathcal{P}_s \equiv Y - \frac{B_s}{1-\varepsilon} > 0$ which is positive due to assumption 2.1 and it captures "pledgeable income" in the same vein as Holmstrom and Tirole (1997). Pledgeable income of the bankers represent the share of the per-unit return of loans that can be pledged to depositors without jeopardizing the incentives of the banker to properly manage the loan portfolio.

This condition also highlights the key role of cash: because cash is riskless, and is available to repay senior claim-holders (depositors) in the event of bank liquidation, the commitment to hold cash has important implications for bankers' incentives toward risk in the future. That commitment affects the way outsiders – who lack information about bank assets and bankers' behavior – view the risk managment of the bank, which has immediate consequences for the bank's access to funding.

The following assumption centres the analysis on the interesting case:

Assumption 4.1. *The bad (good) state, pledgable income is lower (higher) than the leverage ratio when the banker operates in full scale:*

$$\mathcal{P}_b < \frac{\bar{D}}{E_0 + \bar{D}} < \mathcal{P}_g.$$

Under this assumption, the first-best is not attainable when risk-management effort is not observable. In the good state, pledgable income is high enough so that incentive compatibility constraint in that state is not binding and cash is not needed. But in the bad state, cash is needed to induce bankers' effort if they want to operate in full scale, i.e., $D = \bar{D}$. In that case, $C(b) > 0$, which implies that either C_0 or $\Delta C(b)$ has to be positive.

Bankers, however, cannot commit at $t = 0$ to increase their cash holdings at $t = 1$ unless they have the incentives to do so at $t = 1$. In this aspect, a reliable threat by depositors to withdraw their funds and close the bank can have a disciplinary role, since bankers might prefer to do the "right" thing than to lose everything with the bank closure. For this strategy to work, depositors' threat to close the bank must be credible, this is, depositors must be better off closing the bank than not closing.

Formalizing this intuition, if bankers deviate at $t = 1$ and do not increase their cash holdings to meet the incentive compatibility constraint (their optimal deviation is $\Delta\hat{C}(b) = 0$), depositors will close the bank if the liquidation value of the outstanding loans plus the cash is higher than the expected return under no effort:

$$v_b L_0 + C_0 \geq \varepsilon D + (1-\varepsilon) C_0$$

If bankers increase their cash holdings in line with their incentive compatibility constraint, depositors will not have incentives to withdraw early since $min\,\{D, v_s L(s) + C(s)\} \leq D$.

Lemma 4.1. *When effort is not observable, and the first-best is not attainable (assumption (4.1) holds), demandable debt is the optimal financial contract bankers offer to depositors.*

An important insight from the model is that the ability of outsiders to withdraw their claims plays an important role in encouraging banks to maintain adequate amounts of cash, especially when the bad states of the world necessitate a topping up of cash balances. Without the discipline of a withdrawal threat, the bank has no incentive to increase cash holdings in bad states of the world. Instead, it would promise to do so but renege on that promise after outsiders had deposited their funds with the bank. That is, the bank must be exposed to potential liquidity risk in order for cash to be able to improve risk management.

4.1. Banker's maximization problem

Bankers take prices as given, and choose the debt amount, D, their cash holdings at $t = 0$, C_0, and the cash increase at date $t = 1$ and bad state, [11] $\Delta C(b)$, which determine the loan portfolio at the terminal date, $L(g) = L_0$ and $L(b) = L_0 - \frac{1}{p_b}\Delta C(b)$ where $L_0 = E_0 + D - C_0$, to maximize their expected payoffs given by

$$\underbrace{Y E_0}_{\substack{\text{expected payoffs from} \\ \textbf{direct investment}}} + \underbrace{(Y-1)\,D}_{\substack{\text{expected payoffs from} \\ \textbf{issuing debt}}} - \underbrace{\underbrace{(Y-1)C_0}_{\text{forgone net return}} - \underbrace{(1-q)\left(\frac{Y}{p_b} - 1\right)}_{\text{Expected fire-sale cost}}}_{\text{cost of investing in } \textbf{cash} \text{ at } = 0 \text{ and } t = 1} \tag{4}$$

subject to the feasibility constraints, namely, $D \leq \bar{D}$ and participation constraint $D \geq 0$, in addition to the incentive compatibility constraint in equation, taking into account equations (1)-(2), can be re-written as:

$$D - C_0 \leq \frac{\mathcal{P}_b}{1-\mathcal{P}_b} E_0 + \frac{1}{1-\mathcal{P}_b}\left(1 - \frac{\mathcal{P}_b}{p_b}\right)\Delta C(b) \tag{5}$$

[11] The demandable feature of the debt contract serves as commitment device, thus, it is as if $\Delta C(b)$ is chosen at $t = 0$.

Equation (5) highlights that cash at $t = 0$ and extra cash at $t = 1$ both affect the incentive compatibility constraint but do not have the same role. Cash accumulated at $t = 0$ always relaxes the incentive constraint. Instead, an increase in cash holding by liquidating loans early, $\Delta C(b) > 0$, relaxes the incentive constraint if and only if the liquidation return is higher than the pledgeable return, i.e., $\mathcal{P}_b < p_b$. If so, it is more effective in relaxing the constraint compared to C_0 only when $p_b > 1$.

Due to the fact that holding cash is costly, bankers will hold the minimum amount necessary to meet the incentive compatibility constraint, hence, the latter will always be binding – its Lagrangian multiplier is denoted by λ. The key question is when do bankers accumulate the cash holdings needed to satisfy the incentive-compatibility constraint.

First, note that, C_0 is equivalent to negative D. This implies that while the net debt (i.e., $D - C_0$) is uniquely determined in bankers optimality conditions, bankers are indifferent between the combination of D and C_0. Different combinations of debt and initial cash affect the size of bankers' balance-sheet, but the size of the loan portfolio is uniquely determined by net debt. Only when the feasibility constraint of D is binding – its Lagrangian multiplier is ξ, – their optimal debt level is $D = \bar{D}$ and initial cash holdings are zero, $C_0 = 0$.

Second, bankers will top up cash holdings in the bad state, when the following condition is satisfied:

$$\underbrace{\lambda \frac{1}{1-\mathcal{P}_b}\left(1 - \frac{\mathcal{P}_b}{p_b}\right)}_{\text{Benefit of relaxing the ICC}} - \underbrace{(1-q)\left(\frac{Y}{p_b} - 1\right)}_{\text{Expected fire-sale cost}} = 0$$

where $\lambda = Y - 1 + \xi$. This equation tells us that in deciding their cash balances in the bad state, each bank trades off alleviating the incentive compatibility constraint boosting their debt capacity, against the potential for higher fire-sale costs. The lower is p_b, the higher is the fire-sale discount, and the more costly becomes accumulating incentive-compatible cash by selling loans in the bad state. Hence, p_b captures the relative cost of building cash at $t = 1$ as opposed to $t = 0$, thus, it is key in the timing of cash accumulation as summarized in the following proposition.

Proposition 2. *When risk-management effort is not observable and the first-best is unattainable (assumption (4.1)), bankers might be incentivized to exert effort by holding cash. The optimal timing of cash accumulation depends on the market liquidity in the secondary loan market:*

- *If $p_b < \bar{p}_b$, bankers do not build incentive-compatible cash in the bad state as it is too costly. Net debt (i.e., $D - C_0$) is determined by the incentive compatibility constraint in equation (5).*

- *If $p_b > \bar{p}_b$, bankers build incentive-compatible cash only in the bad state, and their cash balances in this state ($\Delta C(b)$) is determined by the incentive compatibility constraint in equation (5). The feasibility constraint of debt is binding ($\xi > 0$), meaning that $D = \bar{D}$ and $C_0 = 0$.*

- *If $p_b = \bar{p}_b$, bankers might build incentive-compatible cash both at $t = 0$ and top it up in the bad state, always meeting the incentive compatibility constraint in equation (5).*

where the threshold is given by $\bar{p}_b \equiv \omega Y = (1-\omega)\mathcal{P}_b$ where $\omega \equiv \frac{1-q}{1-q+\frac{Y-1}{1-\mathcal{P}_b}}$.

After analysing bankers optimal behaviour, we need find the equilibrium price in the secondary loan market, to conclude which of the aforementioned cases prevail in equilibrium. Thus, in the next section the equilibrium in this market will be characterized.

4.2. Equilibrium in secondary loan markets

Arbitrageurs constitute the demand side in the secondary loan markets. They face three investment opportunities at $t = 1$: their productive technology, loans supplied by bankers or cash. They will optimally decide how much to invest in their technology, $K(S)$, and in buying $A(s)$ units of loans from bankers to maximize their expected profits at each state:

$$G(K(s)) + YA(s) \tag{6}$$

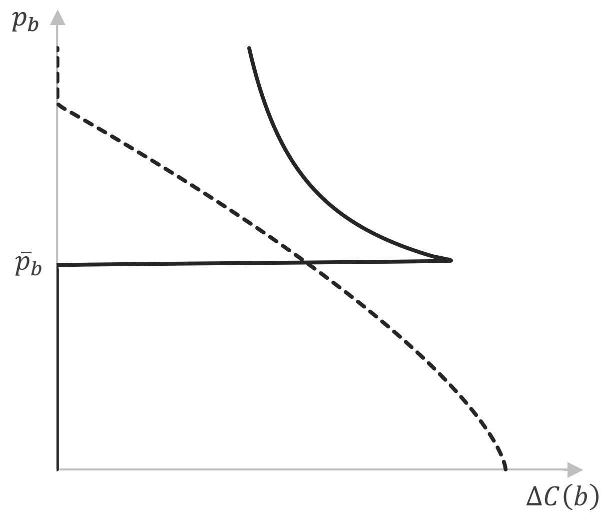
subject to the budget constraint $m = K(s) + p_s A(s)$. This investment decision is particularly interesting in the bad state, in which the banker's supply of loans is positive. [12] In this state, arbitrageurs will never invest in cash as investing in their technology is always more profitable due to assumption (2.2). Their optimal investment in bankers' loan, however, is determined in the following optimality condition:

$$G'(m - p_b A(b)) = \frac{Y}{p_s}$$

The right-hand side is the marginal return on their technology, while the left-hand side is the gross return of buying loans from bankers in secondary markets. Assuming arbitrageurs have the same monitoring technology as bankers, the price that set the net present value of such financial transaction equal to zero is Y. If $p_s < Y$, instead, firms make a positive profit

[12] In the good state, where bankers' supply of loans is zero, the price is simply high enough such that the demand is zero too.

Figure A. *Optimal* $\Delta C(b)$ *as a function of* p_b



Note: $\bar{p}_b = \omega Y + (1-\omega)\mathcal{P}_b < 1$ where $\omega \equiv \frac{(1-q)}{(1-q)+\frac{Y-1}{1-\mathcal{P}_b}}$

when buying loans, which ultimately compensate for the forgone profitable opportunity of investing in $G(.)$. The following lemma summarizes the result.

Lemma 4.2. *From arbitrageurs optimality condition, the supply of resources they are willing to supply (i.e.,* $p_b A(b) = \Delta C^a(b)$*) as a function of loan prices is*

$$p_b^*(\Delta C^d(b)) = \frac{Y}{G'(m - \Delta C^a(b))}$$

where $p_b^*(\Delta C^a(b)) < Y$ *and* $p_b^{*'}(\Delta C^a(b)) < 0$ *for* $\forall \Delta C^a(b)$*.*

The demand function is downward sloping, implying that ultimately, the price level depends on the value of loans transacted. More importantly, loans are sold at a discount from their fundamental value. If firms endowment was abundant, they would invest until the point in which the marginal rate of consumption is equalized across dates, equalling to one. With scarce firms' endowment, the return on secondary market arbitrage opportunities (buying up fire-sold assets) also becomes the hurdle rate for new investment, a point emphasized by Stein (2012). Therefore, liquidating loans early is costly for bankers, as the price is below their reservation value – *fire-sales.*

The following graph illustrates the equilibrium in the cash ex-changed from bankers and arbitrageurs. The graphs illustrates that, when $p_b > \bar{p}_b$, the curve is backward bending. For that range, the feasibility constraint for D is binding, hence, $C_0 = 0$ and $\Delta C(b)$ is determined by the incentive compatibility constraint. Then, the higher is the p_b the proceeds from selling loans is higher, thus, less loans need to be sold to meet the incentive requirement. Similar to Kara and Ozsoy (2020), the following assumption ensure the equilibrium is unique in the backward bending range of the curve.

Assumption 4.2. *The inverse demand function is such that* $\theta \equiv (-p^{*\prime})\frac{\Delta C(b)}{p_b^*} \leq 1$.

Without this assumption, different levels of assets sales would arise the same level of funds, leading to multiple equilibrium. This further implies that $G'() + \Delta C(b) G''() > 0$.

Given the the equilibrium value of $\Delta C(b)$, the following proposition characterizes the equilibrium net debt.

Proposition 3. *When the fire-sale cost is low enough, bankers decide to operate in full scale, i.e.,* $D = \bar{D}$ *and* $C_0 = 0$. *When the fire sales cost is high enough, there is a multiplicity of equilibrium: there are infinite arrangement of* D *and* C_0 *that set net debt,* $D - C_0$ *at the level required by the incentive compatibility constraint.*

The different combinations of D and C_0 that yield a certain level of net debt, affect the size of the balance sheet but it does not affect the bankers net profits. Thus, they could either choose the minimum balance sheet size by setting $C_0 = 0$, or alternatively, they could chose the maximum balance sheet size by setting $D = \bar{D}$ and C_0 correspondingly. In the latter case, bankers store resources on behalf of depositors.

4.3. The role of equity

Insofar, the role of equity has been largely overlooked. But we will show next that, consistent with previous literature, in this environment equity also improves banker's risk-management effort.

In particular, an increase in inside equity, ceteris paribus, increases the resources available to invest either in loans or cash. Since cash is not an attractive investment opportunity per se, the additional units of funds increase the amount of loans granted. Most importantly, bankers increase their skin-in-the-game, as the fraction of the loan portfolio they own increases, ameliorating their incentives to risk-shifting. There is nothing novel in this mechanism, yet, it is interesting to understand how equity interacts with cash in providing adequate incentives. The following proposition summarizes the main result in this aspect:

Proposition 4. *Equity and cash are substitutes in enhancing bankers' risk-management.*

This implies, that higher equity reduces the need for cash, thus, bankers optimally adjust their cash balances accordingly, whatever is the prevalent form of accumulating cash. Hence, higher equity might either reduce the need to invest in a less productive technology or reducing the amount of costly loan liquidation. In the presence of both sources of cash, the former channel prevails.

Hence, we emphasize the need to jointly consider net debt and capital requirements. Both policy tools are substitutes, meaning that one requirement could be relaxed when the other becomes more stringent. In addition, only the net debt requirement needs to be countercyclical. In fact, the equity to loan ratio increases in the bad state – i.e., equity decreases proportionally less than loans in the bad state as opposed to the good state. Hence, in a downturn bankers equity ratio is above the regulator's requirement. On the contrary, net debt to loan ratio decreases in the bad state– i.e., net debt decreases proportionally more than loans in the bad state as opposed to the good state. Thus, net debt requirements must be relaxed in a downturn, although, only the initial requirement is binding for bankers. [13]

5. Welfare analysis

In this section we evaluate if the competitive equilibrium is constrained-efficient, or if alternatively there is room for a Pareto-improvement. [14] To analyze constraint-efficiency, we define a social planner whose optimization problem is subject to the same constraints as the private market (including bankers' incentive constraint), and respecting that asset prices are market-determined. However, unlike private agents, the planner takes into account the effect of the chosen allocations on asset prices.

Similar to Lorenzoni (2008); Dávila and Korinek (2018) and Kara and Ozsoy (2020), the constrained-efficiency concept we use in this paper relies on the planner making compensating transfers; this is, the planner has the means to engage in lump-sum transfers to ensure that efficiency gains are spread around the economy such that all agents in the economy are (weakly) better off. Without loss of generality, we focus on the case in which the constrained

[13]Then, $\frac{E_0}{L_0} < \frac{E(b)}{L(b)} = \frac{E_0 - \left(\frac{1}{p_b} - 1\right)\Delta C(b)}{L_0 - \frac{1}{p_b \Delta C(b)}}$ holds when $\frac{D - C_0}{E_0} < p_b$. Same condition must hold for $\frac{D-C_0}{L_0} > \frac{D-C(b)}{L(b)}$.

[14]Whenever some agents are financially constrained, the market outcome is clearly not first best: removing the frictions that underlie the financial constraints increases efficiency. However, in practice, policymakers frequently must take such frictions as given, which lead to the question of whether decentralized equilibrium allocations are constrained efficient. In other words, can a policymaker subject to the same constraints as private agents improve on the market outcomes?

planner maximizes bankers' expected payoffs subject to the same constraints faced by bankers, in addition to a constraint that, after transfers, depositors and arbitrageurs' utility level equal their expected utility in the competitive equilibrium.

To ensure that depositors are as well of as in the competitive equilibrium, the social planner must ensure a unit-return on their deposits. Arbitrageurs' welfare, however, depends on the amount of sales at $t = 1$, potentially requiring a compensatory transfer if the amount transacted changes. This distributional aspect is a general feature of models that involve a social deadweight cost for fire sales, in the spirit of Shleifer and Vishny (1992). We first ignore the potential distortionary effect of the transfer, and assume that the transfer (if any) occurs in the good state (or it could also occur in the bad state, only if agents do not anticipate it). Then, the effective transfer to arbitrageurs:

$$\begin{aligned} T_2\left(\Delta C(b)\right) \quad =& \frac{(1-q)}{q}\left(\left[G\left(m-\Delta C^*(b)\right)+\frac{Y}{p_b^*(\Delta C^*(b))}\Delta C^*(b)\right]\right. \\ &\left. -\left[G\left(m-\Delta C(b)\right)+\frac{Y}{p_b^*(\Delta C(b))}\Delta C(b)\right]\right) \end{aligned}$$

where $p^*(\Delta C(b)) = \frac{Y}{G'(m-\Delta C(b))}$ for $\forall \Delta C(b)$. Note that, $T > 0$ if $\Delta C^*(b) > \Delta C^{SP}(b)$, $T < 0$ if $\Delta C^*(b) < \Delta C^{SP}(b)$, and $T = 0$ otherwise.

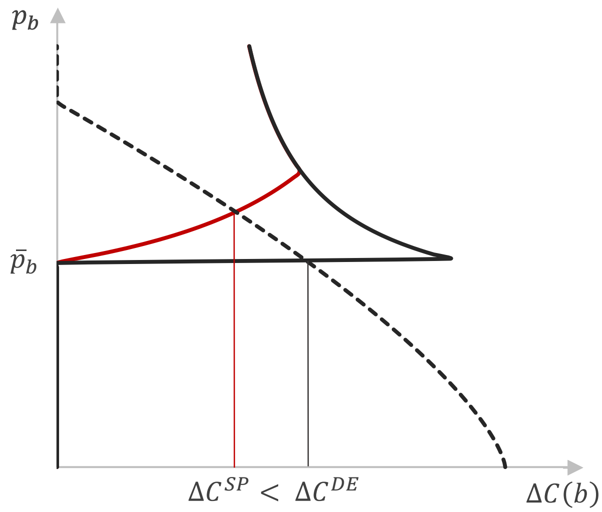
Hence, the social's planner maximization problem is the same as in the previous section, taking the aforementioned transfer function into account, in addition to the price function. The firs-order condition related to cash build-up at $t = 1$, is the only one that differ from bankers':

$$\underbrace{\lambda\frac{1}{1-\mathcal{P}_b}\left(1-\frac{\mathcal{P}_b}{p_b^*}(1+\theta(\Delta C(b))\right)}_{\text{Social benefit of relaxing the ICC}}-\underbrace{(1-q)\left(\frac{Y}{p_b^*}-1\right)}_{\text{expected fire-sale cost}}=0$$

where $\theta(\Delta C(b)) \equiv (-p_b^{*\prime})\frac{\Delta C(b)}{p_b^*} > 0$. These first-order conditions are similar to the first-order condition of the bankers, except that it contains an additional term: $-\lambda\frac{1}{1-\mathcal{P}_b}\frac{\mathcal{P}_b}{p_b^*}\theta(\Delta C(b))$. Banker's private benefit of $\Delta C(b)$ is higher compare to the social benefit, and this wedge arises because, unlike the individual bankers, the constrained planner takes into account how their choices affect the price of assets —pecuniary externality.

Pecuniary externalities might not be welfare-reducing per se; but, in the presence of price-dependent binding-constraints this premise might not hold. In the latter case, any bankers' effect on market prices affects other bankers not only by altering their budget constraints but also by loosening or tightening their incentive compatibility constraints. A suitable change in the behaviour of agents modifies asset prices, relaxing incentive constrains directly and changing the effective financial decisions of those agents for which the constraint

Figure B. *Optimal $\Delta C(b)$ as a function of p_b*



Note: $\bar{p}_b = \omega Y + (1-\omega)\mathcal{P}_b < 1$ where $\omega \equiv \frac{(1-q)}{(1-q)+\frac{Y-1}{1-\mathcal{P}_b}}$

binds. The social planner take the latter effect into consideration, thus, explaining the wedge in the bankers and social planners optimally condition.

Changes in equilibrium price directly affect the tightness of the financial constraint faced by borrowers. The collateral effects capture the direct effect of changes in aggregate state variables on the tightness of the constraint. Unlike distributive effects, collateral effects are generally not zero-sum across agents.

Proposition 5. *When effort is not observable and the first best is not attainable, the competitive equilibrium is constrained inefficient for certain parameter values. Furthermore, the decentralized economy's (DE) allocations compare to the constrained efficient allocations (SP) as follows:*

- *Cash at $t = 1$ and state $s = b$: $\Delta C^{SP}(b) \leq \Delta C^{DE}(b)$*
- *Net debt: $C_0^{DE} \leq C_0^{SP}$*
- *Loans: $L_0^{DE} \geq L_0^{SP}$*

Due to a fire-sale externality, banks' reliance on loans sales to top-up incentive-compatible cash holdings is excessive. Consequently, their initial net debt (debt minus cash) and their investment in loans is inefficiently high.

Similar to the collateral constraint described by Dávila and Korinek (2018), the magnitude of the externality is also determined by the product of three sufficient statistics:

- The shadow value on the binding incentive-constraint: $\lambda = Y - 1$
- The sensitivity of the incentive-constraint to asset price: $-\frac{\mathcal{P}_b}{1-\mathcal{P}_b}\frac{\Delta C(b)}{p_b^2}$
- The sensitivity of the equilibrium asset prices to changes in $\Delta C(b)$: $\frac{\partial p_b}{\partial \Delta C(b)}$

However, bankers profits always increase with regulation and increase in absolute values more than the traditional sector's profits decrease. As a result, total profits increase with regulation. Thus, it is possible to implement capital or liquidity regulations in a Pareto-improving way by taxing banks (in the good state) and transferring resources to arbitrageurs.

Proposition 6. *The social optimum can be implemented with the following policy tools:*

1. *Risk-adjusted capital requirement equal to* $\frac{E_0}{L_0^{SP}}$
2. *Net debt requirement equal to* $\frac{(D-C_0)^{SP}}{L_0^{SP}}$*, but allowing bankers to fall below in the bad state*

6. Conclusion

This paper examines the role that banks' reserves play in the provision of risk-management incentives. Although investing in cash is costly (e.g., forge profitable investment opportunities), it can reduce credit risk by encouraging proper risk-management of the loan portfolio. The welfare analysis unveils that due to a fire-sale externality, the initial investment in cash is socially low, consequently, the fire-sale of loans to top-up the cash in bad state of the economy, is excessive. This calls for policy intervention; liquidity requirement is effective in boosting initial cash, which reduces the need to sale loans ex-post. This liquidity requirement, which ultimately reduces the amount of loans transacted and their price, needs to be accompanied by a compensatory transfer from bankers to arbitrageurs to obtain a Pareto improvement. Although we focus in the particular case in which those transfers do not distort incentives, this opens up the question of how arbitrageurs react in anticipation of those transfers. In work in progress, we endogenize the choice of becoming bankers or arbitrageurs to understand how any policy initiative might ultimately affect this decision.

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7. Mathematical appendix

7.1. Banker's maximization problem

The Lagrangian of the problem is given by

$$\begin{aligned}
\mathcal{L} \quad =& YE_0 + (Y-1)(D-C_0) - (1-q)\left(\frac{Y}{p_b}-1\right)\Delta C(b) \\
&-\lambda\left[D - C_0 - \frac{\mathcal{P}_b}{1-\mathcal{P}_b}E_0 - \frac{1}{1-\mathcal{P}_b}\left(1-\frac{\mathcal{P}_b}{p_b}\right)\Delta C(b)\right] \\
&-\mu\left[(\varepsilon - v_b)(D-C_0) - v_b E_0\right] \\
&-\eta\left[\Delta C(b) - p_b(E_0 + D - C_0)\right] \\
&-\xi\left[D - \bar{D}\right]
\end{aligned}$$

The Kuhn-tucker conditions are given by

$$\begin{aligned}
D: &\qquad Y - 1 - \lambda - \mu(\varepsilon - v_b) + \eta p_b - \xi \leq 0 \\
C_0: &\qquad -(Y-1) + \lambda + \mu(\varepsilon - v_b) - \eta p_b \gtreqless 0 \\
\Delta C(b): &\qquad -(1-q)\left(\frac{Y}{p_b}-1\right) + \lambda\frac{1}{1-\mathcal{P}_b}\left(1-\frac{\mathcal{P}_b}{p_b}\right) - \eta \leq 0 \\
&\lambda \geq 0 \text{ and } \lambda\left[D - C_0 - \frac{\mathcal{P}_b}{1-\mathcal{P}_b}E_0 - \frac{1}{1-\mathcal{P}_b}\left(1-\frac{\mathcal{P}_b}{p_b}\right)\Delta C(b)\right] = 0 \\
&\mu \geq 0 \text{ and } \mu\left[(\varepsilon - v_b)(D-C_0) - v_b E_0\right] = 0 \\
&\eta \geq 0 \text{ and } \eta\left[\Delta C(b) - p_b(E_0 + D - C_0)\right] = 0 \\
&\xi \geq 0 \text{ and } \xi\left[D - \bar{D}\right] = 0
\end{aligned}$$

First of all, note that for $Y > 1$, D will be positive (bear in mind that when $D = 0$, $\lambda = \mu = 0$). This implies that FOC_D holds with equality and FOC_{C_0} holds with $\leq$. In addition, when $\xi = 0$, both FOC_D and FOC_{C_0} are linearly dependent, suggesting that the problem has more than one solution, yet, net debt, i.e., $D - C_0$, is uniquely determined. When $\xi > 0$, instead, $D = \bar{D}$ and $C_0 = 0$.

Hence, from FOC_D the Lagrangian multiplier of the ICC is

$$\lambda = Y - 1 - \mu(\varepsilon - v_b) + \eta p_b - \xi$$

Substituting λ in the $FOC_{\Delta C}$:

$$(Y-1)\,\frac{1}{1-\mathcal{P}_b}\left(1-\frac{\mathcal{P}_b}{p_b}\right)-(1-q)\left(\frac{Y}{p_b}-1\right)-\psi\leq 0$$

where $\psi \equiv (\mu(\varepsilon - v_b)+\xi)\,\frac{1}{1-\mathcal{P}_b}\left(1-\frac{\mathcal{P}_b}{p_b}\right)-\eta\left(\frac{1-p_b}{1-\mathcal{P}_b}\right)$. Since $\eta = 0$ when $p_b > 1$, then $\psi > 0$ when at least one of the constraints is binding (only then the corresponding Lagrangian multiplier is positive).

When $p_b \geq Y$, building cash holdings in the bad state is not costly for bankers, thus, the incentive compatibility constraint is not binding, i.e., $\lambda = 0$. For $p_b > Y$, selling loans at $t = 1$ and bad state is a profitable financial transaction so they will sell all the loan portfolio, $\eta > 0$, and issue as much as debt possible, $\xi > 0$, while costly cash at $t = 0$ will be zero. For $p_b = Y$, the incentive compatibility constraint is not binding either, but there are two potential cases depending on the parameter values:

- If $\frac{\bar{D}}{\bar{D}+E_0} \leq \frac{v_b}{\varepsilon}$, then $\xi > 0$ and $\mu = 0$, which implies that $D = \bar{D}$ and $C_0 = 0$, while $\Delta C(b)$ will ensure that the incentive compatibility constraint is met.
- If $\frac{\bar{D}}{\bar{D}+E_0} > \frac{v_b}{\varepsilon}$, then $\xi = 0$ and $\mu > 0$, which implies that $D < \bar{D}$ and $C_0 = D - \frac{v_b}{(\varepsilon - v_b)}E_0$, while $\Delta C(b)$ will ensure that the incentive compatibility constraint is met ($\Delta C(b)$ is always positive since $\mathcal{P}_b < \frac{\bar{D}}{\bar{D}+E_0}$).

Note that this case will never prevail in equilibrium. But it is important to understand that when $p_b \geq Y$, the incentive compatibility constraint is never binding.

When $p_b < Y$ loans are sold at a discount from their fundamental value, there are different cases depending on the value of p_b:

Case A: $p_b < \omega Y + (1-\omega)\mathcal{P}_b$ where $\omega \equiv \frac{(1-q)}{(1-q)+\left(\frac{Y-1}{1-\mathcal{P}_b}\right)}$

Then, $\Delta C(b) = 0$, and so are $\eta = \mu = 0$. Net debt is determined by the incentive compatibility constraint:

$$D - C_0 = \frac{\mathcal{P}_b}{1-\mathcal{P}_b}E_0$$

for $D \in \left[\frac{\mathcal{P}_b}{1-\mathcal{P}_b}E_0, \bar{D}\right]$ and $C_0 \geq 0$. Due to assumption (4.1), cash holdings at $t = 0$ have to be positive if bankers set $D = \bar{D}$. Then, $\xi = 0$, which implies that only the ICC is binding, this is, $\lambda = Y - 1$.

Case B: $p_b = \omega Y + (1-\omega)\mathcal{P}_b$ where $\omega \equiv \frac{(1-q)}{(1-q)+\left(\frac{Y-1}{1-\mathcal{P}_b}\right)}$

The binding incentive compatibility constraint:

$$D - C_0 = \frac{\mathcal{P}_b}{1-\mathcal{P}_b} E_0 + \frac{1}{1-\mathcal{P}_b}\left(1 - \frac{\mathcal{P}_b}{p_b}\right)\Delta C(b)$$

for $D \in \left[\frac{\mathcal{P}_b}{1-\mathcal{P}_b} E_0, \bar{D}\right]$, $C_0 \geq 0$, $\Delta C(b) \geq 0$, while the rest of the constraint have to be met. Due to assumption (4.1), either cash holdings at $t = 0$ or at $t = 1$ have to be positive if bankers set $D = \bar{D}$. Only the ICC is binding, this is, $\lambda = Y - 1$.

Case C: $p_b > \omega Y + (1-\omega)\mathcal{P}_b$ where $\omega \equiv \frac{(1-q)}{(1-q)+\left(\frac{Y-1}{1-\mathcal{P}_b}\right)}$

In this case, $\psi > 0$ needs to hold, so at least one of the constraints has to be binding. Given that all constraints set a bound on net debt (e.g., $D - C_0 \leq \bar{D}$), they are mutually exclusive and only one of them can be binding. Depending on the parameter values, there are different possible cases:

Case C.i: $\mu > 0$ and $\eta = \xi = 0$

The ICC and the disciplinary role of bank run are binding:

$$\begin{aligned} D - C_0 &= \frac{\mathcal{P}_b}{1-\mathcal{P}_b} E_0 + \frac{1}{1-\mathcal{P}_b}\left(1 - \frac{\mathcal{P}_b}{p_b}\right)\Delta C(b) \\ (\varepsilon - v_b)(D - C_0) &= v_b E_0 \end{aligned}$$

This case prevails when $\frac{v_b}{\varepsilon} \leq p_b$ which ensures that $\eta = 0$, and $\frac{\bar{D}}{\bar{D}+E} \geq \frac{v_b}{\varepsilon}$ which ensures that $\xi = 0$. In addition, $\frac{v_b}{\varepsilon} > \mathcal{P}_b$.

Case C.ii: $\eta > 0$ and $\xi = \mu = 0$

The ICC and the constraint on $\Delta C(b)$ are binding:

$$\begin{aligned} D - C_0 &= \frac{p_b}{1-p_b} E_0 \\ \Delta C(b) &= p_b (E_0 + D - C_0) = D - C_0 \end{aligned}$$

This case prevails when $p_b \leq \frac{\bar{D}}{\bar{D}+E_0} < 1$ which ensures that $\xi = 0$, and $p_b \leq \frac{v_b}{\varepsilon}$ which ensures that $\mu = 0$.

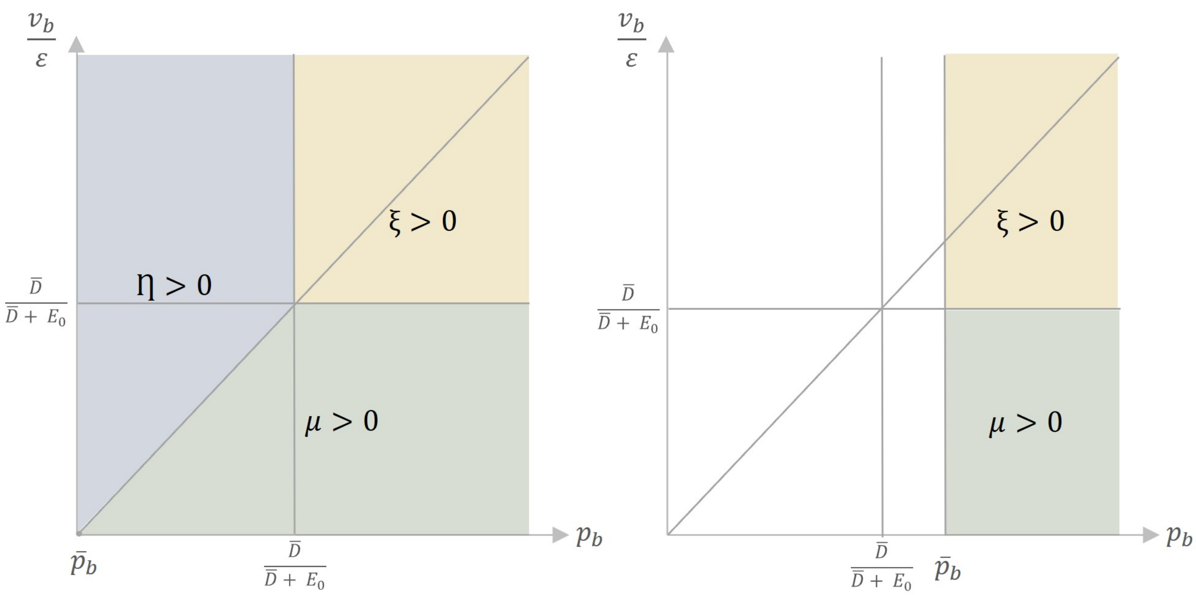
Case C.iii: $\xi > 0$ and $\eta = \mu = 0$

In this case, $D = \bar{D}$ and $C_0 = 0$ and $\Delta C(b)$ is determined by the ICC:

$$\bar{D} = \frac{\mathcal{P}_b}{1-\mathcal{P}_b} E_0 + \frac{1}{1-\mathcal{P}_b}\left(1 - \frac{\mathcal{P}_b}{p_b}\right)\Delta C(b)$$

This case prevails when $p_b \geq \frac{\bar{D}}{\bar{D}+E_0}$ which ensures that $\eta = 0$, and $\frac{\bar{D}}{\bar{D}+E} \leq \frac{v_b}{\varepsilon}$ which ensures that $\mu = 0$.

The last three options are mutually exclusive. Which one prevails depends on $\frac{v_b}{\varepsilon}$ and p_b:



Note: $\bar{p}_b = \omega Y + (1-\omega)\mathcal{P}_b < 1$ where $\omega \equiv \frac{(1-q)}{(1-q)+\frac{Y-1}{1-\mathcal{P}_b}}$

7.2. Equilibrium

In the equilibrium analysis, we will focus on the case in which $\frac{v_b}{\varepsilon} > \frac{\bar{D}}{\bar{D}+E_0}$ (sufficient condition for $\mu = 0$) and $\bar{p} > \frac{\bar{D}}{\bar{D}+E_0}$ (sufficient condition for $\eta = 0$).

Market clearing at $t = 1$, requires that the supply for loans equals the demand for loans at the prevailing price p_b. Then, if the solution is interior:

$$G'(m - p_b \Delta L(b)) = \frac{Y}{p_b}$$

Note that, $\Delta C(b) = p_b \Delta L(b)$, so that the equilibrium price:

$$p_b^*(\Delta C(b)) = \frac{Y}{G'(m - \Delta C(b))} \quad \text{where} \quad \frac{\partial p_b^*}{\partial \Delta C(b)} = \frac{Y G''(m - \Delta C(b))}{G'^2(m - \Delta C(b))} < 0$$

Hence, the price is a negative function of the total value of loans sold at $t = 1$.

Does an equilibrium exist? Yes.

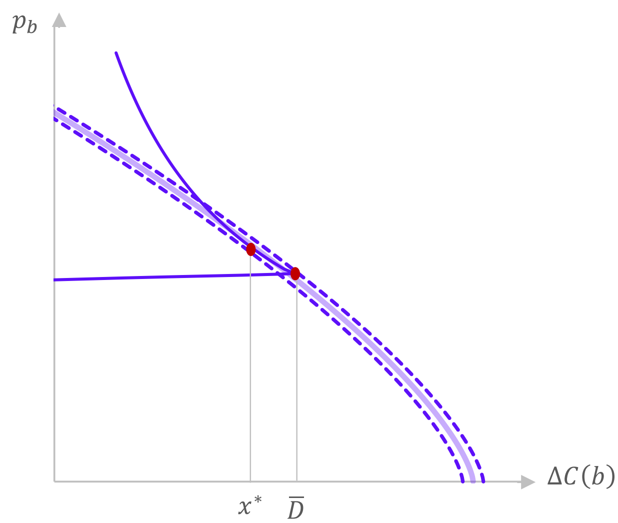
We will denote the demand function by patient investors as $Q^d(p_b)$ and the supply function by bankers as $Q^s(p_b)$. If $p_b^* \geq Y$ then $Q^d(p_b = 0$ and $Q^s(p_b) > 0$. Since for that range $Q^d < Q^s$, then in equilibrium $p_b^* < Y^h$. If $p_b = 0$ then $Q^s(0) = 0$ and $Q^d(0) \approx m$. Since $Q^d(0) > Q^s(0)$, then in equilibrium $p_d^* > 0$. Given that both $Q^s(p_d)$ and $Q^d(p_d)$ are continuous, both curves must cross (the intersection represents the equilibrium).

Is there a unique equilibrium? Only under certain conditions.

There is multiplicity of equilibrium when the inverse demand function, $p_b^*(Q)$, satisfies these conditions:

- For $p_b^{*'}(x^*) = F'(x^*)$ and $p_b^*(x^*) \geq F(x^*)$where $x^* < \bar{D}$
- For $p_b^*(\bar{D}) \leq F(\bar{D})$

where $F(x) = \frac{\mathcal{P}_b x}{x-(1-\mathcal{P}_b)\left(\bar{D}-\frac{\mathcal{P}_b}{1-\mathcal{P}_b}E_0\right)}$ and $p_b^*(x) = \frac{Y^h}{G'(m-x)}$.



To rule out multiple equilibria, a sufficient condition is $p_b^{*'}(\bar{D}) \geq F'(\bar{D})$. This is when the demand curve is elastic enough.(check the exact condition of the rest of the papers). ....

Characterization of equilibrium. Next, we will focus on the case in which the equilibrium is unique. Given that $0 < p^*(x) < Y$ for $\forall x$, the incentive compatibility constraints is always binding which implies that cash is needed to aligned incentives. Bankers can accumulate cash at $t = 0$ or/and at $t = 1$, and their choice between these two option depends on the parameters of the model:

- **Cash accumulation only at $t = 0$ when $p_b^*(0) \leq \bar{p}_b$**

  Then, $\Delta C(b) = 0$, and so are $\eta = \mu = 0$. Net debt is determined by the incentive compatibility constraint:

$$D - C_0 = \frac{\mathcal{P}_b}{1 - \mathcal{P}_b} E_0$$

  for $D \in \left[\frac{\mathcal{P}_b}{1-\mathcal{P}_b} E_0, \bar{D}\right]$ and $C_0 \geq 0$. Due to assumption (4.1), cash holdings at $t = 0$ have to be positive if bankers set $D = \bar{D}$.

- **Cash accumulation both at $t = 0$ and $t = 1$ when $p_b^*(a) \leq \bar{p}_b < p_b^*(0)$** where $a \equiv \left(\bar{D} - \frac{\mathcal{P}_b}{1-\mathcal{P}_b} E_0\right) \times \left(\frac{1-\mathcal{P}_b}{1-\frac{\mathcal{P}_b}{\bar{p}_b}}\right)$.

  Then, cash holding at $t = 1$ is determined by $FOC_{\Delta C}$ (which is equivalent to $p_b^*(\Delta C(b)) = \bar{p}_b$) and net debt by the incentive compatibility constraint

$$(Y-1)\frac{1}{1-\mathcal{P}_b}\left(1 - \frac{\mathcal{P}_b}{p_b^*(\Delta C(b))}\right) - (1-q)\left(\frac{Y}{p_b^*(\Delta C(b))} - 1\right) = 0$$
$$D - C_0 = \frac{\mathcal{P}_b}{1-\mathcal{P}_b} E_0 + \frac{1}{1-\mathcal{P}_b}\left(1 - \frac{\mathcal{P}_b}{p_b^*(\Delta C(b))}\right)\Delta C(b)$$

  In this case too, cash holdings at $t = 0$ have to be positive if bankers set $D = \bar{D}$.

- **Cash accumulation only at $t = 1$ when $\bar{p}_b < p_b^*(a)$**

  In this case, $D = \bar{D}$ and $C_0 = 0$ and $\Delta C(b)$ is determined by the ICC:

$$\bar{D} - \frac{\mathcal{P}_b}{1-\mathcal{P}_b} E_0 - \frac{1}{1-\mathcal{P}_b}\left(1 - \frac{\mathcal{P}_b}{p_b^*(\Delta C(b))}\right)\Delta C(b) = 0$$

  Under the assumed parameter values, the rest of the constraints hold.

7.3. Comparative statics

First, the effect of changes in parameter values will be analyzed in the partial equilibrium — for given values of p_b. Then, the general equilibrium effects will be analysed. To be consistent with the previous section, we will focus on the case in which $\frac{v_b}{\varepsilon} > \frac{\bar{D}}{\bar{D}+E_0}$ (sufficient condition for $\mu = 0$) and $\bar{p} > \frac{\bar{D}}{\bar{D}+E_0}$ (sufficient condition for $\eta = 0$).

Inside equity. Inside equity, if anything, only affects banker's optimality conditions. Next, we will analyze the different cases that might prevail. Note that the treshold that delimits each of the cases is unaffected by changes in inside equity:

Case A: $p_b < \omega Y + (1-\omega)\mathcal{P}_b$ where $\omega \equiv \frac{(1-q)}{(1-q)+\left(\frac{Y-1}{1-\mathcal{P}_b}\right)}$

In this case, $\Delta C(b) = 0$ and it is unaffected by E_0. Instead, optimal net debt increases with inside equity:

$$\frac{\partial(D - C_0)}{\partial E_0} = \frac{\mathcal{P}_b}{1-\mathcal{P}_b} > 0$$

for the particular case in which bankers decide to operate in full scale, i.e., $D = \bar{D}$, for each unit of E_0 increase, C_0 decreases by $\frac{\mathcal{P}_b}{1-\mathcal{P}_b}$ units.

Case B: $p_b = \omega Y + (1-\omega)\mathcal{P}_b$ where $\omega \equiv \frac{(1-q)}{(1-q)+\left(\frac{Y-1}{1-\mathcal{P}_b}\right)}$

When $p_b = \bar{p}_b$, bankers are indifferent between C_0 and $\Delta C(b)$ as an incentive device. Hence, the flat piece of the supply curve remain at the same level p_b, although its length is shorten.

In equilibrium, $\Delta C(b)$ is determined by the $FOC_{\Delta C}$, thus, it is independent of E_0. While net debt, determined by the incentive compatibility constraint increases with equity:

$$\frac{\partial(D - C_0)}{\partial E_0} = \frac{\mathcal{P}_b}{1-\mathcal{P}_b} > 0$$

for the particular case in which bankers decide to operate in full scale, i.e., $D = \bar{D}$, for each unit of E_0 increase, C_0 decreases by $\frac{\mathcal{P}_b}{1-\mathcal{P}_b}$ units.

Case C: $p_b > \omega Y + (1-\omega)\mathcal{P}_b$ where $\omega \equiv \frac{(1-q)}{(1-q)+\left(\frac{Y-1}{1-\mathcal{P}_b}\right)}$

In this case, $C_0 = 0$ and it is unaffected by changes in equity. But the higher equity implies that less cash is needed for incentive purposes, hence, the optimal $\Delta C(b)$ decreases:

$$\frac{\partial \Delta C(b)}{\partial E_0} = -\frac{\mathcal{P}_b}{1-\frac{\mathcal{P}_b}{p_b}} < 0$$

General equilibrium effects amplify that effect due to downward sloping demand, $|\frac{d\Delta C(b)}{dE_0}| > |\frac{\partial \Delta C(b)}{\partial E_0}|$.

To conclude, equity and cash both enhance incentives to exert effort. hence, when equity increases less costly cash is needed.

7.4. Social Planner

$$\begin{aligned}\mathcal{L} \quad =& YE_0 + (Y-1)(D-C_0) + (1-q)\left(G(m-\Delta C(b)) + \Delta C(b)\right) \\ & -\lambda\left[D - C_0 - \frac{\mathcal{P}_b}{1-\mathcal{P}_b}E_0 - \frac{1}{1-\mathcal{P}_b}\left(1-\frac{\mathcal{P}_b}{p_{b^*}}\right)\Delta C(b)\right] \\ & -\mu\left[(\varepsilon - v_b)(D-C_0) - v_b E_0\right] \\ & -\eta\left[\Delta C(b) - p_b^*(E_0 + D - C_0)\right] \\ & -\xi\left[D - \bar{D}\right]\end{aligned}$$

The Kuhn-tucker conditions are given by

$$\begin{aligned} D: \quad & Y - 1 - \lambda - \mu(\varepsilon - v_b) + \eta p_b^* - \xi \leq 0 \\ C_0: \quad & -(Y-1) + \lambda + \mu(\varepsilon - v_b) - \eta p_b^* \gtreqless 0 \\ \Delta C(b): \quad & -(1-q)\left(\frac{Y}{p_b^*} - 1\right) + \lambda \frac{1}{1-\mathcal{P}_b}\left(1 - \frac{\mathcal{P}_b}{p_b^*}\right) - \eta \\ & -\lambda\left(\frac{1}{1-\mathcal{P}_b}\right)\left(\frac{\mathcal{P}_b\left(-p_b^{*'}\right)}{p_b^{*2}}\right)\Delta C(b) - \eta\left(E_0 + D - C_0\right)\left(-p_b^{*'}\right) \leq 0 \\ & \lambda \geq 0 \text{ and } \lambda\left[D - C_0 - \frac{\mathcal{P}_b}{1-\mathcal{P}_b}E_0 - \frac{1}{1-\mathcal{P}_b}\left(1 - \frac{\mathcal{P}_b}{p_b^*}\right)\Delta C(b)\right] = 0 \\ & \mu \geq 0 \text{ and } \mu\left[(\varepsilon - v_b)(D - C_0) - v_b E_0\right] = 0 \\ & \eta \geq 0 \text{ and } \eta\left[\Delta C(b) - p_b^*(E_0 + D - C_0)\right] = 0 \\ & \xi \geq 0 \text{ and } \xi\left[D - \bar{D}\right] = 0 \end{aligned}$$

The aforementioned reasoning applies here, but now the $FOC_{\Delta C}$ boils down to:

$$(Y-1)\frac{1}{1-\mathcal{P}_b}\left(1 - \frac{\mathcal{P}_b}{p_b}(1+\theta)\right) - (1-q)\left(\frac{Y}{p_b} - 1\right) - \tilde{\psi} \leq 0$$

where $\tilde{\psi} \equiv (\mu(\varepsilon - v_b) + \xi)\frac{1}{1-\mathcal{P}_b}\left(1 - \frac{\mathcal{P}_b}{p_b}(1+\theta)\right) - \eta\left(\frac{(1+\theta)-p_b}{1-\mathcal{P}_b}\right)$ and $\theta = (-p_b^{*'})\frac{\Delta C(b)}{p_b^*} > 0$. Thus, $\tilde{\psi} > 0$ when at least one of the constraints is binding (only then the corresponding Lagrangian multiplier is positive). Note that $\theta' > 0$.

Given that $p_b^*(x) < Y$ for $\forall x$, then $\lambda > 0$, and there are different cases depending on the parameter values:

- **Cash accumulation only at $t = 0$ when $p_b^*(0) \leq \bar{p}_b$**

Then, $\Delta C(b) = 0$, and so are $\eta = \mu = 0$. Net debt is determined by the incentive compatibility constraint:

$$D - C_0 = \frac{\mathcal{P}_b}{1-\mathcal{P}_b} E_0$$

for $D \in \left[\frac{\mathcal{P}_b}{1-\mathcal{P}_b} E_0, \bar{D}\right]$ and $C_0 \geq 0$. Compared to the centralized economy, the allocation is the same.

- **Cash accumulation both at $t = 0$ and $t = 1$ when** $p_b^*(a) - (1-\omega)\mathcal{P}_b\theta(a) \leq \bar{p}_b < p_b^*(0)$ where a is determined in $a - \left(\bar{D} - \frac{\mathcal{P}_b}{1-\mathcal{P}_b} E_0\right) \times \left(\frac{1-\mathcal{P}_b}{1-\frac{\mathcal{P}_b}{p_b^*(a)}}\right) = 0$.

  Then, cash holding at $t = 1$ is determined by $FOC_{\Delta C}$ and net debt by the incentive compatibility constraint

$$(Y-1)\frac{1}{1-\mathcal{P}_b}\left(1 - \frac{\mathcal{P}_b}{p_b}(1+\theta(\Delta C(b)))\right) - (1-q)\left(\frac{Y}{p_b(\Delta C(b))} - 1\right) = 0$$

  The first equation can be conveniently re-arrange as

$$p_b^*(\Delta C(b)) = \bar{p}_b + (1-\omega)\mathcal{P}_b\theta(\Delta C(b))$$

  Then $\Delta C(b)$ is higher in the presence of the second term. But equilibrium price is also higher than it otherwise would be.

$$D - C_0 = \frac{\mathcal{P}_b}{1-\mathcal{P}_b} E_0 + \frac{1}{1-\mathcal{P}_b}\left(1 - \frac{\mathcal{P}_b}{p_b(\Delta C(b))}\right)\Delta C(b)$$

  On the one hand, $\Delta C(b)$ is higher but $p{*}_b$ is also higher. However, the first order condition (that holds with equality for this parameter range) ensures that the derivative of the second term with respect to $\Delta C(b)$ is positive, thus, the second term is higher for the private case.

- **Cash accumulation only at $t = 0$ when** $\bar{p}_b < p_b^*(a) - (1-\omega)\mathcal{P}_b\theta(a)$

  In this case, $D = \bar{D}$ and $C_0 = 0$ and $\Delta C(b)$ is determined by the ICC:

$$\bar{D} - \frac{\mathcal{P}_b}{1-\mathcal{P}_b} E_0 - \frac{1}{1-\mathcal{P}_b}\left(1 - \frac{\mathcal{P}_b}{p_b^*(\Delta C(b))}\right)\Delta C(b) = 0$$

  Hence, for this particular case there is no wedge between the social planer and bankers.