The Leverage Ratio Requirement and Banks' Risk-Taking: Why Do Banks' Internal Capital Allocation Practices Matter?*

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Abstract

This paper examines how banks' asset risk changes when banks apply regulatory requirements at a business level lower than the group level in their internal capital allocation process. We develop a theoretical model and calibrate it to UK banks. We find that the practice of applying regulatory requirements at the lower business level may disproportionately affect some low-risk banking services, such as repurchase agreements, as banks reduce their investments in those activities to a larger degree. We also find that this impact differs across bank business models. Our paper offers an explanation of the mechanism behind the reduction of repo intermediation reported in banks not bound by the leverage ratio requirement.

JEL Classification: G21, G28.

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1 Introduction

The Great Financial Crisis of 2007-2009 triggered a substantial overhaul of the prudential framework for banks. One major reform that receives substantial interest from policymakers and academics is the introduction of the leverage ratio (LR) requirement. This requirement is meant to restrict the build-up of leverage in the banking system and act as a backstop for the risk-weighted (RW) requirement. Nonetheless, its risk-insensitive nature may induce banks to reduce low-risk, low-return activities, which raises concerns over its adverse consequences on banks' risk-taking behaviour.

Among low-risk bank activities, repurchase agreements (repos) attract significant attention due to their crucial role in facilitating the flow of cash and securities in the financial system. Empirical evidence finds that the introduction of the LR disincentivised the repo intermediation by banks bound by this regulatory constraint (e.g. Kotidis and Van Horen (2019) and Bicu-Lieb et al. (2020)). Interestingly, a similar effect is also reported in banks for which the LR requirement does not appear to be binding (e.g. Allahrakha et al. (2018)). While it seems reasonable to expect banks bound by the LR to adopt risk-shifting behaviour and reduce repo activities, the main puzzle lies in the underlying mechanism behind the similar impact observed in banks that are not constrained by this requirement.

In this paper, we construct a model to rationalise this empirical fact on the risk-shifting behaviour of banks not bound by the LR. In our model, a banking group with multiple business units selects optimal investments in the presence of multiple regulatory capital requirements. We then show that, even when not constrained by the LR, the banking group can still choose to shift risk by reducing its investment in low-risk businesses. This happens if the bank applies regulatory constraints at a business level lower than the group level in its internal capital allocation process. We also substantiate our analytical findings with a simulation of the UK banking sector.

Capital allocation is the method that banks use to determine the notional amount of equity needed to support each business unit. It is an important internal process that banks employ in practice to support business optimisation decisions.[1] Recent supervisory reviews on banks capital allocation approaches find that some banks choose to apply the LR requirement at the business-unit level, even when the LR is required to be satisfied only at the banking group level (see Bajaj et al. (2018)). We argue in this paper that the practice of applying regulatory capital requirements at lower business levels can be the origin of the risk-shifting behaviour observed in banks not constrained by the LR.

Shedding light on the mechanism behind the above puzzle is useful since it clarifies a potential source for the disproportionate impact that regulatory requirements may have on some specific services provided by banks, especially vital services such as liquidity provision. Our analysis also provides insights for the policy discussion on the optimal application level of regulatory requirements. The more significant impact on low-risk activities that we find in the paper can be seen as a potential cost that policymakers should consider if they decide to apply the LR requirement at lower levels than the consolidated level.

Our model features a risk-neutral banking group that runs two business units. One unit has higher non-risk adjusted returns but is also riskier, while the other has lower returns and is less risky. Motivated by the evidence on the impact of the LR requirement on banks' repo activities, we model the low-risk business as repo while the high-risk business resembles a lending business in our setup. The bank finances its activities with debt and a fixed amount of equity capital that it will allocate across its two business units. The bank is subject to two requirements, namely the RW capital requirement and the LR requirement, which are legally imposed at the group level.

To capture banks' internal capital allocation practices, we consider the case where the banking group allocates equity capital such that each business unit must be assigned enough equity capital to make them comply with both regulatory constraints. We hereafter refer to it as the allocation of regulatory constraints to business units or, interchangeably, the application of regulatory constraints at the business-unit level. We compare the resulting bank's optimal investments in this case to the case where the bank applies the two regulatory

[1] Another internal process that banks use for business optimisation purposes is the Fund Transfer Pricing.

requirements only at the group level. We focus on how the riskiness of the bank's asset portfolio - the bank asset risk - differs between the two cases. This in turn helps to highlight how applying regulatory constraints at the business-unit level can lead to higher risk taking and disproportionately lower investment in the low-risk business.

We first characterise analytically the impact of the application level of regulatory constraints on the banks' asset risk. We then take the model to the data and run numerical simulations using proprietary and confidential regulatory data on UK banks' retail lending and repo activities. The simulation of the calibrated model verifies the empirical relevance of our analytical findings and complements them with insights into different risk-taking responses across banks' business models.

One of the main analytical findings from our model is related to the investment choices of the banking group when it is not bound by the LR but by the RW requirement. In this case, we find that under certain conditions, allocating requirements to business units can increase the bank's asset risk by inducing it to invest less in repo and more in lending. To understand the intuition, note that the incentives to invest in each business depend on the cost in terms of required equity capital for such an investment. This in turn is determined by the binding regulatory constraint. Therefore, when requirements are applied at the group level and the RW requirement binds at group, the bank's incentives to invest in the two business units are determined by the *marginal RW capital cost* - the additional equity capital required by the RW requirement for each additional unit of investment. However, when the bank applies requirements at the business-unit level, there are situations where the binding constraint for the repo business unit is the LR requirement. As such, the cost of investing in repos is now instead determined by the *marginal leverage cost* - the additional equity capital required by the LR requirement for each additional unit of investment. Since repo activities generally incur higher leverage costs than RW capital costs due to their leverage-intensive characteristics, the banking group will invest less in repo when it applies requirements at the business-unit level instead of the group level.

To complement our theoretical analysis, we calibrate the model using data on a sample of UK banks. When simulating the calibrated model, we find that the allocation of constraints leads to an increase in asset risk of the average bank in our sample when only the RW requirement binds at the group level. This result confirms the empirical relevance of our analytical findings described above. Using numerical simulations, we further examine whether banks with different business models adjust their risk-taking behaviour similarly. To do so, we first classify the UK banks in our sample into two types of banks, namely retail and wholesale banks. We then recalibrate the model to each bank type. From our calibration, we observe that the lending business of retail banks is riskier than that of wholesale banks. When we simulate the optimal investment choices for each business model, we find a stark difference in the impact of the allocation of constraints on the asset risk between the two types, especially for the case where only the RW constraint binds at the group level. Precisely, we find that allocating constraints down to business units results in an increase in the asset risk of retail banks but a decrease in the asset risk of wholesale banks.

Related literature Our paper is related to three main strands of literature.

The first strand of literature assesses the impact of the LR requirement on banks' risk-taking.[2] Contributions in the literature include, among others, Kiema and Jokivuolle (2014) and Acosta-Smith et al. (2020). The focus of Kiema and Jokivuolle (2014) is on the model risk argument. They find that the shift in risk-taking does not affect the aggregate risk profile and stability of the whole banking system, as banks re-shuffle the loans: banks focusing on low-risk lending will shift towards more high-risk lending, while the high-risk lending banks will reallocate part of their portfolio to low-risk investments. Acosta-Smith et al. (2020) focus on the complementary role of the LR requirement as compared to risk-based capital requirements. They find that the introduction of the LR leads to an increase in banks' risk-taking if equity is sufficiently costly, or if banks are bound by the LR. They confirm these results empirically for a large panel of European banks. Choi et al. (2018) find similar empirical evidence for the US, where banks shift towards riskier investments but the shift

[2]Other strands of the literature analyse how the LR affects banks' incentives to truthfully report their riskiness (see Blum (2008)) and how it interacts with the RW requirement to affect the business cycle (see Gambacorta and Karmakarb (2018)).

is counterbalanced by increased capital, leading to no change in the overall risk. Our paper adds to this literature by investigating how the impact of the LR on banks' investments depends on the business level at which it is applied by banks.

The second strand of literature examines the optimal capital allocation approach within complex financial institutions. One focus of this literature is on the choice between the Risk-Adjusted Returns on Capital (RAROC) and Economic Value Added (EVA) as measures of relative profitability across different business units (Ita, 2017; Khaykin et al., 2017). Some others analyse how the choice of internal hurdle rate affects banks' investments. For example, Krüger et al. (2015) show empirically that the allocation based on the firm-wide cost of capital leads to under-investment in safer businesses and over-investment in relatively riskier ones. Papers such as Perold (2005) instead discuss how accounting for diversification benefits between different businesses can reduce banks' economic capital needs. The most related to our paper is Goel et al. (2020), which studies how banks' internal capital allocation makes shocks to one banking activity spill over to another activity. They assume that regulatory constraints are applied at the business-unit level. Our paper however examines how the banks' practice of applying regulatory constraints at the lower business level through their internal capital allocation process affects their investment decisions.

Finally, studies are looking at the impact of the LR requirement on banks' incentives to undertake market-making businesses, especially repo activities. In a recent study on the market liquidity in the UK gilt market, Bicu-Lieb et al. (2020) find that a decrease in the repo liquidity coincides with the introduction of the leverage ratio requirement. Kotidis and Van Horen (2019) also analyse the UK gilt repo market. They find that repo dealers constrained by the LR decrease the transacted volumes with smaller clients. Allahrakha et al. (2018) instead look at the US triparty repo market and find that regardless of whether the parent of dealers affiliated with bank holding companies is above or below the US supplementary leverage ratio (SLR) requirement, the announcement of the SLR rule has disincentivised those dealers from borrowing in the triparty repo market. Our paper offers an explanation of the mechanism underlying the impact of the LR on repo business found in these empirical analyses.

The organisation of the paper is as follows. In Section 2 we set out our theoretical model. Section 3 presents our main analytical insights. Then in Section 4 we calibrate our model to the UK banks and explain our numerical simulations in Section 5. In Section 6 we break down our UK bank sample into different business models and discuss our new simulation results in this context. Finally Section 7 concludes.

2 The model

We consider a banking group that is funded by a fixed amount of equity capital K and by debts of gross interest rate R. It runs two business units, one yielding higher non-risk adjusted returns but is riskier than the other. Although our results will hold for any combination of two businesses that have those characteristics, in this paper, we model the riskier business as a lending business and the safer business as a repo business.

Two business units The lending unit grants loans to customers. We denote the bank's ex-ante gross interest income from loans by $G(L)$ where L is the total value of granted loans. We capture the fact that granting loans is a risky business by assuming that ex-post some borrowers default and cannot fully repay their loans. Let $\tilde{Z}$ be the random variable that represents the losses per unit of loans. Therefore, the bank's ex-post lending revenue is equal to $G(L) - ZL$ where Z is the realised value of $\tilde{Z}$. We assume that $\tilde{Z}$ is distributed according to the distribution H_Z, h_Z with expected value equal to μ_Z.

The repo unit owns a stock of government bonds of value X with coupon c. It uses this inventory to raise collateralised funding to finance bond trading activities or to act as an intermediary entering into repo transactions with some counterparties and offsetting reverse repos with others. We denote the ex-ante income from those activities by $F(X)$. We capture the risk of the repo business by assuming that ex-post the bank could suffer losses equal to $\tilde{\varepsilon}X$ due to, for example, unpaid repayments by reverse repo counterparties or losses from trading activities. The distribution of $\tilde{\varepsilon}$ is characterised by $H_\varepsilon, h_\varepsilon$ with expected value μ_ε.

We make the following assumptions on the profitability and riskiness of the two business units.

Assumption 1. *Functions $G(.)$ and $F(.)$ satisfy the following conditions:*

$$G(0) = 0; \quad G'(.) > 0 \quad and \quad G''(.) < 0$$

$$F(0) = 0; \quad F'(.) > 0 \quad and \quad F''(.) < 0$$

Assumption 1 implies that both lending and repo businesses have diminishing marginal returns. For the lending business, this property can be explained by the fact that the loan interest rate is a decreasing function of loan size. For the repo business, this can be because the interest rate of reverse repos is less sensitive to the transactional amount than the repo rate, which in turn can come from the market power of banks in both activities.

Assumption 2. *The rank of profitability between the two business units is as follows:*

$$G'(y) - \mu_Z > F'(y) + c - \mu_\varepsilon \quad for\ all \quad y \leq max(X^*, L^*)$$

where

$$X^* = \underset{y}{\operatorname{argmax}}\,[F(y) + cy - \mu_\varepsilon y - Ry] \quad and \quad L^* = \underset{y}{\operatorname{argmax}}\,[G(y) - \mu_Z y - Ry]$$

Assumption 2 indicates that the lending business is more profitable than the repo business on a non-risk-adjusted basis. Note that X^* and L^* defined in this assumption represent the size of, respectively, the repo and lending businesses so that the expected profits from those activities are maximised when their cost of funding is R. The bank will never grant more loans than L^* and never hold an inventory of government bonds of value higher than X^*.

Assumption 3. *Two random variables $\tilde{Z}$ and $\tilde{\varepsilon}$ are independently distributed and ranked as follows:*

$$VaR_{1-q}(\tilde{Z}) \geq VaR_{1-q}(\tilde{\varepsilon})$$

where $VaR_{1-q}(\tilde{Y})$ *denotes the Value at Risk (VaR) of a random variable* $\tilde{Y}$ *at confidence level* $1-q$*, which is defined as:*

$$VaR_{1-q}(\tilde{Y}) \equiv inf \left\{ y : \mathbb{P}(\tilde{Y} \geq y) \leq q \right\} \tag{1}$$

Assumption 3 states a ranking between two random variables $\tilde{Z}$ and $\tilde{\varepsilon}$ based on the VaR measure. It implies that the lending business is riskier on a stand-alone basis than the repo business.

Regulatory constraints and internal capital allocation The bank is subject to two regulatory constraints, namely the LR requirement and the RW capital requirement. In line with the principle underlying the Basel requirements, we formulate the RW capital requirement using the VaR constraint. Among the total equity capital K that the bank has, K_L will be allocated to support the lending business while K_X is allocated to the repo business.

Before explaining how the regulatory constraints look like depending on the level at which the bank applies them, it is useful to introduce some notations. We denote by $\tilde{\Pi}_L$ and $\tilde{\Pi}_X$ the profit of, respectively, the lending and repo units. $\tilde{\Pi}_L$ and $\tilde{\Pi}_X$ can therefore be written as follows:

$$\tilde{\Pi}_L = G(L) - \tilde{Z}L - R(L - K_L) \quad \text{and} \quad \tilde{\Pi}_X = F(X) + cX - \tilde{\varepsilon}X - R(X - K_X)$$

The overall profit of the whole banking group is thus equal to $\tilde{\Pi}_L + \tilde{\Pi}_X$.

When regulatory constraints are applied at the group level, the RW requirement can be written as follows:

$$\mathbb{P}\left(\tilde{\Pi}_L + \tilde{\Pi}_X \leq 0\right) \leq a \tag{2}$$

In words, Constraint (2) states that the probability for the total losses of the bank's asset portfolio being higher than its equity capital is lower than a. After some algebra, it can be rewritten as[3]:

$$K \geq \frac{VaR_{1-a}(\tilde{Z}L + \tilde{\varepsilon}X) - \Pi(L,X)}{R} \tag{3}$$

where

$$\Pi(L,X) = \underbrace{G(L) - RL}_{\equiv \Pi_L(L)} + \underbrace{F(X) + cX - RX}_{\equiv \Pi_X(X)}$$

In the Basel III framework, the right hand side (RHS) of Constraint (3) is equivalent to the product of the RW capital requirements and the risk-weighted assets (RWAs) of the bank at the group level. We denote the former by γ and the latter by RWA^G. RWA^G can thus be proxied in our model by:

$$RWA^G(L,X) = \frac{VaR_{1-a}(\tilde{Z}L + \tilde{\varepsilon}X) - \Pi(L,X)}{\gamma R}$$

The LR requirement is expressed in terms of the ratio of equity capital over leverage exposure. The leverage exposure of the lending unit is equal to its size L. For the repo unit, because of different possible regulatory treatments of repo activities, its leverage exposure can be a multiple of its size X. For example, when the bank runs a matched repo book, if all reverse repo transactions are not eligible for netting, due to the requirement that securities sold as collateral cannot be removed from the bank's balance sheet, the leverage exposure of the repo business will be equal to $2X$. To capture this characteristic of repo activities, we assume that the leverage exposure of the repo unit equals αX where $\alpha \in [1,2]$.[4] The LR requirement at the group level is therefore as follows:

$$K \geq \chi(L + \alpha X) \tag{4}$$

where χ is the required LR.

[3]See Appendix A.1 for the detailed derivation.

[4]Note that results stated in the following analysis will apply to other types of low risk and low return businesses when α is set to 1.

When the bank chooses to allocate regulatory constraints down to its business units, the allocated capital K_L and K_X are determined so that both business units have enough equity capital to comply with both regulatory constraints individually. Therefore, K_L is such that the lending business has to satisfy:

$$\mathbb{P}\left(\tilde{\Pi}_L \leq 0\right) \leq a \quad \text{and} \quad K_L \geq \chi L$$

while K_X is determined so that the repo business satisfies:

$$\mathbb{P}\left(\tilde{\Pi}_X \leq 0\right) \leq a \quad \text{and} \quad K_X \geq \chi \alpha X$$

The two individual VaR constraints can similarly be expressed in terms of RWA^L and RWA^X - the RWAs of, respectively, the lending and repo businesses on a stand-alone basis - as follows:

$$K_L \geq \gamma RWA^L(L) \quad \text{where} \quad RWA^L(L) = \frac{VaR_{1-a}(\tilde{Z}L) - \Pi_L(L)}{\gamma R} \tag{5}$$

$$K_X \geq \gamma RWA^X(X) \quad \text{where} \quad RWA^X(X) = \frac{VaR_{1-a}(\tilde{\varepsilon}X) - \Pi_X(X)}{\gamma R} \tag{6}$$

Measures of bank asset risk There exist several possible measures for the riskiness of the bank's assets.[5]

In the regulatory world, banks' asset risk is usually measured by the so-called average risk weight (ARW), which is defined as the ratio of RWAs over leverage exposure. In our model, the ARW of the banking group can be computed as

$$ARW^G(L, X) = \frac{RWA^G(L, X)}{L + \alpha X}$$

[5] Note the difference between banks' asset risk and banks' total risk or banks' overall resilience. The overall resilience will decrease when the asset risk increases if the increase in asset risk is not dominated by a decrease in bank funding risk due to, for example, lower leverage.

while the ARW of each business unit calculated as

$$ARW^L(L) = \frac{RWA^L(L)}{L} \quad \text{and} \quad ARW^X(X) = \frac{RWA^X(X)}{\alpha X}$$

In portfolio theory, a simple measure of the riskiness of an asset portfolio is the variance of its returns. In our set-up, since we assume that $\tilde{Z}$ and $\tilde{\varepsilon}$ are independently distributed, the variance of the returns of the bank's asset portfolio, denoted by σ_p^2, is computed as follows

$$\sigma_p^2 = L^2\sigma_Z^2 + X^2\sigma_\varepsilon^2 \tag{7}$$

where σ_Z^2 and σ_ε^2 denote the variance of, respectively, $\tilde{Z}$ and $\tilde{\varepsilon}$.

Given that in our model, the bank runs two businesses with one riskier than the other, an intuitive measure of the riskiness of the bank's assets is the fraction of the bank's total balance sheet devoted to lending business - the riskier one. We denote by w this fraction, i.e.

$$w = \frac{L}{L + X} \tag{8}$$

In the following, we will use w as our main measure of bank asset risk since it also intuitively captures the bank's rebalancing portfolio actions and facilitates the analytical derivations. As will become clear later, in the equilibrium, when w increases, the ARW of the banking group also increases, which implies that our insights are robust across these two measures of bank asset risk. In Appendix A.2, we discuss why it is without loss of generality to focus on the cases where the variance of the returns of the bank's asset portfolio σ_p^2 is increasing with w. Hence, our insights also hold if bank asset risk is measured by the variance of the asset portfolio returns.

3 Analysis

We analyse in this section the bank's optimal investments. Our main objective is to investigate how the bank's asset risk, measured by w is affected by the level at which the two regulatory constraints are applied. To do so, we will compare the bank's optimal investments in each business unit between the case where the two constraints are applied at the group level and the case in which both business units have to individually comply with both regulatory constraints. To get started, we first formulate the bank's profit-maximisation problem for each of these two scenarios. Then, we will examine how the bank's investment decisions differ between them.

3.1 Bank's optimisation problems

Optimisation problem with constraints applied at the group level When all constraints are applied at the group level, the bank's optimisation problem, denoted as $\wp^G$, can be written as follows:

$$\textit{Problem} \quad \wp^G: \qquad \underset{L,X}{\text{Max}} \quad \mathbb{E}\left[\tilde{\Pi}_L + \tilde{\Pi}_X\right]$$

subject to Constraints (3) and (4).

To facilitate the examination of how the bank's asset risk w would change depending on the application level of the two regulatory constraints, we reformulate Problem $\wp^G$ by changing the bank's decision variables from (L, X) to w and the bank's total balance sheet size $S = L + X$. After expressing L and X in terms of S and w, Problem $\wp^G$ could be rewritten as:

$$\underset{S,w}{\text{Max}} \quad \{\Pi(w, S) - \mu_Z wS - \mu_\varepsilon(1 - w)S + RK\}$$

subject to

$$K \geq \gamma RWA^G(w, S) = \frac{VaR_{1-a}(\tilde{Z}w + (1 - w)\tilde{\varepsilon})S - \Pi(w, S)}{R} \tag{9}$$

$$K \geq \chi\left(wS + \alpha(1-w)S\right) \tag{10}$$

Optimisation problem with constraints allocated down to business units When the bank allocates two constraints to its business units, the bank's optimisation problem, denoted as $\wp^B$, is as follows:

$$Problem \quad \wp^B: \qquad \underset{L,X}{\text{Max}} \quad \mathbb{E}\left[\tilde{\Pi}_L + \tilde{\Pi}_X\right]$$

subject to Constraints (5), (6) as well as the following two LR requirements:

$$K_L \geq \chi L \quad \text{and} \quad K_X \geq \chi \alpha X$$

and the internal capital allocation constraint:

$$K \geq K_L + K_X$$

After reformulating Problem $\wp^B$ in terms of w and S, we get:

$$\underset{S,w}{\text{Max}} \quad \left\{\Pi(w,S) - \mu_Z wS - \mu_\varepsilon(1-w)S + RK\right\}$$

subject to

$$K_L \geq \gamma RWA^L(w,S) = \frac{VaR_{1-a}(\tilde{Z}w)S - \Pi_L(w,S)}{R} \tag{11}$$

$$K_X \geq \gamma RWA^X(w,S) = \frac{VaR_{1-a}((1-w)\tilde{\varepsilon})S - \Pi_X(w,S)}{R} \tag{12}$$

$$K_L \geq \chi wS \tag{13}$$

$$K_X \geq \chi\alpha(1-w)S \tag{14}$$

$$K \geq K_L + K_X \tag{15}$$

3.2 Bank's optimal investments

We are now equipped to compare the bank's investments, especially the bank's asset risk, between the two above scenarios. Denote by (w^G, S^G) and (w^B, S^B) the solutions to, respectively, Problem $\wp^G$ and $\wp^B$.

To get a first intuition on how investment decisions of the bank differ between the two cases, let us compare the constraints of Problem $\wp^G$ to those of Problem $\wp^B$. We see that the group-level LR constraint is weakly looser than business unit-level LR constraints. The group-level RW constraint is also looser than the business unit-level one and the gap can be expressed in terms of Div defined as follows:

$$Div = VaR_{1-a}(\tilde{Z}w) + VaR_{1-a}((1-w)\tilde{\varepsilon}) - VaR_{1-a}(\tilde{Z}w + (1-w)\tilde{\varepsilon}) \tag{16}$$

Div represents the diversification benefit per unit of size to the bank if it applies the RW constraint at the group level.

These first observations imply that applying regulatory constraints at the business unit level will weakly reduce the set of investment opportunities available to the bank. The following proposition highlights the efficiency losses resulting from allocating both constraints down to business units.

Proposition 1. *Efficiency losses:*

$$S^B \leq S^G$$

Proof. It is the direct consequence of the fact that constraints of Problem $\wp^B$ are weakly tighter than those of Problem $\wp^G$. □

We now turn to the impact on the bank's asset risk. Before characterising it, we state in the lemma below the relationship between w and ARW^G - two possible measures of the bank's asset risk.

Lemma 1. *In the equilibrium, ARW^G increases with w*

Proof. See Appendix A.4. □

Lemma 1 implies that the following insights, which focus on how w changes depending on the application level of the two regulatory constraints, will also apply to ARW^G as a measure of the bank's asset risk. We will consider two cases where the bank is bound at the group level either by the LR requirement or by the RW requirement. It can happen that both constraints bind at the group level at the same time. But given that that case is a knife-edge case, we do not analyse it in this section.[6]

LR-constrained bank The bank is bound by the LR requirement at the group level when Constraint (10) is tighter than Constraint (9). We state in the following proposition our first results related to the bank's asset risk.

Proposition 2. *When the bank is bound by the LR requirement at the group level (i.e. $ARW^G(w^G, S^G) < \frac{\chi}{\gamma}$), we have:*

1. $w_B = w_G$ *if the following two conditions hold globally*

$$ARW^L(w, S) < \frac{\chi}{\gamma} \quad \textit{and} \quad ARW^X(w, S) < \frac{\chi}{\gamma}$$

2. $w_B < w_G$ *if the following two conditions hold globally*

$$ARW^L(w, S) > \frac{\chi}{\gamma} \quad \textit{and} \quad ARW^X(w, S) > \frac{\chi}{\gamma}$$

Proof. See Appendix A.5 □

Proposition 2 states conditions under which the allocation of constraints down to business units either leaves unchanged the asset risk of the LR-constrained bank or reduces it. Those conditions relate the ARW of each business unit to the so-called critical average risk weight (CRW) that equals $\frac{\chi}{\gamma}$ - the ratio of required LR over the required RW capital ratio.

[6]The case where no constraints bind is also not interesting. That is because in that case, the bank's optimal investments are the unconstrained optimum which won't be affected by the application level of constraints.

Specifically, the first part of the proposition corresponds to the case where both units will be bound by the LR requirement when regulatory constraints are allocated down since their ARW is lower than the CRW. The second part specifies the impact on the bank's asset risk when both units will be bound by the RW constraint. Note that the situation where the group is bound by the LR but business units are bound by the RW constraint can happen if all three inequalities below can be satisfied simultaneously:

$$\frac{VaR_{1-a}(\tilde{Z}w)S - \Pi_L(w,S)}{R} > \chi w S \tag{17}$$

$$\frac{VaR_{1-a}((1-w)\tilde{\varepsilon})S - \Pi_X(w,S)}{R} > \chi\alpha(1-w)S \tag{18}$$

and

$$\chi w S + \chi\alpha(1-w)S > \frac{VaR_{1-a}(\tilde{Z}w + (1-w)\tilde{\varepsilon})S - \Pi(w,S)}{R} \tag{19}$$

The necessary condition for Inequalities (17), (18) and (19) being compatible with each other is as follows:

$$VaR_{1-a}(\tilde{Z}w)S + VaR_{1-a}((1-w)\tilde{\varepsilon})S > VaR_{1-a}(\tilde{Z}w + (1-w)\tilde{\varepsilon})S \tag{20}$$

or, in words, the diversification benefit Div is high enough.

To get the intuition underlying the two results stated in Proposition 2, it is useful to compare the first order conditions (FOC) that characterise w^G and w^B. w^G is indeed determined by:

$$\begin{aligned}\left[G'(w^G S^G) - \mu_Z - R\right] - \left[F'((1-w^G)S^G) + c - \mu_\varepsilon - R\right] = \\ \lambda_{LR}\left(\overbrace{\underbrace{\chi}_{\substack{\text{marginal leverage}\\\text{cost of lending}}} - \underbrace{\alpha\chi}_{\substack{\text{marginal leverage}\\\text{cost of repo}}}}^{\leq 0}\right)\end{aligned} \tag{21}$$

w^B is in turn determined by:

$$\left[G'(w^B S^B) - \mu_Z - R\right] - \left[F'((1-w^B)S^B) + c - \mu_\varepsilon - R\right] = \lambda^L_{LR}\left(\overbrace{\underbrace{\chi}_{\substack{\text{marginal leverage}\\ \text{cost of lending}}} - \underbrace{\alpha\chi}_{\substack{\text{marginal leverage}\\ \text{cost of repo}}}}^{\leq 0}\right) \tag{22}$$

when both ARW^L and ARW^X are below CRW and by:

$$\left[G'(w^B S^B) - \mu_Z - R\right] - \left[F'((1-w^B)S^B) + c - \mu_\varepsilon - R\right] = \lambda^L_{VaR}\left[\overbrace{\underbrace{\frac{\partial(\gamma RWA^L)}{\partial L}}_{\substack{\text{marginal RW}\\ \text{capital cost of lending}}} - \underbrace{\frac{\partial(\gamma RWA^X)}{\partial X}}_{\substack{\text{marginal RW}\\ \text{capital cost of repo}}}}^{\geq 0}\right] \tag{23}$$

when both ARW^L and ARW^X are above CRW.[7]

All three Equations (21), (22) and (23) equate, on the left-hand side (LHS), the marginal benefit of reallocating investment from repo business to lending business with its marginal cost on the right-hand side (RHS). The former is the increase in the bank's expected marginal profit due to higher profitability of the lending business while the latter is measured in terms of marginal changes in required equity capital. Comparing those equations, we see that what drives the difference between w^G and w^B is the marginal cost.

When both units have ARW lower than the CRW, applying regulatory constraints at the business unit level does not affect the bank's asset risk compared to the case of group-level application. That is because, as shown in Equations (21) and (22), the bank's investments are determined by the difference in the marginal leverage cost between two businesses in both cases of application level.

When both units have ARW greater than the CRW, in comparison to the group-level

[7] λ_{LR}, λ^L_{LR} and λ^L_{VaR} are the shadow price of, respectively, the group LR, the business-unit LR and the business-unit RW requirements.

application (Equation (21)), the business unit-level application (Equation (23)) leads to higher marginal costs of rebalancing investment portfolio from repo toward lending. In other words, with the business-level application, investments in repo are relatively more attractive than lending since the repo business incurs lower marginal RW capital costs but higher marginal leverage costs than the lending business. Therefore, when constraints are applied down, the banking group reduces its risk-taking.

RW-constrained bank We now turn to the case where the bank is not bound by the LR but by the RW constraint at the group level. This is equivalent to Constraint (10) being looser than Constraint (9) or $ARW^G(w^G, S^G) > \frac{\chi}{\gamma}$. The following proposition formally states three conditions that will make an increase in the asset risk of the RW-constrained bank more likely to occur when regulatory constraints are allocated down to business units.

Proposition 3. *When the bank is bound by the RW constraint at the group level (i.e. $ARW^G(w^G, S^G) > \frac{\chi}{\gamma}$), it can happen that $w^B > w^G$ if the following conditions are satisfied globally:*

1. $ARW^L \geq \frac{\chi}{\gamma}$ *and* $ARW^X \leq \frac{\chi}{\gamma}$
2. $\chi\alpha \geq \frac{\partial(\gamma RWA^X)}{\partial X}$
3. $\frac{\partial Div}{\partial w} = VaR_{1-a}(\tilde{Z}) - VaR_{1-a}(\tilde{\varepsilon}) - \frac{\partial VaR_{1-a}(\tilde{Z}w+\tilde{\varepsilon}(1-w))}{\partial w} < 0$

Proof. See Appendix A.6 □

Similarly to the case of the LR-constrained bank, the impact of allocating down regulatory constraints to business units on the asset risk of the RW-constrained bank depends on the ARW of each business unit. Proposition 3 considers the case where, as implied by the first condition, the lending unit is bound by the RW requirement while the repo unit is bound by the LR requirement. In this situation, Proposition 3 indicates that if, following the second condition, the marginal leverage cost of the repo business is higher than its marginal RW capital cost and if, as stated by the third condition, the diversification benefit Div is decreasing with the share of the lending business in the bank's total balance sheet, then changing the application level from group to business unit can lead to an increase of the asset risk of the RW-constrained bank.

To understand why the three conditions specified in Proposition 3 make an increase in the asset risk of the RW-constrained bank more likely to occur when regulatory constraints are allocated down to business units, it is again useful to compare the two FOCs that determine w^G and w^B. For a RW-constrained bank, if $ARW^L \geq \frac{\chi}{\gamma}$ and $ARW^X \leq \frac{\chi}{\gamma}$, then w^G and w^B are characterised respectively by:

$$\left[G'(w^G S^G) - \mu_Z - R\right] - \left[F'((1-w^G)S^G) + c - \mu_\varepsilon - R\right] = \lambda_{VaR}\left[\underbrace{\frac{\partial(\gamma RWA^L)}{\partial L}}_{\substack{\text{marginal RW}\\ \text{capital cost of lending}}} - \underbrace{\frac{\partial(\gamma RWA^X)}{\partial X}}_{\substack{\text{marginal RW}\\ \text{capital cost of repo}}} - \frac{\partial Div}{\partial w}\right] \tag{24}$$

and by

$$\left[G'(w^B S^B) - \mu_Z - R\right] - \left[F'((1-w^B)S^B) + c - \mu_\varepsilon - R\right] = \lambda^L_{VaR}\left[\underbrace{\frac{\partial(\gamma RWA^L)}{\partial L}}_{\substack{\text{marginal RW}\\ \text{capital cost of lending}}} - \underbrace{\chi\alpha}_{\substack{\text{marginal leverage}\\ \text{cost of repo}}}\right] \tag{25}$$

Equations (24) and (25) also equate the marginal benefit of reallocating one unit of investment from the repo business to the lending business with its marginal cost. We can see that if the last two conditions of Proposition 3 are satisfied, then the term in the square bracket on the RHS of Equation (24) is higher than the corresponding term on the RHS of Equation (25). This in turn means that the marginal cost of that rebalancing action can be higher in the case of group-level application than in the case of business unit-level application. If so, the bank will prefer to invest relatively more in the lending business in the latter case than in the former one. As a consequence, the bank's asset risk is higher when regulatory constraints are allocated down to business units. Note that the three conditions stated in Proposition 3 are not sufficient conditions for the increase in the bank's asset risk since the RHS of two Equations (24) and (25) also depend on the shadow price of the regulatory constraints.

4 Model calibration

In the previous analysis, we intentionally kept our theoretical setup very general to emphasise the generality of our insights. That generality however implies that we cannot characterise analytically all the possible changes in the bank's investments following the allocation of regulatory constraints to its business units. In this section we calibrate our model to data for banks in the UK, to complement those analytical insights with numerical simulations. The calibration exercise is also helpful to verify the empirical relevance of our previous analytical findings.

We first set out additional parametric assumptions that we make to take the model to the data. We then describes the data used for the calibration and explain our calibration procedure.

Parametric assumptions The bank's ex-ante gross interest income from loans $G(L)$ is naturally the product of the loan volume L and the gross interest rate charged on loans. We assume that the interest rate is a decreasing function of the loan volume: $g_1 + g_2 L$ where $g_1 > 0$ and $g_2 < 0$. Therefore, we have:

$$G(L) = (g_1 + g_2 L)L$$

In line with the literature, we also assume that the losses per unit of loans $\tilde{Z}$ are log-normally distributed with parameter μ_Z^{log} and σ_Z^{log}.

For the repo income, as explained below, since we just have data on short-term repo and reverse repo transactions secured against UK government bonds, we will focus here on this relatively safer type of repo activities and on the role of the repo unit as a market maker. This in turn has two implications. First, $F(X)$ will be the revenue from reverse repo activities net of repo funding cost. We assume that the interest rates charged on both repos and reverse repos depend on the transactional amount, which implies that $F(X)$ can be written as:

$$F(X) = \underbrace{(d_1 + \varepsilon_1 X)X}_{\text{reverse repo revenue}} - \underbrace{(d_2 + \varepsilon_2 X)X}_{\text{repo cost}}.$$

We further denote $\beta_1 = d_1 - d_2$ and $\beta_2 = \varepsilon_1 - \varepsilon_2$ where $\beta_1 > 0$ and $\beta_2 < 0$. Second, we assume that repo business is riskless, i.e. repo losses $\tilde{\varepsilon}$ are equal to zero with probability 1. Table 1 summarises the set of parameters that need to be calibrated.

Table 1: Parameters to be calibrated

Parameters	Description
a	VaR confidence level
χ	Leverage ratio requirement
c	Coupon on government bond
R	Bank's borrowing cost
g_1	Marginal return on loan
g_2	Curvature of loan return
μ_Z^{log}	Lognormal parameter of loan losses
σ_Z^{log}	Lognormal parameter of loan losses
β_1	Marginal return on repo
β_2	Curvature of return on repo

Data To calibrate the model, we use three main data sources. The first dataset is daily yield rates for the 15-year UK government bond retrieved from Factset. Second, we collect information on performance analysis, asset quality and balance sheet of 15 UK banks for which semi-annual data is available in S&P Market Intelligence (S&P MI) from 2015 to 2018. Our last source of data is the confidential Sterling Money Market Data (SMMD) of the Bank of England. It contains daily, transactional level data on repo and reverse repo transactions with a maturity of up to one year that are denominated in GBP and secured against UK government-issued securities. The repo and reverse repo transactions reported in this dataset cover 95% of the total turnover of the market. They are executed by institutions with a significant proportion of total activity in the market among which there are 5 UK banks. Table 2 reports the variables that we use in these datasets for our calibration.

Table 2: Data sources

Variable description	**Timespan**	**Frequency**	**Data source**
Gross loans to customers	2015-2018	Semi-annual	S&P MI
Impaired loans	2015-2018	Semi-annual	S&P MI
Net interest margin	2015-2018	Semi-annual	S&P MI
Cost of funds	2015-2018	Semi-annual	S&P MI
Yield 15Y UK gilt	2015-2018	Daily	Factset
Repo Transaction Nominal Amount	2017-2019	Daily	SMMD
Repo interest rate	2017-2019	Daily	SMMD
Reverse repo Transaction Nominal Amount	2017-2019	Daily	SMMD
Reverse repo interest rate	2017-2019	Daily	SMMD

Calibration methods We set a series of parameters individually. In line with the Basel III risk-weighted capital requirements and the leverage ratio requirement, we set the VaR confidence level a to be equal to 0.001 and the minimum leverage ratio χ equal 3%. We proxy the coupon on government bonds with the 15Y UK gilt yield, as the average of daily yields over the entire period. We set the bank's borrowing cost R to be the average cost of funds of all banks in our sample.

To estimate the distribution parameters μ_Z^{log} and σ_Z^{log} of the random variable $\tilde{Z}$, we proxy its realised value by the amount of impaired loans per unit of total loans. Then we use the maximum likelihood estimation to fit the lognormal distribution of Z with the distribution of impaired loans.

We employ the least square fitting method to derive parameters g_1 and g_2 that underlies the function of gross lending income from the net interest margin reported in our datasets. To do so, we first express the net interest margin of bank i at time t - denoted by $IM_{i,t}$ - via g_1 and g_2 as follows:

$$IM_{i,t} = g_1 + g_2 L_{i,t} - Z_{i,t} - R_{i,t}$$

where $L_{i,t}$ is gross loans to customers; $Z_{i,t}$ is the realised value of impaired loans to total loans and $R_{i,t}$ is the cost of funds - all variables are observed in the data. g_1 and g_2 then can be obtained by estimating the following regression equation:

$$y_{i,t} = g_1 + g_2 L_{i,t} + \eta_{i,t}$$

where $y_{i,t} = IM_{i,t} + Z_{i,t} + R_{i,t}$ and $\eta_{i,t}$ is error term. Both coefficients g_1, g_2 derived from the regression are statistically significant at, respectively, 1% and 5% level.

Similarly, to estimate the repo income, we regress the repo and reverse repo interest rate - denoted by $f_{i,t}^{repo}$ and $f_{i,t}^{reverse}$ respectively - reported for each transaction on the borrowing amount of that transaction using the equations:

$$f_{i,t}^{reverse} = d_1 + \varepsilon_1 X_{i,t}^{reverse} + \nu_{i,t} \quad \text{and} \quad f_{i,t}^{repo} = d_2 + \varepsilon_2 X_{i,t}^{repo} + \upsilon_{i,t}$$

Both regressions give statistically significant coefficients at 1% level. Afterwards, we calculate the marginal return on repo β_1 as equal to $d_1 - d_2$ and the curvature of repo returns β_2 as $\varepsilon_1 - \varepsilon_2$. Table 3 reports the calibrated value for all parameters.[8]

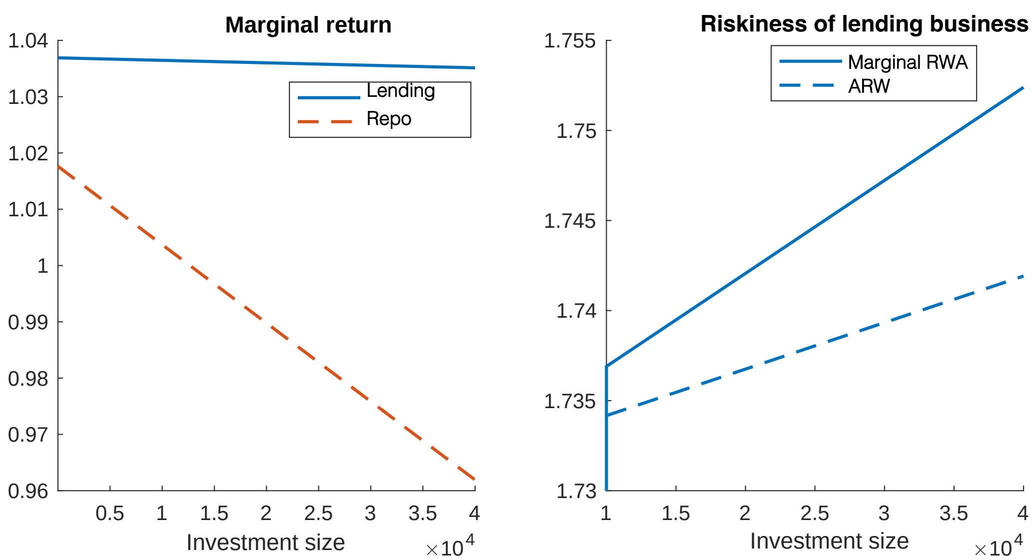
In Figure 1 we display the characteristics of the two business units of our calibrated bank. As seen in the left panel, consistent with Assumption 2, the marginal returns on lending are higher compared to the repo returns. They also decrease at a much slower rate compared to the repo ones, as the investment size increases. In terms of riskiness, we observe from the right panel that the ARW of our calibrated lending business are globally higher than $\frac{\chi}{\gamma}$ which is equal to 0.35.

[8]Values of parameters are reported, when appropriate, in terms of billion GBP.

Table 3: Calibration to UK banks

Description	Parameters	Calibrated Value
VaR confidence level	a	0.001
Leverage requirement	χ	0.03
Coupon on government bond	c	1.0172
Bank's borrowing cost	R	1.0114
Lending unit		
Marginal return on loan	g_1	1.0356
Curvature of loan return	g_2	$-2.22 \cdot 10^{-5}$
Log-normal parameter of Z	μ_Z^{log}	-4.568
(Mean Z)		(0.015)
Log-normal parameter of Z	σ_Z^{log}	0.913
(Standard deviation Z)		(0.018)
Repo unit		
Marginal return on repo business	β_1	0.000427
Diminishing return parameter	β_2	$-6.943 \cdot 10^{-4}$

Figure 1: Characteristics of two business units

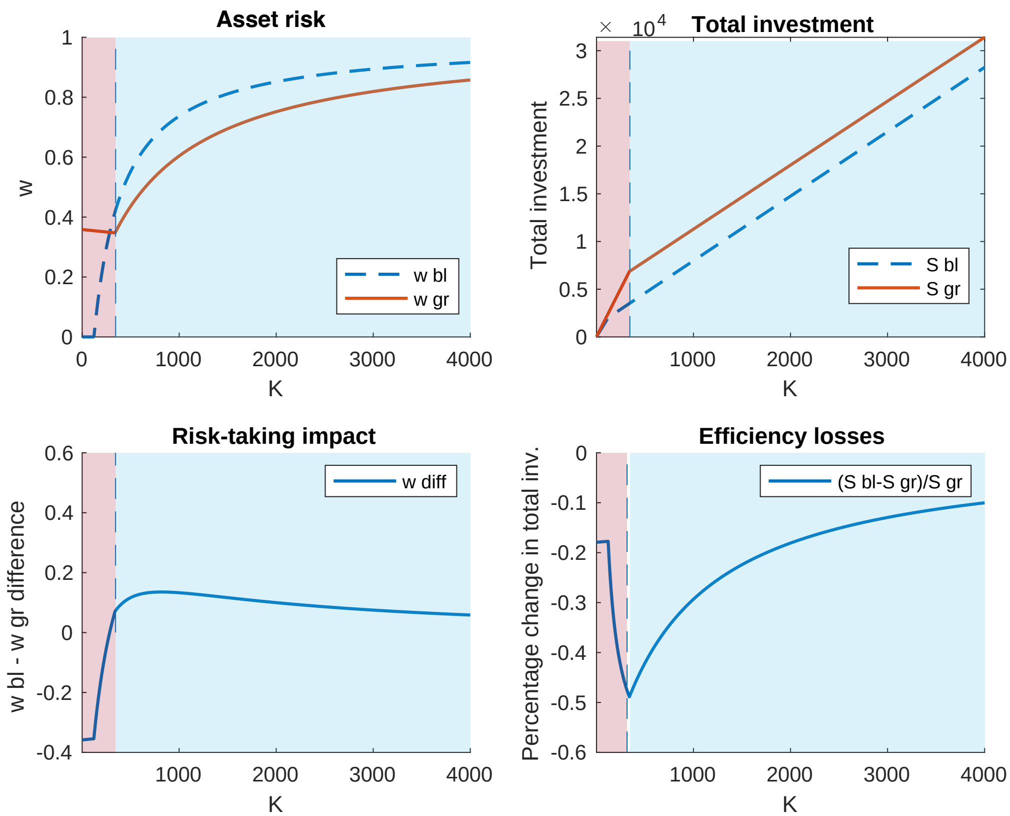


Note: This figure displays the main risk and returns characteristics of the two business units of our calibrated bank. The left panel shows the marginal returns of repo and lending. The right panel shows two riskiness measures of the lending business, namely the marginal RWA and ARW.

5 Numerical simulations

Using the calibrated parameter values, we solve numerically, for different values of the bank's initial equity K, the two optimisation problems $\wp^G$ and $\wp^B$ as defined in Section 3.1. Note that depending on the value of K, the bank can be bound at the group level either by both LR and RW constraints or by only the RW constraint. Figure 2 compares the bank's optimal investments between the case where all constraints are applied at the group level and the case in which both business units have to comply with both constraints individually.

Figure 2: Bank's optimal investments



Note: This figure compares the bank's optimal investments in two cases:(i) when both regulatory constraints are applied at the group level and (ii) when the bank allocates both constraints down to its business units. In the two top panels, the red solid lines represent the bank choices for (i), while the blue dashed lines stand for the bank choices under (ii). The two bottom panels show the difference, between the two cases, in the bank's total investments (bottom right panel) and the bank's asset risk (bottom left panel). For all panels, the dark pink area corresponds to the situation where both LR and RW constraints bind at the group level while in the light blue area, only RW constrain binds at the group level.

We can see that the allocation of constraints leads to efficiency losses since the total investments are reduced (see bottom right panel). As explained in Section 3, these losses are because, when allocating regulatory constraints to its business units, the bank cannot exploit the diversification of its investment portfolio to increase the total size of the portfolio for each level of capital resource.

In terms of asset risk impact, we can observe from the top left panel that this impact depends on whether the bank is constrained only by the RW constraints at the group level (light blue area) or by both RW and LR constraints (dark pink area). When only the RW constraint binds at the group level, requiring all business units to comply with both regulatory constraints will lead to an investment distortion in the sense that the bank will invest relatively more into riskier business - lending. This in turn will increase the overall asset risk of the bank. When both constraints bind at the group level, the impact on the bank's asset risk is somehow ambiguous. When K is very small, the bank's asset risk decreases but when K is above a certain threshold, it increases following the allocation of constraints.

6 Role of business model

As highlighted in the analytical part, the impact of the allocation of constraints on banks' investment decisions depends on the specific characteristics of their businesses such as riskiness. We, therefore, expect that this impact can vary with the banks' business model. In this section, we examine the potential effect of the business model. We first classify the 15 UK banks in our S&P MI dataset into different business models. Then we recalibrate the lending business for each type and run numerical simulations. Note that since the limited number of UK banks in the SMMD database that we use to calibrate the repo unit does not allow us to have a meaningful business model classification, we focus here on the consequences of the difference in lending business characteristics and funding costs between business models.

6.1 Business model classification and calibration

Our business model classification relies on the methodology proposed in Roengpitya et al. (2014). They use a statistical clustering method based on various ratios of banks' balance sheet which are informative on the bank business model. They find that retail-funded banks have a high share of gross loans and rely more on stable sources of funding, such as deposits. The wholesale-funded banks have a lower percentage of funding coming from deposits, but a higher share of inter-bank liabilities compared to retail banks. Lastly, capital markets-orientated banks have a much higher percentage of trading assets and liabilities compared to the previous two types. The last type of bank has the highest ratio of inter-bank borrowing as a percentage of total assets and also displays a lower reliance on stable funding. The paper reports average values of these ratios to total assets, and we use them as a benchmark to construct the selection criteria for our sample.

Due to limited data availability, compared to the Roengpitya et al. (2014), we use a restricted version of their selected ratios, and we adjust downwards the threshold criteria to match our sample. Our criteria include the ratio of customer deposits to total liabilities for the stable source of funding ratio, the ratio of assets held for trading to total assets as a measure of tradable assets, loans to banks as the fraction of total assets for our inter-bank lending measure, and bank deposits to total liabilities as the bank deposit ratio. Classifying these ratios based on observed bank characteristics from the Roengpitya et al. (2014), we find nine retail-funded, five wholesale-funded and one capital markets-oriented bank. Having one bank only in one group would not permit for a meaningful comparison, so we aggregate the wholesale with the capital markets-oriented bank, giving us a sample split into nine retail-funded, and six wholesale-funded and capital markets-orientated banks, to which we refer from now on as wholesale banks. Table 4 reports some characteristics of each business model.

We recalibrate the model to the two different categories of banks. We report the calibrated values of different parameters for each type of banks in Table 5. We observe that wholesale banks have lower costs of funding than retail banks.

Table 4: Business model descriptives

Description	Values	
	Retail	**Wholesale**
Interest rate on unsecured debt	0.0129 (47)	0.009 (31)
Leverage ratio	0.0549 (55)	0.0529 (43)
Fully loaded risk-weighted capital ratio	0.253 (43)	0.185 (32)
Loans to total assets	0.7649 (63)	0.619 (43)
Percentage of impaired loans to total loan size	1.11% (53)	2.04% (28)

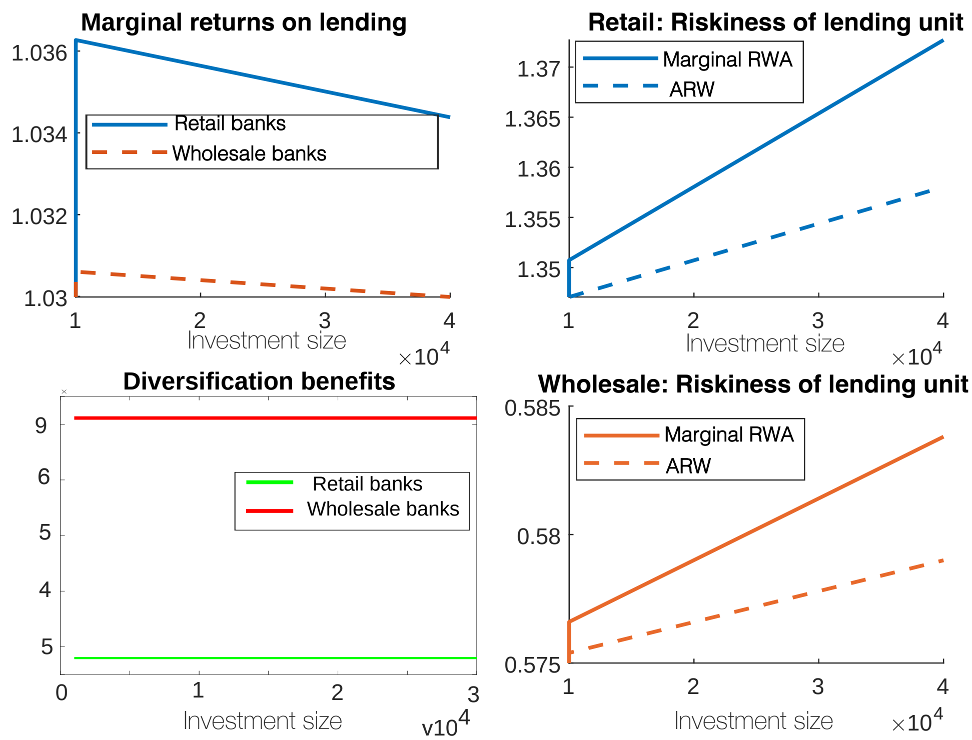
The number of observations is in brackets, unless otherwise stated.

Table 5: Calibration across business models

Description	Parameters	**Retail**	**Wholesale**
Bank's borrowing cost	R	0.0129	0.009
	Lending		
Marginal return on loan	g_1	1.0369	1.03081
Curvature of loan return	g_2	$-3.15 \cdot 10^{-5}$	$-1.03 \cdot 10^{-5}$
Log-normal parameter of Z (Mean Z)	μ_Z^{log}	-4.885 (0.0118)	-3.97 (0.0207)
Log-normal parameter of Z (Standard deviation Z)	σ_Z^{log}	0.945 (0.0142)	0.429 (0.0093)

Further, in Figure 3, we compare the two business models in terms of returns and riskiness of their lending business. As seen in the top left panel, the marginal returns of lending are higher for retail banks and decrease at a higher speed compared to wholesale banks. The two panels on the right show that the lending business of retail banks is riskier than that of wholesale banks.

Figure 3: Characteristics of lending business across business models



Note: This figure compares some main characteristics of lending business across business models. The top left panel shows the marginal returns on lending. The bottom left panel shows, as a function of lending size, the diversification benefits defined as the difference between RWA^L and RWA^G. In the two right panels, we represent the ARW and the marginal RWA as a function of investment in lending.

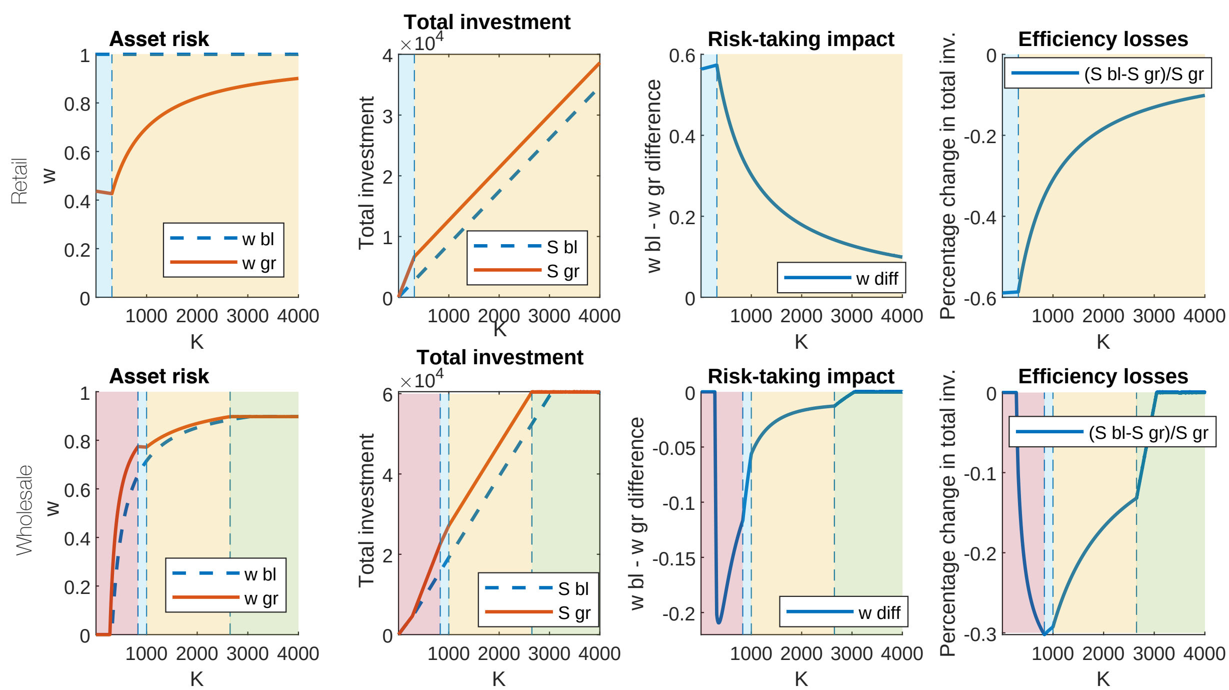
6.2 Numerical simulations for different business models

We now run simulations for each business model. Figure 4 compares the optimal investments of retail and wholesale banks between the case where all constraints are applied at the group level and the case in which both constraints are allocated down to business units.

Two main observations are in order here. First, the situation in which the leverage constraint binds at the group level happens only with wholesale banks, but not for retail banks. Note also that since in these simulations, we assume that the repo business is riskless, the ARW of this business for both types of banks is lower than the CRW which is equal to 0.35. This difference can therefore be explained by the fact that the ARW of the lending business for both types of banks is higher than the CRW but the diversification benefits of wholesale banks are higher than those of retail banks as shown in the bottom left panel of Figure 3.

Second, there is a stark difference in the impact of the allocation of constraints on the bank's asset risk between retail and wholesale banks when only the RW constraint binds at the group level (beige area in Figure 4). Precisely, in this case, while the allocation of constraints increases the asset risk for retail banks, it brings about a decrease in the asset risk for wholesale banks.

Figure 4: Optimal investments: a comparison across business models



Note: This figure compares the optimal investments of retail banks (first row) and wholesale banks (second row) in two cases: (i) both regulatory constraints are applied at the group level and (ii) the bank allocates both constraints to its business units. In the first two columns, the red solid lines represent the bank's choices in the first case while the blue dashed lines stand for the bank's choices in the second case. The panels in the third and fourth columns represent the difference in, respectively, the bank asset risk and banks' total investments between the two cases. For all panels, the dark pink area corresponds to the situation where only the LR constraint binds at the group level; the light blue area to the case in which both LR and RW constraints bind; the beige area to the case where only RW constraint binds and finally the green area is when no constraints bind.

7 Conclusion

This paper assesses how banks' asset risk changes when banks apply regulatory requirements at a business level lower than the group level in their internal capital allocation process. We develop a model where a banking group has two business units: a riskier one that yields higher returns and a less risky one with lower returns. We refer to these units as lending and repo business units, respectively. We also calibrate the model to UK banks and evaluate the implications of bank business models for the impact of application level on asset risk.

We find that when the banking group is not bound by the LR but by the RW requirement, under certain conditions, applying requirements at the business level lower than the group level can lead to an increase in the bank's asset risk by inducing the bank to invest relatively less in repo and more in lending. This finding provides an explanation of the mechanism behind the empirical fact on the reduction of repo intermediation by banks not bound by the LR requirement.

When calibrating the model to UK banks and running simulations, we find that the investments of the RW-constrained bank are distorted by a relatively higher investment in the riskier unit, which is consistent with our analytical finding. Additionally, when calibrating the model to different business models, we observe that applying requirements at the business-unit level increases the risk-taking for retail banks but not for wholesale banks.

In terms of policy implications, our paper offers valuable insights on how the impact of regulatory measures can depend on the level at which banks apply those regulatory constraints. First, we highlight two potential costs when banks apply them at low business levels: it can make banks increase their asset risk and decrease their overall investments. This has immediate implications for low-risk, low-margin markets such as the repo market, as banks will decrease their activities in those markets. Second, we find that one size does not fit all, and the impact on asset risk differs across bank business models.

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A Appendix

A.1 Derivation of Constraint (3) - the RW constraint at the group level

Given that

$$\tilde{\Pi}_L = G(L) - \tilde{Z}L - R(L - K_L)$$

and

$$\tilde{\Pi}_X = F(X) + cX - \tilde{\varepsilon}X - R(X - K_X)$$

we can write $\mathbb{P}\left(\tilde{\Pi}_L + \tilde{\Pi}_X \leq 0\right) \leq a$ as:

$$\mathbb{P}\Big(G(L) + F(X) + cX - R(X + L - K) \leq \tilde{Z}L + \tilde{\varepsilon}X\Big) \leq a \tag{A.1}$$

Using Definition (1), Inequality (A.1) is equivalent to

$$VaR_{1-a}(\tilde{Z}L + \tilde{\varepsilon}X) \leq G(L) + F(X) + cX - R(X + L - K)$$

or

$$K \geq \frac{VaR_{1-a}(\tilde{Z}L + \tilde{\varepsilon}X) - [G(L) - RL + F(X) + cX - RX]}{R} \tag{A.2}$$

A.2 Variance of asset returns as a measure of bank asset risk

Using Expression (7), we could rewrite the variance of the returns of the bank's asset portfolio σ_p^2 as a function of w as follows:

$$\sigma_p^2 = S^2 w^2 \sigma_Z^2 + S^2(1 - w)^2 \sigma_\varepsilon^2$$

We thus obtain:

$$\frac{\partial \sigma_p^2}{\partial w} = 2S^2\left(w\sigma_Z^2 - (1 - w)\sigma_\varepsilon^2\right) \quad \text{and} \quad \frac{\partial^2 \sigma_p^2}{\partial w^2} \geq 0$$

Denote by $\underline{w}$ the value of w when $\frac{\partial \sigma_p^2}{\partial w} = 0$, i.e.

$$\underline{w} = \frac{\sigma_\epsilon^2}{\sigma_Z^2 + \sigma_\varepsilon^2}$$

Since $\sigma_\varepsilon^2 < \sigma_Z^2$, we have $\underline{w} < 0.5$.

We observe that σ_p^2 is decreasing with w when $w \in (0, \underline{w})$ and increasing with w when $w \in [\underline{w}, 1]$. Therefore, the results for w as a measure of the bank asset risk also hold for σ_p^2 when $w \in [\underline{w}, 1]$. We believe that focusing on the interval $[\underline{w}, 1]$ is without loss of generality due to two reasons. First, if σ_ε^2 is low enough, $\underline{w}$ is closing to 0. Second, $[\underline{w}, 1]$ would be the more empirically relevant interval for the share of risky businesses in the banks' balance sheet. Indeed, based on our data about balance sheet information of a sample of UK banks, we find that the share of lending business for those banks is generally above 0.5.

A.3 Derivation of the first order conditions (FOCs) for Problems $\wp^G$ and $\wp^B$

The Lagrangian for Problem $\wp^G$ reads:

$$\begin{aligned}\Lambda_g = \quad & \Pi(w,S) - \mu_Z wS - \mu_\varepsilon(1-w)S + RK + \lambda_{VaR}\Big(K - \frac{VaR_{1-a}(\tilde{Z}w + (1-w)\tilde{\varepsilon})S - \Pi(w,S)}{R}\Big) \\ & + \lambda_{LR}(K - \chi\big(w + \alpha(1-w)\big)S)\end{aligned}$$

where λ_{VaR} and λ_{LR} are the Lagrange multiplier for, respectively, the group-level RW constraint and the group-level LR constraint. The FOC that determines w^G is as follows:

$$\frac{\partial \Pi(S,w)}{\partial w} - \mu_Z S + \mu_\varepsilon S - \lambda_{VaR}\frac{\partial(\gamma RWA^G(w,S))}{\partial w} - \lambda_{LR}\chi(1-\alpha)S = 0 \qquad \text{(A.3)}$$

Similarly, the Lagrangian for Problem $\wp^B$ reads:

$$\begin{aligned}\Lambda_b = \quad & \Pi(S,w) - \mu_Z wS - \mu_\varepsilon(1-w)S + RK + \lambda^L_{VaR}\Big(K_L - \frac{VaR_{1-a}(\tilde{Z}w)S - G(wS) + RwS}{R}\Big) \\ & + \lambda^X_{VaR}\Big(K_X - \frac{VaR_{1-a}(\tilde{\varepsilon}(1-w))S - F\big((1-w)S\big) - c(1-w)S + R(1-w)S}{R}\Big) \\ & + \lambda^L_{LR}(K_L - \chi wS) + \lambda^X_{LR}(K_X - \chi\alpha(1-w)S) + \lambda_K(K - K_L - K_X)\end{aligned}$$

where $\lambda^L_{VaR}, \lambda^X_{VaR}, \lambda^L_{LR}, \lambda^X_{LR}$ and λ_K are the Lagrange multipliers of corresponding constraints. The FOC for w^B is written as follows:

$$\begin{aligned}\frac{\partial\Pi(S,w)}{\partial w} - \mu_Z S + \mu_\varepsilon S - \lambda^L_{VaR}\frac{\partial(\gamma RWA^L(w,S))}{\partial w} \\ - \lambda^X_{VaR}\frac{\partial(\gamma RWA^X(w,S))}{\partial w} - \lambda^L_{LR}\chi S + \lambda^X_{LR}\chi\alpha S = 0 \quad \text{(A.4)}\end{aligned}$$

A.4 Proof of Lemma 1

Note that the ARW of the banking group - ARW^G - is computed as:

$$ARW^G(w,S) = \frac{RWA^G(w,S)}{wS + \alpha(1-w)S} \quad \text{(A.5)}$$

We see that since α is greater than or equal to 1, the denominator of the RHS of Expression (A.5) is weakly decreasing with w. We will now examine how $RWA^G(w,S)$ changes with w.

From Equation (A.3), we see that the FOC that determines w at the group level can be written as follows:

$$\frac{\partial\Pi(w,S)}{\partial w} - (\mu_Z - \mu_\epsilon)S = \gamma\lambda_{VaR}\frac{\partial RWA^G}{\partial w} - \chi\lambda_{LR}(\alpha-1)S \quad \text{(A.6)}$$

Since the bank will always choose w such that the LHS of Equation (A.6) is non negative, in the equilibrium we have:

$$\gamma\lambda_{VaR}\frac{\partial RWA^G}{\partial w} - \chi\lambda_{LR}(\alpha-1)S \geq 0$$

which implies that in the equilibrium $\frac{\partial RWA^G}{\partial w} \geq 0$ since $\alpha \geq 1$.

Hence, in the equilibrium, the numerator of the RHS of Expression (A.5) is increasing with w, which in turn implies that in the equilibrium, ARW^G increases with w.

A.5 Proof of Proposition 2

- *First, we prove that if both business units have ARW below $\frac{\chi}{\gamma}$, then the asset risk of the LR-constrained bank does not change with the application level of requirements.*

To prove the above, we will establish that the solution (w^G, S^G) to Problem $\wp^G$ will also be the solution to Problem $\wp^B$ if the three following conditions are satisfied:

$$\chi(wS + \alpha(1-w)S) > \gamma RWA^G \tag{A.7}$$

as well as

$$\frac{RWA^L}{wS} \leq \frac{\chi}{\gamma} \quad \text{and} \quad \frac{RWA^X}{\alpha(1-w)S} \leq \frac{\chi}{\gamma} \tag{A.8}$$

Indeed, since (w^G, S^G) is the solution to Problem $\wp^G$ when Condition (A.7) is satisfied, we have:

$$K = \chi(w^G S^G + \alpha(1-w^G)S^G) \tag{A.9}$$

When two conditions in (A.8) hold, the relevant constraints for Problem $\wp^B$ will be Constraints (13), (14) and (15). Clearly, (w^G, S^G) that satisfies Equality (A.9) will also satisfy all Constraints (13), (14) and (15) where we simply choose $K_L = \chi w^G S^G$ and $K_X = \chi\alpha(1-w^G)S^G$. This in turn implies that (w^G, S^G) belong to the feasible set of Problem $\wp^B$. Since the feasible set of Problem $\wp^B$ is smaller than that of Problem $\wp^G$, (w^G, S^G) are also the solution to Problem $\wp^B$.

- *We now prove that if both business units have ARW greater than $\frac{\chi}{\gamma}$, then the asset risk of the LR-constrained bank will decrease when regulatory constraints are allocated down to business units.*

To prove this, we will compare the FOCs that determine w^G and w^B. When the LR requirement is the binding constraint at the group level, we have $\lambda_{VaR} = 0$ and $\lambda_{LR} \geq 0$. Therefore, based on Equation (A.3), after some algebra, we see that in this case, w^G is determined by the following equation:

$$\left[G'(w^G S^G) - \mu_Z - R\right] - \left[F'((1-w^G)S^G) + c - \mu_\varepsilon - R\right] = \lambda_{LR}\left(\chi - \alpha\chi\right) \tag{A.10}$$

In relation to w^B, when $ARW^L \geq \frac{\chi}{\gamma}$ and $ARW^X \geq \frac{\chi}{\gamma}$, Constraint (11) is tighter than Constraint (13) and Constraint (12) is tighter than Constraint (14). The binding constraints in Problem $\wp^B$ will thus be Constraints (11) and (12), which implies $\lambda_{LR}^L = \lambda_{LR}^X = 0$.

Based on the Lagrangian for Problem $\wp^B$ explained in Appendix A.3, the two FOCs for K_L and K_X are as follows:

$$\lambda_{VaR}^L + \lambda_{LR}^L - \lambda_K = 0 \quad \text{and} \quad \lambda_{VaR}^X + \lambda_{LR}^X - \lambda_K = 0$$

which means:

$$\lambda_{VaR}^L + \lambda_{LR}^L = \lambda_{VaR}^X + \lambda_{LR}^X = \lambda_K \tag{A.11}$$

Therefore, we obtain

$$\begin{cases} \lambda_{LR}^L = \lambda_{LR}^X = 0 \\ \lambda_{VaR}^L = \lambda_{VaR}^X \geq 0 \end{cases} \tag{A.12}$$

Plugging Result (A.12) into the FOC (A.4), after some algebra, we obtain the equation that characterises w^B as follows:

$$\left[G'(w^B S^B) - \mu_Z - R\right] - \left[F'((1-w^B)S^B) + c - \mu_\varepsilon - R\right] = \lambda_{VaR}^L\left[\frac{\partial(\gamma RWA^L)}{\partial L} - \frac{\partial(\gamma RWA^X)}{\partial X}\right] \tag{A.13}$$

Note that the LHS of Equations (A.10) and (A.13) is a decreasing function of w. Moreover the RHS of Equation (A.10) is non positive while the RHS of Equation (A.13) is non negative. These all together imply that $w^B < w^G$

A.6 Proof of proposition 3

Since the bank is bound by the RW constraint at the group level, we have $\lambda_{LR} = 0$ and $\lambda_{VaR} \geq 0$. From Equation (A.3), we see that w^G is characterised by the following equation:

$$\left[G'(w^G S^G) - \mu_Z - R\right] - \left[F'((1-w^G)S^G) + c - \mu_\varepsilon - R\right] = \lambda_{VaR}\left[\frac{\partial(\gamma RWA^L)}{\partial L} - \frac{\partial(\gamma RWA^X)}{\partial X} - \frac{\partial Div}{\partial w}\right] \quad \text{(A.14)}$$

Regarding w^B, when $ARW^L > \frac{\chi}{\gamma}$ and $ARW^X < \frac{\chi}{\gamma}$, Constraint (11) is tighter than Constraint (13) and Constraint (12) is looser than Constraint (14). The binding constraints in Problem $\wp^B$ will thus be Constraints (11) and (14), which implies $\lambda^L_{LR} = 0$ and $\lambda^X_{VaR} = 0$. Using Result (A.11), we thus have $\lambda^L_{VaR} = \lambda^X_{LR} \geq 0$. This in turn implies that w^B is determined as follows:

$$\left[G'(w^B S^B) - \mu_Z - R\right] - \left[F'((1-w^B)S^B) + c - \mu_\varepsilon - R\right] = \lambda^L_{VaR}\left[\frac{\partial(\gamma RWA^L)}{\partial L} - \chi\alpha\right] \quad \text{(A.15)}$$

Therefore, if the three conditions stated in Proposition 3 are satisfied, it can happen that the RHS of Equation (A.14) is greater than that of Equation (A.15). In that case $w^G < w^B$ since the LHS of the two equations is decreasing with w.