# Secrecy vs. Patenting in Innovation Races*

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**Abstract**

We examine the tradeoff between patenting and secrecy in innovation races, considering a model where two firms simultaneously compete in developing two products. Patenting ensures a claim on the product but discloses information to rivals, while secrecy may delay immediate profits for future technology leadership. In the general case we find that firms have more incentives to patent if they become less patient or when there are lower technological spillovers. Furthermore, we compare the patenting behavior when the goods are substitutes or complements. In a scenario where the R&D spillovers are small and the firms are not moderately patient, they exhibit a greater tendency to patent products acting as perfect complements rather than perfect substitutes. These findings are in line with the empirical evidence by Cohen, Nelson, and Walsh (2000), who argue that firms are more likely to keep the innovation secret in "simple" industries, where goods have many potential substitutes, as opposed to "complex" industries, where a new product involves many complementary components.

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# 1 Introduction

The trade-off between patenting and secrecy plays a central role in innovation races. After obtaining an innovation a firm may decide to patent it or to keep it secret. On the one hand, patenting guarantees the firm a certain level of protection which may translate into a stream of profits. However, patenting requires disclosure, potentially providing valuable information to rivals in the presence of spillovers. This information can then be used to develop other products, which in turn affects the profitability of the original patent. On the other hand, by keeping the innovation secret, the firm forgoes a potential intermediate profit stream, but it can develop the product further in the hope to become a technology leader, thereby earning higher profits in the future. At the same time, it carries a danger that a rival firm will develop and patent the product in the meantime.

The profitability of patenting vs. secrecy depends on the market structure. For instance, in "complex" industries, where a new marketable product involves patents for many components, a firm may prefer rivals to develop complementary products in order to speed up the introduction of the new product into the market. By contrast, in "simple" industries, where a new marketable product involves few patents and usually has many potential substitutes, secrecy might prevent rivals from developing substitutes that do not infringe on the patent, but still lead to competition in the product market. These ideas have already been documented empirically. Cohen, Nelson, and Walsh (2000) argue that patents are commonly used to prevent rivals from developing substitutes in "simple" industries, such as chemicals, while they are used to force rivals into negotiations in "complex" industries, such as telecommunications (see also Reitzig, 2004).

We develop a theoretical model of patent races, where firms undertake R&D for two different products. Innovation is modeled as a Poisson process. Each firm undertakes R&D with the goal to develop two different products. We are particularly interested in cases where the two products are either (perfect) substitutes, corresponding to simple industries, or (perfect) complements, corresponding to complex industries, as defined in Cohen, Nelson, and Walsh (2000). In order to focus on the patenting decision, we abstract from the investments and assume that the arrival of innovations is an exogenous process. After obtaining a breakthrough on one product, the firm decides whether to patent it or not. In order for a product to be taken to the market, the firm needs to possess a patent on it; otherwise the product may be reverse-engineered and copied by rivals. We assume that a breakthrough is not observable by the rival and unless a firms patents it, the firms' progress remains its private information.

At each moment, the firms obtain flow profits that depend on the number of patents each firm has. A firm holding patents on both products becomes a monopolist, winning the race, and enjoying a stream of monopoly profits forever. A firm with no patents does not obtain any flow profit, irrespective of whether it had an innovation success or not. The remaining two flow profits, when one firm has one patent and when each firm has one patent, are determined by the underlying market structure. In case of perfect complements, patents on both innovations are necessary to generate market value. A single patent does not yield any profit, but firms can create a patent pool and share the monopoly profit if each holds one patent. In case of perfect substitutes, even a single patent can generate a profit flow, but firms will compete on the product market if each holds one patent.

As soon as both innovations have been patented (by the same or by different firms), there is no further strategic decision on patenting, ending the race. When only one product has been patented, the firms engage in a race for the other product, benefiting from technological spillovers. Since, by assumption, the innovation process is exogenous, the only relevant strategic decision is, whether to patent a breakthrough or keep it secret, provided nobody has patented yet. Our game thus has the structure of a stopping time game with private information. When deciding between patenting and secrecy the firm faces a trade-off. By patenting the first innovation, the firm secures itself a certain stream of profit, but at the same time decreases a change of obtaining the monopoly profit associated with having both patents. The profitability of patenting then depends on the relation of those two effects.

We identify and characterize three types of equilibria: (1) Instant Patenting equilibrium, where the firms patent the innovations instantly, (2) Full Secrecy equilibrium, where firms never patent the first innovation, and (3) Delayed Patenting equilibrium, where the firms keep the innovation secret until certain time and start patenting with a certain probability afterwards. Delayed Patenting equilibrium is similar to the equilibrium in pre-emption games, since each firm would like to delay patenting but at the same time prefers to patent earlier than the rival. Overall we find that, for a given market structure, firms have more incentives to patent if they are impatient and if there are no significant technological spillovers.

Comparing the patenting behavior under perfect substitutes and perfect complements, we find partial support for the conclusions by Cohen, Nelson, and Walsh (2000). We show that the firms have (weakly) more tendency towards patenting under perfect complements than under perfect substitutes when the technological spillovers are small and when the firms' impatience (or inefficiency of R&D in the second stage) is not moderate.

## Related literature

Our paper builds on the classical theoretical literature on patent races with a single innovation (Loury, 1979; Lee and Wilde, 1980) and subsequent papers that study dynamic environments, where a firm needs to sequentially complete certain stages in order to be able to patent the innovation (Grossman and Shapiro, 1987; Scotchmer and Green, 1990). The innovation is modeled as a Poisson process with arrival rate that depends on the firm's investment. A distinctive feature of these studies is that they assume all information is public, thus excluding secrecy.

A related stream of patent races literature studies the trade-off between patenting and secrecy, but under the assumption that a firm's success in the innovation race is observable by the rivals. Kultti, Takalo, and Toikka (2007) use a different notion of secrecy, namely as a method to reduce technological spillovers. In that case, secrecy only pays off if it is unlikely that a rival develops the same innovation and patents it. Schneider (2008) analyzes a trade-off similar to ours, but in a model of sequential patent races based on Scotchmer and Green (1990). In his model the innovations are publicly observable, whereas under secrecy the rival does not benefit from spillovers. He show that firms relay on secrecy, when the subsequent innovation arrives at high rate relative to the com competitor.

A closely related stream of literature studies sequential patent races, where the innovative activity of firms is not observable. Hopenhayn and Squintani (2016) consider an innovation race and assume that the payoff from patenting grows over time. The firms may then delay patenting an innovation and exposing itself to the risk of losing the race. The authors compare equilibrium patenting strategies with socially optimal strategies and identify forces that yield to too early or too late disclosure. A working paper by Gordon (2011) as well as our companion paper Kocourek and Kováč (2023) analyze a sequential patent race and assume that the firms can patent the innovation and realize profits only upon completing the final stage of the innovation race. A firm can disclosure information about intermediate success and has incentives to do so if it discourages the rival's R&D effort. Gordon (2011) considers linear effort costs and focuses on equilibria in pure strategies. Kocourek and Kováč (2023) consider convex effort cost and provide a more comprehensive analysis of equilibria involving mixed strategies. The structure of mixed strategy equilibria in Kocourek and Kováč (2023), however, differs from the structure in the present paper. The firms randomize over revelation up to certain time $T_D$, after which they do not reveal at all. This resembles the equilibrium known from the war of attrition games, whereas in the present paper the equilibrium resembles the one from pre-emption games. Song and Zhao (2021) analyse model of innovation race with two stages, while it is initially not know whether the first stage can be completed. They decide whether to disclosure or withhold the first innovation. After disclosure, the rival learn that the first stage can indeed be completed. By withholding the success, the rival becomes more pessimistic about first stage and may eventually exit the race. The authors show the equilibrium always exhibits such "disclose-withhold-exit" pattern. In a recent working paper Chatterjee, Das, and Dong (2023) consider a sequential race with two stages and fixed payoffs from completing each stage. They consider linear effort cost that decreases in the second stage. Chatterjee et al. (2023) identify similar types of equilibria as in our paper and show that the final reward has a $U$-shaped effect on when each success is disclosed.

While the present paper is methodologically similar some of the above mentioned papers, we introduce novel economic effects. First, allowing the firms to patent the first (intermediate) innovation, introduces intermediate (but only temporary) profit as an additional motive to reveal and patent a success. Second, assuming simultaneous patent races instead of sequential ones, modifies the incentives of the firms to patent, whereby the firms may obtain the same innovations or different ones. Third, considering patent races in multiple products, enables us to explore how market structure and patenting behavior are related.

The remainder of the paper is organized as follows. In Section 2 we introduce the theoretical model. In Section 3 we characterize equilibrium of the game with general profits. In Section 4 we compare patenting behavior under perfect complements and perfect substitutes. Section 5 discusses our assumptions. Proofs of all Propositions and Lemmas are relegated to the appendix.

# 2 Model

Two ex ante symmetric firms, $A$ and $B$, are engaged in an innovation race in order to develop two products, $L$ and $R$. The R&D is modeled in continuous time with infinite horizon and innovation arrival following a Poisson process. Initially, the breakthrough on each product arrives with the rate $\frac{1}{2}\lambda$ (where $\lambda > 0$). Accordingly, the first innovation then arrives with the rate $\lambda$. Once a firm has developed one product or has observed the rival's patent, this arrival rate increases to $\mu$ due to technological spillovers, i.e., $\mu \geq \lambda$.[1]

Each firm's objective is to maximize its expected discounted value

$$V = \int_0^\infty e^{-rt} r \pi_t \, dt, \tag{1}$$

where $\pi_t$ is the firm's flow profit at time $t \geq 0$ and $r$ (where $r > 0$) is the discount rate. The flow profits depend on the number of patents each firm has. Let us denote $\pi^{kl}$ the flow profit of a firm with $k$ patents if the rival has $l$ patents, where $k, l \in \{0, 1, 2\}$ and $k + l \leq 2$. Furthermore, let us denote $\Pi^{kl}$ the corresponding continuation value in such a case. Note that the profits depend only on the number of patents, not on the number innovations. We assume a firm must patent the product before marketing it.[2] The innovation race ends when both products are patented, either by the same firm or by different firms.[3] A firm with patents on both products wins the race and obtains a stream of monopoly profits (that is normalized to 1) forever, i.e., $\pi^{20} = 1$ with continuation value $\Pi^{20} = 1$. A firm without any patent obtains zero flow profit $\pi^{0l} = 0$ as well as zero continuation value $\Pi^{0l} = 0$ for each $l \in \{0, 1, 2\}$. Given these assumptions, we are left with two distinct flow profits: the profit $\pi^{11} = \alpha$ if each firm has patent on one product and the profit $\pi^{10} = \beta$ of a firm which possesses the only patent on the market. In the former case both firms obtain the flow profit $\alpha$ forever, yielding a continuation value $\Pi^{11} = \alpha$. In the latter case, the flow profit $\beta$ is only temporary until the second product is patented.

The parameters $\alpha$ and $\beta$ are (together with parameters $\lambda$, $\mu$, and $r$) primitives of the model and depend on the market structure. The specification using the general flow profits allows us to capture a range of market structures without the need for particular assumptions on consumer behavior and firms' technology. We assume that $0 \leq \alpha \leq \frac{1}{2}$ and $0 \leq \beta \leq 1$. Flow profit $\beta$ corresponds to the (inverse) degree of complementarity. Under perfect complements, the firms obtain no profit from a single patent, i.e., $\beta = 0$. When the products become less complementary, the flow profit $\beta$ from single patent increases. The upper bound on $\beta$ is the monopoly flow profit 1. Flow profit $\alpha$ corresponds to the intensity of competition. In the extreme case of price competition with perfect substitutes

[1] The interpretation of this assumption is that the firm can pull resources from the already patented product and focus on the other one. It can be obtained, for instance, from a convex R&D production function. An alternative would be a weaker assumption $\mu \geq \frac{1}{2}\lambda$ allowing for concave R&D production functions. In addition, the assumption $\mu \geq \lambda$ significantly simplifies the analysis of equilibria.

[2] As we argue below, all continuation profits except $\Pi^{00}$ are uniquely determined and do not depend on the equilibrium.

[3] Firms continue to receive the corresponding flow profits, but there is no more action to take.

(as in the Bertrand model), the firms obtain no profit when each possesses one patent, i.e., $\alpha = 0$. When competition becomes less intense, the flow profit $\alpha$ increases. The upper bound $\frac{1}{2}$ corresponds to sharing the monopoly profit, for instance in a patent pool or as an outcome of collusion.

In addition, recall that the flow profits $\alpha$ and $\beta$ are jointly determined by the underlying market structure. Therefore, some combinations of those parameters are more reasonable than others. Motivated by the comparison of simple and complex industries (Cohen et al., 2000), in Section 4 we compare two extreme cases: (i) when the goods are are perfect substitutes, characterized by $\alpha = 0$ and $\beta = 1$, and (ii) when the goods are perfect complements, characterized by $\alpha = \frac{1}{2}$ and $\beta = 0$ (see Section 4 for a more detailed discussion).

Over time the firms decide whether to patent their innovations. We assume that the firms do not observe the rival's breakthroughs. Patenting a product is the only observable action. If no product has been patented and a firm obtains breakthroughs on both products, it patents them immediately, ending the race. Similarly, if one product is already patented and a firm obtains a breakthrough on the other product, it patents the product immediately, also ending the race. If a firm has an innovation on a product already patented by the rival, this innovation becomes obsolete. After patenting one product, the firms engage in a race for inventing the other product (with arrival rate $\mu$). The subsequent behavior is clearly determined, as both firms patent the second innovation as soon as they get it. Thus, patenting one product effectively ends the game. Depending on who wins the race the firms attain the continuation values $\Pi^{20} = 1$ and $\Pi^{02} = 0$ or equal continuation values $\Pi^{11} = \alpha$. In addition, during the race for the second innovation, the firm with the patent obtains the flow profit $\beta$. The corresponding continuation values when one product has been patented are[4]

$$\Pi^{10} = \frac{\alpha\mu + \beta r + \mu}{r + 2\mu} \qquad \text{and} \qquad \Pi^{01} = \frac{\alpha\mu}{r + 2\mu}. \tag{2}$$

Based on the above arguments, the only strategic decision of a firm is whether to patent its first innovation or to keep it secret once the firm has it. We refer to those actions simply as *patenting* and *secrecy*. The game thus has a structure of a stopping time game, where *patenting* corresponds to stopping and *secrecy* corresponds to continuing. Let us point out that *patenting* at time $t$ involves patenting any innovation that is either invented at time $t$

[4]See, for instance, Lee and Wilde (1980). We also provide the computation in Appendix.

or that was invented in the past. Based on that there are two strategy profiles that serve as natural candidates for equilibria: *Instant Patenting*, which firms patent any innovation instantly, and a *Full Secrecy*, in which firms always choose secrecy, i.e., never patent only one innovation.

We also allow the firms to use mixed strategies by randomizing over patenting within the next time interval $[t, t + \Delta t]$. In this case, patenting also follows a random process with a hazard rate chosen by the firm. Formally, a mixed strategy (before anyone has revealed) for firm $j$ is represented by a non-negative right-continuous function $x_t^j$ defined for $t \geq 0$, which is equal to the hazard rate with which firm $j$ is expected to patent by his rival, denoted as $-j$. The case where $x_t^j = 0$ then corresponds to secrecy, while the case $x_t^j = \lambda$ to patenting instantly (provided firm $j$ does not have any unpatented invention at time $t$), since the hazard rate of revealing is equal to the arrival rate of the first success.

Characterizing firm $j$'s strategy via $x_t^j$ (where $t \geq 0$) has the advantage that it describes firm $j$'s behavior from the perspective of the rival firm $-j$ and only specifies what is relevant for firm $-j$. One possible interpretation such astrategy (under the additional assumption $x_t^j \leq \lambda$) is that firm $j$ randomizes over patenting the innovation that arrives at time $t$: It patents the newly arrived innovation with the probability $x_t^j/\lambda$, and never patents it otherwise. We will provide other possible interpretations later when discussing specific equilibria involving mixed strategies. Note that $x_t^j$ may also be higher than $\lambda$ and even infinite, if firm $j$ decides to patent an older success.[5] However, as will be discussed later, under the equilibrium refinement that we restrict our attention to, the value of $x_t^j$ does not exceed $\lambda$.

Before any product is patented, each firm's success is considered its private information. Let us denote $p_t^j$ the belief of firm $-j$ about firm $j$ having a breakthrough conditioned on the fact that $j$ has not patented anything. This specification yields to a non-trivial dynamics of beliefs. While the unconditional probability of having a success simply increases over time, there is a countervailing effect on the conditional belief $p_t^j$ due to the fact that firm $j$ did not patent yet. Intuitively, if firm $j$ chooses secrecy all the time, not observing the second patent makes it less likely that firm $j$ indeed has an innovation. If firm $j$ patents its first innovation with a certain probability, not observing the first patent makes it even less likely that firm $j$ indeed has an innovation. In the extreme case, where firm $j$ patents its first innovation immediately, the rival is certain that firm $j$ has no breakthrough if it

[5]The case $x_t^j = +\infty$ means that the probability of firm $j$ patenting in the time interval $[t, t + \Delta t]$ is an arbitrarily large multiple of $\Delta t$ as $\Delta t \to 0$.

has not patented anything.

At time $t = 0$ the belief is $p_0^j = 0$ as firms start with no innovation with certainty. As time progresses, both firms update their beliefs in accordance with Bayes' law. The posterior belief is then governed by the well known law of motion specified in the following lemma.

**Lemma 1.** *Whenever $x_t^j$ is finite, the posterior belief $p_t^j$ follows the law of motion*

$$\dot{p}_t^j = (1 - p_t^j)(\lambda - \mu p_t^j - x_t^j). \tag{3}$$

The formula reflects our intuition about the conditional and unconditional probability. The unconditional probability follows the Poisson process with hazard rate $\lambda$, which satisfies the law of motion $\dot{p}_t^j = (1 - p_t^j)\lambda$. This law of motion corresponds to having no second innovation ($\mu = 0$) and never patenting the first innovation ($x_t^j = 0$). On our case, the hazard rate is lower by $\mu p_t^j$ when conditioned on the fact the firm $j$ has not achieved its second innovation and by additional $x_t^j$ when conditioned by the fact that firm $j$ did not patent its first innovation.

In the extreme case, where firm $j$ always instantly patents its first innovation ($x_t^j = \lambda$), the rival $-j$ also knows with certainty that firm $j$ had no breakthrough, if it did not patent anything. Thus, the belief is constant and equal to zero ($p_t^j = 0$). On the other hand, if firm $j$ never patents its first innovation, the arrival rate of the first patent is zero ($x_t^j = 0$) and the belief follows the law of motion $\dot{p}_t^j = (1 - p_t^j)(\lambda - \mu p_t^j)$. Together with the initial condition $p_0^j = 0$ we obtain the solution $p_t^j = 1 - (\mu - \lambda)/\big(\mu - \lambda e^{-(\mu-\lambda)t}\big)$ when $\mu > \lambda$ and solution $p_t^j = 1 - 1/(1 + \lambda t)$ when $\mu = \lambda$. The belief is then increasing over time and converges to $\lambda/\mu$, which is smaller than 1, in the former case and to 1 in the latter case, as $t \to \infty$.

Finally, let us point out that Instant Patenting is the policy that maximizes the arrival rate of new innovations and patents. Recall that the first innovation arrives with the rate $2\lambda$ for each strategy profile. This innovation is patented immediately under Instant Patenting and only with the second innovation under Full Secrecy. After the first innovation either invents the second invention with hazard rate $\frac{1}{2}\lambda$ or with hazard rate $\mu$. Either way, the total rate at which the second invention is being made is no more than $2\mu$, which is the arrival rate of the second innovation (and patent) under Instant Patenting. The following lemma formalizes those arguments. Recall that dominance in the hazard rate order implies first order stochastic dominance (see, for instance, Theorem 1.B.1 in Shaked and Shanthikumar, 2007).

**Lemma 2.** *Consider an arbitrary strategy profile and denote $\tau_k$ the time of arrival of the k-th patent, where $k \in \{1,2\}$. Denote $\underline{\tau}_k$ and $\bar{\tau}_k$ the corresponding random variables under Instant Patenting and under Full Secrecy. Then $\bar{\tau}_k$ dominates $\tau_k$ and $\tau_k$ dominates $\underline{\tau}_k$ in the hazard rate order for both $k \in \{1,2\}$.*

# 3 Equilibrium

We are interested in symmetric *Markov perfect Bayesian Nash equilibria* (henceforth, equilibria) of the game. The firms condition their actions on the payoff-relevant state, which (for firm $j$) is the posterior posterior belief $p_t^{-j}$. Specifically, the Markov property requires that the firm's strategy $x_t^j$ (where $x_t^j \in \mathbb{R}_+$ for all $t \geq 0$) satisfies $x_{t_1}^j = x_{t_2}^j$ whenever $p_{t_1}^j = p_{t_2}^j$.[6] Note that if the firms follow the same strategy, the beliefs about both firms having an innovation are the same, i.e., $p_t^A = p_t^B$. Since we consider only symmetric equilibria, we drop the index $j$.

The Markov property guarantees that the belief $p_t$ is increasing in equilibrium. Indeed, discrete drops of $p_t$ are not possible in a game where firms have incentive to pre-empt each other (since $\Pi^{10} > \Pi^{01}$). Moreover, continuous decrease of $p_t$ can be excluded by the argument that otherwise firms would need to act differently at different times with equal value of the posterior belief. Thus, conditioning the belief on the fact that the firm did not patent yet, preserves the monotonicity. However, unlike the unconditional probability (which converges to 1 when $t \to \infty$), the steady-state value of the belief is smaller than 1, except for the case of Full Secrecy when $\mu = \lambda$.

As noted earlier, two natural candidates for equilibria are an *Instant Patenting*, where firms patent any innovation instantly, and a *Full Secrecy*, where firms always choose secrecy. We first focus on Instant Patenting, where firms patent their first innovation immediately.[7] The following proposition provides a sufficient and necessary condition when such an equilibrium exists.

[6] Representing firm $j$'s stopping by a finite $x_t^j$ is without loss of generality. Indeed, thanks to the fact $\Pi^{10} > \Pi^{01}$, the firms aim to pre-empt each other in patenting. In a symmetric equilibrium any large value of $x_{t_0}^j$ at some $t_0 > 0$ would incentivize its rival $-j$ to hurry up patenting by increasing $x_t^{-j}$ at some $t < t_0$.

[7] It is worth noting that in equilibrium instant patenting at time $t$ is possible only if $p_t = 0$. Otherwise it would correspond to $x_t = +\infty$ because all the past innovations would be patented at $t$.

**Proposition 1.** *The game has an Instant Patenting equilibrium if and only if*

$$\alpha \cdot (r+\mu)(\mu - \tfrac{1}{2}\lambda) + \beta \cdot r(r+\lambda+\mu) \geq \mu(\mu-\lambda). \tag{4}$$

Condition (4) ensures that patenting is incentive compatible, that is, no firm has incentives to deviate from patenting a success by reverting to secrecy. As we show in the proof, it is sufficient to verify that a deviation to Full Secrecy (i.e., never patenting the first innovation) is not profitable. Condition (4) is then obtained by comparing the corresponding continuation values.

Observe that condition (4) involves a linear combination of the flow profits $\alpha$ and $\beta$, with both coefficients (determined by the parameters $r$, $\lambda$, and $\mu$) positive. Thus, Instant Patenting is an equilibrium if and only if the combined values of $\alpha$ and $\beta$ exceed a specific threshold; see Figure 2 for an illustration. The intuition for this result is as follows. Patenting secures the firm's innovation for commercialization, with the sole disadvantage of increasing the hazard rate at which rivals can invent competing products. Therefore, patenting increases the chance of receiving flow profit $\alpha$ or $\beta$, but because of spillovers decreases the chance of receiving flow profit 1 (the monopoly profit) associated with having both patents. Consequently, the firm is more willing to patent when the flow profits $\alpha$ and $\beta$ are higher.

In the following proposition we assume that the Instant Patenting is not an equlibrium, i.e., (4) does not hold. The proposition postulates that Full Secrecy is an equilibrium when the rewards $\alpha$ and $\beta$ are small. However, there might be a range of parameters, where neither Instant Patenting nor Secrecy equilibrium exist. In such a case the equilibrium involves mixing and the proposition also provides a characterization of such an equilibrium.

**Proposition 2.** *Assume that the game does not have an Instant Patenting equilibrium. Then the equilibrium is unique. The equilibrium is a Full Secrecy equilibrium if*

$$\alpha \cdot [\tfrac{1}{2}\lambda(r+\mu) + \mu^2] + \beta \cdot r(\mu - \tfrac{1}{2}\lambda) \leq \tfrac{1}{2}\lambda\mu + \frac{\mu^2(\mu-\lambda)}{r+\lambda+\mu}. \tag{5}$$

*Otherwise, the equilibrium is a* Delayed Patenting *equilibrium, in which firms choose secrecy until some time $T_D > 0$ and then patent the first innovation with a constant hazard rate.*

Figure 2 illustrates the results of Propositions 1 and 2. The parameter space $(\alpha, \beta) \in [0, \frac{1}{2}] \times [0, 1]$ is split into three regions: a region in which Instant Patenting is an equilibrium (and dominates potential other equilibria); a region in which only Full Secrecy is an equilibrium; and a region in which only Delayed Patenting equilibrium exists.

Condition (5) in the proposition guarantees that maintaining secrecy is incentive compatible and no firm has incentives to deviate by patenting its first innovation. As we show in the proof of the proposition, it is sufficient when the incentive compatibility is satisfied at the beginning (with belief $p = 0$) and at the secrecy steady state (where the belief is $p = \lambda/\mu$). This follows from the fact that both the continuation value of sticking to secrecy as well as deviating to patenting are linear in the posterior $p$, as $p$ weights the probabilities of the outcomes, but has no impact on firms' decisions. Moreover, the condition at $p = 0$ is trivially met, when Instant Patenting is not an equilibrium. The argument is based on the game's pre-emption incentive structure; if a firm lacks the incentive to patent when its rival does so, then it has even less incentive to patent in an equilibrium where the rival opts for secrecy. Thus, verifying that no deviation is profitable in the steady state (i.e., for $p = \lambda/\mu$) is indeed sufficient. The condition in the proposition is then obtained by comparing the corresponding the corresponding continuation values.

Condition (5) for the existence of the Full Secrecy equilibrium again involves a linear combination of flow profits with positive coefficients. Thus, Full Secrecy is an equilibrium, when the flow profits are small. The intuition mirrors the one for the existence of Instant Patenting equilibrium. By sticking to secrecy, a firm lowers the chance of receiving the flow profits $\alpha$ and $\beta$, but by reducing the spillovers increases the chance of winning the monopoly flow profit of 1. Therefore, the firms tend towards secrecy when the flow profits $\alpha$ and $\beta$ are small.

The second part of the proposition states that there might be yet another equilibrium, which we call Delayed Patenting equilibrium. In this equilibrium, the firms start with secrecy. The corresponding posterior belief increases over time and once it attains a certain threshold, denoted $p_D$, at time $T_D$, the firms start to randomize between patenting and secrecy. The hazard rate of patenting then jumps to $x_D = \lambda - \mu p_D$, keeping the rival indifferent between patenting and secrecy. After time $T_D$ the hazard rate $x_D$ remains constant, which maintains the posterior belief fixed at the value $p_D$. Figure 1 illustrates how the patenting hazard rate and the posterior belief evolve over time in the Delayed Patenting equilibrium.

Note that firm $j$'s strategy in the Delayed Patenting equilibrium can be represented by various stopping strategies, all leading to the same behavior from the rival's perspective (the same $x_t^j$). One possible representation of the strategy is that firm $j$ patents innovations arriving after time $T_D$ immediately with probability $x_D/\lambda$, or not at all. Another representation involves firm $j$ postponing the patenting of any innovation it produces by exactly $T_D$.

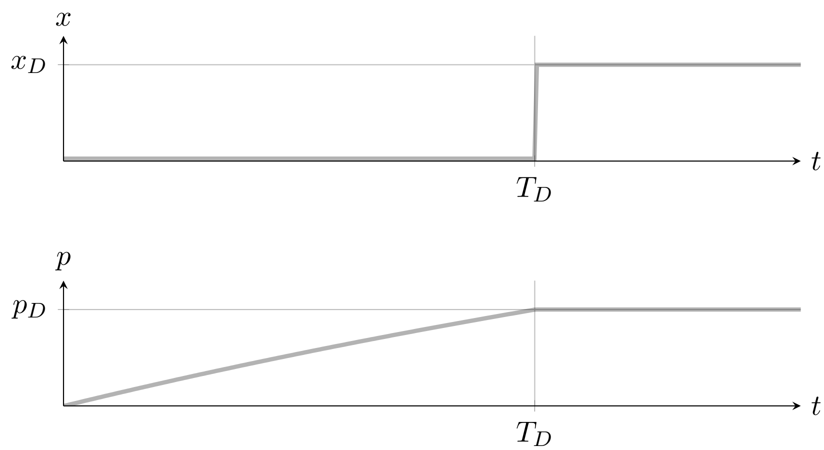


Figure 1: Firm's strategy ($x_t$) and belief ($p_t$) in Delayed Patenting equilibrium (example for $\lambda = r = 1$, $\mu = 2$, $\alpha = 0.38$, $\beta = 0.05$, which yields $T_D \approx 0.128$, $x_D \approx 0.786$, $p_D \approx 0.107$)

We conclude the analysis of the equilibria in the general case by comparative statics with respect to the intertemporal parameters $r$, $\lambda$, and $\mu$. The following corollary shows that the tendency to patent increases with both $r/\mu$ (impatience) and $\lambda/\mu$ (inverse spillovers).

**Corollary 1.** *Consider a fixed pair $(\alpha, \beta) \in \left[0, \frac{1}{2}\right] \times [0, 1]$. Let $(r_1, \lambda_1, \mu_1)$ and $(r_2, \lambda_2, \mu_2)$ be two tripples of parameters such that $r_1/\mu_1 \leq r_2/\mu_2$ and $\lambda_1/\mu_1 \leq \lambda_2/\mu_2 \leq 1$.*

*(i) If Instant Patenting is equilibrium under $(r_1, \lambda_1, \mu_1)$, then Instant Patenting is also equilibrium under $(r_2, \lambda_2, \mu_2)$.*

*(ii) If Full Secrecy is the unique equilibrium under $(r_2, \lambda_2, \mu_2)$, then Full Secrecy is also the unique equilibrium under $(r_1, \lambda_1, \mu_1)$.*

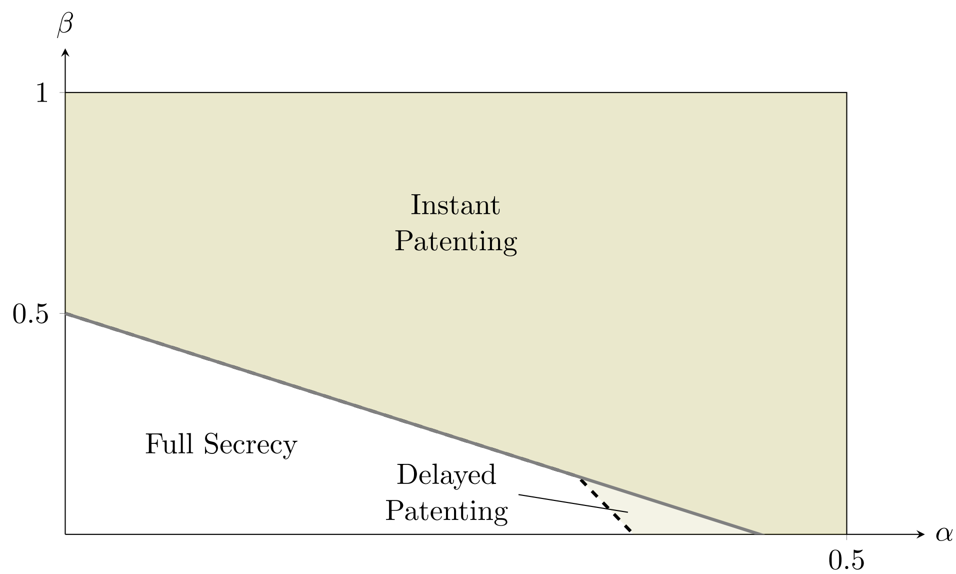


Figure 2: Types of equilibria (for $r = \lambda = 1$ and $\mu = 2$).

# 4 Complements vs. substitutes

Equipped with the general characterization of equilibria, in this section we compare the patenting behavior for perfect complements and perfect substitutes. The motivation behind this comparison is to provide a possible theoretical justification for the results of (Cohen et al., 2000) on simple and complex industries. We associate those two cases with goods being substitutes and complements, respectively. In the model we focus on the extreme cases of perfect substitutes and perfect complements.

If the goods are perfect substitutes, even a single patent yields the monopoly profit, i.e., $\beta = 1$, while each firm having one patent yields to competition. The flow profit $\alpha$ depends on the form and the intensity of competition: we obtain $\alpha = 0$ under price competition, as described by the Bertrand model, and $\alpha = \frac{4}{9}$ under quantity competition, as described by the linear Cournot model.[8] In the following analysis focuses on the case of price competition, in Section 5 we also present results for the quantity competition.[9]

On the other hand, if the goods are perfect complements, there is no profit from having a single patent, i.e., $\beta = 0$. If each firm has one patent, we assume that the firms create a patent pool or joint venture (see, for instance, Kamien et al., 1992; Lerner and Tirole, 2004; Choi, 2010) and share the monopoly profit, i.e., $\alpha = \frac{1}{2}$.[10]

The following two propositions provide a characterization of equilibria for perfect substitutes and perfect complements. Since both conditions (4) and (5) are homogeneous in $(r, \lambda, \mu)$, we can rewrite them as functions of the ratios $\lambda/\mu$ and $r/\mu$.[11] This reduces the dimensionality of the parameter space by 1 and allows us to illustrate the results in a picture with $r/\mu$ and $\lambda/\mu$ on the axes. The ratio $\lambda/\mu$ represents the inverse spillovers; by assumption $\lambda/\mu \leq 1$. The ratio $r/\mu$ can be described as impatience or as inefficiency of R&D for the second breakthrough.

[8] This value corresponds to the textbook case with linear demand function ($P(Q) = a - bQ$) and linear costs functions ($C(q) = cq$), which yields profit $(a-c)^2/(4b)$ in monopoly and profit $(a-c)^2/(9b)$ in the Cournot-Nash equilibrium.

[9] Let us point out that for $\alpha = 0$, once a firm patents the first innovation, the rival attains zero profit independently on who patents the second innovation. In a model with a costly effort (or investment), this would discourage the rival from R&D. In our model R&D is costless and we may interpret it as a limit case when $\alpha$ is small.

[10] When the products are independent, each firm earns monopoly profit from a product after patenting it, thus, $\alpha = \beta = \frac{1}{2}$. Consequently, under imperfect substitutes we would have a continuous transition from the case of perfect substitutes with $\alpha = \hat{\alpha} \in [0, \frac{1}{2}]$ and $\beta = 1$ to the case of independent products with $\alpha = \beta = \frac{1}{2}$. Similarly, under imperfect complements, we would have $\alpha = \frac{1}{2}$ and $\beta = \hat{\beta} \in [0, \frac{1}{2}]$, since the firms still can create a patent pool, but also benefit from a single product on the market.

[11] Formally, this can also be represented by choosing the units of time so that the arrival of the second innovation is normalized to 1.

**Proposition 3.** *Under perfect substitutes ($\alpha = 0$ and $\beta = 1$), Instant Patenting equilibrium exists if and only if $\lambda/\mu \geq \Lambda^s_{IP}(r/\mu)$, where*

$$\Lambda^s_{IP}(\xi) = \frac{1}{1+\xi} - \xi. \tag{6}$$

*Otherwise, Full Secrecy is the unique equilibrium.*

**Proposition 4.** *Under perfect complements ($\alpha = \frac{1}{2}$ and $\beta = 0$), Instant Patenting equilibrium exists if and only if $\lambda/\mu \geq \Lambda^c_{IP}(r/\mu)$, where*

$$\Lambda^c_{IP}(\xi) = 2 - \frac{4}{3-\xi}. \tag{7}$$

*Otherwise, there is a unique equilibrium: Full Secrecy equilibrium if $\lambda/\mu \leq \Lambda^c_{FS}(r/\mu)$, and Delayed Patenting equilibrium otherwise, where*

$$\Lambda^c_{FS}(\xi) = \frac{5+\xi^2 - \sqrt{(5+\xi^2)^2 - 8(1-\xi)^2}}{2(1-\xi)}. \tag{8}$$

Both propositions follow from Propositions 1 and 2 by substituting the appropriate values of $\alpha$ and $\beta$, with an additional result on the existence of the Delayed Patenting equilibrium. The functions $\Lambda^s_{IP}$, $\Lambda^c_{IP}$, and $\Lambda^c_{FS}$ are obtained by solving the corresponding inequalities for $\lambda/\mu$.[12]

Figure 3 illustrates the results of Propositions 3 and 4 and indicates the comparison of equilibria under perfect complements and perfect substitutes. The parameter space $(r/\mu,\ \lambda/\mu) \in (0, +\infty) \times (0, 1]$ is divided into four regions. First, if the spillovers are small ($\lambda/\mu$ is large) and firms are impatient (or R&D for the second innovation is inefficient, i.e., $r/\mu$ is large), then the firms patent instantly regardless of whether the products are complements or substitutes. Second, if there are significant spillovers and the firms are patient, then they choose Full Secrecy regardless of the product synergies. Third, there is a region (indicated on the picture in red), with large spillovers (small $\lambda/\mu$) and intermediately patient firms, where the firms patent instantaneously under perfect substitutes, but choose Delayed Patenting or Full Secrecy under perfect complements. Fourth, if the spillovers are intermediate and firms are patient (indicated by the blue region), we obtain Full Secrecy under perfect substitutes, and Instant or Delayed Patenting under perfect

[12]Interestingly, the intersection of the curve $\Lambda^s_{IP}(\xi)$ with the horizontal axis is $\xi = \frac{1}{2}(-1+\sqrt{5})$, which is the inverse golden ratio.

complements. In the first and the second case, the rate of patenting is the same under both market structures. In the third case (red region) we obtain faster arrival of patenting under perfect substitutes. In the fourth case (blue region) we obtain faster arrival of patents under perfect complements.

Let us now define the following threshold values for $r/\mu$: let $\underline{\xi} \approx 0.25410$ be the solution of $\Lambda^s_{IP}(\xi) = \Lambda^c_{IP}(\xi)$ on $[0,1]$, and let $\bar{\xi} \approx 0.47283$ be the unique solution of $\Lambda^s_{IP}(\xi) = \Lambda^c_{FS}(\xi)$ on $[0,1]$. The following proposition and corollary formalize the above argument and provide a comparison of firms' tendencies to patent under the two extreme scenarios.

**Proposition 5.** *Let $r$, $\lambda$, and $\mu$ such that $r > 0$, $\mu \geq \lambda > 0$. Then the following statements hold:*

*(i) The existence of Instant Patenting equilibrium under perfect substitutes implies its existence under perfect complements if and only if $\lambda/\mu \leq \Lambda^s_{IP}(r/\mu)$ or $\lambda/\mu \geq \Lambda^c_{IP}(r/\mu)$.*

*(ii) If $r/\mu \leq \underline{\xi}$, then the existence of Instant Patenting equilibrium under perfect substitutes implies its existence under perfect complements. Otherwise, the reverse implication holds.*

*(iii) If $r/\mu \geq 1$, then Instant Patenting equilibrium exists under both scenarios.*

*(iv) If $r/\mu \leq \bar{\xi}$, then the existence of Secrecy equilibrium under perfect complements implies the existence of Secrecy equilibrium under perfect substitutes. Otherwise, the reverse implication holds.*

**Corollary 2.** *Assume that the Instant Patenting equilibrium is selected whenever it exists. Then the time of patent arrival of the first and the second patent under perfect substitutes dominates the corresponding time of patent arrival under perfect complements in the hazard rate order if and only if the condition from Proposition 5 (i) holds.*

Statement (i) identifies the region of parameters where Instant Patenting equilibrium under perfect substitutes is a subset of the region with Instant Patenting equilibrium under perfect substitutes. On the picture this corresponds to parameter space excluding the red region. Statements (ii)–(iv) provide sufficient conditions for the comparison on equilibria. Corollary 2 then compares the patenting time. It follows from Lemma 2, since the arrival times of patents in the Instant Patenting equilibrium does not depend on the market structure (i.e., on parameters $\alpha$ and $\beta$). The same holds for the Full Secrecy equilibrium. Based on Proposition 4, the comparison boils down to comparing the equilibrium arrival

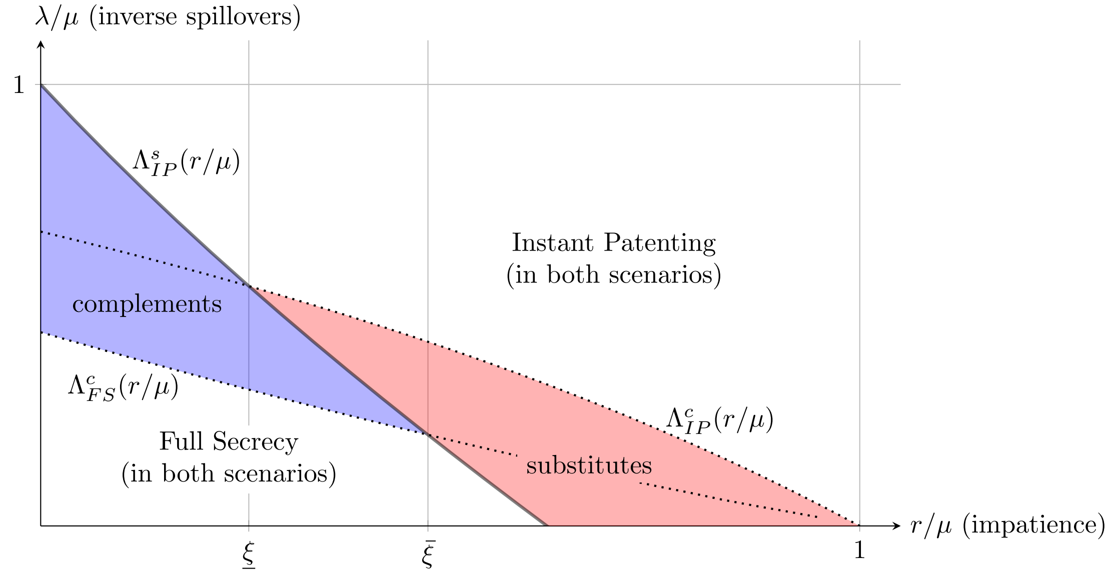


Figure 3: Comparison of patenting under perfect complements ($\alpha = \frac{1}{2}$, $\beta = 0$) and perfect substitutes with price competition ($\alpha = 0$, $\beta = 1$).

time of patents under perfect complements to arrival time under Instant Patenting or Full Secrecy.

# 5 Discussion

Our theoretical model partially conforms to the conclusions by Cohen et al. (2000). We show that the firms have (weakly) more tendency towards patenting under perfect complements than under perfect substitutes for small spillovers and intermediate range of impatience relative to the efficiency of R&D in the second stage. On the contrary, there is also a region of parameters where we obtain more patenting under perfect substitutes.

Our results rely in several important assumptions. First of all, we assume that the R&D process is exogenous. This allows us to disentangle the incentives for patenting from the R&D incentives. Assuming that the arrival rates $\lambda$ and $\mu$ can be determined by effort or investment (like in Gordon, 2011 or Kocourek and Kováč, 2023). The firms may then use patenting in order to discourage the investment, in particular when the additional gain from second innovation is small.

Second, in the comparison, we consider two extreme cases: the case of perfect complements with firms forming a patent pool (or joint venture); and the case of perfect substitutes with firms competing in prices as in the Bertrand model (where $\alpha = 0$). However, the assumption of Bertrand duopoly might be too extreme. Under less intense competition, the value of $\alpha$ would be higher, increasing the profitability of patenting. Consequently, the curve $\Lambda^s_{IP}(r/\mu)$ would shift inwards. Then we may obtain another region, where Delayed Patenting equilibrium exists. For instance, consider the case where the firms compete in quantities as in Cournot duopoly, where $\alpha = \frac{4}{9}$ (see footnote 8). Figure 4 illustrates the comparison of tendency to patent under perfect substitutes versus perfect complements. The new function $\Lambda^s_{FS}(\xi)$ denotes the boundary between Full Secrecy equilibrium and Delayed Patenting equilibrium under perfect substitutes.

Finally, let us point out that the results of Cohen et al. (2000) rely on cross-sectional data. In our comparisons we compare industries *ceteris paribus*. However, not all combinations of parameters are equally likely. For instance, it might be that simple or complex industries tend to be associated with different arrival rates of innovations or different levels of spillovers. In that case only a subset of the parameter space may be relevant making our weakening or strengthening our conclusions.

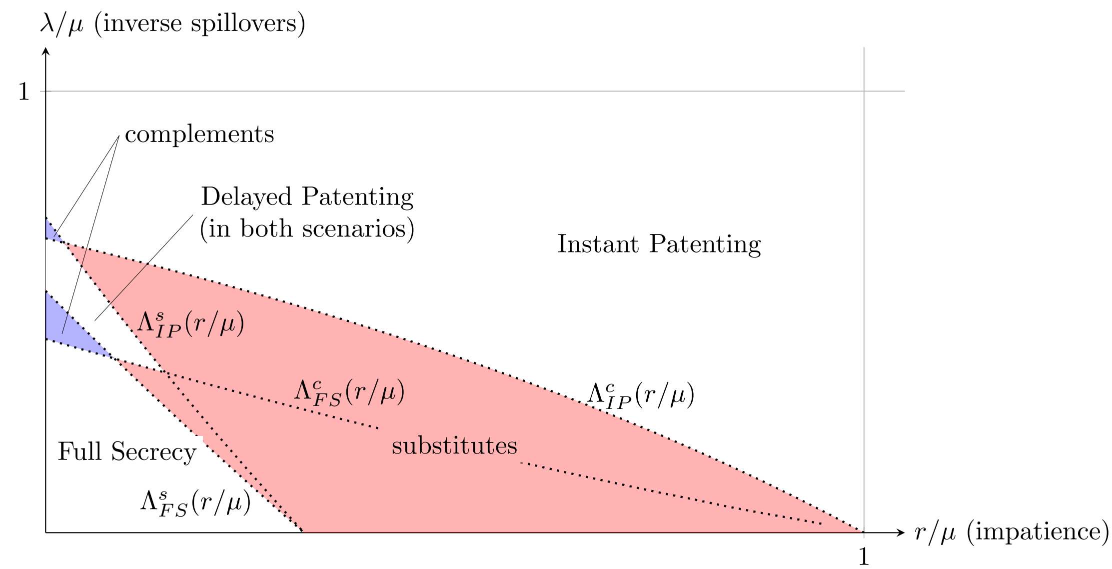


Figure 4: Comparison of patenting under perfect complements ($\alpha = \frac{1}{2}$, $\beta = 0$) and perfect substitutes and quantity competition ($\alpha = \frac{4}{9}$, $\beta = 1$).

# A Appendix

## A.1 Continuation values

Based on the discussion in Section 2, we obtain

$$\begin{aligned}
\Pi^{10} &= \int_0^\infty \mu e^{-\mu t} \left\{ \int_0^t \mu e^{-\mu s} [e^{-rs} \cdot 1 + (1 - e^{-rs})/r \cdot \beta] \, ds + e^{-\mu t} [e^{-rt} \cdot \alpha + (1 - e^{-rt})/r \cdot \beta] \right\} dt \\
&= \int_0^\infty \mu e^{-\mu t} \cdot \frac{\mu(1 - e^{-(r+\mu)t}) + [r + \mu e^{-(r+\mu)t} - (r+\mu) e^{-\mu t}]/r \cdot \beta}{r + \mu} \\
&\qquad + \mu e^{-2\mu t} [e^{-rt} \cdot \alpha + (1 - e^{-rt})/r \cdot \beta] \, dt \\
&= \frac{r \cdot \beta + \mu \cdot 1 + \mu \cdot \alpha}{r + 2\mu}
\end{aligned}$$

and

$$\begin{aligned}
\Pi^{01} &= \int_0^\infty \mu e^{-\mu t} \int_0^t \mu e^{-\mu s} e^{-rs} \cdot \alpha \, ds \, dt \\
&= \int_0^\infty \mu e^{-\mu t} \cdot \frac{\mu(1 - e^{-(r+\mu)t})}{r + \mu} \, dt \\
&= \frac{\mu \cdot \alpha}{r + 2\mu}.
\end{aligned}$$

## A.2 Proofs for Section 3 (Main Results)

Consider any symmetric strategy profile of the players and suppose that firm $j$ patents a product at time $t \geq 0$ and it the first patent made. Let $p$ be $j$'s posterior belief about its rival $-j$ having an invention at time $t$. With probability $1 - \frac{1}{2}p$ the rival has not invented the other product prior to $t$, and so firm $j$'s continuation value after patenting is $\Pi^{10}$. Otherwise, with probability $\frac{1}{2}p$, the rival has invented the other product by time $t$, and observing firm $j$ patenting, it instantly patents its invention as well, thus the continuation value is $\Pi^{11} = \alpha$. Taking expectation over the two scenarios, we conclude that patenting yields continuation value $V^P(p)$ to player $j$, where

$$\begin{aligned}
V^P(p) &= \tfrac{1}{2}p \cdot \Pi^{11} + \left(1 - \tfrac{1}{2}p\right) \cdot \Pi^{10} \\
&= \tfrac{1}{2}p \cdot \alpha + \left(1 - \tfrac{1}{2}p\right) \cdot \frac{\beta r + (1+\alpha)\mu}{r + 2\mu}.
\end{aligned}$$

*Proof of Proposition 1.* In an Instant Patenting Equilibrium each firm has the belief that its rival has not made an invention (i.e., $p = 0$). Accordingly, until one of the firms patents, the game is static. For the equilibrium to exist, none of the firms can be tempted to choose secrecy. Since the game is static, the incentive compatibility can be expressed as that delaying patenting of an invention by $\Delta t$ is not profitable:

$$(r + \lambda + \mu) \cdot \Pi^{10} \geq r \cdot 0 + \lambda \cdot \left(\tfrac{1}{2}\Pi^{01} + \tfrac{1}{2}\alpha\right) + \mu \cdot 1.$$

The left hand side of the above inequality is the opportunity cost of delaying patenting (per $\Delta t$), and its right hand side is the payoff flow of from maintaining secrecy. After dividing by $\mu$ and substituting in the values of $\Pi^{10}$ and $\Pi^{01}$, we obtain

$$(r + \lambda + \mu) \cdot \frac{\beta\, r + (1+\alpha)\mu}{r + 2\mu} \geq \lambda \cdot \left(\frac{\alpha\mu}{2(r + 2\mu)} + \tfrac{1}{2}\alpha\right) + \mu.$$

Multiplying the inequality by $r + 2\mu$ yields

$$(r + \lambda + \mu) \cdot \left(\beta\, r + (1+\alpha)\mu\right) \geq \tfrac{1}{2}\alpha\lambda\,(r + 3\mu) + \mu(r + 2\mu),$$

which can be rearranged into (4). □

Before proceeding with the proof of Proposition 2, we state and prove the following lemma.

**Lemma 3.** *Assume that the game does not have an Instant Patenting Equilibrium. Then it has Secrecy Equilibrium if and only if the inequality* (5) *is satisfied.*

*Proof of Lemma 3.* Define the continuation value $V^S(p)$, where $p \in [0, 1]$, of player $j$ who has made a single invention in the scenario in which both firms follow the full secrecy strategy. In this scenario, each of the firms postpones patenting until it has made both inventions. Provided that the firms follow the full secrecy strategy, all player actions are fully determined, and so information has no additional value. In particular, firm's continuation value would not change if it was about to learn its rival's actual state in the next instance. That allows us to express $V^S(p)$ as the expected value over rival's state become 1 or 0:

$$V^S(p) = pV^S(1) + (1 - p)V^S(0).$$

The continuation values in the extreme states can be evaluated by an analogous computation as in Appendix A.1 as:

$$V^S(1) = \frac{r \cdot 0 + \mu \cdot 1 + \mu \cdot 0}{r + \mu + \mu} = \frac{\mu}{r + 2\mu},$$

and

$$V^S(0) = \frac{r \cdot 0 + \lambda \cdot V^S(1) + \mu \cdot 1}{r + \lambda + \mu} = \frac{\lambda \cdot V^S(1) + \mu}{r + \lambda + \mu} = \frac{(r + \lambda + 2\mu)\mu}{(r + \lambda + \mu)(r + 2\mu)}.$$

In conclusion,

$$V^S(p) = \frac{[r + \lambda + (2 - p)\mu]\mu}{(r + \lambda + \mu)(r + 2\mu)}.$$

Notice that $V^S(p)$ does not depend on $\alpha$ or $\beta$, because in the Secrecy Equilibrium either no patent is made, or one firm patented both products.

Under Full Secrecy, the posterior belief $p_t$ of one player about its rival starts at 0 and it asymptotically approaches its steady-state value $p^* = \lambda/\mu$. The necessary and sufficient condition for the Full Secrecy equilibrium to exist is that no player is ever tempted to patent, i.e. $V^P(p) \leq V^S(p)$ for all $p \in [0, p^*)$. Since both functions $V^S$ and $V^P$ are linear, the condition only needs to be verified at the endpoints of the interval, $p = 0$ and $p = p^*$. At $p = p^*$, the incentive compatibility condition is

$$\frac{\lambda}{2\mu} \cdot \alpha + \left(1 - \frac{\lambda}{2\mu}\right) \cdot \frac{\beta r + (1 + \alpha)\mu}{r + 2\mu} \leq \frac{\mu}{(r + \lambda + \mu)}.$$

Multiplying the inequality by $\mu(r + 2\mu)$, we obtain

$$\tfrac{1}{2}\alpha\lambda(r + 2\mu) + \left(\mu - \tfrac{1}{2}\lambda\right)\left(\beta r + (1 + \alpha)\mu\right) \leq \frac{\mu^2(r + 2\mu)}{(r + \lambda + \mu)},$$

which after rearranging terms gives us the inequality (5).

It remains to verify that the incentive compatibility condition is satisfied at $p = 0$, i.e. that

$$\frac{\beta r + (1 + \alpha)\mu}{r + 2\mu} \leq \frac{(r + \lambda + 2\mu)\mu}{(r + \lambda + \mu)(r + 2\mu)}.$$

Multiplying by $(r + 2\mu)$ and subtracting $\mu$ from both sides of the inequality,

$$\alpha\mu + \beta r \leq \frac{\mu^2}{r + \lambda + \mu}. \tag{9}$$

This inequality is not generally satisfied (consider for example $\beta = r = 1$ and $\mu$ small).

However, knowing that the game does not have Instant Patenting equilibrium, by Proposition 1 the inequality (4) must be violated, and so

$$\alpha \cdot (r + \mu)\left(\mu - \tfrac{1}{2}\lambda\right) + \beta \cdot r(r + \lambda + \mu) < \mu(\mu - \lambda).$$

A direct conclusion of this inequality (considering that $\beta \geq 0$ and $\mu - \frac{1}{2}\lambda < \mu - \lambda$) is that $\alpha \cdot (r + \mu) < \mu$. Adding $\frac{1}{2}\lambda$ multiple of the newly obtained inequality back to the last inline inequality and adding $\frac{1}{2}\lambda\mu$ to both of its sides, we obtain

$$\alpha \cdot \mu(r + \lambda + \mu) + \tfrac{1}{2}\lambda\mu(1 - 2\alpha) + \beta \cdot r(r + \lambda + \mu) < \mu^2.$$

Dividing the inequality by $(r + \lambda + \mu)$, and utilizing thaat $1 - 2\alpha \geq 0$, we obtain (9). $\square$

*Proof of Proposition 2.* Define $D(p, x)\Delta t$ to be the benefit from delaying patenting by $\Delta t$ for a firm whose rival has invention with probability $p$ and patents it with hazard rate $x \geq 0$:

$$D(p, x) = x \cdot \left(\tfrac{1}{2}\Pi^{01} + \tfrac{1}{2}\Pi^{11}\right) + \mu \cdot 1 - (r + x + \mu + p\mu) \cdot V^P(p) + (1 - p)(\lambda - p\mu - x) \cdot (V^P)'(p)$$

Recall that $V^P$ is a linear function, and so $(1 - p)(V^P)'(p) = V^P(1) - V^P(p)$. Thus,

$$D(p, x) = \tfrac{1}{2}x\left(\Pi^{01} - \Pi^{10}\right) + \mu - \Big(r + \lambda + \mu\Big)V^P(p) + (\lambda - p\mu)V^P(1).$$

Notice that the term multiplying $x$ is negative and its value corresponds to the fact that if the rival patents in the meantime whilst the firm postpones patenting, then the firms have the same innovation with 50% chance, in which case the firms aim to preempt each other, and otherwise they have different innovations, in which case it is irrelevant who patents first. We conclude, that the higher the rival firm's rate of patenting is, the higher is firm's incentive to patent early.

Recall that we assume that the game has no Instant Patenting equilibrium. Equivalently, $D(0, \lambda) > 0$, meaning that a firm would be tempted to delay patenting in the under Instant Patenting. Since the coefficient of $x$ is negative, we conclude that $D(0, x) \geq D(0, \lambda) > 0$ for any $x \leq \lambda$. It means that a firm has strict incentive to delay patenting regardless of the rival's behavior. There are two cases to distinguish. If the inequality (5) is satisfied, then by Lemma 3 there exists Secrecy Equilibrium, and accordingly $D(p^*, 0) \geq 0$. Since $D$ is linear in the vector $(p, x)$, we conclude that $D(p, x) > 0$ for all $p \in [0, p^*)$ and

$x \in [0, \lambda - p\mu]$.[13] If to the contrary the inequality (5) is violated, $D(p^*, 0) < 0$, and thus there exists $p_D \in (0, p^*)$ such that $D(p_D, \lambda - p_D\mu) = 0$. What is more, $D(p, x) > 0$ for all $p \in [0, p_D)$ and $x \in [0, \lambda - p\mu]$. It follows that for any $p < p_D$ players necessarily restrain to secrecy. Once the posterior $p_t$ reaches the value $p_D$, players might start patenting with the rate $x = \lambda - p_D\mu$, which maintains the posterior at the constant level and keeps the players indifferent from patenting. This is the Delayed Patenting equilibrium.

Finally, we show that the can not be any other equilibrium. We already know that secrecy is the only incentive compatible action whenever $p < p_D$, it remains to show that the posterior must stop at $p_D$. For contradiction suppose that the posterior grew further, all the way to some limit value $p^{**} > p_D$. Consequently, $x_t \to \lambda - p^{**}\mu$, and so $D(p_t, x_t) \to D(p^{**}, \lambda - p^{**}\mu) < 0$, meaning that the firm has a strict incentive to patent at this point. However, that would imply instant patenting in a situation with $p > 0$, which is not possible.[14] □

*Proof of Corollary 1.* Let us fix the values $\alpha$ and $\beta$ as in the corollary. Assume $\mu_1 = \mu_2 = 1$. Since the inequalities characterizing both Instant Patenting and Full Secrecy equilibria are homogeneous in $\mu$, the assumption is without loss of generality.

(i) Rearranging terms in the inequality (4), we obtain:

$$\alpha \cdot (r+1)\left(1 - \tfrac{1}{2}\lambda\right) + \beta \cdot r \cdot (r + \lambda + 1) - (1 - \lambda) \geq 0.$$

Define the function $I(r, \lambda)$ to represent the left-hand side of the inequality; its partial derivatives are:

$$\begin{aligned} I_r(r, \lambda) &= \alpha\left(1 - \tfrac{1}{2}\lambda\right) + \beta(2r + \lambda + 1), \\ I_\lambda\big(r, \lambda\big) &= -\tfrac{1}{2}\alpha\,(r+1) + \beta r + 1. \end{aligned}$$

The former is clearly positive. The latter is positive unless $\alpha(r+1) > 2\beta r + 2$, in which case $I\big(r, \lambda\big)$ is strictly positive. We conclude that the inequality $I(r_1, \lambda_1) \geq 0$ implies $I(r_2, \lambda_2) \geq 0$. Applying Proposition 1 the claim follows.

(ii) From part (i) of this corollary, it follows that if the game does not have Instant Patenting equilibrium under $(r_2, \lambda_2, \mu_2)$, then it also does not have Instant Patenting equi-

[13] Recall that the Markov condition implies that $\dot{p} \geq 0$, and so $x \leq \lambda - p\mu$.
[14] Recall that it would lead to $x_t = \infty$.

librium under $(r_1, \lambda_1, \mu_1)$. By Proposition 2 it only remains to show that if (5) is satisfied under $(r_2, \lambda_2, \mu_2)$, then it is also satisfied under $(r_1, \lambda_1, \mu_1)$.

Rearranging terms in the inequality (5), we obtain:

$$\alpha \cdot \left[\tfrac{1}{2}\lambda(r+1)+1\right] + \beta \cdot r\left(1-\tfrac{1}{2}\lambda\right) - \tfrac{1}{2}\lambda - \frac{1-\lambda}{r+\lambda+1} \leq 0.$$

Define the function $S(r, \lambda)$ to represent the left-hand side of the inequality; its partial derivatives are:

$$S_r(r,\lambda) = \tfrac{1}{2}\alpha\lambda + \beta\left(1-\tfrac{1}{2}\lambda\right) + \frac{1-\lambda}{(r+\lambda+1)^2},$$
$$S_\lambda(r,\lambda) = \tfrac{1}{2}\alpha(r+1) - \tfrac{1}{2}\beta r - \tfrac{1}{2} + \frac{r+2}{(r+\lambda+1)^2}.$$

The former is clearly positive. Moreover, it can be shown that there whenever the game has Instant Patenting equilibrium and $S(r, \lambda) = 0$, then $S_\lambda(r, \lambda) > 0$. Consequently, $S(r_2, \lambda_2) \leq 0$ implies $S(r_2, \lambda_1) \leq 0$ (otherwise there would need to exist $\lambda^* \in [\lambda_1, \lambda_2]$ such that $S(r_2, \lambda^*) = 0$ and $S(r_2, \lambda^*) < 0$), and since $S_r$ is positive, it also implies that $S(r_1, \lambda_1) \leq 0$. □

## A.3 Proofs for Section 4 (Complements vs. Substitutes)

*Proof of Proposition 3.* We apply Proposition 1 in which we substitute $(\alpha, \beta) = (0, 1)$. The IC condition (4) after being divided by $\mu^2$ simplifies into:

$$r/\mu \cdot (r/\mu + \lambda/\mu + 1) \geq 1 - \lambda/\mu.$$

Rearranging terms,

$$\lambda/\mu \cdot (1 + r/\mu) \geq 1 - r/\mu \cdot (r/\mu + 1),$$
$$\lambda/\mu \geq \frac{1}{1 + r/\mu} - r/\mu,$$

which is equivalent to

$$\lambda/\mu \geq \Lambda^s_{IP}(r/\mu).$$

It remains to show that if the inequality $\lambda/\mu \geq \Lambda^s_{IP}(r/\mu)$ is not satisfied, then the game has Full Secrecy equilibrium. We apply Proposition 2. Substituting $(\alpha, \beta) = (0, 1)$ in the inequality (5), and dividing it by $\mu^2$, we obtain

$$
\begin{aligned}
r/\mu\left(1-\tfrac{1}{2}\lambda/\mu\right) &\leq \tfrac{1}{2}\lambda/\mu + \frac{1-\lambda/\mu}{r/\mu+\lambda/\mu+1}, \\
(r/\mu+1)\left(1-\tfrac{1}{2}\lambda/\mu\right) &\leq \frac{2-r/\mu}{r/\mu+\lambda/\mu+1},
\end{aligned}
$$

which is satisfied whenever $\lambda/\mu < \Lambda^s_{IP}(r/\mu)$. □

*Proof of Proposition 4.* We apply Proposition 1 in which we substitute $(\alpha,\beta) = \left(\frac{1}{2},0\right)$. The IC condition (4) after being divided by $\mu^2$ simplifies into:

$$
\tfrac{1}{2}(r/\mu+1)\left(1-\tfrac{1}{2}\lambda/\mu\right) \geq 1-\lambda/\mu.
$$

Rearranging terms,

$$
\begin{aligned}
\lambda/\mu\cdot\left(\tfrac{3}{4}-\tfrac{1}{4}r/\mu\right) &\geq \tfrac{1}{2}-\tfrac{1}{2}r/\mu, \\
\lambda/\mu &\geq 2-\frac{4}{3-r/\mu}.
\end{aligned}
$$

Thus, the game has Instant Patenting Equilibrium if and only if $\lambda/\mu \geq \Lambda^c(r/\mu)$.

Next, we apply Proposition 2 in the situation that $\lambda/\mu < \Lambda^c(r/\mu)$ in order to determine whether the game has Full Secrecy equilibrium or Delayed Patenting equilibrium. We substitute $(\alpha,\beta) = \left(\frac{1}{2},0\right)$ in the condition (5) and divide it by $\mu^2$:

$$
\tfrac{1}{4}\lambda/\mu\cdot(r/\mu+1)+\tfrac{1}{2} \leq \tfrac{1}{2}\lambda/\mu + \frac{1-\lambda/\mu}{r/\mu+\lambda/\mu+1}.
$$

Multiplying the inequality by $(r/\mu+\lambda/\mu+1)$ and rearranging terms,

$$
\begin{aligned}
\left[\lambda/\mu\cdot\tfrac{1}{4}\left(r/\mu-1\right)+\tfrac{1}{2}\right]\left(\lambda/\mu+r/\mu+1\right) &\leq 1-\lambda/\mu, \\
(\lambda/\mu)^2\cdot(1-r/\mu)-\lambda/\mu\cdot\left[5+(r/\mu)^2\right]+4 &\geq 0.
\end{aligned}
$$

Note that under the assumption of the game having no Instant Patenting equilibrium $r/\lambda < 1$, as otherwise $\Lambda^c_{IC}(r/\lambda)$ would be negative. Regarding the left-hand side of the last inline inequality as a quadratic function of $(\lambda/\mu)$, we conclude that $\Lambda^c_{IP}(\lambda/\mu)$ is its smaller root. Accordingly, the game has Full Secrecy equilibrium if and only if $\lambda/\mu \leq \Lambda^c_{IP}(\lambda/\mu)$. □

*Proof of Proposition 5.* Follows directly by comparing the results of Proposition 3 and Proposition 4. □

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