Rank-Dependent Probability Weighting and the Macroeconomy: Insights from a Model with Incomplete Markets and Aggregate Shocks

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Abstract

A vast experimental literature in both psychology and economics documents that individuals exhibit rank-dependent probability weighting in economic decisions characterized by risk. I incorporate this well-known behavioral bias in a rank-dependent expected utility (RDEU) model. I develop a dynamic general equilibrium model with heterogeneous agents, labor market risk, and aggregate fluctuations, featuring households that are RDEU maximizers in an environment characterized by realistic labor market dynamics. I use the model to quantify the importance of RDEU for a number of macroeconomic outcomes. In a calibration of the model exploiting U.S. data, I find that RDEU plays a quantitatively important role for both the amount of aggregate wealth and the degree of wealth inequality, which are affected by the increased importance of precautionary saving, driven by households' pessimism. As for the outcomes routinely studied in the analysis of business-cycles, such as the volatility of both consumption and investment (relative to income), I find that RDEU improves the fit of the model. Overall, in terms of the discrepancy between model-generated business-cycle statistics and data, the RDEU models attain lower root mean squared errors than the EU one.

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1 Introduction

In the last forty years, a vast literature in the fields of Psychology and Economics has documented the existence of important behavioral biases in the decision making process of individuals faced with economic choices.[1] Kahneman and Tversky (1979), Quiggin (1982), and Prelec (1998) are influential contributions that analyze how individuals assess the probabilities of risky events when taking economic decisions. However, whether these behavioral biases are important for the determination of aggregate economic variables and their evolution over time is still an open question. Backus, Ferriere, and Zin (2015) study the consequences of introducing risk and ambiguity into a standard business cycle model. They find that consumption is typically reduced by increases in uncertainty. However, they argue that this channel cannot account for the magnitude and the persistence of the great recession.

In this paper, I focus on the issue of probability weighting, following the formulation of Rank-Dependent Expected Utility (RDEU) first proposed by Quiggin (1981) and Quiggin (1982), and later generalized by Prelec (1998) and Tversky and Kahneman (1992). RDEU models are particularly attractive, because they are analytically similar to models relying on expected utility, therefore they can be studied with simple extensions to standard dynamic programming methods. In particular, as argued by Segal, Spivak, and Zeira (1988) and Backus, Routledge, and Zin (2005), using weighted probabilities that are rank-dependent still admits a recursive representation, as potential dynamic inconsistency issues do not arise in the framework.

In order to quantify the macroeconomic implications of households that are characterized by probability weighting in their assessment of risk, I develop a model featuring both uninsurable labor market risk and aggregate shocks, in the tradition of Krusell and Smith (1998). However, the model is going to depart from their framework in two important dimensions. First, the agents are not going to be Expected Utility (EU) maximizers, rather they are going to maximize their RDEU. Second, in order to capture realistic labor market dynamics, and the associated degree of labor market risk, I am going to assume that the likelihoods of finding a job when unemployed, and losing one when employed, are time-varying. In this formulation, workers face different probabilities of labor market transitions every quarter, rather than every time the economy moves from a boom to a recession. More formally, I am going to postulate AR(1) stochastic processes for both the Job-Finding Probability (JFP) and the Job-Separation Probability (JSP), which represent the objective risk in the economy.[2] These specifications imply degrees of dispersion for both the JFP and the JSP that match their data counterparts. In turn, these generate empirically credible fluctuations in the aggregate unemployment rate, rather than displaying an economy that jumps between two fixed values, as in Krusell and Smith (1998), Castaneda, Diaz-Gimenez, and Rios-Rull (1998), Krusell, Mukoyama, Sahin, and Smith (2009), and the related literature discussed for example in den Haan, Judd, and Juillard (2010).[3] Moreover, because of the well-

[1] Machina (1987), Machina (1989), Starmer (2000), and Quiggin (2014) are comprehensive surveys that discuss the limitations of Expected Utility theory, the experimental evidence challenging that framework, and the theoretical alternatives that have been proposed in the behavioral literature.

[2] Even though in macroeconomics risk and uncertainty are typically considered as synonyms, I follow the Knightian distinction between them, such that agents are in the presence of risk whenever the probability of different events is known, or knowable.

[3] The same counterfactual implications for the labor market dynamics are also present in the continuous time formulation of models with heterogeneous agents and aggregate shocks, as discussed in Ahn, Kaplan, Moll, Winberry, and Wolf (2017).

documented behavioral biases concerning risky choices, RDEU agents do not base their consumption/saving decisions on these objective probabilities. Rather, they overweight unfavorable low-probability events, such as the probability of losing a job, which is typically interpreted as a form of pessimism. Probability weighting is captured by empirically motivated specifications. Figure (1) plots four probability weighting functions $\omega(p)$ commonly used in the literature. The most popular formulations are the one-parameter function $\omega(p) = (p^{\psi}/(p^{\psi} + (1-p)^{\psi})^{1/\psi}$ and the two-parameter function $\omega(p) = \exp(-\psi_1 * (-\ln(p))^{\psi_2})$. The former has been analysed by Camerer and Ho (1994), Tversky and Kahneman (1992), and Wu and Gonzalez (1996), that obtained the estimates $\widehat{\psi} = \{0.56, 0.61, 0.71\}$, respectively. In the quantitative implementation of the model, I am going to consider two such cases, with $\widehat{\psi} = \{0.56, 0.71\}$. Prelec (1998) proposed and carefully analysed the two-parameter probability weighting function, for which Gonzalez and Wu (1999) estimate $\widehat{\psi}_1 = 0.77$, and $\widehat{\psi}_2 = 0.44$. In the quantitative implementation of the model, I am also going to consider this case. Notice that, for either formulation of the probability weighting function, $\omega(1) = 1$.

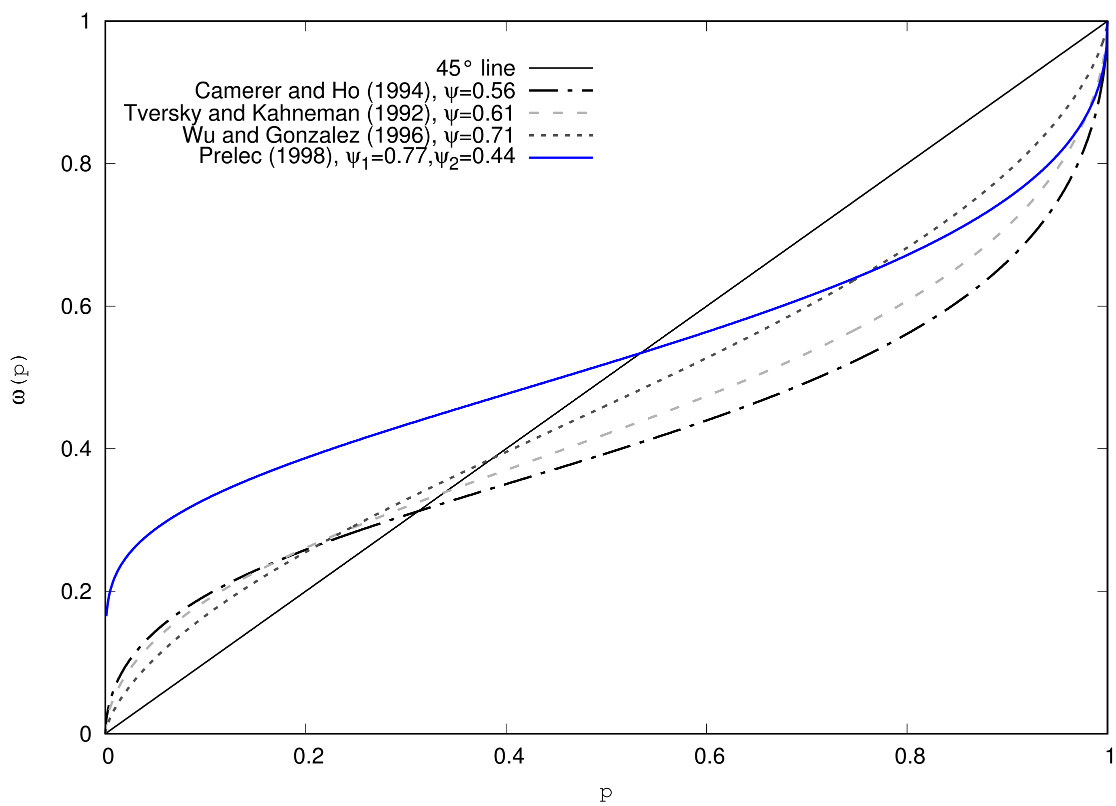

Figure 1: Probability weighting functions $\omega(p)$. Using the one-parameter function $\omega(p) = (p^{\psi}/(p^{\psi} + (1-p)^{\psi})^{1/\psi}$, Camerer and Ho (1994), Tversky and Kahneman (1992), and Wu and Gonzalez (1996) estimate $\psi = \{0.56, 0.61, 0.71\}$, respectively. Using the two-parameter function $\omega(p) = \exp(-\psi_1 * (-\ln(p))^{\psi_2})$ proposed by Prelec (1998), Gonzalez and Wu (1999) estimate $\psi_1 = 0.77$, and $\psi_2 = 0.44$.

A first contribution of the variant of the Krusell and Smith (1998) model that I consider is to rely on a data-driven time evolution of both the JFP and JSP. This is a valuable feature, as the model displays fluctuations in the unemployment rate that are consistent with the U.S. data.

A second contribution is to relax the EU assumption. To solve the model, I use the Krusell and Smith (1998) algorithm, which is a well-know method that this both robust and relatively easy to implement. In my model, this algorithm is not only computationally convenient, but it also represents a coherent formulation with respect to the way agents form expectations. The implied limited rationality assumption fits well with the RDEU set-up, as the agents use forecasting rules to predict future prices that do not rely on the objective probabilities of future events.

In terms of results, I find that the model can capture relevant aspects of the cyclical behavior of the U.S. economy. The time series properties of the simulated unemployment rate are similar to what the data show. The model does a reasonable job at capturing the relative volatilities of output, consumption, investment, and labor. In particular, the model does not suffer from the well-known pitfall of the Real Business Cycle (RBC) model with flexible labor, where the volatility of the labor input is too low.

The quantitative results show that RDEU can drive large differences in some important outcomes. In particular, a formulation of the probability weighting function changes the capital stock by up to 4%. Wealth inequality is also heavily affected, as shown by the changes in the wealth Gini index.

I also perform a decomposition analysis, where idiosyncratic risk is distorted through rank-dependent weighting, while aggregate risk is kept at its objective formulation.

The rest of the paper is organized as follows. Section 2 briefly presents the models and reports some of their quantitative implications. Section 3 discusses the model calibration. Section 4 presents the results, while Section 5 concludes. Three appendices discuss in more detail the numerical and empirical methods used, and present additional results.

2 A Model with Labor Market Risk and Rank-Dependent Expected Utility

I extend the standard incomplete markets model with heterogeneous agents and aggregate shocks in multiple ways. The starting point is the framework in Krusell and Smith (1998), without preference heterogeneity in the discount factors, and with agents that are EU maximizers. Following den Haan, Judd, and Juillard (2010), I assume the existence of a budget-balanced Unemployment Insurance (UI) scheme, which raises contributions by taxing employed workers and distributes unemployment benefits to jobless workers. I model labor market risk relying on an empirically driven formulation, which is based on stochastic job finding and job separation rates. One limitation of the Krusell and Smith (1998) framework, and many of its variants, is that the labor market dynamics are too stylized and unrealistic. In particular, during booms and recessions, the unemployment rate fluctuates between two given values.

The arrangements in the labor market share some features with Gomes, Greenwood, and Rebelo (2001).

The labor market is frictional, as unemployed workers cannot immediately locate the available jobs, and can start working only when given a stochastic opportunity do to so. I assume that firms operate with a constant-returns-to-scale technology and that vacancies can be created at no cost. A free-entry condition guarantees that workers are paid their marginal product.[4] Therefore, production of the price-taking firms can be aggregated into a production function that takes total capital and total labor as inputs.

2.1 Model Set-up

Time is discrete. The model postulates a production economy hit every period by aggregate labor market shocks, which induce aggregate fluctuations. The economy is populated by a measure one of infinitely-lived agents subject to idiosyncratic risk. Agents face different employment histories, and self-insure by accumulating a single risky asset. An exogenous borrowing constraint ($\underline{a}$) prevents households from achieving perfect consumption smoothing.

Technology: Aggregate production takes the form of a constant returns to scale technology of the Cobb-Douglas form, which relies on aggregate capital K_t and labor L_t to generate final output $Y_t = Z_t K_t^{\alpha} L_t^{1-\alpha}$. Aggregate productivity Z_t takes a continuum of values, and it is negatively related to the unemployment rate in a deterministic fashion. This formulation allows to take into account in a parsimonious way the empirical correlation between the unemployment rate and the Solow residual observed in the U.S. data. Firms are price takers, and rely on labor services $L_t = lN_t$, which are the product of the share (l) of the time endowment (normalized to 1) devoted to market activities and the employment level N_t. Firms rent capital from a competitive asset market, and this input depreciates at the exogenous rate δ. The first order conditions to the profit maximization problem, combined with a free-entry assumption, give the usual expressions for the net real return to capital r_t and the wage rate w_t:

$$r_t = \alpha Z_t \left(\frac{L_t}{K_t}\right)^{1-\alpha} - \delta, \tag{1}$$

$$w_t = (1-\alpha)\, Z_t \left(\frac{K_t}{L_t}\right)^{\alpha}. \tag{2}$$

Government: The employed agents' labor income is taxed at rate τ_t, and the associated tax revenues finance a budget-balanced UI scheme. Unemployed agents receive UI benefits equal to a fixed replacement rate χ of the going labor income. The labor input fluctuates exogenously over time, as it is driven by a stochastic process. The unemployment rate U_t can take a continuum of values, and its law of motion is discussed below. The equilibrium tax rate is $\tau_t = \chi(1-N_t)/N_t$, with $N_t = 1 - U_t$.

[4] Assuming a bargaining game between workers and employers would make the model computationally intractable, apart from the special case where the workers have the full bargaining power, whose equilibrium would coincide with the formulation I am using, as firms would have a zero value.

Households: Preferences are represented by a time-separable utility function, and the preference parameters are the discount factor β and the relative risk aversion γ. Households have mass one, and are indexed by $i \in [0,1]$. They choose consumption ($c_{i,t}$) and future asset holdings ($a_{i,t+1}$) to maximize their objective function:

$$\max_{\{c_{i,t},a_{i,t+1}\}_{t=0}^{\infty}} \widetilde{\mathbb{E}}_0 \sum_{t=0}^{\infty} \beta^t \frac{c_{i,t}^{1-\gamma}-1}{1-\gamma}$$

where $\widetilde{\mathbb{E}}_0$ is an operator representing either the standard expectation over all possible histories in the expected utility formulation, or an expectation featuring probability-weighting in the rank-dependent expected utility model.

Recursive formulation of the household problem: Agents can be employed ($\epsilon = e$) or unemployed ($\epsilon = u$). Both the objective job separation probability (s) and the objective job finding probability (ϕ) follow AR(1) stochastic processes, with independent normally distributed shocks. I assume that all workers face the same values for s and ϕ, which therefore are aggregate states. To solve the model, I use recursive methods, and the value function associated with this problem is denoted with $V(a, \epsilon, s, \phi, K, U, Z)$. This represents the (expected) lifetime utility of an agent whose current asset holdings are equal to a, whose current employment status is ϵ, facing objective probabilities of labor market transitions equal to s and ϕ, in an economy with K units of aggregate capital, aggregate unemployment rate U, and aggregate productivity Z. As for aggregate productivity, in principle it should enter the dynamic programming problem as a state variable. However, I am assuming Z and U to be negatively related, using a specification that does not require to keep track of Z independently of U. In particular, I assume that $Z = 1 - \zeta * (U/\overline{U} - 1)$, where $\overline{U}$ is the long-run average of unemployment and ζ is a positive parameter. This specification allows to amplify the response of income in booms and recessions, as productivity decreases (increases) proportionally when the unemployment level is above (below) its long-run average.[5] In this case, the Bellman equation is:

[5] Notice that the long-run average of productivity is $\overline{Z} = 1$. I also considered a formulation where Z was always constant, but the simulated output volatility was too low. Fluctuations in U alone, which are in turn driven by changes in s and ϕ, are not large enough to match the output volatility.

$$V\left(a,\epsilon,s,\phi,K,U\right)=\max_{c,a'}\left\{\frac{c^{1-\gamma}-1}{1-\gamma}+\beta\tilde{\mathbb{E}}_{\epsilon',s',\phi',K',U'|\epsilon,s,\phi,K,U}V\left(a',\epsilon',s',\phi',K',U'\right)\right\} \tag{3}$$

$s.t.$

$$c+a'=\left(1+r\right)a+\left(1-\tau\right)wl,\ \text{ if }\epsilon=e$$

$$c+a'=\left(1+r\right)a+\chi wl,\ \text{ if }\epsilon=u$$

$$c\geq 0,\quad a'\geq \underline{a}$$

$$\ln K'=\theta_{K,0}+\theta_{K,1}\ln K+\theta_{K,2}U+\theta_{K,3}Z \tag{4}$$

$$U'=\theta_{U,0}+\theta_{U,1}U+\theta_{U,2}\pi(u,e')*U+\theta_{U,3}\pi(e,u')*N \tag{5}$$

$$\ln s'=(1-\rho_s)\mu_s+\rho_s\ln s+\varepsilon_s',\varepsilon_s\sim N(0,\sigma_s^2) \tag{6}$$

$$\ln\phi'=(1-\rho_\phi)\mu_\phi+\rho_\phi\ln\phi+\varepsilon_{\phi,t}',\varepsilon_\phi\sim N(0,\sigma_\phi^2) \tag{7}$$

In the dynamic programming problem, $\pi(\epsilon,\epsilon')$ denote the decision weights associated with a labor market transition from the current state ϵ to the future state ϵ'. Their actual expressions depend on whether the model assumes the agents to be EU vis-a-vis RDEU maximizers, and are presented below. To better understand the model, it is informative to spell out the Bellman equations in (3):

$$\begin{aligned}V\left(a,\epsilon=e,s,\phi,K,U\right)=\max_{c,a'}\Bigg\{\frac{c^{1-\gamma}-1}{1-\gamma}+\beta\Bigg[\pi(e,u')\int_0^\infty\int_0^\infty p(s',\phi'|s,\phi)V\left(a',\epsilon'=e,s',\phi',K',U'\right)ds'd\phi'+\\ \pi(e,e')\int_0^\infty\int_0^\infty p(s',\phi'|s,\phi)V\left(a',\epsilon'=u,s',\phi',K',U'\right)ds'd\phi'\Bigg]\Bigg\}\end{aligned} \tag{8}$$

$$\begin{aligned}V\left(a,\epsilon=u,s,\phi,K,U\right)=\max_{c,a'}\Bigg\{\frac{c^{1-\gamma}-1}{1-\gamma}+\beta\Bigg[\pi(u,u')\int_0^\infty\int_0^\infty p(s',\phi'|s,\phi)V\left(a',\epsilon'=e,s',\phi',K',U'\right)ds'd\phi'+\\ \pi(u,e')\int_0^\infty\int_0^\infty p(s',\phi'|s,\phi)V\left(a',\epsilon'=u,s',\phi',K',U'\right)ds'd\phi'\Bigg]\Bigg\}\end{aligned} \tag{9}$$

Notice how, in the two Bellman equations for the employed and the unemployed, the first element in the future employment status is always being unemployed, which satisfies the RDEU requirement of ranking the outcomes (in this case, from worst to best, but Diecidue and Wakker (2001) illustrate that an equivalent analysis could be performed with best to worst ranking and the dual weighting function). Under EU, the decision weights are the objective labor market transition probabilities: $\pi(e,u') = s$, $\pi(e,e') = 1-s$, $\pi(u,u') = 1-\phi$, and $\pi(u,e') = \phi$. Under RDEU, the distorted labor market transition probabilities are a rank-dependent transformation of the objective ones, resulting from the difference in the weighting function $\omega(.)$ evaluated at two points, where the argument is the cumulative probability of two successive (ranked) events: $\pi(e,u') = \omega(s)$, $\pi(e,e') = \omega(1) - \omega(s) = 1 - \omega(s)$, $\pi(u,u') = \omega(1-\phi)$, and $\pi(u,e') = \omega(1) - \omega(1-\phi) = 1 - \omega(1-\phi)$.[6]

Beside individual risk, embedded in the labor market transitions $\pi(\epsilon,\epsilon')$, households are also facing two aggregate risks. The double integrals in (8) and (9) reflect the (possibly rank-dependent) expectation with respect to the JSP and JFP shocks, and $p(s',\phi'|s,\phi)$ denotes the (possibly distorted) joint conditional density of the future JSP and JFP. For tractability, I postulate two independent stochastic processes for s and ϕ, which allows to factorize the two densities. I specify two AR(1) processes in their logs, with both a drift term and normally distributed shocks. I can easily estimate these exogenous stochastic processes relying on the Shimer (2012) data. Figure 2 shows the Kernel densities for both JFP and JSP, indicating that the lognormal specification attains a satisfactory fit, while being parsimonious (which is key to make the model computationally feasible). Empirically, there is a low correlation between the quarterly JFP and JSP. Moreover, the two time series of the residuals resulting from the estimated AR(1) models are virtually uncorrelated (the correlation coefficient is 0.04 in levels, and 0.05 in logs).

Under EU, $p(s',\phi'|s,\phi) = p_s(s'|s)p_\phi(\phi'|\phi)$, where $p_x(x'|x)$ stands for the (conditional) lognormal density function of the random variable x. Under RDEU, in general, $p(s',\phi'|s,\phi) = \frac{d\omega(1-P_s(s'|s))}{dP_s} p_s(s'|s) \frac{d\omega(P_\phi(\phi'|\phi))}{dP_\phi} p_\phi(\phi'|\phi)$, where $\frac{d\omega(P_x(x'|x))}{dP_x}$ stands for the derivative of the probability weighting function $\omega(.)$, and $P_x(x'|x)$ stands for the (conditional) cumulative distribution function of the random variable x.[7] It is worth stressing that, in the RDEU models, rank-dependent weighting can be applied to the transition probabilities of both the idiosyncratic shocks and the aggregate ones. It is straightforward to incorporate rank-dependent probability weighting also in the aggregate dimension, because in the numerical implementation of the model I discretize these transition functions.

Agents set optimally their consumption/savings plans. They enjoy utility from consumption and face several risky events in the future. Notice that, according to the algorithm that I use to solve this model, in the agents' problem the endogenous asset distribution over idiosyncratic states is replaced by its first moment, and the relevant endogenous state variable is just aggregate capital K, rather than the whole distribution. Notice also that the evolution of unemployment is fully driven by exogenous stochastic processes. Agents forecast future

[6] Given that there are only two labor market states, their ranking could be regarded as immaterial, because with two-outcome lotteries RDEU could collapse to a simple probability weighting formulation. However, if one were to neglect their ranking, the two weighted probabilities (say, $\pi(e,u') = \omega(s)$ and $\pi(e,e') = \omega(1-s)$) would not add up to one, a characteristic known as subcertainty. In the literature, this setup has been shelved, because it leads to violations of stochastic dominance.

[7] Notice that, for the JSP, the argument of its weighting function is the complement of the cumulative distribution function, because higher values of s represent worse outcomes, which requires to invert the ranking.

prices relying on the (equilibrium) evolution of the aggregate capital stock and unemployment, the Aggregate Law of Motions (ALM) being specified as the pair of equations (4)-(5). The Krusell and Smith (1998) algorithm is based on a notion of the equilibrium that allows for bounded rationality. In particular, agents base their forecasts on perceived laws of motion. The formulation in Eq. (4) is a simple extension to the Krusell and Smith (1998) one, which here takes into account the effects of other aggregate variables (unemployment and aggregate productivity). The interpretation of this forecasting rule is twofold. Computationally, it is required to approximate a rational expectations equilibrium, as in the set of state variables in principle there is a time-varying infinite-dimensional object (the endogenous distribution of the asset holdings over all possible states). Behaviorally, it postulates a rule-of-thumb used by the agents to make their forecasts.[8] As for the other law of motion, Eq. (5), its formulation encompasses two cases. In a rational expectations equilibrium, it collapses to the law of motion of aggregate unemployment, such that $U' = U + \phi U + sN$. Therefore, this is based on the labor market flows implied by the objective transition probabilities, such that $\pi(u, e') = \phi$ and $\pi(e, u') = s$. In this case, $\theta_{U,0} = 0$ and $\theta_{U,1} = -\theta_{U,2} = \theta_{U,3} = 1$. In a non-RE equilibrium, the parameters $\theta_{U,j}, j = 0, ..., 3$, in the formula can be estimated using a methodology similar to the one applied to obtain the equilibrium ALM for Capital. Namely, OLS estimates on the time series of U, N and the weighted probabilities $\pi(u, e') = 1 - \omega(1 - \phi)$ and $\pi(e, u') = \omega(s)$. However, the equilibrium ALM for Unemployment is much easier to obtain, as it depends only on exogenous stochastic processes. Therefore, its parameters can be obtained considering only one long simulation, rather than having to rely on a guess-and-verify procedure that requires updating the ALM parameters until convergence.

3 Calibration

Table 1 reports the calibrated values of the model parameters.

[Table 1 about here]

Since most studies in the RBC literature use a capital share of 36%, I also set $\alpha = 0.36$. I use a capital depreciation rate $\delta = 0.025$, because it implies an average investment share of output equal to 25%. As for the Relative Risk Aversion parameter, I assume $\theta = 2$, which is consistent with the available empirical evidence on the Elasticity of Intertemporal Substitution, discussed for example by Attanasio and Weber (2010). I set the discount factor to $\beta = 0.988$, because it leads to an average quarterly interest rate of approximately 1%. Following the empirical results obtained from surveys on time use, I set $l = 0.327$, such that the share of the time endowment devoted to market activities is approximately 33%. In particular, this is the figure reported by Juster and Stafford (1991), who averaged the time devoted to market activities between males and females in the U.S. The sensitivity of productivity to changes in unemployment is set to $\zeta = 0.02$, because in the RDEU models this value matches the empirical standard deviation of detrended aggregate income, which is 1.9%. The

[8] A perhaps surprising, yet remarkably robust, feature is the degree of accuracy of these simple forecasting rules, as the forecasting errors are minimal. In the models considered here, the R^2 is always above 99.64%.

UI replacement rate is set to $\chi = 0.4$, therefore UI benefits are 40% of current labor earnings, a common value in the literature. The exogenous borrowing limit is set to $\underline{a} = -0.8$, which implies that households can borrow up to 20% of the average yearly income. Finally, the parameterization of the probability weighting function $\omega(.)$ was already discussed in the Introduction. In particular, I will refer to Model 1 (CH) as the RDEU case with the one-parameter $\omega(.)$ and its parameter set to the Camerer and Ho (1994) estimate $\widehat{\psi} = 0.56$, to Model 2 (WG) as the RDEU case with the one-parameter $\omega(.)$ and its parameter set to the Wu and Gonzalez (1996) estimate $\widehat{\psi} = 0.71$, and Model 3 (Prelec) as the RDEU case with the two-parameter $\omega(.)$ and its parameters set to the Gonzalez and Wu (1999) estimates $\widehat{\psi}_1 = 0.77$, and $\widehat{\psi}_2 = 0.44$.

3.1 Labor market dynamics

The objective labor market risk takes the form of two exogenous stochastic processes, which I estimate relying on the U.S. quarterly data between 1948Q1 and 2007Q1 compiled by Shimer (2012).

The stochastic process for JSP is $\ln(s_t) = (1 - \rho_s)\mu_s + \rho_s \ln(s_{t-1}) + \varepsilon_{s,t}$, $\varepsilon_{s,t} \sim N(0, \sigma_s^2)$, and the estimated parameters are obtained using MLE on the AR(1) model. The estimated persistence parameter of the job separation probability is $\rho_s = 0.9278$, the unconditional mean of the AR(1) process is $\mu_s = -3.398$, while the standard deviation of the i.i.d shock $\varepsilon_{s,t}$ is $\sigma_s = 0.0557$.

The stochastic process for JFP is $\ln(\phi_t) = (1 - \rho_\phi)\mu_\phi + \rho_\phi \ln(\phi_{t-1}) + \varepsilon_{\phi,t}$, $\varepsilon_{\phi,t} \sim N(0, \sigma_\phi^2)$, and the estimated parameters are obtained using MLE on the AR(1) model. The persistence of the job finding probability is $\rho_\phi = 0.9364$, the unconditional mean of the AR(1) process is $\mu_\phi = -0.792$, while the standard deviation of the i.i.d shock $\varepsilon_{\phi,t}$ is $\sigma_\phi = 0.0510$.[9]

Figure (2) plots the probability densities of the labor market transition probabilities obtained from the U.S. quarterly data. The blue lines plot the Gaussian Kernel density estimate. The red lines plot the underlying Gaussian distribution, which represent the limiting distributions of the two estimated AR(1) processes. The plots show that the log-normal specifications provide a somewhat better fit of the data, and achieve a satisfactory fit of the empirical distribution.

[9] I also tried estimating a reduced-form VAR(1) model in the logs of JFP and JSP. Interestingly, in the JSP equation the JFP is not statistically significant at the 30% confidence level, while in the JFP equation the JSP has a small point estimate. This shows that estimating two separate processes is a sensible procedure.

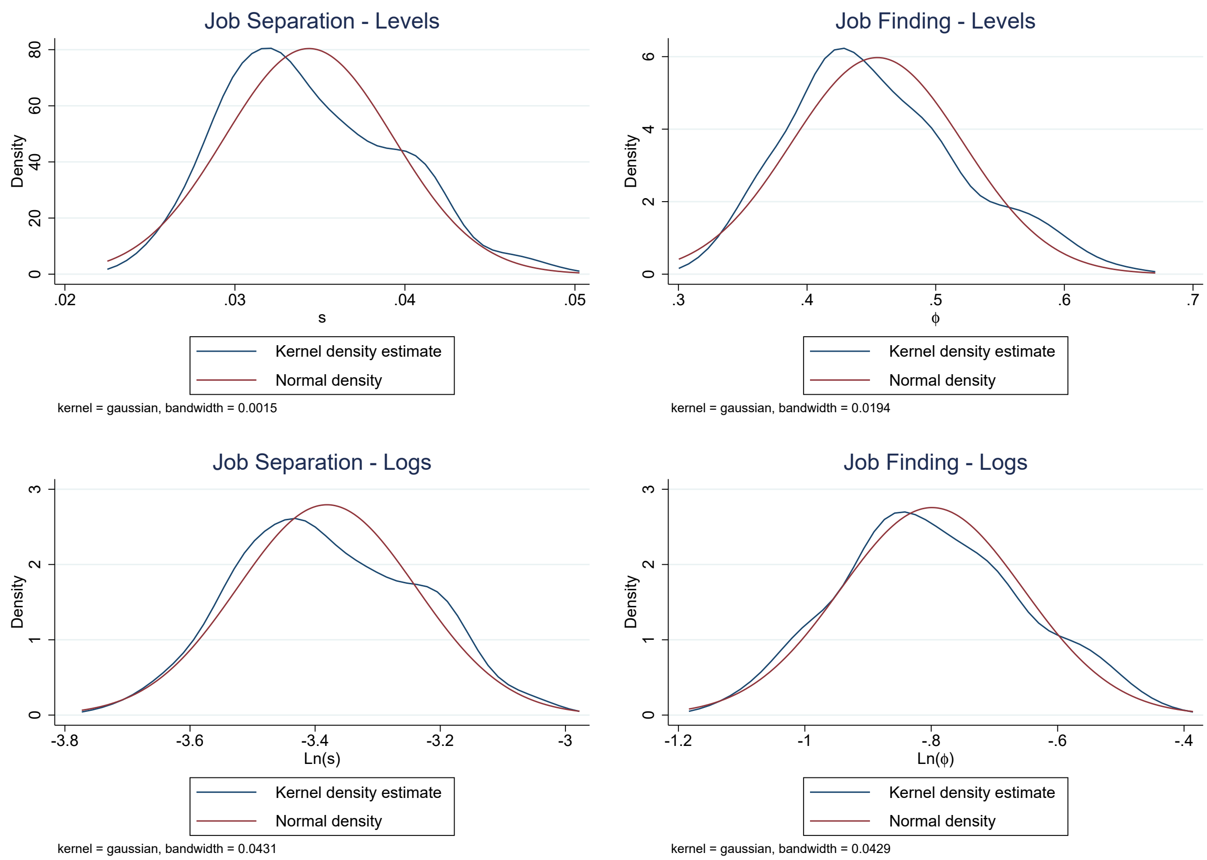

Figure 2: Probability densities of the labor market transition probabilities. The blue line plots the Gaussian Kernel density estimate on the U.S. quarterly data. The red line plots the underlying Gaussian distribution.

The top panel of Figure (3) plots the job separation probabilities computed by Shimer (2012), together with their biased counterparts obtained applying the three weighting functions used in the quantitative implementation of the model. As expected, all formulations imply a form of pessimism, such that the distorted probability of losing a job, a particularly bad outcome, is substantially larger than its objective empirical observation. The act of inflating these probabilities is particularly strong in the Prelec (1998) parameterization.

The bottom panel of Figure (3) plots the job finding probabilities computed by Shimer (2012), together with their biased counterparts obtained applying the three weighting functions used in the quantitative implementation of the model. For the most part, all three formulations imply a form of optimism, which is almost always supported by the Camerer and Ho (1994) parameterization. Conversely, in the Prelec (1998) case, optimism switches to pessimism in six sub-samples. This is due to the fact that, with a quarterly time period, the probabilities of both finding and not finding a job are often around 50%, which is close to the fixed point of that weighting function. Finally, compared to the data, the distorted probabilities display a compression effect, which

Figure 3: Job separations (JSP, top panel) and Job creations (JFP, bottom panel): Times series of the data Vs. distorted (RDEUs) probabilities, obtained with the three formulations of the weighting functions.

reduces the volatility of JFP. With this series, rank-dependent probability weighting works disproportionately in the region where the weighting functions are flat (namely, with a slope lower than 1).[10]

4 Results

I begin this section by discussing the model performance with respect to the labor market dynamics. I then present an analysis of both the business-cycle statistics and the long-run averages generated by the RDEU model, using different versions of the probability-weighting function. These are compared to the same outcomes generated by the EU version of the model.

[10]The plots of the model-generated s and ϕ series, together with their weighted counterparts, are included in Appendix C.

4.1 Labor market outcomes

Figure (4) plots the kernel approximations of the probability densities of quarterly unemployment rates, for both the data and the model. The data span the 1948Q1-2020Q1 period, while the model simulation is obtained drawing a long sequence of shocks from the estimated AR(1) stochastic processes for the probabilities of the labor market transitions, and using the resulting law of motion for the unemployment rate. The model achieves a good fit, especially when compared to the Krusell and Smith (1998) specification, which generates a two-point distribution with a 50% mass at $u = 4\%$ and $u = 10\%$.

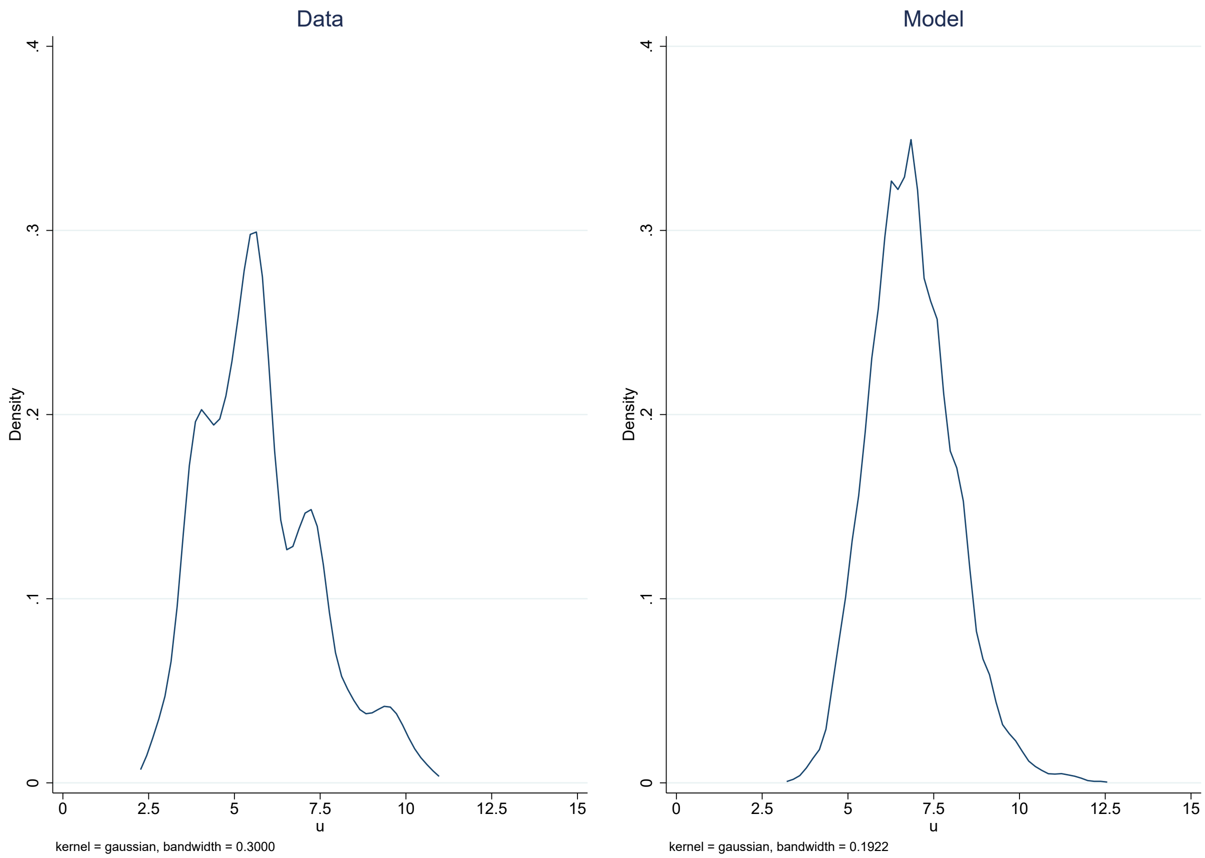

Figure 4: Probability densities of quarterly unemployment rates, Data vs Model. The data span the 1948Q1-2020Q1 period, while the model simulation is obtained drawing a long sequence of shocks from the estimated AR(1) stochastic processes for the probabilities of the labor market transitions, and using the resulting law of motion for the unemployment rate.

Figure (5) plots the quarterly unemployment rate, for both the U.S. data and the model simulation (because of space considerations, I display only 1750 model periods, between quarter 1000 and 2750). The simulated path of the unemployment rate is divided into 7 subsamples of 250 observations each, which make them more directly comparable to the data, whose sample has 288 observations. Unlike Krusell and Smith (1998), whose economy allows for only two values of the unemployment rate (their economy fluctuates between $u = 4\%$ in booms and $u = 10\%$ in recessions, which are equally likely), the model shows unemployment rate fluctuations that are consistent with the patterns in the U.S. data. The data display more clustering, which would perhaps require a stochastic volatility specification, which is computationally intractable for my model. Overall, the parsimonious specification I adopted for the time-varying aggregate shocks delivers a satisfactory fit of the data. Given its computational tractability, this way of modeling aggregate unemployment dynamics can also be used by other researchers in the field of heterogeneous-agent models with aggregate shocks.

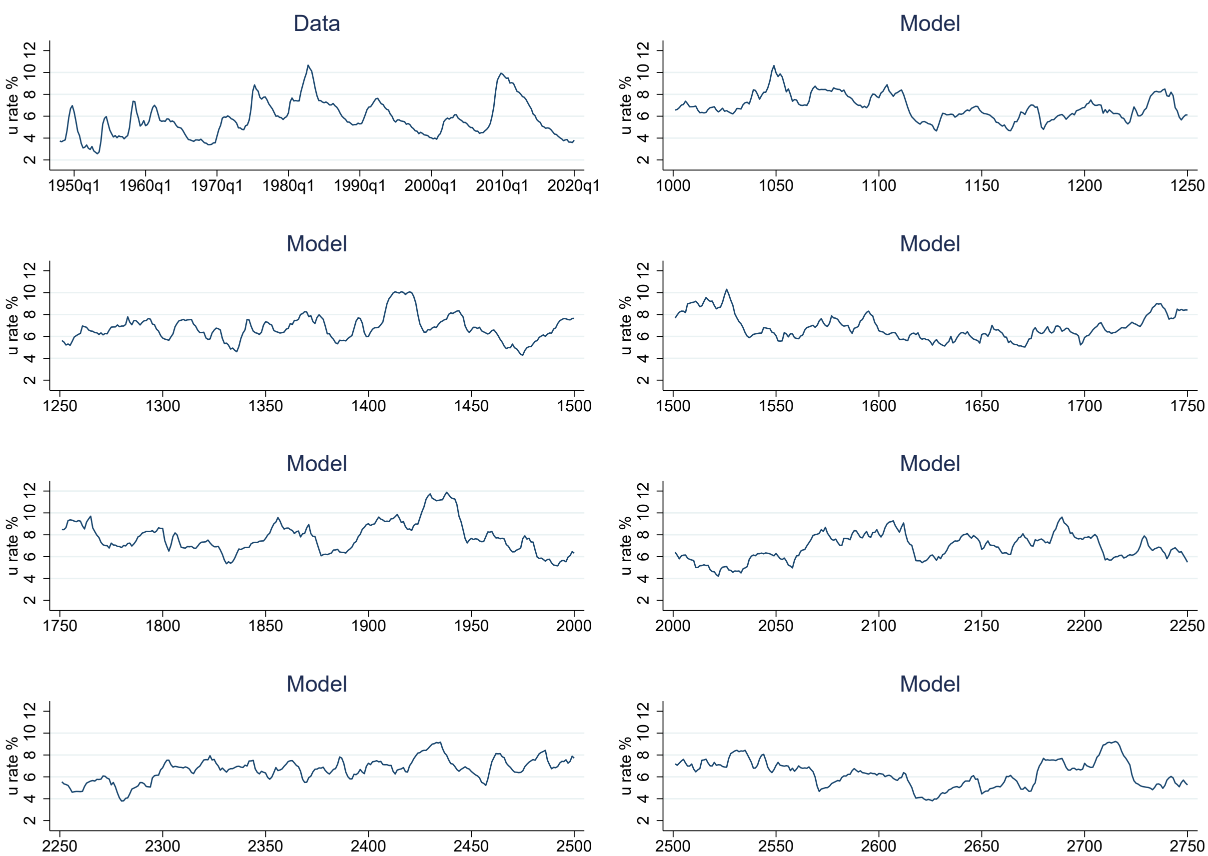

Figure 5: Quarterly unemployment rates, Data Vs. Simulated path. The model simulation is obtained drawing a long sequence of shocks from the estimated AR(1) stochastic processes for the labor market transition probabilities, and using the resulting law of motion for the unemployment rate.

4.2 Business-Cycle statistics and long-run outcomes

Table 2 reports common business-cycle statistics. The top panel lists the statistics computed from the U.S. data, which are HP-filtered with the smoothing parameter set to 1600. The remaining panels present the results obtained from simulating different versions of the model. EU (RDEU) stands for the expected utility (rank-dependent expected utility) formulation of the model. For the RDEU specifications, Models 1 and 2 rely on the one-parameter probability weighting function estimated by Camerer and Ho (1994) and Wu and Gonzalez (1996), respectively. Model 3 relies on the two-parameter (Prelec) probability weighting function estimated by Gonzalez and Wu (1999). Finally, variables denoted with a bar refer to long-run averages.

[Table 2 about here]

Regarding the agents' forecasting rules, they always show a remarkable degree of accuracy. As for the aggregate laws of motion for capital (unemployment), in the EU model the associated measure of fit is $R^2 = 0.999478$ ($R^2 = 0.9970$). In the RDEU models, it is $R^2 = 0.999931$ ($R^2 = 0.9963$) for Model 1, $R^2 = 0.999824$ ($R^2 = 0.9968$) for Model 2, and $R^2 = 0.999961$ ($R^2 = 0.9964$) for Model 3.[11]

A clear finding is that the RDEU formulations display a larger volatility of income, due to a considerably more volatile investment. This in turn reduces the relative volatility of consumption to income, which is still in excess of its empirical counterpart, but it gets closer to the data. Since the volatility of consumption does not vary considerably between the EU and the RDEU models, this result is driven by the increased volatility of income.

Another finding is that RDEU does not help in reducing the autocorrelations of Y, C, and I, which are all higher than both the corresponding values for the EU model, and their data counterparts.

In the RBC literature, it is well-known that models with an endogenous labor supply and flexible prices display a volatility of the labor input that is far too low compared to the data. The models considered here perform better in this dimension, as they account for approximately 80% of both the empirical volatility of the labor input, and its relative volatility with respect to income.

In order to assess the overall fit of the models, I consider the Root Mean Squared Error ($RMSE$) as the measure of distance between the data and the model-generated statistics. The EU model attains a $RMSE_{EU} = 1.57$, while all the RDEU models attain a lower RMSE, being $RMSE_{RDEU,M1} = 0.96$ for the Camerer and Ho (1994) parameterization, $RMSE_{RDEU,M2} = 1.12$ the Wu and Gonzalez (1996) parameterization, and $RMSE_{RDEU,M3} = 1.04$ for the Prelec parameterization.

Regarding the long-run average outcomes, all four models have an average interest rate ($\bar{r}$) that is close to 1%. One dimension where the models differ, is the share of households with negative asset holdings ($\bar{d}$). This

[11] As a robustness check, I also solved the model using the objective law of motion for aggregate unemployment. The results are virtually unaffected, because all the variables in this ALM are driven by exogenous stochastic processes, therefore the difference between the rational expectations formulation and the rule of thumb are minimal. Notice also that the OLS estimates for the unemployment ALM are not subject to a guess-and-verify procedure, because the related simulated paths of U, N, s, and ϕ are purely exogenous, and driven by their objective formulations.

is not surprising, as the distorted probabilities deriving from the rank-dependent weighting lead to pessimism of the employed agents coupled with optimism of the unemployed ones.

Figure (6) shows visually the challenges of comparing the RDEU frameworks to the EU one, when probability weighting is applied to the two sources of aggregate risk.[12] Given a current value of the state, each panel plots the differences between the CDFs of the future states under the objective formulation of labor market risk (which applies to the EU model) and the CDFs of the future states under RDEU. In each panel, there are nine curves, because I am using nine points to discretize the two AR(1) stochastic processes for s and ϕ, a procedure that leads to two 9×9 Markov Chains. Under RDEU, the cumulated value of the transition probabilities in these Markov Chains represent the arguments of the probability weighting function. The plots clarify that the distortion in the probabilities of labor market transitions induced by rank-dependent probability weighting has a monotonic effect only for the highest and lowest values of the current state. For the JSP, we observe pessimism (optimism) when s is at its lowest (highest) realization, because the RDEU CDF is always lower (higher), making higher (lower) rates of job separations more likely. For the seven remaining intermediate values, the distorted CDFs display a mix of pessimism and optimism, which affect in complex ways the overall assessment of risk from the perspective of the households. For the JFP, we observe similar (yet opposite) patterns. There is optimism (pessimism) when ϕ is at its lowest (highest) realization, because the RDEU CDF is always lower (higher), making higher (lower) rates of job creation more likely. Also for this variable, for the seven remaining intermediate values, the distorted CDFs display a mix of pessimism and optimism.

4.3 Decomposition analysis with probability weighting of idiosyncratic shocks and objective aggregate risk

To perform a decomposition analysis, I consider an alternative RDEU formulation. Agents are still distorting the likelihood of idiosyncratic shocks, via rank-dependent probability weighting. However, now I assume that the conditional densities of the future values for the JFP and JSP, $p_s(s'|s)$ and $p_\phi(\phi'|\phi)$, are kept at their objective values. Therefore, this specification assumes objective aggregate risk.

[Table 3 about here]

Table 3 reports the results of this version of the model, listing the same business-cycle and long-run statistics discussed above.

Also in this formulation, all the RDEU models attain lower RMSEs than the EU model, being $RMSE_{RDEU,M1} = 1.29$ for the Camerer and Ho (1994) parameterization, $RMSE_{RDEU,M2} = 1.36$ the Wu and Gonzalez (1996) parameterization, and $RMSE_{RDEU,M3} = 0.97$ for the Prelec parameterization.

[12] Figure (6) displays the Model 3 case, when RDEU takes the Prelec parameterization. Similar plots apply also to the other two RDEU formulations.

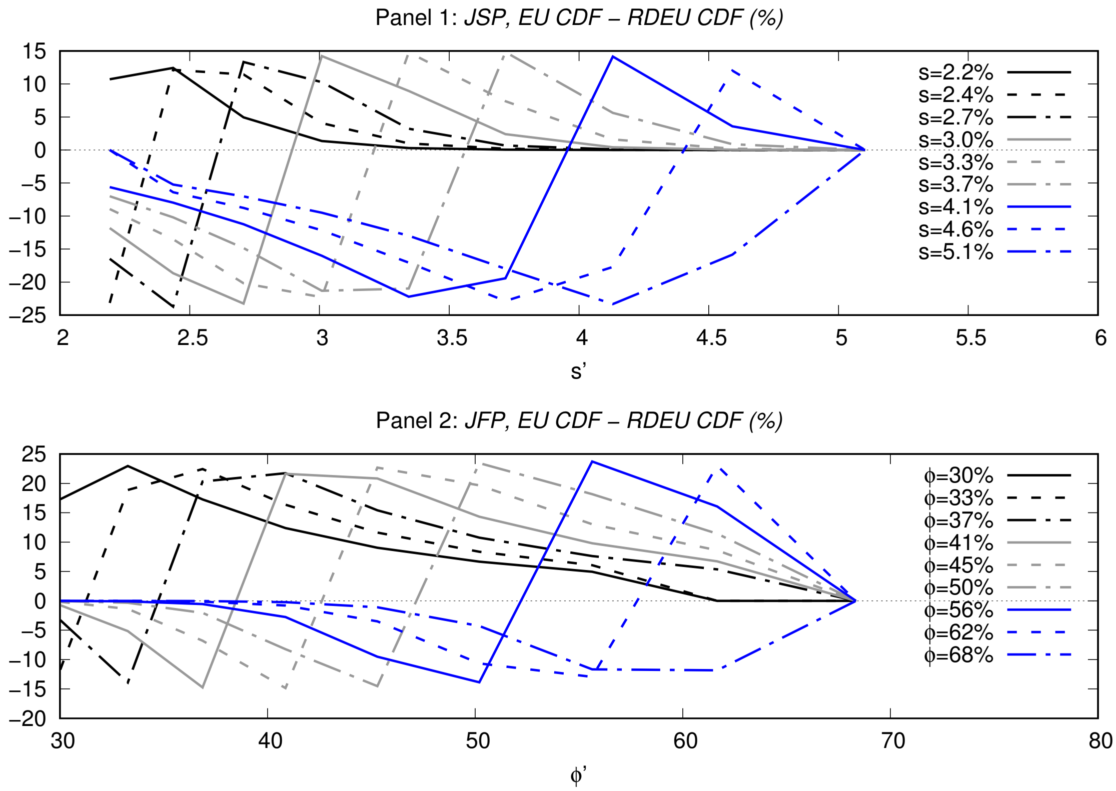

Figure 6: Distorted JSP and JFP for the Prelec case.

5 Conclusions

In this paper, I incorporated rank-dependent probability weighting in a dynamic general equilibrium model with heterogeneous agents, labor market risk, and aggregate fluctuations. I used the model to quantify the importance of Rank-Dependent Expected Utility for a number of macroeconomic outcomes. In a calibration of the model exploiting U.S. data, I found that Rank-Dependent Expected Utility plays a quantitatively important role for both the amount of aggregate wealth and the degree of wealth inequality, which are affected by the increased importance of precautionary saving, driven by households' pessimism. As for the outcomes routinely studied in the analysis of business-cycles, such as the volatility of both consumption and investment (relative to income), I found that Rank-Dependent Expected Utility improves the fit of the model. Given that the introduction of RDEU had a sizable effect on the volatility of both investment and income, rank-dependent probability weighting should be more widely adopted in the study of business-cycles.

A contribution of the paper was to develop a parsimonious time-varying specification for the aggregate shocks, which is computationally tractable while delivering a satisfactory fit of the data. This feature can be adopted by other researchers in the field of heterogeneous-agent models with aggregate shocks.

In terms of future research, the welfare costs of business cycle fluctuations in both the EU and RDEU versions of the model could be computed by applying an extension to the integration principle discussed by

Krusell, Mukoyama, Sahin, and Smith (2009). Finally, a perhaps natural extension would be to apply a similar framework in the presence of uncertainty, rather than risk. However, some recent contributions have shown that dynamic inconsistency can easily arise in that context, which would prevent the use of dynamic programming techniques and other standard tools for macroeconomic analysis.

Parameter	*Value*	*Target*
α - *Capital share*	0.36	*Labor share of output* $= 64\%$
δ - *Capital depreciation rate*	0.025	*Avg. investment share of output* $= 25\%$
θ - *Risk Aversion*	2.0	*Elasticity of Intertemporal Substitution* $= 0.5$
β - *Discount factor*	0.988	*Avg. quarterly interest rate* $= 1\%$
l - *Hours worked*	0.327	*Share of time endowment devoted to market activities* $= 33\%$
ζ - *Productivity sensitivity to unemployment*	0.02	*S.d. of aggregate income* $= 1.9\%$
χ - *UI Replacement Rate*	0.4	*UI benefits are 40% of current labor earnings*
$\underline{a}$ - *Borrowing limit*	-0.8	*Households can borrow up to 20% of avg. yearly income*
ρ_ϕ - *Persistence of the AR(1) JFP (in logs)*	0.936	*MLE estimates on Shimer (2012) data*
σ_ϕ - *S.d. of the JFP shocks (in logs)*	0.051	*MLE estimates on Shimer (2012) data*
μ_ϕ - *Unconditional avg. of the JFP (in logs)*	-0.8	*MLE estimates on Shimer (2012) data*
ρ_s - *Persistence of the AR(1) JSP (in logs)*	0.923	*MLE estimates on Shimer (2012) data*
σ_s - *S.d. of the JSP shocks (in logs)*	0.056	*MLE estimates on Shimer (2012) data*
μ_s - *Unconditional avg. of the JSP (in logs)*	-3.4	*MLE estimates on Shimer (2012) data*
ψ - *Curvature of probability weighting*	$-$	*Estimates in the literature, see text*

Table 1: Calibration of the RDEU Model.

Variable j	σ_j	σ_j/σ_Y	ρ_j	$\rho_{j,Y}$	
Long-run Average	($\overline{K}$)	($\overline{Y}$)	($\overline{C}$)	($\overline{r}$ (%))	($\overline{d}$ (%))
Data					
Y	0.0189	1.000	0.892	1.000	
C	0.0111	0.586	0.850	0.901	
I	0.0658	3.487	0.847	0.937	
L	0.0173	0.916	0.915	0.821	
EU Model	(11.594)	(1.129)	(0.839)	(1.005)	(8.520)
Y	0.0162	1.000	0.975	1.000	
C	0.0138	0.850	0.929	0.911	
I	0.0315	1.948	0.841	0.852	
L	0.0138	0.855	0.910	0.944	
RDEU Model 1 (CH)	(10.828)	(1.101)	(0.830)	(1.162)	(0.171)
Y	0.0195	1.000	0.982	1.000	
C	0.0137	0.702	0.953	0.837	
I	0.0504	2.577	0.901	0.886	
L	0.0138	0.709	0.910	0.900	
RDEU Model 2 (WG)	(11.159)	(1.113)	(0.834)	(1.092)	(1.025)
Y	0.0188	1.000	0.981	1.000	
C	0.0138	0.732	0.949	0.859	
I	0.0453	2.408	0.895	0.883	
L	0.0138	0.735	0.910	0.907	
RDEU Model 3 (Prelec)	(11.785)	(1.135)	(0.841)	(0.968)	(0.001)
Y	0.0192	1.000	0.982	1.000	
C	0.0131	0.681	0.949	0.837	
I	0.0477	2.488	0.901	0.902	
L	0.0138	0.722	0.910	0.902	

Table 2: Business Cycle Statistics and Long-run Averages (the latter are denoted with a bar and reported in parentheses). *Notes*: σ_j denotes the standard deviation of variable j. ρ_j ($\rho_{j,Y}$) denotes the autocorrelation (correlation) of variable j (between variables j and Y). EU (RDEU) stands for the Expected Utility (Rank-Dependent Expected Utility) version of the model. For the RDEU specifications, Models 1 and 2 rely on the one-parameter probability weighting function estimated by Camerer and Ho (1994) and Wu and Gonzalez (1996), respectively. Model 3 relies on the two-parameter (Prelec) probability weighting function estimated by Gonzalez and Wu (1999). The time series data are HP-filtered.

Variable j	σ_j	σ_j/σ_Y	ρ_j	$\rho_{j,Y}$	
Long-run Average	$(\overline{K})$	$(\overline{Y})$	$(\overline{C})$	$(\overline{r}\,(\%))$	$(\overline{d}\,(\%))$
Data					
Y	0.0189	1.000	0.892	1.000	
C	0.0111	0.586	0.850	0.901	
I	0.0658	3.487	0.847	0.937	
L	0.0173	0.916	0.915	0.821	
EU Model	(11.594)	(1.129)	(0.839)	(1.005)	(8.520)
Y	0.0162	1.000	0.975	1.000	
C	0.0138	0.850	0.929	0.911	
I	0.0315	1.948	0.841	0.852	
L	0.0138	0.855	0.910	0.944	
RDEU Model 1b (CH)	(11.666)	(1.131)	(0.839)	(0.991)	(3.841)
Y	0.0174	1.000	0.978	1.000	
C	0.0138	0.791	0.931	0.881	
I	0.0389	2.232	0.870	0.863	
L	0.0138	0.794	0.910	0.935	
RDEU Model 2b (WG)	(11.655)	(1.131)	(0.839)	(0.993)	(2.995)
Y	0.0172	1.000	0.978	1.000	
C	0.0137	0.797	0.931	0.881	
I	0.0371	2.164	0.870	0.863	
L	0.0138	0.807	0.910	0.935	
RDEU Model 3b (Prelec)	(12.034)	(1.144)	(0.843)	(0.923)	(0.001)
Y	0.0194	1.000	0.983	1.000	
C	0.0138	0.711	0.928	0.786	
I	0.0499	2.570	0.890	0.876	
L	0.0138	0.713	0.910	0.898	

Table 3: Business Cycle Statistics and Long-run Averages (the latter are denoted with a bar and reported in parentheses). *Notes*: σ_j denotes the standard deviation of variable j. ρ_j ($\rho_{j,Y}$) denotes the autocorrelation (correlation) of variable j (between variables j and Y). EU (RDEU) stands for the Expected Utility (Rank-Dependent Expected Utility) version of the model, where the JFP and JSP risks are objective. For the RDEU specifications, Models 1 and 2 rely on the one-parameter probability weighting function estimated by Camerer and Ho (1994) and Wu and Gonzalez (1996), respectively. Model 3 relies on the two-parameter (Prelec) probability weighting function estimated by Gonzalez and Wu (1999). The time series data are HP-filtered.

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Appendix A - Computation

- All codes solving the economies were written in the FORTRAN 95 language, relying on the Intel Fortran Compiler, build 18.0.02 (with the IMSL library). They were compiled selecting the O3 option (maximize speed), and without automatic parallelization. They were run on a 64-bit PC platform, running Windows 10 Professional Edition, with either an Intel Core $i7-8700k$ Hexa-core processor clocked at 4.5 Ghz, or an Intel Core $i7-9900k$ Octo-core processor clocked at 4.8 Ghz.

- The different versions of the model can be run in parallel, and their solution takes between 4 and 6 hours to complete. Typically, between 12 to 18 iterations on the ALM are needed to find each equilibrium.

- In the actual solution of the model, I need to discretize the continuous state variables a, ϕ, s, K, and U (the employment status ϵ is already discrete, and aggregate productivity Z is linked deterministically to U). For the household assets a, I rely on an unevenly spaced grid, with the distance between two consecutive points increasing geometrically. This is done to allow for a high precision of the policy functions at low values of a, where the change in curvature is more pronounced. To keep the computational burden manageable, I use 101 grid points on the household assets space, the lowest value being the borrowing constraint b and the highest one being a value $a_{\max}$ high enough not to be binding in the simulations ($a_{\max} = 500$). The two AR(1) processes for the job finding probability ϕ and the job separation probability s are discretized with the Rouwenhorst method, using 9 points, which ensure a high degree of numerical accuracy. For aggregate capital K, I use a fairly dense grid. Over the $[9.5, 14.5]$ interval I use 11 points, which are substantially more than the typical 4-6. In the iterative process on the ALM parameters, the simulations do visit regions for aggregate capital that are relatively far from the support of the ergodic equilibrium distribution, causing convergence issues when using a coarse grid. For aggregate unemployment U, I use a fairly dense grid with 11 evenly-spaced points over the interval $[U_{\min}, U_{\max}]$. The value for the lower bound $U_{\min}$ is obtained by using the formula for the steady state unemployment rate, $U = s/(s + \phi)$, evaluated at the highest value for the job finding probability $\phi_{\max}$, and the lowest value for the job separation probability $s_{\min}$. The value for the upper bound $U_{\max}$ is obtained by using the formula for the steady state unemployment rate evaluated at the lowest value for the job finding probability $\phi_{\min}$, and the highest value for the job separation probability $s_{\max}$. To prevent the simulations from having to extrapolate below or above the chosen interval for U, I further decrease (increase) the lower (upper) bound by a 0.005 factor.

- As for the solution method for the household problem, I use the Endogenous Grid Method (EGM) with linear interpolation in the (a, K, U) dimensions. This method is considerably faster and more stable than the relatively popular value function iteration scheme with cubic spline interpolation.

- The aggregate dynamics are computed by simulating a large panel of individuals for $10,000$ periods, with the first $2,000$ periods discarded as a burn-in. The panel size is $90,000$ agents. As for the approximation method, I rely on a tri-linear approximation scheme for the saving functions, for values of a, K and U falling outside the grid.

Appendix B - Algorithm for the Models Solution

The computational procedure used to solve the incomplete markets model economies can be represented by the following algorithm:

1. Generate a discrete grid over aggregate capital $[K_{\min}, ..., K_{\max}]$.
2. Generate a discrete grid over the AR(1) job-finding probability with the Rouwenhorst method $[\phi_{\min}, ..., \phi_{\max}]$.
3. Generate a discrete grid over the AR(1) job-separation probability with the Rouwenhorst method $[s_{\min}, ..., s_{\max}]$.
4. Generate a discrete grid over aggregate unemployment $[U_{\min}, ..., U_{\max}]$.
5. Generate a discrete grid over the individual asset space $[a_{\min} = \underline{a}, ..., a_{\max}]$.
6. The set of individual labor market states ϵ is already a discrete grid $[u, e]$.
7. Guess a vector of parameters Θ^g representing the two ALMs, for aggregate capital and aggregate unemployment, using in the first iteration the ALM parameters of the representative-agent version of the model.
8. Get the saving functions $a'(a, \epsilon, \phi, s, K, U)$.
9. Simulate the model under the guessed ALMs, and compute an update $\Theta^{g\prime}$ as the parameter estimates of OLS regressions on the simulated data.
10. Repeat steps $8 - 9$ until the all parameters in Θ converge.
11. Compute the aggregate variables and obtain the business-cycle and (detrended) long-run statistics.
12. Move to the next model and repeat all of the steps above.

Appendix C - Additional Results and Plots

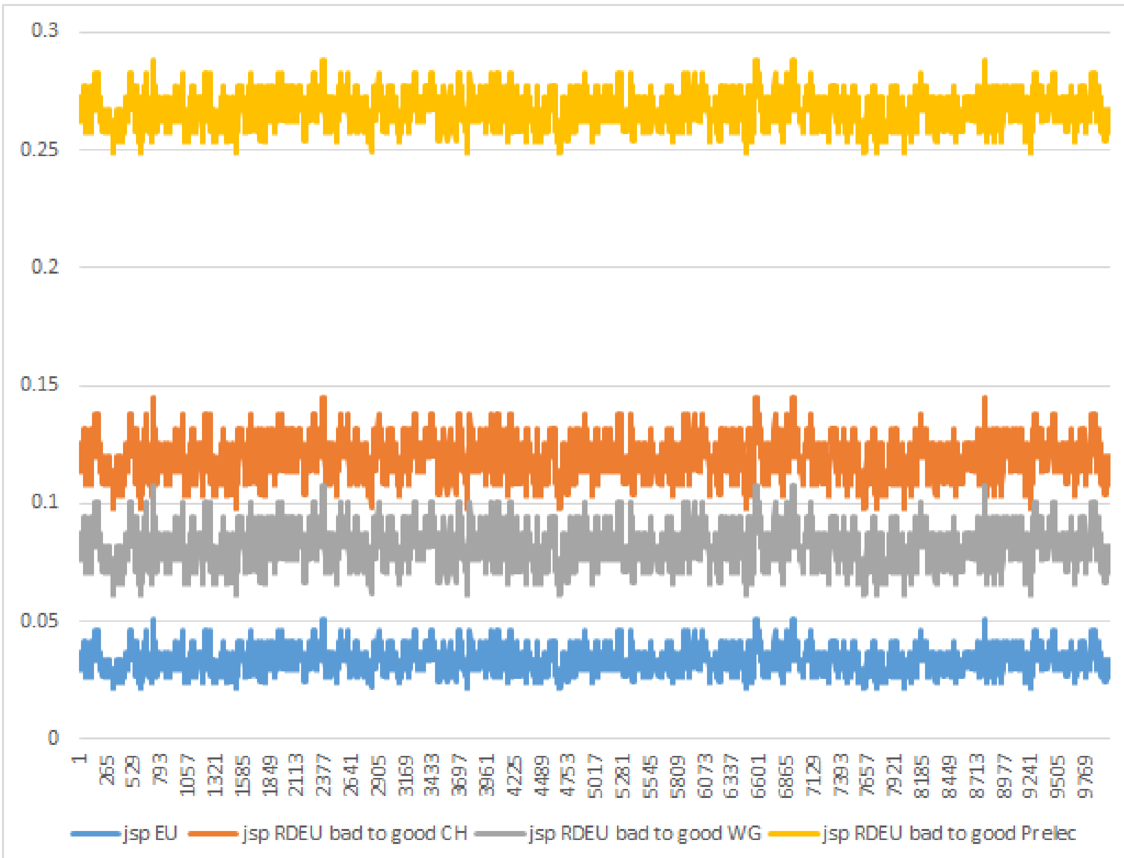

Figure 7: Simulated paths of the objective (EU) Vs. distorted (RDEUs) job separation probability (jsp). The model simulation is obtained drawing a long sequence of shocks from the estimated AR(1) stochastic process for s, and using the three weighting functions used in models 1, 2 and 3.

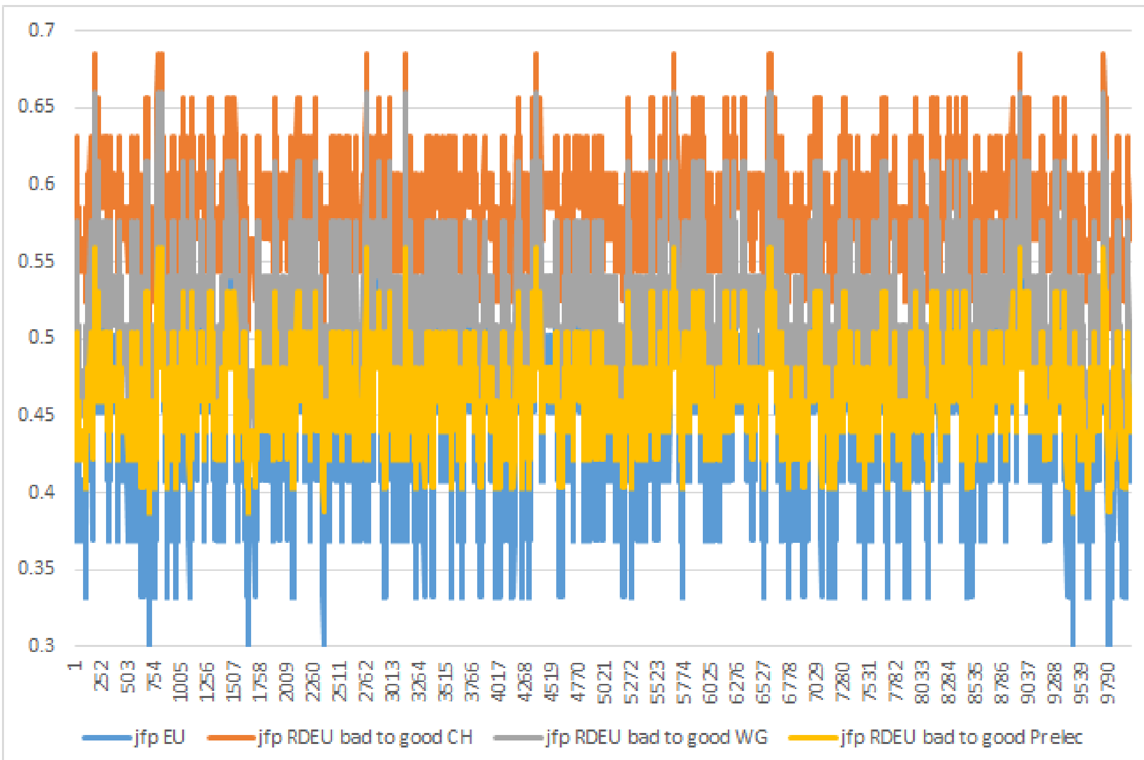

Figure 8: Simulated paths of the objective (EU) Vs. distorted (RDEUs) job finding probability (jfp). The model simulation is obtained drawing a long sequence of shocks from the estimated AR(1) stochastic process for ϕ, and using the three weighting functions used in models 1, 2 and 3.