Income heterogeneity and optimum product diversity*

Sergei Kichko[†] and Pierre M. Picard[‡]

February 01, 2024

Abstract

This paper compares the market and socially optimal allocations in monopolistically competitive markets when consumers have heterogeneous incomes and additive nonhomothetic preferences. If demand elasticity of lower-income households is more sensitive to income change than higher-income demand elasticity, a mean-preserving spread of income distribution reduces the gap between the market and unconstrained optimal (first-best) allocations. The gap between the market and constrained optimal (second-best) allocations shrinks under the same condition if household demand is sufficiently subconvex. An exercise calibrated to the US economy quantifies $10\% - 12\%$ more firms in equilibrium compared to optimum for actual US income distribution, whereas there are about 40% more firms for a representative consumer framework with average US income.

1 Introduction

Income heterogeneity is one of the most salient features observed in many modern economies (Piketty and Saez, 2006). Given that individual income significantly influences consumer demand, income disparities have a strong impact on product demands via the composition of individual demands. Consequently, firms are expected to adjust their pricing strategies, production scales, and market entry decisions according to the level of income inequality. In particular, monopolistically competitive markets likely accommodate a broader array of products if they are confronted with a larger segment of rich consumers because they display stronger preferences for product diversity (love for variety). Income heterogeneity, therefore, alters the well-known tension between competition and appropriation of consumer surplus. As a result, the canonical analysis of equilibrium and optimal product diversity must be revisited to account for consumer income heterogeneity. To this end, we account for consumer income heterogeneity to compare equilibrium and optimal allocation in monopolistically competitive markets.

*We thank Kristian Behrens for helpful comments and suggestions. All remaining errors are ours.

[†]Department of Economics and Management, University of Trento (Italy). E-mail: sergei.kichko@unitn.it, sergey.kichko@gmail.com

[‡]DEM/CREA, University of Luxembourg, Luxembourg. Address: Campus Kirchberg, University du Luxembourg 6, rue Richard Coudenhove-Kalergi L-1359 Luxembourg; e-mail: pierre.picard@uni.lu

In their seminal works, Dixit and Stiglitz (1977) and Spence (1976) compare the equilibrium and socially optimal allocations in the absence of consumer heterogeneity. Under an increasing love for variety, markets are shown to provide an excessive number of varieties while firms' production scale is inefficiently low compared to the optimal allocations. This results from the distortions of imperfectly competitive markets where competition is weakened by product differentiation and firms' incentives to appropriate product surpluses are too strong. On top of the standard distortions, namely, a price distortion leading to pricing above marginal costs and entry distortion associated with the business stealing effect, income inequality and its resulting demand properties can be viewed as an additional mechanism, that may offset these distortions and reverse the canonical result.

To our knowledge, the economic literature does not address the question of whether the presence of income heterogeneity is aligned with the distortions existing in imperfectly competitive markets. Indeed, if income inequality works in the opposite direction, the actual gap between equilibrium and optimal numbers of varieties and firm outputs is lower compared to one predicted by a representative consumer setting. This would qualify policy recommendations. Furthermore, beyond some level of income inequality, the market outcome might provide too low product diversity.

To address this question, we rely on a general equilibrium setting where households differ in their income and consume a set of varieties produced by a monopolistically competitive sector. As in Dixit and Stiglitz (1977, section II), Zhelobodko *et al.* (2012), Mrazova and Neary (2017), and Dhingra and Morrow (2019), households are endowed with non-homothetic additive preferences that feature both variable elasticity of utility and elasticity of demand. We focus on subconvex demands with decreasing elasticity of utility. The latter is considered as the most plausible consumer behavior (Vives 2001, ch. 6) and implies that love for variety increases with consumption level and, therefore, with household income. Subconvex demands are characterized by decreasing demand elasticity and give rise to pro-competitive effects which is consistent with the empirical findings (Syverson, 2007; De Loecker *et al.*, 2016). We assume an arbitrary distribution of household labor productivity which generates income inequality. Then, the *market demand elasticity* faced by firms is a weighted average of the *household demand elasticities*.

Our aim is to analyze how the gap between market outcome and socially optimal allocations (both first- and second-best) varies with the level of income inequality. We capture the latter through the mean-preserving changes of the income distribution. There are two reasons to focus on mean-preserving changes. First, since total labor endowment is preserved, we sterilize market size effects and isolate the pure effect of heterogeneity in labor endowments. Second, it allows us to compare an economy of heterogeneous households with a benchmark economy of homogeneous households who share the average income.

We first study how the gap between market equilibrium and unconstrained optimum (first-best) changes with mean-preservation. We show that the effect of mean-preservation hinges on the property of the convexity of the household demand (Aguirre *et al.*, 2010, Mrazova and Neary, 2017). In particular, an increase in income inequality works in the opposite direction to the imperfect competition distortion when this convexity is an increasing function of consumption. This is the case when low-income households have individual demand elasticity that is more sensitive to income change than high-income ones.[1] Then, as a market demand elasticity is a weighted average of household demand elasticities, an increase in demand elasticity of low-income households outweighs the decrease in the demand elasticity of high-income households. As a result, the market demand elasticity increases implying lower prices and, therefore, the exit of firms. The market outcome is thus shifted towards the unconstrained optimal level of product diversity. This result suggests that the analysis of product market interventions is incomplete if it does not take into account the effect of income inequality. Indeed, the gap between market outcome and social optimum might be narrower than predicted by representative consumer settings. Furthermore, there may exist a level of income heterogeneity such that the market delivers both socially optimal product diversity and firm output, which renders policy intervention void.

In the unconstrained optimum, the planner organizes production in firms. However, since households have identical preferences, it also allocates a symmetric consumption across all households, which creates a redistribution tension between the market equilibrium and social optimum. To lift this tension, we study an *inequality-constrained optimum* in which the planner allocates consumption according to household contributions to production. For instance, this can be justified to reward individual effort, educational achievement, etc. We show that a mean-preserving spread of income distribution also reduces the gap between equilibrium and inequality-constrained optimum under the same condition of an increasing convexity of individual demand and sufficiently subconvex demands. The latter corresponds to "sub-pass-through" (Mrazova and Neary, 2017) meaning that pass-through is less than dollar-for-dollar. We ultimately show that this inequality-constrained optimum is equivalent to the Dixit and Stiglitz' (1977) constrained optimum (second-best) where the social planner chooses the consumption bundles and the number of firms, while firms are required to balance their budget.

To quantify the effects of income distribution, we calibrate the model to the US economy under various additive preferences and alter the income distribution preserving its mean. This exercise confirms the above theoretical findings and demonstrates the strong impacts of the level of income inequality on the gap between equilibrium and optimal allocations. We observe similar gaps for different demand systems. For instance, there are $10\% - 12\%$ more firms in equilibrium compared to optimum for actual US income distribution, whereas there are about 40% more firms if we rely on a representative consumer framework. We also report income inequality levels that imply that the market outcome matches the optimal product diversity. Those levels vary across demand systems, although, for all tested demands, income inequality must be larger than the actual US one to provide optimal product diversity.

To sum up, the paper highlights the importance of income inequality in the assessment

[1]While we find such behavior plausible, it is consistent with empirical findings about the inverse relationship between prices and income inequality (Bekkers *et al.*, 2012).

of firms' behavior and product diversity in monopolistically competitive markets. It qualifies the canonical analysis of industry and industrial policies relying on a representative consumer setting. Those policies can be ill-designed because income inequality significantly affects the gap between the optimal and equilibrium numbers of firms.

Our paper relates to the literature on the welfare implications of monopolistic competition. This literature shows how the efficiency of resource allocation depends on the trade-off between quantity and product variety (Spence, 1976; Dixit and Stiglitz, 1977; Venables, 1985; Mankiw and Whinston, 1986; Stiglitz, 1986). A majority of the economics literature relies on the demand structures with constant elasticity of substitution (CES). Although, the CES demand structure exploits the remarkable analytical properties (Dixit and Stiglitz, 1977, section I), market outcome is socially optimal whereas inequality does not affect prices, firm scales, and product diversity. We then employ a demand structure with the variable demand elasticity or, equivalently, the variable elasticity of substitution (VES) as introduced by Dixit and Stiglitz (1977, section II) and further studied by Zhelobodko *et al*. (2012), Parenti *et al.* (2017), Dhingra and Morrow (2019), and many others. This framework not only captures variation in prices and markups with market size, but also allows to study the effect of income heterogeneity.

On the one hand, the observation of firm heterogeneity has motivated researchers to introduce productivity heterogeneity in the monopolistic competition framework (Melitz, 2003). Dhingra and Morrow (2019) analyze the social trade-off between firm scale and product diversity in the context of such heterogeneity and VES demands. Egger and Huang (2023) extend Dinghra and Morrow's discussion about optimal product diversity for a subset of VES demand in the context of open economies. In a quantitative analysis on China and its trade partner, they spot welfare distortions between 7 and 10%. As to intersectoral distortions, Behrens *et al.* (2020) quantify them to more than 5% losses of the welfare within a multi-sectoral setting with VES demands. On the other hand, the observation of income heterogeneity motivates Foellmi and Zweimüller (2004) and Kichko and Picard (2023) to undertake a positive analysis of income inequality in monopolistically competitive markets and its application to trade. This paper focuses on the normative analysis and discusses the effect of income inequality on the gap between market outcome and socially optimal allocations. It relies on the behavior of the convexity of the direct demand, which is apparent in the literature about the effects and the comparative statics of imperfect competition (Mrazova and Neary, 2017) and welfare analysis of price discrimination (Aguirre *et al.,* 2010).

The paper is organized as follows. Section 2 presents the model. Section 3 discusses the unconstrained optimum while Section 4 examines the two kinds of constrained optima. Section 5 quantifies theoretical results in a calibration based on the US economy. Section 6 concludes. Mathematical details are relegated to Appendices.

2 Model

The economy includes a mass L of households. Each household h is endowed with a $s_h > 0$ labor productivity units that are distributed with a cumulative probability distribution function G. Thus, households differ in their productivity. By the choice of numéraire, we normalize wage per labor unit to one, so that s_h stands for the household's income. In what follows, a variable without subscript h denotes its average over households. The average household productivity is then given by $s = \int s_h \mathrm{d}G$ where we use $\mathrm{d}G$ as a short notation for $\mathrm{d}G(s_h)$ when it does not bring confusion.

2.1 Consumers

Households consume a set of symmetric varieties $\omega \in [0, n]$ where n denotes the endogenous number of varieties, i.e., product diversity. Each household with s_h labor units is endowed with an additively separable utility

$$U = \int_0^n u(x_h(\omega))\mathrm{d}\omega,$$

which it maximizes subject to its budget constraint $\int_0^n p(\omega)x_h(\omega)\mathrm{d}\omega = s_h$, where $p(\omega)$ and $x_h(\omega)$ are the price and its consumption of variety ω. The utility function is increasing and concave, $u''(x_h) < 0 < u'(x_h)$. As in the literature, we assume that the lowest labor productivity is large enough to ensure a positive equilibrium consumption for each available variety.[2] The first order condition yields the inverse demand function $p(\omega) = \lambda_h^{-1} u'(x_h(\omega))$, where λ_h is the multiplier of the household's budget constraint. Then, the household demand is given by

$$x_h(\omega) \equiv v(\lambda_h p(\omega)), \tag{1}$$

where v is the inverse function of u'.

Because of the product symmetry, we omit the reference to ω and define the *household demand elasticity* for each product as

$$\varepsilon_h = \varepsilon(x_h) \equiv -\frac{u'(x_h)}{x_h u''(x_h)}. \tag{2}$$

We focus on subconvex demands (Mrazova and Neary, 2017), $\varepsilon_h' < 0$, which feature the inverse relationship between average consumption and household demand elasticity[3] and

[2]This assumption ensures that equilibrium prices lie below the demand choke prices when the latter exist.

[3]This assumption matches Marshall's Second Law of Demand (1936), which states that demand becomes less elastic at higher prices. It is congruent with the empirical literature (Syverson, 2007; De Loecker *et al.*, 2016).

gives rise to pro-competitive effects.[4] We also define the *elasticity of utility* as

$$\eta_h = \eta(x_h) \equiv \frac{x_h u'(x_h)}{u(x_h)} \in (0,\ 1). \tag{3}$$

With a higher elasticity of utility, households value more quantity than product diversity. The love for variety is, therefore, measured by $1-\eta_h$.[5] In this paper, we assume increasing love for variety so that η_h is a decreasing function of consumption. This is considered the most plausible case in economic theory (Vives, 2001).

2.2 Firms

Labor is the only production factor. Each firm produces a single variety ω and chooses the price $p(\omega)$ that maximizes its profit $\pi(\omega) = L\int(p(\omega)-c)x_h(\omega)\mathrm{d}G - f$. In this expression, c and f are the firm's marginal and fixed labor requirements. Plugging the demand function (1) into profit and differentiating it, we obtain the profit-maximizing price

$$p = \frac{\varepsilon}{\varepsilon - 1}c, \tag{4}$$

where we omit the reference to ω and

$$\varepsilon \equiv \frac{\int x_h \varepsilon_h \mathrm{d}G}{\int x_h \mathrm{d}G}, \tag{5}$$

is the *market demand elasticity*. We assume that the second order condition holds.

2.3 Equilibrium

An equilibrium is defined as the set of consumption x_h, price p, number of firms n, and firm output y that are consistent with the household budget constraints

$$npx_h = s_h, \tag{6}$$

the firm's optimal price

$$p = \frac{\varepsilon}{\varepsilon - 1}c, \tag{7}$$

the zero-profit condition (free entry), the product and labor market clearing conditions

$$p = \frac{f}{y} + c, \qquad y = Lx, \qquad Ls = n(f + cy). \tag{8}$$

[4]Zhelobodko *et al.* (2012) show that market enlargements lead to additional entry and reduce equilibrium prices, therefore, mimic pro-competitive behaviors for subconvex demands.

[5]As known in the literature, utility changes are given by $\mathrm{d}lnU_h = \eta(x_h)\mathrm{d}\ln x_h + \mathrm{d}\ln n = (1-\eta(x_h))\mathrm{d}\ln n$ where the second equality holds because budget constraint: $\mathrm{d}\ln x_h + \mathrm{d}\ln n = 0$. Therefore, a household does not value an increase in product diversity if $\eta(x_h) = 1$ and fully values it if $\eta(x_h) = 0$.

By the Walras law, one identity is redundant. Under subconvex demands, there exists a unique equilibrium if $\varepsilon(0) > 1$ (Kichko and Picard, 2023).

Combining those equilibrium equations yields two identities determining the number of firms and the firm output:

$$n^e = \frac{Ls}{f + cLx^e}, \tag{9}$$

$$\frac{1}{\varepsilon^e} = \frac{f}{f + cLx^e}, \tag{10}$$

where we denote equilibrium variables by the superscript ^{e}.

3 Unconstrained social optimum

Monopolistically competitive markets feature distortions associated with pricing above marginal cost and business-stealing effect. These distortions lead to both a non-optimal number of firms and firm output in the market. In this section, we investigate how heterogeneity of household labor-productivity alters this canonical result. In particular, may such heterogeneity offset imperfect competition distortions and lead to optimal product diversity in equilibrium?

To address this question, we study the problem of a planner who chooses the household consumption $\{x_h\}$ and number of varieties n that maximize aggregate welfare

$$Ln \int u(x_h)\mathrm{d}G, \tag{11}$$

under the resource constraint

$$L \int s_h \mathrm{d}G = n\left(f + cL \int x_h \mathrm{d}G\right). \tag{12}$$

Pointwise maximizing the Lagrangian function of this problem and eliminating the Lagrange multiplier give the following first-order condition:

$$u'(x_h) = \frac{cL \int u(x_l)\mathrm{d}G(s_l)}{f + cL \int x_l \mathrm{d}G(s_l)}.$$

Since the right-hand side is independent of household income s_h, each household receives the same consumption $x_h = x^u \; \forall s_h$. This is because the planner considers the total labor endowment rather than individual endowments of labor units. Therefore, the optimal household consumption does not depend on income distribution. Using $x_h = x^u$ in the previous identity, the unconstrained optimal consumption x^u is given by

$$1 - \eta(x^u) = \frac{f}{f + cLx^u}, \tag{13}$$

Because $\eta' < 0$ and $\eta \in (0,\, 1)$, equation (13) has a unique interior solution. Finally, using the resource constraint, the optimal number of firms is given by

$$n^u = \frac{Ls}{f + cLx^u}. \tag{14}$$

At the household level, as the social planner allocates equal consumption x^u across all households, she implements a redistribution of goods across households compared to equilibrium. At the aggregate level, comparison between equilibrium condition (10) and (13) shows that $x^e < x^u$ if and only if

$$1 - \eta(x^u) < \frac{1}{\varepsilon^e}. \tag{15}$$

Under this condition, each firm output in equilibrium $y^e = Lx^e$ is smaller than the optimal output $y^u = Lx^u$. Then, $x^e < x^u$ implies that equilibrium number of firms (9) exceeds optimal number (14), $n^e > n^u$.

In the absence of household heterogeneity ($s_h = s$), we show in Appendix A that there is excess entry in equilibrium while firm output is below its optimal level (Dixit and Stiglitz, 1977, section II). This result is consistent with (15) as $\eta_h' < 0$ implies $1 - \eta_h < 1/\varepsilon_h$ for any $x_h > 0$.

There are different ways to transform the distribution of household labor endowment in order to study the effects of income inequality. We focus on mean-preserving changes for two reasons. First, because mean-preserving changes preserve total labor endowment, Ls, we sterilize market size effects and isolate the pure effect of heterogeneity in labor endowments. Second, considering mean-preserving changes allows us to compare an economy of households with heterogeneous productivity with a benchmark economy of homogeneous households who share the same labor endowment s. In this case, two economies have the same total labor endowment, Ls, therefore, the differences between the two arise solely due to the presence of inequality in labor productivity.

Since x^u is independent of the distribution of labor productivity, the left-hand side of (15) is also invariant to it. However, the right-hand side of (15) varies with income inequality level because the latter alters household consumption and demand elasticities, ε_h and, therefore, the market demand elasticity, ε^e, as captured by (5). We show in Appendix A that a mean-preserving change in the distribution of labor endowment impacts the right-hand side of (15) according to the properties of the *convexity of individual demand function* (Aguire *et al.*, 2010; Mrazova and Neary, 2017):

$$r_h \equiv \frac{u'(x_h)u'''(x_h)}{(u''(x_h))^2}. \tag{16}$$

We prove the following proposition.

Proposition 1. *If $r_h' > 0$, starting from homogeneous labor endowment, a mean-preserving spread of productivity distribution reduces equilibrium product diversity and augments equilibrium firm output towards the unconstrained optimal levels. The opposite holds for $r_h' < 0$. Finally, if $r_h' = 0$, the level of heterogeneity does not affect the gap between equilibrium and unconstrained optimal* *allocations.*

Proof. See Appendix A.

Proposition 1 provides two contributions. First, if $r_h' > 0$, a mean-preserving spread of the distribution of labor productivity relaxes the condition (15) so that the gap between the equilibrium and socially optimal allocations shrinks. To provide intuition for this result, we differentiate household demand elasticity (5) and obtain

$$\varepsilon_h' = -\frac{1}{x_h}\left(1 + \varepsilon_h - r_h\right) < 0. \tag{17}$$

This expression shows that ε_h' decreases with household consumption x_h, increases with household demand elasticity ε_h, and decreases with convexity of demand r_h. First, the budget constraint $x_h = s_h/np$ implies that consumption x_h increases with household income s_h. Second, subconvexity yields demand elasticity ε_h decreases with income. Therefore, $r_h' > 0$ implies that the right-hand side of (17) unambiguously increases with individual income. In other words, low-income households have individual demand elasticities that are more sensitive to income change than high-income ones. Then, under a mean-preserving spread of income distribution, the increase in the demand elasticity of low-income households is stronger than the drop in the demand elasticity of high-income ones. As the market demand elasticity (5) is a weighted average of household demand elasticities, it increases with the mean-preserving spread. This pushes prices down, and therefore, entices the exit of firms and leads to narrower product diversity. Household heterogeneity then acts as a force working in the direction opposite to the distortions associated with imperfect competition. As a result, those effects may compensate each other for some level of income inequality. In this situation, the market provides both optimal product diversity and firm output. If income heterogeneity further increases, then n^e falls below n^u, so that the market provides too few varieties, while firm output is too large. In Section 5, we provide a quantification exercise that illustrates these theoretical findings and reports households' inequality levels that deliver optimal product diversity.

Second, Proposition 1 shows excess entry for any arbitrary level of inequality if $r_h' < 0$. Indeed, in this case, stronger income heterogeneity strengthens condition (15), and, consequently, pushes the equilibrium number of firms and firm output further away from their optimal levels.

Finally, if $r_h' = 0$, demand functions are locally linear in income (Pollak, 1971). Pollak family includes four classes of utility functions: constant elasticity of substitution (CES), constant absolute risk aversion (CARA), logarithmic, and quadratic. Under CES utility, $u(x_h) = (\alpha/(\alpha-1))x_h^{(\alpha-1)/\alpha}$ with $\alpha > 1$, both sides of (15) are equal to the constant $1/\alpha$. Thus, condition (15) holds as equality for any level of income heterogeneity so that the market equilibrium and unconstrained optimum coincide. For other Pollak preferences, the property $r_h' = 0$ implies that the right-hand side of (15) is not affected by a mean-preserving change of income distribution. Since the market yields excessive entry in the absence of heterogeneity, it does so for any level of heterogeneity in labor endowments. Intuitively, local demand linearity in income implies that mean-preserving changes in the distribution of household labor productivity reshuffle consumption in a way that market demand is unaltered and, therefore, market demand elasticity remains constant. As a result, firms' prices and the number of firms do not change. In other words, the level of heterogeneity does not affect the degree of competition in the market. It, therefore, does not change the gap between equilibrium and unconstrained optimal allocation of resources.

4 Constrained social optima

In the unconstrained optimum framework studied in the previous section, the planner "commands and controls" the production system by gathering all labor resources and allocating them to firms for production. Because households have identical preferences and the planner does not consider individual labor endowments, she allocates the same consumption to every household. By contrast, in the equilibrium, consumption patterns differ across households once they have different labor endowments. The presence of heterogeneity, therefore, creates a redistribution tension between the market equilibrium and unconstrained optimal allocation, which is worth discussing.

Thus, the first objective of this section is to study the planner's choice when this tension is lifted. This is the case when social planner rewards households according to their productivity endowments, as it could be the case for educational investments. We hence study an inequality-constrained optimum in which the social planner is constrained to allocate a consumption level that is proportionate to each household labor endowment.

The second objective is to study the constrained optimum (second-best) without lump-sum transfers to firms as in Dixit and Stiglitz (1977). Since the implementation of the unconstrained optimum requires transfers to firms, it causes conceptual and practical difficulties for policymakers. As a consequence, we study constrained optimum à la Dixit-Stiglitz where the planner requires firms to balance their budgets. We show that the latter is equivalent to the inequality-constrained optimum.

4.1 Inequality-constrained optimum

In the inequality-constrained optimum, a social planner maximizes the total welfare subject to the resource constraint and the distribution of labor endowments. Since goods are symmetric, it allocates the same consumption of each good to households with the same endowment s_h, i.e., $x_h(\omega) = x_h$. The planner chooses household consumption $\{x_h\}$ and number of varieties n that maximize the welfare function

$$Ln \int u(x_h) \mathrm{d}G \tag{18}$$

subject to the resource and inequality constraints

$$L\int s_h \mathrm{d}G = n\left(f + cL\int x_h \mathrm{d}G\right), \qquad \text{and} \qquad \frac{x_h}{x_l} = \frac{s_h}{s_l}, \quad \forall s_h, s_l. \tag{19}$$

Note that the presence of the second constraint in (19) is the only difference with the unconstrained optimum problem studied in Section 3. Then, when households are homogeneous, $x_h^e = x^e$ and $x_h^c = x^c$, the second constraint in (19) vanishes, which yields equivalence of the constrained and unconstrained optimum problems. This implies that $n^u = n^c$ and $x^u = x^c$, where the superscript ^{c} denotes the inequality-constrained optimum. Therefore, as shown in Section 3, the equilibrium with homogeneous households includes too many varieties and too small firms compared to this constrained optimum.

For heterogeneous households, we show in Appendix B that the solution to social planner problem (18)-(19) is given by

$$1 - \eta^c = \frac{f}{f + cLx^c}, \qquad \text{with} \qquad 1 - \eta^c \equiv \frac{\int u(x_h^c)(1 - \eta(x_h^c))\mathrm{d}G}{\int u(x_h^c)\mathrm{d}G}, \tag{20}$$

where x^c is the average consumption over households, and $1 - \eta^c$ an average love for variety. Condition (20) highlight the difference between the optimal allocations in the settings with heterogeneous income and representative consumer. In the latter, the social planner balances variety versus production according to the love for variety of the representative household with average labor endowment. In the former, it makes the balance according to the average love for variety, where contributions of different income groups vary with income.

Comparing (10) and (20) shows that there is excess entry in equilibrium ($n^e > n^c$) and firm output is too low ($x^e < x^c$) compared to the unconstrained optimum, if and only if

$$1 - \eta^c < \frac{1}{\varepsilon^e}. \tag{21}$$

When (21) holds as equality, the equilibrium number of firms coincides with the constrained optimum. In this situation, firm output and household consumption are the same in equilibrium and constrained optimum. The market outcome is optimal at the aggregate level because the number of varieties and the firm output are optimal. It is also optimal at the household level because it entices the optimal consumption bundles. As a result, there is an absence of redistribution tension between the market equilibrium and optimal allocation.

As in the first-best, (21) holds in the absence of income heterogeneity. In this case, a mean-preserving spread of household labor endowment distribution has the following effects on the gap between equilibrium and constrained optimum. On the one hand, as discussed in Section 3, under $r_h' > 0$, the market elasticity ε^e increases, therefore, the right-hand side of (21) diminishes. This situation corresponds to a decreasing equilibrium number of firms n^e.

On the other hand, as the average love for variety $1 - \eta^c$ in (20) varies with mean-preserving changes in income distribution, the constrained optimal allocation also varies with it. We show in Appendix C that a mean-preserving spread of labor endowments increases the average love for variety $1 - \eta^c$ if

$$\varepsilon_h > r_h, \text{ or equivalently, } -x_h\varepsilon'_h > 1. \tag{22}$$

This sufficient condition ensures that a mean-preserving spread increases love for variety of high-income groups more than it decreases it for low income groups, which ensures that the average love for variety rises. The mean-preserving spread therefore implies a reduction in the optimal average consumption x^c due to (20) and, thus, a rise of the optimal number of firms n^c. As a result, it shrinks the gap between equilibrium and optimal numbers of firms. Note that (22) is more stringent than the condition for subconvexity (17) which implies $-x_h\varepsilon'(x_h) > 0$. The former, therefore, requires strong enough demand subconvexity or, put differently, sufficiently strong pro-competitive effects (Zhelobodko *et al.*, 2012). Under (22), pass-through is less than dollar-for-dollar, $\mathrm{d}p/\mathrm{d}c < 1$. Mrazova and Neary (2017) refer to this property as "sub-pass-through" and show that it holds for a wide range of subconvex demands.[6]

To sum up, under (22) and $r'_h > 0$, condition (21) becomes more constraining under mean-preserving spread and the gap between equilibrium and constrained optimum shrinks. In other words, the constrained optimal number of firms rises whereas the equilibrium number falls. Output levels move in opposite directions. Then, the market may provide optimal product diversity and firm output at some level of income inequality.

Note that both (22) and $r'_h > 0$ are sufficient conditions for a gap reduction between equilibrium and constrained optimal allocations. For instance, in our calibration exercises in Section 5, demand functions exhibit both $r'_h > 0$ and $r'_h < 0$, while (22) always holds. It shows, however, that the gap shrinks with higher income inequality for both $r'_h > 0$ and $r'_h < 0$. When $r'_h < 0$, mean-preserving spread reduces the gap because an increase in the optimum number of firms overcomes an increase in the equilibrium number of firms. We summarize our findings in the following proposition.

Proposition 2. *Under* (22) and $r'_h > 0$*, a mean-preserving spread of labor endowment distribution reduces the gap between equilibrium and constrained optimal allocations.*

In the special case of Pollak preferences, changes in income distribution do not affect prices and, therefore, market demand elasticities, which preserves the right-hand side of (21). For CES preferences, the left-hand side also does not vary, so they yield the same allocation in equilibrium and constrained optimum. By contrast, the left-hand side of (21) varies with household inequality for other Pollak preferences. In Appendix C, we show that, under quadratic preferences, a mean-preserving spread of income distribution always diminishes the gap between equilibrium and constrained optimal allocations. Furthermore, this result holds for CARA preferences if $\varepsilon_h > 1\ \forall h$. Finally, the impact of mean-preservation is ambiguous for logarithmic preferences. We complement those theoretical findings by providing a

[6] To be precise, under subconvex demands, a profit-maximizing equilibrium requires $\varepsilon > 1$ and $r < 1 + \varepsilon$, whereas "sub-pass-through" restricts this set to $\varepsilon > 1$ and $r < \varepsilon$.

calibration exercise for all Pollak preferences in Section 5. It also uncovers the levels of income inequality for equilibrium to coincide with the constrained optimum.

4.2 Dixit and Stiglitz' second-best

Dixit and Stiglitz (1977) and followers motivate the constrained optimum (second-best) by the absence of lump sum transfers to firms. In this case, the planner chooses household consumption $\{x_h\}$ and number of varieties n that maximize the welfare function $Ln \int u(x_h)\mathrm{d}G$ subject to household budget constraints and zero-profit condition:

$$npx_h = s_h \qquad \text{and} \qquad \pi = L\int (p-c)x_h \mathrm{d}G - f = 0. \tag{23}$$

This problem is equivalent to the inequality-constrained optimum discussed in Section 4.1. To show this, we proceed in two steps. First, using the household budget constraint (23) to replace p in zero-profit condition yields

$$Ls = n(Lcx + f),$$

which is the resource constraint in (19). Second, taking the ratio of the budget constraints (23) for two income groups, s_h and s_l, leads to

$$\frac{x_h}{x_l} = \frac{s_h}{s_l}, \quad \forall s_h, s_l,$$

which is the endowment distribution constraint in (19). Then, constraints (23) are equivalent to (19), therefore, two constrained optimum problems are also equivalent. In other words, household consumption choices imply proportionality between consumption and labor endowment while the use of labor by firms is the same as the one made by the planner.

Then, we can compare our one-sector economy with the two-sector economy results by Dixit and Stiglitz (1977). As households are homogeneous in the latter, for that comparison we set $s_h = s$ such that households share the same labor endowment s. Then, homogeneity in household income vanishes the endowment distribution constraint in (19). As a result, the constraint and unconstrained optima coincide for households with homogeneous income, $n^c = n^u$. In contrast to that, the number of firms in the constrained optimum is larger than in the unconstrained optimum in Dixit and Stiglitz, $n^c > n^u$. As the low-tier utility is nonhomothetic additive in both settings, the difference stems from the presence of the second sector which creates a miss-allocation of resources across sectors in the constrained optimum compared to the unconstrained one. The reason is that nonhomothetic additive low-tier utility leads to that miss-allocation even for Cobb-Douglas upper-tier utility.

5 Quantification

Our theoretical findings provide conditions for higher income inequality to reduce the gap between equilibrium and social optimal allocations. As a result, there may exist income inequality levels such that the market outcome coincides with social optima (constrained or constrained). Those levels crucially depend on the underlying preferences and income distribution. We then quantify our model for the US income distribution and production characteristics to uncover those levels for various preferences. This exercise also provides estimations for the gap between social optimal and actual allocations and shows how this gap varies with income inequality level. Finally, it estimates biases induced by the representative consumer assumption.

Towards these goals, we calibrate the economy to the US industry structure with 2.22 million firms (above 5 employees) and 148 million workers. The distribution of household income is approximated by a log-normal distribution[7] with a mean of USD $97,192$ and a standard deviation, std, of USD $93,431$ in 2022. We use the standard deviation as a measure of income inequality caused by mean-preserving changes.

We first discuss our findings for Pollak preferences. Table 1 shows how the equilibrium, unconstrained, and constrained optimal numbers of firms vary with mean-preserving changes in the income distribution. The rows show the same economy subject to mean-preservation, where the standard deviation of income distribution (std) is set to 0, 0.25,1, 0.5, 2 and 4 times the calibrated one. The fourth row, therefore, reports the US 2022 benchmark economy with the observed standard deviation of income distribution (1*std). The columns describe the simulated numbers of firms (millions) in the equilibrium, n^{e}, unconstrained optimum, n^{u}, and constrained optimum, n^{c}. The four panels report those numbers for the Pollak utility functions: $u(x_h) = (\alpha/(\alpha-1))x_h^{(\alpha-1)/\alpha}$ for CES, $u(x_h) = 1 - \mathrm{e}^{-\alpha x_h}$ for CARA, $u(x_h) = x_h(\alpha - x_h)$ for quadratic (QUAD), and $u(x_h) = \log(1 + x_h/\alpha)$ for logarithmic (LOG) utility. The last row reports the utility parameter α that matches the benchmark case with an elasticity of substitution equal to 7, as observed in empirical works (Bergstrand *et al.*, 2013).

The results in Table 1 are consistent with our theoretical findings. First, the number of firms is the same for all three allocations under CES preferences. Second, for the other Pollak preferences, the equilibrium and unconstrained optimal numbers of firms are not affected by mean-preserving changes in income distributions. In this case, the market outcome always provides too many varieties (Proposition 1). A mean-preserving spread of income distribution, nevertheless, reduces the gap between equilibrium and constrained optimum. Furthermore, we compute that the equilibrium number of firms exactly matches the constrained optimal number if the standard deviation of the income distribution is 1.41***std** for CARA, 1.04***std** for quadratic utility, and 1.83***std** for logarithmic utility. For all three demand systems, the actual inequality level is too low to provide optimal product diversity (although it

[7]Aitchinson and Brown (1957) and followers show that the log-normal distribution gives a reasonably accurate approximation of income distributions.

	CES	CARA			QUAD			LOG		
St. dev. (std)	$n^e = n^u = n^c$	n^e	n^u	n^c	n^e	n^u	n^c	n^e	n^u	n^c
0*std	2.220	2.220	1.579	1.579	2.220	1.578	1.578	2.220	1.584	1.584
0.25*std	2.220	2.220	1.579	1.614	2.220	1.578	1.622	2.220	1.584	1.612
0.5*std	2.220	2.220	1.579	1.711	2.220	1.578	1.749	2.220	1.584	1.687
1*std	2.220	2.220	1.579	1.991	2.220	1.578	2.178	2.220	1.584	1.897
2*std	2.220	2.220	1.579	2.500	2.220	1.578	3.356	2.220	1.584	2.276
4*std	2.220	2.220	1.579	3.069	2.220	1.578	5.840	2.220	1.584	2.715
α	7.00	3.81			0.60			0.23		

Table 1: Effect of mean-preservation of income distribution for US economy, Pollak utility functions.

is close to it for the quadratic utility). Also, the rate at which the gap between equilibrium and constrained optimum changes significantly varies across the Pollak preferences.

Furthermore, Table 1 illustrates the bias implied by the assumption of a representative consumer in the assessment of market efficiency. For instance, for CARA utility, the market outcome yields an 11.5% excess of firms compared to the second-best for the actual income distribution (the fourth row, 1**std), while economy with homogeneous incomes (the first row, 0**std) shows that the market provides a 40.6% excess of firms. Similar differences exist for logarithmic and quadratic utilities. This suggests that ignoring income inequality leads to significant overestimations of market inefficiencies.

Next, Table 2 presents simulation results for two classes of preferences that generate variations in the equilibrium number of firms under mean-preservation. Namely, we provide quantification for constant proportional pass-through (CPPT) utility defined as $u(x_h) = \int_0^{x_h} \left(\beta + \xi^{-\alpha}\right)^{-1/\alpha} / \xi \mathrm{d}\xi$ (Mrazova *et al.*, 2017) and for constant superelasticity of demand (CSED), described by $u(x_h) = \int_0^{x_h} \exp\left(-\xi^{\alpha}/\alpha\beta\right) \mathrm{d}\xi$ (Gopinath and Itskhoki, 2010). We selected values for the demand parameters α and β to match the benchmark economy with the elasticity of substitution ε of 7 and pass-through elasticity $\mathcal{E}$ of 0.4 (De Loecker *et al.*, 2016) and 0.6 (Amiti *et al.*, 2019).[8] While estimations of pass-through elasticity vary significantly across studies, those two values deliver contrasting results.

Table 2 shows that for CPPT and CSED, equilibrium and constrained optimal numbers of firms vary with income inequality, while it does not in the unconstrained optimum. We start with the pass-through elasticity $\mathcal{E} = 0.4$. This is the case of $r_h' > 0$, therefore, the equilibrium number of firms decreases with mean-preserving spread. As in Proposition 1, starting from the absence of income heterogeneity, the gap between the numbers of firms in equilibrium and unconstrained optimum falls with higher income inequality. Finally, as in Proposition 2, the gap between the numbers of firms in equilibrium and constrained optimum also decreases.

[8]Campa and Golberg (2005) suggest average values of 0.46 and 0.64 for the short and long terms, which closely align with the values selected for this calibration.

	CPPT						CSED					
ε	7						7					
$\mathcal{E}$	0.4			0.6			0.4			0.6		
St. dev. (std)	n^e	n^u	n^c	n^e	n^u	n^c	n^e	n^u	n^c	n^e	n^u	n^c
0*std	2.29	1.62	1.62	2.15	1.58	1.58	2.27	1.60	1.60	2.13	1.56	1.56
0.25*std	2.28	1.62	1.65	2.16	1.58	1.60	2.26	1.60	1.64	2.14	1.56	1.59
0.5*std	2.27	1.62	1.74	2.18	1.58	1.66	2.25	1.60	1.75	2.16	1.56	1.65
1*std	2.22	1.62	1.98	2.22	1.58	1.83	2.22	1.60	2.06	2.22	1.56	1.85
2*std	2.12	1.62	2.40	2.31	1.58	2.15	2.16	1.60	2.61	2.34	1.56	2.32
4*std	2.00	1.62	2.86	2.44	1.58	2.54	2.08	1.60	3.2	2.52	1.56	2.71
α	1.21			0.76			1.12			0.68		
β	9.63			1.91			0.17			0.79		

Table 2: Effect of mean preservation of income distribution for US economy, non-Pollak utility functions.

Furthermore, the benchmark economy with the observed income distribution (1*std) provides too many varieties compared to both constrained and unconstrained optima. From this benchmark, stronger income heterogeneity decreases both the gap between the market and unconstrained optimal numbers of firms and the difference between the market and constrained optimal numbers of firms. However, the former gap decreases more slowly and vanishes for very large degrees of income inequality. To be precise, our computations show that the market outcome coincides with the unconstrained optimum for the standard deviation of 10.5*std for CPPT (14.5*std for CSED). By contrast, the equilibrium yields the constrained optimal number of firms for 1.43*std of income distribution for both CPPT and CSED. Although income inequality is relatively high in the US, it is not high enough to reach constrained optimum product diversity.

Table 2 also highlights that the ordering of equilibrium and optimal numbers of firms varies with income heterogeneity. In the benchmark economy (1*std), we observe $n^u < n^c < n^e$. However, for standard deviations larger 1.43*std, the ordering becomes $n^u < n^e < n^c$. In this case, the market provides an insufficient number of firms compared to the constrained optimum.

The first row of Table 2 illustrates the bias implied by the assumption of a representative individual endowed with the average US labor supply. The gaps between equilibrium and constrained optimal numbers of firms are much larger in an economy with representative individual than with the observed US income distribution. One infers a 41% excessive entry of firms in the former economy and only 12% in the latter for CPPT.[9] Ignoring income inequality therefore leads to strong biases in the assessment of market efficiency and elaboration of market policies.

Finally, we discuss the case with pass-through elasticity $\mathcal{E} = 0.6$. The main difference with the case of $\mathcal{E} = 0.4$ lies in the fact that $r'_h < 0$. Hence, consistent with the above theory, the

[9] One respectively gets 42% and 7% for CSED.

equilibrium number of firms increases with a mean-preserving spread of income distribution. This widens the gap between equilibrium and unconstrained optimum as predicted by Proposition 1. What the quantitative exercise adds is that n^c increases at a greater rate than n^e. As discussed in Section 3.2, this situation suggests significantly more elastic demand elasticity for low-income households. Then, there exists a level of income inequality for which the equilibrium matches the constrained optimum. This occurs when the standard deviation of income distribution is 3.03*std for CPPT (2.57*std for CSED). This inequality level is nevertheless significantly higher than the actual US level.

6 Conclusion

In this paper, we demonstrate the importance of accounting for income inequality for comparison between market outcome and optimum allocations. We investigate in details how equilibrium and social optima respond to changes in income inequality and provide conditions for a decrease in the gap between them. For instance, when richer individuals' expenditures are less sensitive to price change compared to poorer ones, a mean-preserving spread of income distribution reduces the gap between market outcome and unconstrained optimal allocation.

Our quantification exercise calibrated to the US economy shows that this gap strongly depends on the income inequality level. While changes in the gap between market outcome and the first-best allocation crucially depend on the underlying demands, the impact of income inequality on the gap between equilibrium and the second-best is robust across demand systems used in calibration. To be precise, the market provides about $10\% - 12\%$ more firms whereas estimations based on representative consumer setting report a 40% more firms in equilibrium. The latter highlights possible strong biases in the assessment of market efficiency and elaboration of market policies relying on a representative consumer framework.

References

[1] Aitchison J. and J. A. C. Brown: "The Lognormal Distribution, with special reference to its uses in economics." University of Cambridge Department of Applied Economics Monograph No. 5. Cambridge University Press, 1957.

[2] Aguirre, Inaki, Simon Cowan, and John Vickers. 2010. "Monopoly price discrimination and demand curvature." *American Economic Review*, Vol. 100(4): 1601-15.

[3] Alessandria, George, and Joseph P. Kaboski. 2011. "Pricing-to-Market and the Failure of Absolute PPP." *American Economic Journal: Macroeconomics*, Vol. 3(1): 91-127.

[4] Amiti, Mary, Oleg Itskhoki, and Jozef Konings. 2019. "International shocks, variable markups, and domestic prices." *The Review of Economic Studies*, Vol. 86(6): 2356-2402.

[5] Behrens, Kristian, and Yasusada Murata. 2012. "Globalization and individual gains from trade." *Journal of Monetary Economics*, Vol. 59(8): 703-720.

[6] Bekkers, Eddy, Joseph Francois, and Miriam Manchin. 2012. "Import prices, income, and inequality." *European Economic Review*, Vol. 56(4): 848-869.

[7] Bergstrand, Jeffrey H., Peter Egger, and Mario Larch. 2013. "Gravity redux: Estimation of gravity-equation coefficients, elasticities of substitution, and general equilibrium comparative statics under asymmetric bilateral trade costs." *Journal of International Economics*, Vol. 89(1): 110-121.

[8] Bertoletti, Paolo, Federico Etro, and Ina Simonovska. 2018. "International trade with indirect additivity." *American Economic Journal: Microeconomics*, Vol. 10(2): 1-57.

[9] Broda, Christian, and David E. Weinstein. 2006."Globalization and the gains from variety." *The Quarterly journal of economics*, Vol. 121(2): 541-585.

[10] Bulow, Jeremy I., and Paul Pfleiderer. 1983. "A note on the effect of cost changes on prices." *Journal of Political Economy*, Vol. 91(1): 182-185.

[11] Campa, Jose Manuel, and Linda S. Goldberg. 2005. "Exchange rate pass-through into import prices." *Review of Economics and Statistics*, Vol. 87(4): 679-690.

[12] Choi, Yo Chul, David Hummels, and Chong Xiang. 2009. "Explaining import quality: The role of the income distribution." *Journal of International Economics*, Vol. 77(2): 265-275.

[13] Cowan, Simon. 2012. "Third-degree price discrimination and consumer surplus." *The Journal of Industrial Economics*, Vol. 60(2): 333-345.

[14] Dalgin, Muhammed, Vitor Trindade, and Devashish Mitra. 2008. "Inequality, Nonhomothetic Preferences, and Trade: A Gravity Approach." *Southern Economic Journal*: 747-774.

[15] De Loecker, Jan, Goldberg, Pinelopi K., Khandelwal, Amit K., and Pavcnik Nina. 2016. "Prices, markups, and trade reform." *Econometrica*, Vol. 84(2), 445-510.

[16] Dhingra, Swati, and John Morrow. 2019. "Monopolistic competition and optimum product diversity under firm heterogeneity." *Journal of Political Economy*, Vol. 127(1): 196-232.

[17] Dixit, Avinash K., and Joseph E. Stiglitz. 1977. "Monopolistic competition and optimum product diversity." *The American Economic Review*, Vol. 67(3): 297-308.

[18] Dynan, Karen E., Jonathan Skinner, and Stephen P. Zeldes. 2004. "Do the rich save more?" *Journal of Political Economy*, Vol. 112(2): 397-444.

[19] Egger, Peter, and Ruobing Huang. 2023. "Market Versus Optimum Diversity in Open Economies: Theory and Quantitative Evidence." CEPR DP18089.

[20] Egger, Hartmut, and Udo Kreickemeier. 2009. "Firm heterogeneity and the labor market effects of trade liberalization." *International Economic Review*, Vol. 50(1): 187-216.

[21] Falkinger, Josef, and Josef Zweimüller 1996. "The cross-country Engel curve for product diversification." *Structural Change and Economic Dynamics*, Vol. 7(1): 79-97.

[22] Feenstra, Robert C. 2003. "A homothetic utility function for monopolistic competition models, without constant price elasticity." *Economics Letters*, Vol. 78(1): 79-86.

[23] Feenstra, Robert C., Philip Luck, Maurice Obstfeld, and Katheryn N. Russ. 2018. "In search of the Armington elasticity." *Review of Economics and Statistics*, Vol. 100(1): 135-150.

[24] Felbermayr, Gabriel, Julien Prat, and Hans-Jorg Schmerer. 2011. "Globalization and labor market outcomes: Wage bargaining, search frictions, and firm heterogeneity." *Journal of Economic Theory*, Vol. 146(1): 39-73.

[25] Fieler, Ana Cecilia. 2011. "Nonhomotheticity and bilateral trade: Evidence and a quantitative explanation." *Econometrica,* Vol. 79(4): 1069-1101.

[26] Foellmi, Reto, and Josef Zweimüller. 2004. "Inequality, market power, and product diversity." *Economics Letters*, Vol. 82 (1): 139-145.

[27] Friedman, Milton. 1957. "A Theory of the Consumption Function." Princeton, NJ: Princeton University Press.

[28] Gopinath, Gita, and Oleg Itskhoki. 2010. "Frequency of price adjustment and pass-through." *The Quarterly Journal of Economics*, Vol. 125(2): 675-727.

[29] Head, Keith and Thierry Mayer. 2004. "The empirics of agglomeration and trade." *Handbook of Regional and Urban Economics*, Vol. 4, pp.2609-2669.

[30] Head, Keith and John Ries. 2001. "Increasing returns versus national product differentiation as an explanation for the pattern of US-Canada trade." *American Economic Review*, Vol. 91(4): 858–876.

[31] Helpman, Elhanan, Oleg Itskhoki, and Stephen Redding. 2010. "Inequality and unemployment in a global economy." *Econometrica*, Vol. 78(4): 1239-1283.

[32] Holmes, Thomas J. 1989. "The effects of third-degree price discrimination in oligopoly." *The American Economic Review*, Vol. 79(1): 244-250.

[33] Houthakker, Hendrik Samuel. 1957 "An international comparison of household expenditure patterns, commemorating the centenary of Engel's law." *Econometrica*: 532-551.

[34] Hummels, David, and Kwan Yong Lee. 2018. "The income elasticity of import demand: Micro evidence and an application." *Journal of International Economics*, Vol. 113: 20-34.

[35] Jones, Ronald W. 1965. "The structure of simple general equilibrium models." *Journal of Political Economy*, Vol. 73(6): 557-572.

[36] Keynes, John Maynard. 1936. "The general theory of employment, interest, and money", New York: Harcourt, Brace.

[37] Kuhn, Kai-Uwe, and Xavier Vives. 1999. "Excess entry, vertical integration, and welfare." *RAND Journal of Economics*, Vol. 30: 575–603.

[38] Mankiw, N. G., and M. D. Whinston. 1986. "Free Entry and Social Inefficiency." RAND J. Econ. 17 (1): 48–58.

[39] Matsuyama, Kiminori. 2000. "A ricardian model with a continuum of goods under non-homothetic preferences: Demand complementarities, income distribution, and north-south trade." *Journal of Political Economy*, Vol. 108(6): 1093-1120.

[40] McCarthy, Patrick S. 1996. "Market price and income elasticities of new vehicle demands." *The Review of Economics and Statistics*, 543-547.

[41] Mion, Giordano, and Nicholas Jacob. 2020. "On the productivity advantage of cities." CEPR DP 14644.

[42] Mrazova, Monika, and J. Peter Neary. 2017. "Not so demanding: Demand structure and firm behavior." *American Economic Review*, Vol. 107 (12): 3835-74.

[43] Mrazova, Monika, and J. Peter Neary. 2019. "IO for exports (s)." *International Journal of Industrial Organization*: 102561.

[44] Mrazova, Monika, J. Peter Neary, and Mathieu Parenti. 2021. "Sales and markup dispersion: Theory and empirics.", *Econometrica*, Vol. 89(4): 1753-1788.

[45] Nakamura, Emi, and Dawit Zerom. 2010. "Accounting for incomplete pass-through." *The Review of Economic Studies*, Vol. 77(3): 1192-1230.

[46] Neary, J. Peter. 2004. "Monopolistic competition and international trade theory," in The Monopolistic Competition Revolution in Retrospect, ed. by S. Brakman and B. J. Heijdra. Cambridge: Cambridge University Press, 159–184.

[47] Pigou, Arthur C. 1920. "The Economics of Welfare," Palgrave Macmillan.

[48] Pollak, Robert A. 1971. "Additive utility functions and linear Engel curves." *The Review of Economic Studies*, Vol. 38(4): 401-414.

[49] Reinert, Kenneth A., and David W. Roland-Holst. 1992. "Disaggregated Armington elasticities for the mining and manufacturing sectors of the United States." *Journal of Policy Modeling, Vol.* 14(5): 1-28.

[50] Ruhl, Kim J. (2008). The international elasticity puzzle. unpublished paper, NYU.

[51] Simonovska, Ina. 2015. "Income differences and prices of tradables: Insights from an online retailer." *The Review of Economic Studies*, Vol. 82(4): 1612-1656.

[52] Spence, Michael. 1976. "Product Selection, Fixed Costs, and Monopolistic Competition," *Review of Economic Studies*, Vol. 43: 217–235.

[53] Stiglitz, J. E. 1986. "Towards a More General Theory of Monopolistic Competition." In Prices, Competition, and Equilibrium, edited by M. H. Peston and R. E. Quandt. Totowa, NJ: Barnes & Noble.

[54] Syverson, Chad. 2007. "Prices, spatial competition and heterogeneous producers: An empirical test." *Journal of Industrial Economics*, Vol. 55(2): 197–222.

[55] Tarasov, Alexander. 2009. "Income distribution, market structure, and individual welfare." *The BE Journal of Theoretical Economics*, 9(1).

[56] Venables, A. J. 1985. "Trade and Trade Policy with Imperfect Competition: The Case of Identical Products and Free Entry." J. Internat. Econ. 19 (1–2): 1–19.

[57] Vives, Xavier. 2001. Oligopoly pricing: old ideas and new tools. MIT Press.

[58] Yeaple, Stephen Ross. 2005. "A simple model of firm heterogeneity, international trade, and wages." *Journal of International Economics*, Vol. 65(1): 1-20.

[59] Zhelobodko, Evgeny, Sergey Kokovin, Mathieu Parenti, and Jacques-Francois Thisse. 2012. "Monopolistic competition: Beyond the constant elasticity of substitution." *Econometrica*, Vol. 80(6): 2765-2784.

Appendices

Appendix A. Unconstrained optimum

First, for the sake of clarity, we remind the condition (15)

$$1 - \eta(x^u) < \frac{1}{\varepsilon^e} \tag{24}$$

implies $x^e < x^u$ and $n^e > n^u$, where $\varepsilon^e \equiv \int x_h^e \varepsilon(x_h^e) \mathrm{d}G / x^e$ and $x^e \equiv \int x_h^e \mathrm{d}G$. Furthermore, the assumption of increasing love for variety implies that $(1 - \eta(x_h))' = -\eta'(x_h) > 0$. Differentiating $\eta(x_h)$ shows that $\eta'(x_h) < 0$ if and only if

$$1 - \eta(x_h) < 1/\varepsilon(x_h) \tag{25}$$

for any $x_h > 0$.

We then prove statements of Proposition 1 in two steps.

Step 1. For homogeneous income, we have $x_h^e = x^e$ and $\varepsilon^e = \varepsilon(x^e)$. Suppose that $x^u \leq x^e$. Then, by $\eta'(x_h) < 0$ and (25), we simultaneously get the two inequalities $1 - \eta(x^u) \leq 1 - \eta(x^e)$ and $1 - \eta(x^e) < 1/\varepsilon(x^e) = 1/\varepsilon^e$, which contradicts (24). So, it must be that $x^e < x^u$.

Step 2. Consider now a mean-preserving spread of income distribution. The optimum consumption x^u is invariant to this distribution. Kichko and Picard (2023) show that the inverse of market elasticity $1/\varepsilon^e$ decreases, does not change or increase if and only if $r_h' < 0$, $r_h' = 0$, or $r_h' > 0$, respectively. As a consequence, the right-hand side of (24) varies in the same way. As seen in Step 1, (24) holds for homogeneous income. As a result, starting from a homogeneous income distribution, the mean-preserving spread increases the gap between the left-hand and right-hand sides of (24) $r_h' < 0$, keeps the difference between them unaltered if $r_h' = 0$, and finally, reduces this gap under $r_h' > 0$. Therefore, a mean-preserving spread reduces the gap between equilibrium and unconstrained optimum allocations if and only if $r_h' > 0$. Q.E.D.

Appendix B. Inequality-constrained optimum

The inequality and resource constraints in (19) can be written as $x_h = x s_h / s$ and $n = Ls/(f + cLx)$. Plugging x_h and the number of firms n in the planner's program yields

$$\max_x \frac{L^2 s}{f + cLx} \int u\left(\frac{s_h}{s} x\right) \mathrm{d}G.$$

The first-order condition with respect to x simplifies to

$$\frac{\int \frac{s_h}{s} u'\left(\frac{s_h}{s} x\right) \mathrm{d}G}{\int u\left(\frac{s_h}{s} x\right) \mathrm{d}G} - \frac{cL}{f + cLx} = 0. \tag{26}$$

Multiplying both terms of (26) by x and using $x_h = xs_h/s$ implies

$$\frac{\int x_h^c \left(1 - u'(x_h^c)\right) \mathrm{d}G}{\int u(x_h^c) \mathrm{d}G} = \frac{f}{f + cLx^c},$$

where the superscript ^{c} denotes the constrained optimum. Denoting the left-hand side as

$$\eta^c \equiv \frac{\int u(x_h^c)\eta(x_h^c)\mathrm{d}G}{\int u(x_h^c)\mathrm{d}G}$$

yields (20).

Finally, we differentiate (26) to obtain condition for concavity of the objective function:

$$\frac{\int \left(\frac{s_h}{s}\right)^2 u''(x_h)\mathrm{d}G}{\int u(x_h)\mathrm{d}G} - \left(\frac{\int \frac{s_h}{s} u'(x_h)\mathrm{d}G}{\int u(x_h)\mathrm{d}G}\right)^2 + \left(\frac{cL}{f + cLx}\right)^2 < 0.$$

The first term is negative because $u''(x_h) < 0$, whereas the last two terms cancel out due to optimum condition (26). Therefore, this expression is negative for any allocation $\{x_h\}$, therefore, (20) determines a unique maximum of the social planner program.

Appendix C. Constrained optimum

In this appendix, we demonstrate the condition for which a mean-preserving spread of labor unit distribution increases the average love for variety $1 - \eta^c$ in the inequality-constrained optimum. For the sake of conciseness, we dispense all variables with the superscript ^{c}.

First, remind that $\eta_h' < 0$ implies an increasing love for variety. Using $\eta_h = x_h u_h'/u_h$, we get

$$\frac{x_h \eta_h'}{\eta_h} = 1 - \eta_h - \frac{1}{\varepsilon_h} < 0. \tag{27}$$

The inequality constraint (19) can be written as $x_h/s_h = x/s$. Differentiating this yields the following marginal consumption changes

$$\mathrm{dln}x_h = \mathrm{dln}x + \mathrm{dln}s_h \tag{28}$$

since $\mathrm{dln}s = 0$ and where $\mathrm{dln}x_h = \mathrm{d}x_h/x_h$, etc. Also, the optimality condition of the constrained optimum (20) can be written as $\eta = cLx/(f + cLx)$.Taking the logarithm, totally differentiating and simplifying this identity yields

$$\mathrm{dln}\eta = (1 - \eta)\mathrm{dln}x. \tag{29}$$

Totally differentiating the identity (20) and using (27) yield

$$\mathrm{dln}\eta = \frac{1}{\eta \int u_h \mathrm{d}G} \int \left(1 - \eta - \frac{1}{\varepsilon_h}\right) u_h' x_h \mathrm{dln}x_h \mathrm{d}G.$$

Applying (28) and (20) and simplifying yields

$$\mathrm{dln}\eta = (1-\eta)\,\frac{x}{s}\frac{\int \phi(x_h)\mathrm{d}s_h\mathrm{d}G}{\int u_h\eta_h/\varepsilon_h\mathrm{d}G} \tag{30}$$

where

$$\phi(x_h) = \left(1-\eta-\frac{1}{\varepsilon_h}\right)u_h'. \tag{31}$$

An infinitesimally small mean-preserving spread of the distribution of labor units is described by $\mathrm{d}s_h{=}\tilde{s}(s_h)\mathrm{d}\xi$ where $\mathrm{d}\xi$ is an infinitesimally small positive number and $\tilde{s}:[s_0,s_1]\to\mathbb{R}$ is a function that has the integral $\tilde{S}(s_h)\equiv\int_{s_0}^{s_h}\tilde{s}(\varsigma)\mathrm{d}G(\varsigma)$ with the properties $\tilde{S}(s_0)=\tilde{S}(s_1)=0$ and $\tilde{S}(s_h)\leq 0$. In this case, after integration by part of the integral of the numerator of (30) gives

$$\int\phi(x_h)\mathrm{d}s_h\mathrm{d}G = \left[\phi(x_h)\tilde{S}(s_h)\right]_{s_0}^{s_1}\cdot\mathrm{d}\xi - \int_{s_0}^{s_1}\phi'(x_h)\frac{\partial x_h}{\partial s_h}\tilde{S}(s_h)\mathrm{d}G(s_h)\cdot\mathrm{d}\xi.$$

The first term is null due to mean-preservation, while $\partial x_h/\partial s_h > 0$ and $\tilde{S}(s_h)\leq 0$. Therefore, the mean preserving spread increases the average love for variety $1-\eta$ (equivalently, $\mathrm{dln}\eta < 0$), if $\phi'(x_h) < 0$. Differentiating (31) yields

$$\phi'(x_h) = u_h''\left(2-\eta-\frac{r_h}{\varepsilon_h}\right).$$

Since $1-\eta > 0$, $\mathrm{dln}\eta < 0$ under sufficient condition

$$\varepsilon_h \geq r_h. \tag{32}$$

Since $-x_h\varepsilon_h' = 1+\varepsilon_h - r_h$, (32) is equivalent to $-x_h\varepsilon_h' \geq 1$. By a continuity argument, the result holds for any mean-preserving spread.

Note that as the mean-preserving spread reduces η, it decreases the average consumption x by (29), and thus increases the number of firms n by the resource constraint (19).

Finally, Pollak preferences have constant parameter $r_h\equiv r$. The sufficient condition (32) then imposes $\varepsilon(x_h)\geq r$. For quadratic preferences, $r=-1$ and the sufficient condition always holds. For CARA preferences, $r=1$ so that the sufficient condition (32) imposes $\varepsilon(x_h)\geq 1$. Using the definitions in Section 5, CES preferences have $r=1+\alpha$ and $\varepsilon_h=\alpha$, with $\alpha>1$, which breaks condition (32) as $\varepsilon_h < r$. However, because $\eta_h=\eta^c=1-1/\alpha$, these preferences yield $\phi'(x_h)=u_h''\left(2-\eta-\frac{r}{\varepsilon_h}\right)=u_h''\cdot 0=0$, which confirms that mean-preserving spread has no impact on unconstrained optimal allocation under CES. For logarithmic preferences, $r=2$ and the sufficient condition is $\varepsilon_h\geq 2$.