Opening the Black Box: Aggregate Implications of Public Investment Composition*

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Abstract

Aggregate implications of public investment crucially depend on its composition between structures, equipments, and intellectual property products. We show that abstracting from heterogeneity in public investment composition leads to an downward bias of the elasticity of private output to public capital. We introduce the three types of public investment into an otherwise standard New Keynesian economy and find that heterogeneity in public investment composition roughly doubles the optimal level of public investment and the output multiplier. Feeding the model with actual variation in public investment composition both over time and across government levels leads to large changes in the multiplier.

Key Words: Public capital, intellectual property products, equipments, structures, optimal capital, fiscal multiplier.

JEL Classification Codes: E22, E62, H54.

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1 Introduction

Public investment is a multifaceted concept that encompasses a broad spectrum of assets, consisting of equipment (e.g., military hardware, machineries, and vehicles), structures (e.g., residential housing, highways), and intellectual and property products (IPP—e.g., R&D and software). For instance, the Inflation Reduction Act included measures to boost investment in semiconductor equipment, infrastructure for the distribution and storage of clean energy, as well as R&D in high-tech manufacturing. Despite this observation, the macroeconomic literature on public investment predominantly narrows its focus to a single type of public investment, and thus on a homogenous stock of public capital. In this paper, we open up the black box of public investment, by explicitly taking account the composition between equipment, structures, and IPP, and highlight how the aggregate implications of public investment crucially depend on its composition.

We start by providing a simple conceptual framework in which a firm produces output using labor. The government levies a lump-sum tax on the household to finance an exogenous stream of public investment in equipment, structures, and IPP. Public investment raises the productivity of private inputs, to an extent that varies across the three types. This is captured through heterogeneity in the elasticity of private output to public equipment, structures, and IPP.

In this setting, we derive analytically the optimal amount of public investment and the output response to public capital, and find that both depend positively on the *simple sum* of the output elasticities across equipment, structures, and IPP. We then ask what is the output elasticity that an econometrician would estimate using aggregate information on public capital, assuming that our model with three public investment types is the true data generating process. Using the estimation procedure of Bouakez et al. (2017) and Ramey (2021), who regress observed aggregate productivity on the aggregate stock of public capital, we show that the econometrician recovers an aggregate output elasticity which is a *weighted sum* of the output elasticities across equipment, structures, and IPP. Thus, abstracting from heterogeneity in public investment composition leads to a downward bias of the relevant output elasticity to aggregate public capital. As a result, a model with one single type of public investment underestimates both the optimal level of public capital and the output response to public investment.

We then extend the simple framework into a fully-fledged model, which adds the three different types of public investment and public capital into an otherwise standard representative-household New Keynesian economy. In this setting, public equipment, structures, and IPP differ in terms of: (*i*) the elasticity of private output to the stocks of public capital; (*ii*) the depreciation rate of the stocks of public capital; (*iii*) the time-to-build delay, that modulates how long it takes for actual public investment to accrue on the stock of public capital; and (*iv*) the time-to-spend delay, that modulates how long it takes for a legislated public investment decision to translate in actual public investment spending.

We leverage information on the U.S. economy in order to discipline the extent of heterogeneity across public investment types. We set the output elasticity of IPP capital to 0.07, in line with recent evidence of Fieldhouse and Mertens (2023) on the effects of federal government R&D on productivity. The elasticity of structures is 0.05, as in Ramey (2021), while we set that of equipment to 0.01, intended as the lowest productive value possible for this type of public capital.[1] The depreciation rates are set using the data from the U.S. Bureau of Economic Analysis (BEA) on the depreciation and net stock of each public capital type. We find annual depreciation rates of 12.4% for equipment, 1.9% for structures, and 17.0% for IPP. Finally, we set the time-to-build and time-to-spend delays using the information from the disbursement of the American Recovery and Reinvestment Act (ARRA) as documented by Leduc and Wilson (2013) and Ramey (2021), as well as statistics from the U.S. Government Accountability Office and the U.S. Patent Office. We posit delays that are negligible for equipment but particularly relevant for structures.

We compare our baseline model with an economy that abstracts from heterogeneity in public investment composition, and thus features only one type of public investment and public capital. We discipline the output elasticity of public capital by simulating our model, and estimate the elasticity using only aggregate information on observed productivity and total public capital. In line with our analytical derivations, this procedure yields a downward bias: the output elasticity to aggregate public capital of 0.065, which is half of the sum of the output elasticities across the three types. For the depreciation rate, we derive it again using information from the BEA—but this time focusing on total public capital—which yields an annual rate of 4.4%. We then set the time-to-build and time-to-spend delays to the average value across equipment, structures, and IPP.

In the quantitative analysis, we start by computing the steady-state optimal public investment and public capital. Our model with multiple public investment

[1] We also provide empirical evidence supporting our calibration choices, highlighting that, if anything, the calibration of the elasticity of IPP capital is conservative, while that of equipment is overstated.

types implies levels which double those generated by a single type economy. The key determinant of this result is the downward bias in the output elasticity to public capital which is due to abstracting from heterogeneity in public investment composition. As such, this bias of the output elasticity generates an understatement of the optimal levels of public investment and public capital which is economically relevant. The model implies that IPP is most under-invested type: an optimal GDP share of public investment in IPP which is five times that in the data.

We then look at the fiscal multipliers. We find values that are close to zero on impact and large in the medium and long run, consistent with the evidence of Ilzetzki et al. (2013) and Boehm (2020). In the short run, public investment in equipment yields the largest output response, with a 1-year multiplier of 0.30. Instead, the multiplier of structures is mildly negative, as this type of public investment is plagued by prolonged time-to-build and time-to-spend delays.

The differences in the multiplier across types become economically relevant with the horizon of the response to the initial shock. Long-run multipliers equal 0.59 for equipment, 0.35 for structures, and 4.57 for IPP. The rationale of the high multiplier for IPP investment is twofold: (*i*) it features the highest output elasticity to public capital, so that investing in it yields the largest productivity gains for private inputs; and (*ii*) it has the highest depreciation rate, so that any given amount of public investment in IPP changes relatively more its total stock, and, through that, firms' productivity. Importantly, the long-run multiplier of total public investment in our model is 1.56, as it averages out among the multipliers of the three types. This stands in contrast with the long-run multiplier of the single type economy, which equals 0.48. In other words, accounting for heterogeneity in public investment composition more than triples the output multiplier.

Our quantitative results hold through an extensive battery of robustness checks. Specifically, we consider a wide array of sensitivity to the calibrated parameters, with a special focus on the output elasticities of public capital. For instance, we halve all the elasticities, we switch the values of the elasticities of structures and IPP, and we consider a positive output elasticity for equipment. We also consider alternative settings for our economy by including distortionary taxes as in Leeper et al. (2010), sticky wages as in Erceg et al. (2000), and fully flexible prices.

We corroborate the notion that fiscal multipliers crucially depend on the composition of public investment with a final exercise that leverages the observed variation in the composition of public investment between equipment, structures, and IPP both across government levels and over time. We feed the model with shocks that replicate the public investment composition of the general government, federal government, defense federal government, non-defense federal government, and local government in every year between 1950 and 2023. We find substantial variation in the long-run multipliers: they are below one for the local government and range between a minimum of 1.34 in 1953 and a maximum of 3.76 in 2023 for non-defense spending. These differences are uniquely driven by changes in the composition of public investment shocks. While non-defense spending is strongly tilted towards IPP, especially in recent years, structures account for the lion's share of the local government's public investment. Thus, looking at output response to public investment spending aggregating across all government layers conceals a large deal of the heterogeneity in the public investment composition—and thus the associated multipliers—observed both across government levels and over time.

This paper builds on the literature that studies the implications of public investment for the fiscal multiplier, and uncover the optimal amount of public capital (Baxter and King, 1993; Fernald, 1999; Leeper et al., 2010; Leduc and Wilson, 2013; Bouakez et al., 2017, 2020; Boehm, 2020; Ramey, 2021; Roulleau-Pasdeloup, 2022; Malley and Philippopoulos, 2023; Peri et al., 2024). However, this entire strand focuses on a single type of public investment and a homogeneous stock of public capital. We contribute to this literature by showing that incorporating heterogeneity in public investment composition between equipment, structures, and IPP doubles the optimal level of public investment and triples the output multiplier.

We relate to the work that studies the implications of the heterogeneous incidence of public spending across industries (Acemoglu et al., 2016; Bouakez et al., 2023, 2024; Basso and Rachedi, 2024; Cox et al., 2024; Peri et al., 2024). We borrow from this literature the notion that studying public spending at the aggregate level conceals a large deal of heterogeneity, which is key for the aggregate implications of government expenditures. Our approach is complementary as we focus on one single sector, and open up the heterogeneity across multiple types of public investment.

Finally, the relevance of taking into account heterogeneity in private capital was at the core of the Cambridge capital controversy (Solow, 1955; Sraffa, 1960; Fisher, 1965; Jorgenson and Griliches, 1967), and spurred a recent literature that spotlights the key role of private investment and capital heterogeneity for understanding aggregate productivity dynamics, business cycle fluctuations, and secular trends (Gomme and Rupert, 2007; Wilson, 2009; Baqaee and Farhi, 2019; Koh et al., 2020). Although we share a similar spirit, we focus on *public* spending: we emphasize the relevance of explicitly accounting for the composition of public investment and public capital among different types with heterogeneous characteristics.

2 The Composition of Public Investment

The measure of government spending that contributes to GDP according to the expenditure approach consists of the sum of public consumption and public investment expenditures. Public consumption is the sum of the remuneration of public employees, the purchases of goods and services from the private sector, and the depreciation of government-owned fixed capital (see Moro and Rachedi, 2022). Public investment equals the total value of an investment in equipment (e.g., machineries, vehicles, furniture, military hardware), structures (e.g., barracks, highways, hospitals, housing, schools), and intellectual property products (e.g., software, R&D).

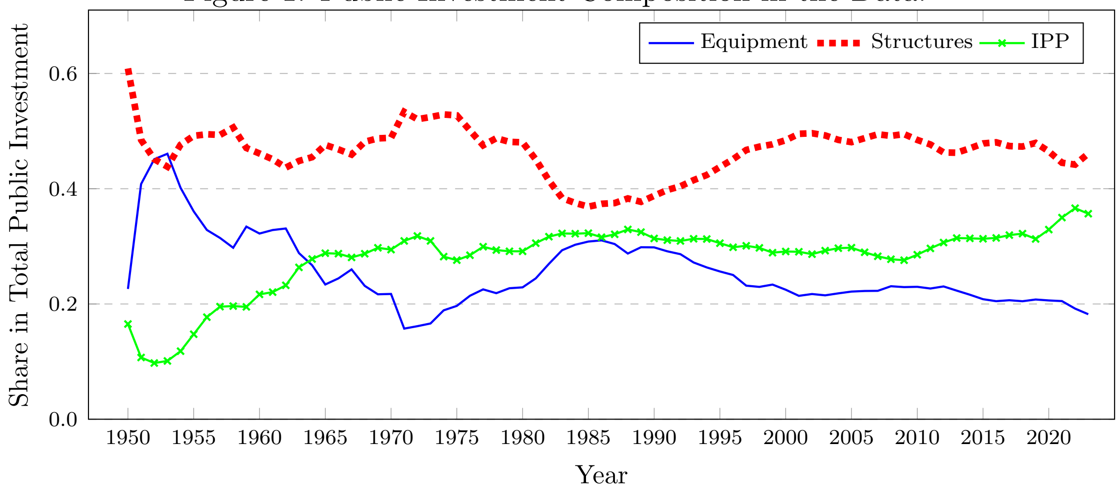
Figure 1 reports the share of public equipment, public structures, and public IPP in total public investment for the general government from 1950 to 2023. This graph indicates that there is no single type that accounts for the lion's share of total public investment. In other words, aggregate public investment is indeed a bundle of different types. In addition, there have been swings in the relative relevance of each type over time. On the one hand, the shares of public structures and equipment have been declining: from 23% in 1950 to 18% in 2023 for equipment, and from 61% in 1950 to 46% in 2023 for structures. On the other hand, IPP has more than doubled its share, going from 17% in 1950 up to 36% in 2023.

Why it is important to account for the composition in public investment between equipment, structures, and IPP? This is because these types of public investment markedly differ from each other. Throughout the paper, we focus on four dimensions of heterogeneity across the different public investment/capital types.

1. *Elasticity of private output to public capital.* This dimension determines to what extent changes in the stock of public capital alter private output. Intuitively, the stock of public capital raises the productivity of private inputs, leading to a higher output for the same amount of production inputs. Structures such as hospitals, schools, and highways raise the productivity of private firms. The same applies—even to a larger extent—for IPP investment such as R&D. However, investment in equipment and other structures (such as warehouses, barracks, offices) has a much more limited impact on the productivity of the private sector, if any.

2. *Depreciation rate.* This dimension captures the extent to which different types of public capital deteriorate over time at different rates. Structures tend to have a very long service life: according to the BEA, the service life of hospitals

Figure 1: Public Investment Composition in the Data.



Note: The figure reports the share of public equipment (continuous blue line), public structures (dotted red line), and public IPP (green crossed line) in total public investment for the general government from 1950 to 2023.

and schools is 50 years. Instead, the service life of equipment is much more limited, with, for example, 20 years for F-16 planes, and 9 years for medical hardware. IPP has the shortest service life, with a duration of 3 years for software and 8 years for R&D.

3. *Time-to-spend delay.* This dimension describes the delays between the moment when the government allocates funds for new investment projects and the moment when these funds are actually being disbursed. This captures any delay in the legislative, administrative, and bureaucratic procedures, as well as any other logistical challenges. While time-to-spend delays may be negligible for equipment, the funds provided for new highway construction by the ARRA in February 2009 were entirely spent only in 2013.

4. *Time-to-build delay.* This dimension describes the delays between the moment when the government disburses the funds to finance new investment and the moment when the new investment becomes a productive part of the stock of public capital. This captures the required duration for completing new investment projects. While it is reasonable to think that installing new equipment and making it fully functional requires little time, the opposite applies to structures, for which the construction of a new highway or complex facilities, such as hospitals, may take a few years.

3 Simple Conceptual Framework

In this section, we build a simple model in which public investment consists of equipments, structures, and IPP, and use it to show that abstracting from heterogeneity across these three types leads to an under-estimation of the elasticity of private output to aggregate public capital. In what follows, we focus exclusively on heterogeneity in the output elasticities. We incorporate the role of heterogeneity in the depreciation rates, the time-to-spend and time-to-build delays when we extend the simple model into a fully-fledged quantitative economy in Section 5. Nonetheless, heterogeneity in the output elasticities to public capital is the key aspect driving the quantitative results of the fully-fledged model.

We consider a static economy in which a risk-neutral representative household—endowed with one unit of time—chooses consumption, C, to solve the problem:

$$\max_{C} \; C \quad \text{s.t. } C + T = WN, \tag{1}$$

where N denotes labor, T denotes lump-sum taxes, and W is the wage. The budget constraint posits that the household uses labor income to finance consumption and taxes.

A perfectly competitive representative firm hires labor to produce output Y with the technology

$$Y = N \, K_{G_e}^{\gamma_e} \, K_{G_s}^{\gamma_s} \, K_{G_i}^{\gamma_i} \tag{2}$$

where K_{G_e}, K_{G_s}, and K_{G_i} denote the per-capita stocks of public equipment, structures, and IPP, respectively. While the technology features constant returns to scale in private inputs, private output is influenced by the stocks of public capital. The elasticities of private output with respect to the three stocks of public capital are γ_e, γ_s, and γ_i. Thus, as in Baxter and King (1993), Leeper et al. (2010), Bouakez et al. (2017), and Ramey (2021), public capital raises the productivity of private inputs. The only difference is that here this effect varies across public capital types.

The government owns the three stocks of public capital, which coincide with the amounts of public investment in equipment, structures, and IPP, I_{G_e}, I_{G_s}, and I_{G_i}, respectively, because we consider a unit depreciation rate:

$$K_{G_e} = I_{G_e}, \tag{3}$$

$$K_{G_s} = I_{G_s}, \tag{4}$$

$$K_{G_i} = I_{G_i}. \tag{5}$$

We consider the three public investment variables as purely exogenous. The government finances public investment expenditures with lump-sum taxes, so that its budget constraint reads

$$I_{G_e} + I_{G_s} + I_{G_i} = T. \tag{6}$$

Finally, the resource constraint of the economy implies that output equals the sum of private consumption and public investment:

$$Y = C + I_{G_e} + I_{G_s} + I_{G_i}. \tag{7}$$

3.1 Single versus Multiple Public Investment

We compare the implications of our simple economy with multiple public investment types (M) with those of a model which abstracts from heterogeneity in public investment composition, and thus considers only a single type (S). The technology of the singly type economy is

$$Y = NK_G^{\bar{\gamma}}, \tag{8}$$

where $\bar{\gamma}$ is the elasticity of private output to *aggregate* public capital. To discipline this comparison, we set the elasticity of the single type economy, $\bar{\gamma}$, to that an econometrician would estimate using aggregate information on public capital, assuming that the multiple type economy is the true data generating process.

Specifically, we derive the implied elasticity following the estimation procedure of Bouakez et al. (2017) and Ramey (2021). This approach regresses the logarithm of aggregate observed productivity, TFP_t, over the logarithm of the per-capita stock of aggregate public capital, $K_{G,t}$, as follows:

$$\log TFP_t = \bar{\gamma} \log K_{G,t}. \tag{9}$$

Insofar public capital affects the productivity of private output, as defined by Equation (8), this regression identifies the implied elasticity of private output to aggregate public capital, $\bar{\gamma}$.

What happens if one estimates regression (9) using data generated by the economy with multiple public investment types? In our economy, aggregate public capital is the sum of equipment, structures, and IPP, that is, $K_G = K_{G_e} + K_{G_s} + K_{G_i}$. This implies that aggregate productivity can be rewritten as

$$TFP_t = K_{G_e,t}^{\gamma_e} K_{G_s,t}^{\gamma_s} K_{G_i,t}^{\gamma_i}. \tag{10}$$

Consequently, for the multiple type economy (M)

$$\log TFP_t = \gamma_e \log K_{G_e,t} + \gamma_s \log K_{G_s,t} + \gamma_i \log K_{G_i,t}. \tag{11}$$

If we recover the implied elasticity of the single type model, $\bar{\gamma}$, using data of the multiple type economy, and thus combine Equation (9) and (11), we find the following

$$\bar{\gamma} = \gamma_e \frac{\log K_{G_e,t}}{\log K_{G,t}} + \gamma_s \frac{\log K_{G_s,t}}{\log K_{G,t}} + \gamma_i \frac{\log K_{G_i,t}}{\log K_{G,t}}. \tag{12}$$

This identity states that the implied elasticity is a weighted sum of the elasticities of equipment, structures, and IPP, in which the weights are the ratios between the logarithm of per-capita stock of each of the three types of public capital and the logarithm of per-capita stock of aggregate public capital.

Observing that the stocks of public capital are defined on a per-capita terms, and thus the log weights are always lower than one, we establish a relationship between the aggregate elasticity and the sum of the three different elasticities. That is,

Lemma 1. $\bar{\gamma} < \gamma_e + \gamma_s + \gamma_i$.

In what follows, we compare the implications of the single and multiple type economy regarding the optimal level of public capital and the output response to public investment.

3.1.1 Optimal Public Capital

We derive the optimal level of public capital, which we define as in Ramey (2021) as the levels of K_{G_e}, K_{G_s}, and K_{G_i} that maximize households' utility. Specifically, when the social planner chooses optimally the three values of public capital, it yields the following first-order conditions:

$$\gamma_e \frac{Y}{K_{G_e}} = 1, \tag{13}$$

$$\gamma_s \frac{Y}{K_{G_s}} = 1, \tag{14}$$

$$\gamma_i \frac{Y}{K_{G_i}} = 1. \tag{15}$$

The optimal level of aggregate public capital in terms of GDP for the multiple type economy (M) is

$$\frac{K_G}{Y}\Big|_M = \frac{K_{G_e} + K_{G_s} + K_{G_i}}{Y} = \gamma_e + \gamma_s + \gamma_i, \tag{16}$$

where the optimal amount of aggregate public capital sums over the optimal levels of equipment, structures, and IPP.

What would the optimal level of public capital be if one abstracts from heterogeneity across public investment types? The single type economy (S) implies an optimal level of aggregate public capital in terms of GDP which equals

$$\frac{K_G}{Y}\Big|_S = \bar{\gamma}. \tag{17}$$

We can then derive our first analytical result, as stated in Proposition 1.

Proposition 1. *Abstracting from heterogeneity in the elasticity of private output to public capital across equipment, structures, and IPP yields to an under-estimation of the optimal amount of public capital.*

Proof. Applying Lemma 1, we have that $\frac{K_G}{Y}\Big|_M > \frac{K_G}{Y}\Big|_S$. □

The optimal public capital of the multiple type economy coincides with that of a single type model only if the output elasticity of aggregate public capital equals to the sum of the elasticities associated with equipment, structures, and IPP, that is, if and only if $\bar{\gamma} = \gamma_e + \gamma_s + \gamma_i$. However, Lemma 1 states that the implied output elasticity to aggregate public capital is strictly lower than the sum of the different elasticities across types. While optimal public capital in a multiple type economy is a function of the *simple sum* of the output elasticities across equipment, structures, and IPP, abstracting from heterogeneity in public investment composition in the single type model yields an optimal public capital which is determined by the *weighted sum* of the output elasticities across equipment, structures, and IPP.

3.1.2 Output Response to Public Investment

What are the implications of abstracting from the heterogeneity in the composition of public investment on the response of aggregate output to a change in aggregate public investment? Our setting allows us to uncover the answer to this question by focusing only on the production side of the economy.

Given aggregate public investment $I_G = I_{G_e} + I_{G_s} + I_{G_i}$, the weights of each public investment type into aggregate public investment are

$$\omega_e = \frac{I_{G_e}}{I_G} = \frac{I_{G_e}}{I_{G_e} + I_{G_s} + I_{G_i}}, \tag{18}$$

$$\omega_s = \frac{I_{G_s}}{I_G} = \frac{I_{G_s}}{I_{G_e} + I_{G_s} + I_{G_i}}, \tag{19}$$

$$\omega_i = \frac{I_{G_i}}{I_G} = \frac{I_{G_i}}{I_{G_e} + I_{G_s} + I_{G_i}}. \tag{20}$$

We can use these weights to rewrite firm's technology in Equation (2) as

$$Y = N\ [\omega_e I_G]^{\gamma_e}\ [\omega_s I_G]^{\gamma_s}\ [\omega_i I_G]^{\gamma_i}. \tag{21}$$

In this multiple type economy (M), the change in output relative to its initial level due to a change in public investment is

$$\frac{\partial Y/\partial I_G}{Y}\Big|_M = \frac{\gamma_e + \gamma_s + \gamma_i}{I_G}. \tag{22}$$

Thus, the output response to public investment increases linearly with the sum of the elasticities of private output to public equipment, structures, and IPP.

In a single type economy (S), the change in output relative to its initial level due to a change in public investment becomes

$$\left.\frac{\partial Y/\partial I_G}{Y}\right|_S = \frac{\bar{\gamma}}{I_G}. \tag{23}$$

We can then established the second analytical result stated in Proposition 2.

Proposition 2. *Abstracting from heterogeneity in the elasticity of private output to public capital across equipment, structures, and IPP yields to an under-estimation of the response of private output to a change in public investment.*

Proof. Applying Lemma 1, we have that $\left.\frac{\partial Y/\partial I_G}{Y}\right|_M > \left.\frac{\partial Y/\partial I_G}{Y}\right|_S$. □

Once again, the output response to public investment in a multiple type economy is a function of the *simple sum* of the output elasticities across equipment, structures, and IPP. In contrast, the single type economy implies an output response which is determined by the *weighted sum* of the output elasticities across equipment, structures, and IPP.

4 Quantitative Model

In this section we extend the simple framework of Section 3 into a fully-fledged model, adding different types of public investment and public capital into an otherwise standard New Keynesian economy. Specifically, we consider a representative-household model in which the fiscal authority finances an exogenous stream of public investment in equipment, structures, and IPP via lump-sum taxes on the household. Public capital stocks positively benefit private firms' production, that assemble output using private physical capital and labor. Firms choose prices subject to price-setting friction.[2] Public equipment, structures, and IPP differ in terms of: (i) the elasticity of private output to the stocks of public capital; (ii) the depreciation rate of the stocks of public capital; (iii) the time-to-build delay (i.e., how long it takes for actual public investment to accrue on the stock of public capita)l; and (iv) the time-to-spend delay (i.e., how long it takes for a legislated public investment decision to translate in actual public investment spending). The model also features a monetary authority that sets nominal interest rate using a standard Taylor rule.

[2]Our baseline economy features price rigidities since we also want to study the heterogeneous short-run implications across the three public investment types. In the quantitative exercises, we also study a version of the economy with fully flexible prices.

4.1 Household

A representative household chooses consumption, C_t, labor, N_t, private investment, I_t, private physical capital, K_{t+1}, and one-period risk-free nominal bonds, B_{t+1}, to maximize its lifetime utility (24), as follows

$$\max_{C_t,N_t,I_t,K_{t+1},B_{t+1}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \theta \frac{N_t^{1+\eta}}{1+\eta} \right] \tag{24}$$

$$\text{s.t. } P_t C_t + P_t I_t + B_{t+1} + T_t = W_t N_t + R_{K,t} K_t + R_t B_t + D_t, \tag{25}$$

where β is the time discount factor, σ is the risk aversion, η is the inverse of the Frisch elasticity, P_t denotes the price of consumption and investment, $R_{K,t}$ is the rental rate of private capital, R_t is the return on bonds, and D_t denotes firms' profits. The budget constraint in Equation (25) posits that households finance expenditures in consumption, investment, bonds, and lump-sum taxes with the sum of labor income, capital income, the revenue on bond holdings, and firms' profits.

Private investment accumulates to the stock of private capital subject to convex adjustment costs, so that physical capital evolves over time with the law of motion:

$$K_{t+1} = (1-\delta_K) K_t + I_t \left[1 - \frac{\Omega}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 \right], \tag{26}$$

where δ_K denotes the depreciation rate of private capital, and Ω captures the magnitude of the adjustment costs.

4.2 Wholesalers

The production side consists of two types of firms: wholesalers and retailers. There is a unit measure of monopolistically competitive wholesalers, indexed by $j \in [0,1]$, which assemble output $Y_{j,t}$ with the technology

$$Y_{j,t} = N_{j,t}^{\alpha} K_{j,t}^{1-\alpha} K_{G_e,t}^{\gamma_e} K_{G_s,t}^{\gamma_s} K_{G_i,t}^{\gamma_i}, \tag{27}$$

where the private inputs are $N_{j,t}$, the labor hired by wholesaler j at time t, and $K_{j,t}$, the stock of physical capital rented from the household. Firms' technology benefits from the stocks of public equipment, $K_{G_e,t}$, public structure, $K_{G_s,t}$, and public IPP, $K_{G_i,t}$. The labor intensity is α, and the parameters γ_e, γ_s, and γ_i denote the elasticity of private output to public equipment, structures, and IPP, respectively.

Wholesalers set prices $P_{j,t}$ subject to a price setting friction à la Calvo (1983), in which wholesalers can reset their prices with a probability $1-\phi$. Wholesalers chose their optimal reset price to maximize the expected discounted stream of future profits, as follows

$$\max_{P_{j,t}} \mathbb{E}_t \left[\sum_{z=t}^{\infty} \beta^z \phi^z \frac{\Lambda_{t+1}}{\Lambda_t} \Big(P_{j,t} Y_{j,z} - W_z N_{j,z} - R_{K,z} K_{j,z} \Big) \right], \tag{28}$$

where Λ_t is the household's stochastic discount factor.

4.3 Retailer

A perfectly competitive retailer purchases the different varieties $Y_{j,t}$ from the wholesalers and assemble them into final output Y_t as follows

$$Y_t = \left[\int_0^1 Y_{j,t}^{\frac{\epsilon-1}{\epsilon}} \mathrm{d}j \right]^{\frac{\epsilon}{\epsilon-1}}, \tag{29}$$

where ϵ is the elasticity of substitution across varieties. This aggregator implies that the price of final goods equals

$$P_t = \left[\int_0^1 P_{j,t}^{1-\epsilon} \mathrm{d}j \right]^{\frac{1}{1-\epsilon}}. \tag{30}$$

The retailer sells the final goods to the household, as private consumption and investment, and to the fiscal authority, as the three different types of public investment. Consequently, the resource constraint reads

$$Y_t = C_t + I_t + I_{G_e,t} + I_{G_s,t} + I_{G_i,t}. \tag{31}$$

4.4 Fiscal Authority

The fiscal authority faces an exogenous stream of planned public investment expenditures in equipment, $A_{G_e,t}$, structures, $A_{G_s,t}$, and IPP, $A_{G_i,t}$, which follow the auto-regressive processes:

$$\log A_{G_e,t} = (1-\rho) \log I_{G_e} + \rho \log A_{e,t-1} + \varepsilon_{e,t} + \varepsilon_t, \tag{32}$$

$$\log A_{G_s,t} = (1-\rho) \log I_{G_s} + \rho \log A_{s,t-1} + \varepsilon_{s,t} + \varepsilon_t, \tag{33}$$

$$\log A_{G_i,t} = (1-\rho) \log I_{G_i} + \rho \log A_{i,t-1} + \varepsilon_{i,t} + \varepsilon_t, \tag{34}$$

where ρ denotes the persistence of the processes, I_{G_e}, I_{G_s}, and I_{G_i} are the steady-state values of public equipment, structures, and IPP, respectively. We consider two types of shocks: a type-specific shock, which raises public investment in only one specific type, say, either only equipment, only structures, or only IPP, and is denoted by $\varepsilon_{e,t}$, $\varepsilon_{s,t}$, and $\varepsilon_{i,t}$, respectively; and a common shock, ε_t, which raises total public investment while preserving constant the composition in planned spending across the three types.

As in Leeper et al. (2010), Ramey (2021), and Peri et al. (2024), planned public investment turns into actual spending only with a lag, the so called time-to-spend delay. Specifically, actual public investment spending is defined as

$$I_{G_s,t} = \frac{1}{\xi_s} \sum_{z=1}^{\tau_s} A_{s,t+1-z}, \tag{35}$$

$$I_{G_e,t} = \frac{1}{\xi_e} \sum_{z=1}^{\tau_e} A_{e,t+1-z}, \tag{36}$$

$$I_{G_i,t} = \frac{1}{\xi_i} \sum_{z=1}^{\tau_i} A_{i,t+1-z}; \tag{37}$$

These specifications imply that current spending in public investment averages planned lagged expenditures. The parameters τ_s, τ_e, and τ_i denote the length of the time-to-spend delays for equipment, structures, and IPP. Importantly, we allow the delays to vary across the three public investment types.

Current public investment accrues to the stock of public capital according to the following law of motion

$$K_{G_s,t+1} = (1 - \delta_s)\, K_{G_s,t} + I_{G_s,t-\zeta_s}, \tag{38}$$

$$K_{G_e,t+1} = (1 - \delta_e)\, K_{G_e,t} + I_{G_e,t-\zeta_e}, \tag{39}$$

$$K_{G_i,t+1} = (1 - \delta_i)\, K_{G_i,t} + I_{G_i,t-\zeta_i}, \tag{40}$$

where δ_s, δ_e, and δ_i are the depreciation rates for equipment, structures, and IPP, and ζ_s, ζ_e, and ζ_i denote the duration of the time-to-build delays for each type of public capital. Both the depreciation rates and the durations of the time-to-build delays are allowed to vary across the three public investment types.

The government finances the stream of public investment via means of a lump-sum tax on the household so that the budget constraint is

$$T_t = I_{s,t} + I_{e,t} + I_{i,t}. \tag{41}$$

4.5 Monetary Authority

The economy features a monetary authority that sets the nominal interest rate according to the Taylor rule that features inertia and responds to inflation, π_t, and the output gap, Y_t/Y_t^f, where Y_t^f denotes the output level in a counterfactual version of the economy with perfectly flexible prices. The Taylor rule reads

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\varphi_r} \left[\pi_t^{\varphi_\pi} \left(\frac{Y_t}{Y_t^f}\right)^{\varphi_y}\right]^{1-\varphi_r} \tag{42}$$

where R denotes the steady-state level of the interest rate, φ_r captures the degree of interest rate inertia, and φ_π and φ_π denote the degree of responsiveness of the nominal interest rate to inflation and the output gap, respectively.

5 Quantitative Analysis

This section presents the results of the analysis based on the quantitative model introduced above. Specifically, Section 5.1 details the calibration of the model and Section 5.2 introduces the alternative economies we compare the baseline model to. Then, Section 5.3 derives the optimal levels of public investment and public capital, and evaluates the role of each dimension of heterogeneity across public investment types in a sequence of counterfactual exercises, while Section 5.4 provides the results of a similar exercise, which instead focuses on fiscal multipliers both in the short run and in the long run. Finally, Section 5.5 measures how feeding the model with actual variation in public investment composition both over time and across government levels leads to large changes in the size of the fiscal multiplier. All in all, while there is uncertainty in how to discipline the model—especially with respect to the magnitude of the differences in the output elasticity to public capital across equipment, structures and IPP—this section provides a proof-of-concept for the fact that the aggregate implications of public investment crucially depend on its composition.

5.1 Calibration

We calibrate the model to the U.S. economy. We log-linearize the model around a zero inflation rate steady-state, and set one time period to coincide with a quarter.

We set the time discount factor to $\beta = 0.99$ so that the annualized real interest rate is 4%. The risk aversion is set to the standard value of $\sigma = 2$, and we fix $\eta = 2$ so that the Frisch elasticity is 0.5, in line with the evidence at the individual level documented by Chetty et al. (2013).[3] We set the preference shifter to $\theta = 0.1090$ so that labor supply in steady state equals 0.33.

We fix the depreciation rate of private physical capital to 0.0205, which is derived by taking information from the BEA, dividing by the current-cost depreciation of private fixed assets on their lagged current-cost net stock, and averaging it between the period 1950-2019. This implies an annual depreciation rate of 7.95%. We set the adjustment cost parameter to 1.774 so that the relative standard deviation of private investment in terms of the standard deviation of output in a model

[3] We study the implications of a specification in which the Frisch elasticity equals 1 in Appendix B.2.

specification featuring aggregate TFP shocks is 2.75, as it is in the data.

For the production side, we set the labour intensity to $\alpha = 0.6$ and the Calvo probability to $\phi = 0.75$ to imply an average duration of prices of 4 quarters. The elasticity of substitution across varieties is $\epsilon = 4$ to be consistent with the markups of around 30% documented in recent decades by De Loecker et al. (2020).

For public investment, we first set the persistence of the auto-regressive processes to $\rho = 0.9$.[4] We discipline the different sources of heterogeneity across public investment types as follows. First, we set the depreciation rates following the same procedure of the depreciation rate of private capital, using information from the BEA, and find $\delta_e = 0.033$, $\delta_s = 0.005$, and $\delta_i = 0.046$. These values imply annual depreciation rates of 12.4% for equipment, 1.9% for structures, and 17.0% for IPP. Second, we choose the output elasticities to public capital starting from that of IPP. A recent paper by Fieldhouse and Mertens (2023) finds that the output elasticity of non-defense R&D is 0.12, whereas that of defense R&D is not statistically different from zero. Since non-defense R&D accounts on average for 55% of total defense and non-defense R&D between 1950 and 2022, we set the output elasticity of IPP to $\gamma_i = 0.07 = 0.55 \times 0.12$. Then, we take the stand that the stock of public equipment is productive but at the minimum level possible, by choosing an elasticity of $\gamma_e = 0.01$. Finally, we set $\gamma_s = 0.05$, so that structures have a large effect on the productivity of private inputs, but relatively lower than that of IPP. This choice coincides with that of Ramey (2021) in the analysis on infrastructure investment, as well as with the calibration of Baxter and King (1993) and Leeper et al. (2010). Appendix A provides empirical evidence in the spirit of Bouakez et al. (2017) and Ramey (2021) in which we estimate output elasticities for equipment, structures, and IPP and show that they are roughly in line with those defined in the calibration of the model. In fact, if anything, our calibration of the elasticity of IPP capital is conservative.[5] Furthermore, in the next section, we show that these calibration choices make our model imply an elasticity of private output to aggregate public capital, which is remarkably in line with empirical estimates provided by the literature.

Regarding the time-to-build and time-to-spend delays, we do the following. We

[4] Appendix B.2 reports the results of a model specification in which we set the persistence up to $\rho = 0.94$, which coincides with the persistence estimated by Leeper et al. (2010).

[5] We study the robustness of our results to alternative calibration choices for the output elasticities in Appendices B.1 and B.2. Specifically, we consider four cases: (*i*) an economy in which we halve the elasticities, so that $\gamma_e = 0.005$, $\gamma_s = 0.025$, and $\gamma_i = 0.035$; (*ii*) an economy in which the stock of public equipment is not productive, so that $\gamma_e = 0.$, $\gamma_s = 0.05$, and $\gamma_i = 0.07$; (*iii*) an economy in which equipment is even more productive, so that $\gamma_e = 0.02$, $\gamma_s = 0.05$, and $\gamma_i = 0.07$; (*iv*) an economy in which we switch the values between structures and IPP, so that $\gamma_e = 0.01$, $\gamma_s = 0.07$, and $\gamma_i = 0.05$.

start with equipment, since this is the investment type which is likely to be less affected by these lags. Accordingly, we set both delays to one quarter so that $\tau_e = 1$ and $\zeta_e = 1$. For IPP, we assume little time-to-spend delays, with $\tau_i = 2$, but prolonged time-to-build ones, with $\zeta_i = 12$. This is consistent with the fact that according to the U.S. Patent Office, the procedures of getting a patent approved take around 22 months. Structures feature the longest delays, with six quarters for time-to-spend, $\tau_s = 6$, and four years for the time-to-build, $\zeta_s = 16$. The first choice is in line with Ramey (2021), who show that time-to-spend delays of 5 quarters can account for the lags in the allocation of ARRA funds, as documented by Leduc and Wilson (2013). The time-to-build lag follows evidence from the U.S. Government Accountability Office, indicating that infrastructure construction may take between 2 and 6 years.

We also need to pin down the value of public investment at a steady state. To do so, we derive in the data the average share of public equipment, public structures, and public IPP for the general government as a fraction of GDP, for the period 1950-2023. We find that public equipments account for 1.28% of GDP, whereas this ratio is 1.31% for public IPP and 2.25% for public structures. We set the steady-state values for each type accordingly.

We close the calibration by setting the Taylor rule parameters following the empirical evidence of Clarida et al. (2000): the degree of inertia is $\phi_r = 0.8$, and the degrees of responsiveness of the nominal interest rate to inflation and the output gap are $\phi_\pi = 1.5$ and $\phi_y = 0.2$, respectively.

5.2 Counterfactual Economies

Before performing the quantitative analysis, let us introduce the counterfactual economies we will use throughout the following sections to disentangle and isolate the channels that make the aggregate implications of public spending crucially depend on its composition.

Throughout the paper, we refer to the baseline model in Section 4 with the calibration choices of Section 5.1 as the "Multiple Type" economy. As in the analytical derivations, we compare our model to a specification which completely abstracts from heterogeneity in the composition of public investment: an economy with only one type of public investment and public capital. We calibrate this case as follows. First, we discipline the output elasticity to public capital by making it to coincide with that the econometrician would recover from observing the aggregate data implied by the "Multiple Type" economy. To do so, we simulate our model, take the values of the observed aggregate total factor productivity and aggregate public capital, and estimate regression (9). We find that the econometrician recovers an implied elasticity of private output to the aggregate stock of public capital of $\bar{\gamma} = 0.065$. This value is remarkably in line with the estimates provided by the literature, which exactly focuses on aggregate information on total public capital. For instance, Bouakez et al. (2017) find a value of 0.065, whereas Peri et al. (2024) estimate an average value across sectors of 0.0575. As we mentioned above, the fact that the model is consistent with estimates based on the aggregate stock of public capital corroborates our calibration choices for the output elasticities to public equipment, structures, and IPP. Second, we compute the depreciation rate as in the baseline model, with the difference that we focus on information on aggregate public capital: we divide by the current-cost depreciation of total public capital on its lagged current-cost net stock and average it between the period 1950-2019. We find a depreciation rate of $\bar{\delta} = 0.011$, that is, an annual depreciation rate of 4.4%. Third, we set the time-to-build delays to 7 quarters, which is the average value of the delays across the three types. Fourth, we fix the time-to-spend delay to 2 quarters, which is the average value of the delays across the three types. Fifth, we set the steady-state level of total public investment to equal 3.9% of GDP, which is the sum of the GDP shares of public equipment, structures, and IPP. We refer to this case as the "Single Type" economy.

We then consider four additional cases, which all feature the three types of public investment but, in turn, abstract from a dimension of heterogeneity among them. Specifically, we set these economies as follows: (*i*) the "Multiple Type with Homogeneous Output Elasticities" economy abstracts from heterogeneity in the elasticities of private output to the different types of public capital, by setting it to 0.065/3 homogeneously across types;[6] (*ii*) the "Multiple Type with Homogeneous Depreciation Rates" economy abstracts from heterogeneity in the depreciation rates, setting it homogeneously so that each type features the same depreciation of the "Single Type" model, that is, $\bar{\delta} = 0.011$; (*iii*) the "Multiple Type with Homogeneous Time-to-Build" economy, which abstracts from heterogeneity in the time-to-build delays, by homogeneously setting the time-to-build delay to the average value of 7 quarters; and (*iv*) the "Multiple Type with Homogeneous Time-to-Spend" economy, which abstracts from heterogeneity in the time-to-spend delays, by homogeneously

[6] As we analytically uncover in Sections 3.1.1 and 3.1.2, a model in which the elasticity equals to 0.065/3 for all the three types implies an amount of optimal public capital and an output response which coincides with that of an economy with only one type of capital, in which the elasticity is 0.065, exactly as in the "Single Type" economy.

setting the time-to-spend delay to the average value of 2 quarters.

5.3 Optimal Public Capital and Public Investment

Similarly to Section 3.1.1, we derive the optimal levels of public capital and public investment, by maximizing household's utility at the steady state. In doing so, we compare the optimal levels implied by our model with those associated with "Single Type" economy, the "Multiple Type with Homogeneous Output Elasticities", and the "Multiple Type with Homogeneous Depreciation Rates". We do not consider the three additional multiple-type economies as they study dimensions of heterogeneity that matter only for the dynamics out of steady state.

We report the results of this exercise in Table 1. Panel (a) shows that the optimal public capital of the "Multiple Type" economy is 217.23% (in terms of annualized GDP), which roughly doubles the level of the "Single Type" model, which equals 120.61%. This comparison highlights that explicitly accounting for heterogeneity in public investment composition doubles the optimal level of public capital. The same applies for optimal public investment: it equals 10.81% in "Multiple Type" economy and 5.26% in the "Single Type" model.

When disentangling the implications of our model on optimal public capital across the three types, we find that the optimal amount of public capital and investment as a fraction of GDP equal 7.45% and 0.92% for equipment, 171% and 3.27% for structures, and 38.78% and 6.61% for IPP.

The variation in the levels of optimal public capital and investment across types depends on heterogeneity in the output elasticities to public capital and the depreciation rates. The very high elasticity and depreciation rate of IPP makes its ratio of public investment to GDP to be the highest. Instead, the very low depreciation rate of structures—coupled with its relatively high output elasticity—implies a large fraction of public capital in its type as a share of GDP.

We disentangle the relevance of these two dimensions by abstracting from heterogeneity in either the output elasticities or the depreciation rate. Variation in the output elasticities is the key feature that accounts for the difference in the optimal levels between the "Single Type" and the "Multiple Type" economies. Indeed, when setting homogeneous output elasticities across types, the economy with multiple public investment types generates ratios almost identical to the model with one single type. Instead, heterogeneity in the depreciation rates plays a marginal role.

Interestingly, there are large differences between the model implications on optimal public investment by type and the actual shares of public investment in GDP

Table 1: Optimal Public Capital and Public Investment.

	Single Type	Multiple Type	Multiple Type with Homogeneous ...		Data
			Output Elasticities	Depreciation Rates	
	(1)	(2)	(3)	(4)	(5)
			Panel A: Optimal Public Capital (% of GDP)		
Total	120.61	217.23	101.92	241.96	73.82
Equipment	-	7.45	16.08	18.61	9.44
Structures	-	171.00	73.87	93.06	57.63
IPP	-	38.78	11.97	130.28	6.75
			Panel B: Optimal Public Investment (% of GDP)		
Total	5.26	10.81	4.96	10.56	4.84
Equipment	-	0.92	1.82	0.81	1.28
Structures	-	3.27	1.29	4.06	2.25
IPP	-	6.61	1.86	5.68	1.31

Note: Panel A reports in Panel (a) the optimal public capital and in Panel (b) the optimal public investment—in percentage terms with respect to annual GDP—for the "Single Type" economy in Column (1), the "Multiple Type" economy in Column (2), the "Multiple Type with Homogeneous Output Elasticities" in Column (3), and the "Multiple Type with Homogeneous Depreciation Rate" in Column (4). Column (5) reports the values in the data averaged over 1950 and 2023. The statistics are computed for total public investment, as well as for each public investment type: equipment, structures, and IPP.

in the data. According to the model, IPP is most under-invested type: the optimal GDP share of public investment in IPP is five times that in the data, 6.61% vs. 1.31%. Public structures are slightly under-invested, with an optimal share of 3.27% compared to the 2.25% in the data. On the contrary, the optimal levels of equipment are remarkably close to those observed in the data.

5.4 Fiscal Multipliers

5.4.1 Measurement

We now turn into the positive implications of the composition of public investment by measuring how the fiscal multiplier varies when abstracting from the different dimensions of heterogeneity between public equipment, public structures, and public IPP. To do so, we follow Mountford and Uhlig (2009) and compute the fiscal multiplier in present-value terms

$$\mathcal{M}_{\mathcal{H}} = \frac{\sum_{t=0}^{\mathcal{H}} \beta^t \left(Y_t - Y\right)}{\sum_{t=0}^{\mathcal{H}} \beta^t \left(X_t - X\right)}, \tag{43}$$

where X_t denotes a generic type of public investment. When we study the effects of the aggregate shock that preserves the composition of public investment, ε_t, the denominator uses the variation in total public investment, $X_t = I_{G_e,t} + I_{G_s,t} + I_{G_i,t}$. Instead, when we focus on the type-specific public investment shocks, $\varepsilon_{e,t}$, $\varepsilon_{s,t}$, and $\varepsilon_{i,t}$, the denominator uses public investment in equipment, that is $X_t = I_{G_e,t}$, structures, that is $X_t = I_{G_s,t}$, and IPP, that is $X_t = I_{G_i,t}$, respectively.

Equation (43) posits that the fiscal multiplier, $\mathcal{M}_{\mathcal{H}}$, equals the ratio between the discounted sum of the deviations from steady state of GDP and the discounted sum of the deviations from steady state of a specific type of public investment up to quarter $\mathcal{H}$. In what follows, we focus on the short-run and long-run implications of public investment by computing the multiplier at the 1-year horizon (i.e., $\mathcal{H} = 4$) and throughout the entire response of output to the public investment shock (i.e., $\mathcal{H} = \infty$), which we refer to as the long-run multiplier.

5.4.2 The Role of the Public Investment Composition

We start by reporting in Columns (1) and (2) of Table 2 the 1-year and long-run multipliers for the "Single Type" and "Multiple Type" economies. In the short run, we find very muted effects of public investment: the multiplier of total public investment at the 1-year horizon is 0.15 in the economy with multiple public investment types, and 0.22 in the model with a single type. This is consistent with the findings of Boehm (2020), pointing out that public investment features a limited stimulus effect on impact. In our case, this limited effect hinges on the presence of the time-to-build and time-to-spend delays.[7] What emerges from these findings is that in the short run, abstracting from heterogeneity in the public investment composition yields an over-statement of the fiscal multiplier, even though the difference in absolute value remains not economically relevant.

When looking at the multipliers for each type of public investment—which are not reported for the "Single Type" economy as this case considers does not feature multiple types—we find that public equipment generates the strongest output response, with a 1-year multiplier of 0.30. Instead, the multiplier for IPP is 0.11, and that of structures is even negative, -0.03. Again, this is due to the heterogeneous incidence of the time-to-build and time-to-spend delays: since they characterize relatively more the investment in structures—and to a lower extent that of IPP—these lags curb substantially their output response. Instead, public investment in equipment is implemented and accrues to the stock of capital almost immediately,

[7] Appendix B.2 shows that the 1-year multiplier for the baseline economy becomes 0.54 when abstracting from the time-to-build and time-to-spend delays.

Table 2: Fiscal Multipliers.

	Single Type	Multiple Type	Multiple Type with Homogeneous ...			
			Output Elasticities	Depreciation Rates	Time-to-Build	Time-to-Spend
	(1)	(2)	(3)	(4)	(5)	(6)
Panel A: 1-Year Multiplier, $\mathcal{M}_4$						
Total	0.22	0.15	0.19	0.17	0.12	0.20
Equipment	-	0.30	0.32	0.29	0.27	0.24
Structures	-	-0.03	-0.02	-0.04	-0.04	0.23
IPP	-	0.11	0.20	0.19	0.07	0.11
Panel B: Long-run Multiplier, $\mathcal{M}_\infty$						
Total	0.48	1.56	0.76	1.07	1.75	1.55
Equipment	-	0.59	1.55	0.26	0.48	0.57
Structures	-	0.35	0.01	0.76	0.45	0.35
IPP	-	4.57	1.24	2.40	5.18	4.57

Note: This table reports fiscal multipliers, at the horizon of 1 year in Panel A, and in the long run in Panel B. Column (1) focuses on the "Single Type" economy, which abstracts from heterogeneity in the public investment composition and considers only one type of public investment, Column (2) shows the "Multiple Type" economy, which is the baseline model with public investment in equipment, structures, and IPP, Column (3) reports the "Multiple Type with Homogeneous Output Elasticities" economy, Column (4) focuses on the "Multiple Type with Homogeneous Depreciation Rates" economy, Column (5) shows the "Multiple Type with Homogeneous Time-to-Build" economy, and Column (6) reports the "Multiple Type with Homogeneous Time-to-Spend" economy.

so that private inputs can benefit even in the short run from this type of spending.

The multiplier increases significantly at longer horizons: the long-run multiplier in the "Multiple Type" economy is 1.56, in line with the values reported in the literature (Boehm, 2020; Ramey, 2021; Peri et al., 2024). Instead, the long-run multiplier for the "Single Type" model is much lower and even below 1, as it equals 0.48. This comparison highlights one of the key quantitative results of the paper: heterogeneity in public investment composition more than doubles the long-run fiscal multiplier.

Again, there is substantial variation in the long-run multipliers between equipment, structures, and IPP: the multiplier is smallest for structures, at 0.35, equipment features a value of 0.59, and it peaks for IPP at 4.57. From this perspective, these results point out that, should the government want to maximize the bang for the buck, public investment plans should be titled toward investment in intellectual property products. Regarding the comparison of the output effects of equipment and structures, in the robustness checks of Appendix B.2 we find that the ordering between these two types of public investment crucially depends on the value of the time discount factor. If we set it to $\beta = 0.995$, then the long-run multiplier of structures is larger than that of equipment, with values of 0.71 and 0.67, respectively. This due to the fact that long delays in implementing investment in structures makes having a negative output response on impact, while it takes time before the positive effects of its high output elasticity kick in. As a result, discounting less the output response to investment in structures capture relatively more its positive effect, raising its long-run multiplier.

5.4.3 Counterfactual Exercises

What drives the differences in the fiscal multipliers between the baseline economy and that which abstracts from heterogeneity in the public investment composition, as well as across the three investment types? To shed light on what are the model features that account for the bulk of these differences, we report series of counterfactual exercises in Columns (3)-(6) of Table 2. Specifically, we compare the results of the baseline economy with the multipliers associated with all the five counterfactual models introduced in Section 5.2.

Since the differences in short-run multipliers across the different economies are negligible, we focus on the long-run multipliers. In this way, we can isolate which model dimension accounts for the differences in the long-run multipliers between the “Single Type” and “Multiple Type” economies.

When abstracting from heterogeneity in the output elasticities, the multiplier of total public investment shrinks by 51%, from 1.56 to 0.77. The relevance of the variation in depreciation rates is more limited, as setting them homogeneously across types reduces the multiplier by 31%, to 1.07. This is because the depreciation rate of aggregate public capital is relatively lower than the average of the depreciation rate across types, as it mainly hinges on structures (given its prominence in total public investment). As discussed in Roulleau-Pasdeloup (2022), a lower depreciation implies that the steady-state level of public capital becomes arbitrarily large with respect to the steady-state level of investment. Consequently, an increase in public investment has a negligible effect in changing the stock of public capital, thus muting the responsiveness of private output. The time-to-build and time-to-spend delays play a minor role. Abstracting from heterogeneity in time-to-build actually raises the multiplier by 12%, up to 1.75. Homogeneous time-to-build delays boost the multiplier as they reduce the lags for the public investment types with relatively larger multipliers, that is, structures and IPP. Instead, heterogeneity in time-to-spend delays does not change the multiplier at all.

Thus, heterogeneity in the output elasticities is the most important determinant of the differences in the multipliers between the "Single Type" and "Multiple Type" economies, as it accounts for 74% of this difference. Heterogeneity in depreciation rates and time-to-build delays account for 45% and 24%, respectively.[8]

Heterogeneity in the output elasticities is also the most relevant factor for rationalizing the variation in the multipliers associated with type-specific spending. Indeed, when abstracting from this dimension, the multiplier of public equipment rises from 0.59 up to 1.55, the multiplier of IPP drops from 4.57 to 1.24, whereas that of structures changes much less, from 0.35 to 0.01. This is because in this case the elasticity of structures—and especially that of IPP—shrinks substantially, whereas the elasticity of equipment increase from the baseline case of no effect of the stock of public equipment on private output.

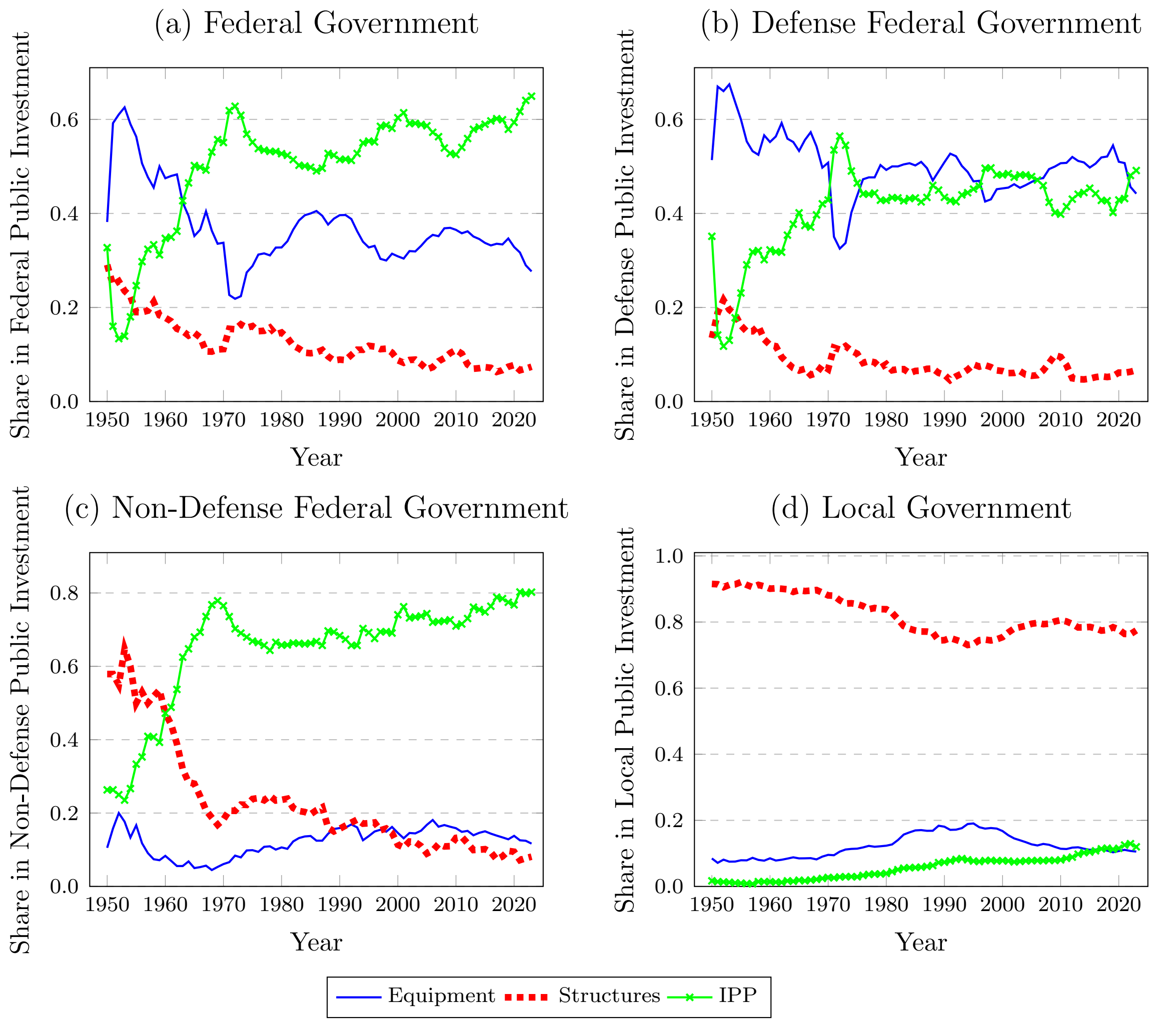
Abstracting from heterogeneity in the depreciation rates does not alter the long-run multiplier of equipment, but doubles the multiplier of structures, from 0.35 to 0.76, while halving the multiplier of IPP, from 4.57 to 2.40. This is because setting a homogeneous depreciation rate across investment types implies that the depreciation rate of structures increases and that of IPP shrinks. As we discuss above, changes in depreciation rates alter the multiplier since the output response to public investment decreases with the magnitude of the depreciation rate. Finally, note that the results of Table 2 indicate that all the other dimensions of heterogeneity—aside the role of the output elasticities and depreciation rates—are virtually inconsequential for the variation in the long-run multipliers across public investment types.

5.5 Changes in the Public Investment Composition

To further substantiate that fiscal multipliers crucially depend on the composition of public investment, in this section, we perform an exercise that leverages the observed variation in the composition of public investment between equipment, structures, and IPP both across government levels and over time. Figure 1 already pointed out that when looking at the public investment of the general government in the U.S., there have been large shifts in its composition, with an overall upward trend in the relevance of IPP between 1950 and 2023. We replicate exactly the

[8]The sum of the contribution of these three factors is larger than 100%, since heterogeneity in time-to-build delays actually reduces the long-run multiplier.

Figure 2: Composition of Public Investment across Government Levels.



Note: The figures show the shares of equipment, structures, and IPP in total public investment across government levels from 1950 to 2023. Panel (a) focuses on the federal government, Panel (b) focuses on the defense federal government, Panel (c) focuses on the non-defense federal government, and Panel (d) focuses on the local government.

same decomposition, this time looking at different government levels.

Figure 2 reports the shares of equipment, structures, and IPP for the federal government (in Panel a), the defense federal government (in Panel b), the non-defense federal government (in Panel c), the local government (in Panel d). These graphs highlight three main facts: (*i*) there are large swings, both cyclical and secular, in all government levels in the relative contribution of each type into total public investment spending; (*ii*) there are structural differences in the composition of public investment across levels, with the IPP being the dominant component for non-defense federal spending, equipment accounting on average for half of total public investment of the defense federal government, and the local government mainly hinging on structures; and (*iii*) the rising relevance of IPP characterizes all government levels, with the non-defense federal government experiencing the largest increase in this component, with a tripling of the IPP share from 26% in 1950 to 80% in 2023, whereas the local government is the level in which IPP increased the least, with its share changing over the entire period from 2% to 12%.

We evaluate the implications of these differences and shifts by feeding them into the model. Specifically, we run a model specification for each combination of government level and year in a three-step process. First, we modify the law of motions of the planned public investment expenditures of Equations (32), (33), and (34) by focusing solely on the common shock, and add type-specific loadings ϑ_e, ϑ_s, and ϑ_i, as follows:

$$\log A_{G_e,t} = (1-\rho)\log I_{G_e} + \rho \log A_{e,t-1} + \vartheta_e \varepsilon_t, \tag{44}$$

$$\log A_{G_s,t} = (1-\rho)\log I_{G_s} + \rho \log A_{s,t-1} + \vartheta_s \varepsilon_t, \tag{45}$$

$$\log A_{G_i,t} = (1-\rho)\log I_{G_i} + \rho \log A_{i,t-1} + \vartheta_i \varepsilon_t, \tag{46}$$

Second, we set the loading parameters to ensure the composition of total public investment coincides with that of the government level of interest in the year of interest. Then, we shock the economy and measure the implied multiplier.[9] We show the results of this exercise in Figure 3.

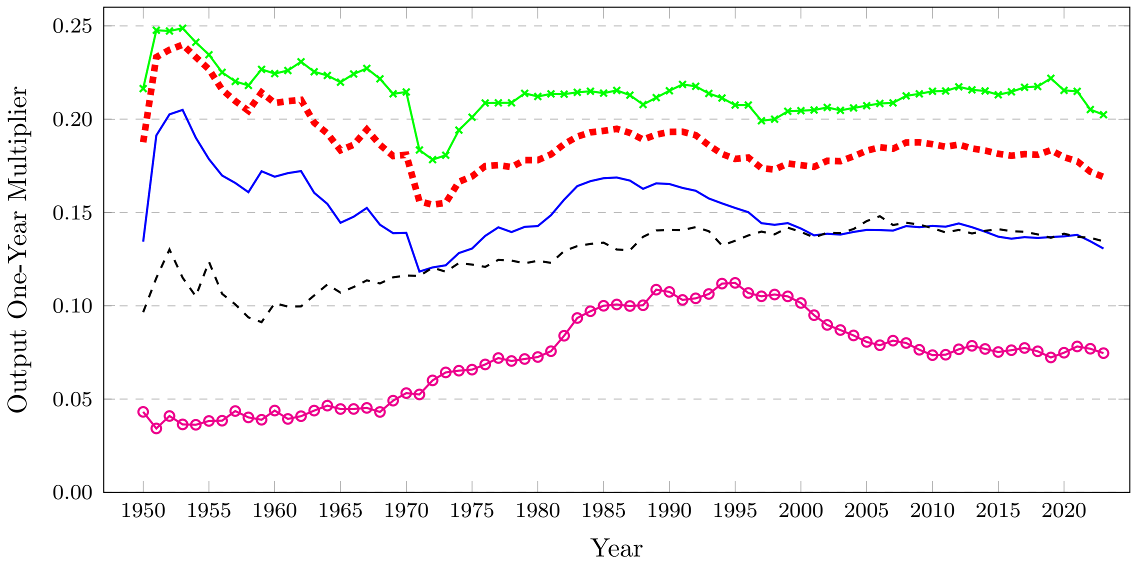
When looking at Panel (a) which reports the 1-year multipliers, we find that although there is variation both across government levels and over time in the size of the multiplier, the dispersion is still quite limited, with an overall range going from 0.03 to 0.24. However, the graphs indicate that in the short run, the public investment in defense spending yields the largest multiplier, whereas the spending of the local government is level with the lowest impact on output.

The overall variation in the long-run multipliers, depicted in Panel (b), is very substantial. On the low end, the public investment multipliers of the local government are low and smoothly increasing over time, from 0.44 in 1950 to 0.89 in 2023. On the high end, the public investment plans of the non-defense federal government have the highest multipliers, and the ability of this government level in triggering a large output response has been varying considerably over time: the long-run multiplier was 1.56 in 1950, shrunk to 1.34 in 1953, and rose steadily up to 3.66 in 1969.

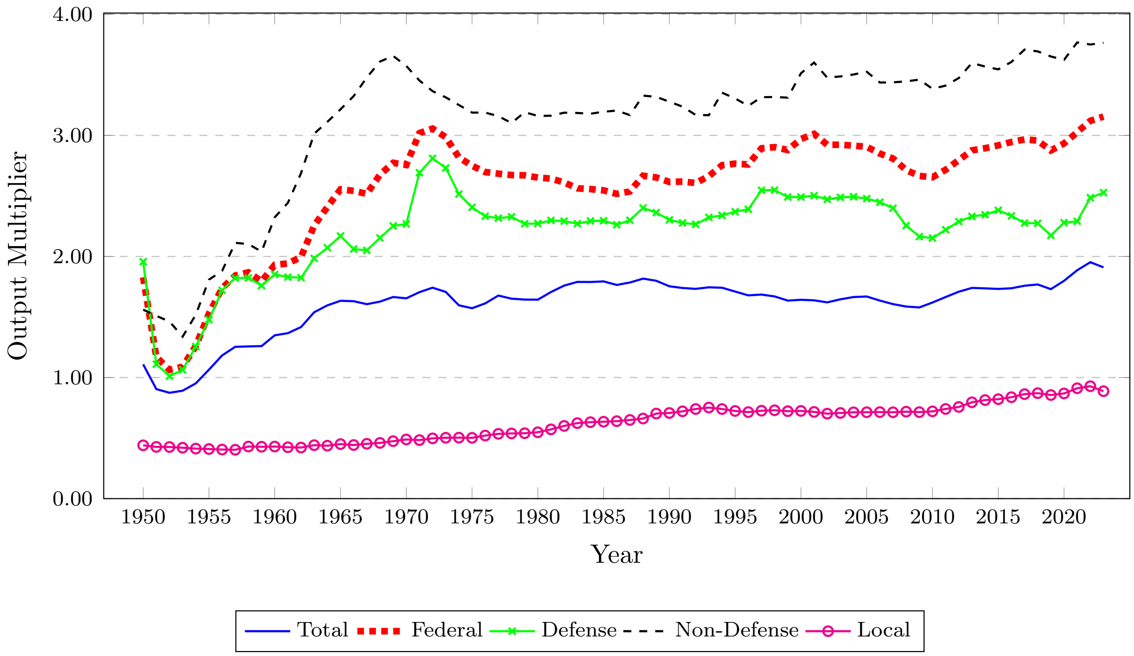
[9]Although the elasticity of output to public capital may differ when comparing spending of different government levels, in this exercise, we keep fixed the quantitative relevance of the different dimensions of heterogeneity across public investment types, as defined in the calibration of Section 5.1.

Figure 3: Fiscal Multipliers with Changing Public Investment Composition.

(a) 1-Year Fiscal Multipliers



(b) Long-run Fiscal Multipliers



Note: The figures show the fiscal multipliers implied by a composition of public investment that resembles that of different government levels, as it varies year by year in the data, between 1950 and 2023. Panel (a) focuses on the 1-year multiplier, and Panel (b) reports the long-run multiplier.

After that, the multiplier has been around 3.2, and only recently increased again, reaching the maximum value of 3.76 exactly in 2023. Consequently, the multiplier of total public investment of non-defense federal spending has been changed by 181%, and $2.4, between 1953 and 2023 just as a function of the observed variation in the composition of its public investment spending between equipment, structures, and IPP.

The multipliers of the general government range between 0.87 and 1.95, indicating that looking at output response of the spending carried out considering all government layers conceal a large deal of the heterogeneity that we have documented between local public investment, and that of the (non-defense) federal government.

Overall, the results of this section corroborate the notion that the composition of public investment has a first-order quantitative bearing on the size of the fiscal multiplier, so that the output response tends to be larger whenever spending is tilted towards IPP investment, as it is the case for the non-defense federal government, especially over the most recent years. Instead, public investment in equipment—and to a lower extent the investment in structures—has a much lower ability in spurring output.

6 Conclusion

This paper argues that the composition of public investment between equipment, structures, and IPP is critical to understand the aggregate implications of this type of spending. First, we show analytically that abstracting from heterogeneity in the composition yields to an under-estimation of the elasticity of private output to public capital. As a result, this under-estimation bias leads to an understatement of both the optimal amount of public capital as well as the size of the fiscal multiplier.

We then introduce the three types of public investment into an otherwise standard New Keynesian economy. We discipline the model by considering heterogeneity between equipment, structures, and IPP in the output elasticities to public capital, depreciation rates, time-to-build and time-to-spend delays, as well as steady-state shares of each type of spending in total GDP. In the baseline case, the output elasticities to public capital are set such that IPP capital has the largest effect on private output, closely followed by structures, whereas equipment is assumed to have no effect whatsoever. We find that heterogeneity in public investment composition doubles both the optimal level of public investment and the output multiplier.

Finally, we feed the model with actual variation in public investment composition both over time, between 1950 and 2023, and across government levels, considering the composition of public investment of the general government, defense federal government, non-defense federal government, and local government.

This exercise highlights that changes in the composition alter substantially the multiplier, and also uncover that the public investment of non-defense federal government has the strongest ability in spurring a surge in output, as this type of spending is tilted more towards IPP investment.

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A Empirical Evidence on the Output Elasticities to Public Capital

In Section 5.1, we discuss the calibration choices of the model regarding the output elasticities to public capital for equipment, structures, and IPP, which are based on values derived in the literature. Section 5.2 corroborates these decisions by showing that the model implies an estimated elasticity of private output to public capital which is remarkably in line with values found in the literature. In this section, we provide further support to our calibration by providing novel empirical evidence in line with our choices for the output elasticities.

To do so, we extent the econometric analysis of Bouakez et al. (2017) and Ramey (2021) by focusing each time on a different type of public capital, rather than considering total public capital. Specifically, we estimate the following regression

$$\log TFP_t = \upsilon_e \log K_{G_e,t} + \mathbf{X'}_t\theta + \epsilon_t, \tag{A.1}$$

$$\log TFP_t = \upsilon_s \log K_{G_s,t} + \mathbf{X'}_t\theta + \epsilon_t, \tag{A.2}$$

$$\log TFP_t = \upsilon_i \log K_{G_i,t} + \mathbf{X'}_t\theta + \epsilon_t, \tag{A.3}$$

where υ_e, υ_s, and υ_i are the values of the elasticity of private output to public investment in equipment, structures, and IPP, respectively, as observed in the data, TFP_t is a measure of capacity-adjusted productivity, $\mathbf{X'}_t$ is a set of covariates, and $K_{G_e,t}$, $K_{G_s,t}$, and $K_{G_i,t}$ are the stocks of public capital for the three types. Note that insofar our model is the true data generator process of the data, then the three stocks of public capital are orthogonal to each other, and thus we can recover the type-specific output elasticities by estimating regressions (A.1), (A.2), and (A.3) one by one, without incurring in any omitted variable bias. Regarding the covariates, Bouakez et al. (2017) and Ramey (2021) argue about the importance of incorporating factors that capture variation in both human capital and private R&D, so that the regressions can uniquely pin down the effect of public capital on productivity.

To run the estimation, we take the very same data used in Bouakez et al. (2017) and Ramey (2021) which refer to the U.S. economy from 1950 on at the annual frequency, and extend them with information on public capital for equipment, structures, and IPP. Specifically, Bouakez et al. (2017) and Ramey (2021) consider data from the U.S. BEA on the current-cost stock of total public capital of the general government, civilian non-institutional population above the age of 16, data from the Federal Reserve Economic Data of the St. Louis Federal Reserve Bank on the GDP deflator, the GDP component of research and development, and the personal consumption expenditures in education services. The series of seasonally-adjusted TFP comes from Fernald (2014). We add to this information by merging it with data from the U.S. BEA on the current-cost stocks of public capital in equipment, structures, and IPP.

We then compute the real stock of public capital for each stock per capita, by dividing the current-cost stocks by the product of the GDP deflator and population. We do the same for both R&D and education expenditures. With this data, we estimate the relationships (A.1), (A.2), and (A.3) as cointegrating regressions using Dynamic Ordinary Least Squares, using two leads and no lags as in Bouakez et al. (2017) and Ramey (2021). We find that the estimate of the output elasticity of equipment is $\hat{\upsilon}_e = -0.0035$ with a standard deviation of 0.0904 and a p-value of 0.969, the estimate of the output elasticity of structures is $\hat{\upsilon}_s = 0.0447$ with a standard deviation of 0.0259 and a p-value of 0.084, and the estimate of the output elasticity of IPP is $\hat{\upsilon}_i = 0.1354$ with a standard deviation of 0.0663 and a p-value of 0.041.

These values imply that our calibration choice for equipment slightly overstates its output elasticity, as we set $\gamma_e = 0.01$ while we cannot reject the null hypothesis that it is zero in the data. The estimated output elasticity for structures is 0.0447, in line with our calibration choice of $\gamma_s = 0.05$. Actually, we cannot reject the null hypothesis that the estimate $\hat{\upsilon}_s$ equals 0.05 at the 10% level (the test yields a p-value of 0.8372). Finally, while the point estimate of the elasticity of IPP is $\hat{\upsilon}_i = 0.1354$ and larger than our calibration choice of $\gamma_s = 0.07$, given the standard error of the estimate we cannot reject the null hypothesis that $\hat{\upsilon}_i$ equals 0.07 at the 10% level (the test yields a p-value of 0.3241). All in all, these empirical results further corroborate the validity of our calibration strategy for the output elasticity to public capital for equipment, structures, and IPP.

B Quantitative Analysis: Robustness Checks

This section provides an extensive battery of robustness checks that corroborate the validity of our main quantitative findings in terms of both the optimal levels of public investment and public capital at the steady state and the fiscal multipliers. We do it in two ways: we consider a sensitivity analysis with respect to a wide sensitivity analysis of different calibration choices, and we study various alternative model specifications.

Importantly, in this section we compare how the quantitative implications of alternative specifications of our multiple type model alters when compared to that of an array of single type economies. For every specification of the multiple type economy that we consider in this robustness analysis, we define the corresponding single type model and discipline the implied output elasticity to aggregate public capital following the approach discussed in Section 5.2. Specifically, we simulate data from the different multiple type economies, compute measures of aggregate TFP and aggregate stock of public capital, and estimate the regression (9) to derive the corresponding implied output elasticity to aggregate public capital, $\bar{\gamma}$. We find that while some alternative economies generate exactly the same implied elasticity of the baseline model, that is, $\bar{\gamma} = 0.061$, there is nonetheless a lot of variation in the estimates of the implied elasticity across model specifications, ranging from 0.029 to 0.078.

Section B.1 reports the analysis on the robustness check on the optimal levels, thus extending the quantitative results in Section 5.3, while Section B.2 evaluates the validity of the findings on fiscal multipliers uncovered in Sections 5.4.2 and 5.4.3.

B.1 Optimal Public Capital: Robustness Checks

To ascertain the robustness of the model implications on the optimal levels of public investment and public capital at the steady state, we consider four alternative calibration choices for the elasticities of private output to the three types of public capital.

In the first case, we halve the three output elasticities from their baseline values, which implies that the values for equipment, structures, and IPP become $\gamma_e = 0.005$, $\gamma_s = 0.025$, and $\gamma_i = 0.035$, respectively. We report this exercise in Table B.1. Consistently with the baseline results of Table 1, moving from the "Single Type" to the "Multiple Type" economy doubles the levels of both the optimal public capital and the optimal public investment (from 56.77% and 2.48% to 108.61% and 5.40%, respectively). Again, this effect is entirely drive by heterogeneity in

Table B.1: Optimal Public Capital and Public Investment - Halved Elasticities.

	Single Type	Multiple Type	Multiple Type with Homogeneous …		Data
			Output Elasticities	Depreciation Rates	
	(1)	(2)	(3)	(4)	(5)
Panel A: Optimal Public Capital (% of GDP)					
Total	56.77	108.61	47.97	120.98	73.82
Equipment	-	3.72	7.57	9.31	9.44
Structures	-	85.50	34.77	46.53	57.63
IPP	-	19.39	5.63	65.14	6.75
Panel B: Optimal Public Investment (% of GDP)					
Total	2.48	5.40	2.57	5.28	4.84
Equipment	-	0.46	0.94	0.41	1.28
Structures	-	1.64	0.67	2.03	2.25
IPP	-	3.30	0.96	2.84	1.31

Note: This table reports the same statistics of Table 1 with the difference that we consider an alternative calibration of the elasticities of private output to the three stocks of public capital in which we halve the values of the baseline elasticities, setting $\gamma_e = 0.005$, $\gamma_s = 0.025$, and $\gamma_i = 0.035$.

the output elasticities across the three types as abstracting from heterogeneity in the depreciation rates barely alter the main findings of the baseline model. While in this case the optimal levels shrink by half with the reduction in the output elasticities, still we find that the type which is mostly under-invested in is IPP.

In the second case, we consider the baseline output elasticities for structures and IPP, but double the output elasticity of equipment, which then changes from from $\gamma_e = 0.01$ to $\gamma_e = 0.02$. We report this exercise in Table B.2. Also in this case, accounting for heterogeneity in public investment composition doubles the optimal levels of public capital (from 127.12% to 224.67%) and public investment (from 5.55% to 11.37%). Since this case does not alter neither the output elasticities nor the depreciation rates of structures and IPP, the implications along these two dimensions exactly coincide with those of the baseline analysis in Table 1. With respect to equipment, its optimal public investment as a share of GDP doubles from the baseline value of 0.92% to 1.85%. Note that the model implies an optimal level of public investment in equipment that matches the value observed in the data if we were to set $\gamma_e = 0.014$.

In the third case, we consider that the stock of public equipment is not produc-

Table B.2: Optimal Public Capital and Public Investment - High Equipment Elasticity.

	Single Type	Multiple Type	Multiple Type with Homogeneous ...		Data
			Output Elasticities	Depreciation Rates	
	(1)	(2)	(3)	(4)	(5)
Panel A: Optimal Public Capital (% of GDP)					
Total	127.12	224.67	107.43	260.57	73.82
Equipment	-	14.89	16.95	37.22	9.44
Structures	-	171.00	77.86	93.06	57.63
IPP	-	38.78	12.61	130.28	6.75
Panel B: Optimal Public Investment (% of GDP)					
Total	5.55	11.73	5.74	11.37	4.84
Equipment	-	1.85	2.11	1.62	1.28
Structures	-	3.27	1.49	4.06	2.25
IPP	-	6.61	2.15	5.68	1.31

Note: This table reports the same statistics of Table 1 with the difference that we consider an alternative calibration of the elasticities of private output to the three stocks of public capital in which we double the output elasticity of equipment, setting $\gamma_e = 0.02$, $\gamma_s = 0.05$, and $\gamma_i = 0.07$.

tive, by setting it to $\gamma_e = 0$. This case is motivated by our empirical evidence of Appendix A, which could not reject the null hypothesis that the output elasticity of the stock of public equipment is zero. Instead, we still preserve the baseline choices for structures and IPP, so that $\gamma_s = 0.05$ and $\gamma_i = 0.07$. We report this exercise in Table B.3. Moving from the "Single Type" to the "Multiple Type" economy changes the levels of both the optimal public capital and the optimal public investment, from 56.77% and 2.48% to 108.61% and 5.40%, respectively. The calibration choice on making the stock of equipment not to be productive immediately implies that it is optimal not to invest at all in it, so that the optimal shares of public investment and public capital in equipment are zero.

Finally, the fourth case keeps the baseline output elasticity of equipment, so that $\gamma_e = 0.02$, but swaps the values of the elasticities of structures and IPP, by setting $\gamma_s = 0.07$ and $\gamma_i = 0.05$. These choices imply that the stock of public structures is the most productive one, followed by that of IPP. We report this exercise in Table B.4. Moving from the "Single Type" economy to the "Multiple Types" one roughly doubles the optimal levels of public capital (from 152.43% to 281.99%, an increase by 85%) and public investment (from 6.65% to 11.15%, an increase by

Table B.3: Optimal Public Capital and Public Investment - Low Equipment Elasticity.

	Single Type	Multiple Type	Multiple Type with Homogeneous …		Data
			Output Elasticities	Depreciation Rates	
	(1)	(2)	(3)	(4)	(5)
Panel A: Optimal Public Capital (% of GDP)					
Total	114.28	209.78	96.57	223.35	73.82
Equipment	-	0.00	15.24	0.00	9.44
Structures	-	171.00	70.00	93.06	57.63
IPP	-	38.78	11.34	130.28	6.75
Panel B: Optimal Public Investment (% of GDP)					
Total	5.55	9.88	5.16	9.74	4.84
Equipment	-	0.00	1.89	0.00	1.28
Structures	-	3.27	1.34	4.06	2.25
IPP	-	6.61	1.93	5.68	1.31

Note: This table reports the same statistics of Table 1 with the difference that we consider an alternative calibration of the elasticities of private output to the three stocks of public capital in which we set the stock of public equipment not to be productive, setting $\gamma_e = 0$, $\gamma_s = 0.05$, and $\gamma_i = 0.07$.

68%). While this case raises the optimal levels of structures, it still implies that the type of public capital which is mostly under-invested is IPP, confirming thus the findings of the baseline analysis in Table 1.

All in all, this section confirms the main findings of the baseline analysis: (*i*) explicitly accounting for heterogeneity in public investment composition doubles both the optimal levels of public capital and public investment; (*ii*) the key model features that accounts for this result is heterogeneity in the output elasticities to public capital, while heterogeneity in depreciation rates play a negligible role; and (*iii*) IPP is the type which is mostly under-invested in.

B.2 Fiscal Multipliers: Robustness Checks

We turn into the robustness checks of the model implications on fiscal multipliers. While the previous section only studied the role of the output elasticities to public capital and the depreciation rates—as these two dimensions are the only ones that matter to pin down the optimal levels of public capital and investment—we now consider a much wider array of alternative calibration choices and model specifications.

Table B.4: Optimal Public Capital and Public Investment - High Structures Elasticity.

	Single Type	Multiple Type	Multiple Type with Homogeneous ...		Data
			Output Elasticities	Depreciation Rates	
	(1)	(2)	(3)	(4)	(5)
Panel A: Optimal Public Capital (% of GDP)					
Total	152.43	281.99	128.82	260.57	73.82
Equipment	-	14.89	20.33	37.22	9.44
Structures	-	239.40	93.37	130.28	57.63
IPP	-	27.70	15.12	93.06	6.75
Panel B: Optimal Public Investment (% of GDP)					
Total	6.65	11.15	6.89	11.37	4.84
Equipment	-	1.85	2.52	1.62	1.28
Structures	-	4.58	1.79	3.27	2.25
IPP	-	4.72	2.58	6.61	1.31

Note: This table reports the same statistics of Table 1 with the difference that we consider an alternative calibration of the elasticities of private output to the three stocks of public capital in which we swap the values associated with structures and IPP, setting $\gamma_e = 0.01$, $\gamma_s = 0.07$, and $\gamma_i = 0.05$.

We start by altering the value of the Frisch elasticity of labor supply. In the baseline analysis, we set it to 0.5 in line with the evidence on the elasticity at the individual level. However, macroeconomic models tend to consider a much more elastic labor supply. For instance, Baxter and King (1993) and Ramey (2021) consider a value of 4. As discussed by Hall (2009), fiscal multipliers increase in the value of the Frisch elasticity so that a more elastic labor supply may be required to generate a relatively stronger output response to public investment. However, in the baseline model we set it to 0.5 because in this way the model can be consistent with the micro-level evidence on the elasticity of labor supply, while still being able to generate a large long-run multiplier thanks to the amplification coming from explicitly accounting for the composition of public investment. In the first robustness check, reported in Columns (1) and (2) of Table B.5, we increase the Frisch elasticity from 0.5 to 1. We still find that the long-run multiplier increases substantially when comparing the "Single Type" and "Multiple Type" economies, that is, from 0.68 to 1.65, an increase of 143%, and \$0.97.

In the baseline model, we choose the elasticity of substitution across wholesaler varieties to $\epsilon = 4$ so that markups equal 33%, in line with the recent evidence by

Table B.5: Fiscal Multipliers - Robustness Checks.

	High Frisch Elasticity		Low Markup		High Persistence	
	Single Type	Multiple Type	Single Type	Multiple Type	Single Type	Multiple Type
	(1)	(2)	(3)	(4)	(5)	(6)
Panel A: 1-Year Multiplier, $\mathcal{M}_4$						
Total	0.34	0.26	0.21	0.15	0.21	0.15
Equipment	-	0.43	-	0.29	-	0.29
Structures	-	0.12	-	-0.02	-	0.01
IPP	-	0.16	-	0.10	-	0.08
Panel B: Long-run Multiplier, $\mathcal{M}_\infty$						
Total	0.68	1.65	0.53	1.61	0.59	1.61
Equipment	-	0.79	-	0.61	-	0.66
Structures	-	0.55	-	0.37	-	0.47
IPP	-	4.34	-	4.66	-	4.47

Note: This table reports the same fiscal multipliers of Table 2 with the only difference that it focuses only on the distinction between the "Single Type" and "Multiple Type" economies, and Columns (1) and (2) consider an alternative calibration of the Frisch elasticity that increases it from 0.5 to 1, Columns (3) and (4) consider an alternative calibration of the elasticity of substitution across varieties that reduces markups from 33% to 20%, and Columns (5) and (6) consider an alternative calibration of the persistence of the auto-regressive processes of planned public investment by type by raising it from 0.9 to 0.94.

De Loecker et al. (2020). We consider an exercise in which we reduce markups to 20%, in line with the estimates of Christiano et al. (2005), so that $\epsilon = 6$. We report this exercise in Columns (3) and (4) of Table B.5. We find that the long-run multiplier is 0.53 for the "Single Type" model and 1.61 for the "Multiple Type" economy, an increase of 204%, and \$1.08.

We then consider the persistence of the auto-regressive processes that govern the law of motion of the planned public investment for each type. In the baseline analysis, we consider a persistence of $\rho = 0.9$. Columns (5) and (6) of Table B.5 report the results of a case in which the persistence equals $\rho = 0.94$, in line with the value estimated by Leeper et al. (2010). This case shows that accounting for heterogeneity in public investment composition raises the long-run multiplier from 0.59 to 1.61, a change by 173%, and \$1.02.

The time discount factor is set to $\beta = 0.99$ in the baseline analysis, so to match an annual real interest rate of 4%. Given the decline in real rates in recent decades, and the fact that the measurement of fiscal multipliers in Equation (43) implies

Table B.6: Fiscal Multipliers - Robustness Checks (cont.).

	High Time Discount Factor		Inflation Response Taylor Rule		No Investment Adj. Cost	
	Single Type	Multiple Type	Single Type	Multiple Type	Single Type	Multiple Type
	(1)	(2)	(3)	(4)	(5)	(6)
			Panel A: 1-Year Multiplier, $\mathcal{M}_4$			
Total	0.20	0.13	0.14	0.09	0.26	0.20
Equipment	-	0.28	-	0.20	-	0.24
Structures	-	-0.05	-	0.04	-	0.66
IPP	-	0.08	-	0.00	-	-0.02
			Panel B: Long-run Multiplier, $\mathcal{M}_\infty$			
Total	0.76	2.16	0.44	1.52	0.45	1.52
Equipment	-	0.67	-	0.54	-	0.51
Structures	-	0.71	-	0.33	-	0.40
IPP	-	6.08	-	4.48	-	4.41

Note: This table reports the same fiscal multipliers of Table 2 with the only difference that it focuses only on the distinction between the "Single Type" and "Multiple Type" economies, and Columns (1) and (2) consider an alternative calibration of the time discount factor by changing it from 0.99 to 0.995, Columns (3) and (4) consider an alternative calibration of the Taylor rule parameter that governs the responsiveness of the nominal interest rates to inflation by raising it from 1.5 to 5, and Columns (5) and (6) consider an alternative model specification that abstracts from the private investment adjustment costs.

that the time discount factor directly affects it, by modulating the discounting of the deviations from steady state of GDP and public investment, we consider a case in which we the time discount factor rises to $\beta = 0.995$, which implies an annual real interest rate of 2%. We report this exercise in Columns (1) and (2) of Table B.6. We find that the long-run multiplier is 0.76 for the "Single Type" model and 2.16 for the "Multiple Type" economy, an increase of 184%, and \$1.40. Interestingly, in this case the long-run multiplier of structures becomes larger of that of equipment, with values of 0.71 and 0.67, respectively. Instead, in the baseline analysis, while the multiplier of equipment is 0.59, the one of structures is much lower, as it equals 0.35. Thus, raising the time discount factor doubles the long-run multiplier of structures. This due to the fact that the long delays in implementing investment in structures makes it to have a negative output response on impact, while it takes time before the positive effects of its high output elasticity kick in. As a result, discounting less the output response to investment in structures capture relatively more its positive effect, raising its long-run multiplier.

We also ascertain the role of the monetary policy stance, as Woodford (2011) demonstrates that the level of the multiplier shrinks if the monetary authority is relatively more reactive to changes in inflation. We address this possibility by raising the inflation sensitivity of nominal interest rates in the Taylor rule from $\phi_\pi = 1.5$ to $\phi_\pi = 5$, and report the results in Columns (3) and (4) of Table B.6. Again, we find that the long-run multiplier increases substantially when comparing the "Single Type" and "Multiple Type" economies, that is, from 0.44 to 1.52, an increase of 246%, and \$1.08. While this case reduces substantially the long-run multiplier of IPP, this is still the public investment type which has the largest effect by far in spurring private output.

We then turn into the convex adjustment costs of private investment, which we calibrate to match the relative volatility of private investment—in terms of that of GDP—in a model version with productivity shocks. The results of Ramey (2021) indicate that the presence of adjustment costs is not innocuous, as they hinder the crowding out effect on investment, raising the multiplier. We study a version of our model that abstracts from the adjustment costs by setting $\Omega = 0$, and report the multipliers in Columns (5) and (6) of Table B.6. In this case, heterogeneity in public investment composition raises the long-run multiplier from 0.45 to 1.52, a change by 238%, and \$1.07.

Next, Table B.7 studies how the multipliers change if we alter the elasticities of private output to public capital for the three types. We start in Columns (1) and (2) by halving the baseline elasticities, so that $\gamma_e = 0.05$, $\gamma_s = 0.025$, and $\gamma_i = 0.035$. Reducing the elasticities dampens substantially the long-run multipliers, with little effect of the short-run effects of public investment. The "Single Type" and "Multiple Type" economies feature long-run multipliers of 0.10 and 0.66, respectively, that is, a change between the two of 560%, and \$0.56. In this case, the long-run multiplier of public investment in structures becomes zero, while IPP still features a value well above one. We consider in Columns (3) and (4) a higher output elasticity for equipment, so that $\gamma_e = 0.02$, $\gamma_s = 0.05$, and $\gamma_i = 0.07$. The long-run of the "Single Type" economy is 0.52, and that of the "Multiple Type" model is 1.78, an increase of 242%, and \$1.26. Interestingly, in this case the long-run multiplier of equipment becomes larger than one, with a value of 1.42. Yet, the type with the largest long-run multiplier by far is IPP; with a value of 4.57. Then, we evaluate the relevance of the output elasticities of structures and IPP by swapping their baseline values, so that $\gamma_e = 0.02$, $\gamma_s = 0.07$, and $\gamma_i = 0.05$. The results in Columns (5) and (6) indicate that accounting for heterogeneity in public

Table B.7: Fiscal Multipliers - Robustness Checks (cont.).

	Halved Elasticities		High Equipment Elasticity		High Structure Elasticity	
	Single Type	Multiple Type	Single Type	Multiple Type	Single Type	Multiple Type
	(1)	(2)	(3)	(4)	(5)	(6)
Panel A: 1-Year Multiplier, $\mathcal{M}_4$						
Total	0.23	0.17	0.22	0.16	0.21	0.16
Equipment	-	0.29	-	0.32	-	0.30
Structures	-	-0.02	-	-0.03	-	-0.04
IPP	-	0.17	-	0.11	-	0.15
Panel B: Long-run Multiplier, $\mathcal{M}_\infty$						
Total	0.10	0.66	0.52	1.78	0.68	1.30
Equipment	-	0.18	-	1.42	-	0.59
Structures	-	0.05	-	0.35	-	0.59
IPP	-	2.16	-	4.57	-	3.19

Note: This table reports the same fiscal multipliers of Table 2 with the only difference that it focuses only on the distinction between the "Single Type" and "Multiple Type" economies, and Columns (1) and (2) consider an alternative calibration of output elasticities to public capital by halving them, Columns (3) and (4) consider an alternative calibration of the output elasticity of equipment by raising it from 0.01 to 0.02, and Columns (5) and (6) consider an alternative calibration of the output elasticities of structures and IPP by swapping their baseline values.

investment composition raises the long-run multiplier from 0.68 to 1.30, a change of 91%, and $0.62. Also in this case, reducing the output elasticity of IPP relative to structures does not alter the conclusion that the largest long-run multiplier is the one associated with public investment in IPP.

Table B.8 evaluates how the multipliers vary when we disregard the time-to-build and time-to-spend delays. In this way, we can isolate how they affect the output response of public investment, both across types as well as with the horizon of the output response to the initial shock. We start by abstracting from the time-to-build delays in Columns (1) and (2), so that $\zeta_e = \zeta_s = \zeta_i = 0$. In this case, the 1-year multiplier of the "Multiple Type" economy surpasses that of the "Single Type", with a marked increase in the short-run output response to IPP investment, since its 1-year multiplier becomes the largest one across the three types with a value of 0.55. When looking at the long-run values, the multipliers of the "Single Type" and "Multiple Type" economies are 0.60 and 2.11, respectively, with a change of 252%, and $1.51. Setting the time-to-spend delays to zero in

Table B.8: Fiscal Multipliers - Robustness Checks (cont.).

	No Time-to-Build		No Time-to-Spend		No Delays	
	Single Type (1)	Multiple Type (2)	Single Type (3)	Multiple Type (4)	Single Type (5)	Multiple Type (6)
Panel A: 1-Year Multiplier, $\mathcal{M}_4$						
Total	0.25	0.32	0.38	0.37	0.42	0.54
Equipment	-	0.32	-	0.42	-	0.44
Structures	-	-0.02	-	0.39	-	0.41
IPP	-	0.55	-	0.31	-	0.85
Panel B: Long-run Multiplier, $\mathcal{M}_\infty$						
Total	0.60	2.11	0.58	1.67	0.70	2.22
Equipment	-	0.61	-	0.67,	-	0.69
Structures	-	0.55	-	0.45	-	0.66
IPP	-	6.22	-	4.75	-	6.40

Note: This table reports the same fiscal multipliers of Table 2 with the only difference that it focuses only on the distinction between the "Single Type" and "Multiple Type" economies, and Columns (1) and (2) consider an alternative model specification that abstracts from any time-to-build delay, Columns (3) and (4) consider an alternative model specification that abstracts from any time-to-spend delay, and Columns (1) and (2) consider an alternative model specification that abstracts from any delay whatsoever.

Columns (3) and (4), so that $\tau_e = \tau_s = \tau_i = 0$, makes almost identical 1-year multipliers across the two economies, with values of 0.38 and 0.37. Interestingly, in this case the 1-year multiplier of structures increases substantially, up to 0.39, almost to coincide with that of equipment. For the long-run multipliers, accounting for heterogeneity in public investment composition raises them from 0.58 to 1.67, a change of 188%, and \$1.09. When we abstract from both time-to-build and time-to-spend delays, so that $\zeta_e = \zeta_s = \zeta_i = \tau_e = \tau_s = \tau_i = 0$, the 1-year multiplier of the "Multiple Type" economy increases up to 0.52, again with the largest contribution coming from IPP, followed up by structures, and equipment. In addition, the long-run multipliers of the "Single Type" and "Multiple Type" economies are 0.70 and 2.22, respectively, with a change of 217%, and \$1.52.

Finally, Table B.9 studies the robustness of our results on the multipliers by looking at alternative model specifications that either alter the degree of nominal rigidities or change the fiscal system. First, we add to the model the presence of nominal wage rigidity as in Erceg et al. (2000). To do so, we consider that house-

Table B.9: Fiscal Multipliers - Robustness Checks (cont.).

	Sticky Wages		Flexible Prices		Distortionary Taxes	
	Single Type	Multiple Type	Single Type	Multiple Type	Single Type	Multiple Type
	(1)	(2)	(3)	(4)	(5)	(6)
Panel A: 1-Year Multiplier, $\mathcal{M}_4$						
Total	0.28	0.18	0.22	0.15	-0.13	-0.24
Equipment	-	0.41	-	0.21	-	-0.02
Structures	-	-0.22	-	0.28	-	-0.60
IPP	-	0.21	-	-0.01	-	-0.24
Panel B: Long-run Multiplier, $\mathcal{M}_\infty$						
Total	0.54	1.64	0.45	1.65	-0.19	0.86
Equipment	-	0.68	-	0.60	-	-0.08
Structures	-	0.36	-	0.43	-	-0.39
IPP	-	4.73	-	4.74	-	3.88

Note: This table reports the same fiscal multipliers of Table 2 with the only difference that it focuses only on the distinction between the "Single Type" and "Multiple Type" economies, and Columns (1) and (2) consider an alternative model specification that introduces nominal wage stickiness, Columns (3) and (4) consider an alternative model specification that abstracts from price rigidity and makes prices fully flexible, and Columns (5) and (6) consider an alternative model specification in which the deviation of public investment from its steady-state level is financed with a distortionary tax that applies to both labor income and capital income.

holds supply differentiated varieties of labor, indexed by $x \in [0, 1]$, which are imperfectly substitutable among themselves. This implies a labor aggregator that reads

$$N_t = \left(\int_0^1 N_{x,t}^{\frac{\epsilon_w - 1}{\epsilon_w}} \mathrm{d}x \right)^{\frac{\epsilon_w}{\epsilon_w - 1}}, \tag{B.4}$$

where $N_{x,t}$ denotes a specific variety of labor, and ϵ_w is the elasticity of substitution. Total labor is then sold to the wholesalers at a wage, which is set according to a Calvo price-setting protocol, in which the probability of adjusting it equals to $1 - \phi_w$. We set the elasticity to substitution to $\epsilon_w = 4$, so that is equals the value of the elasticity of substitution across wholesalers' goods varieties, while the probability of not adjusting the wage is $\phi_w = 2/3$, in line with the evidence on the frequency of wage adjustments documented by Barattieri et al. (2014). We report the multipliers of this case in Columns (1) and (2), and find that the long-run multipliers equal 0.54 for the "Single Type" economy, and 1.64 for the"Multiple Type" model, which implies a difference of 204%, and \$1.10. Next, we abstract from price rigidities, so that $\phi = 0$, which makes the model to feature fully flexible prices. The results of Columns (3) and (4) show that the long-run multipliers become 0.45 and 1.65, among the two economies, with a change of 267%, and \$1.20. In the last case, we alter the fiscal system by positing that changes in public investment from the steady state are financed through a distortionary tax on both labor income and capital income, in the spirit of Leeper et al. (2010). Specifically, we change the household borrowing constraint of Equation (25) as follows

$$P_t C_t + P_t I_t + B_{t+1} + T_t = (1 - \tau_t)\, W_t N_t + (1 - \tau_t)\, R_{K,t} K_t + R_t B_t + D_t, \quad \text{(B.5)}$$

where τ_t denotes the distortionary tax rate. Then, the government budget constraint of Equation (41) becomes

$$T + \tau_t \left[W_t N_t + R_{K,t} K_t\right] = I_{s,t} + I_{e,t} + I_{i,t}. \quad \text{(B.6)}$$

where the lump-sum tax equals total public investment at the steady state, $T = I_s + I_e + I_i$. We report the results of this exercise in Columns (5) and (6). In this case, the multipliers shrink substantially. In the short run, we find negative 1-year multipliers for both economies and all types. For the long-run multipliers, the "Single Type" economy implies a negative value of -0.19, while the value of the "Multiple Type" model is still positive, at 0.86. This implies a change of 553%, and \$1.05. While distortionary taxes make the long-run multipliers of equipment and structures to be negative, that of IPP is still high and positive, at 3.88

All in all, this section confirms the main findings of the baseline analysis: (*i*) explicitly accounting for heterogeneity in public investment composition more than doubles (and roughly triples) the long-run fiscal multiplier associated with an exogenous change in total public investment; (*ii*) public investment in IPP has by far the largest multiplier, with structures having a positive multiplier but below one, whereas equipment tends to feature even negative values; and (*iii*) differences in the short run are limited, and, if anything, heterogeneity in public investment composition dampens the output response to public investment on impact.