Job Search, Entrepreneurship and Labour Market Outcomes of Natives and Immigrants in Germany *

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PRELIMINARY DRAFT

Abstract

We evaluate the effects of low-skilled immigration on small businesses, wages and employment in Germany. We develop a search and matching model with heterogeneous agents, cross-skill matching and endogenous entry into entrepreneurship. The model is calibrated using data from the German Socio-Economic Panel (SOEP). We find that low-skilled immigration benefits high-skill workers while negatively affecting the welfare of low-skilled workers. It leads to the endogenous expansion of immigrant entrepreneurial activities generating positive spillovers for all demographic groups, except native entrepreneurs. Policies restricting immigrant entrepreneurship relax competition for native small businesses but reduce welfare for all other worker groups.

Keywords: entrepreneurship, small business, self-employment, search frictions, immigration
JEL codes: J23, J31, J61, J64

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1 Introduction

In this paper, we revisit the impact of immigration on receiving countries using data from Germany. In the last decade, Germany has experienced one of the largest inflows of immigrants, particularly refugees. These inflows have raised concerns in the country about the possible socio-economic consequences. Several recent papers used quantitative and empirical approaches to quantify the economic implications of these refugee waves (Busch et al., 2020; Scharfbillig and Weissler, 2019; Iftikhar and Zaharieva, 2019). Besides these papers, a large body of literature deals with diverse aspects of the effects of migration on receiving countries. However, most papers from this literature look at the labour market outcomes such as wage (D'Amuri et al., 2010; Ben-Gad, 2008; Borjas, 2003, 1999), employment (Dustmann et al., 2017; Pischke and Velling, 1997) and welfare (Iftikhar and Zaharieva, 2019; Battisti et al., 2018) etc. This literature treats immigrants as workers who can potentially compete with natives for jobs and influence their welfare via effects on native wages, employment opportunities, and fiscal transfers.

Another relevant strand of literature on this topic examines the association between entrepreneurship and immigration.[1] This literature provides evidence that immigrants contribute to job creation as entrepreneurs. Using micro-census data Sachs et al. (2016) find that entrepreneurs with migration background employed at least 1.3 million people in Germany in 2014, while Leicht and Langhauser (2014) find this number between 1.5 to 2 million. The role of immigrants as entrepreneurs has direct implications for the labour market outcomes of natives working as employees or searching for jobs. This role is ignored by the research that treats immigrants as workers only; hence, it may overestimate the competition effects of immigration on native workers. Additionally, we observe in the Socio-Economic Panel Data (SOEP) for Germany that respondents who report self-employment/entrepreneurship at least once in the sample period of 2000-2017 have a much lower unemployment rate than the respondents who never experienced self-employment during this period.

The evidence discussed above raises several questions: 1) does ignoring the role of immigrants as entrepreneurs lead to biased predictions concerning the consequences of immigration for labour market outcomes and the welfare of native workers? 2) whether policies reducing barriers to self-employment may be beneficial in reducing unemployment and increasing income, particularly among low-skilled immigrants. In this paper, we attempt to answer these questions by synthesizing the aforementioned strands of literature.

Our contribution to the literature is manifold. First, we develop a unified framework that allows for a dual role of immigrants as job seekers or job creators. By doing so, we capture both higher competition faced by native workers due to inflow of immigrants

[1](OECD, 2010; Wadhwa, 2007; Hohn et al., 2012; Xavier et al., 2013; Kerr and Kerr, 2020; Riillo and Peroni, 2022; Li, 2001; Lofstrom, 2002; Constant and Schultz-Nielsen, 2004; Constant et al., 2005; Constant and Zimmermann, 2006)

into the German labour market, as well as the better outside options due to jobs created by immigrant entrepreneurs. Second, we provide a quantitative assessment of how these roles impact workers in regular jobs and other small businesses run by native and incumbent immigrants. Third, we evaluate the policies that facilitate immigrants' entry into entrepreneurship. For this purpose, we develop a search and matching (SaM) model with 4 demographic groups: natives/immigrants, high/low skilled. Within each demographic group there are individuals with high entrepreneurial spirit, called potential entrepreneurs, and individuals with low entrepreneurial spirit, called regular workers. Potential entrepreneurs have several options, they can enter solo self-employment, register a small business with co-workers or search and accept a regular paid job. Entry into these states is endogenous and depends on the labour market conditions. Regular workers can be employed in a regular job (large and medium size firms) or in a small business operated by the self-employed entrepreneur. In addition, there is the state of unemployment which is involuntary for all worker groups. Large and medium size firms are modeled in a classical SaM tradition with free-entry and one firm offering one skill-specific vacancy.

Our model has a number of novel features. First, we explicitly allow for the possibility of self-employment "out of necessity", when self-employed entrepreneurs continue searching for regular paid jobs, and self-employment "out of opportunity", when the income from self-employment activities is high enough and there are no gains from job search. Second, the size distribution of small businesses is endogenous. Third, we incorporate the possibility of cross-skill matching on a micro-level, whereby small businesses founded by the high skill entrepreneurs hire workers from both skill groups. The cross-matching captures positive hiring spillovers that low-skilled immigration generates for small businesses operated by the high-skilled entrepreneurs.

The model is calibrated by targeting a large set of data moments from the Socio-Economic Panel (SOEP), which is a comprehensive survey of German households. We combined this data with information on vacancies provided by the Federal Employment Office (BA). The data shows that the unemployment rates of potential entrepreneurs are lower than the unemployment rates of regular workers, suggesting that self-employment out of necessity is a way of avoiding unemployment for these individuals. At the same time, our wage regressions reveal that the earnings of business owners are higher than the wages of comparable individuals employed in regular paid jobs, suggesting an important "opportunity" component of entrepreneurship. We find that high skill immigrant potential entrepreneurs are more frequently observed in operating small businesses, rather than being solo self-employed, compared to all other groups of potential entrepreneurs. Moreover, immigrant businesses (high and low skilled) are substantially more profitable than the businesses of native workers. This is an important observation in the context of immigration analysis since higher profits per coworker make immigrant businesses more sensitive in the sense that their entry reacts stronger to the probability of filling positions. Another relevant empirical fact is that low skill workers are overrepresented in small businesses compared to high skill workers making them more susceptible to the entry and closures of small businesses. All of these empirical facts are well captured by our modelling framework.

We conduct several numerical exercises to evaluate the effects of low skill immigration. First, we use our framework to study the implications of a 20% increase in the number of immigrant workers. This corresponds to the immigration wave observed in Germany in 2012-2017 (during the Syrian war) when the number of immigrant individuals increased from 15 to 18 million. Also the most recent immigrant wave in 2020-2024 (dominated by the Russian-Ukrainian conflict) is associated with a similar increase from 20 to 23 million[2]. Our results show that a 20% increase in low skill immigration is associated with a reduction in welfare of an average incumbent individual equal to 0.4%, however, there is pronounced heterogeneity in this effect across different worker groups. In-line with previous research, we find that rising low skill immigration is beneficial for high skill regular workers with a corresponding gain in welfare equal to 1%. Yet, we document a reduction in the welfare of low skill regular workers equal to -1.3%.

The results for potential entrepreneurs are novel and reveal asymmetric effects. Whereas immigrant entrepreneurs (high and low skilled) experience higher profits and their business entry is reinforced by immigration generating a welfare gain equal to 3.1%, there is a reduction in welfare (-2%) of native low skill entrepreneurs due to the reduced profits and a higher probability of staying solo self-employed without co-workers. Thus, and contrary to the previous studies (e.g. D'Amuri et al. (2010), Ottaviano and Peri (2012)), we find that some incumbent low skill immigrants are gaining from new immigration. The underlying reason for this finding is that immigrant businesses are more profitable on average, making them more sensitive to the improved chances of hiring co-workers in response to the immigration increase. In contrast, we find that the expected profits of native entrepreneurs are less sensitive on the recruitment margin and fall due to the lower marginal productivity of their low skill co-workers.

Second, we conduct a counterfactual experiment by introducing legal barriers to self-employment and entrepreneurship for immigrant individuals. This experiment sheds light on the spillover effects of immigrant entrepreneurship for regular workers in all demographic groups and native entrepreneurs. We find that immigrant entrepreneurship has moderate positive spillovers for regular workers. The corresponding welfare gains are evaluated at $0.3 - 0.4\%$ for low skill regular employees and 0.2% for the high skilled. These gains are twofold. On the one hand, immigrant potential entrepreneurs entering self-employment out of opportunity reduce competition for regular jobs, on the other hand, immigrant entrepreneurs operating small businesses create additional working places for regular workers. On the contrary, our model indicates substantial negative spillovers for

[2]www.destatis.de

native potential entrepreneurs ($1.1 - 1.6\%$ of welfare). This is intuitive since business entry of immigrant entrepreneurs creates additional competition in recruiting and reduces the hiring chances of native small businesses. Summarizing, our findings suggest that policies restricting immigrant entrepreneurship shield native small businesses from competition but reduce welfare for all other worker groups.

The outline of the paper is as follows. Section 2 positions our study in the related literature. Section 3 describes the model and the economic setup. In section 4 we develop and solve the model. Section 5 presents the data and the calibration strategy, while section 6 contains our main results. Finally, section 7 concludes the paper.

2 Related Literature

Recent refugee waves to the developed world have strengthened the debate on the effects of migration on receiving countries. The situation has intrigued the general public, policymakers, and researchers alike. The existing literature on migration tackles diverse aspects of the issue. Our paper contributes to the literature concerning the impact of (low-skilled) migration on the labor market outcomes and welfare of natives in developed countries. The seminal paper by Borjas (1999) shows that an influx of immigrants in the US creates an immigration surplus, given the imperfect substitution between labor and capital. Ben-Gad (2004) extends the analysis by assuming endogenous labor supply and capital accumulation; it reports a smaller immigration surplus for the US. These papers assume homogeneous labour, subsequent papers with heterogeneous labour support varied conclusions regarding wages of competing workers and immigration surplus (Borjas, 2003; Ben-Gad, 2008).

Our work relates to the papers on the wage effects of migration. Literature in this area is vastly empirical and assumes perfectly competitive labor markets (Altonji and Card, 1991; Borjas et al., 1997; Card, 2001; Borjas, 2006; D'Amuri et al., 2010). Despite decades of research, there has yet to be a consensus regarding the conclusions. Borjas et al. (1991), Goldin (1994), Borjas (2003), Aydemir and Borjas (2007), and Aydemir and Borjas (2011) find a negative effect of migration on the average wage of natives. Grossman (1982), Card (1990), Card (2001), Card (2005), LaLonde and Topel (1991), Friedberg (2001), D'Amuri et al. (2010), Felbermayr et al. (2010), Ottaviano and Peri (2012), Busch et al. (2020) find a negligible effect of immigration on native wages. Other papers suggest that the average wage effects of migration mask considerable heterogeneity. For example, the wage effects are considerably larger for younger natives (Dustmann et al., 2017) in Germany, while in the UK, migration negatively affects the wages of low earners but increases the wages of high earners (Dustmann et al., 2013). We study the heterogeneity in migration effects by assuming different skill groups and allowing for occupation choice between paid employment and entrepreneurship. D'Amuri et al. (2010), Felbermayr et al. (2010), and

Ottaviano and Peri (2012) further find that migration adversely affects the wages of the incumbent immigrants. These papers assume imperfect substitution between natives and immigrants in the same skill groups. We also address the differential effect of migration on natives and incumbent immigrants in our analysis. However, we follow a parsimonious approach in assuming perfect substitution between immigrants and natives in the same skill group. Nonetheless, we allow for differences in the productivity between different ethnic groups that affect the profits of job creators and, in turn, allow us to compute the effects of migration on incumbent workers and natives.

Most papers focus on the wage effects of migration, with a few exceptions discussing the employment consequences. The literature on this issue has not been reconciled either, and the dispute arises from disagreement on the strategies employed for causal identification. Some studies find a substantial adverse effect of migration on native employment (Borjas, 2003, 2006; Dustmann et al., 2017). Other papers focusing on the US (Card, 1990; Altonji and Card, 1991), France (Hunt, 1992), Germany (Pischke and Velling, 1997; D'Amuri et al., 2010; Scharfbillig and Weissler, 2019), Israel (Friedberg, 2001), Denmark (Malchow-Møller et al., 2009) find no or negligible effects of migration on native employment. Felbermayr et al. (2010) find positive effects of immigration on native employment in Germany. None of the papers, except for Dustmann et al. (2017), discuss the interaction between wage and employment effects. We formulate a search and matching model incorporating the direct effect of migration on wage and job creation. Borjas and Monras (2017) stresses the distributional consequences of migration. The paper concludes that migration adversely affects the labor market opportunities of competing natives while having positive implications for complementary natives. The quantitative part of our analysis provides insights into the distributional consequences of migration shock to the German labor market.

Recent papers employ search and matching models to analyze the general equilibrium effects of migration. The existence of frictions in this framework allows for investigating the consequences of migration wage, employment, and welfare effects of migration in a unified framework. Ortega (2000) is one of the first papers studying the effects of migration in a frictional labor market. However, this paper is theoretical and lacks a detailed data-based quantitative analysis. Furthermore, it assumes homogeneous labor with constant worker productivity. Hence, it ignores cross-skill complementarities and externalities. Liu (2010) develops a dynamic search and matching model with heterogeneous labor but focuses solely on the consequences of illegal migration for the US economy. Chassamboulli and Palivos (2014) extend Liu's framework with a nested CES production function, low and high-skilled labor, and capital to study the impact of immigration on the US economy. The paper concludes the effects of immigration are moderately positive. Nanos and Schluter (2014) use German administrative data to perform a structural estimation of a search and matching model which is used to decompose the native-immigrant wage gap. Moreno-Galbis and Tritah (2016) focus on the short-run effects of immigration due to worse outside opportunities and lower wages of incoming immigrants in a search and matching model. They find that lower wages of immigrants have a positive effect on the creation of jobs and employment opportunities of native workers in selected European economies. Battisti et al. (2018) calibrate a search and matching model featuring two skill types: search frictions, wage bargaining, and a redistributive welfare state for 20 OECD countries. The paper finds a welfare-enhancing effect of immigration on native welfare in almost all receiving countries, particularly by attenuating the effects of search frictions. Iftikhar and Zaharieva (2019) extend the framework of Battisti et al. (2018) by considering a two-goods economy, allowing for endogenous price setting and mismatch of high-skilled workers with low-skill jobs. Their framework allows them to identify additional effects of immigration on Germany through its impact on domestic demand. Our framework borrows several elements from these frameworks but substantially deviates and innovates by adding entrepreneurship into the model.

Further, our paper addresses how small and medium-sized entrepreneurs respond to migration shocks. One strand of existing literature on this topic examines the effects of immigration on business creation and entrepreneurship. It suggests positive contributions of migrants to entrepreneurship in receiving countries (OECD, 2010; Wadhwa, 2007; Hohn et al., 2012; Xavier et al., 2013; Kerr and Kerr, 2020; Riillo and Peroni, 2022). Other strand focuses on comparing characteristics of native and immigrant entrepreneurs in terms of earnings, human capital, likelihood, and reasons to start a business (Borjas, 1986; Evans and Leighton, 1989; Yuengert, 1995; Fairlie and Meyer, 1996; Clark and Drinkwater, 1998; Basu, 1998; Borooah and Hart, 1999; Li, 2001; Lofstrom, 2002; Constant and Schultz-Nielsen, 2004; Constant et al., 2005; Constant and Zimmermann, 2006) and how migration affects the entrepreneurship opportunities of natives Duleep et al. (2021). Most of these papers are focused on the US, and to the best of our knowledge, none of these studies consider the spillovers entrepreneurship has on the job market for workers in paid jobs.[3] Entrepreneurs contribute to job creation (Kerr and Kerr, 2020; Azoulay et al., 2022) as well as job destruction (Georgarakos and Tatsiramos, 2009), resulting in direct consequences of immigrant entrepreneurship for employment opportunities of workers in regular employment. Provided this evidence, we propose a unified framework for the entrepreneurial pursuits of natives and immigrants and regular employment activities.

Finally, our paper builds on a traditional search and matching framework (Mortensen and Pissarides (1994), Pissarides (2000)) and extends it to account for self-employment and entrepreneurship. An early search model with occupational choice between paid employment and entrepreneurship is by Fonseca et al. (2001). Several later studies (Rissman (2007), Kredler et al. (2014)) extend this setup to a “business idea ladder”, where self-

[3]We refer to it as regular employment throughout the text.

employed individuals generate business ideas arriving randomly over time and implement those of them which are more profitable. We contribute to this literature by combining the approach of a "business idea ladder" with endogenous creation of small businesses in the spirit of Masters (2016). Thus, we account for a double impact of search frictions on individuals with high entrepreneurial potential by influencing their opportunities in paid employment and by affecting their recruitment chances when managing small businesses.

3 The Setup

The labour market is populated by four demographic groups: low skill immigrants d^{IL} , low skill natives d^{NL}, high skill immigrants d^{IH} and high skill natives d^{NH}. All individuals are risk-neutral and discount future income flows at rate r. Labour supply is inelastic, so the total labour force is fixed and normalized to 1:

$$d^{IL} + d^{NL} + d^{IH} + d^{NH} = 1$$

We use the first superscript i to denote the origin of the individual $\{I, N\}$ and the second superscript j to indicate the skill level $\{L, H\}$. Within each demographic group ij there are two subgroups: regular workers with low entrepreneurial spirit $d^{ij} - l^{ij}$ and potential entrepreneurs with high entrepreneurial spirit l^{ij}. This group distinction is exogenous and based on innate ability, attitudes and personality traits, which we do not model explicitly.[4] Being a *potential* entrepreneur does not imply being an *active* entrepreneur at a given point in time. Potential entrepreneurs can also be observed in paid employment or unemployment but their entrepreneurial spirit may lead them to switch from these states to starting a solo or with co-workers business. There are two employment states for regular workers. First, regular workers can be employed in a regular job $\bar{e}^{ij}$, producing output $\bar{y}^{ij}$ and receiving a wage $\bar{w}^{ij}$. Regular jobs are positions in medium and large size firms. Second, they can be employed in a small business e_0^{ij} by potential entrepreneurs. Productivity and wages in small businesses depend on the type of the worker and the type of the business owner, so we introduce them later in the paper. Hence, we get:

$$\bar{e}^{ij} + e_0^{ij} + \bar{u}^{ij} = d^{ij} - l^{ij}$$

where $\bar{u}^{ij}$ is a state of involuntary unemployment for regular workers associated with a flow income $\bar{z}^{ij} - h$. Variable $\bar{z}^{ij}$ is the unemployment benefit, whereas h – is the disutility from unemployment (e.g. stigma of failure). The exogenous shock of job destruction

[4]Intuitively, individuals with high managerial and organizational skills, extroverted and communicative are more likely to be potential entrepreneurs. Empirically we classify individuals to the group of regular workers if they were never observed in self-employment or entrepreneurship over the sample period 2000-2017. On the contrary, individuals observed at least once in self-employment and/or entrepreneurship are classified as potential entrepreneurs.

arrives to all employees in regular jobs at rate $\bar{\gamma}^{ij}$ rendering them unemployed.

Potential entrepreneurs have high entrepreneurial spirit, so they can be solo self-employed s^{ij} or operating small businesses with coworkers b^{ij}. The decision to become self-employed and start a business is endogenous and driven by market forces. The stock of potential entrepreneurs in paid employment in a regular job is denoted by e^{ij} and is associated with a flow output y^{ij} and a wage w^{ij}, this yields:

$$s^{ij} + b^{ij} + e^{ij} + u^{ij} = l^{ij}$$

where u^{ij} is a state of unemployment for potential entrepreneurs with a flow income $z^{ij} - h$.

Potential entrepreneurs generate business ideas associated with a product/service quality α, which follows a distribution $F(\alpha)$, $\alpha \in [0..\bar{\alpha}]$. New realizations of α arrive to potential entrepreneurs at the Poisson rate δ^{ij}. The intuitive idea behind this process is that potential entrepreneurs may invent new products and decide to work in self-employment, or improve the quality of the existing product and register a small business with coworkers. Note that the idiosyncratic productivity shocks δ^{ij} are positive, since potential entrepreneurs have the option of ignoring the new realization of α if it is not beneficial for them. We follow the "business idea ladder" approach (Rissman (2007), Kredler et al. (2014)) and assume that new business ideas can not be stored for future use. In addition, we assume that there are no productivity shocks δ^{ij} in the state of business ownership.[5]

To be more specific, consider the state of unemployment. If the new realization of α is such that $\alpha \in [\alpha_u^{ij}..\alpha_0^{ij}]$, then unemployed potential entrepreneurs move into solo self-employment $s^{ij}(\alpha)$ with a corresponding flow output $\sigma^{ij}\alpha$. Thus, α_u^{ij} is the endogenous lower cut-off for solo self-employment, whereas α_0^{ij} is the endogenous upper cut-off. This yields:

$$s^{ij} = \int_{\alpha_u^{ij}}^{\alpha_0^{ij}} s^{ij}(\alpha)$$

For very high realizations of α, such that $\alpha \in [\alpha_0^{ij}..\bar{\alpha}]$ potential entrepreneurs register a small business and may hire co-workers. The stock of business owners for every α is denoted by $b^{ij}(\alpha)$. In this case the entrepreneur is incurring an additional flow cost of capital c_k^{ij} and the cost of posting a vacancy and hiring c_h^j. Let the two costs together be denoted by $c^{ij} = c_k^{ij} + c_h^{ij}$, so the individual net output of the entrepreneur becomes

[5] Alternatively, one could assume that positive productivity shocks (new business ideas) arrive to business owners at rate δ_0^{ij}. It is intuitive to think that $\delta_0^{ij} < \delta^{ij}$ since entrepreneurs managing small business have less time for developing new ideas and are less flexible in implementing them compared to the solo self-employed. We set $\delta_0^{ij} = 0$, this simplifies the model without changing its intuition. Moreover, our empirical data is not sufficiently rich to separate parameters δ^{ij} and δ_0^{ij}.

$\sigma^{ij}\alpha - c^{ij}$. Profits generated by the coworkers and the firm size distribution of small businesses are introduced and discussed later in the paper. For business owners we get:

$$b^{ij} = \int_{\alpha_0^{ij}}^{\bar{\alpha}} b^{ij}(\alpha)$$

Small businesses become bankrupt at the Poisson arrival rate γ^{ij}, where ij is the demographic group of the business owner. In the case of bankruptcy all coworkers and the business owner become unemployed. The same shock arrives to solo self-employed and also leads to involuntary unemployment.

All unemployed workers actively search for jobs in medium and large firms while only regular workers also look for jobs in small businesses. In order to model matching between searching workers and open vacancies we follow the matching function approach. First, consider matching in the regular market $j = L, H$, these are skill-specific positions in medium and large firms. Posting a vacancy in submarket j is associated with a posting cost $\bar{c}_h^j$ and a flow capital cost $\bar{c}_k^j$. Let $M^j(\Sigma^j)^\zeta(v^j)^{1-\zeta}$, $j = L, H$ denote the corresponding Cobb-Douglas matching function, where v^j denotes the number of open vacancies and Σ^j denotes the number of searching workers in efficiency units. Σ^j consists of unemployed regular workers $\bar{u}^{ij}$, unemployed potential entrepreneurs u^{ij} and searching solo self-employed entrepreneurs weighted by the exogenous search intensity X^{ij}. Note that solo self-employed entrepreneurs can search for regular jobs if it is optimal for them. We refer to searching self-employed (s_u^{ij}) as self-employed "*out of necessity*" since they would strictly prefer to have a regular job. In contrast, we refer to non-searching self-employed (s_e^{ij}) as self-employed "*out of opportunity*" as they would not accept a regular job even if they get an offer. Given this notation we get:

$$\Sigma^j = X^{Ij}(\bar{u}^{Ij} + u^{Ij} + s_u^{Ij}) + X^{Nj}(\bar{u}^{Nj} + u^{Nj} + s_u^{Nj})$$

$s^{ij} = s_u^{ij} + s_e^{ij}$ implies that every self-employed worker can be assigned into a searching or non-searching group based on the optimal job search decision. The job-finding rate $\lambda(\theta^j)$ and the vacancy filling rate $q(\theta^j)$ in a regular submarket j can then be written as:

$$\lambda(\theta^j) = M^j(\theta^j)^{1-\zeta} \qquad q(\theta^j) = M^j(\theta^j)^{-\zeta} \qquad \theta^j = \frac{v^j}{\Sigma^j}$$

where θ^j is the market tightness in the submarket j.

Second, consider matching between searching workers and small businesses operated by the entrepreneurs. Small businesses post vacancies sequentially, meaning that there is exactly one vacancy per business posted at any time. The next vacancy is posted only after the previous was filled with a worker. This means that small businesses grow over time, and that the total number of posted vacancies is given by $b^{Nj} + b^{Ij}$ since both native and immigrant small businesses post vacancies. Let $\bar{M}^j(\Sigma_0^j)^\zeta(b^{Ij}+b^{Nj})^{1-\zeta}$, $j = L, H$ denote the corresponding matching function. Σ_0^L consists of unemployed regular low skill workers $\bar{u}^{iL}$ weighted by their search intensities x^{iL}. This means that small businesses operated by low skill entrepreneurs attract low skill applicants, native and immigrant. One example would be a retail shop or a restaurant hiring low skill sellers and waiters respectively.

In contrast, high skill small businesses attract applicants from both skill groups, thus, we explicitly allow for the possibility of *cross-skill matching*. One example, would be a medical doctor working privately, hiring a high skill assistant, a secretary and a nurse. Accounting for cross-skill matching allows us to fit the empirical data better than it would be with separated skill-specific labour markets. In particular, we account for the fact that there are relatively many low skill workers and relatively few high skill workers employed in small businesses. Thus, variable Σ_0^H consists of unemployed regular high skill workers $\bar{u}^{iH}$ weighted by their search intensities x^{iH} as well as regular low skill workers weighted by their search intensities κx^{iL}. Here parameter κ reflects the intensity of cross-skill matching. With this notation we get:

$$\Sigma_0^j = x^{Ij}\bar{u}^{Ij} + x^{Nj}\bar{u}^{Nj} + \mathbb{1}_H\kappa(x^{IL}\bar{u}^{IL} + x^{NL}\bar{u}^{NL})$$

where $\mathbb{1}_H$ takes value 1 for $j = H$ and value 0 for $j = L$. Given this setup, the job-finding rate $\bar{\lambda}(\theta_0^j)$ and the vacancy filling rate $\bar{q}(\theta_0^j)$ in submarket j become:

$$\bar{\lambda}(\theta_0^j) = \bar{M}^j(\theta_0^j)^{1-\zeta} \qquad \bar{q}(\theta_0^j) = \bar{M}^j(\theta_0^j)^{-\zeta} \qquad \theta_0^j = \frac{b^{Nj}+b^{Ij}}{\Sigma_0^j}$$

where θ_0^j is the market tightness associated with small businesses in submarket j. Later in the paper we use the search intensity parameters X^{ij} and x^{ij} as well as the multipliers of the matching function M^j and $\bar{M}^j$ to fit the empirically observed unemployment rates for all worker groups.

The final good Y is produced using capital K and a composite labour good Z with a CRS technology. For the composite good Z we follow the literature (Acemoglu (2001) and Battisti et al. (2018)) and use a CES production function with two intermediate skill-specific inputs, Y_L and Y_H, that is:

$$Y = AK^\eta Z^{1-\eta}, \quad \eta \in (0..1) \tag{1}$$

$$Z = [aY_L^\rho + (1-a)Y_H^\rho]^{\frac{1}{\rho}}, \quad \rho < 1 \tag{2}$$

where $1/(1-\rho)$ is the elasticity of substitution between high and low skill intermediate inputs. Each intermediate input Y_j is a linear aggregate of the output units produced in submarket j. More specifically, individual output of every employee in paid or self-employment is a product of a fixed number of output units and the price of the intermediate good j, which we denote by P^j. Let variable $\bar{\varphi}^{ij}$ denote the units of output produced by regular workers in group ij. Then the value of their output becomes $\bar{y}^{ij} = \bar{\varphi}^{ij} P^j$. In a similar way we define variables φ^{ij} and ς^{ij}, so that $y^{ij} = \varphi^{ij} P^j$ and $\sigma^{ij} = \varsigma^{ij} P^j$. The units of output are exogenous and specific to each worker group reflecting individual productivity heterogeneity, however, the price of the intermediate good P^j is endogenous and common to all workers in the skill category $j = L, H$. Later in the paper we use the units of output to match the earnings data for all worker groups. Intermediate goods are sold in a competitive market to the final good producer, so the price P^j is equal to the marginal productivity of the intermediate good Y_j. In addition, the final good producer buys capital K up to the point where the marginal productivity of capital equals the international (exogenous) price of capital R.

Finally, all employees in paid and self-employment pay an income tax t. In addition, regular firms and small businesses pay the corporate tax τ on their profits.

4 The Model

4.1 Potential entrepreneurs

In this section we analyze the decisions of potential entrepreneurs. There are three binary decisions to be considered: entering solo self-employment, searching for a regular paid job, starting a business with coworkers. All the following equations are specific to the entrepreneurs of the type ij, however, we drop the upper index where possible for the ease of exposition and recover full-scale notation later in the paper. Consider unemployed potential entrepreneurs and let U denote the present value of income in unemployment. Potential entrepreneurs generate business ideas associated with product/service quality α. If the new realization of α crosses the endogenous boundary α_u then potential entrepreneurs enter the state of self-employment and start producing the flow output $\sigma\alpha$. The threshold value α_u follows from the following optimization problem:

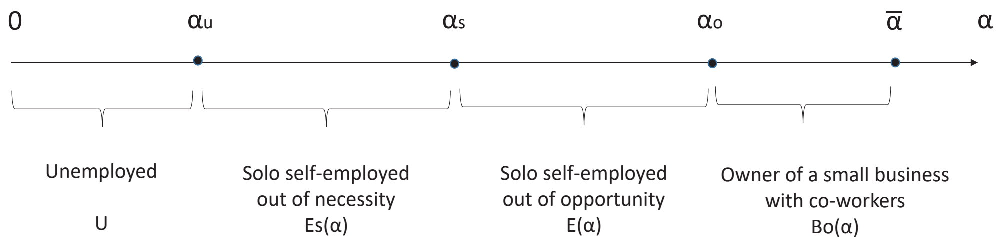
$$\max[U, E_s(\alpha)] = \begin{cases} E_s(\alpha) & \text{for} \quad \alpha > \alpha_u \\ U & \text{for} \quad \alpha < \alpha_u \end{cases} \qquad \text{and} \qquad E_s(\alpha_u) = U$$

where $E_s(\alpha)$ denotes the present value of income in self-employment *out of necessity*, if the entrepreneur continues searching for a regular job. We show later that $E_s(\alpha)$ is an increasing function of α. Further, let $W > U$ denote the present value of income in a regular job. If the new realization of α exceeds the next threshold α_s, then self-employed entrepreneurs stop searching for regular jobs, and with $\alpha \in [\alpha_s..\alpha_0]$ they are solo self-employment *out of opportunity*. This is because staying in self-employment in this state is associated with a higher present value of income than regular employment. The present value of income in this state is denoted by $E(\alpha)$, so we get:

$$\max[E_s(\alpha), E(\alpha)] = \begin{cases} E(\alpha) > W & \text{for} \quad \alpha > \alpha_s \\ E_s(\alpha) < W & \text{for} \quad \alpha < \alpha_s \end{cases} \qquad \text{and} \qquad E_s(\alpha_s) = W = E(\alpha_s)$$

where again $E(\alpha)$ should be increasing in α. The sequence of labour market decisions for potential entrepreneurs is illustrated on figure 1.

Figure 1: Self-employment states for different values of α



Finally, for very high realizations of α, such that $\alpha \in [\alpha_0..\bar{\alpha}]$ entrepreneurs register a small business and post vacancies. The present value of profits in this state is denoted by $B_0(\alpha)$, where the subindex refers to the number of coworkers. Any new business starts with 0 coworkers. Later we show that $B_0(\alpha)$ is increasing in α, so we get:

$$\max[E(\alpha), B_0(\alpha)] = \begin{cases} B_0(\alpha) & \text{for} \quad \alpha > \alpha_0 \\ E(\alpha) & \text{for} \quad \alpha < \alpha_0 \end{cases} \qquad \text{and} \qquad E(\alpha_0) = B_0(\alpha_0)$$

Applying the optimal decision strategies of potential entrepreneurs we can write down the present value of unemployment U in the following way:

$$\begin{aligned} rU &= z - h + X\lambda(\theta)(W - U) \qquad (3) \\ &+ \underbrace{\delta \int_{\alpha_u}^{\alpha_s} (E_s(x) - U)dF(x)}_{\text{solo-entr. out of necessity}} + \underbrace{\delta \int_{\alpha_s}^{\alpha_0} (E(x) - U)dF(x)}_{\text{solo-entr. out of opportunity}} + \underbrace{\delta \int_{\alpha_0}^{\bar{\alpha}} (B_0(x) - U)dF(x)}_{\text{business with co-workers}} \end{aligned}$$

where $z - h$ is the flow utility in unemployment and X is the search efficiency parameter, so that $X\lambda(\theta)$ is the job-finding rate associated with regular jobs. Moreover, δ denotes the Poisson arrival rate of new business ideas. This equation shows that depending on the product quality associated with the new business idea α unemployed potential entrepreneurs may decide to stay unemployed, move into solo self-semployment, or even register a business hiring coworkers.

Next we describe the present value of income in solo self-employment out of necessity, which applies for $\alpha \in [\alpha_u..\alpha_s]$:

$$
\begin{aligned}
rE_s(\alpha) &= \sigma\alpha(1-t) + \underbrace{\delta\int_{\alpha}^{\alpha_s}(E_s(x)-E_s(\alpha))dF(x)}_{\text{improving profits}} + \underbrace{\delta\int_{\alpha_s}^{\alpha_0}(E(x)-E_s(\alpha))dF(x)}_{\text{solo-entr. out of opportunity}} \\
&+ \underbrace{\delta\int_{\alpha_0}^{\bar{\alpha}}(B_0(x)-E_s(\alpha))dF(x)}_{\text{business with co-workers}} + X\lambda(\theta)(W-E_s(\alpha)) - \gamma(E_s(\alpha)-U) \qquad (4)
\end{aligned}
$$

where $\sigma\alpha(1-t)$ is the after tax income in self-employment, whereas γ is the entrepreneurship destruction rate. Even though solo self-employed out of necessity continue searching for a regular job, they do not receive the unemployment benefit, however, they also do not experience the pressure/stigma of failure h. This equation shows that the quality of the service/product offered by the self-employed entrepreneur is improving over time, which is associated with increasing income/profits. So when the income achieves the level $\sigma\alpha_s(1-t)$, so that $E_s(\alpha_s) = W$, self-employed entrepreneurs optimally stop searching for regular jobs and fully focus on their entrepreneurial activities.

The present value of income associated with entrepreneurship out of opportunity $E(\alpha)$ is given by:

$$
\begin{aligned}
rE(\alpha) &= \sigma\alpha(1-t) + \underbrace{\delta\int_{\alpha}^{\alpha_0}(E(x)-E(\alpha))dF(x)}_{\text{improving profits}} + \underbrace{\delta\int_{\alpha_0}^{\bar{\alpha}}(B_0(x)-E(\alpha))dF(x)}_{\text{business with co-workers}} \\
&- \gamma(E(\alpha)-U) \quad \text{for} \quad \alpha_s < \alpha < \alpha_0 \qquad (5)
\end{aligned}
$$

where again the first integral on the right hand side accounts for the improving quality of the entrepreneurial product α, whereas the second integral refers to the possibility of registering a business and posting vacancies associated with a present value $B_0(\alpha)$.

When modeling small businesses with co-workers we follow the approach in Masters (2016). Registering a start-up business is associated with a capital cost c_k (e.g. rental cost) and a vacancy posting cost c_h, jointly denoted by $c = c_k + c_h$. Let variable $B_0(\alpha)$, $\alpha > \alpha_0$ denote the present value of a start-up business with 0 co-workers and one open vacancy. Recall that small businesses post vacancies sequentially, so there is one open position per small business at any time. At rate $\bar{q}(\theta_0)$ small businesses are matched with potential candidates. Consider first the low skill submarket, so that all applicants have low skills. With probability μ, which we derive later in the paper, the applicant has immigrant background, while it is a native worker with probability $1-\mu$. The present value of a start-up business $B_0(\alpha)$ is then given by:

$$
(r+\gamma)B_0(\alpha) = \sigma\alpha(1-t) - c + \bar{q}(\theta_0)[\mu(B_{10}(\alpha)-B_0(\alpha)) + (1-\mu)(B_{01}(\alpha)-B_0(\alpha))] + \gamma U
$$

where $B_{km}(\alpha)$ denotes the present value of a small business with k immigrant co-workers and m native co-workers. Let y_0^i and w_0^i, $i = I, N$ denote productivity and wages of workers employed in small businesses. We assume that the output of workers in small firms is additive (constant returns to scale), but it is not a very restrictive assumption because the total output of workers in different labour market segments is further aggregated by means of the CES production function (equation 2). Thus, the present value $B_{km}(\alpha)$ can be written as:

$$\begin{aligned} rB_{km}(\alpha) &= \sigma\alpha(1-t) - c + k(y_0^I - w_0^I)(1-\tau) + m(y_0^N - w_0^N)(1-\tau) + \\ &+ \bar{q}(\theta_0)[\mu(B_{k+1m}(\alpha) - B_{km}(\alpha)) + (1-\mu)(B_{km+1} - B_{km}(\alpha))] - \gamma(B_{km}(\alpha) - U) \end{aligned}$$

In the appendix A, we show that the value gain from hiring one more co-worker, denoted by $\Delta^I = B_{k+1m}(\alpha) - B_{km}(\alpha)$ and $\Delta^N = B_{km+1}(\alpha) - B_{km}(\alpha)$ is constant $\forall k, m = 0..\infty$ and can be written as:

$$\Delta^I = \frac{y_0^I - w_0^I}{r+\gamma}(1-\tau) \qquad \Delta^N = \frac{y_0^N - w_0^N}{r+\gamma}(1-\tau)$$

This gives rise to the following expression for a low skill start-up business:

$$(r+\gamma)B_0(\alpha) = \sigma\alpha(1-t) - c + \bar{q}(\theta_0)\pi + \gamma U, \quad \text{where} \quad \pi = \mu\Delta^I + (1-\mu)\Delta^N \tag{6}$$

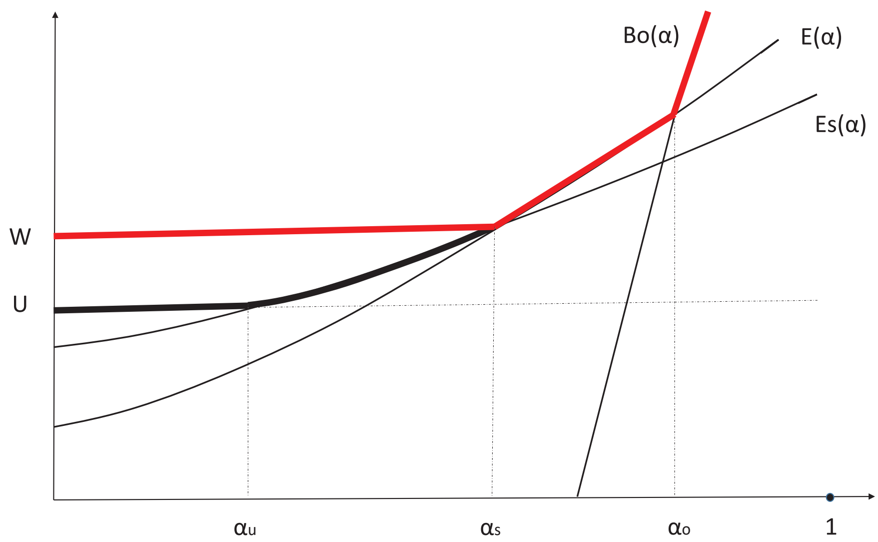
Note that variable π denotes the expected after-tax profit from hiring co-workers in small businesses and that posting a vacancy is only profitable if $c_h < \bar{q}(\theta_0)\pi$. Differentiating $B_0(\alpha)$ with respect to α, one can see that $B_0(\alpha)$ is a linear increasing function of α with a slope $\sigma(1-t)/(r+\gamma)$. Next we consider high skill start-up businesses. Recall that high skill entrepreneurs hire both types of employees, high and low skill ones. Let μ^{ij} denote the probability of matching and hiring a co-worker of type ij, $i = I, N$, $j = L, H$. Applying the same approach as above to high skill businesses we can show that $\pi = \mu^{IH}\Delta^{IH} + \mu^{NH}\Delta^{NH} + \mu^{IL}\Delta^{IL} + \mu^{NL}\Delta^{NL}$.

Rewriting the present value equation for $E(\alpha)$ using integration by parts and differentiating it with respect to α (see appendix A) gives rise to the following final expression for the present value of entrepreneurship out of opportunity $E(\alpha)$:

$$(r+\gamma)E(\alpha) = \sigma\alpha(1-t) + \delta\int_{\alpha}^{\alpha_0} \frac{(1-F(x))\sigma(1-t)}{r+\gamma+\delta(1-F(x))}dx + \delta(1-t)\int_{\alpha_0}^{\bar{\alpha}} \frac{(1-F(x))\sigma}{r+\gamma}dx + \gamma U$$

This equation shows that $E(\alpha)$ is an increasing convex function of α with a slope $E'(\alpha) = \sigma(1-t)/(r+\gamma+\delta(1-F(\alpha)))$. Both functions $E(\alpha)$ and $B_0(\alpha)$ are illustrated on figure 2. Comparing the slopes, we can see that $B_0(\alpha)$ is steeper than $E(\alpha)$ for every $\alpha < 1$. The intuition for this result is that solo self-employed entrepreneurs discontinue their activities and register a small business for very high realizations of α above α_0, so the duration of solo self-employment $1/(\gamma + \delta(1 - F(\alpha_0))$ is shorter on average than the duration of a small business $1/\gamma$. Building on this result we can show that there exists a unique cut-off value α_0 making entrepreneurs indifferent between remaining in solo self-employment $E(\alpha_0)$ and starting a small business $(B_0(\alpha_0))$, which yields Proposition 1:

Figure 2: Endogenous self-employment cut-offs



Proposition 1: *The endogenous quality cut-off α_0, which is driving the creation of small businesses, can be found from the following* ***indifference condition*** $E(\alpha_0) = B_0(\alpha_0)$*, and is implicitly given by:*

$$\delta(1-t)\int_{\alpha_0}^{\bar{\alpha}} \frac{(1-F(x))\sigma}{r+\gamma}dx + c = \bar{q}(\theta_0)\pi \qquad (7)$$

$$\text{where} \quad \pi = \begin{cases} \mu\Delta^I + (1-\mu)\Delta^N & \text{for low skill businesses} \\ \sum_{j=L,H} \mu^{Ij}\Delta^{Ij} + \mu^{Nj}\Delta^{Nj} & \text{for high skill businesses} \end{cases}$$

The cut-off value α_0 exists and is unique if $0 < (-c+\bar{q}\pi)(r+\gamma) < \delta(1-t)\sigma\int_0^{\bar{\alpha}}(1-F(x))dx$. For given values of endogenous variables $\bar{q}(\theta_0))$ and π, α_0 is increasing in δ, σ, and c.
Proof: appendix A.

The left-hand side of equation (7) is the total cost of starting a business. It includes the direct cost of capital and recruitment c and the indirect cost reflected by the integral term. This is the option value of staying in solo self-employment and waiting for another product idea of better quality (above α_0). The right hand side of equation (7) is the expected (after tax) profit from hiring co-workers. Intuitively, it means that every new beneficial realization of α reduces the option value of staying in solo self-employment, so the business entry becomes optimal when the option value of waiting equalizes with the expected gain from registering a start-up.

Rewriting the present value equation for $E_s(\alpha)$ using integration by parts and differentiating it with respect to α, we find the final expression for $E_s(\alpha)$, which is the present value of income in self-employment out of necessity:

$$\begin{aligned}(r+\gamma)E_s(\alpha) &= \sigma\alpha(1-t)+X\lambda(\theta)(W-E_s(\alpha))+\delta\int_{\alpha}^{\alpha_s}\frac{(1-F(x))\sigma(1-t)}{r+\gamma+X\lambda(\theta)+\delta(1-F(\alpha))}dx \\ &+ \delta\int_{\alpha_s}^{\alpha_0}\frac{(1-F(x))\sigma(1-t)}{r+\gamma+\delta(1-F(x))}dx+\delta(1-t)\int_{\alpha_0}^{\bar{\alpha}}\frac{(1-F(x))\sigma}{r+\gamma}dx+\gamma U\end{aligned}$$

This expression indicates that $E_s(\alpha)$ is an increasing convex function of α with a slope $E_s'(\alpha)=\sigma(1-t)/(r+\gamma+X\lambda(\theta)+\delta(1-F(\alpha)))$ as shown on figure 2. We observe that the curve $E_s(\alpha)$ is flatter than $E(\alpha)$ since $E_s'(\alpha)<E'(\alpha)$ in the whole range of α. The reason is that the duration of self-employment out of necessity $1/(\gamma+X\lambda(\theta)+\delta(1-F(\alpha_0))$ is shorter on average than the duration of self-employment out of opportunity $1/(\gamma+\delta(1-F(\alpha_0))$ since in the former state entrepreneurs are still searching for a regular job and interrupt their self-employment activities once they find it.

The present value of employment in a regular job W which pays a wage w is given by:

$$rW = w(1-t)+\underbrace{\delta\int_{\alpha_s}^{\alpha_0}(E(x)-W)dF(x)}_{\text{solo-entr. out of opportunity}}+\underbrace{\delta\int_{\alpha_0}^{\bar{\alpha}}(B_0(x)-W)dF(x)}_{\text{business with co-workers}}-\bar{\gamma}(W-U) \quad (8)$$

where $w(1-t)$ is the after tax labour income and $\bar{\gamma}$ is the negative job destruction rate. This relationship shows that even when employed potential entrepreneurs continue generating new business ideas and may voluntarily quit the job and enter self-employment (out of opportunity) or start a new business. Note that self-employment out of necessity is a dominated state for individuals already employed in a regular paid job.

Following the same approach we find the final equation for the present value of income in a regular paid job W:

$$(r+\bar{\gamma})W = w(1-t)+\delta\int_{\alpha_s}^{\alpha_0}\frac{(1-F(x))\sigma(1-t)}{r+\gamma+\delta(1-F(x))}dx+\delta(1-t)\int_{\alpha_0}^{\bar{\alpha}}\frac{(1-F(x))\sigma}{r+\gamma}dx+\bar{\gamma}U$$

As we could show above, self-employed entrepreneurs stop searching and applying for regular paid jobs when the quality of their product/service becomes sufficiently high, which is associated with a relatively high income $\sigma\alpha(1-t)$. This happens for $\alpha>\alpha_s$, so the endogenous cut-off value is given by the following indifference condition: $E_s(\alpha_s)=W$. It then holds that $E_s(\alpha_s)=E(\alpha_s)$ since the option of searching for a regular job has zero value for $\alpha=\alpha_s$. We derive this cut-off in proposition 2:

Proposition 2: *The endogenous threshold α_s separating solo self-employed out of necessity and opportunity can be obtained from the following indifference condition: $E_s(\alpha_s)=$*

$E(\alpha_s) = W$, *which yields:*

$$\sigma\alpha_s \quad = \quad w - \frac{(\bar{\gamma} - \gamma)}{r + \bar{\gamma} + X\lambda(\theta)} \Big(w - \frac{z - h}{1 - t} - \delta \int_{\alpha_u}^{\alpha_s} \frac{(1 - F(x))\sigma}{r + \gamma + X\lambda(\theta) + \delta(1 - F(x))} dx \Big) \quad (9)$$

The cutoff α_s is increasing in the wage w with the following slope:

$$\frac{\partial \alpha_s}{\partial w} = \frac{(r + \gamma + X\lambda(\theta) + \delta(1 - F(\alpha_s)))}{\sigma(r + \bar{\gamma} + X\lambda(\theta) + \delta(1 - F(\alpha_s)))} \quad (10)$$

Proof: appendix A

Equation 9 is an explicit form of a shorter expression $\sigma\alpha_s(1-t) = w(1-t) - (\bar{\gamma} - \gamma)(W - U)$. First, self-employed entrepreneurs compare their flow incomes $\sigma\alpha(1-t)$ and $w(1-t)$. Second, they also consider the risk of dropping back to the state of unemployment, which is $\bar{\gamma}$ in a regular job and γ in self-employment. In the special case $\bar{\gamma} = \gamma$ we would get $\alpha_s = w/\sigma$. Later in the calibration we find that $\bar{\gamma} < \gamma$ for all groups of entrepreneurs, meaning that the risk of unemployment is smaller in regular paid jobs. This property raises the value of a regular job for the entrepreneurs and leads to the case $\alpha_s > w/\sigma$, meaning that α_s is relatively high and self-employed entrepreneurs do not give up a valuable option of entering a regular job that quickly. This intuition also leads to finding (10), that a higher wage w makes regular jobs even more valuable and gives rise to a higher threshold α_s.

In the final step we consider the threshold value α_u. It is given by the indifference condition $E_s(\alpha_u) = U$, where the final equation for U can be written as:

$$\begin{aligned} rU \quad &= \quad z - h + X\lambda(\theta)(W - U) + \delta \int_{\alpha_u}^{\alpha_s} \frac{(1 - F(x))\sigma(1 - t)}{r + \gamma + X\lambda(\theta) + \delta(1 - F(x))} dx \\ &+ \quad \delta \int_{\alpha_s}^{\alpha_0} \frac{(1 - F(x))\sigma(1 - t)}{r + \gamma + \delta(1 - F(x))} dx + \delta(1 - t) \int_{\alpha_0}^{\bar{\alpha}} \frac{(1 - F(x))\sigma}{r + \gamma} dx \end{aligned} \quad (11)$$

Comparing the present value from solo self-employment $E_s(\alpha_u)$ and the present value of unemployment gives rise to proposition 3:

Proposition 3: *The endogenous threshold α_u driving the entry into solo self-employment can be obtained from the indifference condition $E_s(\alpha_u) = U$, which yields: $\sigma\alpha_u(1 - t) = z - h$.*

Proof: appendix A

Potential entrepreneurs entering self-employment keep the option of searching for regular jobs, and they also keep the option of starting a small business, so there are no costs of starting self-employment apart from losing the flow income in unemployment $z - h$. The additional gain from self-employment is the flow income $\sigma\alpha(1-t)$, which leads to the finding $\alpha_u = (z - h)/(\sigma(1 - t))$. Even though parameters z, h, t are exogenous, the productivity in self-employment is a product of the number of output units and the endogenous price ($\sigma = \varsigma P$), making the entry cut-off α_u endogenous as well. Hence, if the output price P is increasing more potential entrepreneurs decide to enter self-employment.

4.2 Steady-state distribution

In this section we consider the steady-state distribution of the number of employees in small businesses (i.e. their size), as well as the distribution of potential entrepreneurs across different labour market states. We start by analyzing low skill small businesses hiring native and immigrant low skill co-workers, which gives rise to the two-dimensional discrete probability density. Let $b_{km}(\alpha)$ – be the number of small businesses with exactly k immigrant and m – native co-workers, and $p_{km} = b_{km}(\alpha)/b(\alpha)$ be the corresponding two-dimensional probability density. Here $b(\alpha) = \sum_{k=0}^{\infty}\sum_{m=0}^{\infty} b_{km}(\alpha)$ is a total number of businesses of type α. We show later that p_{km} does not depend on α. While the proof is delegated to the appendix A, proposition 4 presents the distribution p_{km} and summarizes its properties.

Proposition 4: *The two-dimensional distribution of immigrant and native co-workers in small businesses p_{km} is given by:*

$$p_{km} = \frac{\gamma}{\gamma + \bar{q}(\theta_0)} \frac{(k+m)!}{k!m!} \frac{(\bar{q}(\theta_0)(1-\mu))^m (\bar{q}(\theta_0)\mu)^k}{(\gamma + \bar{q}(\theta_0))^{m+k}} \tag{12}$$

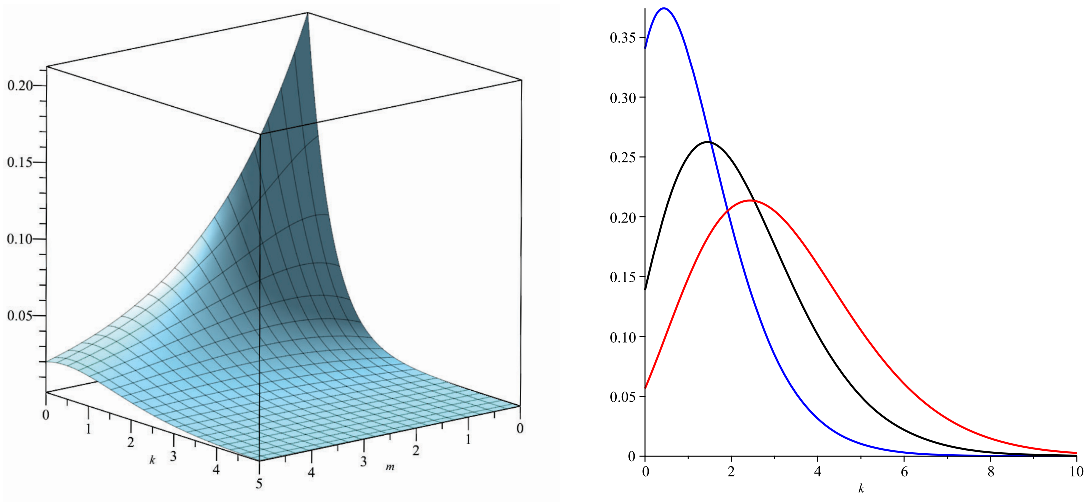
The conditional distributions of the number of immigrant/native co-workers $p_{k|m}$ and $p_{m|k}$ are given by the Negative Binomial densities with parameters $(r_I = m+1, p_I = \bar{q}(\theta_0)\mu/(\gamma + \bar{q}(\theta_0)))$ and $(r_N = k+1, p_N = \bar{q}(\theta_0)(1-\mu)/(\gamma + \bar{q}(\theta_0)))$:

$$\begin{aligned} p_{k|m} &= \frac{(k+m)!}{k!m!}\Big(\frac{\bar{q}(\theta_0)\mu}{\gamma + \bar{q}(\theta_0)}\Big)^k \Big(\frac{\gamma + \bar{q}(\theta_0)(1-\mu)}{\gamma + \bar{q}(\theta_0)}\Big)^{m+1} \\ p_{m|k} &= \frac{(k+m)!}{k!m!}\Big(\frac{\bar{q}(\theta_0)(1-\mu)}{\gamma + \bar{q}(\theta_0)}\Big)^m \Big(\frac{\gamma + \bar{q}(\theta_0)\mu}{\gamma + \bar{q}(\theta_0)}\Big)^{k+1} \end{aligned}$$

Thus the average number of immigrant co-workers in a firm with m natives is given by $\bar{q}(\theta_0)\mu(m+1)/(\gamma + \bar{q}(\theta_0)(1-\mu))$ and the average number of native co-workers in a firm with k immigrants is given by $\bar{q}(\theta_0)(1-\mu)(k+1)/(\gamma + \bar{q}(\theta_0)\mu)$, indicating that the two variables are positively correlated.

The left panel of figure 3 shows the distribution of employees in small businesses operated by native entrepreneurs p^N_{km} (generated using parameters calibrated in section 5.1). We can see that start-up businesses without co-workers p_{00} are quite frequent since the probability of filling positions in small firms $\bar{q}(\theta_0)$ is relatively low and the rate of business destruction γ is relatively high. In addition, this figure reveals that the two variables k and m are positively correlated. This is intuitive since a large number of native co-workers in the firm suggests that the business survived for a long period of time, meaning that also the number of immigrant workers should be proportionally high. For example, the right panel of figure 3 shows the conditional distribution of immigrant employees (Negative Binomial) for $m = [5, 10, 15]$. It is the hump-shaped curve, and the average number of immigrant employees in such a business is increasing with m: $E^N[k|m = 5] = 1.18$, $E^N[k|m = 10] = 2.16$ $E^N[k|m = 15] = 3.15$.

Figure 3: Left: joint distribution of the number of immigrant k and native m co-workers in native-owned small businesses p^N_{km}. Right: conditional densities $p^N_{k|m}$ for $m = [5, 10, 15]$.



Notes: parameter values $\gamma_N = 0.03$, $\bar{q}(\theta_0) = 0.11$, $\mu = 0.21$

Next, we derive the unconditional distributions of the number of employees. For this purpose let $b^N_m(\alpha)$ be the number of small businesses with exactly m native co-workers and p^N_m be the share of such businesses, so that $p^N_m = b^N_m(\alpha)/b(\alpha)$. In a similar way, let $b^I_k(\alpha)$ be the number of small businesses with exactly k immigrant co-workers and p^k_I be the corresponding share, so that $p^k_I = b^I_k(\alpha)/b(\alpha)$. The distributions p^N_m and p^k_I are obtained in proposition 5:

Proposition 5: *The unconditional distributions of native and immigrant workers in small businesses have geometric densities with parameters $\gamma/(\gamma + \bar{q}(\theta_0)(1 - \mu))$ and $\gamma/(\gamma + \bar{q}(\theta_0)\mu)$ respectively:*

$$p^N_m = \left(\frac{\bar{q}(\theta_0)(1-\mu)}{\gamma + \bar{q}(\theta_0)(1-\mu)}\right)^m \frac{\gamma}{\gamma + \bar{q}(\theta_0)(1-\mu)} \qquad p^I_k = \left(\frac{\bar{q}(\theta_0)\mu}{\gamma + \bar{q}(\theta_0)\mu}\right)^k \frac{\gamma}{\gamma + \bar{q}(\theta_0)\mu}$$

The average number of native employees in small businesses is given by $\bar{q}(\theta_0)(1 - \mu)/\gamma$, while the average number of immigrant employees is $\bar{q}(\theta_0)\mu/\gamma$. The number of all em-

ployees in small businesses also has a geometric density with parameter $\gamma/(\gamma + \bar{q}(\theta_0))$*:*

$$\frac{b_n(\alpha)}{b(\alpha)} = p_n = \Big(\frac{\bar{q}(\theta_0)}{\gamma + \bar{q}(\theta_0)}\Big)^n \frac{\gamma}{\gamma + \bar{q}(\theta_0)}$$

so the average size of small businesses is given by $\bar{q}(\theta_0)/\gamma$*.*

These probability densities are decreasing in the whole support implying that a vast majority of businesses are small and only very few of them survive for a long period of time and become large. We find that this decreasing pattern is consistent with the empirical evidence observed in Germany. For example, according to Leicht and Langhauser (2014) the share of native-owned businesses with less than 5 employees is 59%, the share of businesses with 6-10 employees is 21%, with $11 - 19$ employees – 10%, with $20 - 49$ employees – 6% and more than 50 employees – 3%. A similar decreasing pattern is also reported for immigrant businesses and it is well captured by the geometric distribution function derived above.

Considering high skill businesses, we show that the average number of type ij employees, $i = I, N$, $j = L, H$, is given by $\bar{q}(\theta_0)\mu^{ij}/\gamma$. Moreover, the unconditional distribution of employees ij in these businesses also has a geometric density with parameter $\gamma/(\gamma + \bar{q}(\theta_0)\mu^{ij})$.

In the next step, we turn to analyzing the steady state distribution of potential entrepreneurs across their labour markets states. Recall that s_u and s_e are the numbers of necessity and opportunity entrepreneurs, respectively, so that $s = s_u + s_e$ is a total number of solo-entrepreneurs. Given that the total number of potential entrepreneurs (in the group ij, which we omit for the ease of exposition) is denoted by l, we get: $s_u + s_e + e + u + b = l$. Potential entrepreneurs, who are not business owners $l - b$, register start-up firms at rate $\delta f(\alpha)$ if $\alpha > \alpha_0$. At the same time, businesses are destroyed at rate γ, which gives rise to the following dynamics: $\dot{b}(\alpha) = \delta f(\alpha)(l - b) - \gamma b(\alpha)$. Imposing the steady state condition $\dot{b}(\alpha) = 0$ and integrating $b(\alpha)$ we can find the total number of small businesses b:

$$b = \int_{\alpha_0}^{\bar{\alpha}} b(\alpha)d\alpha = \frac{\delta(1 - F(\alpha_0))(l - b)}{\gamma} \quad \Rightarrow \quad b = \frac{\delta(1 - F(\alpha_0))l}{\gamma + \delta(1 - F(\alpha_0))}$$

This condition postulates a negative monotonous relationship between the entry threshold α_0 and the stock of small businesses b. Intuitively, a higher cut-off α_0 leads to a lower probability of starting a business $1 - F(\alpha_0)$, so the stock of small businesses b declines.

Let $G(\alpha)$ be the equilibrium distribution of solo self-employed out of opportunity with respect to the quality of their product α. More precisely it is the accumulated stock of self-employed entrepreneurs with a product quality in the range $[\alpha_s..\alpha]$. The dynamic equation for $G(\alpha)$ is given by:

$$\dot{G}(\alpha) = \underbrace{(s_u + e + u)}_{=l-b-s_e} \delta(F(\alpha) - F(\alpha_s)) - (\gamma + \delta(1 - F(\alpha)))G(\alpha) \tag{13}$$

Unemployed entrepreneurs u, and those self-employed out of necessity s_u, stop searching for regular jobs, if they develop a product/service with quality above α_s. Also employees in regular employment e quit their jobs and become self-employed out of opportunity if they get a realization of α above α_s. So the first term in equation 13 is the inflow of entrepreneurs into solo self-employment out of opportunity. The second term is the outflow consisting of unsuccessful entrepreneurs dropping back into unemployment (at rate γ) and those who develop a better product (at rate $\delta(1 - F(\alpha))$). Imposing the steady state condition $\dot{G}(\alpha) = 0$ and taking into account that $s_e = G(\alpha_0)$ we get:

$$s_e = G(\alpha_0) = \frac{\delta(F(\alpha_0) - F(\alpha_s))}{[\gamma + \delta(1 - F(\alpha_0))]} \frac{\gamma l}{[\gamma + \delta(1 - F(\alpha_s))]} \tag{14}$$

This equation shows that *ceteris paribus* the stock s_e is increasing in the threshold α_0, but decreasing in α_s. This is intuitive since a lower probability of starting a business $(1 - F(\alpha_0))$ is associated with a larger number of entrepreneurs remaining in solo self-employment out of opportunity s_e. On the contrary, if solo self-employed keep the option of moving into a regular job for higher values of α, meaning a higher threshold α_s, then there would be fewer solo self-employed out of opportunity s_e. Next consider the steady-state equation for employed potential entrepreneurs:

$$\dot{e} = (u + s_u)X\lambda(\theta) - (\bar{\gamma} + \delta(1 - F(\alpha_s)))e = 0$$

where the first term is the inflow which corresponds to all searching potential entrepreneurs $(u + s_u)$ starting a regular job at rate $X\lambda(\theta)$. Again, the second term is the outflow consisting of those losing regular jobs at rate $\bar{\gamma}$ or quitting their jobs voluntarily and becoming solo self-employed out of opportunity. Taking into account that $u + s_u = l - b - s_e - e$ we get the equilibrium expression for e:

$$e = \frac{X\lambda(\theta)}{[\bar{\gamma} + X\lambda(\theta) + \delta(1 - F(\alpha_s))]} \frac{\gamma l}{[(\gamma + \delta(1 - F(\alpha_s))]} \tag{15}$$

Relation 15 shows that *ceteris paribus* the number of employed potential entrepreneurs e is increasing in the job-finding rate $X\lambda(\theta)$ and also in the threshold variable α_s. The second effect implies that solo self-employed keep the option of searching for regular jobs and moving into paid employment for higher values of α.

Further, let $H(\alpha)$ be the equilibrium distribution of solo self-employed out of necessity with respect to the quality of their product or service α. It is the accumulated stock of self-employed with product qualities in the range $[\alpha_u..\alpha]$. So we get:

$$\dot{H}(\alpha) = u\delta(F(\alpha) - F(\alpha_u)) - (\gamma + \delta(1 - F(\alpha)) + X\lambda(\theta))H(\alpha)$$

The first term is the inflow and includes unemployed individuals developing product qualities above α_u and starting self-employment. The second term is the outflow consisting of self-employed entrepreneurs taking a regular job (at rate $X\lambda(\theta)$), dropping back into unemployment (at rate γ) and developing a better product quality (at rate $\delta(1 - F(\alpha))$). Imposing again the steady state condition $\dot{H}(\alpha) = 0$ and taking into account that $s_u = H(\alpha_s)$ we can find the equilibrium stock of potential entrepreneurs out of necessity s_u and in unemployment u. We summarize these results in proposition 6:

Proposition 6: *The equilibrium number of solo self-employed out of necessity s_u and the number of unemployed potential entrepreneurs u are given by:*

$$\begin{aligned} s_u &= \frac{\delta(F(\alpha_s) - F(\alpha_u))}{[\gamma + +X\lambda(\theta) + \delta(1 - F(\alpha_u))]} \frac{(\bar{\gamma} + \delta(1 - F(\alpha_s)))}{[\bar{\gamma} + X\lambda(\theta) + \delta(1 - F(\alpha_s))]} \frac{\gamma l}{[\gamma + \delta(1 - F(\alpha_s))]} \\ u &= \frac{\gamma + X\lambda(\theta) + \delta(1 - F(\alpha_s))}{[\gamma + +X\lambda(\theta) + \delta(1 - F(\alpha_u))]} \frac{(\bar{\gamma} + \delta(1 - F(\alpha_s)))}{[\bar{\gamma} + X\lambda(\theta) + \delta(1 - F(\alpha_s))]} \frac{\gamma l}{[\gamma + \delta(1 - F(\alpha_s))]} \end{aligned}$$

Propoition 6 shows that, other things being equal, the stock s_u is decreasing in the cut-off α_u. On the contrary, a higher α_u is associated with a higher stock of unemployed entrepreneurs u. Such a shift could happen, for example, if the price for output P falls, meaning a lower productivity of the self-employed entrepreneurs $\sigma = \varsigma P$ and a higher cut-off $\alpha_u = (z - h)/(\sigma(1 - t))$.

In the final step we analyze the distributions of solo self-employed with respect to the quality of their products α. These can be found due to the fact that $s_u(\alpha) = H'(\alpha)$ and $s_e(\alpha) = G'(\alpha)$. This leads to the following proposition:

Proposition 7: *The distributions of solo self-employed with respect to the quality of their product are given by $s_e(\alpha)/s_e$ for $\alpha \in [\alpha_s..\alpha_0]$ and $s_u(\alpha)/s_u$ for $\alpha \in [\alpha_u..\alpha_s]$, where:*

$$\begin{aligned} \frac{s_e(\alpha)}{s_e} &= \frac{\delta f(\alpha)(\gamma + \delta(1 - F(\alpha_s)))}{[\gamma + \delta(1 - F(\alpha))]^2} \frac{(\gamma + \delta(1 - F(\alpha_0)))}{[\delta(F(\alpha_0) - F(\alpha_s))]} \\ \frac{s_u(\alpha)}{s_u} &= \frac{\delta f(\alpha)(\gamma + \delta(1 - F(\alpha_u)) + X\lambda(\theta))}{[\gamma + \delta(1 - F(\alpha)) + X\lambda(\theta)]^2} \frac{(\gamma + X\lambda(\theta) + \delta(1 - F(\alpha_s)))}{[\delta(F(\alpha_s) - F(\alpha_u))]} \end{aligned}$$

Note that $G(\alpha_s)/s_e = 0$, $G(\alpha_0)/s_e = 1$, $H(\alpha_u)/s_u = 0$ and $H(\alpha_s)/s_u = 1$.

4.3 Wage setting

Regular firms can be matched with workers or potential entrepreneurs seeking to take a paid job.

Potential Entrepreneur: Consider a match between a regular firm and a potential entrepreneur and let the corresponding present value of a job be denoted by J, it becomes:

$$rJ \;=\; y - \bar{c}_k - w - \bar{\gamma}(J - V) - \delta(1 - F(\alpha_s(w)))(J - V)$$

where $\bar{c}_k$ corresponds to the flow cost of capital for regular firms. This equation show that the total job destruction rate associated with potential entrepreneurs is relatively high since there is a positive probability $\delta(1 - F(\alpha_s(w)))$ that they will quit the job and enter self-employment. This reduces the value of the job surplus to the firm.

In order to model wage bargaining between job applicants and regular firms we follow the approach in Gautier (2002). This approach assumes that applicants matched with an employer and negotiating over the wage stop searching for alternative jobs. In addition, we assume that they also disregard their activities in self-employment out of necessity, since paid employment is a superior state of income (i.e $E_s(\alpha) < W \quad \forall \quad \alpha < \alpha_s$). However, they continue considering self-employment out of opportunity and the possibility of starting a business (which happens for $\alpha > \alpha_s$) while negotiating with a regular employer. The advantage of this approach is that labour contracts are renegotiation proof, meaning that potential entrepreneurs receive the same wage w in paid employment irrespective of their previous income in self-employment. The disagreement payoff for potential entrepreneurs becomes:

$$rW_D = z + \delta \int_{\alpha_s}^{\alpha_0} \frac{(1 - F(x))\sigma(1 - t)}{r + \gamma + \delta(1 - F(x))} dx + \delta(1 - t) \int_{\alpha_0}^{\bar{\alpha}} \frac{(1 - F(x))\sigma}{r + \gamma} dx - \bar{\gamma}(W_D - U)$$

Here we assume that potential entrepreneurs still enjoy the unemployment benefit in the state W_D, however, the disutility of unemployment h does not enter the equation. Intuitively, this means that having the job offer from a potential employer and bargaining over the wage alleviates the stigma of failure for the person. Taking difference $W - W_D$ and using β for the bargaining power of workers negotiating with regular firms we can write down the Nash bargaining problem:

$$\max_{w} \Big(\frac{w(1 - t) - z}{r + \bar{\gamma}}\Big)^{\beta} \Big(\frac{(y - \bar{c}_k - w)}{r + \bar{\gamma} + \delta(1 - F(\alpha_s(w)))}\Big)^{1-\beta}$$

This equation shows that higher wages have an ambiguous effect on the present value of firms' profits. On the one hand, there is the direct effect of higher labour costs reducing profits. But on the other hand, paying a higher wage reduces the probability that the potential entrepreneur employed in a regular job will quit into self-employment (i.e. higher $\alpha_s(w)$). This reduces the overall job destruction rate from the perspective of the firm and has a positive effect on the present value of profits.

Proposition 8: *The wage of potential entrepreneurs in paid employment is given by:*

$$w(1-t) = \beta(y-\bar{c}_k)(1-t) + (1-\beta)z + \underbrace{(1-\beta)\frac{(y-\bar{c}_k-w)(w(1-t)-z)\delta f(\alpha_s)}{r+\bar{\gamma}+\delta(1-F(\alpha_s))}\frac{\partial \alpha_s}{\partial w}}_{\text{no-quit premium}}$$

where $\partial\alpha_s/\partial w$ is given by equation 10 *and* $z < w(1-t) \leq (y-\bar{c}_k)(1-t)$.

This equation shows that firms pay a no-quit premium to potential entrepreneurs in order to reduce their quitting probability. The size of the no-quit premium depends crucially on the sensitivity of the quitting probability to wage. If the threshold α_s is not sensitive to wages, $\partial\alpha_s/\partial w \approx 0$, then we get a standard Nash bargaining result that the wage is a weighted average between the net productivity and the outside option of the employee. This happens when the productivity multiplier of self-employed entrepreneurs σ is relatively high. On the other hand, if $\partial\alpha_s/\partial w$ is relatively high, which happens for low values of σ, than the equilibrium wage equation may contain a substantial no-quit premium for potential entrepreneurs taking a regular paid job. This, however, does not imply that their wages are higher than the wages of regular workers employed in the same firm, since in the first place, wages depend on the productivity.

Regular Worker: Next, consider regular workers. Let $\bar{W}$ denote their present value of employment, $\bar{W}_D$ be the payoff in disagreement and $\bar{J}$ be the present value of a job given by:

$$\begin{aligned} r\bar{W} &= \bar{w}(1-t) - \bar{\gamma}(\bar{W}-\bar{U}) \qquad r\bar{W}_D = \bar{z} - \bar{\gamma}(\bar{W}_D - \bar{U}) \\ r\bar{J} &= (\bar{y}-\bar{c}_k-\bar{w})(1-\tau) - \bar{\gamma}(\bar{J}-V) \end{aligned}$$

Formulating the Nash bargaining problem we get: $\bar{w}(1-t) = \beta(\bar{y}-\bar{c}_k)(1-t) + (1-\beta)\bar{z}$. In addition, these workers can be employed in small businesses run by self-employed entrepreneurs. Jobs in small businesses pay wages w_{0N} if the business is operated by the native owner, and w_{0I} if the business is operated by the immigrant owner.[6] Let W_{0N} and W_{0I} be the corresponding present values of employment in small businesses:

$$rW_{0N} = w_{0N}(1-t) - \gamma^N(W_{0N}-\bar{U}) \qquad rW_{0I} = w_{0I}(1-t) - \gamma^I(W_{0I}-\bar{U})$$

The job destruction rates are specific to the type of the business owner. Thus, all workers in a small firm are subject to the same job destruction rate γ^N if the business owner is a native entrepreneur, and γ^I if the owner is an immigrant entrepreneur. The disagreement

[6] With full scale notation the two wages are written as: w_{0N}^{ij} for workers of type ij, $i = I, N$, $j = L, H$ employed in small businesses operated by the native owner, and w_{0I}^{ij} for workers of type ij employed in small businesses operated by the immigrant owner.

payoffs in the bargaining state are denoted by W_{DN} and W_{DI} and can be written as:

$$rW_{DN} = \bar{z} - \gamma^N (W_{0N} - \bar{U}) \qquad rW_{DI} = \bar{z} - \gamma^I (W_{0I} - \bar{U})$$

Workers then seek to maximize their job rent $W_{0N} - W_{DN}$ or $W^i_{0I} - W^i_{DI}$ depending on the type of the business. Let β_0 denote the bargaining power of workers negotiating with small businesses. The Nash bargaining problem then becomes:

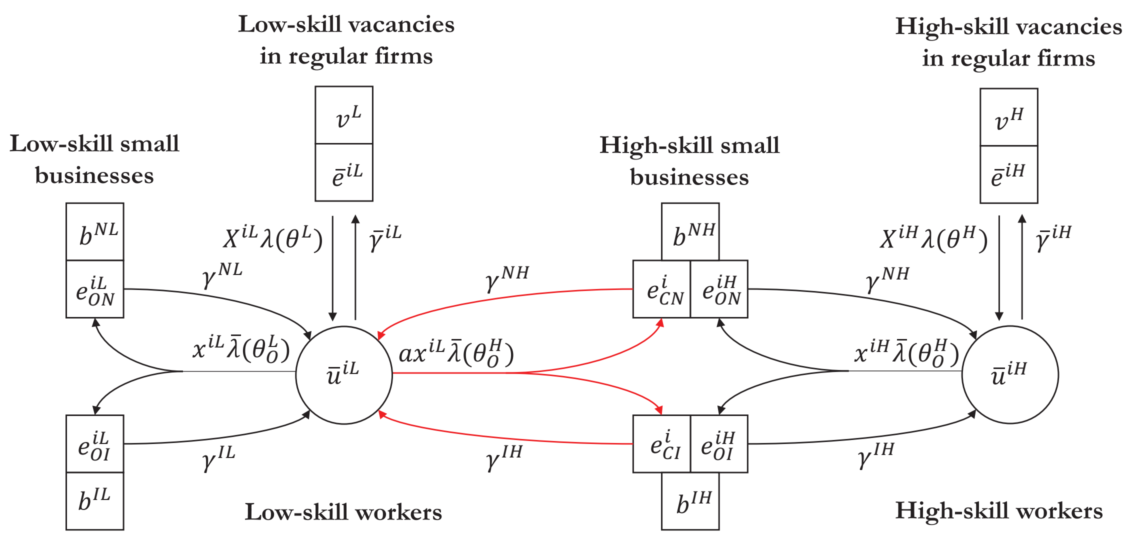
$$\max_{w_{0N}} (w_{0N}(1-t) - \bar{z})^{\beta_0} (y_{0N} - w_{0N})^{1-\beta_0} \quad \Rightarrow \quad w_{0N}(1-t) = \beta_0 y_{0N}(1-t) + (1-\beta_0)\bar{z}$$

and the same for immigrant business owners, so we get $w_{0I}(1-t) = \beta_0 y_{0I}(1-t) + (1-\beta_0)\bar{z}$. In addition, wages of low skill workers employed in high skill small businesses can be obtained as $w_{CI}(1-t) = \beta_0 y_{CI}(1-t) + (1-\beta_0)\bar{z}$, where the subindex C stands for cross-skill matching.

4.4 Matching and free-entry

In this section we describe matching between searching workers and vacancies in the market for regular jobs, as well as matching between searching workers and openings in small businesses. Figure 4 illustrates all labour market states and transitions of regular workers. The middle part of the diagram (red colour) corresponds to the cross-skill matching whereby low skill regular workers apply for positions in high skill small businesses. These could be, for example, nurses or secretaries working for a (high skill) medical doctor or a tax consultant respectively.

Figure 4: Labour market states and transitions of regular workers



First, consider low skill small businesses. From proposition 5 we already know that the average number of immigrant employees per small business is given by $\bar{q}(\theta^L_0)\mu/(\gamma^{iL})$, where

i reflects the ethnic group of the business owner. At the same time, the average number of native employees is $\bar{q}(\theta_0^L)(1-\mu)/(\gamma^{iL})$. Given that the number of small businesses in this group is equal to b^{iL}, we can find how many immigrant and native workers are employed in these businesses e_{0i}^{IL} and e_{0i}^{NL}:

$$\frac{\bar{q}(\theta_0^L)}{\gamma^{iL}} b^{iL} = \frac{e_{0i}^{IL}}{\mu} = \frac{e_{0i}^{NL}}{1-\mu} \quad \text{where} \quad \mu = \frac{x^{IL}\bar{u}^{IL}}{x^{NL}\bar{u}^{NL} + x^{IL}\bar{u}^{IL}} \tag{16}$$

Here the superscript $i = N, I$ refers to the ethnic background of the worker, while the subscript refers to the ethnic background of the entrepreneur $i = N, I$ and 0 refers to small businesses. Next consider high skill businesses. Here the average number of immigrant high skill employees is given by $\bar{q}(\theta_0^H)\mu^{IH}/(\gamma^{iH})$, while the average number of native high skill workers is $\bar{q}(\theta_0^H)\mu^{NH}/(\gamma^{iH})$. In a similar way, we can use information about the average number of immigrant and native low skill employees. Multiplying the average number of employees per business with the number of businesses b^{iH} yields the following:

$$\frac{\bar{q}(\theta_0^H)}{\gamma^{iH}} b^{iH} = \frac{e_{0i}^{IH}}{\mu^{IH}} = \frac{e_{0i}^{NH}}{\mu^{NH}} = \frac{e_{Ci}^{I}}{\mu^{IL}} = \frac{e_{Ci}^{N}}{\mu^{NL}} \tag{17}$$

where the probabilities of hiring an employee of type iH and iL from the perspective of the business owner are given by:

$$\mu^{iH} = \frac{x^{iH}\bar{u}^{iH}}{\sum_{i=I,N} x^{iH}\bar{u}^{iH} + ax^{iL}\bar{u}^{iL}} \quad \text{and} \quad \mu^{iL} = \frac{ax^{iL}\bar{u}^{iL}}{\sum_{i=I,N} x^{iH}\bar{u}^{iH} + ax^{iL}\bar{u}^{iL}}$$

These probabilities do not depend on the ethnic background of the business owner.

Further, we write down the steady state equation for employees in regular jobs $\bar{e}^{ij}$, recall the distribution of native and immigrant workers in the skill submarket j. This yields:

$$\begin{aligned} 0 &= X^{ij}\lambda(\theta^j)\bar{u}^{ij} - \bar{\gamma}^{ij}\bar{e}^{ij} \\ d^{ij} - l^{ij} &= \bar{u}^{ij} + \bar{e}^{ij} + e_{0N}^{ij} + e_{0I}^{ij} + \mathbb{1}_L(e_{CN}^{i} + e_{CI}^{i}) \end{aligned} \tag{18}$$

where the indicator function $\mathbb{1}_L$ takes value one for $j = L$ and zero otherwise. Solving equations (18) jointly with (16), (17) allows us to find the equilibrium unemployment rates of regular workers $\bar{u}^{ij}$, $i = I, N$, $j = L, H$.

Proposition 9 *The equilibrium numbers of unemployed regular workers* $\bar{u}^{ij}$, $i = I, N$, $j = L, H$ *are given by:*

$$\bar{u}^{ij} = \Big[d^{ij} - l^{ij} - \mu^{ij}\bar{q}(\theta_0^H)\big(\frac{b^{NH}}{\gamma^{NH}} + \frac{b^{IH}}{\gamma^{IH}}\big) - \mathbb{1}_L \mathbb{M}\bar{q}(\theta_0^L)\big(\frac{b^{NL}}{\gamma^{NL}} + \frac{b^{IL}}{\gamma^{IL}}\big)\Big]\frac{\bar{\gamma}^{ij}}{\bar{\gamma}^{ij} + X^{ij}\lambda(\theta^j)}$$

where variable $\mathbb{M}$ *takes value* μ *for immigrant low skill workers and* $1-\mu$ *for native low* *skill workers.*

The square bracket of this equation contains the number of workers not employed in small businesses. The share of unemployed among them is $\bar{\gamma}^{ij}/(\bar{\gamma}^{ij} + X^{ij}\lambda(\theta^j))$, whereas the share of those, who are employed in regular jobs, is $X^{ij}\lambda(\theta^j)/(\bar{\gamma}^{ij} + X^{ij}\lambda(\theta^j))$. This indicates that *ceteris paribus* the unemployment rate of regular workers is increasing in the probability of losing a job $\bar{\gamma}^{ij}$ but decreasing in the job-finding rate $X^{ij}\lambda(\theta^j)$.

Finally, we consider the free-entry conditions for regular jobs. Denote $\bar{c} = \bar{c}_h + \bar{c}_k$ – the total cost of capital and posting facing regular firms. Using this notation we write down the present value equation for V, which is the expected discounted profit flow associated with an open vacancy as follows:

$$rV = -\bar{c} + q(\theta)\Big[\sum_{i=I,N} \underbrace{\frac{X^i \bar{u}^i}{\sum_{i=I,N} X^i(\bar{u}^i + u^i + s_u^i)}(\bar{J}^i - V)}_{\text{matching with regular workers}} + \underbrace{\frac{X^i(u^i + s_u^i)}{\sum_{i=I,N} X^i(\bar{u}^i + u^i + s_u^i)}(J^i - V)}_{\text{matching with pot. entrepreneurs}}\Big]$$

Imposing the free-entry condition $V = 0$ we get the standard result that regular firms create vacancies up to the point, where the expected cost of an open vacancy is equal to the expected present value of profits from a filled job. Writing down these equations for $j = L, H$ allows us to find the equilibrium stocks of vacancies v^L and v^H.

4.5 Production of the final good and welfare

The final output good is produced using capital K and a composite good Z using the constant returns to scale technology (1). The composite good Z is produced using two intermediate inputs, Y_L and Y_H according to the CES production function (2), where Y_j is a linear aggregate of all the units of output produced by regular workers and potential entrepreneurs of type j, so it can be calculated as:

$$\begin{aligned} Y_j &= \sum_{i=I,N}\Bigg(\varsigma^{ij}\int_{\alpha_u^{ij}}^{\alpha_s^{ij}} \alpha s_u^{ij}(\alpha)d\alpha + \varsigma^{ij}\int_{\alpha_s^{ij}}^{\alpha_0^{ij}} \alpha s_e^{ij}(\alpha)d\alpha + \varsigma^{ij}\int_{\alpha_0^{ij}}^{\bar{\alpha}} \alpha b^{ij}(\alpha)d\alpha \\ &+ \varphi^{ij}e^{ij} + \bar{\varphi}^{ij}\bar{e}^{ij} + \varphi_{0N}^{ij}e_{0N}^{ij} + \varphi_{0I}^{ij}e_{0I}^{ij} + \mathbb{1}_L(\varphi_{CN}^i e_{CN}^i + \varphi_{CI}^i e_{CI}^i)\Bigg) \end{aligned}$$

where the first three terms include the output of self-employed entrepreneurs and business owners, $\varphi^{ij}e^{ij}$ is the output produced by potential entrepreneurs employed in regular jobs, $\bar{\varphi}^{ij}\bar{e}^{ij}$ – output produced by workers in regular jobs, and the remaining terms constitute the output produced by workers employed in small businesses. Finally, the indicator variable takes value 1 for $j = L$ and value zero for $j = H$. This allows us to capture the output of low skill workers employed in small businesses operated by the high skilled owners.

Since capital as well as the two intermediate goods are supplied in competitive markets, their prices R and P^j are equal to the marginal productivities:

$$R = A\eta K^{\eta-1}[aY_L^\rho + (1-a)Y_H^\rho]^{\frac{1-\eta}{\rho}} \tag{19}$$

$$P_L = a(1-\eta)AK^\eta Y_L^{\rho-1}[aY_L^\rho + (1-a)Y_H^\rho]^{\frac{1-\eta-\rho}{\rho}} \tag{20}$$

$$P_H = (1-a)(1-\eta)AK^\eta Y_H^{\rho-1}[aY_L^\rho + (1-a)Y_H^\rho]^{\frac{1-\eta-\rho}{\rho}} \tag{21}$$

where R is the exogenous price of capital including the risk-free interest rate and the cost of capital depreciation.

In the final step we consider the welfare of workers in every demographic group. These variables will become important later on for understanding the impact of an immigration shock on incumbent individuals. Let $\bar{\Omega}^{ij}$, $i = I, N$, $j = L, H$ denote social welfare of regular workers in the demographic group ij. In a similar way, let Ω^{ij} denote the welfare of potential entrepreneurs. Since potential entrepreneurs are characterized by various qualities of the product α, we calculate an average product quality in each group:

$$\bar{\alpha}_{us} = \int_{\alpha_u}^{\alpha_s} \alpha \frac{s_u(\alpha)}{s_u} d\alpha \qquad \bar{\alpha}_{s0} = \int_{\alpha_s}^{\alpha_0} \alpha \frac{s_e(\alpha)}{s_e} d\alpha \qquad \bar{\alpha}_{01} = \int_{\alpha_0}^{\bar{\alpha}} \alpha \frac{b(\alpha)}{b} d\alpha$$

Variable $\bar{\alpha}_{us}$ stands for the average product quality of solo self-employed out of necessity distributed in the range $[\alpha_u ... \alpha_s]$. Variable $\bar{\alpha}_{s0}$ denotes the average product quality of solo self-employed out of opportunity distributed in the range $[\alpha_s ... \alpha_0]$ and, finally, $\bar{\alpha}_{01}$ corresponds to the average product quality of business owners distributed in the range $[\alpha_0 ... \bar{\alpha}]$. Based on these definitions, the welfare values for all worker groups can be obtained as (we suppress the upper index ij for the ease of exposition):

$$\bar{\Omega} = \frac{1}{d-l}(\bar{u}(\bar{z}-h) + (1-t)(\bar{e}\bar{w} + e_{0N}w_{0N} + e_{0I}w_{0I} + \mathbb{1}_L(e_{CN}w_{CN} + e_{CI}w_{CI}))) + T$$

$$\Omega = \frac{1}{l}\Big(u(z-h) + (1-t)(ew + s_u\sigma\bar{\alpha}_{us} + s_e\sigma\bar{\alpha}_{s0} + b\sigma\bar{\alpha}_{01}) + \underbrace{b\Big(-c + \frac{\bar{q}(\theta_0)}{\gamma}\pi(r+\gamma)\Big)}_{\text{net profit from co-workers}}\Big) + T$$

where the indicator function $\mathbb{1}_L$ takes value one for $j = L$ and zero otherwise and variable T is the lump-sum transfer from the public budget. The cash inflow into the public budget consists of revenues from labour income taxes of regular workers and potential entrepreneurs BR_t and corporate taxes paid on flow profits by small businesses and regular jobs BR_τ. The outflow from the public budget consists of expenses for unemployment benefits for all demographic groups BE. Given that total revenues exceed substantially the expenses for unemployment benefits, the surplus of the budget is equally split and distributed as a lump-sum transfer T across all worker groups. The revenues and expenses are calculated as follows:

$$
\begin{aligned}
BR_t &= t\sum_{ij}\Big(\underbrace{\bar{e}\bar{w} + e_{0N}w_{0N} + e_{0I}w_{0I} + \mathbb{1}_L(e_{CN}w_{CN} + e_{CI}w_{CI})}_{\text{income taxes of regular workers}} \\
&\qquad + \underbrace{ew + s_u\sigma\bar{\alpha}_{us} + s_e\sigma\bar{\alpha}_{s0} + b\sigma\bar{\alpha}_{01}}_{\text{income taxes of potential entrepreneurs}}\Big) \\
BR_\tau &= \tau\sum_{ij}\Big(\underbrace{b\frac{\bar{q}(\theta_0)}{\gamma}\frac{\pi(r+\gamma)}{1-\tau}}_{\text{small businesses}} + \underbrace{(\bar{y}-\bar{c}_k-\bar{w})\bar{e} + (y-\bar{c}_k-w)e}_{\text{regular jobs}}\Big) \\
BE &= \sum_{ij} zu + \bar{z}\bar{u} \qquad T = \frac{1}{\sum_{ij} d}\Big(BR_t + BR_\tau - BE\Big)
\end{aligned} \tag{22}
$$

where $\sum_{ij} d$ is the total size of the population. It is normalized to 1 in the benchmark calibration, but will be larger than 1 upon the immigration shock.

5 Data

We use data from the German Socio-Economic Panel (SOEP) waves 2000-2017. SOEP is a representative panel of households and individuals in Germany. It provides detailed information about the ethnicity of the individuals, their qualification, wage, size of the employer firm as well as their employment status including the possibility of self-employment and entrepreneurship. Moreover, it contains information about the business size (i.e. the number of coworkers) for the self-employed entrepreneurs. We restrict the sample to labour force participants working full/part time or actively searching for jobs (workers who report to be registered as unemployed at the federal employment agency). Non-participants, retired, marginally employed or military personnel as well as those below 17 years of age are dropped from the sample. This yields an unbalanced panel with a total of 302686 observations over 18 years.

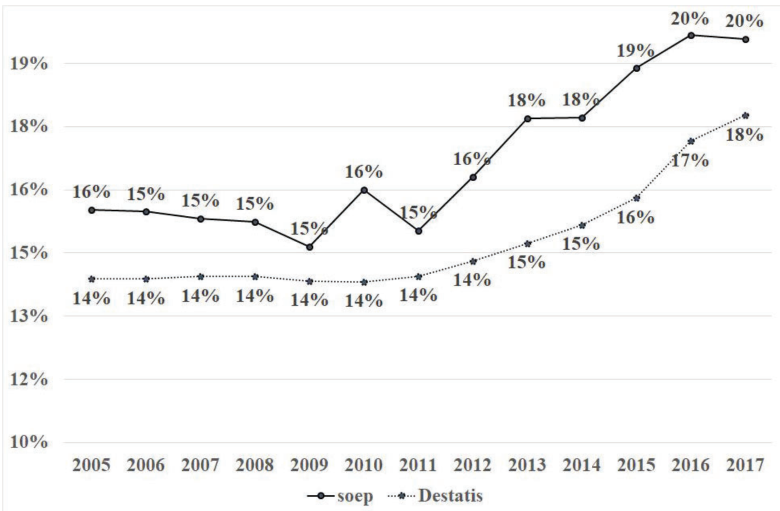
We define as ***native*** (N) an individual who is born in Germany or has German nationality since birth and is not an ethnic German from Eastern Europe. All second generation immigrants born in Germany are considered natives. Figure 5 compares our estimates (from SOEP) of the share of immigrants in the total population with the Destatis estimates using our definition of an immigrant. The difference in the two series is 2 percentage points on average. We drop all non-participants and exclude workers age 16 or younger, which may have lead to a slightly higher fraction of immigrants in our sample.

Workers with at least 13 years of schooling are defined as ***high skilled (H)*** while others are considered ***low skilled (L)***. High skilled immigrants form the smallest fraction of the sample population. The average share of high and low skilled natives in the sample is 32.6% and 51.0% respectively. While the high and low skilled immigrants form 13.7% and 2.7% of the sample. Table 1 provides the sample profile.

We define ***potential entrepreneurs*** the individuals, who are observed at least once in the data as self-employed, freelance professionals or as small business owners (with or without co-workers). Their stock is denoted by l^{ij} in the model. Individuals who are never observed as business owners or in self-employment are considered ***regular workers*** (low entrepreneurial spirit). Their stock is denoted by $d^{ij} - l^{ij}$. We may underestimate the share of potential entrepreneurs as some individuals may have been in entrepreneurship before participating in the survey while other potential entrepreneurs may have dropped out of the sample before observed in entrepreneurial state. The attrition rates could be particularly high for immigrants due to return migration. However, it is a reasonable measure given the fact that there is no other way to deduce entrepreneurial abilities due to the lack of detailed information about the interest in entrepreneurship or efforts (both successful and unsuccessful) to open up a business.

Table 1 shows the distribution of all workers in the sample across the four demographic groups and labour market states. It is observed that high skill native workers are least likely to be employed in small businesses (15.3%). This share is slightly higher for high skill immigrant workers (18.3%), whereas it is much higher for low skill workers ($22.5 - 23.5\%$). It is interesting to see that the potential entrepreneurs have much lower unemployment rates compared to workers without entrepreneurial abilities. Low skilled immigrant workers have the highest unemployment rates. Both natives and immigrants in the high-skill group have the highest shares of active entrepreneurs $(b^{ij} + s^{ij})/l^{ij}$.

Figure 5: Share of immigrants in the total population, Germany, 2005-2017.



Note: Authors' calculations based on SOEP: 2000-2017 and Destatis available at: https://www.destatis.de

Further, SOEP provides information on solo self-employed entrepreneurs $s^{ij} + b_0^{ij}$ and business owners with co-workers $b^{ij} - b_0^{ij}$. The last group is further split into business owners with less than 9 co-workers $\sum_{n=1}^{9} b_n$ and those with more than 9 co-workers. $59-$

60% of native and immigrant potential businessmen with low skills are solo-entrepreneurs and $36 - 37\%$ of them are managing small businesses with 1 to 9 co-workers. This is different in the high skill group where only 46% of immigrant entrepreneurs are solo self-employed and 43% are managing small businesses.

Table 1: Sample Profile. SOEP, waves 2000-2017

Natives $d^{NL} + d^{NH} = 0.836$			
Low skill $d^{NL} = 0.510$		High skill $d^{NH} = 0.326$	
Regular workers $d^{NL} - l^{NL} = 0.449$	Pot. Entrepreneurs $l^{NL} = 0.061$	Regular workers $d^{HN} - l^{NH} = 0.261$	Pot. Entrepreneurs $l^{NH} = 0.065$
$\frac{\bar{u}^{NL}}{d^{NL}-l^{NL}} = 0.085$	$\frac{u^{NL}}{l^{NL}} = 0.035$	$\frac{\bar{u}^{NH}}{d^{NH}-l^{NH}} = 0.022$	$\frac{u^{NH}}{l^{NH}} = 0.014$
$\frac{\bar{e}^{NL}}{d^{NL}-l^{NL}} = 0.680$	$\frac{b^{NL}+s^{NL}}{l^{NL}} = 0.547$	$\frac{\bar{e}^{NH}}{d^{NH}-l^{NH}} = 0.825$	$\frac{b^{NH}+s^{NH}}{l^{NH}} = 0.568$
	$\frac{b_0^{NL}+s^{NL}}{b^{NL}+s^{NL}} = 0.586$		$\frac{b_0^{NH}+s^{NH}}{b^{NH}+s^{NH}} = 0.591$
	$\frac{\sum_{n=1}^{9} b_n^{NL}}{b^{NL}+s^{NL}} = 0.366$		$\frac{\sum_{n=1}^{9} b_n^{NH}}{b^{NH}+s^{NH}} = 0.326$
Immigrants $d^{IL} + d^{IH} = 0.164$			
Low skill $d^{IL} = 0.137$		High skill $d^{IH} = 0.027$	
Workers $d^{IL} - l^{IL} = 0.124$	Po. Entrepreneurs $l^{IL} = 0.013$	Workers $d^{IH} - l^{IH} = 0.022$	Pot. Entrepreneurs $l^{IH} = 0.005$
$\frac{\bar{u}^{IL}}{d^{IL}-l^{IL}} = 0.141$	$\frac{u^{IL}}{l^{IL}} = 0.049$	$\frac{\bar{u}^{IH}}{d^{IH}-l^{IH}} = 0.091$	$\frac{u^{IH}}{l^{IH}} = 0.039$
$\frac{\bar{e}^{IL}}{d^{IL}-l^{IL}} = 0.634$	$\frac{b^{IL}+s^{IL}}{l^{IL}} = 0.525$	$\frac{\bar{e}^{IH}}{d^{IH}-l^{IH}} = 0.726$	$\frac{b^{IH}+s^{IH}}{l^{IH}} = 0.577$
	$\frac{b_0^{IL}+s^{IL}}{b^{IL}+s^{IL}} = 0.607$		$\frac{b_0^{IH}+s^{IH}}{b^{IH}+s^{IH}} = 0.459$
	$\frac{\sum_{n=1}^{9} b_n^{IL}}{b^{IL}+s^{IL}} = 0.362$		$\frac{\sum_{n=1}^{9} b_n^{IH}}{b^{IH}+s^{IH}} = 0.425$

Note: The sample has 302686 observations. We have 98488 observations on low skilled natives, 154566 on high skilled natives, 8172 on high skilled immigrants and 41460 observations on low skilled immigrants. All estimates are generated using individual sampling weights.

5.1 Calibration

Our model includes a vector of 13 parameters $\{x, X, \gamma, \delta, c, \varsigma, \bar{\gamma}, \varphi_{0I}, \varphi_{0N}, \bar{\varphi}, \varphi, \bar{z}, z\}^{ij}$, which are specific to each of the four demographic groups $i = I, N$ and $j = L, H$. Recall that variables φ and ς denote the exogenous units of output for employed and self-employed individuals, for example, $y_{0I}^{ij} = \varphi_{0I}^{ij} P^j$ and $y_{0N}^{ij} = \varphi_{0N}^{ij} P^j$. This should be combined with 4 parameters φ_{CI}^{i}, φ_{CN}^{i}, $i = I, N$ corresponding to the output units of low skill workers employed in high skill small businesses (cross-skill matching). Further, there are 5 parameters, which are skill-specific, but common to native and immigrant workers in the same skill group, these parameters include $\{M, \bar{M}, \bar{c}, \beta_0, t\}^j$ for $j = L, H$. Finally, there are 11 parameters, which are common to all demographic groups, these are $\{r, \rho, \eta, R, \kappa, \zeta, a, A, h, \tau, \beta\}$.

Pre-eset Parameters: One period of time is taken to be one quarter, so we set $r = 0.0125$ corresponding to the 5% annual interest rate. Following Ottaviano and Peri (2012) and Battisti et al. (2018), we set $\rho = 0.5$, which is a parameter governing the elasticity of substitution between high and low skill labour. This corresponds to the elasticity of substitution equal to 2. In addition, we follow Chassamboulli and Palivos (2014) and Battisti et al. (2018) and set the quarterly cost of capital for the final goods producer and the capital share as follows: $R = 0.03$, $\eta = 0.35$. We use the findings by Rebien et al. (2020) that the average recruitment cost for positions with vocational training qualification (i.e. low skill) is equal to 521 EUR, whereas filling high skill positions requiring a university degree is much more expensive with an average recruitment cost equal to 2046 EUR. This corresponds to the normalized values 0.48 and 1.88 in our calibration. Based on this information we set $\bar{c}_h^L/q(\theta^L) = 0.48$ and $\bar{c}_h^H/q(\theta^H) = 1.88$, which yields parameters $\bar{c}_h^j$ for $j = L, H$. The flow cost of capital is then obtained as a residual category $\bar{c}_k^j = \bar{c}^j - \bar{c}_h^j$.

Calibrated Parameters: The search intensity parameters x^{ij}, X^{ij} and the multipliers of the matching function $M^j, \bar{M}^j$ can not be identified separately from the data, so we normalize $x^{NL} = 1$, $x^{NH} = 1$, $X^{NL} = 1$, $X^{NH} = 1$. Targeting the empirically observed unemployment rates of regular workers and potential entrepreneurs $\bar{u}^{ij}$, u^{ij} (8 values) we can find parameters x^{Ij}, X^{Ij}, M^j and $\bar{M}^j$. We use the remaining 4x4 data moments from table 1 to find parameters $\gamma, \delta, c, \varsigma$ for each worker group ij. In addition, we use information about tenure of native and immigrant workers in regular jobs from table 9 to identify the job-destruction rates $\bar{\gamma}$. Job tenure is traditionally long in Germany, even in small firms the average tenure amounts to 8.2 years. We target this value to obtain the cross-skill matching parameter $\kappa = 0.170$. Our results indicate that job destruction rates of small businesses γ vary between 0.029 and 0.036 depending on the type of the business owner, this corresponds to the 2-year survival probabilities in the range $71-76\%$. These probabilities are very close to the average 2-year survival probability reported for Germany by the OECD and equal to 70.7%, which provides another indirect validation of our calibration.[7]

To estimate ζ, the elasticity parameter in the Cobb-Douglas matching function, we use statistical information of the Federal Employment Office (Bundesagentur für Arbeit) for the period between January 2000 and December 2018. This data includes information on the absolute number of unemployed, vacancies and vacancy durations which are not

[7]OECD.stat SDBS Business Demography Indicators (ISIC Rev. 3): Enterprise survival rates

available in the SOEP survey. This yields 228 monthly observations on unemployment, vacancies and the average vacancy duration (in days). We use this data to construct an economy wide average market tightness ($\theta_t = \frac{V_t}{U_t}$) and the hiring rate ($q(\theta_t)$. These variables are illustrated in Fig 6. The upper left panel of this figure shows a sharp increase of the average vacancy duration between the years 2000 and 2018. Whereas it was only 41.5 days on average in the year 2000, it almost tripled and reached a level 112 days by 2018. This is important for our analysis because the impact of immigration may be different depending on whether the labour market has high unemployment or a shortage of labour. The top right panel of figure 6 shows the corresponding Beveridge curve for Germany showing a stable negative relationship between unemployment and vacancies. The bottom left panel shows the market tightness, while the aggregate job-filling rate $q(\theta)$ is illustrated in the bottom right panel of figure 6. Using this information we estimate the following regression:

$$\ln q(\theta_t) = cst - \zeta \ln \theta_t + Year + \iota_t \tag{23}$$

We control for time fixed effects by introducing a dummy for each year. This regression gives us a value of the slope parameter ζ equal to 0.47. In addition, we use information about the average vacancy duration, which was 64 days or 64/92 quarters in the considered period. Targeting this data moment, and using the free-entry conditions for regular jobs we estimate the flow cost parameters in regular jobs $\bar{c}$.

We have three types of tax rates in the model. There is income tax t^L for low-skilled workers, t^H for high-skilled workers, and a corporate tax τ for small businesses and large firms. According to the German law, solo self-employed individuals and freelancers are subject to the tax on labour income. The regular income tax varies between 14-45% depending on the taxpayer's income. We, therefore, use an average tax rate of 25% for low-skilled workers and solo self-employed entrepreneurs and a tax rate of 35% for the high-skilled group. We use a 15% tax rate for corporate taxes inline with the German law.

In addition, we use official information on the share of net wages in the total output (GDP), which was stable over the years 2000-2017 with an average equal to 28%.[8] We target this value to obtain the bargaining power of workers in regular jobs ($\beta = 0.9$). The relatively high bargaining power suggests that a substantial share of the GDP consists of labour taxes, corporate taxes and non-labour costs, whereas the share of firm profits is relatively low. Note that accounting for both types of taxes, as well as capital and hiring costs allows us to achieve an adequate fit between the theoretical model and the empirical data.

Productivities and prices are linked since multiplying the price by a given factor

[8]www.destatis.de Volkswirtschaftliche Gesamtrechnungen, Fachserie 18, Reihe 1.4, Tabelle 2.1.8

would raise the productivities of workers in a given skill group by the same factor of proportionality. Hence, without loss of generality we normalize the output produced by native regular workers in a given skill group to 1 unit, so that $\bar{y}^{NL} = P_L$ and $\bar{y}^{NH} = P_H$. Using these identification equations and combining them with the fact that the marginal product of a given intermediate good should be equal to its price (equations 20and 21) we find parameters $A = 2.563$ and $a = 0.555$. A is a total factor productivity, while a is a share of the low skill intermediate good in the composite labour product.

In order to calibrate the output units for all worker groups, we estimate a Mincer earnings regression with a logarithm of gross monthly wages as dependent variable for regular workers and a similar Mincer regression for the earnings of potential entrepreneurs. For regular workers we control for ethnicity ($i = I, N$), skill ($j = L, H$), firm size and the year fixed effects. The exact regression specifications and output tables are presented in the appendix B. The average wage of native low skill workers employed in small businesses is normalized to 1, that is $w_0^{NL} = 1$.[9] Not surprisingly, the wages are quite dispersed with the lowest value observed for immigrant low skill workers employed in small businesses ($w_0^{IL} = 0.87$) and the highest value observed for native high skill workers employed in regular jobs $\bar{w}^{NH} = 2.56$.

In the Mincer regression for potential entrepreneurs we control for ethnicity ($i = I, N$), skill ($j = L, H$) and the economic state distinguishing between paid employment, solo self-employment and business ownership. In particular, SOEP data includes information about the profits of small businesses with less (more) than 9 co-workers. Our regressions reveal that the predicted profits of immigrant businesses with less than 9 coworkers are substantially higher then the predicted profits of native businesses of the same size. This holds for both skill groups, for example, for high skill businesses we get $4.52 > 3.85$. We highlight this data property as it makes the profits of immigrant businesses more sensitive to the number of coworkers and the probability of hiring a coworker, which will be important later in the analysis. Another important observation is that the profits from business ownership are higher than wages in paid employment for all potential entrepreneurs. In the next step we calibrate the unemployment benefits z^{ij} and $\bar{z}^{ij}$. According to the German regulation unemployed individuals eligible for unemployment insurance obtain 60% of the previous netto wage for a maximum period of 12 months (ALG I).[10,11] After the unemployment spell of 12 months the person becomes eligible for social support (ALG II). This support was introduced during the labour market reform in 2005 and remained at the level of 400 EUR until it started increasing in 2017. In order to account for the sharp reduction of the unemployment benfit after the first year of payments we calculated a permanent flow value such that it's present value is equivalent

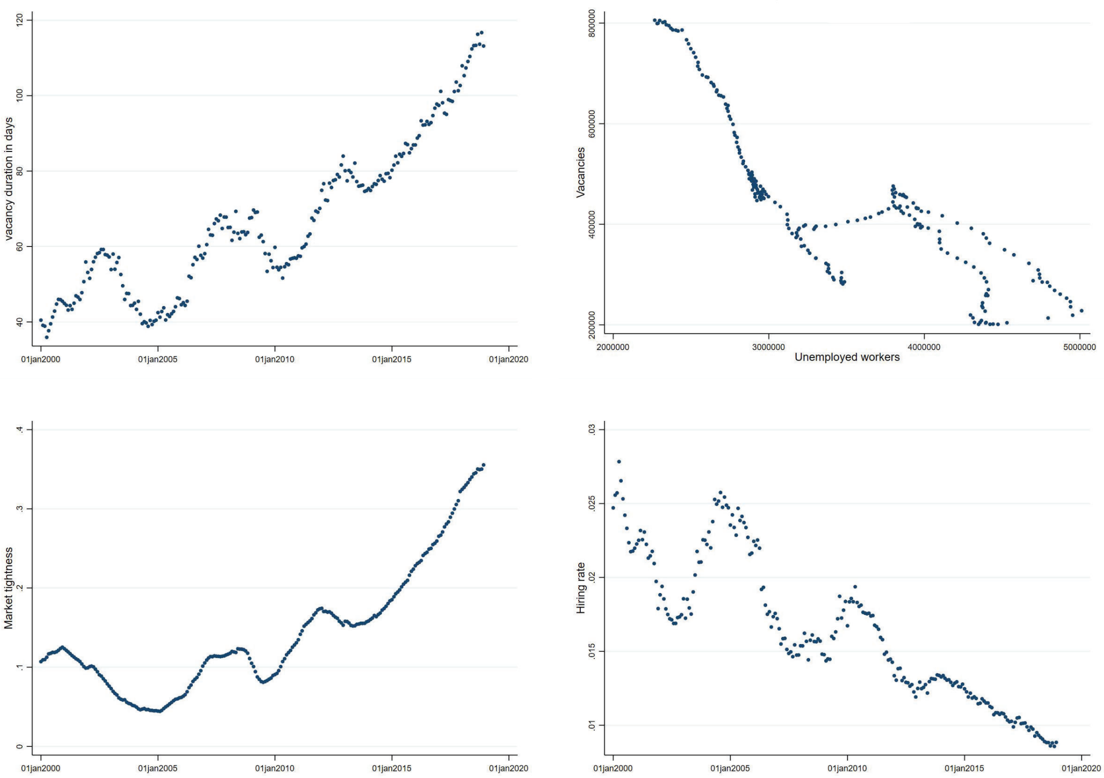
[9] This means that 1 monetary unit in the model corresponds to 1086 EUR.

[10] www.arbeitsagentur.de

[11] The amount is slightly higher for families with children and the duration of benefits can be longer for older individuals above the age of 50, but we use the replacement rate of 60% as a benchmark value.

to 60% of the average net wage for a period of 4 quarters and the normalized income 0.368 thereafter. Using wages predicted by the Mincer earnings regression we find parameters z^{ij} and $\bar{z}^{ij}$ (8 values). Further, we take $h = 0.3$ to be the disutility from unemployment (e.g. stigma of failure).

Figure 6: Vacancy duration and market tightness.



Note: Authors' calculations using statistical information of the Federal Employment Office (Bundesagentur für Arbeit) between January 2000 and December 2018.

Combining predicted earnings with the Nash bargaining equations for wages allows us to indirectly infer the productivities of employees in paid employment $\{\bar{y}, y\}^{ij}$. Dividing these values by the endogenous prices of intermediate goods P^j we find the units of output $\{\bar{\varphi}, \varphi\}^{ij}$, which we keep fixed throughout the paper. Finally, combining information on wages of workers in small firms and the corresponding profits of small businesses we obtain the bargaining power parameter β_0. More specifically, for low skill businesses we set the following expressions equal to the pre-tax profits predicted from a Mincer regression:

$$\underbrace{\sigma \int_{\alpha_0}^{1} \frac{f(\alpha)}{1 - F(\alpha_0)} d\alpha - c}_{\text{individual earnings}} + \frac{\gamma + \bar{q}(\theta_0)}{\gamma} \underbrace{\frac{(1-\beta_0)}{\beta_0(1-t)} \Big(\mu(w_0^I(1-t) - \bar{z}^I)) + (1-\mu)(w_0^N(1-t) - \bar{z}^N) \Big)}_{\text{average profit per co-worker}}$$

Note that $(\gamma + \bar{q}(\theta_0))/\gamma$ is the average number of co-workers conditional on hiring at least one. For high skill businesses we extend the expression to account for cross-skill matching. Based on these target values for profits we find $\beta_0^L = 0.456$ and $\beta_0^H = 0.422$. Both parameters are relatively close to 50% since bargaining between the self-employed entrepreneur and a worker in a small business is likely to be individual, whereas workers in medium and large firms are likely to be covered by collective agreements. Combining these values of β_0 with the Nash bargaining equations and predicted wages of workers in small businesses we find the productivities $\{y_{0I}^{ij}, y_{0N}^{ij}, y_{CI}^{i}, y_{CN}^{i}\}$ and the corresponding output units $\{\varphi_{0I}^{ij}, \varphi_{0N}^{ij}, \varphi_{CI}^{i}, \varphi_{CN}^{i}\}$ $i = I, N$, $j = L, H$.

Table 11 in the appendix C contains a full list of the calibrated parameter values and targeted data moments. In order to check the plausibility of our parameter values we compare the non-target moments (the average size of micro-enterprises) generated by our model to those provided by the official statistics.[12] The calibration exercise shows that high skill businesses are larger with 5.1-5.4 employees on average, whereas the size of low skill businesses is smaller with 3.1-3.7 employees on average. In particular, micro-enterprises are defined as businesses with less than 9 coworkers and their average size equals 2.7 over the period 2008-2017. For the purpose of comparison, we calculated conditional average numbers of employees (less or equal to 9) in our setting. They are equal to $2.4-2.7$ for low skill businesses and $3.1-3.2$ for high skill businesses. We did not target these numbers when setting parameters, yet they are very close to those reported in the official statistics, which indirectly validates our parameter values.

6 Results

In this section we analyze the response of the labour market to an exogenous immigration shock, similar to the one observed in Germany in the years 2012-2017. Aggregate statistics (see figure 5 and the underlying data link) reveal that the registered increase in the number of immigrant individuals over this period was approx. 3 million, whereas the starting stock of individuals with immigration background in 2012 was 15 million. This corresponds to the additional immigration inflow of 20%. Whereas it is clear that immigration to Germany in the considered period was low skill biased, the exact skill shares of the incoming immigrant workers can not be precisely measured. The reason is the skill degradation upon arrival (Dustmann et al. (2013)), as many high skilled workers take low skill positions, at least in the short and medium run. Hence, we study the implications of a 20% increase in immigration based on two scenarios: scenario (1) corresponds to low skill immigration, whereas scenario (2) corresponds to skill-neutral immigration.

Scenario (1): Low skill immigration

In this experiment we study consequences of a 20% increase in the immigrant labour force, such that all new immigrant workers are considered low skilled. Thus, a 20% increase in immigration corresponds to a combination of a 24% increase in the number

[12] www.destatis.de

of low skill immigrants and no change in the stock of high skill immigrants. This change is equivalent to an increase in the share of low skill population from 64.6% to 65.9%. Table 2 summarizes our findings and presents %-changes in welfare for all worker groups compared to the benchmark equilibrium in row (0). At the same time, table 3 contains the key labour market indicators, such as aggregate outputs Y_j, prices P^j, market tightness θ^j and θ_0^j, the capital cost rK, and the lump-sum transfer from the budget T. We decompose the total effect into the following four steps:

- (A) In this step we keep the capital stock K, the lump-sum transfer T and the business entry thresholds α_0^{ij} for all potential entrepreneurs constant. Thus, the main effect of immigration at this stage is the change in the prices of intermediate goods P^j, $j = L, H$, which are directly linked to the productivities and wages of all workers. The market tightness variables θ^j and θ_0^j also respond endogenously.
- (B) In this step we allow for an endogenous adjustment of the capital stock K, but the lump-sum transfer T and the business entry thresholds α_0^{ij} for all potential entrepreneurs remain constant. The demand for capital is given by equation (19).
- (C) In this step we update the lump-sum transfer T, but the business entry thresholds α_0^{ij} are still constant for all potential entrepreneurs. The new value of T follows from the budget constraint (22).
- (D) In the final step we allow for the endogenous adjustment of the entry thresholds α_0^{ij} based on the business entry condition from proposition 1.

Table 2: Increase in immigration by 20%, low skill scenario

	Low skilled				High skilled			
	Regular workers		Pot. entrepreneurs		Regular workers		Pot. entrepreneurs	
	N $\bar{\Omega}^{NL}$	I $\bar{\Omega}^{IL}$	N Ω^{NL}	I Ω^{IL}	N $\bar{\Omega}^{NH}$	I $\bar{\Omega}^{IH}$	N Ω^{NH}	I Ω^{IH}
(0)	1.552	1.405	1.523	1.497	1.988	1.641	1.970	1.810
	Keeping K, T and α_0^{ij}							
(A)	-2.05%	-2.21%	-1.09%	-0.92%	+0.28%	+0.29%	+0.34%	+0.76%
	Keeping T and α_0^{ij} fixed							
(B)	-0.94%	-1.03%	-0.54%	-0.36%	+1.26%	+1.39%	+0.77%	+0.83%
	Keeping α_0^{ij} fixed							
(C)	-1.36%	-1.50%	-0.97%	-0.80%	+0.78%	+0.93%	+0.44%	+0.47%
	Full adjustment							
(D)	-1.27%	-1.37%	-2.04%	+3.15%	+1.00%	+1.08%	+0.34%	+3.10%

Table 3 (row-A) shows that low skill immigration is associated with a lower price of the low skill intermediate good P^L and a higher price of the high skill intermediate good

P^H. This is a classical effect of immigration reducing productivities and wages in the low skill group and raising productivities and wages of the high skill group of workers. Table 2 (row-A) shows that this price adjustment comes along with a lower welfare of low skill individuals Ω^{iL} and $\bar{\Omega}^{iL}$ and an increase in welfare of high skill individuals Ω^{iH} and Ω^{iH}, $i = I, N$. The losses in welfare of low skill entrepreneurs are smaller compared to regular workers because the probability of hiring employees $\bar{q}(\theta^L)$ is increasing, which moderates the negative effect of lower productivity on business profits. For the same reason, the gains of high skill entrepreneurs are increasing more compared to high skill regular workers. In step (B) we allow for the endogenous adjustment of capital K. Firms in the final goods production borrow more capital K, which has a positive effect on the productivity and welfare of all worker groups via changes in prices and market tightness. The low skill workers still lose welfare but the losses are smaller compared to the ones in step (A). In step (C) we update the lump-sum budget transfer T and welfare values, but there are no changes in other variables at this step. This transfer is measured per person and is lower in the post-immigration equilibrium since the contribution of an average immigrant worker is lower compared to an average native worker. A lower lump-sum transfer reduces welfare of all worker groups.

Table 3: Detailed changes of variables upon a 20% increase in low skill immigration.

	Low skilled				High skilled				Aggregate	
	P^L	Y_L	θ_0^L	θ^L	P^H	Y_H	θ_0^H	θ^H	rK	T
(0)	5.221	0.459	0.443	0.103	5.303	0.287	1.356	0.626	0.879	0.493
(A)	5.171	0.471	0.417	0.091	5.307	0.288	1.280	0.632	0.879	0.493
(B)	5.199	0.476	0.426	0.096	5.340	0.291	1.312	0.673	0.903	0.493
(C)	–	–	–	–	–	–	–	–	–	0.486
(D)	5.198	0.476	0.427	0.096	5.341	0.291	1.316	0.675	0.904	0.487

Note: Row (C) only includes an adjustment of the lump-sum transfer T.

In the final step (D) we allow for the endogenous adjustment of the entrepreneurial business entry cutoffs α_0^{ij}. This is the novel part of our model, so we zoom in on the process of job creation and the business entry decisions of potential entrepreneurs in table (4). Increasing numbers of low skill immigrants imply that there are more applicants for low skill positions posted by regular firms making it easier for firms to hire workers. This effect boosts job creation in the low skill submarket, thus, regular firms respond by creating more vacancies. This is a positive job creation effect that happens despite a drop in the marginal productivity of low skill workers, reflected in a lower price of one unit of the low skill intermediate good P^L (3, row-D). Yet, a larger number of applicants for positions in regular jobs creates competition among workers, which is not compensated by the increase in vacancies. As a consequence of this process the market tightness θ_L

and the job-finding rate in the low skill submarket fall making it more difficult to find a job. This leads to lower employment rates in regular jobs for all low skill individuals and fosters the reallocation of native potential entrepreneurs from regular paid jobs to solo self-employment. Table 4 column (2) shows that the fraction of native potential entrepreneurs employed in regular jobs falls by 5.7% from a starting value of 41.8%, also their wages fall substantially in column (3). At the same time, their fraction in solo self-employment (column (4)) is increasing by 8.9% from a starting value of 25.9%.

Table 4: Detailed changes for pot. entrepreneurs upon 20% increase in low skill immigration

Group	(1) u/l	(2) e/l	(3) w	(4) $(s_u + s_e)/l$	(5) $\sigma\bar{\alpha}_{u0}$	(6) b/l	(7) $\sigma\bar{\alpha}_{01}$	(8) $\bar{q}(\theta_0)\pi/\gamma$
Low skill								
Natives	0.035	**0.418**	1.222	**0.259**	1.145	**0.288**	1.291	1.971
Change	+0.001	**-0.057**	-0.019	**+0.089**	+0.019	**-0.032**	-0.004	-0.001
Low skill								
Immigr.	0.049	**0.426**	1.245	**0.251**	1.100	**0.274**	1.314	2.077
Change	+0.001	**-0.054**	-0.019	**-0.077**	-0.102	**+0.130**	-0.019	+0.01
High Skill								
Natives	0.014	**0.418**	1.884	**0.290**	1.883	**0.277**	1.953	3.763
Change	-0.001	**+0.072**	+0.030	**-0.070**	+0.010	**-0.001**	+0.014	-0.000
High Skill								
Immigr.	0.039	**0.384**	1.309	**0.206**	1.199	**0.370**	1.374	5.270
Change	-0.001	**+0.056**	+0.026	**-0.093**	-0.092	**+0.037**	+0.006	+0.004

Note: $\bar{\alpha}_{u0}$ is the average entrepreneurial ability in the range $[\alpha_u..\alpha_0]$, calculated as $(s_u\bar{\alpha}_{us} + s_e\bar{\alpha}_{s0})/(s_u + s_e)$. $\bar{\alpha}_{01}$ is the average entrepreneurial ability in the range $[\alpha_0..\bar{\alpha}]$.

The positive immigration inflow also has implications for small businesses. On the one hand, hiring workers for open positions becomes easier, but on the other hand, a drop in the price of the low skill intermediate good makes workers in small businesses less productive. Table 4 shows that the first effect is dominating for businesses operated by immigrant entrepreneurs. Their profits in column (8) increase making such businesses more attractive for their owners. As a response we observe a 13% increase in the fraction of immigrant potential entrepreneurs operating small businesses in column (6), inducing a reallocation of immigrant potential entrepreneurs away from regular jobs and solo self-employment towards operating small businesses. These effects are associated with a sizable increase in welfare of immigrant potential entrepreneurs Ω^{IL} in table 2.

Considering the situation of small businesses operated by native entrepreneurs, we find that the drop in the productivity is dominating and leads to a slight decrease in profits in column (8). Thus, operating small businesses becomes less attractive for native entrepreneurs and their fraction in column (6) is falling by 3.2% inducing a reallocation of native potential entrepreneurs away from regular employment and businesses towards solo self-employment. These effects are associated with a decrease in welfare of native potential entrepreneurs Ω^{NL}. The asymmetric response of small businesses towards the immigration shock can be explained by the fact that immigrant businesses are more profitable on average, meaning that the expected profits per recruitment are also higher. This makes immigrant businesses more sensitive to the job-filling rate, that is the probability of hiring new coworkers. The profits of native entrepreneurs are lower (inline with the empirical data), making them less sensitive and less responsive to changes in the probability of filling positions.

Detailed wage and employment changes for regular workers are presented in table 10 in the appendix C. Wages of workers employed in regular low skill jobs fall (column (3)), moreover, finding jobs becomes more difficult since both market tightness variables θ_0^L and θ^L are falling due to the stronger competition among workers. This leads to slightly higher unemployment rates in column (1) of table 10 and a moderate reduction in welfare $\bar{\Omega}^{NL}$ and $\bar{\Omega}^{IL}$.

Next we consider, the high skill submarket. High skill workers become more productive, which is reflected in the higher price of the high skill intermediate good P_H. This boosts job creation in the high skill submarket leading to a higher market tightness θ_H and a higher job-finding rate. Thus, there is a sizable increase in the fraction of high skill potential entrepreneurs employed in regular paid jobs in column (2): 7.2% for natives and 5.6% for immigrant potential entrepreneurs. This process is accompanied by increasing wages in column (3) of table 4 and leads to the increased welfare of regular high skill workers $\bar{\Omega}^{IH}$ and $\bar{\Omega}^{NH}$ in table 2.

Higher productivity of coworkers is also beneficial for high skill small businesses, but the productivity of low skill coworkers employed in high skill businesses is decreasing. We find that the second effect is dominating, so the average productivity of coworkers in small businesses operated by high skill entrepreneurs decreases moderately. But at the same time, filling positions becomes easier, especially low skill positions, thus the market tightness θ_0^H is decreasing, while the job-filling rate is increasing. Again, we find that immigrant and native entrepreneurs operating businesses are asymmetrically affected. Column (8) of table 4 shows that the positive effect of a higher job-filling rate is dominating for immigrant entrepreneurs, and there is a small increase in their profits. This makes immigrant businesses more attractive for their owners and induces the reallocation of immigrant entrepreneurs from solo self-employment to operating small businesses equal to 3.7%. Overall, we observe a sizable increase in welfare of high skill immigrant entrepreneurs Ω^{IH} in table 2. It is partially due to higher profits from operating small businesses, but also due to the reallocation away from low income activities in solo self-employment towards regular jobs. More specifically, table 4 shows that the income of immigrant self-employed (1.199) is well below their wages in regular paid jobs (1.309).

For native high skill entrepreneurs the situation is different. The positive effect of a higher job-filling rate is neutralized by the lower productivity of an average coworker. Thus, the profits and the fraction of native high skill business owners is hardly changing and the only important change is a shift towards regular jobs. Yet, the difference in income between these two states is negligibly small, compare columns (5) and (3) in table 4, so there is only a small increase in the welfare of native entrepreneurs Ω^{NH} in table 2.

Our results so far suggest a moderate decrease in the welfare of incumbent workers equal to -0.37%. This number is calculated as an average welfare change in row (D) of table 2. However, we find that the average estimate hides substantial heterogeneity of the effect across worker groups. Contrary to the previous studies reporting wage and welfare losses for the incumbent immigrant population (D'Amuri et al. (2010), Ottaviano and Peri (2012)), we find that immigrant entrepreneurs experience sizable welfare gains from a new immigration wave equal to 3.1%.

Scenario (2): Skill-neutral immigration

In this part of the paper we consider the implications of skill-neutral immigration. That is we increase proportionally the number of low- and high-skill immigrants by 20%. Our results are summarized in table 5

Table 5: Increase in immigration by 20%, skill-neutral scenario

Low skilled				High skilled			
Regular workers		Pot. entrepreneurs		Regular workers		Pot. entrepreneurs	
N	I	N	I	N	I	N	I
$\bar{\Omega}^{NL}$	$\bar{\Omega}^{IL}$	Ω^{NL}	Ω^{IL}	$\bar{\Omega}^{NH}$	$\bar{\Omega}^{IH}$	Ω^{NH}	Ω^{IH}
1.552	1.405	1.523	1.497	1.988	1.641	1.970	1.810
-0.77%	-0.83%	-1.56%	+2.95%	+0.52%	+0.53%	-0.08%	+4.24%

If the numbers of immigrant workers increase proportionally, the changes in wages and productivities of regular workers are less pronounced compared to scenario (1). This holds for the losses of regular low skill workers (approx. -0.8% instead of -1.3%) and for the gains of the regular high skill workers (approx. $+0.5\%$ instead of $+1\%$). Considering the group of potential entrepreneurs, we find that the changes for low skill entrepreneurs are not substantially different from those under scenario (1), but the effect is much more pronounced for the high skill entrepreneurs. This is intuitive since hiring high skill co-workers becomes easier in the case of skill-neutral immigration as some of the newly arriving immigrant workers confirm their high skill degrees. This is particularly important for the hiring chances and profits of the high skill immigrant entrepreneurs and boosts their welfare gains up to $+4.2\%$ under scenario (2).

Overall, our findings in table 5 indicate that welfare gains of immigrant potential entrepreneurs are robust to varying the intensity of low skill immigration from fully low skilled (scenario (1)) to proportional changes (scenario (2)).

6.1 Value of immigrant entrepreneurship

In this section we conduct a counterfactual experiment that mimics the situation of legal barriers to self-employment and entrepreneurship for immigrant individuals. This experiment sheds light on the consequences of policies restricting immigrant self-employment and entrepreneurship. The experiment is conducted by setting $\delta^{IL} = 0$ and $\delta^{IH} = 0$. This corresponds to a situation when the legal entry barrier into self-employment and business ownership is infinitely high for immigrant potential entrepreneurs, so that none of them are observed in the states s^{Ij} and b^{Ij}, $j = L, H$ in the counterfactual equilibrium. The goal of this experiment is twofold. On the one hand, it allows us to estimate the option value of entrepreneurship for immigrant individuals. On the other hand, we can estimate the spillover effects of immigrant entrepreneurship for all other groups producing insights about the role of entry barriers for immigrant small businesses. Our modeling framework is well suited for this analysis and the estimates are novel in the literature. The results are summarized in table 6, where line (0) corresponds to the benchmark equilibrium and line (E) contains welfare changes for all worker groups in the counterfactual setting.

Table 6: Welfare changes in the counterfactual scenario without immigrant entrepreneurship

	Low skilled				High skilled			
	Regular workers		Pot. entrepreneurs		Regular workers		Pot. entrepreneurs	
	N	I	N	I	N	I	N	I
	$\bar{\Omega}^{NL}$	$\bar{\Omega}^{IL}$	Ω^{NL}	Ω^{IL}	$\bar{\Omega}^{NH}$	$\bar{\Omega}^{IH}$	Ω^{NH}	Ω^{IH}
(0)	1.552	1.405	1.523	1.497	1.988	1.641	1.970	1.810
(E)	-0.35%	-0.43%	+1.62%	-15.2%	-0.17%	-0.18%	+1.08%	-30.5%

Table 6 shows that the welfare losses of high skill potential entrepreneurs are dramatic and amount to 30.5% of their welfare. This is primarily driven by higher unemployment increasing from 3.9% to 11%. Also the losses of low skill potential entrepreneurs are large reaching a level of 15.2% and caused by the corresponding increase in unemployment from 4.9% to 18%. These findings indicate a very high value of the self-employment option for immigrant individuals with high entrepreneurial spirit. They also suggest that self-employment is an efficient way of reducing unemployment for immigrants. Further, we observe moderate negative spillovers for regular workers with a reduction in welfare. In contrast, the externality for native potential entrepreneurs is positive, meaning that native entrepreneurs gain welfare due to the reduced competition in recruitment if there are policies in place preventing immigrant entrepreneurship.

Overall, our results indicate that policies reducing entry barriers for immigrant entrepreneurs are likely to be beneficial for regular workers (native and immigrant). These worker groups experience moderate welfare gains equal to $0.3-0.4\%$ for the low skilled and

0.2% for the high skilled. The gains are twofold: first, immigrant potential entrepreneurs entering self-employment reduce competition for regular jobs, second, immigrant small businesses create new working places for all worker groups. Yet, native potential entrepreneurs face stronger competition in the presence of immigrant businesses generating moderate welfare losses for this group equal to $1.1 - 1.6\%$. Germany introduced several amendments in its residence act for non-German entrepreneurs between 2004 to 2012. These amendments relaxed the conditions required for entrepreneurial ventures and offered incentives to foreigners to invest in Germany.[13] Our experiment is helpful in evaluating the effectiveness of these policies.

7 Conclusions

In this paper, we develop a unified framework that incorporates the dual role of immigrants as workers and entrepreneurs. As workers, migrants compete with native workers for jobs in the regular job market. In contrast, as entrepreneurs, they compete with native entrepreneurs but simultaneously create employment opportunities for other natives and migrants looking for jobs. The theoretical setup combined with survey data from the SOEP allows us to quantify the impact of low-skill immigration on small businesses, unemployment rates, wage structure, and welfare of native and incumbent immigrants in Germany.

Consistent with the literature, we find that the low-skilled migration similar in intensity to the refugee wave 2012-2017 increased unemployment and reduced wages for all low-skilled workers, negatively affecting their welfare. Similarly, the high-skilled workers gained from such an inflow. Nonetheless, we extend the literature by providing a more detailed picture of the effects of migration. Contrary to the existing literature, suggesting a negative impact of migration on incumbent migrants, we find that incumbent immigrant entrepreneurs in both skill groups gain from immigration and expand their entrepreneurial activities. This amplifies the welfare gains of high skill regular workers and reduces the welfare losses of the low skilled. Yet, we document an adverse effect on native entrepreneurs, especially the low skilled, losing welfare from the lower productivity of their co-workers and facing stronger recruitment competition from immigrant small businesses.

We conduct a counterfactual experiment that restricts the entry of immigrants into entrepreneurship. This experiment has important implications for labour market policies in the context of migration. We conclude that policies restricting entrepreneurial activities of immigrants lead to an overall welfare loss for the economy. All demographic groups incur welfare losses except for the native entrepreneurs gaining from reduced competition

[13]Residence Act 2004, Act to Implement Residence- and Asylum-Related Directives of the European Union 2007, Labour Migration Control Act 2009, Residence Act 1st August 2012

with immigrant businesses.

References

Altonji, J. G. and D. Card (1991). *The Effects of Immigration on the Labor Market Outcomes of Less-skilled Natives*, pp. 201–234. University of Chicago Press.

Aydemir, A. and G. J. Borjas (2007). Cross-country variation in the impact of international migration: Canada, Mexico, and the United States. *Journal of the European Economic Association 5*(4), 663–708.

Aydemir, A. and G. J. Borjas (2011). Attenuation bias in measuring the wage impact of immigration. *Journal of Labor Economics 29*(1), 69–113.

Azoulay, P., B. F. Jones, J. D. Kim, and J. Miranda (2022). Immigration and entrepreneurship in the United States. *American Economic Review: Insights 4*(1), 71–88.

Basu, A. (1998). An exploration of entrepreneurial activity among Asian small businesses in Britain. *Small Business Economics 10*(4), 313–326.

Battisti, M., G. Felbermayr, G. Peri, and P. Poutvaara (2018). Immigration, search, and redistribution: A quantitative assessment of native welfare. *Journal of the European Economic Association 16*(4), 1137–1188.

Ben-Gad, M. (2004). The economic effects of immigration: A dynamic analysis. *Journal of Economic Dynamics and Control 28*(9), 1825–1845.

Ben-Gad, M. (2008). Capital–skill complementarity and the immigration surplus. *Review of Economic Dynamics 11*(2), 335–365.

Borjas, G. J. (1986). The self-employment experience of immigrants. *Journal of Human Resources 21*(4), 485–506.

Borjas, G. J. (1999). The economic analysis of immigration. In O. Ashenfelter and D. Card (Eds.), *Handbook of Labor Economics* (1 ed.), Volume 3, Part A, Chapter 28, pp. 1697–1760. Elsevier.

Borjas, G. J. (2003, 11). The labor demand curve is downward sloping: Reexamining the impact of immigration on the labor market. *The Quarterly Journal of Economics 118*(4), 1335–1374.

Borjas, G. J. (2006). Native internal migration and the labor market impact of immigration. *Journal of Human Resources 41*(2).

Borjas, G. J., R. B. Freeman, and L. F. Katz (1991). On the labor market effects of immigration and trade. NBER Working Papers 3761, National Bureau of Economic Research, Inc.

Borjas, G. J., R. B. Freeman, L. F. Katz, J. DiNardo, and J. M. Abowd (1997). How much do immigration and trade affect labor market outcomes? *Brookings Papers on Economic Activity 1997*(1), 1–90.

Borjas, G. J. and J. Monras (2017). The labour market consequences of refugee supply shocks. *Economic Policy 32*(91), 361–413.

Borooah, V. and M. Hart (1999). Factors affecting self-employment among Indian and Black Caribbean men in Britain. *Small Business Economics 13*(2), 111–29.

Busch, C., D. Krueger, A. Ludwig, I. Popova, and Z. Iftikhar (2020). Should Germany have built a new wall? Macroeconomic lessons from the 2015-18 refugee wave. *Journal of Monetary Economics 113*(C), 28–55.

Card, D. (1990). The impact of the mariel boatlift on the Miami labor market. *Industrial and Labor Relations Review 43*(2), 245–257.

Card, D. (2001). Immigrant inflows, native outflows, and the local labor market impacts of higher immigration. *Journal of Labor Economics 19*(1), 22–64.

Card, D. (2005). Is the new immigration really so bad? *The Economic Journal 115*(507), 300–323.

Chassamboulli, A. and T. Palivos (2014). A search-equilibrium approach to the effects of immigration on labor market outcomes. *International Economic Review 55*(1), 111–129.

Clark, K. and S. Drinkwater (1998). Ethnicity and self-employment in Britain. *Oxford Bulletin of Economics and Statistics 60*(3), 383–407.

Constant, A. and M. Schultz-Nielsen (2004). *Immigrant Self-employment and Economic Performance*. University Press of Southern Denmark.

Constant, A., Y. Shachmurove, and K. Zimmermann (2005). The role of Turkish immigrants in entrepreneurial activities in Germany. PIER Working Paper Archive, Penn Institute for Economic Research, Department of Economics, University of Pennsylvania.

Constant, A. and K. Zimmermann (2006). The making of entrepreneurs in Germany: Are native men and immigrants alike? *Small Business Economics 26*(3), 279–300.

D'Amuri, F., G. Ottaviano, and G. Peri (2010). The labor market impact of immigration in Western Germany in the 1990s. *European Economic Review 54*(4), 550–570.

Duleep, H., D. A. Jaeger, and P. McHenry (2021). On immigration and native entrepreneurship. Technical Report 14188, IZA Institute of Labor Economics.

Dustmann, C., T. Frattini, and I. Preston (2013). The effect of immigration along the distribution of wages. *Review of Economic Studies 80*(1), 145–173.

Dustmann, C., U. Schönberg, and J. Stuhler (2017). Labor supply shocks, native wages, and the adjustment of local employment. *The Quarterly Journal of Economics 132*(1), pp. 435–483.

Evans, D. S. and L. S. Leighton (1989). Some empirical aspects of entrepreneurship. *The American Economic Review 79*(3), 519–535.

Fairlie, R. W. and B. D. Meyer (1996). Ethnic and racial self-employment differences and possible explanations. *The Journal of Human Resources 31*(4), 757–793.

Felbermayr, G., W. Geis, and W. Kohler (2010). Restrictive immigration policy in Germany: Pains and gains foregone? *Review of World Economics 146*(1), 1–21.

Fonseca, R., P. Lopez-Garcia, and A. Pissarides (2001). Entrepreneurship, start-up costs and employment. *European Economic Review 45*, 692–705.

Friedberg, R. M. (2001). The impact of mass migration on the Israeli labor market. *The Quarterly Journal of Economics 116*(4), 1373–1408.

Georgarakos, D. and K. Tatsiramos (2009). Entrepreneurship and survival dynamics of immigrants to the U.S. and their descendants. *Labour Economics 16*(2), 161–170.

Goldin, C. (1994). The political economy of immigration restriction in the United States, 1890 to 1921. In *The Regulated Economy: A Historical Approach to Political Economy*, NBER Chapters, pp. 223–258. National Bureau of Economic Research, Inc.

Grossman, J. B. (1982). The substitutability of natives and immigrants in production. *The Review of Economics and Statistics 64*(4), 596–603.

Hohn, M., L. Atkins, and M. Waslin (2012). Immigrant entrepreneurs: Creating jobs and strengthening the economy. Technical report, Immigration Policy Center of the American Immigration Council.

Hunt, J. (1992). The impact of the 1962 repatriates from Algeria on the French labor market. *Industrial and Labor Relations Review 45*(3), 556–572.

Iftikhar, Z. and A. Zaharieva (2019). General equilibrium effects of immigration in Germany: Search and matching approach. *Review of Economic Dynamics 31*, 245–276.

Kerr, S. P. and W. Kerr (2020). Immigrant entrepreneurship in America: Evidence from the survey of business owners 2007 2012. *Research Policy 49*(3), 103918.

Kredler, M., A. B. Millan, and L. Visschers (2014). Great opportunities or poor alternatives: self-employment, unemployment and paid employment over the business cycle. Meeting Paper 597, Society for Economic Dynamics.

LaLonde, R. and R. Topel (1991). Labor market adjustments to increased immigration. In *Immigration, Trade, and the Labor Market*, pp. 167–199. National Bureau of Economic Research, Inc.

Leicht, R. and M. Langhauser (2014). Ökonomische bedeutung und leistungspotenziale von migrantenunternehmen in deutschland. Technical report, Friedrich-Ebert-Stiftung, Abteilung Wirtschafts- und Sozialpolitik. https://www.fachportal-paedagogik.de/literatur/vollanzeige.html?FId=3210227.

Li, P. S. (2001). Immigrants' propensity to self-employment: Evidence from Canada. *International Migration Review 35*(4), 1106–1128.

Liu, X. (2010). On the macroeconomic and welfare effects of illegal immigration. *Journal of Economic Dynamics and Control 34*(12), 2547–2567.

Lofstrom, M. (2002). Labor market assimilation and the self-employment decision of immigrant entrepreneurs. *Journal of Population Economics 15*(1), 83–114.

Malchow-Møller, N., J. Munch, and J. Skaksen (2009). Do immigrants take the jobs of native workers? *SSRN Electronic Journal*.

Masters, A. (2016). Job creators, job creation, and the tax code. *Journal of Public Economic Theory 19*, 674–691.

Moreno-Galbis, E. and A. Tritah (2016). The effects of immigration in frictional labor markets: Theory and empirical evidence from eu countries. *European Economic Review 84*, 76–98.

Mortensen, D. and A. Pissarides (1994). Job creation and job destruction in the theory of unemployment. *Review of Economic Studies 61*, 397–415.

Nanos, P. and C. Schluter (2014). The composition of wage differentials between migrants and natives. *European Economic Review 65*, 23–44.

OECD (2010). Entrepreneurship and migrants. OECD Working Papers, OECD Working Party on SMEs and Entrepreneurship.

Ortega, J. (2000). Pareto-improving immigration in an economy with equilibrium unemployment. *The Economic Journal 110*(460), 92–112.

Ottaviano, G. and G. Peri (2012). Rethinking the effect of immigration on wages. *Journal of the European Economic Association 10*(1), 152–197.

Pischke, J.-S. and J. Velling (1997). Employment effects of immigration to Germany: An analysis based on local labor markets. *The Review of Economics and Statistics 79*(4), 594–604.

Pissarides, A. (2000). *Equilibrium Unemployment History.* The MIT Press.

Riillo, C. F. A. and C. Peroni (2022). Immigration and entrepreneurship in Europe: Cross-country evidence. MPRA Paper 114580, University Library of Munich, Germany.

Rissman, E. (2007). Labor market transitions and self-employment. Working Paper 2007-14, Federal Reserve Bank of Chicago.

Sachs, A., M. Hoch, C. Münch, and H. Steidle (2016). Migrant entrepreneurs in Germany from 2005 to 2014. Technical report. `http://aei.pitt.edu/102515/`.

Scharfbillig, M. and M. Weissler (2019). Heterogeneous displacement effects of migrant labor supply: Quasi-experimental evidence from Germany. *SSRN Electronic Journal*.

Wadhwa, V. (2007). America's new immigrant entrepreneurs. Technical report, UC Berkeley, School of Information.

Xavier, S., D. Kelley, J. Kew, M. Herrington, and A. Vorderwulbecke (2013). Global Entrepreneurship Monitor global report. Technical report, GEM consortium.

Yuengert, A. M. (1995). Testing hypotheses of immigrant self-employment. *Journal of Human Resources 30*(1), 194–204.

Appendices

A Appendix

$$(r+\gamma)\Delta_k^I = (y_0^I - w_0^I)(1-\tau) + \bar{q}(\theta_0)\mu\Delta_{k+1}^I - \bar{q}(\theta_0)\mu\Delta_k^I + \bar{q}(\theta_0)(1-\mu)\Delta_k^I - \bar{q}(\theta_0)(1-\mu)\Delta_k^I$$

In a similar way, let $\Delta_m^N = B_{tm}(\alpha) - B_{tm-1}(\alpha)$, $\forall t = 0..\infty$ $\alpha > \alpha_0$ then we get:

$$(r+\gamma)\Delta_m^I = (y_0^N - w_0^N)(1-\tau) + \bar{q}(\theta_0)\mu\Delta_m^I - \bar{q}(\theta_0)\mu\Delta_m^I + \bar{q}(\theta_0)(1-\mu)\Delta_{m+1}^I - \bar{q}(\theta_0)(1-\mu)\Delta_m^I$$

Considering a stationary case $\Delta^I = \Delta_{k-1}^I = \Delta_k^I = \Delta_{k+1}^I$ and $\Delta^N = \Delta_{m-1}^I = \Delta_m^I = \Delta_{m+1}^I$:

Proof of proposition 1:

Consider the first integral in (5), using integration by parts it can be rewritten as:

$$\begin{aligned}
& \int_\alpha^{\alpha_0} (E(x) - E(\alpha))dF(x) = (E(x) - E(\alpha))F(x)\Big|_\alpha^{\alpha_0} - \int_\alpha^{\alpha_0} F(x)E'(x)dx \\
= & \ (E(\alpha_0) - E(\alpha))F(\alpha_0) - \int_\alpha^{\alpha_0} F(x)E'(x)dx \\
= & \ (E(\alpha_0) - E(\alpha)) + (E(\alpha_0) - E(\alpha)) + (E(\alpha_0) - E(\alpha))F(\alpha_0) - \int_\alpha^{\alpha_0} F(x)E'(x)dx \\
= & \int_\alpha^{\alpha_0} E'(x)dx - (1 - F(\alpha_0))(E(\alpha_0) - E(\alpha)) - \int_\alpha^{\alpha_0} F(x)E'(x)dx \\
= & \int_\alpha^{\alpha_0} (1 - F(x))E'(x)dx - (1 - F(\alpha_0))(E(\alpha_0) - E(\alpha))
\end{aligned}$$

Consider the second integral in (5), using integration by parts it can be rewritten as:

$$\begin{aligned}
& \int_{\alpha_0}^{\bar{\alpha}} (B_0(x) - E(\alpha))dF(x) = (B_0(x) - E(\alpha))F(x)\Big|_{\alpha_0}^{\bar{\alpha}} - \int_{\alpha_0}^{\bar{\alpha}} F(x)B_0'(x)dx \\
= & \ (B_0(\alpha_0) - E(\alpha)) - (B_0(\alpha_0) - E(\alpha))F(\alpha_0) - \int_{\alpha_0}^{\bar{\alpha}} F(x)B_0'(x)dx \\
= & \ (B_0(\alpha_0) - B_0(\alpha_0)) + (B_0(\alpha_0) - E(\alpha)) - (B_0(\alpha_0) - E(\alpha))F(\alpha_0) - \int_{\alpha_0}^{\bar{\alpha}} F(x)B_0'(x)dx
\end{aligned}$$

$$\begin{aligned}
= & \int_{\alpha_0}^{\bar{\alpha}} B_0'(x)dx + (1 - F(\alpha_0))(B_0(\alpha_0) - E(\alpha)) - \int_{\alpha_0}^{\bar{\alpha}} F(x)B_0'(x)dx \\
= & \int_{\alpha_0}^{\bar{\alpha}} (1 - F(x))B_0'(x)dx + (1 - F(\alpha_0))(B_0(\alpha_0) - E(\alpha))
\end{aligned}$$

Inserting both expressions back into (5) and taking into account that $E(\alpha_0) = B_0(\alpha_0)$ we get equation 7. □

Proof of proposition 2 Proof: Using integration by parts to rewrite the first integral in equation (4) we get:

$$\int_\alpha^{\alpha_s} (E_s(x) - E_s(\alpha))dF(x) = \int_\alpha^{\alpha_s} (1 - F(x))E_s'(x)dx - (1 - F(\alpha_s))(E_s(\alpha_s) - E_s(\alpha))$$

Next we rewrite the second integral in equation (4):

$$
\begin{aligned}
& \int_{\alpha_s}^{\alpha_0} (E(x) - E_s(\alpha)) dF(x) = (E(x) - E_s(\alpha)) F(x) \Big|_{\alpha_s}^{\alpha_0} - \int_{\alpha_s}^{\alpha_0} F(x) E'(x) dx \\
= \; & (E(\alpha_0) - E_s(\alpha)) F(\alpha_0) - (E(\alpha_s) - E_s(\alpha)) F(\alpha_s) - \int_{\alpha_s}^{\alpha_0} F(x) E'(x) dx \\
= \; & E(\alpha_0) F(\alpha_0) - E(\alpha_s) F(\alpha_s) + E_s(\alpha)(F(\alpha_s) - F(\alpha_0)) - \int_{\alpha_s}^{\alpha_0} F(x) E'(x) dx \\
= \; & (E(\alpha_0) - E(\alpha_s)) - E(\alpha_0)(1 - F(\alpha_0)) + E(\alpha_s)(1 - F(\alpha_s)) \\
+ \; & E_s(\alpha)(F(\alpha_s) - F(\alpha_0)) - \int_{\alpha_s}^{\alpha_0} F(x) E'(x) dx \\
= \; & \int_{\alpha_s}^{\alpha_0} (1 - F(x)) E'(x) dx - E(\alpha_0)(1 - F(\alpha_0)) + E(\alpha_s)(1 - F(\alpha_s)) + E_s(\alpha)(F(\alpha_s) - F(\alpha_0))
\end{aligned}
$$

Using integration by parts to rewrite the first integral in equation (4) we get:

$$
\int_{\alpha_0}^{\bar{\alpha}} (B_0(x) - E_s(\alpha)) dF(x) = \int_{\alpha_0}^{\bar{\alpha}} (1 - F(x)) B_0'(x) dx + (1 - F(\alpha_0))(B_0(\alpha_0) - E_s(\alpha))
$$

Inserting all three expressions back into (4) and taking into account that $E_s(\alpha_s) = E(\alpha_s)$ and $B_0(\alpha_0) = E(\alpha_0)$ we get equation **??**. Next we consider the first integral in equation (8):

$$
\begin{aligned}
& \int_{\alpha_s}^{\alpha_0} (E(x) - W) dF(x) \\
= \; & \int_{\alpha_s}^{\alpha_0} (1 - F(x)) E'(x) dx - E(\alpha_0)(1 - F(\alpha_0)) + E(\alpha_s)(1 - F(\alpha_s)) + W(F(\alpha_s) - F(\alpha_0))
\end{aligned}
$$

Considering the second integral in equation (8) we get:

$$
\int_{\alpha_0}^{\bar{\alpha}} (B_0(x) - W) dF(x) = \int_{\alpha_0}^{\bar{\alpha}} (1 - F(x)) B_0'(x) dx + (1 - F(\alpha_0))(B_0(\alpha_0) - W)
$$

Inserting both expressions back into (8) and taking into account that $E_s(\alpha_s) = E(\alpha_s) = W$ and $B_0(\alpha_0) = E(\alpha_0)$ we get equation 9. Next, we show that α_s is increasing in the wage w by differentiating equation 9 and dividing both parts by $(1 - t)$:

$$
\sigma(r + \bar{\gamma} + X\lambda(\theta)) \frac{\partial \alpha_s}{\partial w} = r + \gamma + X\lambda(\theta) + (\bar{\gamma} - \gamma)\delta \frac{\sigma(1 - F(\alpha_s))}{r + \gamma + X\lambda(\theta) + \delta(1 - F(\alpha_s))} \frac{\partial \alpha_s}{\partial w}
$$

$$
\sigma \frac{(r + \gamma + X\lambda(\theta))(r + \bar{\gamma} + X\lambda(\theta) + \delta(1 - F(\alpha_s)))}{(r + \gamma + X\lambda(\theta) + \delta(1 - F(\alpha_s))} \frac{\partial \alpha_s}{\partial w} = r + \gamma + X\lambda(\theta)
$$

$$\frac{\partial \alpha_s}{\partial w} = \frac{(r+\gamma+X\lambda(\theta)+\delta(1-F(\alpha_s)))}{\sigma(r+\bar{\gamma}+X\lambda(\theta)+\delta(1-F(\alpha_s)))}$$

□

Proof of proposition 3: Using integration by parts we can rewrite the first integral in equation (3):

$$\begin{aligned}
& \int_{\alpha_u}^{\alpha_s} (E_s(x)-U)dF(x) \\
= & \int_{\alpha_u}^{\alpha_s} (1-F(x))E_s'(x)dx - E_s(\alpha_s)(1-F(\alpha_s)) + E_s(\alpha_u)(1-F(\alpha_u)) + U(F(\alpha_u)-F(\alpha_s))
\end{aligned}$$

Next we can rewrite the second integral in equation (3):

$$\begin{aligned}
& \int_{\alpha_s}^{\alpha_0} (E(x)-U)dF(x) \\
= & \int_{\alpha_s}^{\alpha_0} (1-F(x))E'(x)dx - E(\alpha_0)(1-F(\alpha_0)) + E(\alpha_s)(1-F(\alpha_s)) + U(F(\alpha_s)-F(\alpha_0))
\end{aligned}$$

and the third integral in equation (3):

$$\begin{aligned}
& \int_{\alpha_0}^{\bar{\alpha}} (B_0(x)-U)dF(x) \\
= & \int_{\alpha_0}^{\bar{\alpha}} (1-F(x))B_0'(x)dx + B_0(\alpha_0)(1-F(\alpha_0)) + U(F(\alpha_0)-1)
\end{aligned}$$

Inserting both expressions back into (3) we get equation (11). □

Proof of proposition 5: In order to reduce the notation we suppress the dependence of all b variables on α and use $\bar{q}$ for $\bar{q}(\theta_0)$. Consider the steady state equations for $b_{01}..b_{04}$:

$$\begin{aligned}
\dot{b}_{01} &= \bar{q}(1-\mu)b_{00} - (\gamma+\bar{q})b_{01} \qquad & \dot{b}_{02} = \bar{q}(1-\mu)b_{01} - (\gamma+\bar{q})b_{02} \\
\dot{b}_{03} &= \bar{q}(1-\mu)b_{02} - (\gamma+\bar{q})b_{03} \qquad & \dot{b}_{04} = \bar{q}(1-\mu)b_{03} - (\gamma+\bar{q})b_{04}
\end{aligned}$$

which means:

$$b_{01} = \frac{\bar{q}(1-\mu)}{\gamma+\bar{q}}b_{00} \qquad b_{02} = \frac{(\bar{q}(1-\mu))^2}{(\gamma+\bar{q})^2}b_{00} \qquad b_{03} = \frac{(\bar{q}(1-\mu))^3}{(\gamma+\bar{q})^3}b_{00} \qquad b_{04} = \frac{(\bar{q}(1-\mu))^4}{(\gamma+\bar{q})^4}b_{00}$$

$$b_{0m} = \frac{(\bar{q}(1-\mu))^m}{(\gamma+\bar{q})^m}b_{00} \qquad b_{k0} = \frac{(\bar{q}\mu)^k}{(\gamma+\bar{q})^k}b_{00}$$

Next consider the steady state equations for $b_{11}..b_{14}$:

$$\begin{aligned}\dot{b}_{11} &= \bar{q}\mu b_{01} + \bar{q}(1-\mu)b_{10} - (\gamma+\bar{q})b_{11} \quad \dot{b}_{12} = \bar{q}\mu b_{02} + \bar{q}(1-\mu)b_{11} - (\gamma+\bar{q})b_{12} \\ \dot{b}_{13} &= \bar{q}\mu b_{03} + \bar{q}(1-\mu)b_{12} - (\gamma+\bar{q})b_{13} \quad \dot{b}_{14} = \bar{q}\mu b_{04} + \bar{q}(1-\mu)b_{13} - (\gamma+\bar{q})b_{14}\end{aligned}$$

which means that:

$$b_{11} = 2\frac{\bar{q}\mu[\bar{q}(1-\mu)]}{(\gamma+\bar{q})^2}b_{00} \quad b_{12} = 3\frac{\bar{q}\mu[\bar{q}(1-\mu)]^2}{(\gamma+\bar{q})^3}b_{00} \quad b_{13} = 4\frac{\bar{q}\mu[\bar{q}(1-\mu)]^3}{(\gamma+\bar{q})^4}b_{00} \quad b_{14} = 5\frac{\bar{q}\mu[\bar{q}(1-\mu)]^4}{(\gamma+\bar{q})^5}b_{00}$$

$$b_{1m} = (m+1)\frac{\bar{q}\mu[\bar{q}(1-\mu)]^m}{(\gamma+\bar{q})^{m+1}}b_{00} \qquad b_{k1} = (k+1)\frac{\bar{q}(1-\mu)[\bar{q}\mu]^k}{(\gamma+\bar{q})^{k+1}}b_{00}$$

Next consider the steady state equations for $b_{21}..b_{24}$:

$$\begin{aligned}\dot{b}_{21} &= \bar{q}\mu b_{11} + \bar{q}(1-\mu)b_{20} - (\gamma+\bar{q})b_{21} \quad \dot{b}_{22} = \bar{q}\mu b_{12} + \bar{q}(1-\mu)b_{21} - (\gamma+\bar{q})b_{22} \\ \dot{b}_{23} &= \bar{q}\mu b_{13} + \bar{q}(1-\mu)b_{22} - (\gamma+\bar{q})b_{23} \quad \dot{b}_{24} = \bar{q}\mu b_{14} + \bar{q}(1-\mu)b_{23} - (\gamma+\bar{q})b_{24}\end{aligned}$$

which means that:

$$b_{22} = 6\frac{[\bar{q}\mu]^2[\bar{q}(1-\mu)]^2}{(\gamma+\bar{q})^4}b_{00} \quad b_{23} = 10\frac{[\bar{q}\mu]^2[\bar{q}(1-\mu)]^3}{(\gamma+\bar{q})^5}b_{00} \quad b_{24} = 15\frac{[\bar{q}\mu]^2[\bar{q}(1-\mu)]^4}{(\gamma+\bar{q})^6}b_{00}$$

$$b_{2m} = \frac{(2+m)!}{2!m!}\frac{[\bar{q}\mu]^2[\bar{q}(1-\mu)]^m}{(\gamma+\bar{q})^{2+m}}b_{00} \qquad b_{k2} = \frac{(k+2)!}{k!2!}\frac{[\bar{q}\mu]^k[\bar{q}(1-\mu)]^2}{(\gamma+\bar{q})^{k+2}}b_{00}$$

Continuing by induction we get:

$$b_{km} = \frac{(k+m)!}{k!m!}\frac{[\bar{q}\mu]^k[\bar{q}(1-\mu)]^m}{(\gamma+\bar{q})^{k+m}}b_{00}$$

Adding up all values for $k = 0..\infty$ and for $m = 0..\infty$ we get:

$$b = \sum_{k=0}^{\infty}\sum_{m=0}^{\infty} b_{km} = \frac{\gamma+\bar{q}}{\gamma}b_{00} \quad \rightarrow \quad p_{km} = \frac{b_{km}}{b} = \frac{\gamma}{\gamma+\bar{q}}\frac{(k+m)!}{k!m!}\frac{(\bar{q}(1-\mu))^m(\bar{q}\mu)^k}{(\gamma+\bar{q})^{m+k}}$$

Recovering detailed notation we get equation 12. Next we derive the two conditional distributions:

$$\begin{aligned}
p_k^I &= \sum_{m=0}^{\infty} p_{km} = \frac{\gamma}{\gamma+\bar{q}} \sum_{m=0}^{\infty} \frac{(k+m)!}{k!m!} \Big(\frac{\bar{q}(1-\mu)}{\gamma+\bar{q}}\Big)^m \Big(\frac{\bar{q}\mu}{\gamma+\bar{q}}\Big)^k \cdot \Big(\frac{\gamma+\bar{q}\mu}{\gamma+\bar{q}}\Big)^{k+1} \Big(\frac{\gamma+\bar{q}}{\gamma+\bar{q}\mu}\Big)^{k+1} \\
&= \Big(\frac{\bar{q}\mu}{\gamma+\bar{q}\mu}\Big)^k \Big(\frac{\gamma}{\gamma+\bar{q}\mu}\Big) \underbrace{\sum_{m=0}^{\infty} \frac{(k+m)!}{k!m!} \Big(\frac{\bar{q}(1-\mu)}{\gamma+\bar{q}}\Big)^m \Big(\frac{\gamma+\bar{q}\mu}{\gamma+\bar{q}}\Big)^{k+1}}_{=1}
\end{aligned}$$

$$\begin{aligned}
p_{m|k} &= \frac{p_{km}}{\sum_{m=0}^{\infty} p_{km}} = \frac{\gamma}{\gamma+\bar{q}} \frac{(k+m)!}{k!m!} \frac{(\bar{q}(1-\mu))^m (\bar{q}\mu)^k}{(\gamma+\bar{q})^{m+k}} \Big(\frac{\bar{q}\mu}{\gamma+\bar{q}\mu}\Big)^{-k} \Big(\frac{\gamma}{\gamma+\bar{q}\mu}\Big)^{-1} \\
&= \frac{(k+m)!}{k!m!} \Big(\frac{\bar{q}(1-\mu)}{\gamma+\bar{q}}\Big)^m \Big(\frac{\gamma+\bar{q}\mu}{\gamma+\bar{q}}\Big)^{k+1}
\end{aligned}$$

$$\begin{aligned}
p_m^N &= \sum_{k=0}^{\infty} p_{km} = \frac{\gamma}{\gamma+\bar{q}} \sum_{k=0}^{\infty} \frac{(k+m)!}{k!m!} \Big(\frac{\bar{q}(1-\mu)}{\gamma+\bar{q}}\Big)^m \Big(\frac{\bar{q}\mu}{\gamma+\bar{q}}\Big)^k \cdot \Big(\frac{\gamma+\bar{q}(1-\mu)}{\gamma+\bar{q}}\Big)^{m+1} \Big(\frac{\gamma+\bar{q}}{\gamma+\bar{q}(1-\mu)}\Big)^{m+1} \\
&= \Big(\frac{\bar{q}(1-\mu)}{\gamma+\bar{q}(1-\mu)}\Big)^m \Big(\frac{\gamma}{\gamma+\bar{q}(1-\mu)}\Big) \underbrace{\sum_{k=0}^{\infty} \frac{(k+m)!}{k!m!} \Big(\frac{\bar{q}\mu}{\gamma+\bar{q}}\Big)^k \Big(\frac{\gamma+\bar{q}(1-\mu)}{\gamma+\bar{q}}\Big)^{m+1}}_{=1}
\end{aligned}$$

$$\begin{aligned}
p_{k|m} &= \frac{p_{km}}{\sum_{k=0}^{\infty} p_{km}} = \frac{\gamma}{\gamma+\bar{q}} \frac{(k+m)!}{k!m!} \frac{(\bar{q}(1-\mu))^m (\bar{q}\mu)^k}{(\gamma+\bar{q})^{m+k}} \Big(\frac{\bar{q}(1-\mu)}{\gamma+\bar{q}(1-\mu)}\Big)^{-m} \Big(\frac{\gamma}{\gamma+\bar{q}(1-\mu)}\Big)^{-1} \\
&= \frac{(k+m)!}{k!m!} \Big(\frac{\bar{q}\mu}{\gamma+\bar{q}}\Big)^k \Big(\frac{\gamma+\bar{q}(1-\mu)}{\gamma+\bar{q}}\Big)^{m+1}
\end{aligned}$$

Proof of proposition 6: Let the total firm size be $n = m + k$, so $m = n - k$, for fixed value of n we get:

$$\begin{aligned}
p_n &= \sum_{k+m=n} p_{km} = \frac{\gamma}{\gamma+\bar{q}} \sum_{k=0}^{n} \frac{n!}{k!(n-k)!} \Big(\frac{\bar{q}(1-\mu)}{\gamma+\bar{q}}\Big)^{n-k} \Big(\frac{\bar{q}\mu}{\gamma+\bar{q}}\Big)^k \\
&= \frac{\gamma}{\gamma+\bar{q}} \Big(\frac{\bar{q}}{\gamma+\bar{q}}\Big)^n \underbrace{\sum_{k=0}^{n} \frac{n!}{k!(n-k)!} (1-\mu)^{n-k} \mu^k}_{=1} = \frac{\gamma}{\gamma+\bar{q}} \Big(\frac{\bar{q}}{\gamma+\bar{q}}\Big)^n
\end{aligned}$$

B Appendix

Mincer earnings regressions

The information on gross monthly wage/income for regualr workers and potential entrepreneurs is available in SOEP. We use data for 2000-2017 and run the following Mincer earnings regressions for regular workers:

$$\begin{aligned}\ln w_{it} &= \phi_0 + \phi_1 Nat_{it} + \phi_2 Fsize_{it} + \phi_3 HSkill_{it} + \phi_4 Nat_{it} \times Fsize_{it} + \phi_5 Nat_{it} \times HSkill_{it} \\ &+ \phi_6 HSkill_{it} \times Fsize_{it} + \phi_7 Nat_{it} \times HSkill_{it} \times Fsize_{it} + \phi_8 t + \varsigma_{it}\end{aligned}$$

Where w_{it} is the wage of worker i observed in year t. Nat_{it} is the indicator function that takes the value 1 for natives, $Fsize_{it}$ is the indicator function with value 1 for large firms, $HSkill_{it}$ takes value 1 for high skill workers. We introduce interaction terms $(Nat_{it} \times Fsize_{it})$ and $(HSkill_{it} \times Fsize_{it})$ in order to capture variation in the effect of firm size on wages by ethnicity and skill. Similarly, $(Nat_{it} \times HSkill_{it})$ captures the fact that the return to schooling could be different for native and immigrant workers.

For the potential entrepreneurs we run the following regression

$$\begin{aligned}\ln w_{it} &= \phi_0 + \phi_1 Nat_{it} + \phi_2 HSkill_{it} + \phi_3 Nat_{it} \times HSkill_{it} + \sum_{l=1}^{3} \phi_{4l} PEtype^l_{it} + \\ &+ \sum_{l=1}^{3} \phi_{5l} Nat_{it} \times PEtype^l_{it} + \sum_{l=1}^{3} \phi_{6l} HSkill_{it} \times PEtype^l_{it} + \\ &+ \sum_{l=1}^{3} \phi_{7l} HSkill_{it} \times PEtype^l_{it} \times Nat_{it} + \phi_8 t + \epsilon_{it}\end{aligned}$$

where $PEtype^l_{it}$ is the indicator function, such that $l = 1$ for potential entrepreneurs in solo self-employment, $l = 2$ for business owners with less than 9 co-workers, $l = 3$ for business owners with more than 9 co-workers. Potential entrepreneurs in paid employment serve as a reference category. The interaction terms $Nat_{it} \times PEtype^l_{it}$ and $HSkill_{it} \times PEtype^l_{it}$ capture the ethnicity specific and skill specific fixed effects for entrepreneurs if different economic states. $Nat_{it} \times HSkill_{it}$ captures the skill fixed effects specific to the two ethnic groups. The coefficients from both regressions are summarized in tables 7 and 8 respectively. We use these results to predict gross earnings of different worker groups. These are used as target moments for the calibration of productivities of workers and the related parameters.

Table 7: Wage regression for regular workers

Variable	Coefficient	Predicted ln(w)	Normalized wage
Native	0.142***	e_0^{NH}: 7.25	$w_0^{NH} = 1.295$
	(0.012)		
Fsize	0.609***	e^{NH}: 7.93	$\bar{w}^{NH} = 2.557$
	(0.012)		
Native × Fsize	-.064***	e_0^{NL}: 6.99	$w_0^{NL} = 1$
	(0.014)		
Skill	0.251***	$\bar{e}^{NL}$: 7.53	$\bar{w}^{NL} = 1.715$
	(0.029)		
Native × Skill	0 .008	e_0^{IH}: 7.10	$w_0^{IH} = 1.120$
	(0.030)		
Fsize × Skill	0.048	$\bar{e}^{IH}$: 7.76	$\bar{w}^{IH} = 2.158$
	(0.330)		
Native × Fsize × Skill	0.086**	e_0^{IL}: 6.85	$w_0^{IL} = 0.870$
	(0.034)		
Constant	6.779***	$\bar{e}^{IL}$: 7.46	$\bar{w}^{IL} = 1.599$
	(0.013)		
Observations	180699		
Period	2000-2017		

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

$Fsize = 1$ for large firm with more than 20 workers

Table 8: Wage regression for potential entrepreneurs

Variable	Coefficient	Predicted ln(w)	Normalized wage/profit
PE $type_1$	-0.009	s^{NH}: 7.43	1.551
	(0.043)		
PE $type_2$	0.770***	s^{NL}: 7.26	1.309
	(0.048)		
PE $type_3$	1.078***	$b_{<9}^{NH}$: 8.34	3.854
	(0.119)		
Skill	0.049	$b_{<9}^{NL}$: 7.89	2.458
	(0.060)		
PE $type_1$ × Skill	0 .131	$b_{>9}^{NH}$: 8.71	5.580
	(0.082)		
PE $type_2$ × Skill	0 .460***	$b_{>9}^{NL}$: 8.37	3.972
	(0.091)		
PE $type_3$ × Skill=1	0.182	e^{NH}: 7.62	1.876
	(0.191)		
Native	-0.018	e^{NL}: 7.19	1.220
	(0.034)		
PE $type_1$ × Native	0.081 *	s^{IH}: 7.38	1.476
	(0.046)		
PE $type_2$ × Native	-0.077	s^{IL}: 7.20	1.232
	(0.052)		
PE $type_3$ × Native	0.101	$b_{<9}^{IH}$: 8.50	4.523
	(0.126)		
Skill × Native	0.383***	$b_{<9}^{IL}$: 7.99	2.716
	(0.063)		
PE $type_1$ × Skill × Native	-0.398***	$b_{>9}^{IH}$: 8.52	4.614
	(0.085)		
PE $type_2$ × Skill × Native	-0.443***	$b_{>9}^{IL}$: 8.29	3.666
	(0.095)		
PE $type_3$ × Skill × Native	-0.272	e^{IH}: 7.26	1.309
	(0.199)		
Constant	7.111***	e^{IL}: 7.21	1.245
	(0.038)		
Observations	37831		
Period	2000-2017		

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Where PE $type_1$ = Solo self-employed or helpers in family business
PE $type_2$ = self-employed with less than 9 coworkers
PE $type_3$ = self-employed with more than 9 coworkers
PE $type_4$ = in paid employment

Table 9: Average tenure for different demographic groups of workers

Group	Tenure
Never Entrepreneurs immigrants average tenure	8.781
Never Entrepreneurs native average tenure	11.547
Never entrepreneurs average tenure (native and immigrants combined)	8.237
Never entrepreneurs small firms natives average tenure	8.504
Never entrepreneurs small firms natives average tenure	6.750

C Appendix

We define $\bar{e}_0 = e_{0N} + e_{0I}$ – total employment of workers in small businesses within their own skill group with the corresponding average wage $\bar{w}_0 = (w_{0N}e_{0N} + w_{0I}e_{0I})/(e_{0N} + e_{0I})$. Further, $\bar{e}_c = e_{cN} + e_{cI}$ – total employment of low skill workers in high skill small businesses with the corresponding average wage $\bar{w}_c = (w_{cN}e_{cN} + w_{cI}e_{cI})/(e_{cN} + e_{cI})$.

Table 10: Detailed changes for workers without entrepreneurial abilities upon a 20% increase in low skill immigration.

Group	(1) $\bar{u}/(d-l)$	(2) $\bar{e}/(d-l)$	(3) $\bar{w}$	(4) $\bar{e}_0/(d-l)$	(5) $\bar{w}_0$	(6) $\bar{e}_c/(d-l)$	(7) $\bar{w}_c$
Low skill Natives	0.085	0.680	1.718	0.135	1.002	0.100	0.997
Change	+0.003	-0.003	-0.021	-0.001	+0.009	+0.002	-0.002
Low skill Immigr.	0.141	0.634	1.588	0.129	0.871	0.096	0.862
Change	+0.004	-0.004	-0.021	-0.002	+0.016	+0.001	-0.001
High Skill Natives	0.022	0.825	2.544	0.153	1.295		
Change	-0.001	+0.007	+0.034	-0.007	+0.009		
High Skill Immigr.	0.091	0.726	2.163	0.182	1.120		
Change	-0.002	+0.010	+0.031	-0.007	+0.007		

Table 11: Parameters calibrated using SOEP data 2000-2017

Prm.	Empirical moment/target	NL	IL	NH	IH	Definition and calibrated values	NL	IL	NH	IH
$\bar{\gamma}$	Average tenure of regular workers	11.55	8.78	11.55	8.78	Regular job destruction rate	0.0198	0.0270	0.0206	0.0269
γ	Share of workers in small businesses $e_0/(e_0+\bar{e})$	0.265	0.270	0.165	0.212	Business destruction/exit rate	0.0296	0.0358	0.0305	0.0292
δ	Share of active entrepreneurs $(b+s)/l$	0.547	0.525	0.568	0.577	Transition rate to entrepreneurship	0.5893	0.5355	1.1149	0.5173
x	Unempl. rate of immigrant workers $\bar{u}/(d-l)$	-	0.141	-	0.091	Search intensity for small businesses	1	0.5713	1	0.2851
X	Unempl. rate of immigrant pot. entrepreneurs u/l	-	0.049	-	0.039	Search intensity for regular jobs	1	0.7593	1	0.2750
c	Share of solo-entrepreneurs $(b_0+s)/(b+s)$	0.586	0.607	0.591	0.459	Flow cost of a small business	1.1636	1.2919	2.2448	3.0985
ς	Businesses with 1-9 coworkers $\sum_{n=1}^{9} b_n/(b+s)$	0.366	0.362	0.326	0.425	Entrepreneurial productivity	0.2499	0.255	0.3703	0.2635
φ_{0N}	Profits of small businesses with coworkers	2.63	2.77	4.20	4.52	Output units in small (native) businesses	0.2973	0.2398	0.4109	0.3363
φ_{0I}	Av. wages in small businesses $\bar{w}_0$	1	0.8674	1.2950	1.1144	Output units in small (immigrant) businesses	0.3437	0.3097	0.5293	0.4227
$\bar{\varphi}$	Av. wages of workers in regular jobs $\bar{w}$	1.625	1.492	2.940	2.630	Output units of workers in regular jobs	0.9727	1	0.9157	1
φ	Wages of pot. entrepreneurs in regular jobs w	1.222	1.246	1.884	1.309	Output units of pot. entrepr. in regular jobs	0.8954	0.9004	0.8619	0.7440
z	60% replacement rate in the 1st year and ALG II assistance afterwords					Unemployment benefits of pot. entrepreneurs	0.3719	0.3723	0.3757	0.3709
$\bar{z}$	60% replacement rate in the 1st year and ALG II assistance afterwords					Unemployment benefits of regular workers	0.3838	0.3809	0.3948	0.3869
M	Unempl. rate of native entrepreneurs u/l	0.035	-	0.014	-	Matching multiplier, regular jobs	0.5319		0.9955	
$\bar{M}$	Unempl. rate of native workers $\bar{u}/(d-l)$	0.085	-	0.022	-	Matching multiplier, small businesses	0.0748		0.1809	
$\bar{c}_k$	Average job-filling rate $q(\theta)$	1.5500		1.2398		Capital cost in regular firms	3.3821		2.5642	
c_h	Rebien, Stops, Zaharieva (2020)					Vacancy posting cost	0.7431		2.3344	
β_0	Assumption $w_{0I}^{N}/w_{0I}^{I}=\bar{w}^{N}/\bar{w}^{I}$	1.0819		1.1905		Bargaining power of workers in small firms	0.4564		0.4222	
r	Annual discount rate = 5%					(Quarterly) Discount rate	0.0125			
η	Chassamboulli and Palivos (2014) Battisti et al. (2018)					Elasticity of subst. between K and Z	0.350			
ρ	Ottaviano and Peri (2012) Battisti et al. (2018)					Elasticity of subst. between Y^H and Y^L	0.500			
R	Chassamboulli and Palivos (2014) Battisti et al. (2018)					(Quarterly) cost of capital	0.030			
A	Normalization $\bar{y}^{NH}=P_H$					Total factor productivity	2.5630			
a	Normalization $\bar{y}^{NL}=P_L$					Income share of Y_L	0.5547			
κ	Average tenure in small firms	8.2369				Cross-skill matching parameter	0.1700			
ζ	Regression $\ln q(\theta)$ on $\ln\theta$					Elasticity of the matching function	0.47			
β	Ratio of nominal wages to GDP	0.28				Bargaining power of workers in regular jobs	0.9084			