Stochastic Volatility in Interest Rates and Trend Cycles *

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Abstract

We use estimated time series models to demonstrate that an increase in the volatility of the U.S. interest rate leads to a decline in the common trends of GDP, consumption, and investment. This effect is more pronounced in emerging economies than in advanced ones. We construct a small open economy model that incorporates endogenous growth, financial crises, and shocks to the volatility of the international interest rate. While small interest rate shocks have symmetric effects on firm's value, large adverse interest rate shocks trigger larger absolute decreases in firms' values. The root of this asymmetry is that firm's value serves as collateral, then, when the borrowing constraints tighten, it leads to further reductions in firm values and economic growth. Due to this asymmetry, increases in the volatility of the international interest rate decrease innovation and growth in the economy as the absolute magnitude of future interest rate shocks is expected to be larger.

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1 Introduction

Policymakers from emerging market economies (EMEs) and the financial media often express concerns about the effects that rising interest rate in the U.S. have on these economies - a view supported by recent empirical work. In addition, a growing literature has argued that not only rising interest rates, but also changes in the variability and uncertainty surrounding the global interest has contractionary effects on EMEs.[1] This interest rate pass-through likely contributes to the excess volatility in GDP and consumption in EMEs relative to developed economies.

In this paper we argue that an increase in the volatility of the U.S. interest rate not only causes business cycle fluctuations in EMEs, but also reduces the trend around which these economies are growing, while, simultaneously making the trend itself more volatile. Using a multi-variate time series model we establish that a rise in stochastic volatility of the real interest rate in the United States forecasts decreases in the growth rate of the trend of GDP, which are more pronounced in EMEs (in line with Aguiar and Gopinath, 2007). We present a parsimonious small open economy model featuring endogenous growth and occasionally binding borrowing constraints. In the model, households can collateralize the value of their investment in intermediate good producers. Endogenous technical change links the collateral value to innovation decisions of incumbent and entrant firms. Shocks to the first and second moments of the world interest rate affect innovation decisions, firm dynamics, and the trend growth rate, which happen to be more pronounced when financial frictions are more severe. This model is able to replicate the empirical patterns we uncovered, while standard, 'off-the-shelf' alternatives need to assume that exogenous TFP trend shocks are correlated with the U.S. interest rate. Because, every shock in the model is stationary, trend fluctuations arise endogenously due to the internal propaga-

[1] See, for example, Rey (2015), Iacoviello and Navarro (2019), Miranda-Agrippino and Rey (2020), Bräuning and Ivashina (2020) for empirical evidence on U.S. monetary spillovers on EMEs, and Fernández-Villaverde et al. (2011) and Reyes-Heroles and Tenorio (2020) for the importance of volatility shocks in emerging markets.

tion of the model. The focus on stationary shocks and the central role of financial frictions allows us to reconcile the results in Aguiar and Gopinath (2007) and Garcia-Cicco et al. (2010).

In our empirical analysis we estimate the effect of stochastic volatility shocks of the U.S. interest rate on the growth rate of a common trend of GDP, consumption, and investment. First, we use the particle filter (Fernández-Villaverde et al., 2011) to estimate a time series of stochastic volatility for the U.S. interest rate. Second, we use the filtered stochastic volatility time series in a multi-variate model that jointly estimates the common trend of GDP, consumption, and investment while uncovering the effects of volatility innovations in the common trend component. A one standard deviation increase in the volatility of U.S. interest rates predicts a decline in the trend growth rate of EMEs twice as large as the one of advanced economies on impact. This differential effect persists for several quarters.

To study the nature of the novel pass-through of interest rate volatility shocks into trend, we develop a simple small open economy model. The goal of the model is to present the simplest framework that can provide a micro foundation for this connection. A representative household borrows from international markets to smooth consumption and buys shares of a continuum of intermediate good producers. The household needs to post the value of her shares as collateral, so borrowing is limited by how pledgeable this financial asset is. The productivity of each intermediate good increases when a new or an incumbent firm successfully innovates, becoming a monopolist of that variety. In making their investment decision entrepreneurs trade off the cost of engaging in a project with the expected reward, which is given by the discounted value of future monopoly rents. A rise in the foreign real rate increases the discount factor of the household, triggering a reduction in the present discounted value of innovating. Innovation therefore decreases, slowing down the growth rate of productivity. Because the value of firms directly influences the collateral value of households, this slowdown triggers a further tightening in the borrowing constraint, providing a feedback mechanism that exacerbates the negative effect on growth. Thus, despite its simplicity, the framework can deliver a negative correlation between interest rate and productivity growth. Moreover, the occasionally binding nature of the borrowing constraint triggers state dependence in the effects of interest rate shocks. In fact, interest rate shocks of moderate size have symmetric effects on firm's values. In contrast, large interest rate shocks feature a strong asymmetry between interest rate increases and interest rate decreases. For instance, the decrease in firm values caused by a large interest rate increase is 3 times the size of the increase in firm values caused by an interest rate decrease of the same absolute magnitude. Thus, when interest rate volatility increases, the agents expect larger absolute swing on interest rate in the future, triggering a decrease in the expected value of firms due to the asymmetry. Lower expected firm's value depresses firm's entry and expansion efforts by incumbents, persistently slowing the endogenous trend growth in the economy.

We calibrate the model to the Mexican economy in order to match long-run productivity growth, the entry rate of new firms, and the relative size of incumbent firms. We use the interest rate and TFP process estimated from our empirical analysis to discipline the stochastic elements in the model. Despite its parsimony, the model is able to replicate the patterns observed in our empirical analysis. We find that roughly one-half of the differential pass-through of interest rate volatility on trend growth between emerging and advanced economies can be explained by lower financial development (i.e. lower pledgeability of assets), while the remaining portion is explained by higher financial intermediation costs faced by EMEs during episodes of high interest rate volatility.

Related Literature: Allowing for endogenous growth can help modern macroeconomic models to better account for business cycle and medium term dynamics by allowing the productivity trend to respond to stationary shocks (Comin and Gertler, 2006). Because business cycles are more volatile and trend fluctuations in productivity growth are more prominent in emerging market economies (Cerra and Saxena, 2008), the addition of endogenous technological change has been particularly successful in this context. Queralto (2020) shows that a model with financial frictions and endogenous productivity helps to explain the medium term effects of the Asian crisis on growth in the region. Ates and Saffie (2021) develop a model of endogenous growth with selection and firm dynamics and provide granular support for selection at entry in the case of Chile's sudden stop in 1998. Gornemann (2014) shows the importance of accounting for medium term dynamics in the context of sovereign defaults in EMEs. Croce et al. (2012) show that a rise in fiscal uncertainty can lead to a growth slowdown in the U.S. We contribute to this literature by providing new time series evidence and, especially, by demonstrating the important role that global interest rates volatility plays for these endogenous movements in the trend in EMEs.

Our focus on the role of global interest rates and their volatility connects us to a large body of work in small open economy models. Mendoza (1991), Neumeyer and Perri (2005), Uribe and Yue (2006), study the importance of changes in the level of interest rates for fluctuations in small open economies. In order to match the empirical effects of interest rates on output, this literature often assumes working capital constraints and an exogenous correlation between TFP and interest rate fluctuations. Our simple model abstracts from both elements, but the internal propagation of the model delivers both, the importance of interest rate shocks and the correlation between productivity and interest rates. Fernández-Villaverde et al. (2011), Gruss and Mertens (2009), Reyes-Heroles and Tenorio (2020) and Johri et al. (2022) provide evidence for the role of changes in the volatility of interest rate on the business cycle of emerging economies. We contribute to this literature by showing that the rate fluctuations not only affect short run dynamics, but through their effect on trend productivity growth, also trigger hysteresis and low frequency fluctuations. Another branch of this literature discusses the role of trend shocks to productivity for EME business cycles with mixed results (see, for instance, Aguiar and Gopinath (2007), Garcia-Cicco et al. (2010), and Chang and Fernández (2013)). By connecting movements in trend productivity to stationary shocks, models with endogenous growth can abstract from exogenous trend movement when generating excess volatility. This micro founded approach seems to be preferred by the data and opens avenues for policy analysis (Gornemann et al., 2020).

The paper is structured as follows. Section 2 uses Mexican data to document the link between U.S. interest rate volatility and dynamics in Mexican productivity growth. Section 3 presents a simple model linking interest rate dynamics to productivity growth. Section 4 calibrates the model to the Mexican economy and shows that the model can generate a sizable pass-through between interest rate volatility and endogenous productivity trend dynamics. Finally, Section 5 concludes and discusses future extensions.

2 Empirical Analysis

This section provides some evidence for a relation between changes in the volatility of global interest rates, which we approximate as US real rates, and persistent changes in economic activity in small open economies based on some time series models. In particular, we show that this link is stronger for emerging market economies. After describing the data used, we outline how we obtain a series for the volatility of U.S. real rates. We then outline the multi-variate time series model we take to the data for a set of emerging and developed small economies and present estimation results for it.

2.1 Data and Stochastic Volatility in U.S. Real Rates

For the United States we use data on PCE and CPI inflation as well as real GDP and interest rate on Treasuries (3-month, 2-years, 5-years) from FRED. In addition, we obtain data on the excess bond premium. We pair this data with time series for GDP, consumption, investment, and the trade balance-to-GDP ratio for our emerging and developed economies from the IMF's International Financial Statistics and JP Morgan's EMBI spread time series for the former set of countries.[2] After transformations of the data, we focus on the time period 1985Q1 to 2019Q4, ending our investigation before the outbreak of COVID-19 affects macroeconomic conditions. Due to missing data for some countries, the data panel is unbalanced.

We construct a demeaned quarterly time series for the US real interest rate (measured as the real 3-month T-bill rate) for our target period.[3] Following Fernández-Villaverde et al. (2011), we adjust nominal rates by the total change in the PCE index over the current and past three quarters. We then proceed to estimate the stochastic volatility of the U.S. real interest rate using the particle filter Fernández-Villaverde et al. (2011). Specifically, we filter a sequence σ_t from the following model after estimating it using maximum likelihood:

$$r_t - \bar{r} = \phi r_{t-1} + \exp(\sigma^r)\varepsilon_t^r, \tag{1}$$

$$\sigma_t^r = (1 - \rho_\sigma^r)\mu_\sigma^r + \rho_\sigma^r \sigma_{t-1}^r + \sigma_r \nu_t^r. \tag{2}$$

where $\bar{r}$ denotes the mean of the real interest rate in our sample.

Table 1 shows our parameter estimates for the model.[4] As we see both the interest rates and its hidden volatility state are highly persistent. The following figure shows the path of the real rate and the extracted mean path of the smoothed state of volatility.[5] We see large changes in the hidden state over time. Volatility was falling between 1985 and 2000, in line with the relatively stable real rates themselves over this time period. It then starts moving up over the next ten years or so, in line with, both, a persistent decline in

[2]Current results are for Australia, Canada, Netherlands, Sweden, Chile, Colombia, Korea, and Mexico. Future iterations will extend them to a larger set of countries.

[3]Using the 2-year and the 5-year interest rate instead does not alter our core results as the three series show a high degree of correlation over our sample period.

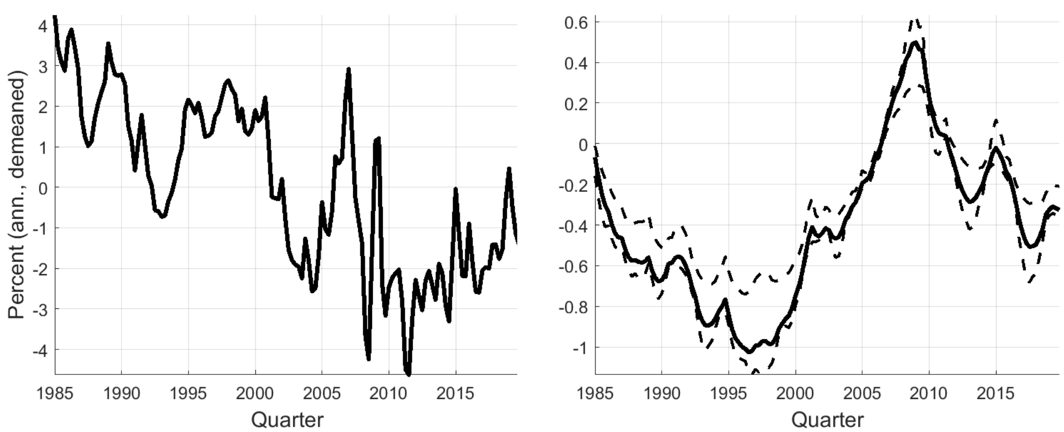
[4]Standard errors are obtained through Monte Carlo simulations. While we later bound persistence parameters to be below 0.999, here we allow estimates to be larger than 1.

[5]The error bands are obtained by repeatedly computing the smoothed path in a Monte Carlo simulation. The asymmetry in the standard error bands reflects the asymmetry in the confidence sets for the persistence parameters.

the average rate and large movements around this secular decline. After 2010, real rates become more stable again, a dynamic picked up by the stochastic volatility process.

ϕ	ρ_σ^r	$10 * \mu_\sigma^r$	$10 * \sigma_r$
0.941	0.967	1.60	0.50
(0.86,1.00)	(0.87,1.01)	(0.72,1.90)	(0.45,0.59)

Table 1: US Interest Rate Model: Estimated Parameters



(a) U.S. Real Interest Rate

(b) U.S. Real Interest Rate Stochastic Volatility

Figure 1: Estimated Series

2.2 Trend Growth Estimation and Stochastic Volatility Effects

Having obtained our measure of the time varying volatility of U.S. interest rates, our next goal is to be able to estimate the trend component of GDP, as well as its growth rate. In order to provide structure for identification we assume that GDP, consumption, and investment share a joint stochastic trend. The reasoning behind this assumption is that we impose a structure that resembles many exogenous and endogenous growth models, where these macroeconomic variables grow at the same rate once temporary disturbances have settled.

Let $X_t = \tilde{X}_t \cdot \exp(A_t)$, with X being GDP (Y), consumption (C) or investment (I) and where A_t denotes the stochastic component of the trend. Here, $\tilde{X}$ is the stationary part of the observed time series. We assume that the growth rate of the trend $a_t = A_t - A_{t-1}$ responds to country and global variables with a lag, due to the need to perform R&D.[6] We then set up a state space model and estimate the response of the trend growth rate a_t to changes in interest rate volatility using these restrictions. We assume that the economies are small and open so the dynamics of their macro variables do not influence the U.S. interest rate or its volatility.

The state space representation of our model is the following. The observation equation is

$$\begin{pmatrix} y_t \\ c_t \\ i_t \\ tb_t \end{pmatrix} = \begin{pmatrix} \mu \cdot t + A_t \\ \mu \cdot t + A_t \\ \mu \cdot t + A_t \\ 0 \end{pmatrix} + \begin{pmatrix} \tilde{y}_t \\ \tilde{c}_t \\ \tilde{i}_t \\ tb_t \end{pmatrix}, \tag{3}$$

where μ represents the deterministic trend. The transition equations are

$$a_t = M_1 \begin{pmatrix} \tilde{y}_{t-1} \\ \tilde{c}_{t-1} \\ \tilde{i}_{t-1} \\ \tilde{tb}_{t-1} \end{pmatrix} + M_2 \begin{pmatrix} r^{US}_{t-1} \\ \sigma^{r,US}_{t-1} \\ \Delta y^{US}_{t-1} \end{pmatrix} + \eta^a_t \tag{4}$$

$$\begin{pmatrix} \tilde{y}_t \\ \tilde{c}_t \\ \tilde{i}_t \\ tb_t \end{pmatrix} = M_3 \begin{pmatrix} \tilde{y}_{t-1} \\ \tilde{c}_{t-1} \\ \tilde{i}_{t-1} \\ \tilde{tb}_{t-1} \end{pmatrix} + M_4 \begin{pmatrix} r^{US}_t \\ \sigma^{r,US}_t \\ \Delta y^{US}_t \end{pmatrix} + \eta^s_t. \tag{5}$$

[6] As we will show later, this is consistent with our model setup.

The U.S. variables follow a standard VAR process. We assume that the contemporary U.S. interest rate or its stochastic volatility has no impact on the stochastic trend growth. In particular, we assume that the trend growth responds with a lag, as in the endogenous growth literature. This timing assumption on U.S. variables helps us separate their effect on the stochastic trend and on the stationary component of GDP, consumption, and investment.

An additional assumption we make is that countries in each group (emerging and developed) differ only in μ, and we impose that M_3 is stationary. We then proceed to estimate the state space representation of the model using Maximum Likelihood, and we use Monte Carlo for the standard errors. We identify the volatility shock with Cholesky and trace response.

Figure 2 presents the impulse response function of the trend growth component to a one standard deviation innovation in the US stochastic volatility. We show the responses for emerging and developed economies. We see that emerging economies experience a substantially larger decline in trend economic activity relative to developed ones.

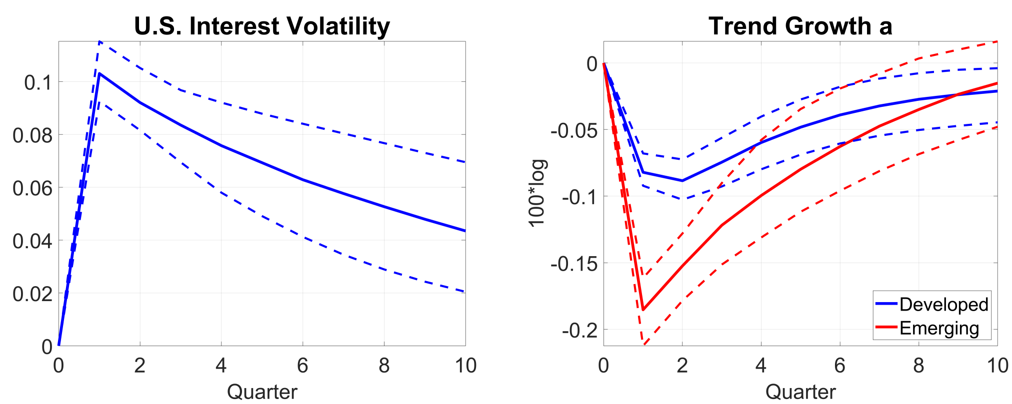


Figure 2: Trend Growth Rate Responses to Stochastic Volatility Shocks

3 A Simple Model of Endogenous Trend Cycles

3.1 Model Overview

This section develops a small open economy model with an occasionally binding borrowing constraint and endogenous growth subject to an aggregate supply shock, and first and second moment shocks to the international interest rate. We extend the workhorse endogenous sudden stop models in three directions. First, we allow the representative household to collateralize the value of its investment in intermediate good producers. Second, we include endogenous technical change that link the collateral value to the innovation decision of incumbent and entrant firms. Third, we include first and second moment interest rate shocks that affect firm dynamics, endogenous technical change, and the severity of the financial friction.

Figure 3: Diagram of the Model

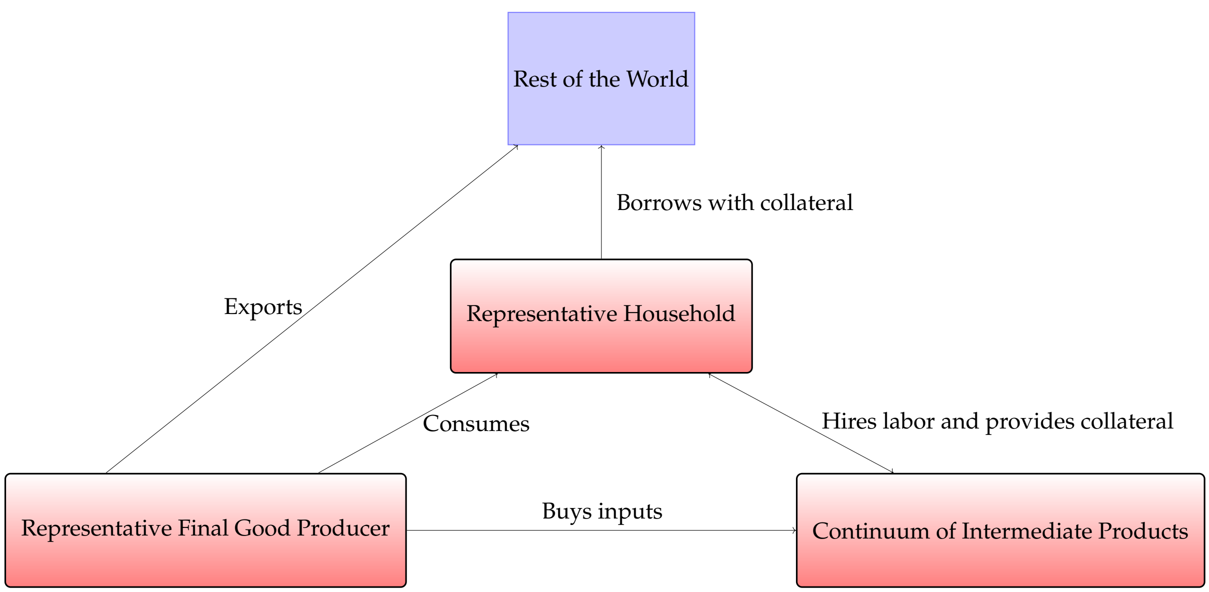


Figure 3 provides a summary of the key building blocks in the model. A representative household hold claims on a continuum of intermediate good producers. The household can post the value of these claims as collateral and borrow from the rest of the world subject to a stochastic interest rate process. The entry, exit, and expansion of intermediate producers (Klette and Kortum, 2004) is the engine of long-run growth in this economy. Intermediate goods are sold to a representative final good producer that is subject to an aggregate efficiency shock. The final good is the only tradable good in the economy.

3.2 Households

The representative household solves the following problem:

$$\max_{C(s^t),B(s^t)^H,d(s^t)} \sum_{t=0}^{\infty} \beta^t \mathbb{E}\left[\frac{1}{1-\gamma}\left[C(s^t)\right]^{1-\gamma} |s_0\right] \tag{6}$$

subject to:

$$C(s^t) + d(s^t)q(s^t) \leq w(s^t) + B^H(s^{t-1}) + d(s^{t-1})(q(s^t) + e(s^t)) - \hat{q}(s^t)B^H(s^t) \tag{7}$$

$$\hat{q}(s^t)B^H(s^t) \geq -\phi q(s^t)d(s^t), \tag{8}$$

where C is consumption of the only final and tradable good in the economy, B^H represents holding of the non-contingent bonds, d corresponds to shares of firms in the economy that are purchased at a price q, e denotes dividends to shareholders from firms, and w is the wage rate that the household earn by supplying a unit of labor inelastically, and $\hat{q}$ denotes the price of debt, which is the reciprocal of the gross interest rate set by financial intermediaries in the economy. The financial constraint prevents households from borrowing more than a fraction ϕ of the value of their current claims $(q(s^t)d(s^t))$. The key deviation from the literature of endogenous financial crises (Bianchi and Mendoza, 2018) is the use of the household's investment in production as collateral.

3.3 Financial Intermediaries

Households borrow from international financial intermediaries. These intermediaries are competitive and risk-averse, they have a gross funding cost R and lend to households at a price $\hat{q}$. The gross interest rate in international markets follows an AR(1) subject to stochastic volatility given by:

$$R(s^t) - \bar{R} = \phi_R \left(R(s^{t-1}) - \bar{R} \right) + \exp(\sigma(s^t))\omega(s^t), \quad \text{where} \quad \omega(s^t) \sim N(0,1), \tag{9}$$

$$\log(\sigma(s^t)) = (1-\rho_\sigma)\mu_\sigma + \rho_\sigma \log(\sigma(s^{t-1})) + \eta_\sigma \nu(s^t), \quad \text{with} \quad \nu(s^t) \sim N(0,1), \tag{10}$$

where $\bar{R}$ is the average cost of funds, ϕ_R the persistence of the cost of funds, and σ the volatility of the process. The volatility of R follows an AR(1) with mean μ_σ, persistence ρ_σ, and volatility ν_σ.

Intermediaries lend at a price $\hat{q}$, derived from the following optimization problem:

$$\max_{b(s^t)} \mathbb{E} \left[\hat{m}(s^{t+1}) b(s^t) \middle| s^t \right] - \hat{q}(s^t) b(s^t), \tag{11}$$

where $\hat{m}(s^t, s_{t+1})$ corresponds to the intermediaries' stochastic discount factor. To reflect lenders risk aversion, we follow Johri et al. (2022) and assume that the discount factor of financial intermediaries is given by $\hat{m}(s^{t+1}) = \exp(-(R(s^{t+1}) + \iota\sigma(s^{t+1})))$. Thus, the solution to (11) yields the following pricing function:

$$\hat{q}(s^t) = \mathbb{E} \left[\exp(-(R(s^{t+1}) + \iota\sigma(s^{t+1}))) \middle| s^t \right], \tag{12}$$

where $\iota > 0$ reflects that lenders charge more during high volatility times. This pricing function can reflect risk aversion or increasing costs for financial intermediaries when volatility is high.[7]

[7] A similar structure for the discount factor of financial intermediaries has also been used in the literature. See also Arellano and Ramanarayanan (2012), Bianchi et al. (2018) and Hegarty et al. (2023).

3.4 Final Good Producer

The final good producer aggregates a mass Λ of differentiated intermediate goods according to the following technology:

$$\ln Y(s^t) = z(s^t) + \frac{1}{\Lambda}\int_0^{\Lambda} \ln y_i(s^t) di,, \tag{13}$$

where $Y(s^t)$ is the unique final good, $y_i(s^t)$ is the final good producer's demand for variety i, and $z(s^t)$ is an aggregate efficiency shock that follows:

$$\ln z(s^t) = \rho_z \ln z(s^{t-1}) + \epsilon_t, \quad \text{with} \quad \epsilon_t \sim N(0, \eta_z^2),$$

where ρ_z is the persistence of the aggregate efficiency shock and η_z its volatility.

3.5 Intermediate Good Producers

There is a continuum of intermediate goods indexed by i uniformly distributed in $[0, \Lambda]$. Each intermediate good is produced by the firm with the lowest marginal cost according to the following production function:

$$y_i(s^t) = q_i(s^t) l_i(s^t), \tag{14}$$

where q_i is the efficiency in the production of intermediate goods i, and l_i represents the labor used in production, respectively. The leader of each intermediate good is the producer with the lowest marginal cost, this producer enjoys monopoly rents and competes á la Bertrand with the producer with the second lowest marginal cost. A firm can own and produce several intermediate goods. Firms can expand by becoming leaders in other product lines, they can also contract when they lose their advantage in the production of some intermediary good, and even exit by losing all of their products.

3.6 Innovation by Incumbents

The productivity of each intermediate variety evolves endogenously with each innovation. Innovations arise when either an incumbent or a potential entrant successfully improves the existing technology for producing an intermediate good. Thus, when an innovation occurs in variety i the new technological leader has access to a productivity $q_i(s^{t+1}) = (1+\sigma)\tilde{q}_i(s^t)$, where $\tilde{q}_i(s^t)$ is the technology that the former leader had and $\sigma > 0$ is the innovation step size.

An incumbent firm can attempt to innovate and become the new leader in an intermediate product that she does not currently own. To model the innovation technology, we follow Ates and Saffie (2021)'s discrete time version of Klette and Kortum (2004). In particular, a firm with n products can use research labor $L_r(s^t)$ to generate a success probability $x(s^t)$ per product line according to the following production function:

$$x(s^t) = \xi \left(\frac{L_r(s^t)}{n} \right)^{\theta} \equiv \xi \left(l_r(s^t) \right)^{\theta}. \tag{15}$$

The expansion and destruction probabilities are governed by binomial processes. Therefore, a firm with n products that optimally chooses an innovation rate $x(s^t)$ will win k new products with probability:

$$\mathbb{B}(k, n, x(s^t)) = \binom{n}{k} x(s^t)^k \left(1 - x(s^t)\right)^{n-k}, \tag{16}$$

and lose $\tilde{k}$ products when facing the aggregate creative destruction rate $\Delta(s^t)$ according to:

$$\mathbb{B}(\tilde{k}, n, \Delta(s^t)) = \binom{n}{\tilde{k}} \Delta(s^t)^{\tilde{k}} \left(1 - \Delta(s^t)\right)^{n-\tilde{k}}. \tag{17}$$

The discrete time problem of an incumbent firm is given by:

$$\begin{aligned} V_n(s^t) &= \max_{x_n(s^t)} n \left[\pi(s^t) - w(s^t) \left(\frac{x_n(s^t)}{\xi} \right)^{\frac{1}{\theta}} \right] \\ &+ \mathbb{E} \left[m(s^{t+1}) \sum_{\tilde{k}=0}^{n} \mathbb{B}\left(\tilde{k}, n, \Delta(s^t)\right) \sum_{k=0}^{n} \mathbb{B}\left(k, n, x_n(s^t)\right) V_{n-\tilde{k}+k}(s^{t+1}) \middle| s_t \right], \end{aligned} \quad (18)$$

where $m(s^{t+1})$ denotes the stochastic discount factor of the household that owns the firms.[8] We denote firm's profits by $\pi(s^t)$ and the equilibrium wage by $w(s^t)$. Note that, because firms are atomistic we can assume that the two binomial processes are independent and, therefore, separable. The combination of the two binomial processes characterizes the probability distribution of a firm with n products to transition in only one period to any number of products in $[0, 2n]$ instead.[9]

We also allow for an efficiency loss in each variety by depreciating the technology of leaders and followers in every product line at a rate of δ. This depreciation can capture either actual losses in production efficiency or changes in the taste of the consumer as the time passes. Because the depreciation affects every producer within a variety, the relative technological gap remains unaffected.

3.7 Entry of New Firms

To capture entry of new firms, define $M(s^t) \in [0, 1]$ as the innovation effort to start a new business. We assume that when an entrepreneur invests $\kappa M(s^t)$ units of labor she succeeds with probability $M(s^t)^\nu$ in becoming a single-product firm next period. The entrepreneur solves the following problem:

$$\max_{M(s^t)} \left(M(s^t)\right)^\upsilon \mathbb{E}\left[m(s^{t+1}) V_1(s^{t+1}) \middle| s_t \right] - \kappa M(s^t) w(s^t). \quad (19)$$

[8]The innovation investment at time t materializes in product transitions at time $t+1$. Entry of new firms also takes one period to materialize.

[9]See Appendix A for a detailed description of how to calculate the firm size distribution of the economy.

The result of this optimization determines the number of new single-product firms in the economy.

3.8 Equilibrium Characterization

The parsimony of the model allows us to unveil the link between interest rate volatility, innovation, and the endogenous trend of productivity. In order to characterize the innovation decisions of firms we need to determine the relevant discounting and the production decision of intermediate good producers. To start, the stochastic discount factor of the household is given by

$$m(s^t, s_{t+1}) = \beta \frac{C(s_{t+1})^{-\gamma}}{C(s^t)^{-\gamma} - \phi\mu(s^t)}, \tag{20}$$

where $\mu(s^t)$ represents the Lagrange multiplier associated with the household's borrowing constraint (8). Household consumption is a function of the labor and asset income of households, in addition to bond-holding allocations, which are influenced by the borrowing constraint that the representative household faces. We show below that the value of the collateral that households can post is a function of the value of firms in the economy.

Because the intermediate goods producers act as monopolists, we need the demand for intermediate goods in order to solve for the production decision of intermediate firms. The demand for each variety is given by:

$$y_i(s^t) = \frac{Y(s^t)}{\Lambda p_i(s^t)}. \tag{21}$$

Using this unit elastic demand and assuming Bertrand monopolistic competition at the intermediate good level, we can show that profits and labor per product are independent of the product-specific efficiency $q_i(s^t)$. In particular, we have:

$$\pi(s^t) = \frac{\sigma}{1+\sigma}\frac{Y(s^t)}{\Lambda} \tag{22}$$

$$l(s^t) = \frac{Y(s^t)}{\Lambda w(s^t)\,(1+\sigma)} \tag{23}$$

Because firm's decisions are independent of the productivity level of the intermediate products in their portfolio, this framework preserves the proportionality of the value function that has made the Klette and Kortum (2004) so popular in the creative destruction literature. In fact, we can guess and verify that $V_n(s^t) = nV_1(s^t)$. This means that every firm invests the same amount per product. By replacing the guess, factoring the future value of a product and using the properties of the binomial distribution, Equation (18) simplifies to:

$$V_1(s^t) = \max_{x(s^t)} \pi(s^t) - w(s^t)\left(\frac{x(s^t)}{\xi}\right)^{\frac{1}{\theta}} + \mathbb{E}\left[m(s^{t+1})\left(1-\Delta(s^t)+x(s^t)\right)V_1(s^{t+1})\Big|\, s_t\right] \tag{24}$$

Therefore, the optimal innovation effort per product line is independent of the total number of products that the firm has:

$$x^*(s^t) = \left[\theta\xi^{\frac{1}{\theta}}\frac{\mathbb{E}\left[m(s^{t+1})V_1(s^{t+1})\big|\, s_t\right]}{w(s^t)}\right]^{\frac{\theta}{1-\theta}}. \tag{25}$$

Having characterized the value of each intermediate product line, we can solve for the mass of new firms:

$$M^*(s^t) = \left[\frac{\upsilon}{\kappa}\frac{\mathbb{E}\left[m(s^{t+1})V_1(s^{t+1})\big|\, s_t\right]}{w(s^t)}\right]^{\frac{1}{1-\upsilon}}, \tag{26}$$

The endogenous creative destruction rate is composed by the innovation of incumbent firms and new entrants, therefore:

$$\Delta(s^t) = \frac{\left(M^*(s^t)\right)^v}{\Lambda} + x^*(s^t) \tag{27}$$

Having the aggregate rate of creative destruction we can derive the endogenous productivity growth in the economy. In fact, replacing Equation (14) in Equation (13) we obtain:

$$Y(s^t) = e^{z(s^t)} A(s^t) l(s^t), \tag{28}$$

where $A(s^t)$ is the endogenous productivity level defined by:

$$\ln A(s^t) = \frac{1}{\Lambda} \int_0^\Lambda \ln q_i(s^t) di. \tag{29}$$

Endogenous productivity growth is generated by creative destruction. Moreover, the productivity of an intermediate good production only increases by a factor $(1+\sigma)$ when it is subject to the creative destruction rate $\Delta(s^t)$

$$\begin{aligned} &\ln(A(s^t, s_{t+1})) - \ln(A(s^t)) = \Delta(s^t) \ln(1+\sigma) + \ln(1-\delta) \\ \text{or} \quad &\frac{A(s^t, s_{t+1})}{A(s^t)} = 1 + g\left(s^t, s_{t+1}\right) = (1+\sigma)^{\Delta(s^t)} (1-\delta), \end{aligned} \tag{30}$$

where $g\left(s^t, s_{t+1}\right)$ is the growth rate of the endogenous productivity index. When a shock triggers fluctuations in $\Delta(s^t)$, we will observe fluctuations in the endogenous productivity growth rate. Note that, endogenous productivity increases if and only if creative destruction overtakes the depreciation rate δ.

We can further characterize the equilibrium by combining Equations (28) and (23) to solve for the equilibrium wage:

$$w(s^t) = \frac{e^{z(s^t)} A(s^t)}{\Lambda (1+\sigma)}. \tag{31}$$

Appendix B renders the model stationary by normalizing the growing variables using the productivity index. It also summarizes the full system of dynamic equations that characterizes the stochastic solution of the model.

Lastly, it remains to characterize the optimality conditions for the representative household's problem. The first order conditions yield the following equilibrium conditions:

$$C(s^t)^{-\gamma} = \mu(s^t) + \beta\hat{R}(s^t)\mathbb{E}\left[C(s^{t+1})^{-\gamma}|s^t\right] \tag{32}$$

$$C(s^t)^{-\gamma} = \phi\mu(s^t) + \beta\mathbb{E}\left[C(s^{t+1})^{-\gamma}R^q(s^{t+1})|s^t\right] \tag{33}$$

$$C(s^t) \leq w(s^t) + B^H(s^{t-1}) + \Lambda e(s^t) - \hat{q}(s^t)B^H(s^t), \tag{34}$$

with $R^q(s^{t+1}) = \frac{q(s^{t+1})+e(s^{t+1})}{q(s^t)}$ and $\hat{R}(s^t) = 1/\hat{q}(s^t)$. Equation (32) corresponds to the Euler equation of bond holdings, Equation (33) is the Euler equation of firm shares and Equation (34) describes the budget constraint of the household after imposing the aggregate consistency condition that aggregate shares are equal to the mass of intermediate goods Λ. Note that Equation (24) implies $e(s^t) = \pi(s^t) - w(s^t)\left(\frac{x(s^t)}{\xi}\right)^{\frac{1}{\theta}}$.

We use Equation (33) to determine the equilibrium price of firm shares in the economy. Rearranging terms, we have that

$$q(s^t) = \mathbb{E}\left[m(s^{t+1})(q(s^{t+1}) + e(s^{t+1}))|s^t\right] \tag{35}$$

Replacing recursively, and assuming that the transversality condition for the present value of dividends holds, we have that $q(s^t) = V_1(s^t)$. Hence, the price of a share corresponds to the value of owning an intermediate-good-producing firm with one product line.

Using the equilibrium conditions described above, we provide an alternative expression for the borrowing constraint of the representative household:

$$\hat{q}(s^t)B^H(s^t) \geq -\phi\Lambda(V_1(s^t) - e(s^t)). \tag{36}$$

Equation (36) shows that in equilibrium the collateral value of households is a fraction of the value of all firms in the economy net of dividends.[10]

3.9 Trend Dynamics and Interest Rates

Equation (30) shows that stationary distortions affecting the endogenous rate of creative destruction (Δ) can trigger permanent losses in the level of the productivity index defined in Equation (29). Because that index determines the long-run growth rate of the economy, every non-stationary variable is normalized by it. Thus, fluctuations in creative destruction generate hysteresis in output and consumption. In fact, in this class of models, stationary shocks can trigger non-stationary dynamics due to the internal propagation of the model. To understand the drivers of these endogenous trend dynamics we need to study how different shocks affect the determinants of creative building blocks of creative destruction and, thus, how aggregate shocks drive firm entry and the expansion decision of incumbents. Equations (25) and (26) provide guidance when studying these drivers.

The dynamics of the two components of creative destruction are due to two endogenous objects. First, the dynamics of the stochastic discount factor of the household. Second the relative future value of a product normalized by the current wage of the economy. Let's focus first on the effect of a TFP shock. Because TFP shocks are persistent, the permanent income hypothesis implies that they entail little effect on the stochastic discount factor of the household. Therefore, TFP shocks cannot affect productivity growth significantly through this channel. As seen in Equation (31), a TFP shock directly increases

[10] Note that from iteratively replacing Equation (35) we have that $q(s^t) = V_1(s^t) - e(s^t) = \mathbb{E}\left[m(s^{t+1})\left(1 - \Delta(s^t) + x(s^t)\right)V_1(s^{t+1})|s_t\right]$.

wages, increasing the denominator of the relative future value of a product normalized by the current wage. Nevertheless, as seen in Equation (24) the value of owning a product is given by the expected discounted value of profits nets of innovation costs, and both profits and R&D costs increase with a TFP shock. Once again, given the persistence of TFP shock, the ratio of future profits and future output to current wages conditional on a TFP shock is relatively stable. Thus, TFP shocks are not a promising channel to generate endogenous trend dynamics.

In contrast to a TFP shock, an interest rate shock does not directly affect profits or the cost of innovation.[11] Therefore, the numerator and denominator of the relative future value of a product normalized by the current wage of the economy are mostly unaffected by an interest rate shock. Nevertheless, the interest rate shock is the main driver of the stochastic discount factor of the household. Thus, an interest rate shock has a direct impact on the innovation rate of the economy and constitutes the main driver of the endogenous productivity trend. Moreover, the expected discount factor is persistently affected by the stochastic volatility shock on the interest rate, thus, generating a pass-through between stochastic volatility and trend dynamics.

Note that, both shocks can have further effects by triggering a current or future violation of the constraint in Equation (36). In fact, lower firm values trigger a decrease in the borrowing capacity of the household. If the constraint binds today, this can trigger a sharp recession. Moreover, when the expectation of a future binding constraint arises, Equation (20) shows that the stochastic discount factor increases, further affecting the rate of creative destruction in the economy and creating permanent effects on productivity. Thus, while TFP shocks generally are not a key driver of hysteresis, large shocks affecting the likelihood of future crises can have permanent effects mediated by the discount factor. Furthermore, a Fisherian deflation mechanism is also at work in our framework, as

[11] Because we abstract from working capital constraints. interest rate shocks do not affect profits or wages. With a working capital constraint, an interest rate shock acts as a TFP shock in wages and values, thus, interest rate shocks mediated by working capital constraints have little effect on endogenous trend dynamics.

higher discount factors further decrease the firm's value exacerbating the tightness of the borrowing constraint.

4 Quantitative Analysis

In this section, we calibrate the model to quarterly Mexican data and illustrate how stochastic volatility in interest rates can generate volatile trend dynamics and economic hysteresis.

4.1 Calibration

The model has 18 parameters to calibrate. We divide these parameters into two groups. The first set of 11 parameters is externally calibrated to common values in the literature and used to match time series dynamics of the Mexican economy. Table 2 shows the externally calibrated values. First, we estimate the process of the U.S. interest rate directly from the data, as described in Section 2. We set the average interest rate to 4% annually, which is equivalent to roughly 1% every quarter. We directly estimate the productivity process for Mexican data.

We set the innovation curvature of incumbents and entrants equal to 0.5, a value standard in the endogenous growth literature (Akcigit and Kerr, 2018). We assume a CRRA utility function with a curvature of $\gamma = 2$, which is also standard in the macro literature. The last externally calibrated parameter is the depreciation of ideas, which is set to be equal to 1% every quarter. From a growth perspective, this value implies a lower bound for productivity growth of -1% per quarter. This is, without any investment in the creation of new firms and expansion by incumbents, the efficiency index would shrink at rate δ.

Parameter	Symbol	Value	Source
Average US Interest Rate	$\bar{R}$	$(1.04)^{0.25}$	Estimation, US data
Persistence of Interest Rate	ϕ_R	0.94	Estimation, US data
Average Interest Rate Volatility	μ_σ	0.16	Estimation, US data
Persistence of Interest Rate Volatility	ρ_σ	0.97	Estimation, US data
Volatility of Interest Rate Volatility	η_σ	0.05	Estimation, US data
Persistence of TFP Shock	ρ_z	0.95	Estimation, Mexican data
Volatility of TFP Shock	η_z	0.02	Estimation, Mexican data
Innovation curvature cost by incumbents	θ	0.50	Akcigit and Kerr (2018)
Innovation curvature cost by entrants	υ	0.50	Akcigit and Kerr (2018)
Utility function curvature	γ	2	Standard
Depreciation of ideas	δ	0.01	Max A contraction

Table 2: Externally Calibrated Parameters

Table 3 presents a summary of our 7 internally calibrated parameters. The step size σ is set to 0.22, which implies an average annualized growth rate of 2%, consistent with Mexican data. We set the cost level of entry κ to be equal to 16, which generates an annualized 9% entry rate (again, consistent with the Mexican case). The R&D cost level ξ is set to 0.49 so that the average incumbent firm size relative to entrants is 3. The mass of products Λ is normalized to 3, so we have a unit mass of firms in the balanced growth path. We set the discount factor β to be 0.96, which generates an implied net-foreign-assets-to-GDP ratio close to 33% annually, following the long-run average for Mexico presented in Lane and Milesi-Ferretti (2017). We calibrate the collateral coefficient of the borrowing constraint ϕ to be 0.59, so that the model delivers a 3% annual probability of a crisis, in line with the Sudden Stops literature.[12] Lastly, we set ι, the parameter that rules the representative financial intermediary's discount factor, to be 2.47 in order to generate an average spread of 3%, consistent with the observed Mexican data.

Because of the nonlinear nature of a model featuring occasionally binding constraints, we rely on global methods to solve it. We employ a hybrid method that combines the time-iteration method and value function iteration. Appendix C presents a detailed description of the solution method.

[12]We define a crisis or a Sudden Stop as an event where the current account-to-GDP ratio is two standard deviations above its long-run mean and when the borrowing constraint binds. This definition is standard and employed extensively in the literature (Bianchi et al., 2016; Rojas and Saffie, 2022).

Parameter	Symbol	Value	Main Identification	Target
Step Size	σ	0.22	Annual Growth rate	2%
Cost level Entry	κ	16	Annual Entry rate	9%
Cost level R&D	ξ	0.49	Avg. Firm Size relative to entrants	3
Mass of Products	Λ	3	Unit mass of firms	N/A
Discount Factor	β	0.96	Annual Debt-to-GDP Ratio	33%
Borrowing Constraint	ϕ	0.59	Annual Prob. of Crisis	3%
Financial Intermediary Discount Factor	ι	2.47	Avg. Spread	3%

Table 3: Internally Calibrated Parameters

4.2 Stochastic Volatility and Trend Cycles

Our purpose is to assess whether our model can generate patterns like the ones described in Section 2. In order to achieve this, we use model-simulated data to estimate impulse response functions, where the shock is an increase in the volatility of the world interest rate. We estimate these responses by simulating the model for 1,140 periods and discarding the first 1,000 to eliminate any dependence from initial conditions. The purpose is to generate simulated data with the same length as its empirical counterpart.[13] We follow the same estimation approach used in Section 2.

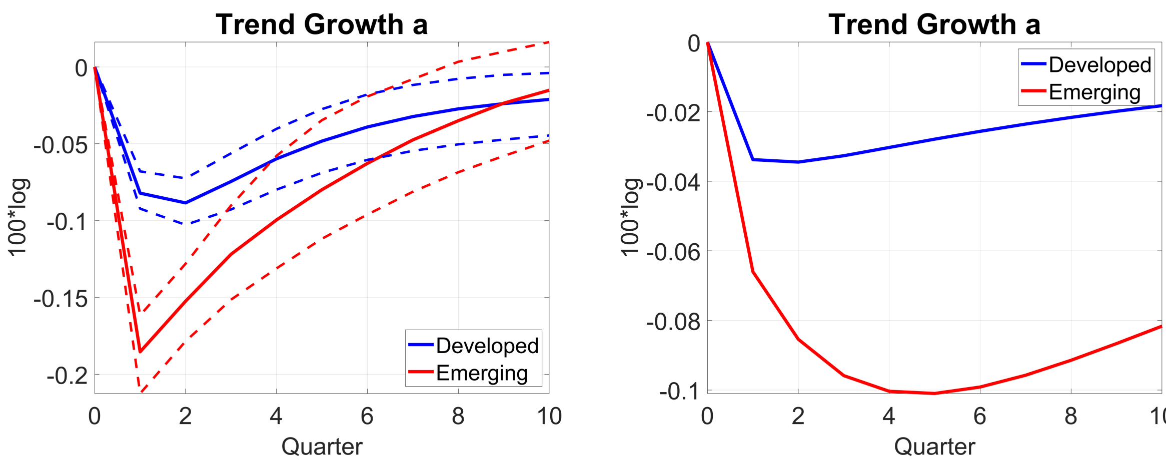
Since we calibrate the model to match moments of the Mexican economy, we still need an advanced economy to contrast the responses of the trend growth to stochastic volatility shocks. In order to do so, we consider an alternative economy which is an *advanced* version of the Mexican economy. Specifically, we take the baseline model and recalibrate the fraction of collateral that can be pledged when borrowing ϕ and the pass-through of volatility to borrowing costs ι. We set $\phi = 0.7$ so that the economy experiences a financial crisis with a 1% probability, roughly consistent with developed economies (Bianchi and Mendoza, 2020), and we set $\iota = 0.85$ so that the average spread is 1%, in line with empirical observations.

Figure 4 presents the impulse response functions of the trend growth component to

[13] We include the following variables in the simulated data exercise: GDP, consumption trade balance, entry costs, world interest rate, world interest rate volatility, and country spread (difference between effective borrowing cost for households net of the world interest rate). Since our model does not have capital and hence investment, we proxy the latter by entry costs.

a volatility shock of the world interest rate, for the empirical and model-simulated approaches (in order to directly contrast them).

The responses illustrated in panel (b) Figure 4 shows 2 important patterns: (1) the trend component in the developed economy (this is, where the average spread is lower and where the financial sector is more developed) experiences substantially less negative effects from a stochastic volatility shock than its counterpart (where trough vs. trough we have a roughly 150% amplification), and (2) the magnitudes of the declines are consistent with the ones found in the data, despite not being a target of the calibration. These two findings speak about the relative success that our model has in terms of explaining the dynamics of the trend growth in response to stochastic volatility shocks in the U.S. interest rates.



(a) Data Estimated Responses (b) Model Estimated Responses

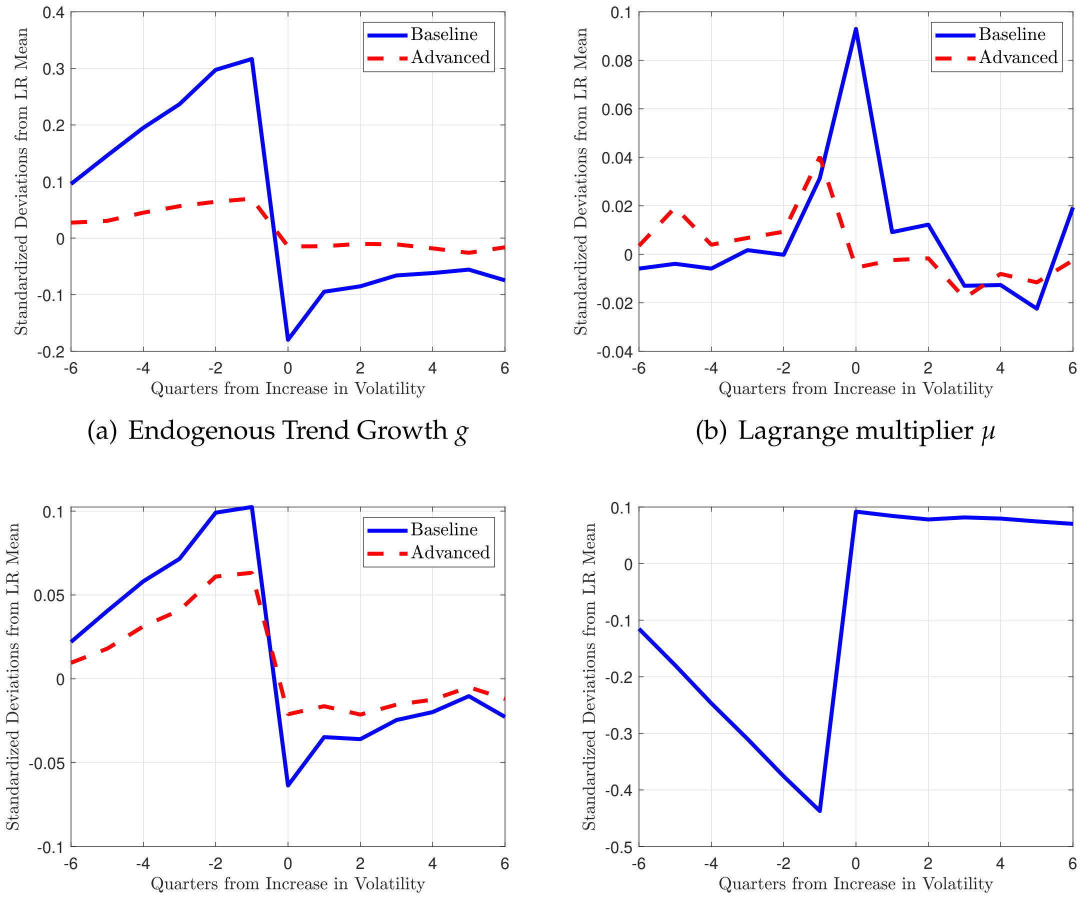
Figure 4: Impulse Response Functions

Notes: Panel (a) presents the impulse response functions of the trend growth component to a one standard deviation stochastic volatility shock of the U.S. interest rate, for advanced and emerging economies. These responses are estimated following the approach described in Section 2. Panel (b) estimates responses following a similar approach but using model-simulated data.

How do increases in volatility translate into worse macroeconomic outcomes for an emerging economy like Mexico? We now proceed to answer this question. Since there are macroeconomic outcomes that are not present in our empirical specification (and are hardly observed empirically), we use model-simulated data to perform an event analysis. We define an event as an increase in volatility.[14] Given the large persistence in the stochastic volatility process, this implies that an event occurs nearly 3.3% of the time (note that the frequency is quarterly). Once we define the event, we average the series for the endogenous trend growth, the Lagrange multiplier, the value of a product line, and stochastic volatility, for 13 quarters (6 before the event, at the event, and 6 after the event). Figure 5 presents the dynamics of the aforementioned variables around the event for the baseline and advanced economy calibrations.

Panel (a) in Figure 5 shows the dynamics of the endogenous trend growth. A similar pattern to the one observed in Figure 4 arises, where the endogenous trend growth declines at the moment of the volatility hike. Note that the size of the decline is much sharper for the baseline case, calibrated to an emerging economy, where a volatility hike implies a drop of nearly 0.5 standard deviations in comparison to a decline of 0.1 standard deviations for advanced economies. The average volatility hike is of nearly half a standard deviation of σ^r, so the implied "elasticity" of the trend growth to volatility hikes is 1 versus 0.5 in the case of advanced economies. Thus, we can say that the trend growth component in emerging economies is twice as sensitive to volatility hikes than in advanced economies. Another noticeable feature observed in Panel (a) is the long-lasting effects on the trend growth. Six quarters after the volatility hike, the trend growth is roughly 0.1 standard deviations below its long-run mean (for both economies). Given the high persistence of the volatility process, a hike implies higher uncertainty in the borrowing costs for a prolonged period of time, as shown in Panel (d). This persistent increase depresses the trend growth of the economy for a similar time period.

[14] The average increase in volatility in our simulated data is roughly half a long-run standard deviation. Appendix D.1 presents a robustness analysis where we define an event as an increase in volatility of more than one standard deviation. The likelihood of these events is substantially lower, around 0.5% of the time, because of the large persistence in the volatility process. Our results show that the effect of volatility is more pronounced than the one we observe in this section.



(a) Endogenous Trend Growth g

(b) Lagrange multiplier μ

(c) Value of a Product Line V_1

(d) Stochastic Volatility σ^r

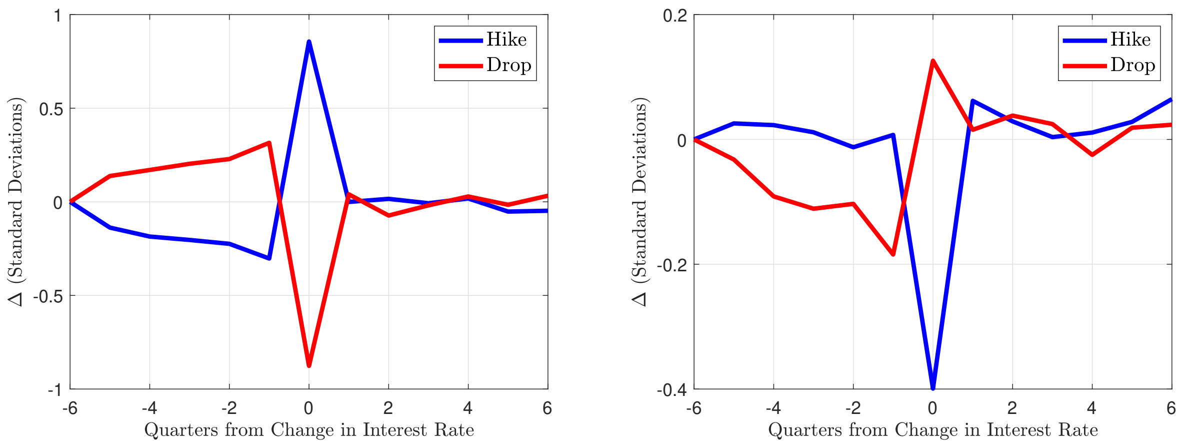
Figure 5: Volatility Hike Dynamics - Event Study

Notes: A volatility hike is defined as an increase in volatility σ^r. The average volatility hike is 0.14 percentage points, roughly 50% of the standard deviation of the volatility process. Standardized deviations from the long-run (LR) mean correspond to deviations scaled by the long-run standard deviation of the corresponding process.

The mechanism described in Section 3.9 can help us understand the reason why prolonged periods of volatility lead to persistent growth losses in the long run, and why it affects emerging economies the most. To see this more clearly, we study the dynamics of the Lagrange multiplier associated with the borrowing constraint (Panel (b)) and the value of a product line (Panel (c)). In the baseline calibration, we see that the volatility hike causes a deterioration of outcomes via two channels. First, the hike generates a sharp tightening in the borrowing constraint, which leads to an abrupt deleveraging of households and heavier discounting of the future (Equation (20)). Second, the increase in volatility raises borrowing costs in the economy, which causes a further deterioration in the debt position of households in the economy. In the case of an emerging economy, the two effects are present, while for the advanced only the latter is as the multiplier of the borrowing constraint does not experience any changes (Panel (b)). The effect of the tighter constraint explains in part the larger adjustment in the value of a product line in an emerging economy (nearly twice the relative adjustment), which ultimately leads to fewer innovation efforts (Equations (25) and (26)) and thus a weaker trend growth (Equation (27)).

Increases in the volatility of the US interest rate lead to a decline in the trend growth, caused by a decline in the value of owning a product line along with a tightening of borrowing conditions. To further understand the connection between interest rate volatility and trade dynamics we study the effects of interest rate fluctuations of different sizes and signs. In periods of high volatility, the average size of an interest rate increase (decrease) is larger conditional on observing an interest rate hike (drop).[15] In other words, during periods of high volatility, interest rate shocks are larger on average, but symmetric (same size in absolute value for hikes and drops). Interestingly, this symmetry in interest rate fluctuations does not generate symmetric responses in our model. Figure 6 illustrates this key asymmetry. Panel (a) presents the (model-simulated) dynamics of the world interest rate 6 quarters before and after a hike (blue line) or a drop (red line), where a hike (drop) is defined as an increase (decrease) in the interest rate. Panel (b) presents the dynamics of the trend growth rate for interest rate hikes and drops as well. For illustration purposes, we focus on interest rate changes that occur during periods of high volatility (interest rate volatility is one standard deviation above its long-run mean).

[15] To see this more clearly, we have that the average hike conditional on $\sigma(s^t)$ is $\mathbb{E}[\omega(s^t)\exp(\sigma(s^t))|\omega(s^t) > 0, \sigma(s^t), s^t]$. Note that we are using the same notation as the one presented in Section 3.3. Since $\omega(s^t) \sim N(0,1)$, we can use the truncated normal formula for a conditional expectation and have that $\mathbb{E}[\omega(s^t)\sigma(s^t)|\omega(s^t) > 0, s^t] = \sigma(s^t)\frac{\phi(0)}{1-\Phi(0)}$. Conditional on observing an interest rate hike, the expected hike is increasing in interest rate volatility $\sigma(s^t)$. A similar reasoning can be applied to interest rate cuts.



(a) Interest Rate R (b) Endogenous Trend Growth g

Figure 6: Interest Rate Fluctuations and Asymmetric Responses of the Trend Growth Rate

Notes: Variables are expressed as changes with respect to their previous period value. Changes are scaled by their corresponding long-run standard deviations. The figure considers increases and decreases of the interest rate that are conditional on interest rate volatility being one standard deviation above its long-run mean.

The observed dynamics of interest rates are symmetric: the magnitude of hikes and drops is identical. The symmetry does not hold for the endogenous trend growth. Specifically, an interest rate hike leads to a much larger adjustment in the trend growth rate than the adjustment generated by an interest rate cut of the same absolute size. Financial frictions are key when explaining this asymmetry. When the constraint binds, an interest rate hike leads to a deleveraging of households, which translates into lower present consumption, a heavier discounting of the future, and hence a decline in the trend growth rate and in the value of a product line. Our model features a Fisherian deflationary mechanism, which amplifies this cycle: as the value of the product line decreases, so does the collateral value of the household. This triggers a further tightening of the borrowing constraint, feeding back into the cycle and generating substantial nonlinearities that lead to the aforementioned amplification. When households experience an interest rate cut, the amplification mechanism is much weaker as the multiplier is bounded by 0. Thus, a more volatile interest rate foretells a persistent period of large swings in interest rates. Because of the asymmetric effect of large swings, the expected value of a firm decreases when the volatility regime changes. A lower expected firm value explains the slow down in innovation effort that triggers the trend dynamics depicted in Figure 5.

A prediction of our model is that the asymmetry between interest rate hikes and cuts should weaken or disappear as interest rate volatility shrinks because the average size of an interest rate shock decreases. Table 4 presents changes between the period where the interest rate hike or cut occurred and the period preceding it, for the endogenous trend growth, the value of a product line, and the Lagrange multiplier of the borrowing constraint, for different levels of volatility. The Table shows that the asymmetry between interest rate hikes and drops is the highest in periods of high volatility. When the volatility is close to its long-run average, interest rate hikes and drops generate symmetric effects. When volatility is low, interest rate shocks have weak effects on the reported variables.[16]

Scenario	Δg		ΔV_1		$\Delta \mu$		ΔR	
	Hike	Drop	Hike	Drop	Hike	Drop	Hike	Drop
Low Volatility	0.07	0.04	0.03	0.02	-0.02	0.02	0.39	-0.39
Average Volatility	-0.13	0.12	-0.05	0.04	0.04	-0.03	0.52	-0.52
High Volatility	-0.40	0.13	-0.14	0.05	0.06	-0.01	0.86	-0.88

Table 4: Interest Rate Fluctuations and Stochastic Volatility

Notes: Changes in the trend growth rate g, value of a product line V_1, Lagrange multiplier of the borrowing constraint μ, and world interest rate R are expressed in standard deviations. We identify periods with interest rate hikes and drops in our model-simulated data and average changes in the reported variables at the moment of the interest rate movement. Low (high) volatility regimes are defined as those in which the volatility of the interest rate is lower (higher) than its long-run mean minus (plus) one standard deviation.

In conclusion, we observe that increases in volatility lead to declines in the growth rate of the trend of the economy, consistent with our empirical analysis. Higher volatility leads to a worsening in the financial conditions faced by households in the economy, which leads to deleveraging, heavier discounting of the future, and a subsequent decline in the value of a product line and trend growth rate. The transmission of higher volatility to these outcomes occurs via interest rate fluctuations, which happen to be larger in these episodes. Interest rate hikes have much larger effects than interest rate drops when

[16] An additional prediction of our model is that if financial frictions are ameliorated, the effects of interest rate hikes and drops should be more symmetric in periods of high volatility. We repeat the same exercise in this section but for an economy whose collateral constraint coefficient is the one of an advanced economy, holding every other parameter constant. Results are presented in Appendix D.2. We see that the prediction is consistent with the observed behavior of this alternative economy.

volatility is high due to the presence of the collateral constraint and Fisherian deflation mechanism associated with it.

4.3 Interaction Between Stochastic Volatility and Financial Frictions

The previous section highlighted the asymmetrical responses of the trend growth to hikes in the volatility of the world interest rate across economies. The main source of these asymmetries is the interaction between the tightness of the borrowing constraint and borrowing costs (higher spreads). In this section, we attempt to disentangle the role that these two features play in the dynamics of the trend growth.

We start by generating an alternative economy which is an intermediate point between the baseline (targeted to an emerging economy) and the advanced economy calibrations. We relax the borrowing constraint of the baseline economy so that the collateral coefficient ϕ is equal to the one of the advanced economy calibration, but the rest of the parameters remain the same. We denominate this calibration the "relaxed constraint" case. Thus, the main difference between the baseline and the relaxed constraint calibration is that in the latter we set the collateral pledgeability to be the one of an advanced economy, so discrepancies in the long-run moments can be attributed to the borrowing constraint. Differences in the long-run moments of the relaxed constraint scenario and the advanced economy can be attributed to the role of higher spreads.

Table 5 presents the long-run moments of the three economies. Relaxing the borrowing constraint brings substantial changes in the long-run moments of the economy: growth volatility, measured by the coefficient of variation, is 30% lower; the debt-to-GDP ratio increases by nearly ten percentage points of GDP and more stable (less volatile); the likelihood of a Sudden Stop is reduced modestly. Noticeably, the constraint binds much more often in the relaxed constraint economy. Since a crisis happens to be less likely in this setting, agents borrow more and face fewer negative consequences from being at the borrowing limit.

Moment	Baseline	Relaxed Constraint	Advanced
$\sigma(g)/\mathbb{E}(g)$	0.10	0.07	0.05
$\mathbb{E}(B^H/Y)$ %	-33.01	-42.98	-45.56
$\sigma(CA/Y)$ %	3.39	2.28	1.03
$\mathbb{E}$(Spread) %	3.00	3.00	1.04
$\Pr(\mu_t > 0)$ %	16.71	62.84	99.16
Pr. Crisis %	2.83	2.60	0.97

Table 5: Long-run Moments Across Specifications

Notes: Baseline corresponds to the base calibration model, the relaxed constraint model considers a collateral coefficient equal to the one employed in the advanced economy (all other parameters are those of the base calibration), and advanced is the advanced economy calibration. Moments related to g, B^H/Y, and CA/Y are annualized, as well as the probability of a crisis.

The comparison between the relaxed constraint and the advanced economies sheds light on the role that higher spreads play in the long run. The second and third columns of Table 5 show that higher spreads play a crucial role in explaining differences between emerging and advanced economies. Lowering the financial cost of borrowing leads to a more stable growth in the long run (30% less volatile). Also, lower spreads contribute to larger debt positions and a less volatile current account, which is intuitive. We also see that lower spreads are associated with lower probabilities of a financial crisis.

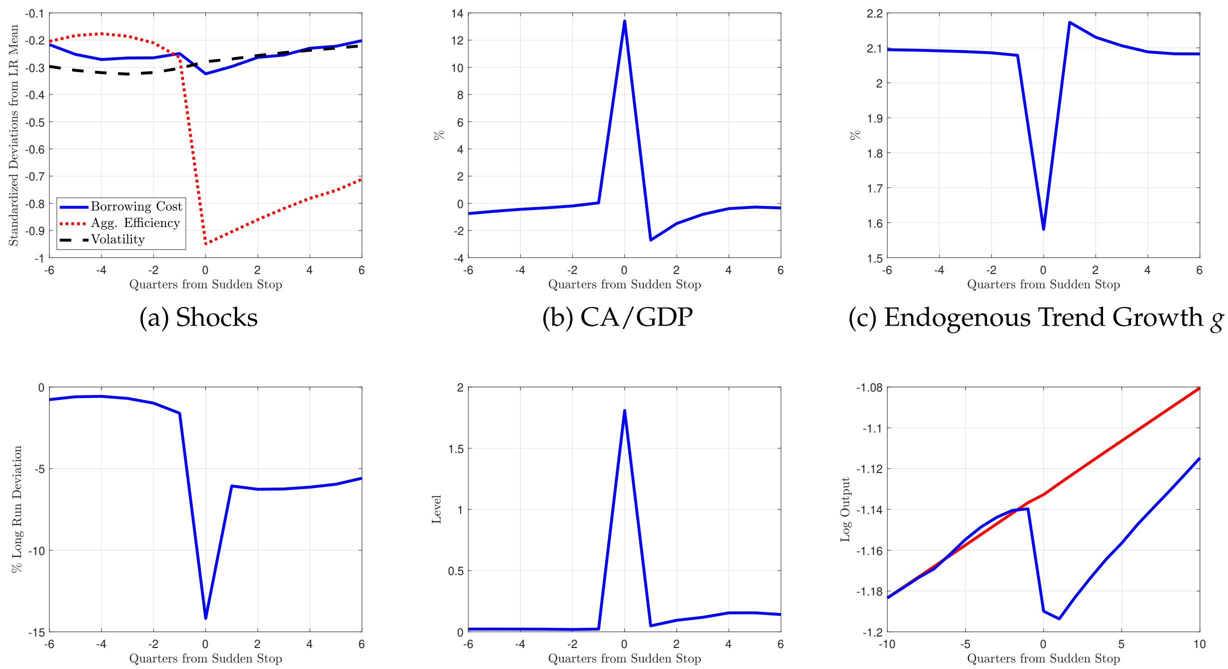
4.4 Persistent Effects of Sudden Stops

Our model exhibits a nonlinear propagation of shocks to macroeconomic outcomes. This is a feature of models with occasionally binding collateral constraints and endogenous collateral values, where a Fisherian deflation mechanism takes place (Mendoza, 2005; Mendoza, 2010). In this section, we show the dynamics of Sudden Stops. Moreover, given the endogenous growth component, we show that Sudden Stops have persistent effects on output.[17]

We perform an event analysis around Sudden Stops that the model endogenously generates, for our baseline calibration. We consider a time window of 10 quarters before and after the event, and we average out all relevant time series. Figure 7 presents the

[17] This is a feature that a few endogenous growth models with financial frictions can also deliver (Queralto, 2020; Ates and Saffie, 2021).

Sudden Stop dynamics.



(a) Shocks (b) CA/GDP (c) Endogenous Trend Growth g

(d) Value of a Product Line V_1 (e) Lagrange multiplier μ (f) Log GDP

Figure 7: Sudden Stop Dynamics

Notes: Sudden Stops are defined as events where the borrowing constraint binds and where the current account-to-GDP ratio is two standard deviations above its long-run mean. The three shocks of the model are standardized by their corresponding long-run means and standard deviations. CA/GDP denotes the current account-to-GDP ratio, and log GDP corresponds to the log of GDP in levels. The red solid line in the log GDP panel denotes the average trend, which is constructed by averaging the linear growth using the trend growth rate 10 quarters before a Sudden Stop.

Our analysis focuses on the Sudden Stop dynamics of 5 variables: the value of a product line, the Lagrange multiplier, the current account-to-GDP ratio, the endogenous trend, and log GDP (in levels). Panel (a) presents the dynamics of the three shocks of the model around the event. We see that, in our model, Sudden Stops are events that tend to occur when 3 conditions are met: (1) borrowing costs are below their long-run average, (2) interest rate volatility is also below its long-run average, and (3) large and sudden contractions in aggregate efficiency occur. Low and weakly dispersed borrowing costs fuel episodes of increased borrowing. Households in the economy do not fully internalize the consequences of their borrowing decisions so they overborrow.[18] This especially happens in periods where borrowing conditions are favorable. Efficiency shocks tend to be long-lived, so the combination of highly levered agents with protracted declines in productivity leads to sharp adjustments in borrowing, as shown in panel (b), where a sharp current account reversal takes place.

Panel (c) illustrates what happens with the trend growth: a decline of roughly 0.5 percentage points (annualized), followed by a quick recovery. Note, however, that the value of a product line (panel (d)) experiences a sharp decline but it does not return back to its pre-Sudden Stop level. Low aggregate efficiency as well as a borrowing constraint that binds with a higher likelihood post-event (panel (e)) explain this behavior. A depressed value of a product line also implies that the collateral value will remain lower than its long-run average, generating a slow recovery.[19] Finally, panel (f) shows very clearly the long-lasting consequences (i.e. hysteresis) of a Sudden Stop: output experiences an abrupt and permanent deviation from its pre-Sudden Stop trend.

5 Conclusion

In this paper we provide new facts linking temporary surges in uncertainty in world interest rates to permanent losses in the level of economic activity. We show that the uncertainty hikes have a more pronounced effect on the trend growth of EMEs relative to advanced economies. We conjecture that financial frictions are key in explaining this asymmetry. We propose a small open economy model that features endogenous growth and occasionally binding borrowing constraints, where households post intangible assets as collateral for borrowing. We assume that EMEs and advanced economies (AEs) differ

[18] This is a traditional feature of models with occasionally binding borrowing constraints and endogenous collateral values. See Bianchi (2011) and Bianchi and Mendoza (2018) for an extensive characterization of the pecuniary externality in settings with endowment and production models, respectively.

[19] It is important to note that our framework is able to generate slow recoveries and a borrowing constraint that binds for several periods *after* the Sudden Stop. This is a novel result since this was a feature of models with capital accumulation, where the value of capital is used as collateral for borrowing.

in two dimensions: (1) the fraction of their assets that is pledgeable as collateral, and (2) the degree of risk aversion that lenders experience at the time of issuing loans.

We show that our framework is able to replicate the patterns observed in the data without relying on exogenous shocks to the trend or an artificially correlated TFP to the U.S. real interest rate. Moreover, the occasionally binding constraint that the model features generates state-dependent responses to interest rate fluctuations. In particular, we see that large increases in the interest rate generate a much larger adjustment in terms of the trend growth rate relative to the observed adjustment when a decline of a similar magnitude in the interest rate is materialized. This asymmetry is stronger when volatility is high, as individuals in the economy expect larger future fluctuations in financing costs.

We use our model to assess the magnitude of the main channels that generate the asymmetry in the response of the trend growth to uncertainty hikes. Specifically, we decompose the trend growth dynamics into a financial development component and a spread component. We find that financial development explains nearly 50% of the observed gap in the trend growth to volatility between EMEs and AEs, while the remaining fraction is explained by EMEs facing higher spreads than their advanced counterparts.

The fact that financial development explains a fraction of the asymmetric response of the trend growth to volatility hikes opens an avenue for exploring the role that macroprudential policy can play in helping EMEs become more resilient to this kind of shocks. We plan on pursuing this direction in future work. In addition, we plan to investigate how differences in the distributions of entrance and firms across countries might contribute to explaining the heterogeneous responsiveness of trend growth.

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Appendices

Appendix A Calculating the Firm Size Distribution

Because there is a continuum of products we can use the law of large numbers to track the distribution of firms. In particular, denote by $\Omega_n(s^t)$ the mass of firms with n products. The law of motion of this distribution is characterized by a system of dynamic equations that only depend on the innovation rate of incumbents and the mass of entrants:

$$
\begin{aligned}
\Omega_1(s^t) &= \left(M^*(s^{t-1})\right)^v + \sum_{n=1}^{\infty} \Omega_n(s^{t-1}) \sum_{k=0}^{1} \mathbb{B}\left(k, n, x^*(s^{t-1})\right) \mathbb{B}\left((k+n-1, n, \Delta(s^{t-1})\right) \\
\Omega_{\tilde{n}>1}(s^t) &= \sum_{n=\mathbb{I}^+(\frac{\tilde{n}}{2})}^{\tilde{n}} \left\{\Omega_n(s^{t-1}) \sum_{k=\tilde{n}-n}^{n} \mathbb{B}\left(k, n, x^*(s^{t-1})\right) \mathbb{B}\left(k-(\tilde{n}-n), n, \Delta(s^{t-1})\right)\right\} \\
&+ \sum_{n=\tilde{n}+1}^{\infty} \left\{\Omega_n(s^{t-1}) \sum_{k=0}^{\tilde{n}} \mathbb{B}\left(k, n, x^*(s^{t-1})\right) \mathbb{B}\left(k-(\tilde{n}-n), n, \Delta(s^{t-1})\right)\right\},
\end{aligned}
$$

where $\mathbb{I}^+(a)$ refers to the integer closest to a such that $\mathbb{I}^+(a) \geq a$.[20] The first equation has two terms, the first one tracks the entry of new firms with one product while the second term tracks firms that used to have more than 1 product and contracted to exactly 1 product. The second line shows an analogous law of motion for categories of firms with more than one product where the first component are firms that started with less than $\tilde{n}$ products and had net gains that left them at $\tilde{n}$, while the second term reflects firms that were above $\tilde{n}$ and experienced net losses that left them exactly at $\tilde{n}$.[21] The BGP distribution can be found by iterating in these laws of motion until the mass of firms in each product category is constant.

[20] A firm with n products can become a firm with $n' \in [0, 2n]$ in one period, so, at most it can double its size in one period.

[21] The condition that ensures that the mass of products is always equal to one is given by $\sum_{n=1}^{\infty} n\Omega_n(s^t) = \Lambda$.

Appendix B Dynamic System of Equations

B.1 Representative Household

$$
\begin{aligned}
\tilde{m}(s^t, s_{t+1}) &= \beta \frac{\tilde{C}(s_{t+1})^{-\gamma}}{\tilde{C}(s^t)^{-\gamma} - \phi\tilde{\mu}(s^t)}(1 + g(s^t, s_{t+1}))^{-\gamma} \\
\tilde{C}(s^t)^{-\gamma} &= \tilde{\mu}(s^t) + \frac{\beta}{\hat{q}(s^t)}\mathbb{E}\left[\tilde{C}(s^{t+1})^{-\gamma}|s^t\right](1 + g(s^t, s_{t+1}))^{-\gamma} \\
\tilde{C}(s^t)^{-\gamma} &= \phi\tilde{\mu}(s^t) + \beta\mathbb{E}\left[\tilde{C}(s^{t+1})^{-\gamma}\tilde{R}^q(s^{t+1})|s^t\right](1 + g(s^t, s_{t+1}))^{1-\gamma} \\
\tilde{C}(s^t) &= \tilde{w}(s^t) + \tilde{B}^H(s^{t-1}) + \Lambda\tilde{e}(s^t) - \hat{q}(s^t)\tilde{B}^H(s^t)(1 + g(s^t, s_{t+1})) \\
\hat{q}(s^t)\tilde{B}^H(s^t)(1 + g(s^t, s_{t+1})) &\geq -\phi\Lambda(\tilde{V}_1(s^t) - \tilde{e}(s^t)).
\end{aligned}
$$

B.2 Final Good Producer

$$
\tilde{Y}(s^t) = e^{z(s^t)}l(s^t)
$$

B.3 Intermediate Good Producer

$$
\begin{aligned}
l(s^t) &= \frac{\tilde{Y}(s^t)}{\tilde{w}(s^t)\Lambda(1+\sigma)} \\
\tilde{\pi}(s^t) &= \frac{\sigma}{\Lambda(1+\sigma)}\tilde{Y}(s^t) \\
\tilde{V}_1(s^t) &= \tilde{\pi}(s^t) - \tilde{w}(s^t)\left(\frac{x(s^t)}{\xi}\right)^{\frac{1}{\theta}} + \mathbb{E}\left[\tilde{m}(s^{t+1})(1 - \Delta(s^t) + x(s^t))\left(1 + g\left(s^t, s_{t+1}\right)\right)\tilde{V}_1(s^{t+1})|s^t\right] \\
x(s^t) &= \left[\theta\xi^{\frac{1}{\theta}}\frac{\mathbb{E}\left[\tilde{m}(s^{t+1})\left(1 + g\left(s^t, s_{t+1}\right)\right)\tilde{V}_1(s^{t+1})\middle| s_t\right]}{\tilde{w}(s^t)}\right]^{\frac{\theta}{1-\theta}} \\
l_r(s^t) &= \left(\frac{x(s^t)}{\xi}\right)^{\frac{1}{\theta}}
\end{aligned}
$$

B.4 Entry

$$M^*(s^t) \quad = \quad \left[\frac{v}{\kappa}\frac{\mathbb{E}\left[\tilde{m}(s^{t+1})\left(1+g\left(s^t,s_{t+1}\right)\right)\tilde{V}_1(s^{t+1})|s^t\right]}{\tilde{w}(s^t)}\right]^{\frac{1}{1-v}}$$

B.5 Aggregate Variables

$$\begin{aligned}
1+g(s^t,s_{t+1}) &= (1+\sigma)^{\Delta}(1-\delta) = (1+\sigma)^{x^*(s^t)+\frac{(M^*(s^t))^v}{\Lambda}}(1-\delta) \\
\tilde{t}(s^t) &= \Lambda\left(\tilde{\pi}(s^t)-\tilde{w}(s^t)l_r(s^t)\right)-\kappa M^*(s^t)\tilde{w}(s^t) \\
\tilde{w}(s^t) &= \frac{e^{z(s^t)}}{\Lambda(1+\sigma)} \\
1 &= \Lambda\left(l(s^t)+l_r(s^t)\right)+\kappa M^*(s^t)
\end{aligned}$$

B.6 Exogenous Shocks

$$\begin{aligned}
\ln z(s^t) &= \rho_z \ln z(s^{t-1})+\epsilon_t, \quad \text{with} \quad \epsilon_t \sim N(0,\eta_z^2) \\
R(s^t)-\bar{R} &= \phi_R\left(R(s^{t-1})-\bar{R}\right)+\exp(\sigma(s^t))\omega(s^t), \quad \text{with} \quad \omega(s^t)\sim N(0,1), \\
\log(\sigma(s^t)) &= (1-\rho_\sigma)\mu_\sigma+\rho_\sigma\log(\sigma(s^{t-1}))+\eta_\sigma\nu(s^t), \quad \text{with} \quad \nu(s^t)\sim N(0,1),
\end{aligned}$$

Appendix C Solution Method

Our solution algorithm for the decentralized equilibrium follows a combination of the time-iteration method employed in Bianchi et al. (2016) with value function iteration. We extend it in order to update guesses for the continuation value of a product line.

We discretize the processes for aggregate efficiency z, interest rate R and volatility σ_r using following Tauchen (1986). We consider grids of 7 points for aggregate efficiency, 11 points for the interest rate, and 9 points for interest rate volatility. Our bond holdings grid consists of 50 points, and is skewed towards larger debt holdings.

In what follows we drop the superscript H from bond holdings B^H in order to save notation. All functions presented below are stationary, unless otherwise noted. We start with a conjecture for the bond holdings policy function, B', defined over the state space (z, R, σ_r, B).[22] We also make a guess for innovation intensity $x(z, R, \sigma_r, B)$.

The steps of the solution algorithm are the following:

1. Start iteration j with a guess for $B'_j(z, R, \sigma_r, B)$ and innovation intensity $x_j(z, R, \sigma_r, B)$. Using these guesses construct:

$$M_j(z, R, \sigma_r, B) = \left[\frac{x_j(z, R, \sigma_r, B)^{\frac{1-\theta}{\theta}}}{\xi^{\frac{1}{\theta}}\theta}\frac{\nu}{\kappa}\right]^{\frac{1}{1-\nu}} \tag{37}$$

$$g_j(z, R, \sigma_r, B) = (1+\sigma)^{x_j(z,R,\sigma_r,B)+\frac{M_j(z,R,\sigma_r,B)^{\nu}}{\Lambda}}(1-\delta) - 1 \tag{38}$$

$$\Delta_j(z, R, \sigma_r, B) = x_j(z, R, \sigma_r, B) + \frac{M_j(z, R, \sigma_r, B)^{\nu}}{\Lambda} \tag{39}$$

$$l_{rj}(z, R, \sigma_r, B) = \left(\frac{x_j(z, R, \sigma_r, B)}{\xi}\right)^{\frac{1}{\theta}} \tag{40}$$

[22]Note that this guess corresponds to a matrix with dimensions $N_z \times N_R \times N_\sigma \times N_B$, where N_z, N_R, N_σ and N_B correspond to the number of elements in the grid of aggregate efficiency, interest rate, volatility of interest rate and debt, respectively.

$$l_j(z,R,\sigma_r,B) = \frac{(1-\kappa M_j(z,R,\sigma_r,B))}{\Lambda} - l_{rj}(z,R,\sigma_r,B) \tag{41}$$

$$Y_j(z,R,\sigma_r,B) = z l_j(z,R,\sigma_R,B) \tag{42}$$

$$\pi(z,R,\sigma_r,B) = \frac{\sigma}{\Lambda(1+\sigma)} Y_j(z,R,\sigma_r,B) \tag{43}$$

$$t_j(z,R,\sigma_r,B) = \Lambda(\pi_j(z,R,\sigma_r,B) - w(z) l_{rj}(z,R,\sigma_r,B)) - \kappa M_j(z,R,\sigma_r,B) w(z) \tag{44}$$

$$w(z) = \frac{\exp(z)}{\Lambda(1+\sigma)} \tag{45}$$

$$c_j(z,R,\sigma_r,B) = w(z) + B - B'_j(z,R,\sigma_r,B)(1+g_j(z,R,\sigma_r,B))\hat{q}(R,\sigma_r) + t_j(z,R,\sigma_R,B) \tag{46}$$

Lastly, compute the discounted expected marginal utility

$$\beta \frac{\beta}{\hat{q}(R,\sigma_r)} \mathbb{E}\left[u_j(z',R',\sigma_r',B'_j(z,R,\sigma_r,B))\right](1+g_j(z,R,\sigma_r,B))^{-\gamma}, \tag{47}$$

where $u_j(z,R,\sigma_r,B) = c_j(z,R,\sigma_r,B)^{-\gamma}$.

2. Using the above guesses compute guess for the value of a product line and the stochastic discount factor. For the value or a product line we iterate over the following value function:

$$\begin{aligned} V_{1,k+1}(z,R,\sigma_r,B) &= \pi_j(z,R,\sigma_r,B) - w(z)\left(\frac{x_j(z,R,\sigma_r,B)}{\xi}\right)^{\frac{1}{\theta}} \\ &+ \mathbb{E}\left[m_j(z,R,\sigma_r,B)(1-\Delta_j(z,R,\sigma_r,B) + x_j(z,R,\sigma_r,B))\left(1+g_j(z,R,\sigma_r,B)\right) V_{1,k}(z,R,\sigma_r,B)\right] \end{aligned} \tag{48}$$

Until $||V_{1,k+1}(z,R,\sigma_r,B) - V_{1,k}(z,R,\sigma_r,B)|| <$ tol. $V_{1,j}(z,R,\sigma_r,B)$ is the converged value function.

3. Assume the borrowing constraint binds. Note that when the constraint binds we

have that consumption is

$$c_{j+1}(z,R,\sigma_r,B) = w(z) + B + \phi\Lambda(V_{1,j}(z,R,\sigma_r,B) - (\pi_j(z,R,\sigma_r,B) - w(z)l_{rj}(z,R,\sigma_r,B))) \\ + t_j(z,R,\sigma_R,B) \tag{49}$$

We then check whether this assumption holds by calculating the residual of the Euler equation:

$$\mathcal{R}(z,R,\sigma_r,B) = u_{j+1}(z,R,\sigma_r,B) \\ - \beta\frac{\beta}{\hat{q}(R,\sigma_r)}\mathbb{E}\left[u_j(z',R',\sigma_r',B_j'(z,R,\sigma_r,B))\right](1+g_j(z,R,\sigma_r,B))^{-\gamma}. \tag{50}$$

If $\mathcal{R}(z,R,\sigma_r,B) > 0$, we keep the values for $c_{j+1}(z,R,\sigma_r,B)$, and we set $B_{j+1}'(z,R,\sigma_r,R) = -\frac{\phi\Lambda V_{1,j}(z,R,\sigma_r,B)}{(1+g_j(z,R,\sigma_r,B))\hat{q}(R,\sigma_r)}$, $\mu_{j+1}(z,R,\sigma_r,B) = \mathcal{R}(z,R,\sigma_r,B)$. Otherwise, the constraint does not bind for that point of the state space and we discard $c_{j+1}(z,R,\sigma_r,B)$ and $B_{j+1}'(z,R,\sigma_r,R)$. We then numerically solve for the value of $c_{j+1}(z,R,\sigma_r,B)$ that satisfies

$$u_{j+1}(z,R,\sigma_r,B) = \beta\frac{\beta}{\hat{q}(R,\sigma_r)}\mathbb{E}\left[u_j(z',R',\sigma_r',B_j'(z,R,\sigma_r,B))\right](1+g_j(z,R,\sigma_r,B))^{-\gamma} \tag{51}$$

We then set $B_{j+1}'(z,R,\sigma_r,B) = \frac{w(z)+B+t_j(z,R,\sigma_R,B)-c_{j+1}(z,R,\sigma_r,B)}{(1+g_j(z,R,\sigma_r,B))\hat{q}(R,\sigma_r)}$ and $\mu_{j+1}(z,R,\sigma_r,B) = 0$.

4. Using the updated guesses $c_{j+1}(z,R,\sigma_R,B)$ and $B_{j+1}'(z,R,\sigma_R,B)$, compute an updated guess for the stochastic discount factor:

$$m_{j+1}(z,R,\sigma_r,B) = \beta\frac{c_{j+1}(z,R,\sigma_r,B'(z,R,\sigma_r,B))^{-\gamma}}{c_{j+1}(z,R,\sigma_r,B)^{-\gamma} - \phi\mu_{j+1}(z,R,\sigma_r,B)}(1+g_j(z,R,\sigma_r,B))^{-\gamma} \tag{52}$$

5. Recompute the value of a product line using the updated stochastic discount factor.

This is, use value function iteration over the value of a product line:

$$V_{1,k+1}(z, R, \sigma_r, B) = \pi_j(z, R, \sigma_r, B) - w(z) \left(\frac{x_j(z, R, \sigma_r, B)}{\xi} \right)^{\frac{1}{\theta}}$$
$$+ \mathbb{E} \left[m_{j+1}(z, R, \sigma_r, B)(1 - \Delta_j(z, R, \sigma_r, B) + x_j(z, R, \sigma_r, B)) \left(1 + g_j(z, R, \sigma_r, B)\right) V_{1,k}(z, R, \sigma_r, B) \right] \tag{53}$$

Until $||V_{1,k+1}(z, R, \sigma_r, B) - V_{1,k}(z, R, \sigma_r, B)|| <$ tol. $V_{1,j+1}(z, R, \sigma_r, B)$ is the converged value function.

6. Update guess for innovation intensity of incumbents:

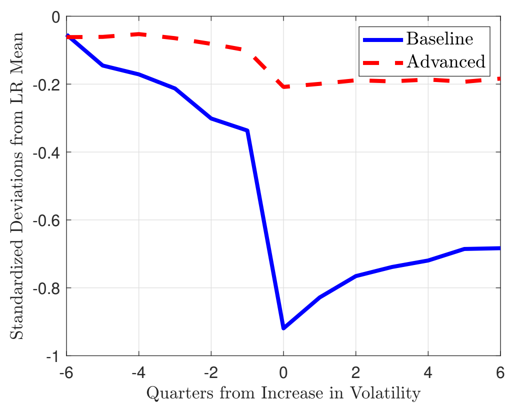
$$x_{j+1}(z, R, \sigma_r, B) = \left[\theta \xi^{\frac{1}{\theta}} \frac{\mathbb{E} \left[m_{j+1}(z, R, \sigma_r, R) \left(1 + g_j(z, R, \sigma_r, R)\right) V_{1,j+1}(z, R, \sigma_R, B) \middle| s_t \right]}{w(z)} \right]^{\frac{\theta}{1-\theta}} \tag{54}$$

7. Check for convergence. If $||B'_{j+1}(z, R, \sigma_r, B) - B'_j(z, R, \sigma_r, B)|| < \epsilon$ and $||x_{j+1}(z, R, \sigma_r, B) - x_j(z, R, \sigma_r, B)|| < \epsilon$ then the problem is solved. Otherwise, discard $B'_j(z, R, \sigma_r, B)$ and $x'_j(z, R, \sigma_r, B)$, and use $B'_{j+1}(z, R, \sigma_r, B)$ and $x'_{j+1}(z, R, \sigma_r, B)$ as new guesses for the problem (go back to step 1).

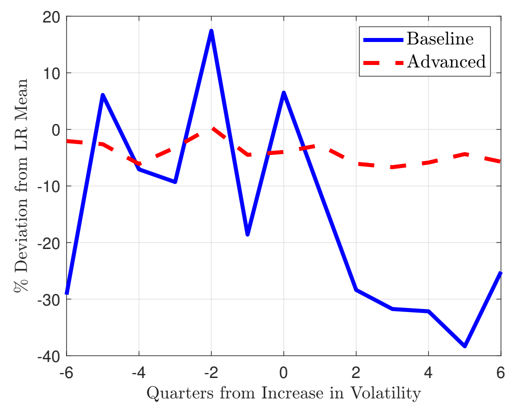
Appendix D Numerical Appendix

D.1 Robustness Analysis

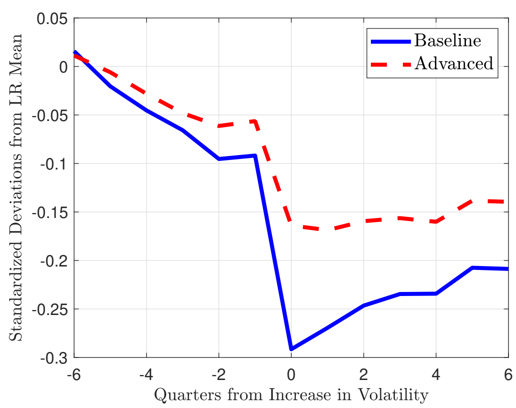
In this section presents a robustness analysis where we consider an alternative definition for the volatility hike event study. Specifically, we define a volatility hike event as one where the standard deviation of the interest rate is one standard deviation above its long run mean.[23] Figure 8 presents the event analysis.



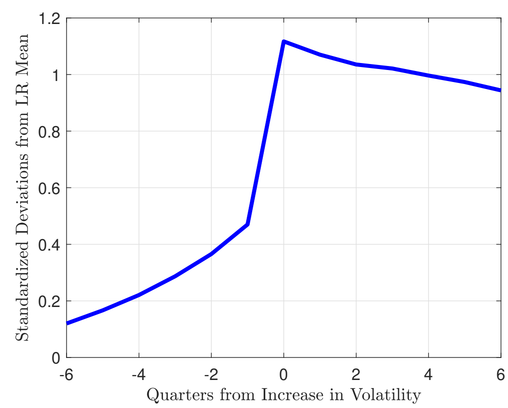
(a) Endogenous Trend Growth g



(b) Lagrange multiplier μ



(c) Value of a Product Line V_1



(d) Stochastic Volatility σ^r

Figure 8: Volatility Hike Dynamics - Event Study - Alternative Definition

Notes: A volatility hike is defined as an increase in volatility σ^r that exceeds the long-run standard deviation of σ^r. The average volatility hike is 0.17 percentage points, roughly 0.65 standard deviations of the volatility process. Standardized deviations from the

[23]Given the distribution of $\sigma(s^t)$, a one standard deviation corresponds to roughly a 1 percentage point increase in volatility.

long-run (LR) mean correspond to deviations scaled by the long-run standard deviation of the corresponding process.

We see similar qualitative features to the ones observed in Section 4.2: volatility hikes lead to larger declines in the trend growth rate of emerging economies, which is caused in part by (asymmetrically) tighter borrowing conditions and more aggressive collateral value adjustments. Notice that the trend growth and the value of a product line were already below their corresponding long-run averages, while the Lagrange multiplier fluctuates substantially before the event. The reason for this is that events where volatility hikes of this nature are situations in which volatility was already high, as we can see in Panel (d). Nevertheless, we see that we can draw similar conclusions as before, except that now adjustments seem to be even more pronounced across the two calibrations of interest.

D.2 Interest Rate Fluctuations - Relaxed Constraint Economy

In this section, we present the effects of interest rate hikes and drops and volatility for an economy with a relaxed borrowing constraint. In particular, we assume that the economy has a collateral coefficient equal to the one of the advanced economy calibration, but with all of the rest of the parameters being equal to those of the baseline calibration. Table 6 presents the results of the quantitative exercise.

Scenario	Δg		ΔV_1		$\Delta \mu$		ΔR	
	Hike	Drop	Hike	Drop	Hike	Drop	Hike	Drop
Low Volatility	0.05	0.01	0.03	0.03	-0.04	0.05	0.39	-0.39
Average Volatility	-0.07	0.06	-0.05	0.04	-0.01	0.02	0.52	-0.52
High Volatility	-0.25	0.07	-0.15	0.04	-0.00	0.05	0.86	-0.88

Table 6: Interest Rate Fluctuations and Stochastic Volatility

Notes: Changes in the trend growth rate g, value of a product line V_1, Lagrange multiplier of the borrowing constraint μ, and world interest rate R are expressed in standard deviations. We identify periods with interest rate hikes and drops in our model-simulated data and average changes in the reported variables at the moment of the interest rate movement. Low (high) volatility regimes are defined as those in which the volatility of the interest rate is lower (higher) than its long-run mean minus (plus) one standard deviation.

Consistent with the predictions of the model, we see that the asymmetry between interest rate hikes and drops during episodes of high volatility weakens with milder financial frictions. For example, the observed gap in the response of the endogenous trend growth to interest rate hikes and drops is 0.32 standard deviations, substantially lower than the gap of 0.53 standard deviations observed in the baseline economy.