Endogenous Bargaining Power and Declining Labor Compensation Share

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Abstract: We study the role of labor market tightness and workers' bargaining power in explaining the labor share decline in the long run. Using a search and matching model with endogenous bargaining power, we find that the decline in labor demand accounts for 30 percent of the decline in the U.S. labor share between 1980 and 2007. In the model, decrease in labor demand decreases the labor market tightness and implies a lower job-finding rate of workers. To mitigate the effects of lower demand on the job-finding rate, workers' optimal bargaining weight decreases, leading to a lower labor share. We use the model to recover changes in workers' implied bargaining power and find that it has declined about 17 percent over the same period.

Keywords: Labor share, Endogenous bargaining power, Search and matching, CES matching function

JEL Codes: E25, J30, J50

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1 Introduction

Recent literature has documented a decline in the U.S. labor share since late 1970s and has provided various explanations for the decline.[1] One of the explanations suggests that this long-run decline in the U.S. labor share is linked to reduced workers bargaining power, in particular via declining unionization and other labor market institutions (Bental and Demougin, 2010; Stansbury and Summers, 2020). In this paper, we study the role of labor market tightness (vacancies over job seekers) and bargaining power in explaining the decline in the labor share. In contrast with the existing literature focusing on the role of unions and other institutions, our hypothesis is that workers' bargaining power has declined as an equilibrium response to a decrease in labor demand captured by labor market tightness. We find that a decline in labor market tightness has decreased workers' bargaining power, and this decline can account for about 30 percent of the drop in the labor share between 1980 and 2007.

We formalize our hypothesis using a search and matching model with endogenous bargaining power. Our approach involves two key modifications to the conventional Diamond-Mortensen-Pissarides (DMP) framework. First, we replace the standard Cobb-Douglas (CD) matching function with a more general constant elasticity of substitution (CES) function. Second, we impose the condition that bargaining weights must be efficient. Specifically, according to the well-known Hosios condition (Hosios, 1990), efficiency dictates that workers' bargaining weight must be equal with matching elasticity. While this elasticity remains constant in the CD model, it is a variable in the CES framework leading to endogenous bargaining weights. By disciplining the model by matching the various labor market outcomes in the data in two business cycle peaks, 1976–80 and 2003-07, we found that the bargaining channel is quantitatively important in explaining the decrease in the labor share. Moreover, using the model to recover changes in the workers' bargaining weights, we find that workers' bargaining power has declined about 17 percent, on average.

[1] Common explanations for the decline include technological change and globalization in the forms of cheaper relative prices of investments goods relative to labor (Karabarbounis and Neiman, 2014), increased concentration of production towards firms with lower labor shares (Autor et al., 2020), increasing international competition (Elsby et al., 2013) and changes in the effective capital-to-labor ratios (Lawrence, 2015).

We also show that decentralizing the efficient planner's solution using a directed search with a bargaining weight posting delivers exactly same outcomes. In this case, when tightness decreases, workers face a trade-off between directing their search toward jobs with higher workers' surplus share (a higher bargaining weight) or a higher job-finding rate (and lower unemployment). By directing search toward lower bargaining weight jobs, workers improve their likelihood of finding a job, and thus mitigate the effect of lower demand on the job-finding rate.[2]

Our paper makes four contributions. First, we begin the paper by presenting novel empirical evidence on the long-run covariability between labor market tightness and the labor share. Using state-of-the-art methods introduced by Müller and Watson (2018) and U.S. data from 1964 to 2019, we show that there is a positive and statistically significant long-run correlation between the two series, long-run cycles of tightness leading the labor share cycles by approximately two years. These results thus suggest that long-run labor demand is linked to the long-run labor share, and that the labor share is in fact affected by labor demand rather than the other way around. The correlation is stronger when we define tightness as vacancies over unemployed and nonparticipants, rather than vacancies over unemployed, as is standard.[3]

Second, we build a DMP model with endogenous bargaining power and nonparticipation, and show that the model can generate a meaningful decline in the labor share when the model is augmented with endogenous bargaining power. The role of the CES matching function is a key for delivering the results. When vacancies and job seekers are *complements* in the matching process, matching elasticity with respect to job seekers is increasing in

[2]Wages tend to be sticky in the short run, maybe because it takes time for workers to learn about changes in the labor demand, or there are other frictions preventing wage adjustments. However, as we focus on long-run relationship between labor demand, bargaining power, and the labor share, this short-run stickiness of wages is less of a concern.

[3]A job seeker measure including both unemployed and nonparticipants, or a subset of nonparticipants, is consistent with recent literature highlighting that vacancies over unemployment may not be the best approximation of tightness because it ignores large employment flows from nonparticipation and between jobs (see, for example, Abraham et al., 2020; Hall and Schulhofer-Wohl, 2018); and Hornstein et al., 2014).

tightness. This result means that the number of matches is more sensitive to the number of job seekers when there are many available vacancies relative to job seekers, and vice versa. For that reason, compared with a CD case where matching elasticity is constant, it is efficient to decrease workers' bargaining weight when tightness is low to increase the number of vacancies and bring the number of vacancies and job seekers closer together. When disciplining the model by matching various labor market outcomes in the data, we find that the model can explain about 30 percent in the labor share decline. In contract, an otherwise similar model with fixed bargaining power or a Cobb-Douglas (CD) matching function generates an increase in the labor share. Thus, the bargaining power channel is a key ingredient in the model to produce a decrease in the labor share.

Third, as our theoretical results rely on the functional form and properties of the matching function, we provide empirical evidence for the CES matching function and for the complementarity of matching function inputs. We estimate CD and CES matching functions following an estimation strategy that controls for the endogeneity in the matching efficiency, introduced in Şahin et al. (2014). Using monthly U.S. data from 2000 to 2023 on vacancies and job seekers, we find robust evidence for the CES matching function and complementarity between vacancies and job seekers when we measure job seekers using *both unemployed and nonparticipants*.[4] The results hold whether we use all unemployed and nonparticipating individuals, or the Hornstein-Kudlyak-Lange Non-Employment Index from Richmond Fed, a measure that includes all non-employed individuals but takes into an account differences in different groups labor market attachment. Our estimates point to a negative value of matching elasticity with respect to job seekers indicating complementarity between these inputs, the estimates varying between -0.5 and -1.1.

Fourth, using the model, we quantify the decline in workers' bargaining power and look at the determinants of the decline. Our results suggest that, on average, the bargaining power

[4]Previous research have not found strong evidence for the CES matching function when using unemployed as job seekers (Blanchard and Diamond,1989; Shimer, 2005; and Şahin et al., 2014). We confirm this result and thus find that estimating the CES matching function with this more realistic measure of job seekers is crucial for finding support for the CES matching function.

has declined by about 17 percent between 1980 and 2007 because of a decline in tightness. Previous literature has found that bargaining power varies across different demographic groups.[5] For that reason, we also look at how bargaining power has evolved for four distinct demographics: males and females with at least some college education, as well as males and females without a college education. We find that the bargaining power of males has decreased more than that of females, leading to a decrease in the gender bargaining power gap. The bargaining power of both college and non-college males has declined around 24 percent, while the declines have been 4 percent for college-educated females and 10 percent for non-college females. Lastly, while the gender bargaining power gap has diminished, the opposite is true for the education gap—especially for females—as college-educated workers' bargaining power has decreased less relative to non-college workers.

Finally, we decompose the decline in bargaining power and the labor share. We conclude that an increase in κ, the relative vacancy-posting cost, has driven the decline in tightness, and thus bargaining power and the labor share. While our calibration results point to an improved matching efficiency and higher productivity, increasing tightness, we find that vacancy-posting costs have risen. This rise is necessary for the model to match the observed decline in tightness along with the observed employment and wage trends.

Relation to the literature. On the theory side, the most related paper is Mangin and Sedláček (2018). They study business cycle fluctuations of the labor share by building a search and matching model where heterogeneous firms compete over workers and in which the division of output between firms and workers in endogenous. Specifically, a tighter labor market increases labor's share of output—a mechanism like the one in our model. However, Mangin and Sedláček (2018) focus on explaining the business cycle dynamics of the labor share, while our focus is on longer-term changes in bargaining power and the labor share.

[5] Recent literature has documented a gender bargaining power gap, and the gap can explain a fraction of the gender wage gap (Biasi and Sarsons, 2021; Blau and Kahn, 2017; Card et al., 2016; Harding et al., 2003). Some literature also uses bargaining power differences to explain a wage gap between older workers and young workers (Farmand and Ghilarducci, 2019 and Glover and Short, 2020). The literature thus suggests that assuming a constant bargaining power across groups is problematic and that accounting for the noted differences in bargaining power is important to understand the dynamics of the labor market.

We are not the first to use the CES matching function in search and matching models. den Haan et al. (2000) use a special case of the CES matching function and highlight the preferable properties of the function that guarantee matching probabilities between zero and one. A similar CES matching function is also used, for example, by Hagedorn and Manovskii (2008) and Petrosky-Nadeau et al. (2018). Stevens (2007) microfounds a matching function by showing that a "telephone line" Poisson queuing process implies a CES matching function. Recently, Bernstein et al. (2022) studied how a CES matching function and cyclicality of matching efficiency affect nonlinear business cycle properties of search and matching models and found quantitatively important effects. While these papers allow variation in matching elasticities, they assume constant bargaining weights.

Our paper also relates to the literature that studies workers' bargaining power—both the long-run trends and the differences across different worker groups—and the relationship between bargaining power and the labor share.[6] We contribute to the efforts to measure changes in bargaining power by indirectly inferring changes in efficient bargaining power for different demographic groups using a general equilibrium model with endogenous bargaining power. Consistent with the previous literature, we find that bargaining power has declined in the past four decades and that there are gaps in bargaining power across gender and education.

While previous literature has focused on studying a decline bargaining power arising from changes in labor market institutions (Stansbury and Summers, 2020 and Ratner and Sim, 2022), we focus on studying changes in bargaining power arising from labor demand. Stansbury and Summers (2020) argue that three factors have caused the decline in worker power in the U.S. over recent decades: (1) institutional changes like decreased unionism, (2) within-firm changes like an increase in shareholder power that has led to pressure to cut labor costs, and (3) changes in economic conditions—like increased globalization and technology— that have improved employers' outside options. While Stansbury and

[6] See for example, Bental and Demougin (2010); Biasi and Sarsons (2021); Blau and Kahn (2017); Card et al. (2016); Farmand and Ghilarducci (2019); Glover and Short (2020); Ratner and Sim (2022); Roussille (2022); Stansbury and Summers (2020).

Summers (2020) focus on studying and presenting supporting evidence for the first two factors, we complement their work by focusing on the third. In addition, the declining unionization documented by Stansbury and Summers (2020) can also our relate tp our hypothesis: Workers may be less inclined to join unions for fear of jobs disappearing quickly.

Charles et al. (2021) find a similar result when looking at a decline in unionization. They estimate the causal effect of increased import competition from China on the accelerated decline in the rate of union elections between 1990 and 2007. They find that the "China shock" contributed to 4.5 percent of the decline among workers in directly exposed industries, while the shock contributed to 8.8 percent of the decline among workers indirectly exposed through weaker local relative labor demand. In other words, workers in industries that were not directly exposed to the China shock unionized less because the shock weakened their outside employment options in the face of a job loss.

We find a similar mechanism using a structural general equilibrium model, but we focus on studying the effect on bargaining power. An increased vacancy cost, which can capture the China shock, reduces rents from any match and leads to weaker labor demand via lower tightness. This increases the cost of job loss because workers' job-finding rate goes down. We also show that workers' efficient bargaining power decreases. The decreases in the job-finding rate and bargaining power then lead to a decrease in the labor share.

The rest of the paper is organized as follows. Section 2 presents the results about the long-run covariability between the labor share and tightness. Section 3 summarizes the properties of the CD versus CES matching functions. Section 4 introduces the model, and Section 5 describes (1) the matching function estimation; (2) how we parameterize the model; and (3) the calibration results. We then move on to reporting results from counterfactuals exercises (Section 6) and concluding the paper (Section 7).

2 Long-Run Covariability between the Labor Share and Tightness

In this section, we study low-frequency trends in the U.S. labor share and labor market tightness and their long-run covariability. If tightness affects workers' bargaining power in the long run and changes in bargaining power affect the labor share, we expect these series to move in tandem. This is, in fact, what we find: The labor share and tightness co-vary positively in the long run, tightness leading the labor share by one to three years.

To study the long-run trends in the labor share and tightness and the co-movement between the two series, we use methods introduced in Müller and Watson (2018). We first extract low-frequency trends in the labor share and tightness data. We then compute the low-pass correlation and related confidence intervals between the filtered series. The method first calculates low-frequency averages of the series and then uses these averages to measures the long-run variability and covariability of the series. These low-frequency transformations can be thought of as low-pass filtered versions of the data. The noteworthy contribution of Müller and Watson (2018) is that they provide a method to construct confidence intervals for long-run correlation coefficients, linear regression coefficients, and standard deviations of regression errors that are robust for the series being $I(0)$, $I(1)$, near unit root, fractionally integrated models, and linear combinations of variables with these types of persistence. See Müller and Watson (2018) for technical details.

We use the quarterly labor share data for the U.S. nonfarm business sector between 1965 and 2019 from the Bureau of Labor Statistics (BLS) (U.S. Bureau of Labor Statistics, 2022). To construct a measure of labor market tightness between years 1965 and 2019, we use historical vacancy-rate data from Petrosky-Nadeau and Zhang (2021) up to year 2017 and then from 2018 onward the seasonally adjusted total nonfarm job openings from the Job Openings and Labor Turnover Survey (JOLTS) at BLS. Our tightness measure is constructed such that the job seekers include both the unemployed and nonparticipating persons. The measure is consistent with recent literature highlighting that vacancies over unemployment may not be the best approximation of tightness because it ignores large employment flows from nonparticipation and between jobs (See, for example, Abraham et al., 2020; Hall and Schulhofer-Wohl, 2018; and Hornstein et al., 2014).). We get unemployment and nonparticipation data from U.S. Bureau of Labor Statistics obtained via the Federal Reserve Economic Data (FRED). Specifically, our tightness measure includes the number of unemployed and nonparticipating persons over age 16 in the denominator. The underlying

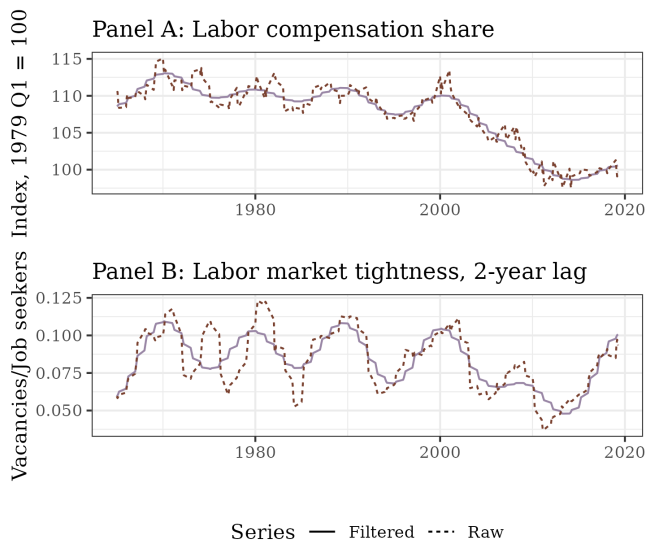

Figure 1. Labor market tightness and labor compensation share, 1965–2019
Note: Panel A shows the quarterly, seasonally adjusted labor share for all employed persons in the nonfarm business sector, and panel B shows the quarterly labor market tightness series, lagged by two years. The dashed lines plot the raw series, while the purple solid lines plot the low-pass filtered series using Müller and Watson (2018) filter with cycles longer than 10 years.
Source: Bureau of Labor Statistics; Authors' calculations based on Petrosky-Nadeau and Zhang (2021) and IPUMS-CPS.

data series are reported on a monthly basis, so we calculate quarterly rates by averaging monthly values.[7]

We first present the results showing the long-run filtered trends for both series. Panel A in figure 1 shows the evolution of the quarterly labor share using both raw and filtered series, while Panel B shows the results for the tightness series, lagged by two years. We plot the lagged tightness series to highlight that the long-run peaks and troughs of the lagged tightness series coincides with the peaks and troughs of the labor share, indicating that the long-run cycles in tightness lead the cycles in the labor share.

As previously documented, the U.S. labor share has declined in the long run: Between two

[7]Our tightness measure is also closely positively correlated with a tightness rate constructed using the Hornstein-Kudlyak-Lange Nonemployment Index (Hornstein-Kudlyak-Lange Non-Employment Index, 2023) from the Richmond Fed as the denominator. The correlation coefficient is .99 when using data from 1994 to 2022. Index starts in 1994.

Table 1. Low-pass correlation coefficients and confidence intervals, labor share and tightness, periods longer than 10 years

		$\hat{\rho}$	67% CI	90% CI	95% CI
Sample: 1965:Q1-2019:Q4					
Labor share$_t$	θ_t	**0.307***	0.034, 0.504	-0.124, 0.667	-0.255, 0.733
	θ_{t-4}	**0.506*****	0.386, 0.715	0.131, 0.776	0.063, 0.807
	θ_{t-8}	**0.640*****	0.462, 0.744	0.386, 0.813	0.317, 0.874
	θ_{t-12}	**0.697*****	0.648, 0.804	0.428, 0.877	0.401, 0.892
	θ_{t-16}	**0.619*****	0.429, 0.786	0.301, 0.864	0.161, 0.890
Sample: 1965:Q1-2007:Q4					
Labor share$_t$	θ_t	**0.395***	0.063, 0.703	-0.115, 0.796	-0.267, 0.822
	θ_{t-4}	**0.832*****	0.718, 0.943	0.559, 0.962	0.450, 0.968
	θ_{t-8}	**0.803*****	0.707, 0.921	0.564, 0.954	0.500, 0.960
	θ_{t-12}	**0.597****	0.408, 0.805	0.130, 0.885	-0.036, 0.911
	θ_{t-16}	**0.386****	0.061, 0.643	-0.082, 0.778	-0.199, 0.825

* significance at 67% level, ** 90% level, *** 95% level

Source: Petrosky-Nadeau & Zhang (2021); IPUMS-CPS; BLS; Authors' estimations.

five-year periods preceding two business cycle peaks, 1976–80 and 2003–07, the decline has been 4.0 percent. The majority of the decline has occurred after 2000, and, overall, the labor share shows significant variation over time and across business cycles. Like the labor share, labor market tightness has also trended downward.[8] The decline between the two periods 1976–80 and 2003–07 has been 21.5 percent.

Table 1 reports the estimated long-run correlation coefficients between the labor share and tightness together with the 67, 90, and 95 percent confidence intervals.[9] We report the posterior mean correlation and focus on periods longer than 10 years.

The results show a positive and highly significant long-run correlation between the series,

[8] Hall (2017) also documents the declining trend in the rate of labor market tightness.

[9] We present 67 percent confidence intervals following Müller and Watson (2018). The number of observations for long-run averages is small, so less strict means of rejecting the null hypothesis is reasonable.

supporting the hypothesis that tightness and the labor share are linked in the long run, potentially through workers' bargaining power. The correlation tends to be the strongest when tightness leads the labor share by one to three years. For these lags, the correlation is statistically significant at 5 percent level, and varies between .506 and .697 in the full sample and between .597 and .832 in the sample excluding data after 2007.[10] These results thus confirm that the peaks and troughs of the two filtered series are not aligned in time.

We report additional robustness results in the Appendix A. These results include testing for a different choice of the minimum-length period of the low-frequency component (periods longer than 15 years), extending the sample of data to start from year 1952 and end in 2022, using first-differenced data, testing whether the labor share leads tightness, and testing the correlation between the standard measure of labor market tightness and the labor share. The results are broadly robust to different specifications. The correlation coefficients are smaller and only significant at the 67 percent level and with longer lags when we look at the longer sample from 1952 to 2022. However, the correlations coefficients over the same sample are large and significant at at least 10 percent level when using first-difference data.

3 The Properties of the CD and the CES Matching Functions

This section highlights some properties of the CES matching function that speak to its use. First, under the CES matching function, matching probabilities are between zero and one (den Haan et al., 2000 and Petrosky-Nadeau et al., 2018). Define the CES matching function as

$$M(u,v) = \left\{ \begin{array}{c} A\left(\alpha u^{\rho} + (1-\alpha)\, v^{\rho}\right)^{1/\rho} \text{ if } \rho \leq 1, \rho \neq 0 \\ A u^{\alpha} v^{(1-\alpha)} \text{ if } \rho = 0 \end{array} \right\}, \tag{1}$$

where u refers to job seekers, v refers to vacancies, $\alpha \in (0,1)$ is the share parameter, and A is the matching efficiency. The elasticity of substitution is $\sigma \equiv \frac{1}{1-\rho} \in (0,\infty)$. The case of the CD matching function with $\rho = 0$ implies that $\sigma = 1$. The value of ρ is negative when u and v are complements.

Given the matching function, a firm's vacancy-filling rate $q(\theta) = M(u,v)/v = M(1/\theta, 1)$ is

[10] This relationship also holds if we use the standard measure of labor market tightness (vacancies over unemployed), but the correlation is less strong.

given by

$$q(\theta)=\left\{\begin{array}{c}A\left(\alpha\theta^{-\rho}+(1-\alpha)\right)^{1/\rho}\text{ if }\rho\leq 1,\rho\neq 0\\ A\theta^{-\alpha}\text{ if }\rho=0\end{array}\right\}. \tag{2}$$

Notice that $q'(\theta)<0$. Furthermore,

$$q(0)=\left\{\begin{array}{c}A\left(1-\alpha\right)^{1/\rho}\text{ if }\rho<0\\ \infty\text{ if }\rho\geq 0\end{array}\right\},\ q(\infty)=\left\{\begin{array}{c}0\text{ if }\ \rho<0\\ A(1-\alpha)^{1/\rho}\text{ if }\ \rho>0\\ 0\text{ if }\rho=0\end{array}\right\}.$$

Therefore, the probability of filling a vacancy is well behaved when $\rho<0$ if $1\geq A(1-\alpha)^{1/\rho}$. When that is the case, $q(\theta)\in[0,1]$ for all $\theta\geq 0$. In contrast, $q(\theta)$ is not well behaved when $\rho\geq 0$, as $q(0)=\infty$.

In a similar manner, a job-finding rate $f(\theta)=M(u,v)/u=M(1,\theta)$ is

$$f(\theta)=\left\{\begin{array}{c}A\left(\alpha+(1-\alpha)\,\theta^{\rho}\right)^{1/\rho}\text{ if }\rho\leq 1,\rho\neq 0\\ A\theta^{1-\alpha}\text{ if }\rho=0\end{array}\right\}. \tag{3}$$

A job-finding rate is increasing in tightness, $f'(\theta)>0$ and,

$$f(0)=\left\{\begin{array}{c}A\alpha^{1/\rho}\text{ if }\rho>0\\ \text{ if }\rho\leq 0\end{array}\right\},\ f(\infty)=\left\{\begin{array}{c}A\alpha^{1/\rho}\text{ if }\ \rho<0\\ \infty\text{ if }\rho\geq 0\end{array}\right\}.$$

Therefore, the job-finding rate is well behaved when $\rho<0$ if $1\geq A\alpha^{1/\rho}$. In that case, $f(\theta)\in[0,1]$ for all $\theta\geq 0$. When $\rho\geq 0$, $f(\theta)$ is not well behaved, as $f(\infty)=\infty$.

To conclude, the CES matching function produces sensible job-finding and vacancy-filling probabilities when $1\geq max\left\{A(1-\alpha)^{1/\rho},A\alpha^{1/\rho}\right\}$. This is not the case with the CD matching function.

Second, we show how the CES matching function generates intuitively reasonable matching elasticities. Note first that with the CES matching function, $q'(\theta)=-A\alpha[\alpha\theta^{-\rho}+(1-\alpha)]^{\frac{1}{\rho}-1}(\theta^{-\rho-1})$. Therefore, we can write the matching elasticity with respect to job seekers as a function of θ:

$$M_u(u,v)\frac{u}{M}=\alpha(\theta)=-\frac{q'(\theta)\theta}{q(\theta)}=\frac{\alpha}{\alpha+(1-\alpha)\theta^{\rho}}. \tag{4}$$

This expression includes the CD result with $\rho = 0$. As is well known, the CD matching function implies a constant matching elasticity α. Moreover, notice that $\alpha'(\theta) > 0$ whenever $\rho < 0$—that is, the matching elasticity is increasing in tightness when v and u are complements in the matching process. As $\alpha(\theta)$ represents the elasticity of the matching function with respect to u, a lower θ means that there are relatively more job seekers compared with vacancies. This means that the number of successful matches is less sensitive to the number of job seekers, a result that highlights the complementarity of job seekers and vacancies in the matching process.

Third, we show that the CES matching function generates efficient bargaining power dynamics consistent with both micro-evidence and macro-evidence. The well-established Hosios condition (Hosios, 1990) states that the decentralized solution of the standard DMP model is constrained efficient as long as the elasticity of the matching function with respect to the number of job seekers equals the bargaining weight ϕ of the worker,

$$\phi(\theta) = \alpha(\theta) = \frac{\alpha}{\alpha + (1-\alpha)\theta^{\rho}}. \tag{5}$$

This simple formulation shows that the bargaining power of workers is increasing in α, but more importantly, bargaining power is increasing in the endogenous tightness rate θ whenever ρ is negative. We now have efficient bargaining power that depends on labor market tightness.

Proposition 1. *Under the CES matching function $M(u, v) = A(\alpha u^{\rho} + (1-\alpha)v^{\rho})^{1/\rho}$, the efficient bargaining power of workers decreases with labor market tightness if $\rho > 0$; the efficient bargaining power of workers increases with labor market tightness if $\rho < 0$.*

From the point of view of the social planner, proposition 1 means that the large relative number of job seekers reduces the potential for forming a match because of the complementarity of u and v. To increase the number of vacancies, it is optimal to reduce the surplus share of workers to spur vacancy creation.

It is also easy to see that the expression nests the Cobb-Douglas case: When $\rho = 0$, the bargaining power is exactly α. The Cobb-Douglas case also means that the bargaining power of workers does not depend on workers' characteristics or labor market conditions, contrasting with both micro-evidence as well as casual observations, as noted in the introduction.[11]

Let's further derive the bargaining power elasticity with respect to tightness θ:

$$\begin{aligned}\varepsilon_{\phi,\theta} = \frac{\partial \phi}{\partial \theta}\frac{\theta}{\phi} &= -\alpha[\alpha + (1-\alpha)\theta^{\rho}]^{-2} \times [\rho(1-\alpha)\theta^{\rho-1}] \times \frac{\theta[\alpha + (1-\alpha)\theta^{\rho}]}{\alpha} \\ &= -[\alpha + (1-\alpha)\theta^{\rho}]^{-1} \times [\rho(1-\alpha)\theta^{\rho}] = -\rho\frac{(1-\alpha)\theta^{\rho}}{\alpha + (1-\alpha)\theta^{\rho}}.\end{aligned} \tag{6}$$

The above expression implies that the elasticity of bargaining power with respect to tightness is positive whenever $\rho < 0$, and that bargaining power increases with tightness more whenever ρ gets smaller and the complementarity between vacancies and job seekers in the matching process gets higher. Again, the above formula includes the Cobb-Douglas case: When $\rho = 0$, $\varepsilon_{\phi,\theta} = 0$.

We use the expression for bargaining power to discuss bargaining power gaps documented in the literature. What can explain lower bargaining power of females? Females generally have higher separation rates compared with males over the life cycle (Choi et al., 2015 and Córdoba et al., 2021). Does that imply that females will be in a weaker position when bargaining with firms? The answer depends on bargaining power elasticity. We derive the effect of a separation rate, π_{EN}, on the efficient bargaining power:

$$\frac{\partial \phi}{\partial \pi_{EN}} = \frac{\partial \phi}{\partial \theta}\frac{\partial \theta}{\partial \pi_{EN}} = \frac{-\alpha(1-\alpha)\rho\theta^{\rho-1}}{[\alpha + (1-\alpha)\theta^{\rho}]^2} \times \frac{\partial \theta}{\partial \pi_{EN}} = \varepsilon_{\phi,\theta} \times \frac{\phi}{\theta} \times \frac{\partial \theta}{\partial \pi_{EN}}.$$

The sign of $\frac{\partial \phi}{\partial \pi_{EN}}$ depends on the signs of ρ and $\frac{\partial \theta}{\partial \pi_{EN}}$. $\frac{\partial \theta}{\partial \pi_{EN}}$ is negative, and the intuition for $\frac{\partial \theta}{\partial \pi_{EN}}$ being negative is that hiring workers with higher separation rates will lower the match continuation value, and the lower continuation value needs to be compensated by a higher chance of successfully hiring such workers. Then the sign of $\frac{\partial \phi}{\partial \pi_{EN}}$ can be fully

[11] Intuitively, when $\rho < 0$, bargaining power responds to labor demand and supply. Labor market tightness reflects the relative demand for labor. As a larger θ means a shift in the demand curve to the right, the labor market endogenously gives a larger production share to workers through larger bargaining power ϕ. This relationship is also in line with the finding of Fortin (2006), who shows that the college wage premium is negatively related to the supply of highly educated workers.

pinned down by ρ. When $\rho < 0$, groups with higher separation rates have lower bargaining power compared with groups with lower separation rates, all else equal.

To conclude, we argue that there are four reasons why the CES matching function with $\rho < 0$ is a sounder choice for a matching function: (i) it is theoretically sounder as the CD introduces discontinuities and requires truncation; (ii) it generates intuitively sensible matching elasticities and efficient bargaining power; (iii) it is consistent with micro-evidence that shows that groups with weaker labor markets (for example, women) have lower bargaining power; (iv) it is consistent with casual evidence—for example during and after COVID-19 pandemic—of workers' bargaining power increasing with labor scarcity.

4 Search and Matching Model with Nonparticipation and Endogenous Bargaining Power

Consider a textbook search and matching model à la Diamond (1982), Mortensen (1982), and Pissarides (1990) extended to include (i) CES matching function, (ii) efficient bargaining, and (iii) a non-participation state. There is a continuum of infinitely lived job seekers indexed by i, where i captures any characteristics of workers, like education level and gender. At any point in time, a worker is either employed, $\bar{E}$, unemployed, $\bar{U}$, or nonparticipating $\bar{N}$. Let $s \in S \equiv \{\bar{E}, \bar{U}, \bar{N}\}$ denote the labor market status of an individual and $t \in \{\bar{U}, \bar{N}\}$ the last labor status before becoming employed. If s $\in \{\bar{U}, \bar{N}\}$ then $t = s$. Keeping track of the last labor status is important for the efficiency of the decentralized markets, as shown below. Denote the state of a worker by $x = (s, t, i)$. Importantly, labor markets are assumed to be segmented across x types.[12]

Let $m(x)$ be the mass of workers of type x. Workers transition into unemployment and nonparticipation at exogenous rates $\pi_{EU}(x)$, $\pi_{EN}(x)$, $\pi_{UN}(x)$, and $\pi_{NU}(x)$, and into employment at endogenous rates $f(x)$. Workers seek to maximize their expected present value of consumption. They are risk neutral and discount the future according to the discount

[12] An alternative to the DMP model with efficient Nash bargaining and random search is directed search as in Moen (1997). We show in Appendix C that the results are exactly the same. This result implies that workers and firms would choose efficient bargaining weights in a decentralized equilibrium with rich enough segmentation.

factor $\beta \in (0,1)$. There are no savings. Wages of employed workers are determined by Nash bargaining between workers and firms, while consumption of non-employed workers is given by $\bar{c}(x)$, an exogenous parametric form.

Each worker is endowed with one unit of labor, but workers differ in terms of their labor productivity. We refer to the productivity of a worker as human capital, y_i, and it is of the general type.

The continuum of infinitely lived firms seek to maximize their expected present value of profits net of hiring costs. Firms are risk-neutral and discount the future at the same rate as workers do. Labor markets are assumed to be perfectly segmented across worker types. Firms can freely enter any segmented markets. Firms post vacancies for long-term positions at a cost of $\kappa(x)$ per vacancy, a cost that may depend on a worker's type. Once a firm is matched with a worker, a worker produces y_i units of output per period, while gross per-period profits of the firm are $y_i - w(x)$. A match is destroyed exogenously at a rate of $d(x) = \pi_{EU}(x) + \pi_{EN}(x)$.

Matching Technology*:* A worker and a firm with a vacant position are randomly matched in each submarket according to the matching technology $M(u(x), v(x); x)$, where $u(x)$ and $v(x)$ are the masses of workers and firms, respectively, searching in a labor market. We assume that (1) all unemployed workers search for a job, (2) employed workers do not search, and (3) a fraction $\psi(x) \leq 1$ of nonparticipants search. Thus, the mass of workers searching at a given employment status can be defined as follows:

$$u(x) \equiv \left\{ \begin{array}{c} m(x), \text{ if } s = \overline{U}, \\ \psi(x)\, m(x), \text{ if } s = \overline{N}. \end{array} \right\} \tag{7}$$

Labor market tightness for each market x is defined as $\theta(x) \equiv v(x)/u(x)$, the vacancy-filling rate as $q(\theta(x)) = M(u(x), v(x))/v(x)$, and the job-finding rate as $f(\theta(x)) = \theta(x)q(\theta(x))$. We assume that the matching function takes the CES form, introduced in Section 2, equation 2, which implies that $q(x) = A(\alpha\theta^{-\rho} + (1-\alpha))^{1/\rho}$ and $f(x) = A(\alpha + (1-\alpha)\theta^{\rho})^{1/\rho}$.

Labor flows*:* In a steady state, the share of workers entering unemployment must equal the share of workers exiting unemployment. The same condition holds for nonparticipants. The steady state masses of unemployment and nonparticipation, $m_i^{\bar{U}} \equiv m(U,U,i)$ and $m_i^{\bar{N}} \equiv m(N,N,i)$ are thus determined by the following two equations:

$$\begin{aligned} f_i^{\bar{U}} \times m^{\bar{U}} + \pi_{UN} \times m^{\bar{U}} &= \pi_{NU} \times m^{\bar{N}} + \pi_{EU} \times (1 - m^{\bar{U}} - m^{\bar{N}}), \\ f_i^{\bar{N}} \times m^{\bar{N}} + \pi_{NU} \times m^{\bar{N}} &= \pi_{UN} \times m^{\bar{U}} + \pi_{EN} \times (1 - m^{\bar{U}} - m^{\bar{N}}). \end{aligned} \tag{8}$$

where $f_i^{\bar{U}} \equiv f(U,U,i)$ and $f_i^{\bar{N}} \equiv f(N,N,i)$.

4.1 *Value Functions of Firms and Workers*

A firm's value of filled job J and a vacancy V can be written as

$$V(x) = \max\{-\kappa(x) + \beta[q(x)J(x') + (1 - q(x))V(x)], 0\}.$$

Free entry of firms into any labor market guarantees that the values of unfilled vacancies must all be equal to zero: $V(x) = 0$ for all feasible x.

The problem of a firm with a worker is then

$$J(x) = \left\{ y(x) - w(x) + \beta(1 - d(x))J(x) \right\}, \tag{9}$$

which simplifies to

$$J(x) = \frac{y(x) - w(x)}{1 - \beta(1 - d(x))}. \tag{10}$$

The value of firms that post vacancies simplifies to

$$J(x) = \frac{\kappa(x)}{\beta q(x)}. \tag{11}$$

The last equation states that the expected present value of filling a vacancy must be just enough to recover the costs of posting the vacancy. Combining the previous equation with equation (10), we get

$$w(x) = y(x) - \frac{[1 - \beta(1 - d(x))]}{\beta q(x)}\kappa(x). \tag{12}$$

The wage rate guarantees that firms are able to recover the average discounted costs of creating a vacancy.

A worker's value functions can be written as follows:

$$
\begin{aligned}
E(x) &= w\,(x) + \beta\left[\pi^i_{EU}U(x') + \pi^i_{EN}N(x') + (1-d\,(x))E(x)\right], \\
U(x) &= \bar{c}\,(x) + \beta\left[f^{\bar{U}}_i E(x') + \pi^i_{UN}N(x') + (1-f^{\bar{U}}_i - \pi^i_{UN})U(x)\right], \\
N(x) &= \bar{c}\,(x) + \beta\left[f^{\bar{N}}_i E(x') + \pi^i_{NU}U(x') + (1-f^{\bar{N}}_i - \pi^i_{NU})N(x)\right].
\end{aligned}
\tag{13}
$$

A worker's share of the surplus of the match is the difference between the value of employment and the value of the outside option. For the unemployed, the surplus is $E(x) - U(x)$ and for the nonparticipant $E(x) - N(x)$.

These expressions can be used to calculate surpluses $S_{EU}(x) \equiv E(x)) - U(x)$, $S_{EN}(x) \equiv E(x)) - N(x)$ and $S_{UN}(x) \equiv U(x) - N(x)$. As shown in Appendix A, the expressions for the surpluses are:

$$
S_{EU}(x) = \frac{w(x) - \bar{c}\,(x)}{1-\beta\left(1-d(x)-f^{\bar{U}}_i\right)} + \frac{\beta\,(\pi^i_{UN} - \pi^i_{EN})}{1-\beta\left(1-d(x)-f^{\bar{U}}_i\right)} S_{UN}(x) \tag{14}
$$

$$
S_{EN}(x) = \frac{w(x)) - \bar{c}\,(x)}{1-\beta\left(1-d(x)-f^{\bar{N}}_i\right)} + \frac{\beta\,(\pi^i_{EU} - \pi^i_{NU})}{1-\beta\left(1-d(x)-f^{\bar{N}}_i\right)} S_{UN}(x) \tag{15}
$$

$$
S_{UN}(x) = \frac{\bar{c}\,(i,U) - \bar{c}\,(i,N)}{1-\beta\,(1-\pi^i_{UN} - \pi^i_{NU})} + \frac{\beta f^{\bar{U}}_i}{1-\beta\,(1-\pi^i_{UN} - \pi^i_{NU})} S_{EU}(x) - \frac{\beta f^{\bar{N}}_i}{1-\beta\,(1-\pi^i_{UN} - \pi^i_{NU})} S_{EN}(x). \tag{16}
$$

4.2 *Nash Bargaining*

Wages are negotiated through Nash bargaining. A firm and a worker share the match surplus $S(x) = S_{Es}(x) + J(x)$, s $\in$ S $\equiv \{E, U, N\}$, $S_{EU}(x) = E(x) - U(x)$ for an unemployed, and $S_{EN}(x) = E(x) - N(x)$ for a nonparticipant. Given the bargaining weights $\phi\,(x)$ for the worker and $1 - \phi\,(x)$ for the firm, the maximization problem is written as:

$$
\underset{S_{Es},J}{max} (S_{Es}(x))^{\phi(x)} J\,(x)^{\,1-\phi(x)} \text{ subject to } S(x), \tag{17}
$$

and the solution for each labor market satisfies

$$
J(x) = \Theta\,(x) \times (S_{Es}(x)) \text{ where } \Theta\,(x) = \frac{1-\phi\,(x)}{\phi\,(x)}. \tag{18}
$$

Model solution: We can use equations (12) and (18) along with equations (14)-(16) and (10) to solve for $w(x)$ and $\theta(x)$ for each x. Steady state levels of employment, unemployment, and nonparticipation can then be solved using (8).

4.3 *Efficient bargaining*

The following proposition states that the Hosios condition guarantees labor market efficiency as long as markets are sufficiently segmented.

Proposition 2. *Decentralized markets are efficient if the following conditions hold:*

$$\phi(x) = -\frac{q'(\theta(x))\theta(x)}{q(\theta(x))}. \tag{19}$$

Proof. See Appendix B. □

In Appendix C, we further show that the Hosios condition arises endogenously if we write the model as a directed search problem and assume that firms post a menu of bargaining powers and workers choose to apply for jobs that offer the bargaining power that maximizes their utility.

4.4 *Labor share*

We can use equation (12) to define the labor share in the model:

$$s_L(x) = \frac{w(x)}{y(x)} = 1 - \frac{[1 - \beta\,(1 - d\,(x))]}{\beta y(x) q(x)}\kappa(x). \tag{20}$$

The equation above shows that the labor share is not constant, but varies in the model parameters according to the second term.

We can investigate how the labor share responds to changes in exogenous parameters. Let's first look at the variation with respect to changes in the job destruction rate $d(x)$:

$$\begin{aligned}\frac{\partial s_L(x)}{\partial d(x)} &= -\left\{\frac{\beta\kappa}{\beta y(x)q(\theta)} - \frac{[1-\beta(1-d(x))]\kappa}{\beta y(x)} \times \frac{q\prime(\theta)}{q(\theta)^2} \times \frac{\partial\theta}{\partial d(x)}\right\} \\ &= -\frac{\kappa(x)}{\beta y(x)q(x)}\left\{1 - \frac{[1-\beta(1-d(x))]}{\beta d(x)} \times \frac{q\prime(\theta)\theta}{q(\theta)} \times \frac{\partial\theta}{\partial d(x)}\frac{d(x)}{\theta}\right\} \\ &= -\frac{\kappa(x)}{\beta y(x)q(x)}\left\{1 + \frac{[1-\beta(1-d(x))]}{\beta d(x)} \times \phi(x) \times \epsilon_{\theta,d}\right\}. \end{aligned} \tag{21}$$

The labor share is decreasing in $d(x)$ if the term in the braces is positive. As $\epsilon_{\theta,d} < 0$, the term is positive as long as the elasticity $\epsilon_{\theta,d}$ and the bargaining weight $\phi(x)$ are small enough.

Let's then look at the variation with respect to changes in the vacancy creation cost $\kappa(x)$:

$$\begin{aligned}\frac{\partial s_L(x)}{\partial \kappa(x)} &= -\left\{\frac{[1-\beta(1-d(x))]}{\beta y(x)q(\theta)} - \frac{[1-\beta(1-d(x))]\kappa(x)}{\beta y(x)} \times \frac{q\prime(\theta)}{q(\theta)^2} \times \frac{\partial \theta}{\partial \kappa(x)}\right\} \\ &= -\frac{[1-\beta(1-d(x))]}{\beta y(x)q(x)}\left\{1 - \frac{q\prime(\theta)\theta}{q(\theta)} \times \frac{\partial \theta}{\partial \kappa(x)}\frac{\kappa(x)}{\theta}\right\} \\ &= -\frac{[1-\beta(1-d(x))]}{\beta y(x)q(x)}\left\{1-\phi(x)\times\epsilon_{\theta,\kappa}\right\}. \end{aligned} \tag{22}$$

Again, the labor share is decreasing in $\kappa(x)$ as long as the elasticity $\epsilon_{\theta,\kappa}$ and the bargaining weight $\phi(x)$ are small enough. We can also easily show that the labor share is increasing in y and A as long as the corresponding elasticities $\epsilon_{\theta,y}$ and $\epsilon_{\theta,A}$ are small enough.

Note that the magnitude of the labor share change tends to be larger when the matching function is CES. In the efficient solution, $\phi(x) = \alpha$ when the matching function is CD, and $\phi(x) = \frac{\alpha}{\alpha+(1-\alpha)\theta^{\rho}}$ when the matching function is CES. The bargaining weight in the CD case is larger whenever $\theta < 1$ if $\rho < 0$. Thus, the labor share declines more in the CES case when these conditions hold. Likewise, the elasticities between tightness and exogenous parameters (e.g. $\epsilon_{\theta,d}$) should be larger in the case of CD as any change in θ for a given parameter value is muted due to the response in $\phi(x)$ (see simple illustrative case in Section 4.5). These results imply that the labor share declines more in the CES model.

4.5 *A simple illustrative case*

It is useful to illustrate the model's solution and its new implications relative to the standard model. When bargaining weights are exogenous and there are only two labor statuses, employment and unemployment, the equilibrium tightness rate is fully characterized by the equation:

$$q(\theta)(1-\phi)(y-\overline{c}) = (r + d + \phi f(\theta))\kappa, \tag{23}$$

where $r = \frac{1-\beta}{\beta}$.

The equilibrium tightness rate balances the gross expected profits of posting a vacancy,

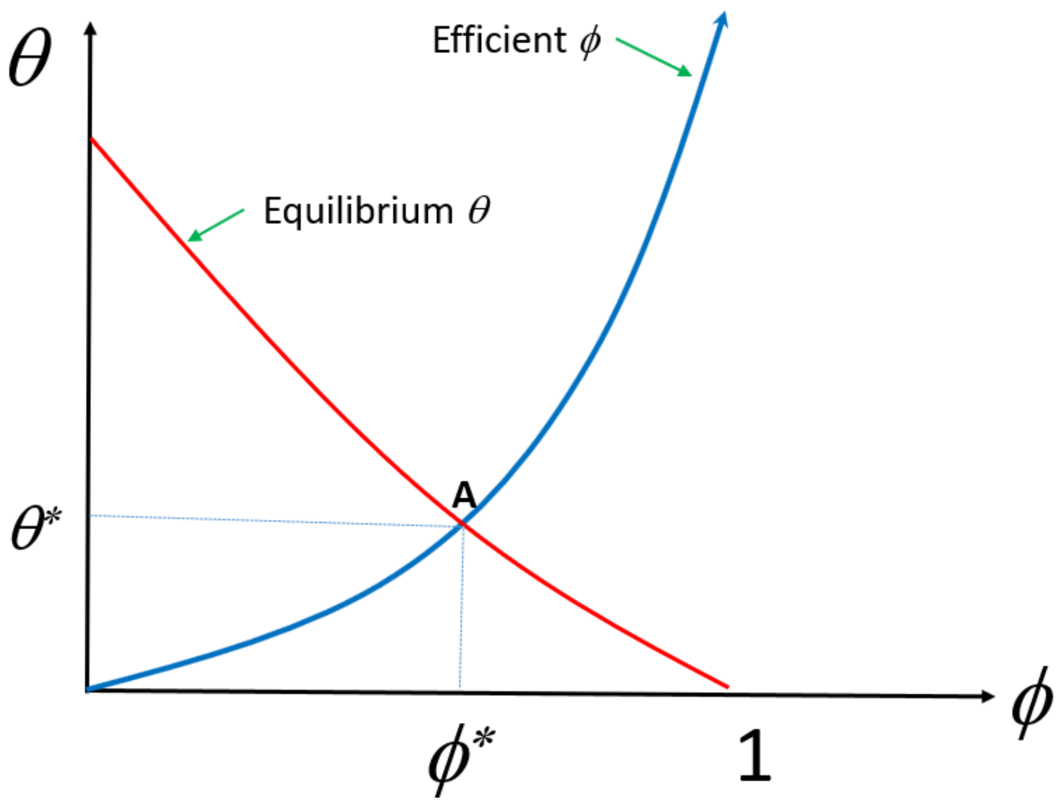

Figure 2. Joint determination of the tightness rate and bargaining power

the term on the left-hand side of the equation, to its annuitized cost, the term on the right-hand side. Under efficient bargaining, equation (23) is not enough to characterize the equilibrium. Equation (5) is also required. These two equations can be used to solve for θ and ϕ. The solution is illustrated in figure 2. Equation (23) describes a negative relationship between tightness and bargaining power since a higher worker's bargaining power reduces net profits and the incentives to post vacancies. Equation (23) describes a positive relationship when vacancies and unemployment are complements, as discussed in Section 2.

Figure 3 illustrates the effect of an increase in the job destruction rate, d, on tightness and bargaining power In the standard model, with exogenous bargaining power, the equilibrium would shift from point A to B. Only the tightness rate falls to θ_0 reflecting the lower incentives to post vacancies when the duration of the match is reduced. But point B implies inefficiently high bargaining power for that low level of tightness. In that case,

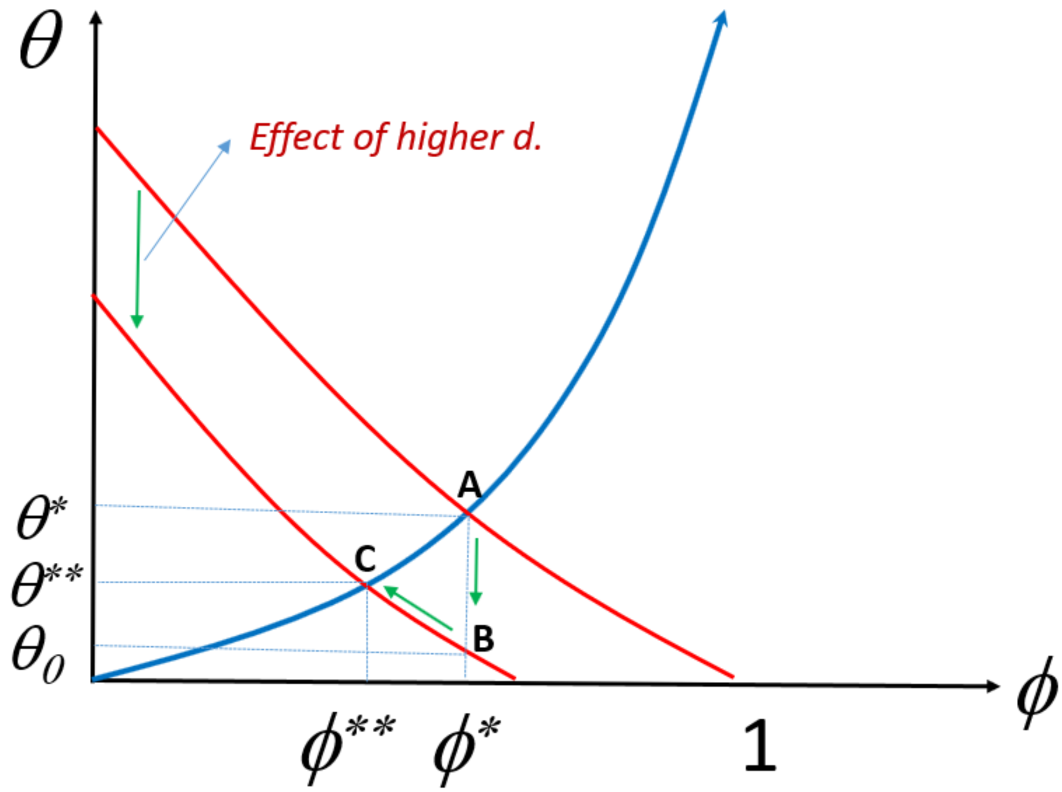

Figure 3. Effect of an increase in the job destruction rate in tightness and bargaining power

there is an underlying arbitrage opportunity for firms to offer lower bargaining power to workers, one that workers would accept in exchange for higher chances of getting a job. The efficient solution is at point C. The final outcome is a reduction in both tightness and bargaining power. The endogenous response of bargaining dampens the response of the tightness rate.

5 Parameterization

5.1 *Estimation of the CES matching function parameters*

In the model, we assume that the matching function is of the CES form where inputs are complements and $\rho < 0$. Does data support this assumption? We find robust support for the CES matching function with $\rho < 0$ in the U.S. data when expanding the set of job seekers to include not only unemployed job seekers but also nonparticipants, or a subset of nonparticipants.[13] Including other non-employed groups in the job seeker pool seems sensible given that a large fraction of the open jobs in the United States are filled by individuals coming from nonparticipation rather than unemployment.

Specifically, we estimate the matching function using two alternative measures of job seekers: our first measure, "*UN*", includes all unemployed and nonparticipating individuals, as defined in the Bureau of Labor Statistics' official measures; and our second measure, "*NEI*", is the Hornstein-Kudlyak-Lange Non-Employment index obtained from Richmond Fed, which includes all non-employed individuals but takes into an account differences in the labor market attachment of different groups. Specifically, non-employed groups in *NEI* are weighted based on their probability of transitioning back into the labor market relative to the highest transition rate group, the short-term unemployed. To measure matches and vacancy rates, we rely on U.S. data on total nonfarm vacancy and hire rates from the Job Openings and Labor Turnover Survey (JOLTS). All data is monthly, seasonally adjusted, and the sample period starts in December 2000 and ends in August 2023.

To estimate the parameters of the CES matching function, we follow the estimation strategy introduced in Şahin et al. (2014) and Borowczyk-Martins et al. (2013). Throughout different specifications, we assume that matching functions are constant returns to scale, as is standard in the literature. As is widely known, the estimation of matching functions suffers from an endogeneity problem: matching efficiency is not necessarily independent from vacancies and unemployment. To tackle this endogeneity issue, we model the dynamics of matching efficiency through time-varying polynomials as in Şahin et al. (2014). We estimate the following non-linear least squares model:

$$log(hires_t) = const + \gamma^{'} QTT_t + \frac{1}{\rho} log\left[(1-\alpha)v_t^{\rho} + \alpha u_t^{\rho}\right] + \epsilon_t,$$

where v is the vacancy rate, u is a measure of job seekers (either *UN* or *NEI*), and QTT_t is

[13] This approach resembles the one in Sedláček (2016), who shows that excluding non-unemployed job-seekers from matching function estimation leads to biased estimates unless the numbers of unemployed and non-unemployed job seekers are perfectly correlated in the data. While Sedláček (2016) only estimates the parameters in the CD matching function, we focus on estimating the CES matching function and compare the results with the CD matching function.

a vector of four elements for the quartic time trend capturing the dynamics of matching efficiency.

Table 2 presents the results.[14] The results point to a strongly significant and negative value for ρ, estimates ranging from -.49 to -1.1. The specifications excluding year 2020 generate smaller estimated values for ρ, -.49 in the *UN* model and -.58 in the *NEI* model. In contrast, estimating the same model but using only unemployed as a measure of job seekers leads to estimates of ρ that are not statistically significantly different from zero at 5 percent level (Table E.6 in Appendix D).

[14] Figure E.1 in Appendix D plots the model fit for specifications using the full data sample. Figure E.2 plots the estimated quartic time trend for specifications using the full data sample and table E.5 shows the results for the quartic time trend parameter estimates in all specifications.

Table 2. Estimation results—Nonlinear least squares estimation

	Dependent variable: log(hires)					
Sample	Full		Excl. 2020		Excl. 2009-10, 2020	
Job seeker	*UN* (1)	*NEI* (2)	*UN* (3)	*NEI* (4)	*UN* (5)	*NEI* (6)
α	0.945***	0.583***	0.827***	0.620***	0.932***	0.723***
	(0.034)	(0.042)	(0.062)	(0.040)	(0.035)	(0.041)
ρ	−0.941***	−0.514***	−0.485***	−0.582***	−0.936***	−1.140***
	(0.253)	(0.153)	(0.163)	(0.138)	(0.221)	(0.192)
Obs.	273	273	261	261	237	237
Residual Std. Error	0.04302 (df = 266)	0.04472 (df = 266)	0.02715 (df = 254)	0.03317 (df = 254)	0.02555 (df = 230)	0.03094 (df = 230)

Note: Columns 1 and 2 show the results for both *UN* and *NEI* measures using the full sample. Columns 3 and 4 exclude observations from year 2020 to eliminate COVID-19 pandemic-related large jumps in the vacancy and job seeker data, while columns 5 and 6 eliminate year 2020 and years 2009 and 2010 to evaluate robustness of the results for excluding both COVID-19 and years related to the Global Financial Crisis (GFC). All specifications include a vector of four elements for the quartic time trend, capturing shifts in aggregate matching efficiency A_t. *p<0.1; **p<0.05; ***p<0.01

The results also point to a variation in the job-seeker share parameter α. In the regressions, the estimated parameter values are larger in the *UN* specifications and vary between 0.83 and 0.95. The estimated α gets values between 0.58 and 0.72 in the *NEI* specifications.

Our findings contrast the findings in the existing literature. The papers estimating the CES matching function have found values for ρ ranging from small negative to small positive values, but mostly values not statistically significant from zero, supporting the CD matching function (Blanchard and Diamond,1989; Shimer, 2005; and Şahin et al., 2014). However, these research have solely included unemployed job seekers in their estimation, which we find to be a crucial for finding support for the CES matching function. Thus, the exclusion of other job seekers from the matching function estimation may explain why earlier results have not found a strong support for the CES matching function.

While we find that the CES matching function provides a great fit of the data, we can further compare whether the CES matching function or the CD matching function provides a better fit of the data. As the CES matching function nests the CD case with $\rho = 0$, we can use the F-test to compare the models. We run the estimation using a Cobb-Douglas matching function and focus on two specifications: one excluding year 2020, and one excluding years 2009–10 and 2020. The p-values (Table 3) show that the two models are statistically significantly different, and that the CES model provides a better fit of the data. The F-test statistics thus provide support for the use of the CES matching function.[15]

5.2 *Calibration: Preliminaries*

We calibrate our model using data from two periods, $t \in \{1976–80, 2003–07\}$. Both periods reflect the peak of the business cycle, and we refer to these two periods using simply the years 1980 and 2007. We calibrate the model for four disaggregated groups: men and women with and without college education.[16] Moreover, we also calibrate otherwise similar model but with a CD matching function to the same targets and compare the models' performance in generating a decline in the labor share.

[15]We also use AIC test statistics to compare the CES and CD models, and the results also show that the AIC value is lower for the CES models.

[16]See Appendix D.1. for data details.

Table 3. Model comparison—F-test statistics

Sample	Excl. 2020		Excl. 2009-10, 2020	
Specification	*UN*, CD	*UN*, CES	*NEI*, CD	*NEI*, CES
Residual DF	255	254	255	254
Residual Sum Sq	0.1944	0.1872	0.2999	0.2794
Df		1		1
Sum Sq		0.0071		0.0205
F value		9.671		18.596
Pr(>F)		0.0021**		0.0000***

Note: · $p < 0.1$; * $p < 0.05$; ** $p < 0.01$; *** $p < 0.001$.

5.3 *Calibration: Constant Parameters*

We calibrate parameters at a quarterly frequency and set the discount rate β equal to 0.9902, which implies that the real interest rate equals 4 percent annually. We assume that unemployed workers consume a fixed fraction of their human capital: $\bar{c}\,(x)= \gamma \cdot y_i^t$. We set the replacement-rate parameters for unemployed workers to $\gamma = 0.35$. Under these parameter values, in the model, the average consumption during unemployment is about 40 percent of the average consumption of the employed.

We choose the matching function parameters ρ and α based the estimation results. To be consistent with their lower-end estimates, we set ρ = -.5 and α = 0.6. We also provide robustness results using different values for α and ρ in Appendix 7.

5.4 *Calibration: Time-Specific Parameters*

We observe average labor market flows and average wages in the data. We directly estimate the exogenous flows $(\pi^t_{EU}(i), \pi^t_{EN}(i), \pi^t_{UN}(i), \pi^t_{NU}(i))$ for each i and t using IPUMS-CPS data.

We use the model solution to recover the matching efficiency and human capital parameters for each demographic group i. Specifically, we calibrate the human capital y_i^t such that we exactly match the model wage to the average wage rate of i observed in the Center for

Economic and Policy Research (CEPR) data for workers between ages 25 and 64.

We allow different matching efficiencies A_i^t, $A_{i,s}^t$ for job-seekers from the unemployment pool and the nonparticipation pool. Specifically, we recover the matching efficiencies for the unemployed by exactly matching the estimated job-finding rates. Then we calibrate ψ_i^t such that we match the unexplained differences in job-finding rates between unemployed and nonparticipants of otherwise similar workers. Hence, $A_{i,s}^t = \psi_i^t A_i^t$.

We set the consumption for nonparticipants $\bar{c}(x) = \gamma_i^{N,t} \cdot y_i^t$ and calibrate the replacement rate $\gamma_i^{N,t}$ for each group such that the wage rates for workers coming from unemployment or nonparticipation are equal in the model.

Vacancy-Posting Cost κ_i^t: We separately calibrate the values of $\bar{\kappa}_i^t$ for each gender–education group. Specifically, we set the $\bar{\kappa}_i^{1980}$ to 0.33 for non-college males—a standard value in the literature—and set $\bar{\kappa}_i^{1980}$ for other groups such that the average tightness gaps between non-college males and other groups are matched. We then calibrate the $\bar{\kappa}_i^t$ for all groups such that the changes in tightness rates between 1980 and 2007 are matched. In particular, the tightness target in the data that we use for each group is vacancies per group over the non-employed between the ages of 25 and 64 (unemployed + nonparticipants), a target that can easily be mapped to our model.

Constructing group-specific vacancies is not straightforward as vacancy postings are not targeted to specific demographic groups. We rely on a simple assumption that current employment shares of each group in each year provide an estimate of the number of vacancies available for each group. Thus, we calculate group-specific vacancies v_i^t for by multiplying the number of vacancies with the employment share of group i at time t. The tightness-rate denominators for each group are simply the sum of their unemployment and nonparticipation levels at t. These data are directly observed in IPUMS-CPS data. Group-specific measures of tightness are then

$$\theta_i^t \equiv \frac{s_{E,i}^t \times v^t}{u_i^t + n_i^t}, \tag{24}$$

where v_i^t is the number of vacancies, u_i^t is the number of unemployed, n_i^t is the number of nonparticipants, and $s^t_{E,i}$ is the share of group i of the total employment at t.

Labor market tightness has decreased for all groups, but specifically for males (see table 7 for details). For both college and non-college males, tightness in 2007 was about one-third of the tightness in 1980. The decline for females was more subdued: The labor market tightness has decreased about 27 percent for non-college females and about 13 percent for college-educated females.

5.5 *Calibration results*

Tables 5 and 6 sum up the CES model parameters for the four groups. For men, the results (Table 5) indicate that the matching efficiency, captured by A and ψ, has improved, but while productivity of college-educated men has increased, the opposite is true for non-college men. Overall, men are less attached to labor force reflected in the labor market flows: the flows into nonparticipation (π_{EN}, π_{UN}) has increased for both education groups while the flow from nonparticipation to unemployment (π_{NU}) has decreased. However, the nonparticipation state does not seem to be more attractive in terms of utility, as the consumption value γ^N has only slightly increased for college-educated males and decreased for non-college males. Finally, the calibration results indicate that the vacancy creation cost κ has increased remarkably for both education groups, reflected as a decline in tightness.

The calibration results for women (Table 6) have similarities and differences when comparing with the results of men. Similar to men, both education groups have faced increasing matching efficiencies between 1980 and 2007. However, productivity of both college and non-college females have increased while productivity of non-college males actually decreased. Overall, within education group, a gender gap in productivity has declined. Moreover, both female groups have increased their attachment to the labor force, reflected in the decreased flows out of labor force and increased flows out of nonparticipation. When it comes to vacancy creation costs and consumption value during nonparticipation, the changes have been similar to the ones of males.

The calibration results reflect the observed changes in wages during the same period. First, an increase in y reflects the fact that real wages have increased for all groups except

Table 4. Parameter values for 1976–80 and 2003–07 steady states—common parameters across the CD and CES models

Parameter	Explanation	Value	Source
β	Discount factor	.9902	Quarterly rate
γ	Replacement rate	.35	Average consumption during unemployment
α	Matching function: share	.6	Own estimates
ρ	Matching elasticity: CES model	-.5	Own estimates

Table 5. Calibrated parameter values, men, CES model, steady states

	1980	2007	1980	2007
	Male, College	Male, College	Male, Non-college	Male, Non-college
Parameter				
y	1.337	1.537	1.043	0.968
A	0.499	0.678	0.585	0.899
ψ	0.614	0.649	0.517	0.634
π_{EU}	0.014	0.016	0.029	0.030
π_{EN}	0.020	0.032	0.034	0.053
π_{UN}	0.147	0.195	0.158	0.233
π_{NU}	0.069	0.048	0.045	0.043
$\bar{\kappa}$	0.162	0.710	0.333	1.347
γ^N	0.793	0.795	0.856	0.799

Source: Authors' estimations.

non-college males. Second, the decline in the gender gap in y goes hand in hand with the significant decline in the gender wage gap during the same period, as documented in Blau and Kahn (2017) and observed in the CEPR data.

Our calibration results show that matching efficiency is higher in 2007 than in 1980 for all groups. A validation of our calibration results for matching efficiency between the late 1970s and 2000s comes from the corresponding shifts in the Beveridge curve.[17] Shifts in the Beveridge curve are typically interpreted as changes in matching efficiency, with outward shifts reflecting lower matching efficiency. Michaillat and Saez (2021) and Diamond and

[17] The Beveridge curve reflects the negative relationship between unemployment and vacancy rates over the business cycle (Beveridge, 1944).

Table 6. Calibrated parameter values, women, CES model, steady state

	1980	2007	1980	2007
	Female, College	Female, College	Female, Non-college	Female, Non-college
Parameter				
y	0.966	1.237	0.742	0.827
A	0.761	0.863	0.760	1.007
ψ	0.646	0.670	0.625	0.691
π_{EU}	0.015	0.014	0.023	0.024
π_{EN}	0.100	0.063	0.122	0.096
π_{UN}	0.335	0.286	0.403	0.376
π_{NU}	0.028	0.031	0.026	0.030
$\bar{\kappa}$	1.018	1.519	1.347	2.767
γ^N	0.762	0.781	0.754	0.736

Source: Authors' estimations.

Şahin (2015) study long-term movements of the Beveridge curve in the U.S., and their findings show that the Beveridge curve in the late 1970s was located to the right of the curve in the late 2000s. This result supports the finding that matching efficiency was lower in the 1970s.

All the calibrated values of $\bar{\kappa}_i^t$ are shown in tables 5 and 6. We find that vacancy costs vary by gender and education, likely capturing the differences in representative occupations and industries for each group. More interestingly, we find that $\bar{\kappa}$ has increased for every group between 1980 and 2007.

What are the potential reasons for the increased $\bar{\kappa}$, and why has this increase varied between groups? We interpret the changes in the vacancy posting costs to broadly reflect the changes in relative costs of creating jobs for certain groups.[18] Any outside factor that raises the relative cost of opening a vacancy in the U.S., given the vacancy value, will be captured by $\bar{\kappa}$.

Hence, natural candidates are automation, increased globalization, and import competition. As described in Section 2, a large share of the drop in both the labor share and tightness

[18]Another way to interpret the cost of posting a vacancy is to interpret it as a fixed entry cost, either in the units of capital or labor, as in Mangin and Sedláček (2018).

Table 7. Tightness and efficient bargaining power

	Tightness, θ			Bargaining power, ϕ		
Group	1980	2007	% Change	1980	2007	% Change
Male, C	2.48	0.73	-70.50	0.61	0.47	-23.79
Female, C	0.36	0.32	-12.50	0.38	0.37	-3.67
Male, NC	1.00	0.36	-64.00	0.52	0.39	-24.35
Female, NC	0.21	0.15	-26.80	0.32	0.29	-10.15
Weighted average	0.46	0.19	-59.56	0.47	0.39	-16.87

Note: Weighted averages are calculated using employment shares of each group in each period as weights.

Source: Authors' estimations.

occurred after 2000. The sluggish employment growth in the U.S. in the 2000s is tightly linked to increased import competition (Acemoglu et al., 2016; Charles et al., 2019). While import competition has directly depressed employment in the most affected industries, these effects have transmitted to other industries through input-output and aggregate demand linkages, further elevating employment losses (Acemoglu et al., 2016; Autor et al., 2016). Moreover, there is no strong evidence of offsetting employment gains in other industries in the long-term: Out-migration from the most affected local labor markets has been modest, manufacturing job losses have translated to declines in employment-to-population ratios, and the negative effects of the China shock have persisted until the late 2010s (Autor et al., 2021). In our model, these negative employment effects are captured by $\bar{\kappa}$, leading to lower vacancy posting and a drop in labor demand.

Moreover, evidence shows that the described negative employment effects from rising import competition have had heterogeneous effects on different gender and education groups, potentially explaining the heterogeneous changes in the calibrated vacancy costs. First, while both female- and male-dominated manufacturing industries have faced negative employment and wage consequences from import competition, a larger share of males work in manufacturing, leading to a larger effect on men (Autor et al., 2019). Second, the negative effect of trade exposure on employment have concentrated in local labor markets with a smaller share of college-educated workers (Autor et al., 2021). The lower decline in college-educated females' $\bar{\kappa}$ could arise from the fact that, compared with males, college-educated females are more often working on health-care- and education-related occupations, which are less affected by import competition.

Finally, the increase in the cost of creating a new job also lines up with the findings of Wolcott (2020). She concludes that the decline in employment rates of U.S. male workers, especially those without a college education, since the late 1970s is driven by demand factors rather than supply factors.

To sum up, we find that in order to match the observed wage trends (increases for other groups except non-college males), the increases in job-finding rates for females and the decreases for males, and the decreases in tightness rates, there must be counteracting forces that can jointly generate these trends. First, increased real wages indicate an increase in the productivity captured by human capital parameters. Second, while tightness rates have decreased, job-finding rates either have decreased less or have increased, meaning that matching efficiencies must have increased. Both factors increase demand for workers by increasing the value of opening a vacancy, leading to higher tightness rates. To match the declines in tightness rates, vacancy costs have grown, capturing the fact that while the vacancy value has also grown, there has been a counter force that has lowered labor demand.

Decline in Workers' Bargaining Weights: We study changes in bargaining power of workers between 1980 and 2007 by gender and education (table 7). Our calibration results suggest a decline in bargaining power. Using employment shares of each group as weights, we find that aggregate bargaining power of workers has declined about 17 percent between 1980 and 2007.

The decline has not been uniform across groups: the decrease has been larger for males and for workers without a college education. Based on our results, both male groups have faced a decline of one fourth in their bargaining power, while the decline for non-college females has been about 10 percent and for college females about 4 percent. The results are fairly robust to different values of ρ: the decline in bargaining power is higher when the value of ρ gets smaller (Table E.3).

Using equation (5), the calibrated parameter values for α and ρ, and observed time series of aggregate tightness, we construct an aggregate bargaining power series between 1976 and 2016. Based on the aggregate data, we find that the bargaining power of workers has decreased around 8.5 percent between the two business cycle peaks of 1979 and 2007 (panel B in figure 4). The decline in bargaining power is smaller than the weighted average from table 7, indicating that relying on aggregate tightness can lead to underestimating the overall decline.

Changes in the Labor Share in the endogenous bargaining power vs. fixed bargaining power models. To gauge the importance of endogenous bargaining power in explaining the labor share decline, we calibrate two alternative models. The models are otherwise similar to the baseline model, but the first alternative model replaces the CES matching function with the CD matching function, and the second alternative model uses fixed bargaining weights (while still assuming that the matching function is CES). In both alternative models, the bargaining weight of workers is set to be equal to $\alpha = \phi = 0.6$, reflecting the standard calibration strategy in the DMP models.

We define the labor share as $\frac{w(x)}{y_i}$ and study its changes in the three different models. We find that the CES model generates a 1.21 percent decline in the labor share between 1980 and 2007, while the CD model generates an increase of 0.32 percent (Table 8). The labor share has dropped 4.0 percent during the same period.[19] The CES model thus accounts for 30 percent of the decline in the labor share, while the CD model would imply an increase in the labor share.[20] This result implies that introducing endogenous bargaining weights is critical for generating the labor share decline in the model.

We also test whether the combination used in the earlier literature—a CES matching function and a constant bargaining weight—delivers a decline in the labor share. The

[19]The decline is calculated by comparing the average labor shares in the periods of 1976–80 and 2003–07.
[20]Note that we do not target the labor share decline in our calibration.

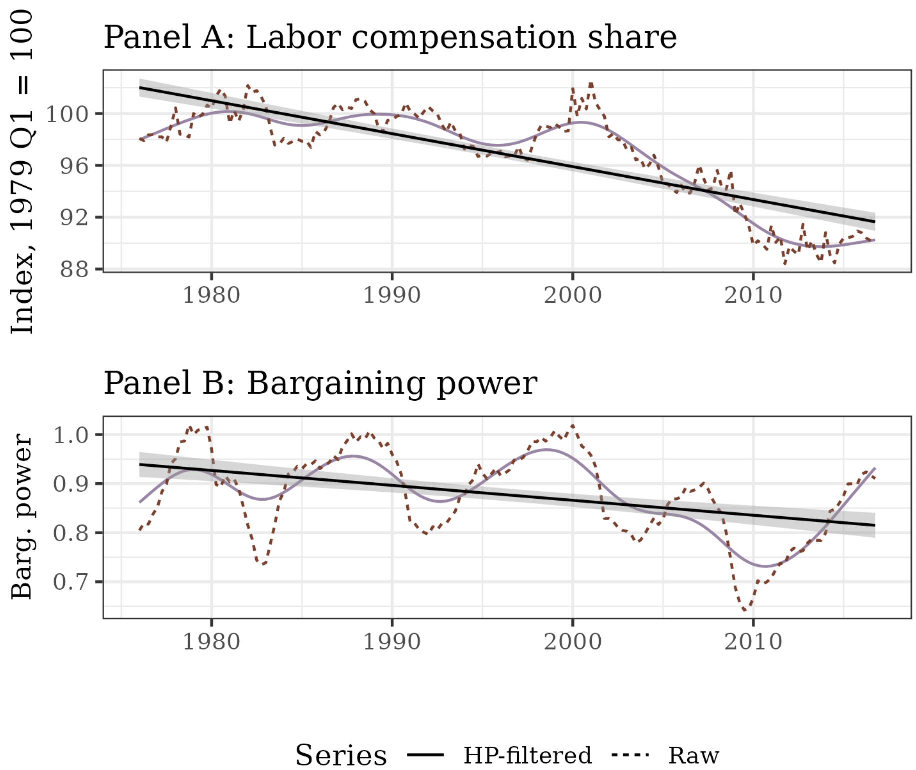

Figure 4. Efficient bargaining power has declined between 1979 and 2007, along with the labor share

Note: Panel A includes the quarterly, seasonally adjusted labor share for all employed persons in the nonfarm business sector, and panel B shows the quarterly bargaining power series normalized to 1 in 1979:Q1. The dashed lines plot the raw series, while the purple solid lines plot the HP-filtered series with lambda set to 1,600. Both figures also include a linear trend line with 95 percent confidence bounds.

Source: Bureau of Labor Statistics; Authors' estimations.

results are shown in the third column of Table 8: the results are exactly the same as in the CD model, indicating that the decline in the labor share is fully driven by the changes in the bargaining weight.

6 Counterfactual Analysis

We assess the effect of each exogenous parameter on the model-generated changes in bargaining power. We do that by giving each parameter its 2007 value one at a time and by keeping all the other parameters at their 1980 levels. Table 9 shows the results.

Table 8. Decrease in the labor share from 1980 to 2007

	Data	CD model	CES + fixed barg.	CES model
Percent decline in labor share	4.00	-0.32	-0.32	1.21
Percent of decline in data	100.00	-8.00	-8.00	30.25

Note: Table 8 presents the decrease in the labor share in the data, in the CD model, and in the CES model.

Source: Bureau of Labor Statistics; Authors' estimations.

We find that three parameters have driven changes in bargaining power through changes in tightness. First, the increased vacancy-posting cost $\bar{\kappa}$ can explain the majority of the bargaining power decline for all groups. Second, improved matching efficiency has mitigated the decline in the bargaining power of all groups. Third, females have benefited from a higher likelihood to participate measured by less frequent flows to nonparticipation (lower π_{EN} and π_{UN}), while a lower likelihood to participate has strengthened the decline in bargaining power for males. These three factors also play a key role in explaining the declines in the labor share for males and noncollege females (Table E.2).

Intuitively, better matching efficiencies increase labor market tightness by increasing a vacancy value. To match the observed *decline* in the tightness, along with the observed wages and job-finding rates, the model predicts that the vacancy-posting cost has increased. An increase in the relative cost of opening a vacancy has decreased demand for labor and the number of vacancies.

To summarize our findings, we find that κ, a proxy for a decline in labor demand, has driven the decline in tightness, and thus bargaining power, between 1980 and 2007.

7 Conclusion

In this paper, we used a DMP search and matching model with endogenous bargaining power to study how the labor share and workers' bargaining power have changed over the past four decades. Specifically, we assumed that the matching function takes the CES form, which implies that bargaining power increases with labor market tightness whenever the Hosios condition holds and there is enough complementarity between vacancies and job

Table 9. Counterfactuals—bargaining power, weighted average

	Male, C	Female, C	Male, NC	Female, NC
Total change—model	-23.8	-3.7	-24.8	-10.1
y	0.0	-0.2	0.0	-0.1
A	-1.9	-46.7	-8.7	-65.8
ψ	-0.2	-11.7	-1.4	-19.1
π_{EU}	-0.9	-1.6	-0.3	1.1
π_{EN}	7.2	-140.3	14.1	-48.1
π_{UN}	1.9	-8.6	5.4	-2.6
π_{NU}	2.7	-8.5	0.7	-5.9
κ	90.9	272.6	102.7	261.4
γ^N	0.3	44.9	-12.7	-20.9
Total contribution	100.0	99.9	99.8	100.0

Source: Authors' estimations.

seekers in the matching process.

First, we find that a DMP model with endogenous bargaining power generates a significantly larger drop in the labor share compared with the model with fixed bargaining power. Second, our calibration results suggest that workers' efficient bargaining power has decreased about 17 percent between 1980 and 2007. This decline can be attributed to a higher vacancy-posting cost, which has driven down the labor demand. We also find that the decline in bargaining power has been larger for men and workers without a college education.

Overall, the decline in bargaining power based on our model has been modest. However, as we abstract away from the reported decline in both union membership and coverage, we are likely underestimating the decline in bargaining power. Consider the import-competition shock that reduces demand for U.S. labor and thus labor market tightness. Unionized workers affected by the shock now face a tradeoff: Staying unionized with higher bargaining power and better benefits while facing even higher risk of jobs moving abroad. Existing evidence points out that unionization has decreased because of the same shocks that have decreased tightness, and it is possible that tightness would have decreased more without unionization declining, leading to lower bargaining power in our model. It would be interesting to extend our model to include endogenous decisionmaking on union membership and study the dynamics of bargaining power, union membership, and tightness in the face of demand shocks for labor. This is left to future work.

Also, we leave it to future work to establish what exactly has driven the decline in tightness in the U.S.

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Table A.1. Low-pass correlation and confidence intervals, labor share and tightness, periods longer than 15 years

		$\hat{\rho}$	67% CI	90% CI	95% CI
Sample: 1965:Q1-2019:Q4					
Labor share$_t$	θ_t	**0.320***	0.013, 0.603	-0.209, 0.776	-0.331, 0.807
	θ_{t-4}	**0.485***	0.211, 0.772	-0.011, 0.841	-0.130, 0.890
	θ_{t-8}	**0.646*****	0.421, 0.846	0.168, 0.912	0.031, 0.947
	θ_{t-12}	**0.678*****	0.461, 0.877	0.253, 0.922	0.115, 0.947
	θ_{t-16}	**0.603*****	0.404, 0.814	0.129, 0.894	0.001, 0.916

* significance at 67% level, ** 90% level, *** 95% level

Source: Petrosky-Nadeau & Zhang (2021); IPUMS-CPS; BLS; Authors' estimations.

Table A.2. Low-pass correlation and confidence intervals, labor share and tightness, periods longer than 15 years

		$\hat{\rho}$	67% CI	90% CI	95% CI
Sample: 1952:Q1-2022:Q4					
Labor share$_t$	θ_t	**0.032**	-0.226, 0.311	-0.461, 0.462	-0.493, 0.540
	θ_{t-4}	**0.109**	-0.130, 0.386	-0.321, 0.512	-0.462, 0.637
	θ_{t-8}	**0.190**	-0.034, 0.450	-0.269, 0.637	-0.379, 0.667
	θ_{t-12}	**0.254***	0.001, 0.493	-0.211, 0.659	-0.301, 0.712
	θ_{t-16}	**0.302***	0.030, 0.597	-0.150, 0.680	-0.266, 0.724

* significance at 67% level, ** 90% level, *** 95% level

Source: Petrosky-Nadeau & Zhang (2021); IPUMS-CPS; BLS; Authors' estimations.

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Table A.3. Low-pass correlation and confidence intervals, labor share and tightness, first difference, periods longer than 15 years

		$\hat{\rho}$	67% CI	90% CI	95% CI
Sample: 1952:Q1-2022:Q4					
Labor share$_t$	θ_t	**0.099**	-0.160, 0.401	-0.404, 0.594	-0.470, 0.653
	θ_{t-4}	**0.192**	-0.081, 0.504	-0.319, 0.653	-0.408, 0.721
	θ_{t-8}	**0.742*** **	0.577, 0.912	0.380, 0.945	0.332, 0.957
	θ_{t-12}	**0.580** **	0.365, 0.786	0.079, 0.874	-0.011, 0.903
	θ_{t-16}	**0.324***	0.004, 0.639	-0.160, 0.777	-0.302, 0.824

* significance at 67% level, ** 90% level, *** 95% level

Source: Petrosky-Nadeau & Zhang (2021); IPUMS-CPS; BLS; Authors' estimations.

Table A.4. Low-pass correlation and confidence intervals, labor share and tightness, first difference, periods longer than 10 years

		$\hat{\rho}$	67% CI	90% CI	95% CI
Sample: 1965:Q1-2019:Q4					
Labor share$_t$	θ_t	**-0.045**	-0.364, 0.160	-0.533, 0.379	-0.630, 0.432
	θ_{t-4}	**0.136**	-0.082, 0.380	-0.333, 0.586	-0.385, 0.645
	θ_{t-8}	**0.482** **	0.304, 0.703	0.011, 0.794	-0.045, 0.825
	θ_{t-12}	**0.687*** **	0.550, 0.848	0.364, 0.903	0.304, 0.922
	θ_{t-16}	**0.617*** **	0.432, 0.781	0.296, 0.871	0.095, 0.902

* significance at 67% level, ** 90% level, *** 95% level

Source: Petrosky-Nadeau & Zhang (2021); IPUMS-CPS; BLS; Authors' estimations.

Appendix A: Long-Run Covariability between the Labor Share and Tightness—Robustness

Appendix B: Workers' surpluses

The values of being employed, E, unemployed, U, or non-participant, N, are given by:

$$\begin{aligned} E(i,U) &= w(x) + \beta\left[\pi^i_{EU}U(x) + \pi^i_{EN}N(x) + \left(1 - \pi^i_{EU} - \pi^i_{EN}\right)E(i,U)\right] \\ &= w(x) + \beta\left[E(i,U) - \pi^x_{EU}\left(E(i,U) - U(x)\right) - \pi^i_{EN}\left(E(i,U) - U(x) + U(x) - N(x)\right)\right] \\ &= w(x) + \beta\left[E(i,U) - \left(\pi^i_{EU} + \pi^i_{EN}\right)\left(E(i,U) - U(x)\right) - \pi^i_{EN}\left(U(x) - N(x)\right)\right], \end{aligned}$$

Table A.5. Low-pass correlation and confidence intervals, tightness and labor share, periods longer than 10 years

		$\hat{\rho}$	67% CI	90% CI	95% CI
Sample: 1965:Q1-2019:Q4					
θ_t	Labor share$_t$	**0.307***	0.034, 0.504	-0.124, 0.667	-0.255, 0.733
	Labor share$_{t-4}$	**0.187**	-0.028, 0.443	-0.269, 0.538	-0.401, 0.569
	Labor share$_{t-8}$	**0.044**	-0.226, 0.269	-0.418, 0.456	-0.480, 0.491
	Labor share$_{t-12}$	**-0.047**	-0.340, 0.178	-0.495, 0.408	-0.603, 0.447
	Labor share$_{t-16}$	**-0.045**	-0.345, 0.206	-0.513, 0.428	-0.597, 0.460

* significance at 67% level, ** 90% level, *** 95% level

Source: Petrosky-Nadeau & Zhang (2021); IPUMS-CPS; BLS; Authors' estimations.

Table A.6. Low-pass correlation and confidence intervals, labor share and standard tightness (vacancies/unemployed), periods longer than 10 years

		$\hat{\rho}$	67% CI	90% CI	95% CI
Sample: 1965:Q1-2019:Q4					
Labor share$_t$	θ_t	**0.179**	-0.115, 0.461	-0.350, 0.648	-0.474, 0.712
	θ_{t-4}	**0.311***	0.013, 0.596	-0.209, 0.716	-0.319, 0.798
	θ_{t-8}	**0.445***	0.161, 0.712	-0.027, 0.825	-0.178, 0.866
	θ_{t-12}	**0.473***	0.211, 0.716	-0.003, 0.829	-0.124, 0.883
	θ_{t-16}	**0.425***	0.151, 0.712	-0.036, 0.813	-0.184, 0.857

* significance at 67% level, ** 90% level, *** 95% level

Source: Petrosky-Nadeau & Zhang (2021); IPUMS-CPS; BLS; Authors' estimations.

$$\begin{aligned} E(i,N) &= w(x) + \beta \left[\pi^i_{EU} U(x) + \pi^i_{EN} N(x) + \left(1 - \pi^i_{EU} - \pi^i_{EN}\right) E(i,N)\right] \\ &= w(x) + \beta \left[E(i,N)) - \pi^i_{EU}\left(E(i,N) - N(x) + N(x) - U(x)\right) - \pi^i_{EN}\left(E(i,N) - N(x)\right)\right] \\ &= w(x) + \beta \left[E(i,N) - \left(\pi^i_{EU} + \pi^i_{EN}\right)\left(E(i,N) - N(x)\right) + \pi^i_{EU}\left(U(x) - N(x)\right)\right], \end{aligned}$$

$$\begin{aligned} U(x) &= \bar{c}(i,U) + \beta \left[f^{\bar{U}}_i E(i,U) + \pi^i_{UN} N + \left(1 - f^{\bar{U}}_i - \pi^i_{UN}\right) U(x)\right] \\ &= \bar{c}(i,U) + \beta \left[U(x) + f^{\bar{U}}_i \left(E(i,U) - U(x)\right) + \pi^i_{UN}\left(N(x) - U(x)\right)\right], \end{aligned}$$

and

$$
\begin{aligned}
N(x) &= \bar{c}(i,N) + \beta\left[f_i^{\bar{N}} E(i,N) + \pi_{NU}^i U(x) + \left(1 - f_i^{\bar{N}} - \pi_{NU}^i\right) N(x)\right] \\
&= \bar{c}(i,N) + \beta\left[N(x) + f_i^{\bar{N}}\left(E(i,N)) - N(x)\right) + \pi_{NU}^i\left(U(x) - N(x)\right)\right].
\end{aligned}
$$

These expressions can be used to calculate surpluses $S_{EU}(x) \equiv E(i,U) - U(x)$, $S_{EN}(x) \equiv E(i,N) - N(x)$ and $S_{UN}(x) \equiv U(x) - N(x)$ as follows:

$$
\begin{aligned}
S_{EU}(x) =& w(x) + \beta\left[E(i,U)) - d(x)S_{EU}(x) - \pi_{EN}^i S_{UN}(x)\right] - \bar{c}(i,U) \\
& - \beta\left[U(x) + f_i^{i\bar{U}} S_{EU}(x) + \pi_{UN}^i S_{UN}(x)\right] \\
=& w(x) - \bar{c}(i,U) + \beta\left[E(i,U) - d(x)S_{EU}(x) - \pi_{EN}^i S_{UN}(x) - U(x) - f_i^{\bar{U}} S_{EU}(x) - \pi_{UN}^i S_{UN}(x)\right] \\
=& w(x) - \bar{c}(i,U) + \beta\left[\left(1 - d(x) - f_i^{\bar{U}}\right) S_{EU}(x) + \left(\pi_{UN}^i - \pi_{EN}^i\right) S_{UN}(x)\right] \\
\Longleftrightarrow & S_{EU}(x) = \frac{w(x) - \bar{c}(i,U)}{1 - \beta\left(1 - d(x) - f_i^{\bar{U}}\right)} + \frac{\beta\left(\pi_{UN}^i - \pi_{EN}^i\right)}{1 - \beta\left(1 - d(x) - f_i^{\bar{U}}\right)} S_{UN}(x)
\end{aligned}
$$

$$
\begin{aligned}
S_{EN}(x) =& w(x) + \beta\left[E(i,N)) - d(x)S_{EN}(x) + \pi_{EU}^i S_{UN}(x)\right] - \bar{c}(i,N) \\
& - \beta\left[N(x) + f_i^{\bar{N}} S_{EN}(x) + \pi_{NU}^i S_{UN}(x)\right] \\
=& w(x) - \bar{c}(i,N) + \beta\left[E(i,N) - N(x) - d(x)S_{EN}(x) + \pi_{EU} S_{UN}(x) - f_i^{\bar{N}} S_{EN}(x) - \pi_{NU}^i S_{UN}(x)\right] \\
=& w(x) - \bar{c}(i,N) + \beta\left[\left(1 - d(x) - f_i^{\bar{N}}\right) S_{EN}(x) + \left(\pi_{EU}^i - \pi_{NU}^i\right) S_{UN}(x)\right] \\
\Longleftrightarrow & S_{EN}(x) = \frac{w(x) - \bar{c}(i,N)}{1 - \beta\left(1 - d(x) - f_i^{\bar{N}}\right)} + \frac{\beta\left(\pi_{EU}^i - \pi_{NU}^i\right)}{1 - \beta\left(1 - d(x) - f_i^{\bar{N}}\right)} S_{UN}(x)
\end{aligned}
$$

$$
\begin{aligned}
S_{UN}(x) =& \bar{c}(i,U) + \beta\left[U(x) + f_i^{\bar{U}} S_{EU}(x) - \pi_{UN}^i S_{UN}(x)\right] - \bar{c}(i,N) - \beta\left[N(x) + f_i^{\bar{N}} S_{EN}(x) + \pi_{NU}^i S_{UN}(x)\right] \\
=& \bar{c}(i,U) - \bar{c}(i,N) + \beta\left[\left(1 - \pi_{UN}^i - \pi_{NU}^i\right) S_{UN}(x) + f_i^{\bar{U}} S_{EU}(x) - f_i^{\bar{N}} S_{EN}(x)\right] \\
\Longleftrightarrow & S_{UN}(x) = \frac{\bar{c}(i,U) - \bar{c}(i,N)}{1 - \beta\left(1 - \pi_{UN}^i - \pi_{NU}^i\right)} + \frac{\beta f_i^{\bar{U}}}{1 - \beta\left(1 - \pi_{UN}^i - \pi_{NU}^i\right)} S_{EU}(x) \\
& - \frac{\beta f_i^{\bar{N}}}{1 - \beta\left(1 - \pi_{UN}^i - \pi_{NU}^i\right)} S_{EN}(x).
\end{aligned}
$$

Appendix C: A Social Planner's Problem and Hosios Condition in a DMP Model with Nonparticipation and Segmented Markets

Appendix C.1: Social Planner's Problem

The planner maximizes the sum of flows of market and home productions net of search costs. The planner chooses the optimal full sequences of vacancies $\left\{v_i^{\bar{U},t}, v_i^{\bar{N},t}\right\}$, and employment, unemployment, and nonparticipation masses $\{M_i^{\bar{E},t+1}, M_i^{\bar{U},t+1}, M_i^{\bar{N},t+1}\}$. Denote the control variables by

$X = \left\{v_i^{\bar{U},t}, v_i^{\bar{N},t}, M_i^{\bar{E},t+1}, M_i^{\bar{U},t+1}, M_i^{\bar{N},t+1}\right\}$. We assume that labor markets are segmented for each i, which means that the planner faces a different matching technology for workers from different statuses. This also simplifies the planner's problem: It can treat each problem as a separate optimization problem. Moreover, the planner also observes the current labor market status of a job-seeker, $\bar{U}$ and $\bar{N}$. The social planner's problem is

$$\max_X \sum_{t=0}^{\infty} \beta^t \left[y_i M_i^{\bar{E},t} + \bar{c}_i^{\bar{U}} M_i^{\bar{U},t} + \bar{c}_i^{\bar{N}} M_i^{\bar{N},t} - \kappa_i^{\bar{U}} v_i^{\bar{U},t} - \kappa_i^{\bar{N}} v_i^{\bar{N},t} \right] \tag{25}$$

subject to

$$M_i^{\bar{E},t+1} = \left[1 - \pi_{EU}^i - \pi_{EN}^i\right] M_i^{\bar{E},t} + q\left(\frac{v_i^{\bar{U},t}}{M_i^{\bar{U},t}}\right) v_i^{\bar{U},t} + \widehat{q}\left(\frac{v_i^{\bar{N},t}}{M_i^{\bar{N},t}}\right) v_i^{\bar{N},t}, \tag{26}$$

$$M_i^{\bar{U},t+1} = M_i^{\bar{U},t} + \pi_{EU}^i M_i^{\bar{E},t} - q\left(\frac{v_i^{\bar{U},t}}{M_i^{\bar{U},t}}\right) v_i^{\bar{U},t} - \pi_{UN}^i M_i^{\bar{U},t} + \pi_{NU}^i M_i^{\bar{N},t}, \tag{27}$$

$$M_i^{\bar{N},t+1} = M_i^{\bar{N},t} + \pi_{EN}^i M_i^{\bar{E},t} - \widehat{q}\left(\frac{v_i^{\bar{N},t}}{M_i^{\bar{N},t}}\right) v_i^{\bar{N},t} - \pi_{UN}^i M_i^{\bar{U},t} + \pi_{NU}^i M_i^{\bar{N},t}, \tag{28}$$

for $t = 0, \ldots, \infty$.

The first constraint represents employment dynamics between two periods, t and $t-1$ for each i, while the last two constraints represent the evolution of unemployment and nonparticipation masses.

As labor market tightness is defined as $\theta_i^{s,t} = \frac{v_i^{s,t}}{M_i^{s,t}}$, where $s \in \left\{\bar{U}, \bar{N}\right\}$, the planner's problem can be written in terms of tightness. The Lagrangian becomes

$$
\begin{aligned}
L = \max_X \sum_{t=0}^{\infty} \beta^t &\Big\{ y_i M_i^{\bar{E},t} + M_i^{\bar{U},t}\left(\bar{c}_i^{\bar{U}} - \kappa_i^{\bar{U}}\theta_i^{\bar{U},t}\right) + M_i^{\bar{N},t}\left(\bar{c}_i^{\bar{N}} - \kappa_i^{\bar{N}}\theta_i^{\bar{N},t}\right)\Big\} \\
+ \sum_{t=0}^{\infty} \beta^t \lambda_i^t &\Big\{ \left(1 - \pi_{EU}^i - \pi_{EN}^i\right) M_i^{\bar{E},t} f\left(\theta_i^{\bar{U},t}\right) M_i^{\bar{U},t} + \widehat{f}\left(\theta_i^{\bar{N},t}\right) M_i^{\bar{N},t} - M_i^{\bar{E},t+1}\Big\} \\
+ \sum_{t=0}^{\infty} \beta^t \mu_i^t &\Big\{ (1 - f\left(\theta_i^{\bar{U},t}\right) - \pi_{UN}^i) M_i^{\bar{U},t} \pi_{EU}^i M_i^{\bar{E},t} + \pi_{NU}^i M_i^{\bar{N},t} - M_i^{\bar{U},t+1}\Big\} \\
+ \sum_{t=0}^{\infty} \beta^t \eta_i^t &\Big\{ (1 - \widehat{f}\left(\theta_i^{\bar{N},t}\right) - \pi_{UN}^i) M_i^{\bar{N},t} \pi_{EN}^i M_i^{\bar{E},t} + \pi_{UN}^i M_i^{\bar{U},t} - M_i^{\bar{N},t+1}\Big\}
\end{aligned}
$$

The first order conditions with respect to $M_i^{\bar{E},t+1}$, $M_i^{\bar{U},t+1}$, $M_i^{\bar{N},t+1}$, $\theta_i^{\bar{U},t}$ and $\theta_i^{\bar{N},t}$ are written as follows:

$$
\begin{aligned}
M_i^{\bar{E},t+1} :& \beta^{t+1} y_i - \beta^t \lambda_i^t + \beta^{t+1}\lambda_i^{t+1}\left(1 - \pi_{EU}^i - \pi_{EN}^i\right) + \beta^{t+1}\mu_i^{t+1}\pi_{EU}^i + \beta^{t+1}\eta_{t+1}^i \pi_{EN}^i = 0 \\
M_i^{\bar{U},t+1} :& \quad \beta^{t+1}(\bar{c}_i^{\bar{U}} - \kappa_i^{\bar{U}}\theta_i^{\bar{U},t+1}) + \beta^{t+1}\lambda_i^{t+1} f\left(\theta_i^{\bar{U},t+1}\right) + \beta^{t+1}\mu_i^{t+1}\left[1 - f\left(\theta_i^{\bar{U},t+1}\right) - \pi_{UN}^i\right] \\
&+ \beta^{t+1}\eta_i^{t+1}\pi_{UN}^i - \beta^t \mu_i^t = 0 \\
M_i^{\bar{N},t+1} :& \quad \beta^{t+1}(\bar{c}_i^{\bar{N}} - \kappa_i^{\bar{N}}\theta_i^{\bar{N},t+1}) + \beta^{t+1}\lambda_i^{t+1}\widehat{f}\left(\theta_i^{\bar{N},t+1}\right) + \beta^{t+1}\eta_i^{t+1}\left[1 - \widehat{f}\left(\theta_i^{\bar{N},t+1}\right) - \pi_{NU}^i\right] \\
&+ \beta^{t+1}\mu_i^{t+1}\pi_{NU}^i - \beta^t \eta_i^t (e,a) = 0 \\
\theta_i^{\bar{U},t} :& - \beta^t M_i^{\bar{U},t}\kappa_i^{\bar{U}} + \beta^t \lambda_i^t f'\left(\theta_i^{\bar{U},t}\right) M_i^{\bar{U},t} - \beta^t \mu_i^t f'\left(\theta_i^{\bar{U},t}\right) M_i^{\bar{U},t} = 0 \\
\theta_i^{\bar{N},t} :& - \beta^t M_i^{\bar{N},t}\kappa_i^{\bar{N}} + \beta^t \lambda_i^t \widehat{f}'\left(\theta_i^{\bar{N},t}\right) M_i^{\bar{N},t} - \beta^t \eta_i^t \widehat{f}'\left(\theta_i^{\bar{N},t}\right) M_i^{\bar{N},t} = 0.
\end{aligned}
$$

In the steady state, $t = t + 1$ for all t, and we can reorganize and write the following:

$$
\begin{aligned}
\frac{\lambda_i}{\beta} =& y_i + \lambda_i\left[1 - \pi_{EU}^i - \pi_{EN}^i\right] + \mu_i \pi_{EU}^i + \eta_i \pi_{EN}^i; \\
\frac{\mu_i}{\beta} =& (\bar{c}_i^{\bar{U}} - \kappa_i^{\bar{U}}\theta_i^{\bar{U}}) + \lambda_i f\left(\theta_i^{\bar{U}}\right) + \mu_i\left[1 - f\left(\theta_i^{\bar{U}}\right) - \pi_{UN}^i\right] + \eta_i \pi_{UN}^i; \\
\frac{\eta_i^t}{\beta} =& (\bar{c}_i^{\bar{N}} - \kappa_i^{\bar{N}}\theta_i^{\bar{N}}) + \lambda_i \widehat{f}\left(\theta_i^{\bar{N}}\right) + \eta_i\left[1 - \widehat{f}\left(\theta_i^{\bar{N}}\right) - \pi_{UN}^i\right] + \mu_i \pi_{NU}^i;
\end{aligned}
$$

$$
-\kappa_i^{\bar{U}} + \lambda_i f'\left(\theta_i^{\bar{U}}\right) - \mu_i f'\left(\theta_i^{\bar{U}}\right) = 0;
$$

$$-\kappa_i^{\bar{N}} + \lambda_i \widehat{f}' \left(\theta_i^{\bar{N}}\right) - \eta_i \widehat{f}' \left(\theta_i^{\bar{N}}\right) = 0.$$

Moreover, define $S_i^{\bar{U}*} = (\lambda_i - \mu_i)/\beta, S_i^{\bar{N}*} = (\lambda_i - \eta_i)/\beta$, and write

$$\begin{aligned} \lambda_i/\beta = &\quad y_i + \lambda_i - (\pi_{EU}^i + \pi_{EN}^i)\beta S_i^{\bar{U}*} - \pi_{EN}^i(\mu_i - \eta_i) \\ = &\quad y_i + \lambda_i - (\pi_{EU}^i + \pi_{EN}^i)\beta S_i^{\bar{N}*} - \pi_{EU}^i(\eta_i - \mu_i); \end{aligned} \tag{29}$$

$$\mu_i/\beta = \quad (\bar{c}_i^{\bar{U}} - \kappa_i^{\bar{U}} \theta_i^{\bar{U}}) + \beta f \left(\theta_i^{\bar{U}}\right) S_i^{\bar{U}*} + \mu_i + \pi_{UN}^i (\eta_i - \mu_i); \tag{30}$$

$$\eta_i/\beta = \quad (\bar{c}_i^{\bar{N}} - \kappa_i^{\bar{N}} \theta_i^{\bar{N}}) + \beta \widehat{f} \left(\theta_i^{\bar{N}}\right) S_i^{\bar{N}*} + \eta_i + \pi_{NU}^i (\mu_i - \eta_i); \tag{31}$$

$$\kappa_i^{\bar{U}} = \beta f' \left(\theta_i^{\bar{U}}\right) S_i^{\bar{U}*}; \tag{32}$$

$$\kappa_i^{\bar{N}} = \beta \widehat{f}' \left(\theta_i^{\bar{N}}\right) S_i^{\bar{N}*}. \tag{33}$$

Then, subtract equation (30) from equation (29), and (31) from (29), and insert (32) and (33):

$$\begin{aligned} S_i^{\bar{U}*} \equiv &\quad \frac{\lambda_i - \mu_i}{\beta} = y_i - \bar{c}_i^{\bar{U}} + \beta S_i^{\bar{U}*} \left[\theta_i^{\bar{U}} f' \left(\theta_i^{\bar{U}}\right) - f \left(\theta_i^{\bar{U}}\right)\right] - \pi_{UN}^i (\eta_i - \mu_i) \\ & -\pi_{EN}^i (\mu_i - \eta_i) + \beta \left(1 - \pi_{EU}^i - \pi_{EN}^i\right) S_i^{\bar{U}*}; \end{aligned} \tag{34}$$

$$\begin{aligned} S_i^{\bar{N}*} \equiv &\quad \frac{\lambda_i - \eta_i}{\beta} = y_i - \bar{c}_i^{\bar{N}} + \beta S_i^{\bar{N}*} \left[\theta_i^{\bar{N}} \widehat{f}' \left(\theta_i^{\bar{N}}\right) - \widehat{f} \left(\theta_i^{\bar{N}}\right)\right] - \pi_{NU}^i (\mu_i - \eta_i) \\ & -\pi_{EU}^i (\mu_i - \eta_i) + \beta \left(1 - \pi_{EU}^i - \pi_{EN}^i\right) S_i^{\bar{N}*}; \end{aligned} \tag{35}$$

$$S_i^{\bar{U}*} - S_i^{\bar{N}*} = \frac{\eta_i - \mu_i}{\beta}. \tag{36}$$

Thus, the planner's solution is summarized by the following seven equations:

$$\begin{aligned} S_i^{\bar{U}*} \equiv &\quad \frac{\lambda_i - \mu_i}{\beta} = y_i - \bar{c}_i^{\bar{U}} + \beta S_i^{\bar{U}*} \left[\theta_i^{\bar{U}} f' \left(\theta_i^{\bar{U}}\right) - f \left(\theta_i^{\bar{U}}\right)\right] \\ & -\pi_{UN}^i (\eta_i - \mu_i) - \pi_{EN}^i (\mu_i - \eta_i) + \beta \left(1 - \pi_{EU}^i - \pi_{EN}^i\right) S_i^{\bar{U}*}; \end{aligned} \tag{37}$$

$$\begin{aligned} S_i^{\bar{N}*} \equiv &\quad \frac{\lambda_i - \eta_i}{\beta} = y_i - \bar{c}_i^{\bar{N}} + \beta S_i^{\bar{N}*} \left[\theta_i^{\bar{N}} \widehat{f}' \left(\theta_i^{\bar{N}}\right) - \widehat{f} \left(\theta_i^{\bar{N}}\right)\right] \\ & -\pi_{NU}^i (\mu_i - \eta_i) - \pi_{EU}^i (\mu_i - \eta_i) + \beta \left(1 - \pi_{EU}^i - \pi_{EN}^i\right) S_i^{\bar{N}*}; \end{aligned} \tag{38}$$

$$S_i^{\bar{U}*} - S_i^{\bar{N}*} = \frac{\eta_i - \mu_i}{\beta} \tag{39}$$

$$\mu_i/\beta = \quad (\bar{c}_i^{\bar{U}} - \kappa_i^{\bar{U}}\theta_i^{\bar{U}}) + \beta f\left(\theta_i^{\bar{U}}\right) S_i^{\bar{U}*} + \mu_i + \pi_{UN}^i\left(\eta_i - \mu_i\right); \tag{40}$$

$$\eta_i/\beta = \quad (\bar{c}_i^{\bar{N}} - \kappa_i^{\bar{N}}\theta_i^{\bar{N}}) + \beta \widehat{f}\left(\theta_i^{\bar{N}}\right) S_i^{\bar{N}*} + \eta_i + \pi_{NU}^i\left(\mu_i - \eta_i\right); \tag{41}$$

$$\kappa_i^{\bar{U}} = \beta f'\left(\theta_i^{\bar{U}}\right) S_i^{\bar{U}*}; \tag{42}$$

$$\kappa_i^{\bar{N}} = \beta \widehat{f'}\left(\theta_i^{\bar{N}}\right) S_i^{\bar{N}*}. \tag{43}$$

Appendix C.2: Decentralized Problem

Next, we characterize the decentralized problem and its solution. We assume markets are segmented, which implies that firms can choose how many vacancies to post for each type of worker across i, and s.

Workers' value functions are written as follows:

A value of an employed worker from the unemployment pool is $E_i^{\bar{U}}$ and from the nonparticipation pool is $E_i^{\bar{N}}$:

$$\begin{aligned} E_i^{\bar{U}} = &\quad w_i^{\bar{U}} + \beta\left[\pi_{EU}^i U_i + \pi_{EN}^i N_i + \left(1 - \pi_{EU}^i - \pi_{EN}^i\right) E_i^{\bar{U}}\right] \\ = &\quad w_i^{\bar{U}} + \beta\left[E_i^{\bar{U}} - (\pi_{EU}^i(e,a) + \pi_{EN}^i) D_i^{\bar{U}} - \pi_{EN}^i(U_i - N_i)\right]; \end{aligned}$$

$$E_i^{\bar{N}} = \quad w_i^{\bar{N}} + \beta\left[E_i^{\bar{N}} - (\pi_{EU}^i(e,a) + \pi_{EN}^i) D_i^{\bar{N}} + \pi_{EU}^i(e,a)(U_i - N_i)\right];$$

where $D_i^{\bar{U}} = E_i^{\bar{U}} - U_i$; $D_i^{\bar{N}} = E_i^{\bar{N}} - N_i$.

The value functions for unemployed and nonparticipants are the following:

$$\begin{aligned} U_i = & c_i^{\bar{U}} + \beta\left\{f\left(\theta_i^{\bar{U}}\right) E_i^{\bar{U}} + \left(1 - f\left(\theta_i^{\bar{U}}\right) - \pi_{UN}^i\right) U_i + \pi_{UN}^i N_i\right\} \\ = & c_i^{\bar{U}} + \beta\left\{U_i + f\left(\theta_i^{\bar{U}}\right) D_i^{\bar{U}} + \pi_{UN}^i(N_i - U_i)\right\}. \end{aligned}$$

$$\begin{aligned} N_i = & c_i^{\bar{N}} + \beta\left\{\widehat{f}\left(\theta_i^{\bar{N}}\right) E_i^{\bar{N}} + \left(1 - \widehat{f}\left(\theta_i^{\bar{N}}\right) - \pi_{UN}^i\right) N_i + \pi_{UN}^i N_i\right\} \\ = & c_i^{\bar{N}} + \beta\left\{N_i + \widehat{f}\left(\theta_i^{\bar{N}}\right) D_i^{\bar{N}} - \pi_{NU}^i(N_i - U_i)\right\}. \end{aligned}$$

To get worker surpluses, subtract the value of unemployment (nonparticipation) from $E_i^{\bar{U}}$ ($E_i^{\bar{N}}$):

$$\begin{aligned} D_i^{\bar{U}} =& w_i^{\bar{U}} - c_i^{\bar{U}} + \beta\Big\{(1-\pi_{EU}^i - \pi_{EN}^i)D_i^{\bar{U}} + U_i - f\left(\theta_i^{\bar{U}}\right) D_i^{\bar{U}} - \pi_{EN}^i(U_i - N_i) \\ & - U_i - \pi_{UN}^i(U_i - N_i)\Big\}; \\ D_i^{\bar{N}} =& w_i^{\bar{N}} - c_i^{\bar{N}} + \beta\Big\{(1-\pi_{EU}^i - \pi_{EN}^i)D_i^{\bar{N}} + N_i - \widehat{f}\left(\theta_i^{\bar{N}}\right) D_i^{\bar{N}} + \pi_{EU}^i(U_i - N_i) \\ & - N_i + \pi_{NU}^i(N_i - U_i)\Big\}. \end{aligned}$$

Regarding the firms' problem, firms' value functions can be written as follows. First, the value of filling the vacancy from the unemployment pool is

$$J_i^{\bar{U}} = y_i - w_i^{\bar{U}} + \beta\left(1 - \pi_{EU}^i - \pi_{EN}^i\right) J_i^{\bar{U}},$$

and from the nonparticipant pool is

$$J_i^{\bar{N}} = y_i - w_i^{\bar{N}} + \beta\left(1 - \pi_{EU}^i - \pi_{EN}^i\right) J_i^{\bar{N}}.$$

The values of unfilled vacancies are written as

$$V_i^{\bar{U}} = max\{-\kappa_i^{\bar{U}} + \beta[q(\theta_i^{\bar{U}}).J_i^{\bar{U}} + (1 - [q(\theta_i^{\bar{U}}))\bar{V}], 0\} \quad \text{and}$$

$$V_i^{\bar{N}} = max\{-\kappa_i^{\bar{N}} + \beta[\widehat{q}[\theta_i^{\bar{N}}]J_i^{\bar{N}} + (1 - [\widehat{q}[\theta_i^{\bar{N}}])\bar{V}], 0\}.$$

With free entry, the values of unfilled vacancies are zero, so the previous two equations simplify to

$$\kappa_i^{\bar{U}} = \beta q(\theta_i^{\bar{U}})J_i^{\bar{U}} \quad \text{and}$$

$$\kappa_i^{\bar{N}} = \beta\widehat{q}(\theta_i^{\bar{N}})J_i^{\bar{N}}.$$

Wages are determined through Nash bargaining. The match surpluses, $E_i^{\bar{U}} - U_i + J_i^{\bar{U}}$ and $E_i^{\bar{N}} - N_i + J_i^{\bar{N}}$, are shared according to the Nash product:

$$\max_{E_i - U_i, J_i}(E_i^{\bar{U}} - U_i)^{\phi_i^{\bar{U}}(e,a)} J_i^{\bar{U}\,1-\phi_i^{\bar{U}}(e,a)} \text{ subject to } S_i^{\bar{U}} = D_i^{\bar{U}} + J_i^{\bar{U}} \quad \text{and}$$

$$\max_{E_i - N_i, J_i} (E_i^{\bar{N}} - N_i)^{\phi_i^{\bar{N}}} J_i^{\bar{N}\,1-\phi_i^{\bar{N}}} \text{ subject to } S_i^{\bar{N}} = D_i^{\bar{N}} + J_i^{\bar{N}}.$$

The solution for unemployed satisfies

$$\frac{\phi_i^{\bar{U}}}{1-\phi_i^{\bar{U}}} = \frac{E_i^{\bar{U}} - U_i}{J_i^{\bar{U}}} \quad \text{or}$$

$$E_i^{\bar{U}} - U_i = \phi_i^{\bar{U}} S_i^{\bar{U}} \text{ and } J_i^{\bar{U}} = \left(1 - \phi_i^{\bar{U}}\right) S_i^{\bar{U}},$$

and for nonparticipants

$$\frac{\phi_i^{\bar{N}}}{1-\phi_i^{\bar{N}}} = \frac{E_i^{\bar{N}} - N_i}{J_i^{\bar{N}}} \quad \text{or}$$

$$E_i^{\bar{N}} - N_i = \phi_i^{\bar{N}} S_i^{\bar{N}} \text{ and } J_i^{\bar{N}} = \left(1 - \phi_i^{\bar{N}}\right) S_i^{\bar{N}}.$$

Thus, the decentralized solution is defined by the following six equations:

$$\begin{aligned} S_i^{\bar{U}} \equiv & J_i^{\bar{U}} + D_i^{U} = y_i - c_i^{\bar{U}} + \beta\left(1 - \pi_{EU}^i - \pi_{EN}^i\right) S_i^{\bar{U}} \\ & - \beta f\left(\theta_i^{\bar{U}}\right) D_i^{\bar{U}} - \beta\left\{\pi_{EN}^i\left(U_i - N_i\right) + \pi_{UN}^i\left(N_i - U_i\right)\right\} \end{aligned} \tag{44}$$

$$\begin{aligned} S_i^{\bar{N}} \equiv & J_i^{\bar{N}} + D_i^{N} = y_i - c_i^{\bar{N}} + \beta\left(1 - \pi_{EU}^i - \pi_{EN}^i\right) S_i^{\bar{N}} \\ & - \beta \widehat{f}\left(\theta_i^{\bar{N}}\right) D_i^{\bar{N}} + \beta\left\{\pi_{EU}^i\left(U_i - N_i\right) + \pi_{NU}^i\left(N_i - U_i\right)\right\} \end{aligned} \tag{45}$$

$$U_i = c_i^{\bar{U}} + \beta\left\{U_i + f\left(\theta_i^{\bar{U}}\right) D_i^{\bar{U}} + \pi_{UN}^i (N_i - U_i)\right\} \tag{46}$$

$$N_i = c_i^{\bar{N}} + \beta\left\{N_i + \widehat{f}\left(\theta_i^{\bar{N}}\right) D_i^{\bar{N}} - \pi_{NU}^i (N_i - U_i)\right\} \tag{47}$$

$$\kappa_i^{\bar{U}} = \beta q(\theta_i^{\bar{U}}) J_i^{\bar{U}}, \quad \text{and} \tag{48}$$

$$\kappa_i^{\bar{N}} = \beta \widehat{q}(\theta_i^{\bar{N}}) J_i^{\bar{N}}. \tag{49}$$

Appendix C.3: Proof of Proposition 1

Let's now compare the social planner's solution and the decentralized solution. When comparing equations (37)–(43) and (44)–(49), we see that the two systems are equivalent when $\phi_i^{\bar{U}} = -\frac{q'(\theta_i^{\bar{U}})\theta_i^{\bar{U}}}{q(\theta_i^{\bar{U}})}$ and $\phi_i^{\bar{N}} = -\frac{q'(\theta_i^{\bar{N}})\theta_i^{\bar{N}}}{q(\theta_i^{\bar{N}})}$, when also noting that $\frac{\mu_i}{\beta} = U_i$ and $\frac{\eta_i}{\beta} = N_i$. □

Appendix D: Competitive Equilibrium with Bargaining Posting in DMP Model with Nonparticipation

The wage setting in the model follows the one commonly used in the competitive search theory: While competitive search theory assumes that firms directly post wage rates, we assume that firms post bargaining weights and workers direct their search towards their utility-maximizing bargaining weight. As noted in Wright et al. (2021) on page 131, these approaches are fundamentally the same. Competitive search equilibrium implies that the match surplus shares are not constant but respond to market conditions. The only exception is the special case of the Cobb-Douglas matching function, which guarantees constant surplus shares.

In our case, explicitly focusing on bargaining power posting allows us to use the model to discuss how bargaining power might have changed in response to labor market conditions, given that bargaining power is not directly observed in the data. Once a firm and a worker are matched in a submarket determined by the bargaining weight $\phi(x)$, the firm and the worker share the match surplus using the optimal bargaining weight, as in Nash bargaining. Firms and workers update wages every period. We assume that bargaining weights respond to current market conditions: Whenever market tightness changes, bargaining weights react accordingly, even if a worker and a firm are already matched.

Following Moen (1997) competitive search equilibrium, we show that the profit- and utility-maximizing behaviors of firms and job seekers determine the optimal bargaining power. While Moen (1997) shows how firms and workers optimally choose a wage from a menu of wages, we assume instead that both parties choose their optimal bargaining weights. This assumption leads to competitive allocation, which we confirm coincides with the socially optimal allocation.

Assume again a similar DMP model with nonparticipation. Assume that for each labor market x there is a set of Φ equilibrium sub-labor markets, where Φ stands for all the possible workers' bargaining powers.[21] Then the values of being either a job seeker or

[21] Again, x captures the type i and status s of a worker.

employed in a submarket with ϕ and state x are given as

$$E^s(x;\phi) = w(x;\phi) + \beta \left[E^s(x;\phi) - (\pi_{EU}(x) + \pi_{EN}(x))D^s(x;\phi) - \pi_{EN}(x)(U(x;\phi) - N(x;\phi))\right];$$

$$U(x;\phi) = c(x) + \beta \left[U(x;\phi) + f(\theta(x;\phi))\, D^{\bar{U}}(x;\phi) + \pi_{UN}(x)(N(x;\phi) - U(x;\phi))\right];$$

$$N(x;\phi) = c(x) + \beta \left[N(x;\phi) + \widehat{f}(\theta(x;\phi))\, D^{\bar{N}}(x;\phi) - \pi_{NU}(x)(N(x;\phi) - U(x;\phi))\right],$$

where $D^{\bar{U}}(x;\phi) = E^{\bar{U}}(x;\phi) - U(x;\phi)$ and $D^{\bar{N}}(x;\phi) = E^{\bar{N}}(x;\phi) - N(x;\phi)$.

We can also define the value of posting a vacancy and the value of having a vacancy filled as:

$$V^{\bar{U}}(x,\phi) = \max\left\{-\kappa(x) + \beta\left[q(\theta(x;\phi))\, J^{\bar{U}}(x;\phi) + (1 - q(\theta(x;\phi)))\, V^{\bar{U}}(x;\phi)\right], 0\right\};$$

$$V^{\bar{N}}(x,\phi) = \max\left\{-\kappa(x) + \beta\left[\widehat{q}(\theta(x;\phi))\, J^{\bar{N}}(x;\phi) + (1 - \widehat{q}(\theta(x;\phi)))\, V^{\bar{N}}(x;\phi)\right], 0\right\};$$

$$J^s(x,\phi) = h(x) - w(x;\phi) + \beta\left\{(\phi_{EU}(x) + \phi_{EN}(x))V^s(x;\phi) + (1 - \phi_{EU}(x) - \phi_{EN}(x))\, J^s(x;\phi)\right\}.$$

Because of the free-entry condition and the workers' search behaviors, we have

$$V^s(x;\phi) = 0\ \forall \phi \in \phi \text{ and for any } x\ ,$$

$$U(x,\phi) = \overline{U}(x)\ \forall \phi \in \phi \text{ and for any } x,$$

$$N(x,\phi) = \overline{N}(x)\ \forall \phi \in \phi \text{ and for any } x.$$

Now we differentiate the value functions $U(x;\phi)$, $N(x;\phi)$, $V^{\bar{U}}(x;\phi)$, and $V^{\bar{N}}(x;\phi)$, and we get

$$\frac{dU(x;\phi)}{d\phi} = f'(\theta(x;\phi))\frac{d\theta(x;\phi)}{d\phi}(D^{\bar{U}}(x;\phi)) + f(\theta(\phi))\frac{d(D^{\bar{U}}(x;\phi))}{d\phi}) = 0; \tag{50}$$

$$\frac{dN(x;\phi)}{d\phi} = \widehat{f}'(\theta(x;\phi))\frac{d\theta(x;\phi)}{d\phi}(D^{\bar{N}}(x;\phi)) + \widehat{f}(\theta(\phi))\frac{d(D^{\bar{N}}(x;\phi))}{d\phi}) = 0; \tag{51}$$

$$\frac{dV^{\bar{U}}(x;\phi)}{d\phi} = q'(\theta(x;\phi))\frac{d\theta(x;\phi)}{d\phi}(J^{\bar{U}}(x;\phi)) + q(\theta(\phi))\frac{dJ^{\bar{U}}(x;\phi)}{d\phi}) = 0; \tag{52}$$

$$\frac{dV^{\bar{N}}(x;\phi)}{d\phi} = \widehat{q}'(\theta(x;\phi))\frac{d\theta(x;\phi)}{d\phi}(J^{\bar{N}}(x;\phi)) + \widehat{q}(\theta(\phi))\frac{dJ^{\bar{N}}(x;\phi)}{d\phi}) = 0. \tag{53}$$

Let's solve the optimal bargaining power for the unemployed. The same solution applies to the optimal bargaining power for nonparticipants. Rearrange equations (50) and equation (52) and get

$$f'(\theta(x;\phi))\frac{d\theta(x;\phi)}{d\phi}D^{\bar{U}}(x;\phi) = -f(\theta(\phi))\frac{dD^{\bar{U}}(x;\phi)}{d\phi})$$

$$q'(\theta(x;\phi))\frac{d\theta(x;\phi)}{d\phi}(J(x;\phi)) = -q(\theta(\phi))\frac{dJ(x;\phi)}{d\phi})$$

$$\Rightarrow \frac{f'(\theta(x;\phi))}{q'(\theta(x;\phi))}\frac{D^{\bar{U}}(x;\phi)}{J(x;\phi)} = \frac{f(\theta(x;\phi))}{q(\theta(x;\phi))}\frac{dD^{\bar{U}}(x;\phi)/d\phi}{dJ(x;\phi)/d\phi}. \tag{54}$$

Combining the equation (54) with the fact that a surplus is $S(\phi) = J(x;\phi) + E(x;\phi) - U(x;\phi)$[22] and the sharing rule of surplus is $E(x;\phi) - U(x;\phi) = \phi S(x;\phi)$, $J(x;\phi) = (1-\phi)S(x;\phi)$, we get

$$\frac{f'(\theta(x;\phi))}{q'(\theta(x;\phi))}\frac{\phi S(x;\phi)}{(1-\phi)S(x;\phi)} = \frac{f(\theta(x;\phi))}{q(\theta(x;\phi))} \times \frac{d\phi S(x;\phi)}{d\phi} \times \left[\frac{d(1-\phi)S(x;\phi)}{d\phi}\right]^{-1}$$

$$\Rightarrow \frac{\theta q'(\theta(\phi)) + q(\theta(\phi))}{q'(\theta(\phi))}\frac{\phi}{1-\phi} = -\theta(\phi).$$

When the equation is simplified, the solution for the system becomes

$$\phi = -\frac{q'(\theta(x;\phi))\theta(x;\phi)}{q(\theta(x;\phi))}. \tag{55}$$

Thus, equation (55) is exactly the socially efficient condition for bargaining power, and the efficient condition for bargaining power holds for any given set Φ of submarkets that exists in the equilibrium. In other words, the bargaining power ϕ serves as a price device to adjust the relative demand and supply of labor. In equilibrium, firms are picking the efficient submarkets to pursue the highest profit, and so do the utility-maximizing job seekers. The "price" of bargaining power must follow the efficient rule $\phi = -\frac{q'(\theta(\phi))\theta(\phi)}{q(\theta(\phi))}$.

The decentralized equilibrium is efficient. Equivalently, the efficient bargaining condition arises endogenously.

[22] Note here the surplus created is irrelevant of the bargaining power. The reason is that once the worker entered the bargaining process, the surplus is fixed, the bargaining is just dividing this surplus between two parties. Because of this, the surplus has already been maximized before determining the share of each party. Thus, it has no effect on the surplus.

Appendix E: Data and Detailed Calibration Results

Appendix E.1: Description of Data

We use the basic monthly IPUMS-CPS data from 1976 to 1980 and 2003 to 2007 (Center for Economic and Policy Research, Center for Economic and Policy Research (CEPR) and Flood et al., 2020). Our sample includes workers between ages 25 and 64, and it includes both full- and part-time U.S. workers. We disaggregate the data based on an individual's gender (male or female) and education status (college or non-college). An individual is assigned to the college group if she has completed at least some college and to the non-college group, if her highest level of completed education is high school or less. We then calculate average wages, and employment, unemployment, and non-participation rates for each of the described demographic groups.

We rely on the hourly wage rates obtained from the CEPR, while the other data are obtained from the raw CPS data files from IPUMS (Flood et al., 2020). The advantage of using the CEPR wage data instead of the raw CPS data is that the CEPR adjusts the raw CPS wage data such that the constructed wage data series are consistent and comparable over time and are especially suitable for research uses.[23]

We also estimate monthly transition probabilities between employment $(\bar{E})$, unemployment $(\bar{U})$, and nonparticipation $(\bar{N})$ separately for each group, following the method in Choi et al. (2015). In practice, the transition probability estimates are weighted-average flows between labor market states when controlling for birth cohorts. For a given cohort and survey year, we observe the fraction of individuals that transfers from one labor market state to another. Denote this variable as $\pi_{ss'}(c,t)$, where ss' denotes the transition from a status $s \in \{\bar{E}, \bar{U}, \bar{N}\}$ to a status $s' \in \{\bar{E}', \bar{U}', \bar{N}'\}$, c denotes the cohort (the birth year) an individual belongs to, and t denotes the survey year.

We obtain the final transition probabilities by calculating weighted average transitions over cohorts and survey year by using CPS weights. We denote these estimated transition

[23] For a detailed description, please refer to the CEPR-CPS documentation found at https://ceprdata.org/cps-uniform-data-extracts/cps-basic-programs/.

probabilities as $\pi_{ss'}(i) \equiv \pi_{ss'}(x)$.

To remove high-frequency reversals of transitions between unemployment and nonparticipation, we follow the method suggested by Elsby et al. (2015) called "deNUNification." The key idea is to correct for a possible classification error of an individual's labor market state: An individual who moves from nonparticipation to unemployment and back to nonparticipation within a short period of time is likely to be a nonparticipant—including these high-frequency transitions between states may lead to spurious transition estimates. The correction method thus recodes the high-frequency transitions, NUN, as NNN. The same method is applied to high-frequency transitions from unemployment to non-participation and back.

The estimated flows between different labor market states are flow probabilities from employment to unemployment and to nonparticipation—$\pi_{EU}(i)$ and $\pi_{EN}(i)$, respectively; unemployment to nonparticipation, $\pi_{UN}(i)$; nonparticipation to unemployment, $\pi_{NU}(i)$; and unemployment and nonparticipation to employment—$\pi_{UE}(i) \equiv f_i\left(\bar{U}\right)$ and $\pi_{NE}(i) \equiv f_i\left(\bar{N}\right)$, respectively. As the period in our model calibration will be set to a quarter instead of a month, we calculate quarterly transition probability matrices, $\Lambda_Q(i)$, as $\Lambda_Q(i) = (\Lambda_M(i)^{\wedge 3}$, where $\Lambda_M(i)$ equals

$$\begin{pmatrix} 1-\pi_{EU}(i)-\pi_{EN}(i) & \pi_{EU}(i) & \pi_{EN}(i) \\ \pi_{UE}(i) & 1-\pi_{UE}(i)-\pi_{UN}(i) & \pi_{UN}(i) \\ \pi_{NE}(i) & \pi_{NU}(i) & 1-\pi_{NE}(i)-\pi_{NU}(i) \end{pmatrix}.$$

Appendix E.3: Calibration Algorithm

We solve the model using backwards induction. Given the set values of β, $\bar{\gamma}$, α, ρ, and $\pi_{ss'}(i)$, the calibration algorithm to recover y_i, A_i, ψ_i, γ_i^N, $\bar{\kappa}_i$ and bargaining weights ϕ_i^s is the following:

Step 1: Make a reasonable guess of the bargaining weights of workers ϕ_i^s and vacancy-posting costs $\bar{\kappa}_i$.

Step 2: At given bargaining weights, vacancy-posting costs, and other parameter values, solve the model and use model solutions to reverse engineer the group-specific human capital parameters (y_i) and matching efficiencies (A_i, ψ_i) to fit the observed wage rate and job-finding rates. We obtain γ_i^N by equalizing the wage rates for the unemployed and nonparticipants.

Step 3: We use group-specific tightness rates with Proposition 2 (the Hosios condition) to update the guess of bargaining power and group-specific tightness to update guesses for $\bar{\kappa}_i$.

Step 4: We repeat steps 2 and step 3 until the bargaining power series converge and the tightness rates hit their targets.

Appendix E.4: Changes in the Disaggregated Labor Shares and Counterfactual Analysis

Table E.1 summarizes the model-generated changes in the labor share for different groups. First, we find that the labor share has declined for men and non-college females in the CES model. Male workers have faced a larger decline: Non-college males experienced a 4.1 percent decline in their labor share, while college-educated males experienced a 2.6 percent decline. The labor share decreased by 1.3 percent for non-college females and increased by 1.9 percent for college females. Consistent with the aggregate results, the labor shares have declined less in the CD model—or have increased more, as is the case for college-educated women.

Table E.1. Simulated labor compensation shares: 1976–80 and 2003–07

Group	% change, CD model	% change, CES + fixed barg.	% change, CES model
Male, college	-1.2	-1.2	-2.6
Female, college	2.1	2.1	1.9
Male, non-college	-1.2	-1.2	-4.1
Female, non-college	1.2	1.2	-1.3
Weighted average	0.3	0.3	-1.2

Note: Table E.1 presents the simulated change in the labor share for each group under the CD and the CES models, and the CES model with fixed bargaining weights.

Source: Authors' estimations.

Table E.2. Counterfactuals—labor share

	Male, C	Female, C	Male, NC	Female, NC
Total change—model	-2.6	1.9	-4.1	-1.3
y	0.0	0.1	-0.1	-0.1
A	-18.0	69.4	-27.1	-125.7
ψ	-2.0	17.7	-7.5	-38.4
π_{EU}	3.2	5.3	1.4	2.6
π_{EN}	18.3	187.0	18.6	-83.3
π_{UN}	-0.3	-1.1	-0.7	0.6
π_{NU}	-0.4	-0.9	-0.2	1.5
κ	99.3	-199.5	108.8	322.1
γ^N	-0.2	21.8	6.8	20.8
Total contribution	99.9	99.8	100.0	100.1

Source: Authors' estimations.

Appendix E.4.1: Sensitivity analysis

As in shown in Table E.3, both the labor share and bargaining power are higher when ρ decreases and complementarity between vacancies and job seekers in the matching process increases. In contrast, the labor share and bargaining power for any given ρ are less sensitive to changes in θ when α increases (Table E.4).

Table E.3. Sensitivity of the results to different values of ρ

ρ	%-change, labor share	%-change, bargaining power
-0.3	-0.5	-9.1
-0.5	-1.2	-16.9
-0.7	-1.9	-25.1

Note: We recalibrate the model using different values of ρ and present the model-generated changes in the average labor share and the average bargaining weight. All other externally set parameters (β, α, γ) are set to the same values as in the baseline calibration.

Source: Authors' estimations.

Table E.4. Sensitivity of the results to different values of α

α	%-change, labor share	%-change, bargaining power
0.5	-1.5	-19.0
0.6	-1.2	-16.9
0.7	-0.9	-14.4

Note: We recalibrate the model using different values of α and present the model-generated changes in the average labor share and the average bargaining weight. All other externally set parameters (β, ρ, γ) are set to the same values as in the baseline calibration.

Source: Authors' estimations.

Appendix E.5: CES matching function estimation - additional results

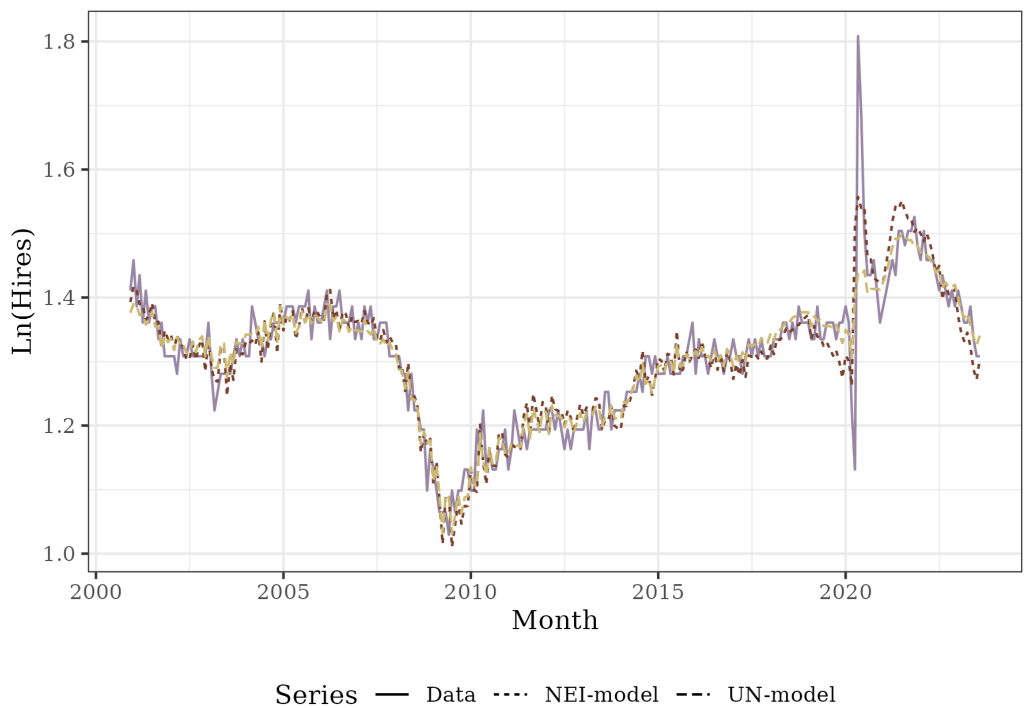

Figure E.1. CES matching function estimation - model fit

Note: The solid line plots natural logarithm of seasonally adjusted hires rate from December 2000 to August 2023 as observed in the JOLTS data. The two dashed lines plot the predicted hires rates from the CES matching function estimations using *UN* (dot-dash lines) and *NEI* (dotted line) as job-seeker measures.

Source: JOLTS; Authors' estimations.

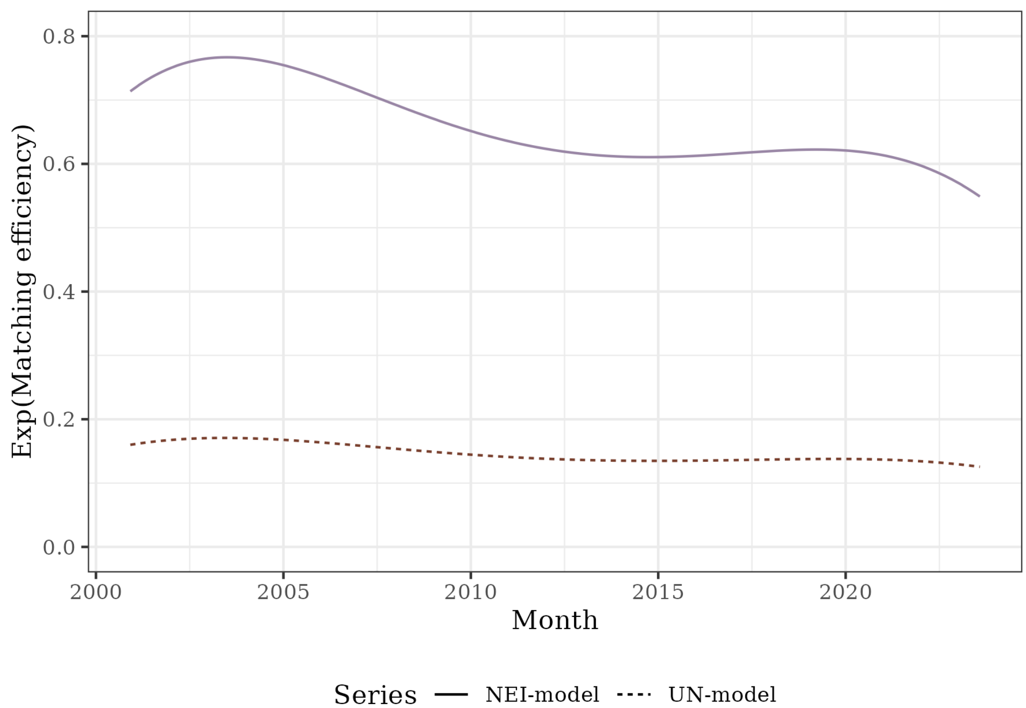

Figure E.2. CES matching function estimation - Estimated quartic time trend

Note: The solid line plots the exponential of the estimated quartic time trend measuring matching efficiency in the *NEI* model. The dashed line plots the exponential of the estimated quartic time trend measuring matching efficiency in the *UN* model. Both estimations use JOLTS data from December 2000 to August 2023. Source: Authors' estimations.

Table E.5. Estimation results—Quartic time trend parameters

	Dependent variable: log(hires)					
Sample	Full		Excl. 2020		Excl. 2009-10, 2020	
Job seeker	*UN* (1)	*NEI* (2)	*UN* (3)	*NEI* (4)	*UN* (5)	*NEI* (6)
A	−1.838***	−0.309***	−1.572***	−0.343***	−1.748***	−0.382***
	(0.096)	(0.029)	(0.107)	(0.029)	(0.087)	(0.028)
t	0.005***	0.005***	0.004***	0.005***	0.004***	0.005***
	(0.001)	(0.001)	(0.0004)	(0.001)	(0.0004)	(0.001)
t^2	−0.0001***	−0.0001***	−0.0001***	−0.0001***	−0.0001***	−0.0001***
	(0.00001)	(0.00001)	(0.00001)	(0.00001)	(0.00001)	(0.00001)
t^3	0.00000***	0.00000***	0.00000***	0.00000***	0.00000***	0.00000***
	(0.00000)	(0.00000)	(0.00000)	(0.00000)	(0.00000)	(0.00000)
t^4	−0.000***	−0.000***	−0.000***	−0.000***	−0.000***	−0.000***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Obs.	273	273	261	261	237	237
Residual Std. Error	0.04302 (df = 266)	0.04472 (df = 266)	0.02715 (df = 254)	0.03317 (df = 254)	0.02555 (df = 230)	0.03094 (df = 230)

Note: *p<0.1; **p<0.05; ***p<0.01

Table E.6. Estimation results—Unemployed as job seekers

	Dependent variable: log(hires)		
Sample	Full	Excl. 2020	Excl. 2009-10, 2020
	(1)	(2)	(3)
α	0.247***	0.232***	0.267***
	(0.019)	(0.021)	(0.023)
ρ	0.124	0.218*	−0.226
	(0.126)	(0.128)	(0.172)
Obs.	273	261	237
Residual Std. Error	0.0556 (df = 266)	0.04755 (df = 254)	0.04685 (df = 230)

Note: All specifications include a vector of four elements for the quartic time trend, capturing shifts in aggregate matching efficiency A_t. *p<0.1; **p<0.05; ***p<0.01

Appendix F: Wage Posting and Share Posting in Competitive Search

Appendix F.1: Wage Posting in Competitive Search

To be consistent, here we use discrete time. Staring with wage posting, suppose that a set $W^a = (w_1, ..., w_n)$ of wages is announced in equilibrium, with a measure of $v_1, ..., v_n$ of vacancies. Unemployed workers are assumed to search in one of the subsets of jobs indexed by wage rate each period. They, together with the hiring firms, form the corresponding submarket. Let $u_1, ..., u_n$ be the masses of unemployed workers searching in each submarket. We know the tightness rate can be defined as $\theta_j = \frac{v_j}{u_j}$. Let J_j and V_j be the firm's payoff to having a worker and open vacancy in submarket j, in the steady state, we have

$$J_j = y - w_j + \beta[(1-s)J_j + sV_j],$$

$$V_j = -\kappa + \beta[q(\theta_j)J_j + (1 - q(\theta_j))V_j].$$

where κ is the cost of a vacancy, β the discount factor, and s the job destruction rate.

Similarly, for workers, we can define E and U as the value of being employed and unemployed in submarket j as

$$E_j = w_j + \beta[(1-s)E_j + s \max_k(U_k)].$$

$$U_j = c + \beta[f(\theta_j)E_j + (1 - f(\theta_j)) \max_k(U_k)].$$

In equilibrium, we know unemployed workers should be indifferent about which submarket to enter, which means $U_j = \bar{U} \forall j$, thus we have

$$\bar{U} = c + \beta[f(\theta_j)E_j + (1 - f(\theta_j))\bar{U}].$$

In the meantime, firms knowing the reaction of workers, will act accordingly.

$$\max_{w_j} V_j = -\kappa + \beta[q(\theta_j)J_j + (1 - q(\theta_j))V_j] s.t. \bar{U} = c + \beta[f(\theta_j)E_j + (1 - f(\theta_j))\bar{U}]$$

Assumed interior solution, it is nontrival to get

$$\frac{E_j - \bar{U}}{J_j - V_j} = -\frac{f(\theta_j)/f'(\theta_j)}{q(\theta_j)/q'(\theta_j)}$$

*Here are the steps needed to get the above results. From the constraint, we can get tightness as an implicit function of wage rate,

$$\frac{\partial \bar{U}}{\partial w_j} = \beta f'(\theta)(E_j - \bar{U})\frac{\partial \theta_j}{\partial w_j} + \beta f(\theta)\frac{\partial (E_j - \bar{U})}{\partial w_j} + \beta \frac{\partial \bar{U}}{\partial w_j} = 0$$

*Rearrange terms

$$\beta f'(\theta_j)(E_j - \bar{U})\frac{\partial \theta_j}{\partial w_j} + \beta f(\theta_j)\frac{\partial (E_j - \bar{U})}{\partial w_j} = 0$$

*For the first order conditions, we get

$$\frac{\partial V_j}{\partial w_j} = \beta q'(\theta_j)(J_j - V_j)\frac{\partial \theta_j}{\partial w_j} + \beta q(\theta_j)\frac{\partial (J_j - V_j)}{\partial w_j} + \beta \frac{\partial V_j}{\partial w_j} = 0$$

*Rearrange terms

$$\beta q'(\theta_j)(J_j - V_j)\frac{\partial \theta_j}{\partial w_j} + \beta q(\theta)\frac{\partial (J_j - V_j)}{\partial w_j} = 0$$

*Utilize the definition of J_j and E_j, we can get $\frac{\partial (J_j - V_j)}{\partial w_j}$ and $\frac{\partial (E_j - \bar{U})}{\partial w_j}$, these two yields

$$\frac{\partial (J_j - V_j)}{\partial w_j} = -\frac{\partial (E_j - \bar{U})}{\partial w_j}$$

*Combine these three equations,

$$\frac{E_j - \bar{U}}{J_j - V_j} = -\frac{f(\theta_j)/f'(\theta_j)}{q(\theta_j)/q'(\theta_j)}$$

Impose the free entry condition, we can get

$$\frac{E_j - \bar{U}}{J_j} = -\frac{f(\theta_j)/f'(\theta_j)}{q(\theta_j)/q'(\theta_j)} = \frac{-q'(\theta_j)\theta_j}{q(\theta_j) + q'(\theta_j)\theta_j},$$
$$\kappa = \beta q(\theta_j) J_j.$$

This two conditions uniquely pin down the solution of θ and w.

Under the Hosios condition, $\phi = -\frac{q'(\theta)\theta}{q(\theta)}$, we get the share of surplus to be exactly the Nash bargaining solution.

$$\frac{E - \bar{U}}{J} = \frac{\phi}{1 - \phi},$$

Appendix F.2: Share Posting in Competitive Search

Now we can turn our eye to share posting in competitive search, here, we assume a set $\Gamma^a = (\gamma_1, ..., \gamma_n)$ of sharing rule is announced in equilibrium for the surplus of a successful match, with a measure of $v_1, ..., v_n$ of vacancies. Unemployed workers are assumed to search in one of the subsets of jobs indexed by the sharing number each period. They, together with the hiring firms, form the corresponding submarket. Let $u_1, ..., u_n$ be the masses of unemployed workers searching in each submarket. Using the same notation system,

$$J_j = (1 - \gamma_j)S_j + V_j,$$

$$V_j = -\kappa + \beta[q(\theta_j)J_j + (1 - q(\theta_j))V_j].$$

Similarly, for workers, we can define E and U as the value of being employed and unemployed in submarket j as

$$E_j = \gamma_j S_j + U_j.$$

$$U_j = c + \beta[f(\theta_j)E_j + (1 - f(\theta_j))\max_k(U_k)].$$

Again, workers are indifferent about which submarket to enter, thus,

$$\bar{U} = c + \beta[f(\theta_j)E_j + (1 - f(\theta_j))\bar{U}].$$

we can first define the match surplus as

$$S_j = E_j - U_j + J_j - V_j = y - c - \kappa + \beta[(1 - s)S_j - f(\theta_j)(E_j - \bar{U}) - q(\theta_j)(J_j - V_j)],$$

$$S_j = E_j - U_j + J_j - V_j = y - c - \kappa + \beta[(1 - s)S_j - f(\theta_j)\gamma_j S_j - q(\theta_j)(1 - \gamma_j)S_j],$$

Firms, act accordingly, will determine the best sharing rule, this turns into

$$\max_{\gamma_j} V_j = -\kappa + \beta[q(\theta_j)J_j + (1 - q(\theta_j))V_j] s.t. \bar{U} = c + \beta[f(\theta_j)E_j + (1 - f(\theta_j))\bar{U}]$$

Assumed interior solution, the solution delivers

$$\frac{E_j - \bar{U}}{J_j - V_j} = -\frac{f(\theta_j)/f'(\theta_j)}{q(\theta_j)/q'(\theta_j)} = \frac{-q'(\theta_j)\theta_j}{q(\theta_j) + q'(\theta_j)\theta_j}$$

With the free entry condition, again, there is only one submarket that exists in the equilibrium.

$$\frac{E - \bar{U}}{J} = \frac{\gamma}{1-\gamma} = -\frac{f(\theta)/f'(\theta)}{q(\theta)/q'(\theta)} == \frac{-q'(\theta)\theta}{q(\theta) + q'(\theta)\theta},$$
$$\kappa = \beta q(\theta) J.$$

The key is that firms can marginally adjust factors that influence the value of posting a vacancy, whether through the wage rate or the sharing rule. The competitive market drives the sharing rule to be efficient, which is the Hosios condition.