Weitzman Meets Taylor: EU Allowances Futures Price Drivers and Carbon Cap Rules *

Ghassane Benmir† Josselin Roman‡ Luca Taschini§

January 28, 2024

Abstract

We examine the determinants of European Union Allowances (EUA) futures prices and the market design of the European Trading System (ETS) using a macro-finance framework. We introduce an innovative approach to estimate the impact of abatement and climate sentiment shocks, leveraging the information contained in the price of EUA futures. Our analysis reveals that during the third phase of the ETS, the price has been primarily influenced by energy fluctuations, climate sentiment, and abatement shocks. When compared to the social cost of carbon (SCC), representing the optimal policy scenario, we find that the ETS price is 100 times more volatile. Furthermore, we observe that volatility in ETS prices generates monthly losses of 0.22% in consumption-equivalent terms compared to the SCC case. We conclude by illustrating how implementing a carbon cap rule can significantly reduce this price volatility and welfare losses, bringing it closer to the first-best policy.

Keywords: EU ETS, Contingent Allowance Allocation, Social Cost of Carbon, Non-Linear Bayesian Estimation, Carbon Policy Rules.

JEL: Q58, G12, E32.

---

*This draft has benefited from comments and suggestions by M. Barnett, S. Dietz, L. P. Hansen, A. Michaelides, R. van der Ploeg, and G. Vermandel and the participants of University of Chicago Booth seminar.

†IE University and Business School, P. de la Castellana, 259, Madrid, and London School of Economics and Political Science, Houghton Street, London WC2A 2AE, UK, E-mail: ghassane.benmir@ie.edu.

‡European Commission - Joint Research Centre, JRC Ispra, 1, 21027 Ispra VA, IT, and PSL Research University - Paris-Dauphine, Pl. du Maréchal de Lattre de Tassigny, 75016 Paris, FR, E-mail: josselin.roman@ec.europa.eu.

§University of Edinburgh Business School, 29 Buccleuch Place, Edinburgh EH8 9JS, and Grantham Research Institute – LSE , Houghton Street, London WC2A 2AE, UK. E-mail: l.taschini1@lse.ac.uk

1 Introduction

In this paper, we estimate the drivers of the carbon price of the European Union Emission Trading System (EU ETS) and shed light on the impact of various shocks on the carbon price over the studied period. To achieve this, we develop a macro-finance model that comprises two sectors: i) an energy sector and ii) a non-energy final sector. The model also incorporates climate and emission dynamics. We begin by estimating our model using Bayesian techniques and subsequently extracting the shock decomposition of the carbon price. We then compare the theoretical social cost of carbon (SCC)–the first-best policy–to the estimated EU ETS carbon price. Finally, we introduce a novel mechanism for determining the supply of emission permits, termed the 'carbon cap rule'. This approach narrows the gap with the first best optimal carbon price and presents a feasible implementation strategy for policy makers.

Our primary finding highlights that the EU ETS carbon cap policy leads to increased price volatility, primarily driven by abatement cost shocks and climate sentiment shocks, in contrast to the first-best carbon price policy. Given that the SCC inherently exhibits minimal price volatility, closing this gap essentially involves minimizing price uncertainty, which we find to be 100 times more volatile than the SCC. Additionally, our analysis reveals that price volatility and economic uncertainty incur costs at the business cycle, resulting in 0.22% welfare losses in consumption equivalent terms (CE) compared to the first-best policy. To address this, we introduce an innovative method to infer abatement shock series, utilizing information from the market price of carbon. Furthermore, we propose a new carbon cap rule designed to achieve reduced price volatility and welfare losses, bringing them down from 0.22% to 0.03% with respect to the SCC scenario.

To internalize the effects of the carbon externality, public economists have consistently advocated for setting a carbon price equivalent to the social cost of carbon – the shadow value of CO_2 emissions. Such a price would guide the economy towards a welfare-enhancing trajectory while incentivizing emissions reduction. Yet, determining the appropriate level of the SCC has sparked significant debates, as seen in works by Stern (2008) and Nordhaus (2008). The correct level of the SCC remains ambiguous, given its dependence on factors such as climate damages, climate dynamics, and the discount rate.

In response to this challenge, numerous governments and public authorities, spanning both developed and developing nations, have adopted either a carbon tax or a cap-and-trade system. These strategies aim to curtail emissions by either directly setting a carbon price or allowing market participants to determine the carbon price through the trading of carbon permits. However, such policies do not ensure that the actual carbon price will result in a first-best optimal allocation. Moreover, they might introduce market inefficiencies due to policy structures and market designs, as discussed in Goulder (2013), Jenkins (2014), and Benmir and Roman (2020).

The challenge faced by environmental and public authorities in establishing a carbon policy mirror those confronted by monetary and financial policy authorities. For example, monetary authorities determine interest rates based on specific rules, such as the Taylor (1993) rule, rather than adhere to the natural interest rate, which remains unobservable. A similar analogy can be made with the SCC. The SCC is fraught with uncertainties, making it challenging to estimate and monitor over time. To address this issue, Grosjean, Acworth, Flachsland, and Marschinski (2016) introduce the concept of a central bank of carbon, envisioned to operate akin to a monetary central bank. This regulatory body would set the carbon cap and monitor the implicit carbon price, taking into account business cycle fluctuations, which are emphasized as significant in Benmir, Jaccard, and Vermandel (2020). The adoption of such a framework could potentially alleviate market inefficiencies inherent in cap-and-trade market structures and foster a tighter alignment with the SCC.

Theory predicts that the price of emission permits should reflect market fundamentals associated with the marginal costs of emissions abatement (e.g. Montgomery (1972) and Rubin (1996)).[1] Shifts in business-as-usual emissions, determined by changes in demand for emission permits (e.g. weather, economic activity, and energy intensity of their products), and shifts in abatement supply (e.g. supply of fossil fuels, the response of consumers to fuel prices, and the cost of new technologies for production), modify market fundamentals expectations. Predominantly, existing cap-and-trade programs target major domestic industries with high energy consumption (e.g. electricity and heat production, cement manufacture, iron, and steel production). Fluctuations in both emission demand and abatement supply are anticipated to be the primary sources of uncertainty in the emission permit market.

A collection of research studies has empirically examined the significance of the theoretically suggested permit price determinants in the California cap-and-trade program (e.g. Borenstein, Bushnell, Wolak, and Zaragoza-Watkins (2019)) and the EU ETS (e.g. Hintermann, Peterson, and Rickels (2016), Friedrich, Mauer, Pahle, and Tietjen (2020), and Batten, Maddox, and Young (2021)). Regarding permit demand, a consistent observation across these studies is the prominent role of fossil fuels. Specifically, while many of these

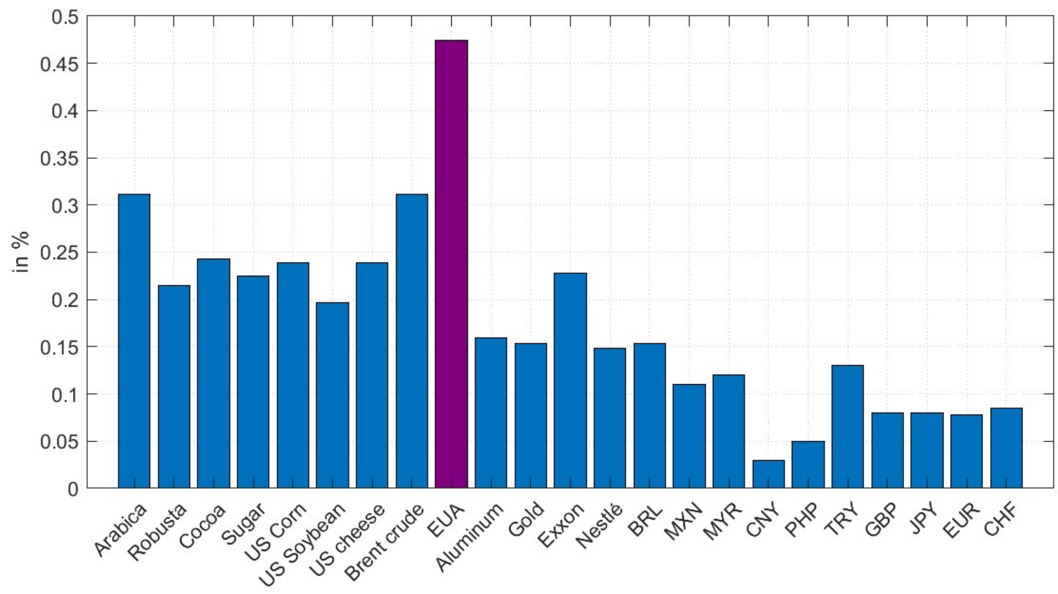
[1] For a recent survey of permit pricing theory, see Weitzman (1974), Hoel and Karp (2002), Newell and Pizer (2003), Wood and Jotzo (2011), and Karp and Traeger (2018).

papers highlight the significance of oil and gas, coal appears to be a less influential factor. In the majority of these studies, economic activity and growth announcements emerge as clear price drivers too. On the permit supply front, a challenge faced by empirical research is that several price determinants are *not directly observable*. For instance, while we can observe the supply of fossil fuels, technological advancements and innovation, along with expectations about them, remain *unobservable*. Consequently, changes in the costs of abatement technology have been scarcely addressed in empirical studies, even though they hold significant importance in the theoretical forecasting of permit prices.

Operational insights from cap-and-trade programs underscore that the permit supply schedule is not fixed, but subject to potential policy revisions. The proposed regulatory amendments in California in 2013[2] and the EU's decision in 2021 regarding 2030 targets serve as instances of mid-term cap adjustments during the periodic updates of the long-term cap. Simultaneously, due to the rigidity of most cap-and-trade frameworks in modifying the legislated caps within each commitment phase based on current situations (e.g. severe economic shocks), there have been discussions about supply management mechanisms that render the cap endogenous. Some of these mechanisms have even been implemented. A notable instance of such market intervention is the so-called EU Market Stability Reserve. While policy program interventions are observable, sudden chances of shocks in policy are not, leading to policy uncertainty. The European cap-and-trade program is particularly apt for analyzing the impact of policy uncertainty. Reacting to intense demand shocks during the EU ETS's third phase, a series of proposals and decisions were unveiled, aiming to reinstate the stringency of the EU ETS cap. This presents us with a unique period characterized by significant policy events and associated carbon price volatility that exceeds that of several typically volatile agricultural commodities and currencies; see Figure 1.

[2] The proposed revisions cover several areas of the regulation, including allocation and distribution of allowances, see: https://www.edf.org/sites/default/files/content/carbon-market-california-year_two.pdf

Figure 1: 2011-2022 Price Volatility (avg of annualised st. dev.)



Note: The figure was constructed using data from Bloomberg.

These fluctuations can arise from various factors, including changes in economic conditions, energy inputs, technological advancements, and policy shifts. Crucially, the volatility in carbon prices can create uncertainty for businesses, making it difficult for them to plan long-term investments in emission reduction strategies (Martin, Muuls, and Wagner (2011), and European Parliament (2022)).

Our methodology is grounded in both theoretical and empirical insights and addresses a unique research question that distinguishes it from a significant portion of the environmental economics literature, both theoretical and empirical. Previous studies, including Fowlie (2010), Acemoglu, Aghion, Bursztyn, and Hemous (2012), Fowlie and Perloff (2013), Aghion, Dechezleprêtre, Hemous, Martin, and Van Reenen (2016), Pommeret and Schubert (2018), and Acemoglu, Hemous, Barrage, Aghion, et al. (2019) have examined the effects of emission permit prices in the EU ETS impact macro-financial aggregates like clean technology investment. Yet, there is a noticeable gap in research focusing on the fundamental determinants influencing the implicit carbon price in the EU ETS market.[3]

[3] The primary emphasis of environmental and climate economists over the last decade, as outlined in the literature review conducted by Schubert (2018) has centered on the pricing of the environmental externality and the global macroeconomic impacts of climate change. There is a dearth of studies examining the connection between macro-finance and environmental policy frameworks such as the interactions between

Our contribution to the literature is twofold. First, we present a novel strategy for estimating and decomposing the drivers of the EU ETS. Second, we introduce a carbon cap rule that mitigates business cycle fluctuations in relation to the estimated EU ETS policy, aligning more closely with the social cost of carbon, which represents the first-best optimal policy.

Drawing from the empirical literature mentioned earlier, we investigate the relationship between the EU-wide allowance price and a set of observable determinants that capture shifts in market fundamentals of key regulated sectors and changes in economic activity across EU ETS countries. This approach allows us to micro-found our macro-finance framework. To this end, we estimate a panel Vector Autoregression (VAR) to analyze how the EU emission permit price reacts to key demand and supply aggregate shocks. Our findings underscore the significance of energy as a crucial component when evaluating carbon prices. Many of the early business cycle environmental-macro models, known as E-DSGE (e.g., Fischer and Springborn (2011) and Heutel (2012)), which probe the connections between environmental policy and macroeconomic aggregates, do not explicitly model energy production as an intermediary input nor they focus on impacts of implied volatility on macro-financial aggregates and prices.[4] Thus, examining the influence of energy inputs and prices on cap-and-trade prices enhances the micro-founding of our framework. We then utilize a unique estimation strategy to explore the factors driving the inherent market volatility within the implicit carbon price, volatility which is highly important for business cycle welfare costs.

2 Data

We assembled our dataset by integrating multiple sources. This includes macroeconomic goods productivity data and consumption patterns obtained from National Statistical Offices and Eurostat; energy production and pricing information sourced from Bloomberg; carbon dioxide emissions data from the Emissions Database for Global Atmospheric Research (EDGAR); and data on European Union Allowance (EUA) futures prices from the Intercontinental Exchange (ICE).[5] We restrict our empirical study to countries within the

carbon markets (e.g., EU ETS) and macro-financial aggregates.

[4]However, it's worth noting that business cycle environmental-macro frameworks have began recently to incorporate energy as an input (*e.g.* Golosov, Hassler, Krusell, and Tsyvinski (2014)).

[5]In the EU ETS, EUAs represent the primary form of carbon allowance, granting permission to emit one tonne of carbon dioxide (CO_2) or an equivalent amount of another greenhouse gas. "EUA Futures" refers to futures contracts based on these allowances.

European Union Emission Trading System (EU ETS) framework. Specifically, we utilize data from countries that participated in the system from January 2013 to December 2019.[6] This period aligns with phase 3 of the EU ETS and includes the UK, which remained a part of the European carbon market until 2020. Due to data constraints, we omit Norway and Liechtenstein, resulting in a total of 28 countries for our analysis (including the UK).[7] Below, we describe each data source used.

Goods productivity and consumption patterns From Eurostat, we have compiled data on the consumption preference index for each country to capture the evolving trends in consumer behavior and preferences. Additionally, we have gathered data on the industrial production index for each European state, providing a measure of the extent of industrial activity.

Energy supply data From Bloomberg, we compile data on energy production, focusing on both the volume of energy produced and the corresponding price levels. In line with the empirical literature discussed earlier, we consider three critical energy sources: Brent crude oil, natural gas, and coal. This data collection enables us to closely monitor the supply of energy, an essential determinant of the price of emission allowances.

Carbon dioxide emissions The Emissions Database for Global Atmospheric Research (EDGAR) provides estimates for emissions of the three main greenhouse gases (CO_2, CH_4, N_2O) per sector and country. This comprehensive dataset enables us to study emission dynamics at a high frequency.

Emission allowance prices From the Intercontinental Exchange (ICE), we retrieve data on daily carbon futures contracts, the European Union Allowance (EUA) futures contracts. Our data collection includes the daily prices of these EUA futures contracts, which we then convert from a daily to a monthly frequency. By examining EUA prices, we gain valuable insights into the market's response to innovations in abatement technologies, a less well-observed driver of the EU ETS.

[6] Our study does not include the COVID-19 period.

[7] The EU ETS currently operates in 30 countries: the 27 EU member states plus Iceland, Liechtenstein, and Norway. The United Kingdom left the EU on 31 January 2020 but remained subject to EU rules until 31 December 2020. In our analysis, we consider the 27 EU member states and the United Kingdom.

3 Empirical Results: EUA Futures Prices Responses To Macro and Energy Supply Aggregates

In this section, we present our analysis of the impact of macroeconomic and energy supply factors on EUA futures prices, using aggregate data from the countries included in our dataset. To estimate these effects, we employ a panel Vector Autoregressive (VAR) model. This approach enables us to explore the behavior of the EU-wide allowance permit price in response to various aggregate shocks.

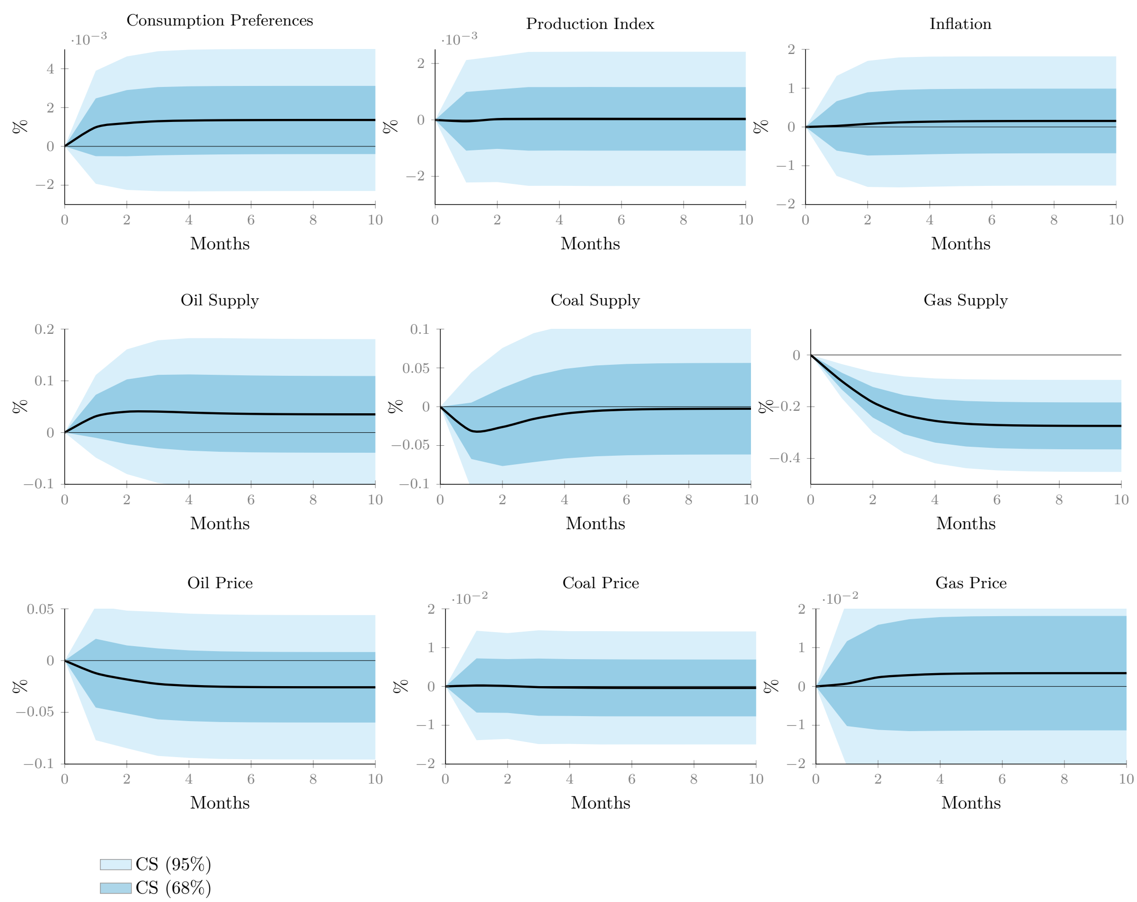
In practice, inspired by Friedrich et al. (2020), Hintermann et al. (2016), andBorenstein et al. (2019), we consider 9 *observable* determinants and define the vector of endogenous variables for each country i in month-year t containing: consumption preference index, industrial production index, inflation, oil supply, coal supply, gas supply, oil price, coal price, and gas price. Then, we specify the following panel VAR:

$$\mathbf{Y_{i,t}} = \sum_{\mathbf{j=1}}^{\mathbf{p}} \mathbf{A_{i,t}} \mathbf{Y_{i,t-j}} + \mathbf{\Gamma_i} \varepsilon_{\mathbf{i,t}}, \tag{1}$$

where Γ is a non-singular 9 x 9 structural impact matrix, and $\varepsilon_{i,t}$ is the vector of strucutral shocks.

Figure 2 plots the cumulative impulse response functions (IRFs) of the EUA futures prices when subjected to various macroeconomic and energy supply shocks. The solid black lines represent the estimated paths, while the shaded blue regions indicate the confidence intervals, with the inner and outer bands representing 68% and 95% confidence levels, respectively.

Figure 2: EUA Futures Response To Macro and Price Aggregates



Notes: The figure shows EUA futures price cumulative impulse responses to different macroeconomic and energy price aggregates over monthly periods.

The analysis reveals that when there is a shock in consumption preference index, it triggers a prolonged rise in EUA futures prices. This is likely because such a shock indicates a surge in the demand for goods, which in turn can drive up the prices of carbon allowances as industries ramp up production to meet this demand. A shock in the industrial production index appears to have no impact on EUA futures prices, reinforcing the limited role of unexpected change in the output of production activities as observed in the empirical studies discussed earlier. With respect to inflation, its role in influencing EUA price movements seems to be minimal during the period under study. This could mean that the general price level in the economy, represented by inflation, does not have a direct and strong correlation with the fluctuations in EUA futures prices. These observations underscore that the relationship between EUA futures prices and certain macroeconomic indicators is limited, consistent with patterns highlighted in previous empirical research (Friedrich et al. (2020) and Batten et al. (2021)).

Regarding the energy drivers, we find that oil and gas significantly influence EUA futures price levels. After an oil supply shock, the EUA futures price increases persistently, whereas it declines following a gas supply shock. This aligns with the findings of Aatola, Ollikainen, and Toppinen (2013) and Friedrich et al. (2020), given that energy supply based on gas is less emission-intensive than that based on oil. A similar pattern is evident when the EUA futures prices face gas and oil price shocks (Rickels, Görlich, and Peterson (2015)). Specifically, the futures prices tend to persistently increase after a gas price shock and decrease after an oil price shock. The latter two findings could be further substantiated if renewable energy data were accessible for all 28 EU countries throughout the examined period. Lastly, the outcomes remain inconclusive when considering coal energy generation and prices, consistent with Friedrich et al. (2020).

The panel VAR analysis emphasizes the relevance of observable variables like energy generation and prices, particularly oil and gas, corroborating existing literature that underscores the crucial role of energy dynamics in shaping allowance prices (Friedrich et al. (2020), Hintermann et al. (2016) and Borenstein et al. (2019)). However, this empirical analysis overlooks critical factors such as innovations in abatement technologies and shifts in policy and regulations, which are essential for a more comprehensive understanding of the determinants of allowance prices.

Abatement technologies, which are key to reducing emissions, play a vital role in theoretical models of emission control and pricing, as discussed in studies like Rubin (1996), Newell and Pizer (2003), and Schennach (2000). In these models, emission allowance prices are primarily influenced by marginal abatement costs. However, these costs are often not directly observable. Moreover, while the advancement of low-carbon technologies does affect marginal abatement costs, and thereby the prices, the technologies themselves and the market's expectations about them are not readily observable. This lack of visibility into technological developments and market expectations poses a challenge in identifying abatement shocks and accurately decomposing their impact on emission allowance prices within the market.

Regulatory uncertainty has been identified in the literature as another crucial factor (Koch, Grosjean, Fuss, and Edenhofer (2016) and Deeney, Cummins, Dowling, and Smeaton (2016)). The growing attention toward policy and regulatory changes emerges in light of the challenges in explaining recent allowance price developments by solely relying on demand-side fundamentals (Friedrich et al. (2020)). The regulatory landscape of these government-created markets is dynamic. Changes in political leadership, public opinion, international agreements, and other factors can prompt shifts in the regulatory framework over time. Such shifts inject a level of uncertainty into the market, as participants are unable to accurately predict future regulatory trajectories. Indeed, there is evidence that the allowance market responds to various policy and regulatory news with fluctuations in price and price volatility (Koch et al. (2016) and Deeney et al. (2016)).

Thus, while the allowance market evidently reacts to observable shocks such as changes in energy supply and energy prices, it is apparent it also responds to less well-observable factors. These include innovations (shocks) in abatement technology and shifts in regulatory frameworks, which, despite their significant impact on allowance prices, are not adequately captured in empirical studies due to their less observable nature. To address this gap, we introduce a comprehensive model in the next section. This model integrates energy dynamics, as indicated by the panel VAR, with both theoretically significant abatement shocks and empirically evident policy uncertainty shocks. Designed to provide a deeper and more insightful analysis, this model aims to enhance our understanding of the diverse factors influencing EUA futures prices.

4 The model

We consider an infinite-horizon, closed economy comprising two production sectors (energy producers and final firms), households, a government, and an environmental regulator. The households in this economy are identical, infinitely lived, and collectively account for a measure of one. Energy producers generate an environmental externality through carbon dioxide emissions (hereafter CO_2 emissions). These emissions impact the utility (or alternatively the productivity of final firms via a damage function) and subsequently affect the welfare of the representative households. Energy producers do not internalize the broader environmental consequences of their CO_2 emissions, resulting in a market failure.

We start by outlining the climate dynamics. This is followed by a detailed description of the energy and non-energy firms' problems. Next, we introduce households in the model. Lastly, we delve into the roles of the government and the environmental regulator.

4.1 Climate change and emission dynamics

Following standard integrated assessment models (IAMs) (see Nordhaus (1991) and Nordhaus and Yang (1996)), we cast environmental externality within a macro-finance framework. A significant portion of the accumulation of carbon dioxide and other Greenhouse Gases (GHGs) in the atmosphere is attributed to the human activity of economic production. We describe the temperature and concentration process of carbon dioxide in the atmosphere as follows. Firstly, the global temperature T_t^o is linearly proportional to the CO_2 emission stock – the cumulative amount of emissions – as posited by Matthews, Gillett, Stott, and Zickfeld (2009):

$$T_{t+1}^o = \zeta_1^o(\zeta_2^o X_t - T_t^o) + T_t^o, \tag{2}$$

with ζ_1^o and ζ_2^o chosen following Dietz and Venmans (2019).[8]

Second, cumulative CO_2 emissions, denoted as X_t, follow a law of motion:[9]

$$X_{t+1} = \eta X_t + E_t^T + E_t^*, \tag{3}$$

where X_{t+1} is the concentration of gases in the atmosphere, $E_t^T \geq 0$ the total inflow of CO_2 at time t (both from the energy E_t^E and non-energy E_t^{NE} sectors in the EU)[10], E_t^* represents the rest of the world emissions,and $0 < \eta < 1$ represents the persistence of CO_2 emissions, which is chosen to be very close to 1, as argued by Dietz and Venmans (2019). Anthropogenic emissions of CO_2 are comprised of both energy and non-energy emissions.

The energy emissions E_t^E arise from energy production denoted as Y_t^E, and are influenced by an exogenous trend Γ_t^X. This trend encapsulates the decoupling between CO_2 emissions and production. The relationship can be expressed as:

$$E_t^E = (1 - \mu_t)\, \varphi_{\mathrm{E}} Y_t^E \Gamma_t^X, \tag{4}$$

where $\varphi_{\mathrm{E}} Y_t^E$ represents the total CO_2 influx resulting from production prior to the imple-

[8] We observe that although variations in climate dynamics and damage modeling over the long horizon (be it à la Golosov et al. (2014), à la Nordhaus (2017), or à la Matthews et al. (2009), among others) lead to subsequent effects on macroeconomic aggregate equilibria, over the business cycle horizon (and under equivalent calibrations), these modeling specifications do not result in significant changes to macroeconomic aggregate equilibria.

[9] To ensure convergence in the auto-regressive law of motion for the stock of emissions process, and without a loss of generality, we deviate slightly from the transient climate response to cumulative CO_2 emissions theory by setting $\eta \neq 1$. However, we select η to be sufficiently close to one so that $X_t \approx X_0 + \sum_{i=0}^{t}(E_i^T + E_i^*)$.

[10] Where $E_t^T = E_t^E + E_t^{\mathrm{NE}}$.

mentation of any abatement measures. The variable $0 \leq \mu_t \leq 1$ represents the fraction of emissions that are mitigated (abated) by the energy firms, while $\varphi_{\mathrm{E}} \geq 0$ is a carbon-intensity parameter that defines the steady-state relationship between emissions and output.

CO_2 emissions from the non-energy sector follow a similar law of motion to the emissions from the energy sector. The key distinction lies in the fact that the non-energy sector is not subject to any environmental policy or carbon price. Consequently, firms in the non-energy sector do not undertake any abatement efforts:

$$E_t^{\mathrm{NE}} = \varphi_{\mathrm{NE}} Y_t^{\mathrm{NE}} \Gamma_t^X \tag{5}$$

with φ_{NE} the emission intensity in the non-energy sectors.

4.2 Energy Firms

The energy producers aim to maximize their profit by balancing the desired levels of capital and labor, taking into account the energy price. Energy production follows a Cobb-Douglas production function:

$$\tilde{Y}_t^E = \varepsilon_t^{A_E} A_t^E (K_t^E)^{\alpha_E} (\Gamma_t^Y l_t^E)^{1-\alpha_E} \Gamma_t^{Y^E}, \tag{6}$$

where K_t^E represents the capital stock utilized by the energy firms with an intensity parameter $\alpha_E \in [0,1]$, l_t^E denotes labor, $A_t^E > 0$ denotes the productivity level, and $\varepsilon_t^{A^E}$ is a total energy productivity shock that evolves as follows:

$$\log\left(\varepsilon_t^{A^E}\right) = \rho_{A^E} \log\left(\varepsilon_{t-1}^{A^E}\right) + \eta_t^{A^E}, \text{ with } \eta_t^{A^E} \sim N(0, \sigma_{A^E}^2).$$

To align with the trends in the EU's energy and industrial production sectors, we incorporate an energy transition trend $\Gamma_t^{Y^E} = \gamma^{y^E} \Gamma_{t-1}^{Y^E}$, enabling the capture of the energy sector's growth rate, distinct from that of the overall economy, denoted as γ^y. Consequently, the trend-corrected energy production is expressed as $Y_t^E = \tilde{Y}_t^E \Gamma_t^{Y^E -1}$. This is further discussed in the balanced growth path (BGP) section.

Energy producers maximize profits:

$$\Pi_t^E = \varepsilon_t^p p_t^E Y_t^E - w_t^E l_t^E - I_t^E - Z_t - \tau_t E_t^E. \tag{7}$$

The relative price of energy and the real wage are denoted by p_t^E and w_t^E, respectively. ε_t^p is an AR(1) shock to the energy price.

$$\log\left(\varepsilon_t^p\right) = \rho_p \log\left(\varepsilon_{t-1}^p\right) + \eta_t^p, \text{ with } \eta_t^p \sim N(0, \sigma_p^2).$$

The function $Z_t = f(\mu_t) Y_t^E$ represents the abatement-cost function per unite of energy production. Additionally, $\tau_t \geq 0$ is a price on CO_2 emissions, reflecting the carbon policy set forth by the environmental regulatory authority, which will be detailed later. Investment is denoted by I_t^E, and the accumulation of physical capital follows the law of motion:

$$K_{t+1}^E = (1-\delta)K_t^E + I_t^E, \tag{8}$$

where $\delta \in [0, 1]$ is the depreciation rate of physical capital.

The abatement-cost function is adapted from Nordhaus (2008), where $f(\mu_t) = \theta_1 \mu_t^{\theta_2} \varepsilon_t^z$. In this expression, $\theta_1 \geq 0$ determines the steady state of the abatement, while $\theta_2 > 0$ represents the elasticity of the abatement cost concerning the fraction of abated emissions. This function $f(\mu_t)$ connects the fraction of emissions abated to the fraction of output allocated to abatement, with the price of abatement normalized to one. Lastly, the abatement shock (ε_t^z) that captures market uncertainties about both abatement investment cost and technology, evolves as follows:

$$\log\left(\varepsilon_t^z\right) = \rho_z \log\left(\varepsilon_{t-1}^z\right) + \eta_t^z \text{ with } \eta_t^z \sim N(0, \sigma_z^2).$$

4.3 Final goods firms

Final firms aim to maximize profit by balancing the desired levels of capital, energy consumption, and labor. Following Bachmann, Baqaee, Bayer, Kuhn, Löschel, Moll, Peichl, Pittel, and Schularick (2022), output is generated using a Constant Elasticity of Substitution (CES) production function[11], allowing for the capture of the non-linear dynamics involved in substituting away from energy to different production inputs.

$$Y_t = \left((1-\chi)^{\frac{1}{\sigma}} (Y_t^{\mathrm{NE}})^{\frac{\sigma-1}{\sigma}} + \chi^{\frac{1}{\sigma}} Y_t^{\mathrm{E}\,\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} \tag{9}$$

with

$$Y_t^{\mathrm{NE}} = \varepsilon_t^{A^{\mathrm{NE}}} A_t^{\mathrm{NE}} (K_t^{\mathrm{NE}})^{\alpha_{\mathrm{NE}}} (\Gamma_t^Y l_t^{\mathrm{NE}})^{1-\alpha_{\mathrm{NE}}} \tag{10}$$

[11]In the robustness section, we also consider the simple case of a Cobb-Douglas aggregator: $Y_t = \varepsilon_t^{A_y} A_t^y d(T_t^o)(K_t^{\mathrm{NE}})^{\alpha_{\mathrm{NE}}}(Y_t^E \Gamma_t^{E^{-1}})^{\alpha_{\mathrm{E}}}(\Gamma_t^Y l_t^{\mathrm{NE}})^{1-\alpha_{\mathrm{NE}}-\alpha_{\mathrm{E}}}$, where α_{E} represents the energy share and α_{NE} the non-energy capital share.

where K_t^{NE} is the capital stock utilized by the final firms with an intensity parameter $\alpha_{\mathrm{NE}} \in [0,1]$, l_t^{NE} is non-energy labor, $A_t^{\mathrm{NE}} > 0$ the productivity level of the non-energy final sector, σ the elasticity of substitution between the energy and non-energy factors of production, χ the energy share in total production, $\Gamma_t^{Y^E}$ the exogenous corrective trend applied to the energy sector in order to match the growth dynamics in the EU while maintaining a BGP, and $\varepsilon_t^{A^{\mathrm{NE}}}$ a total factor productivity shock that evolves as follows:

$$\log\left(\varepsilon_t^{A^{\mathrm{NE}}}\right) = \rho_{A^{\mathrm{NE}}} \log\left(\varepsilon_{t-1}^{A^{\mathrm{NE}}}\right) + \eta_t^{A^{\mathrm{NE}}} \quad \text{with } \eta_t^{A^{\mathrm{NE}}} \sim N(0, \sigma^2_{A^{\mathrm{NE}}})$$

While a significant portion of the climate economics literature models environmental damages through the production side following Nordhaus (1991) in the form of $Y_t = D_p(T_t^o)\left((1-\chi)^{\frac{1}{\sigma}}(Y_t^{\mathrm{NE}})^{\frac{\sigma-1}{\sigma}} + \chi^{\frac{1}{\sigma}} Y_t^{\mathrm{E}\,\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}$, we adopt the approach of Barnett, Brock, and Hansen (2020) by incorporating environmental damages within the utility function. Although modeling environmental damages via disutility can be isomorphic to production damages given specific functional forms and calibration, it enables us to maintain a BGP without imposing restrictive assumptions on the form of the damage function and its parametrization. We delve into this point in detail in the BGP section. Furthermore, in a robustness exercise, we demonstrate that both utility damages and production damages yield similar results.[12]

Final firms producers maximize profits as follows:

$$\Pi_t^F = Y_t - w_t^{\mathrm{NE}} l_t^{\mathrm{NE}} - I_t^{\mathrm{NE}} - \varepsilon_t^p p_t^{\mathrm{E}} Y_t^{\mathrm{E}}. \tag{11}$$

where the real wage is denoted by w_t^{NE} and capital investment by I_t^{NE}. The accumulation of physical capital is given by a similar law of motion to the energy firms:

$$K_{t+1}^{\mathrm{NE}} = (1-\delta)K_t^{\mathrm{NE}} + I_t^{\mathrm{NE}}, \tag{12}$$

where $\delta \in [0,1]$ is the depreciation rate of physical capital.

4.4 Households

Households make consumption and savings decisions, supplying labor inelastically. They hold government bonds and possess ownership of firms in the corporate sector, from which

[12] See results in Appendix C.

they receive dividends or profits. Additionally, our households encounter climate damages, denoted as $D_u(T_t^o)$, which result from a disutility associated with rising temperatures, akin to the approaches in Barnett et al. (2020) and Barrage (2020).

Households maximize their life-time utility:

$$\mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \varepsilon_t^B u \left(C_t - H_{t-1} - D_u(T_t^o) \right), \tag{13}$$

where $\mathbb{E}_t$ is the expectations operator conditioned on information at time t, β is the time discount factor, C_t represents consumption while H_{t-1} represents consumption habits, and ε_t^B is the preference shock

$$\log \varepsilon_t^B = \rho_B \log \varepsilon_{t-1}^B + \eta_t^B \quad \text{with } \eta_t^B \sim N(0, \sigma_B^2)$$

Climate damages are linear in temperature:

$$D_u(T_t^o) = \Theta_t^T T_t^o$$

where Θ_t^T is the damage sensitivity of households to temperature increase.

The law of motion for the habit stock is determined in accordance with Campbell and Cochrane (1999), specifically as $H_{t-1} = hC_{t-1}$. In contrast to the approach taken by Cai and Lontzek (2019), who employ recursive utility à la Epstein and Zin (1989) to capture long-run risk associated with climate change, we opt for consumption habits for two reasons: i) our focus is on the business cycle (i.e., phase 3 of the EU ETS from 2013 to 2019), where long-run risk related to climate change is not as significant as in long-run climate policy analysis, and ii) consumption habits are demonstrated to play a crucial role in generating higher volatility levels within business cycle fluctuations of the social cost of carbon, contrary to the recursive utility framework (Benmir et al. (2020)), while still aligning with consumption and output volatility obsevered in real data.

The budget constraint of the representative household is as follows:

$$w_t^{\text{NE}} l_t^{\text{NE}} + w_t^{\text{E}} l_t^{\text{E}} + r_t B_t + \Pi_t^E + \Pi_t^F - T_t = C_t + B_{t+1} \tag{14}$$

where the left-hand side represents the household's various sources of income. The total income primarily consists of labor earnings. In each period, the household also receives returns from holding a long-term government bond, denoted as B_t, with a return rate of r_t. Additionally, since the representative household owns firms in the corporate sector, they receive dividend income from both the energy firms Π_t^E, and the final firms Π_t^F. On the spending front, the representative household primarily allocates its income towards consumption goods, denoted as C_t and the purchase of long-term government bonds B_t. Additionally, it's assumed that the government imposes a lump-sum tax, represented by T_t.

4.5 Government and market clearing

The government funds its expenditures through tax collection. The budget constraint for the government is given by:

$$G_t = T_t + \tau_t E_t. \tag{15}$$

Here G_t represents public expenditure, and T_t is a lump-sum tax. The second revenue component $\tau_t E_t$ represents the earnings derived from the imposition of a cost on the environmental externality. In this expression, E_t and τ_t denote respectively emissions and the carbon price – the price of the right to emit one unit of CO_2 emission, respectively.

As is standard in most business-cycle models, the government's expenditure is a proportion of the total output. The resource constraint of the economy reads as follows:

$$Y_t = C_t + I_t^{\mathrm{NE}} + I_t^{\mathrm{E}} + G_t + Z_t. \tag{16}$$

4.6 Emission cap

Considering a cap-and-trade framework, we focus on a scenario where emissions in the economy are regulated by an environmental authority to ensure they remain within a predefined limit, commonly known as "the cap." Ideally, the environmental regulator sets this cap by equating the marginal costs to the marginal benefits of emission reduction, essentially reflecting the social cost of carbon that a social planner would pick in a centralized economy.[13] In a decentralized economy and without uncertainty, setting the carbon policy equal to the social cost of carbon would retrieve the first-best optimal solution. However, the existence of uncertainties in production, consumer behavior, and abatement processes presents significant challenges in setting a fixed cap (or a comparable constant tax) at the optimal level after the fact. These uncertainties render it challenging to accurately assess the marginal value of emission reductions in advance. In fact, it is widely understood that policies should be conditioned on the available information (Ellerman and Wing (2003), Jotzo and Pezzey

[13] A complete derivation of the theoretical social cost of carbon can be found in Appendix A.

(2007), Newell and Pizer (2008), and Doda (2016)). In an ideal scenario, policies would be adaptive, responding to the actual occurrence of these shocks, thereby necessitating a cap that is *dynamically adjusted* over time (Kollenberg and Taschini (2016) and Karp and Traeger (2023)).

In practice, setting the cap is typically a complex political process aimed at balancing environmental goals with socio-economic considerations. As new information emerges and societal priorities shift, policymakers may adjust the emission limits to align with these evolving conditions. These adjustments, however, can lead to policy and regulatory uncertainties. Consequently, we treat the cap, denoted as $\bar{E}^E$, as an exogenous variable, and we will outline the associated per-period quota as follows:

$$E_t^E = Q_t \varepsilon_t^{CS}, \tag{17}$$

where ε_t^{CS} evolves as:

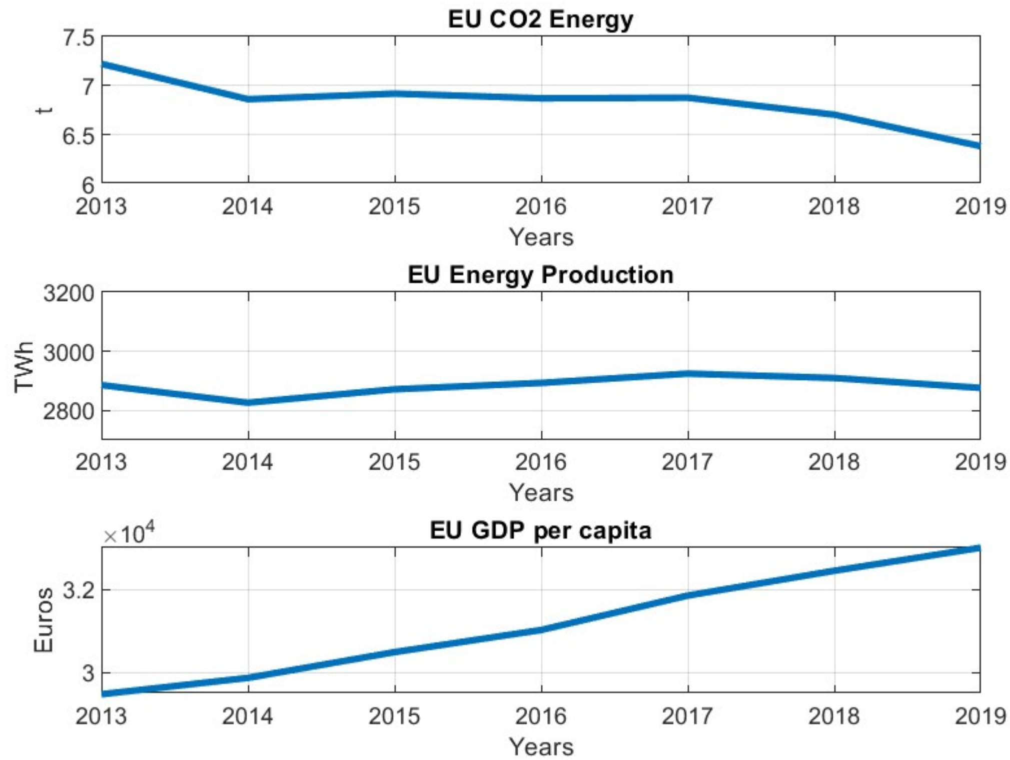
$$\log \varepsilon_t^{CS} = \rho_{CS} \log \varepsilon_{t-1}^{CS} + \eta_t^{CS} \quad \text{with } \eta_t^{CS} \sim N(0, \sigma_{CS}^2).$$

The innovation ε_t^{CS} captures the policy uncertainty surrounding the cap target or corresponding shifts in the availability of allowances. This is referred to as the *climate sentiment shock*.

4.7 Balanced growth

Given that the primary focus of the paper is to estimate the drivers of the carbon permit price and the permit market, we derive the de-trended model concerning its balanced growth path. Additionally, we account for the scenario where emissions and energy production exhibit growth rates different from that of the overall output. As illustrated in Figure 3, EU energy emissions, energy production, and output all experience distinct growth rates.

Figure 3: EU Trends in CO2 Energy Emissions, Energy Production (Electricity), and GDP Per Capita



Notes: The figure was generated utilizing data on CO2 emissions and energy production from https://ourworldindata.org/, along with GDP per capita data sourced from FRED.

In the context of our model, we assume that the energy sector is stationary due to the zero growth rate observed in the data (i.e., $\Gamma_t^E = \Gamma_t^{Y^{-1}}$). The disparity in growth rates between energy emissions and output is attributed to the introduction of a rate of green technological progress.

Consistent with the literature, macroeconomic variables are also assumed to grow along the balanced growth path. This growth is facilitated by labor-augmenting technological progress, represented by Γ_t^Y. The growth rate of this labor-augmenting technological progress is denoted by γ^y, where:

$$\frac{\Gamma_{t+1}^Y}{\Gamma_t^Y} = \gamma^y.$$

We represent green technological progress in the growing economy by Γ_t^X. The growth rate of this green progress, denoted as γ^x, is defined as follows:

$$\frac{\Gamma^X_{t+1}}{\Gamma^X_t} = \gamma^x.$$

This trend is crucial for capturing the long-term process of decoupling output growth from emission growth. As documented by Newell, Jaffe, and Stavins (1999), this trend can be interpreted as an energy-saving technological shift that reflects the adoption of less energy-intensive technologies in capital goods. An improvement in this technology would result in a value for γ^x that is below 1. Following Nordhaus (1991), we assume that this trend is deterministic.

Turning to climate damages, a model choice following that of Nordhaus (1991) where climate damages $D_p(T^o_t)$ are represented via a convex function relating the temperature level to a deterioration in output given by $D_p(T^o_t) = ae^{-b_t T^{o2}_t}$ poses some BGP challenges (especially as we are estimating our model over the business cycle and thus requiring stationarity). One potential solution is to correct for the damage sensitivity $b_t = \frac{b}{\Gamma^{Y2}_t}$ in order to adjust it with the economy's growth rate. Therefore, introducing Γ^{Y2}_t to the damage sensitivity parameter such that $D_p(T^o_t) = ae^{-\frac{b}{\Gamma^{Y2}_t} T^{o2}_t} = ae^{-bT^{o2}_t}$. ensure the existence of a BGP. This however could be problematic over the long-run as b_t would be declining overtime, which means as temperature increases we have less damages. However, such an assumption should not be restrictive over the studied period $b_t \approx \frac{b}{(\Gamma^Y_t)^2}$.[14]

In the appendix, we introduce the de-trended economy, providing a detailed derivation. Additionally, we discuss both the social planner problem and the decentralized problem in details.

5 Bringing the Model to Data

Bringing our model to data is essential to disentangle the drivers of the EUA futures. A major challenge, however, stems from the unobservable nature of abatement and climate sentiment dynamics at the monthly frequency. To overcome this limitation and fill the data gap in these areas, we have devised an innovative methodology for estimating the shocks associated with both abatement and climate sentiment changes. We also pay particular attention to match a broad spectrum of statistics, including the share of EU emissions, emissions per

[14] As we are looking at 6 years with an average low growth rate in the EU.

sector within the EU, and the energy intensity of each sector.

5.1 Strategy

While we can reliably estimate consumption patterns, goods productivity, energy prices, and energy supply dynamics using data from Eurostat and Bloomberg, the same approach is not feasible for abatement and climate sentiment dynamics. This discrepancy arises because the latter involves more complex and less tangible elements that are not directly captured in standard economic databases.

We begin our analysis by focusing on policy change dynamics and the estimation of *climate sentiment* shocks, which are pivotal as they mirror shifts in the regulatory environment and alterations in firms' perceptions of policy stringency. These shocks are indicative of how regulatory changes and policy updates influence market sentiment, particularly in the context of emission control. To estimate these shocks, we leverage the intrinsic design of the cap-and-trade system, which is structured to achieve a consistent reduction in emissions over the duration of our study. This system's design is key because it provides a framework where emission targets are incrementally tightened, indicative of a progressively decreasing per-period cap. In our methodology, we integrate an emissions dataset from EDGAR into our model, which has been specifically modified to exclude long-term trends. This adjustment is critical as it enables us to effectively isolate the climate sentiment shock series. This series provides insight into the evolution of market sentiment and expectations in the face of ongoing regulatory and policy changes. This technique enables us to uncover the often unobservable effects of policy shifts on market dynamics, thereby allowing us to represent how regulatory changes impact carbon allowance prices.

Our model's structural framework, which links economic activity, energy factors, climate sentiment changes, abatement costs, and the carbon price, provides a comprehensive basis for analyzing residual volatility in the allowance permit market. This residual volatility is of particular importance in our analysis, as it encompasses market fluctuations that remain unaccounted for by factors such as energy dynamics, economic activity, and policy shifts. In our model, we can ascribe them to abatement shocks. These shocks refer to unforeseen changes in the costs associated with strategies and technologies that firms employ to reduce emissions.

We estimate our model using Bayesian methods on monthly EU data from January 2013 to December 2018. To map our model to the data, we augment our equilibrium equations with observation equations as follows:

$$\begin{bmatrix} \text{Production Index Growth} \\ \text{Consumption Index Growth} \\ \text{Per Capita Emissions Growth} \\ \text{Per Capita Energy Production Growth} \\ \text{Energy Production Price} \\ \text{Real } CO_2 \text{ Price Growth} \end{bmatrix} = \begin{bmatrix} (\gamma^y y_t - y_{t-1})/y_{t-1} \\ (\gamma^y c_t - c_{t-1})/c_{t-1} \\ \log \gamma^s + \Delta \log (e_t) \\ \Delta \log \left(y_t^E\right) \\ \Delta \log \left(p_t^E\right) \\ \Delta \log (\tau_t) \end{bmatrix}, \tag{18}$$

where γ^s represents the trend in emissions[15] and γ^y denotes the trend growth rate of the economy. Considering the model's stationary nature, it is imperative to transform the data series into a stationary form before integrating them into the model. In line with the foundational approach established by Smets and Wouters (2007), we address data that exhibit a unit root by rendering them stationary. This is achieved by taking the logarithmic difference of the series as necessary.

5.2 Calibration

We summarise in this section the parametrisation of the model. For parameters for which the time interval is relevant, the calibration is monthly. Consistent with standard practice, we have tailored the model's calibration to align with certain observed key aggregates. These include temperature, the share of EU emissions, emissions per sector within the EU, the energy intensity of each sector, and the average value of the EU ETS allowance price, all specifically within the context of the European Union. This calibration ensures that our model accurately reflects the real-world dynamics and trends of these critical environmental and economic indicators.

The parameters pertaining to the business cycle structure of our model are conventional. For the standard parameters in these models, such as the discount factor β and the risk aversion σ^U, we align to typical values used in macroeconomic modeling.[16] Specifically, the capital intensity parameters are set at $\alpha_N = \alpha_{NE} = 0.333$, while the depreciation rate δ is fixed at 0.008. The discount factor β is set at 0.9986 and the risk aversion σ^U at 1.5.

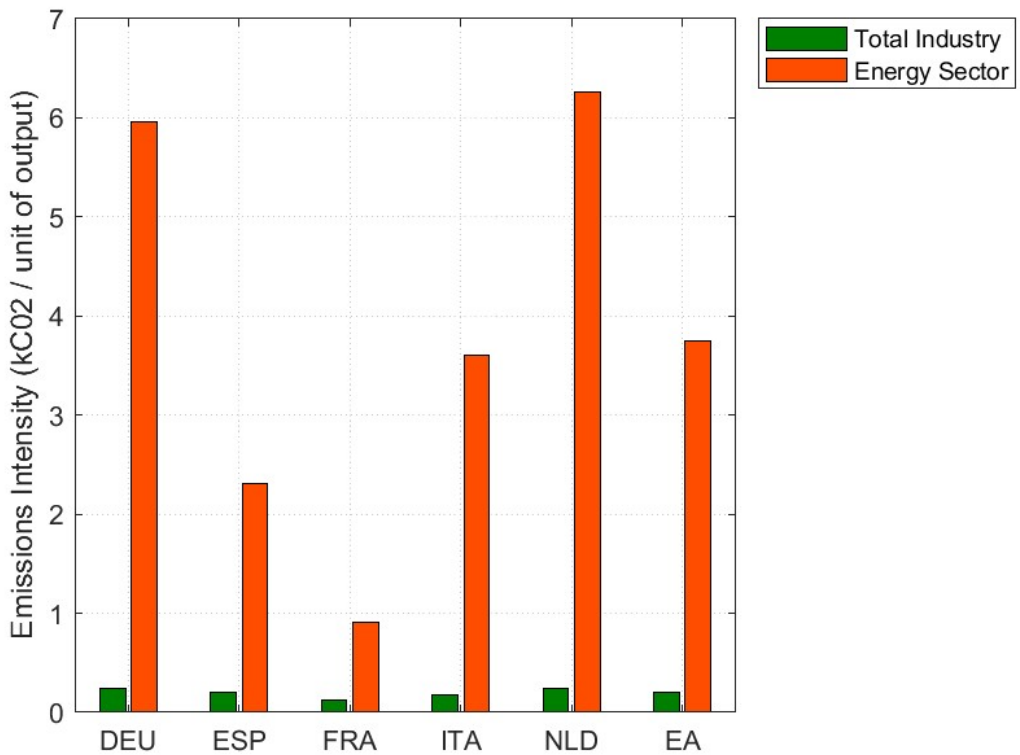
In calibrating the climate block of the model, we follow Dietz and Venmans (2019) and set the parameters for the global temperature function $\zeta_1^o = 0.50$ and $\zeta_2^o = 0.00125$.

[15] Refer to appendix for the full description of the BGP.

[16] Notice that we calibrated all the parameters to a monthly frequency.

We use the remaining parameters to match a number of relevant statistics for the EU. Specifically, the share of the energy sector in the economy χ is fixed at 2%, while the elasticity in the CES function σ is 0.2, consistent with estimates in Labandeira, Labeaga, and López-Otero (2017). The emission intensity parameters φ_E and φ_{NE} are calibrated to match emission to production in both sectors. As depicted in Figure 4, the energy sector is approximately thirty times more emission intensive than the industry as a whole.

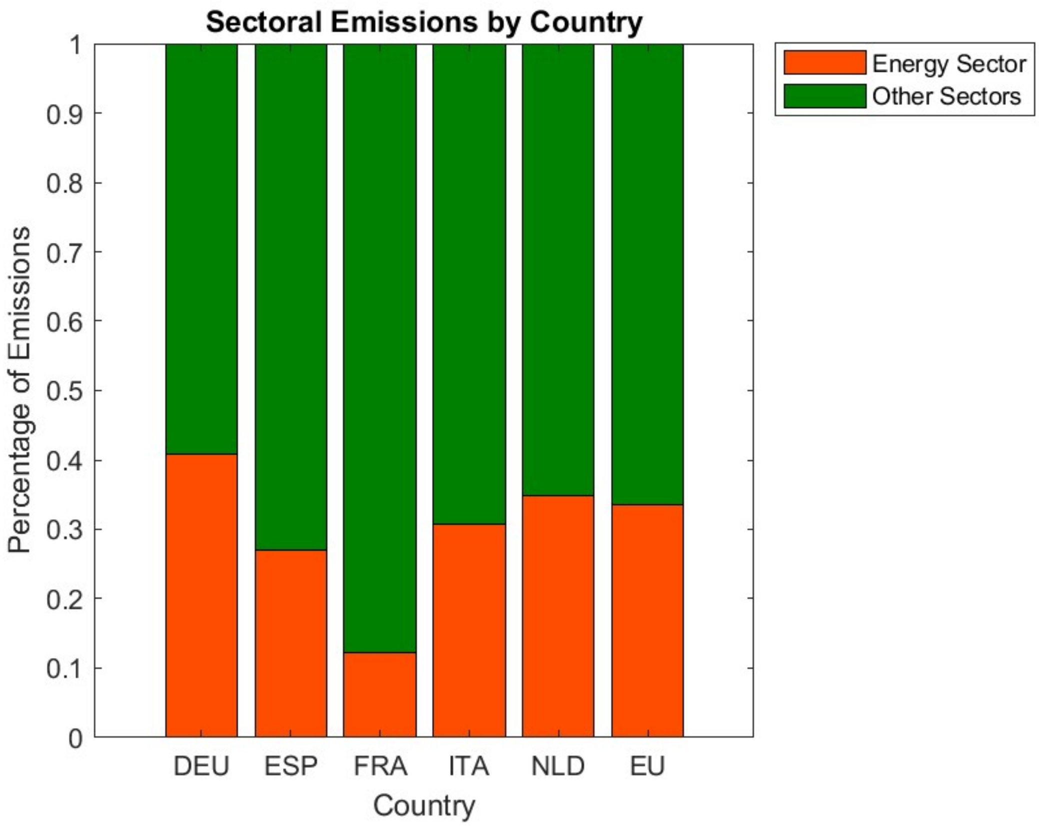
Figure 4: Emission Intensity in the EA



Notes: The figure depicts the emissions intensity in the energy sector and in the total industry for the top 5 EA economies, along with the EA mean over the estimated period (2013 – 2019).

As for the price of carbon, we proceed in two steps. We first find the value of the abatement function level θ_1 that is consistent with the observed mean EUA price of 7.54 euros. This also takes into account the split of emissions across sectors (see Figure 5) and the emission intensity of the energy sector. Then, we assume that the implied level of the EUA price was optimal over the 2013-2019 period and retrieve the value of Θ^T. More precisely, we find the value of Θ^T that equates the steady state level of the welfare in the model to the level of the welfare in the counterfactual optimal case. As we will show later, this does not imply that the economy in the estimated model behaves optimally. In particular, high volatility in the EUA price will generate losses in consumption for risk-averse agents that are more severe in the estimated case than in the optimal case.

Figure 5: Sectoral Emissions in the EA



Notes: The figure depicts the emissions split between the energy sector and the rest of the industry for the top 5 EA economies, along with the EA mean over the estimated period (2013 – 2019).

Finally, we use the decay rate of emissions η to ensure that the stock of emissions in the atmosphere is consistent with the mean level of emissions observed during the studied period and we set the public consumption to GDP ratio $\bar{g}/\bar{y}$ at 0.22.

The comprehensive list of calibrated parameters, along with the targeted economic and environmental moments they allow us to replicate, can be found in Table 3 and Table 4, respectively.

5.3 Estimation

Our model's shock processes and trends are estimated using a generalized Kalman Filter, specifically chosen to effectively handle the model's non-linear characteristics.[17] We employ the Metropolis-Hastings algorithm to approximate the posterior distribution, constructing our results based on four distinct chains. The estimation outcomes are concisely presented in Table 5, where we display both the prior and posterior densities of the estimated parameters.

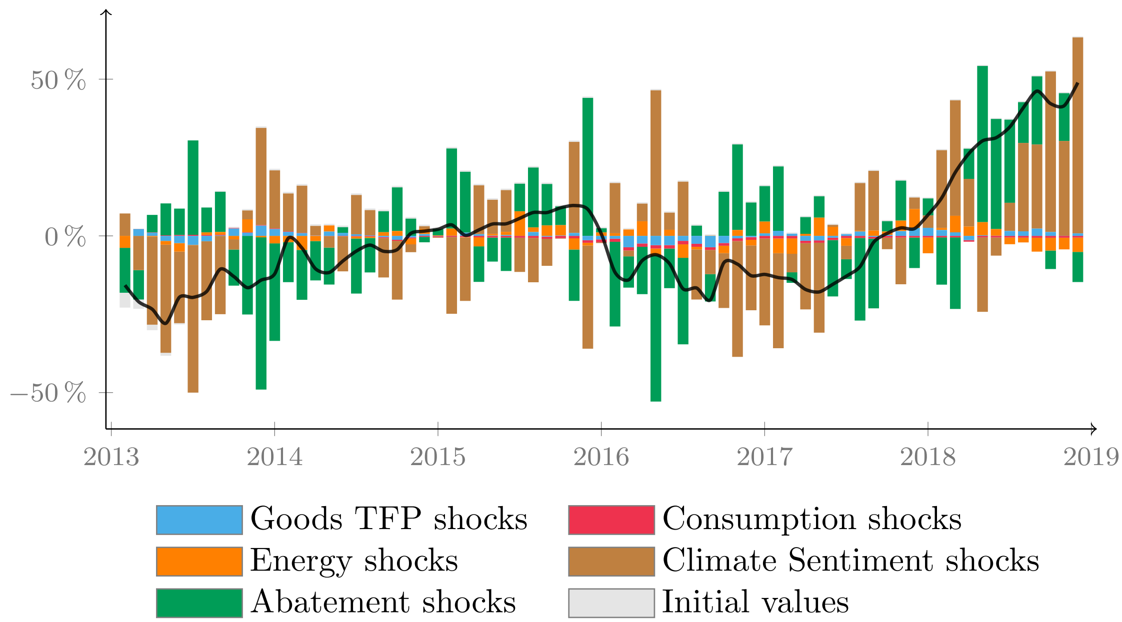
The robust identification of the majority of these parameters indicates the informativeness of the data used. Despite some constraints in pinpointing their exact values, the trends in emissions and output are clearly discernible. Notably, our model's estimation is able to capture the decoupling between output and emissions. This is evidenced by the negative value of γ^x and the positive value of γ^y, a pattern that persists even with the application of normal priors centered around zero for both trends.

6 EUA Futures Drivers and Optimal Policy

In this section, we first analyse the contribution of the different structural shocks to fluctuations in the allowance price in the EU ETS. To do so, we utilize the parameters and shock series that we have previously estimated. We then undertake a comparative study. We compare the EUA futures price with the case where the environmental regulator sets the carbon price equal to the social cost of carbon (SCC), as determined by our estimated parameters and shock series. This comparison is designed to measure the additional volatility in the EU ETS market compared to the SCC over the studied period.

[17] A comparative analysis between first-order Bayesian estimation and second-order estimation highlights the significance of non-linearities, especially during periods of increased market volatility (refer to Figure 11 and Figure 12).

Figure 6: EUA Futures Price Historical Decomposition



Notes: The figure depicts the path of the EUA futures price (black line) broken down into different drivers over the estimated period (2013 – 2019).

Figure 6 illustrates the trajectory of the de-trended EUA futures price from 2013 to 2019, broken down by various influencing factors. The primary drivers of the EUA futures during this period have been abatement shocks, climate sentiment, and, to a lesser extent, energy shocks. In contrast, the other two factors impacting firms' demand for emission permits—goods supply and consumer demand shocks—have contributed less to market volatility. This aligns with expectations, as these factors indirectly affect energy firms' production rather than their emissions directly. Goods Total Factor Productivity (TFP) shocks, for instance, represent typical variations in productivity among final goods firms, such as the adoption of a new manufacturing technique that increases output. Similarly, consumption shocks are indicative of changes in consumer demand patterns, often reflecting shifts in consumer preferences. An example of this could be a heightened environmental awareness due to a climate change campaign, leading to reduced demand for products with high emission footprints. In our model's context, this translates to consumers becoming more patient and deferring consumption, thereby influencing the demand for emission permits.

Thus, EUA prices are primarily influenced by shifts in abatement technology, policy uncertainty, and energy market dynamics. Energy shocks play a crucial role in energy generation, directly impacting the equilibrium energy price. For instance, a significant technological advancement in the energy sector that enhances efficiency in energy production exemplifies a positive energy TFP shock. Empirical studies discussed earlier highlight the strong connection between the ETS market and energy markets, emphasizing the influence of energy shocks.

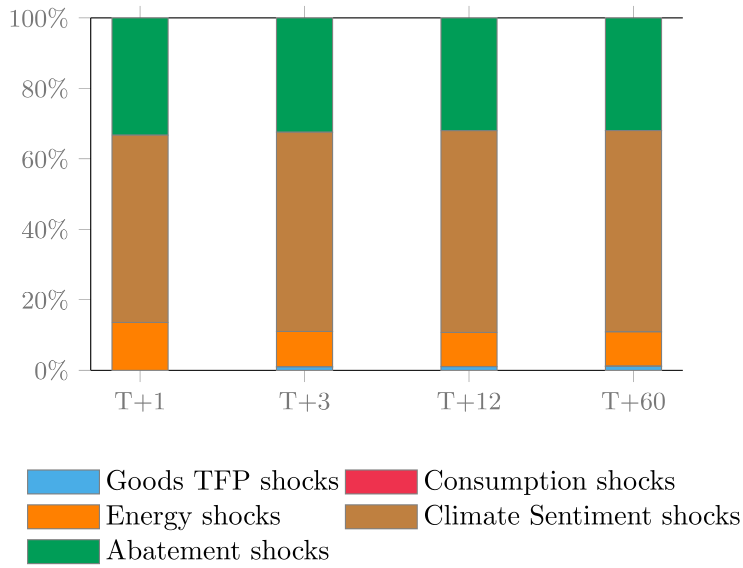
Climate sentiment shocks are driven by changes in the regulatory landscape and shifts in firms' perceptions of policy strictness. As presented in the model section, these shocks reflect the impact of regulatory and policy changes on market sentiment, especially regarding the per-period cap. Our analysis indicates that uncertainty in allowance allocation significantly contributes to the variance in carbon prices. This finding is consistent with the research presented in Koch et al. (2016), Deeney et al. (2016), and more recently, Känzig (2021), which all underscore the substantial role of policy and regulatory shocks in historical EUA price fluctuations.

Finally, abatement shocks, representing unexpected changes in the costs for energy companies to reduce emissions, are also a key factor. These shocks might arise from groundbreaking innovations in low-emission technologies or increased adoption of existing solutions. While such changes are not directly observable in macro data, our innovative approach, as detailed in **??**, enables us to identify these shocks.

Together, these observations effectively bridge the gap between empirical findings and the theory about cap-and-trade, offering a comprehensive view of the factors driving EUA prices.

Figure 7 illustrates the contribution of each driver to the variance of the EUA futures price across various horizons, highlighting the shocks with a prolonged influence on the EU ETS market.

Figure 7: EUA Futures Price Variance Decomposition



Notes: The figure displays the variance decomposition of the EUA futures price based on different horizons: one month, three months, one year, and five years. This represents the theoretical variance decomposition of the permit price, taking into account the estimated variances of shocks.

The three primary factors influencing EUA futures prices–energy TFP, climate sentiment, and abatement shocks–account for a relatively constant proportion of the variance across different time horizons. In the long run, the significance of energy shocks appears to slightly diminish, giving way to climate sentiment and abatement shocks.

6.2 How does our estimated series compare to the actual data?

From our prior analysis, the primary drivers influencing the EUA future price and the overall EU ETS market appear to be climate sentiment and abatement shocks. To ensure the validity of these findings, we cross-referenced them with closely related real-world data to ascertain their accuracy and correlation.

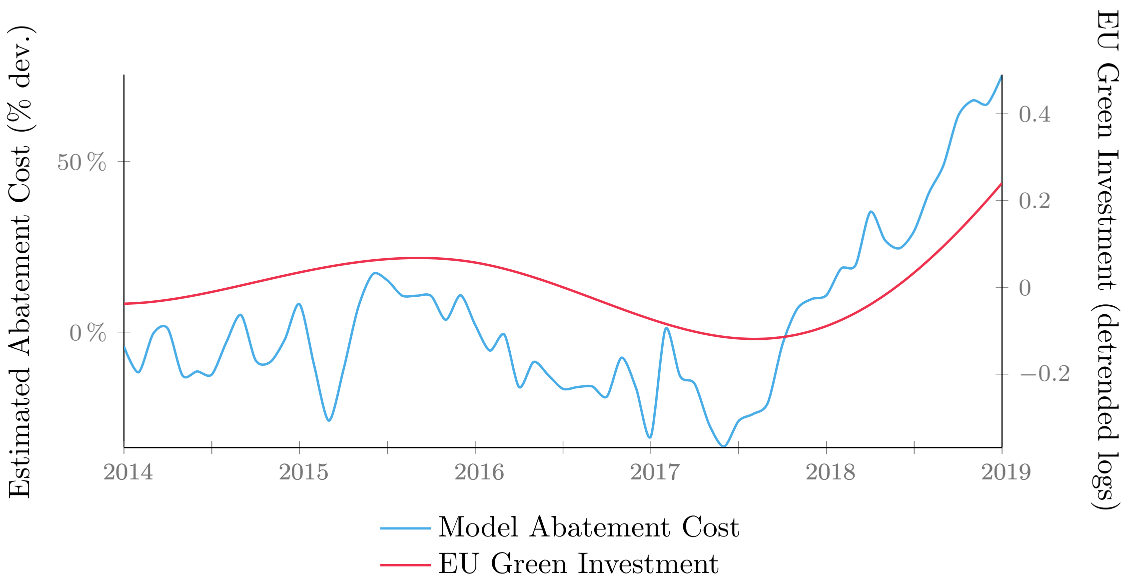
Starting from abatement, we compare our derived estimated series for abatement against the annual data reflecting the EU's net-zero emission total expenditure. It's essential to note that this data primarily showcases the EU's overall commitment to green investments[18] rather than the explicit abatement costs featured in our model.[19] Nevertheless, this com-

[18]Data: Contribution to the international 100bn USD commitment on climate-related expending (source: DG CLIMA, EIONET)

[19]It measures the total amount spent from the annual budget of the EU Member States as well as of the European Commission and the European Investment Bank, in order to contribute to the international 100bn

parison provides a valuable perspective on whether our model's estimations are in sync with the actual green investment expenditures. To facilitate a more detailed comparison, we first transformed the annual data into a monthly format using Cubic Spline Interpolation. This allowed us to align it with our monthly abatement estimation series. Figure 8 presents both the interpolated EU data on total climate mitigation expenditure and our model's estimated abatement investment. Both series exhibit similar trends and business cycle fluctuations.

Figure 8: Estimated Abatement Costs and Climate Mitigation Investment Data



Notes: The figure displays the estimated abatement costs as a deviation of their steady state, alongside the actual data on climate mitigation investment for the EU in detrended log million euros.

Moving on to the climate sentiment, we turned to the US Sentometric data, as presented by Ardia, Bluteau, Boudt, and Inghelbrecht (2022).[20] This data served as a benchmark to gauge the correlation with our deduced climate sentiment shock series. For the estimated climate sentiment/policy shock series, we observe a negative correlation of 14% with the US climate Sentometric index. This correlation suggests that when the climate sentiment index rises, climate concerns intensify, prompting the environmental regulator to tighten the cap

USD commitment for climate finance under the United Nations Framework Convention on Climate Change (UNFCCC).

[20]Sentometrics is a term that refers to the quantitative analysis of sentiment derived from textual data. This approach is often used in finance and economics to analyze sentiment in news articles, financial reports, and other textual sources to predict financial market movements or economic indicators. See Algaba, Ardia, Bluteau, Borms, and Boudt (2020) for a comprehensive literature review.

on emissions. Consequently, this leads to an increase in carbon prices.

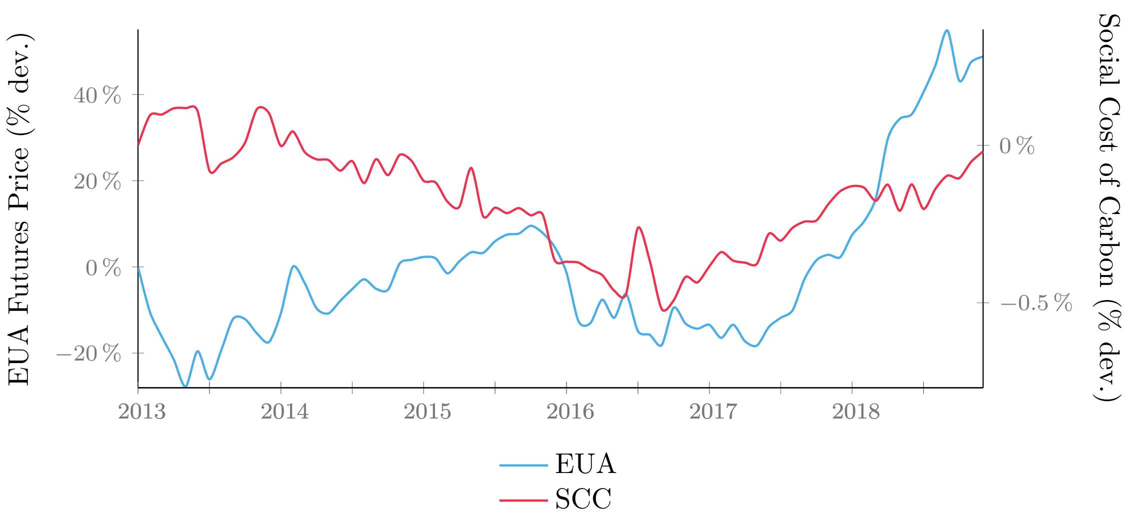
Our findings from these cross-references reveal that our shock estimates align with the characteristics of the actual observed data. This alignment affirms the validity of our analytical approach and the robustness of its results.

6.3 EU ETS cap-and-trade and Optimal Policy

To gauge the extra volatility in the EU ETS market, establishing a baseline is essential. For this, we create a hypothetical scenario where the environmental regulator aligns the carbon price with the estimated SCC, thereby achieving what is considered the first-best allocation in a decentralized economy. This scenario serves as our baseline. In this approach, we utilize the parameters and shock series estimated earlier, replacing our carbon price equation with the SCC.[21] In this hypothetical model, policy uncertainty is non-existent. There are no political interferences or alterations; once a policy is set, it remains unaltered. Consequently, under the SCC framework, unforeseen climate sentiment shifts, which could otherwise influence the market, are effectively nullified. This setup enables us to isolate and assess the extra market volatility in the EU ETS in comparison to a scenario governed by the SCC, where the SCC is responsive to goods supply and consumer demand shocks, energy shocks, and abatement shocks.

[21] The SCC is formally derived in the Appendix.

Figure 9: EUA Futures Price vs SCC Variations



Notes: The figure shows the deviations of the estimated EUA futures price and the SCC in percentage deviations from their respective steady states.

Figure 9 displays percentage deviations from the steady-state for both the estimated EUA futures price and the estimated SCC. Although the trajectories of the EUA futures price and the SCC appear to mirror each other to some extent, the magnitude of the SCC's fluctuations is notably smaller–approximately one hundred times less than that of the EUA futures price. This significant disparity highlights the presence of considerable additional volatility in the EU ETS market when compared to the SCC.

Table 1 contrasts several moments in the estimated EU ETS cap policy case with the counterfactual SCC. By continuously aligning the marginal costs with the marginal benefits of emission reduction, the SCC stabilizes abatement costs and marginal abatement costs, leading to a carbon price that has virtually zero volatility, though at the expense of more fluctuating emissions. Additionally, the SCC tends to result in a higher carbon price level. While a higher carbon price could potentially yield welfare gains for households, it's crucial to acknowledge the substantial uncertainty surrounding the estimation of the social cost of carbon, as discussed in studies like Cai and Lontzek (2019) and Barnett et al. (2020).

Considering the inherent complexity in determining the SCC, it is unlikely that an environmental regulator would rely exclusively on a single estimation of the SCC to set the emission cap. A more pragmatic approach might involve agreeing on a specific emission trajectory through a cap-and-trade market (or, equivalently, a defined path for the carbon price) and then adjusting the number of permit allowances allocated in response to the shocks in the economy. In the subsequent section, we explore the potential of such adaptive carbon cap rules, where the allocation of permit allowances is conditioned on the available information.

6.4 The cost of business cycle fluctuations

Furthermore, the implied uncertainty and price volatility incorporate business cycle welfare costs (Lucas (1987)). We calculate the cost of business cycle fluctuations in consumption equivalence (CE) terms for both the SCC and ETS price cases. To perform this calculation, we first compute the lifetime utility given by the value function of our representative agent for the deterministic case. Subsequently, we compute the value for the parameter Δ such that the welfare in the stochastic framework is equal to the welfare in the deterministic case. We denote the deterministic welfare function by Welfare_t^D and the stochastic counterpart by Welfare_t^S.

$$\text{Welfare}_t^D = u(C_t, D_u(T_t^o)) + \beta E_t\{\text{Welfare}_{t+1}\}$$

$$\text{Welfare}_t^S = u((1-\Delta)C_t, D_u(T_t^o)) + \beta E_t\{\text{Welfare}_{t+1}\}$$

As risk-averse agents dislike fluctuations, welfare in the stochastic economy is lower than what would be obtained in the same economy where the standard deviation of all shocks is set to zero. Therefore, we have $E(\text{Welfare}t^S) < \bar{\text{Welfare}}t^D$.

We then find the necessary compensation needed in terms of CE to close the distance between the welfare in the stochastic case and the deterministic case. In other words, we find the value of Δ such that $E(\text{Welfare}t^S) = \bar{\text{Welfare}}t^D$.

We obtain a value for $|\Delta|$ of 0.01915 in the case where we follow a SCC and value for $|\Delta|$ of 0.01919 in the case where we implement an ETS price. This implies a cost of business cycle fluctuations amounting to .22% in CE term per month between the first best policy and ETS carbon cap.

7 Responsiveness of Carbon Cap Rules

In the preceding section, we highlighted the pronounced volatility in the permit price observed during phase 3 (2013-2019) of the EU ETS market. This volatility is not merely a statistical observation but carries significant real-world implications. It is important to understand that while some degree of volatility is anticipated in any cap-and-trade system, the levels observed in the EU ETS market during this phase were particularly high. Such volatility can be a double-edged sword. On one hand, it can be seen as a reflection of a market's responsiveness to changing conditions, but on the other, it can introduce a level of unpredictability that can be detrimental.

For policymakers, this volatility presents challenges. It makes it difficult to set long-term policies and can undermine the very goals the cap-and-trade system is designed to achieve. If prices are too volatile, firms might hesitate to invest in long-term emission reduction strategies, fearing that the costs might outweigh the benefits if prices swing too widely (Martin et al. (2011), and European Parliament (2022), among others). Market participants, especially firms, bear the brunt of this volatility. Excessive price fluctuations introduce a level of market uncertainty that can be challenging to navigate. Such unpredictability makes it difficult for firms and investors to commit to substantial long-term investments, especially when these investments are potentially irreversible (Calel (2020) and Taschini (2021)). The fear of making a costly mistake due to volatile prices can deter companies from investing in capital-intensive projects or adopting new technologies. This hesitancy can slow down innovation and progress, particularly in sectors where upfront investments are crucial for future decarbonization. Moreover, the inability to accurately forecast returns on these investments due to price volatility can lead to missed opportunities and hinder strategic planning.

Furthermore, as Benmir and Roman (2020) points out, this volatility can seep into the financial markets. When firms face higher risks due to unpredictable carbon prices, it can lead to higher risk premia. This means that firms might face higher borrowing costs, as lenders demand a higher return to compensate for the increased risk. In the long run, this can impact firms' investment decisions, potentially slowing down the transition to greener technologies.

To counteract the uncertainties stemming from energy drivers, abatement innovations, and perceived policy stringency, it is essential to adopt a strategy where the cap is not static but dynamically adjusted over time to reflect these changing factors (Kollenberg and Taschini (2016), Karp and Traeger (2023)).[22] Implementing a conditional supply of permit allowances that allows for a dynamic per-period cap could effectively manage price volatility in the allowance market while keeping emissions on the desired trajectory. To achieve this, we introduce a Carbon Cap Rule (CCR) designed to address the uncertainties primarily

[22] It is widely understood that the cap should be conditioned on the realized shocks (Ellerman and Wing (2003), Jotzo and Pezzey (2007), Newell and Pizer (2008), and Doda (2016)).

associated with two critical factors: innovation in abatement and perceived regulatory uncertainty. The CCR we propose can be likened to the environmental policy equivalent of the Taylor rule, a well-known monetary policy guideline. Just as the Taylor rule provides a formulaic approach for central banks to adjust interest rates in response to changes in economic conditions like inflation and output gaps, the CCR offers a structured method for adjusting emission caps in response to both abatement and sentiment/regulatory shocks. The CCR adjusts the quantity of emission permits (Q_t) in the market. This adjustment is based on deviations from the de-trended steady-state emissions ($\bar{e}^E$) and abatement costs ($\bar{z}$):

$$\mathrm{Q}_t = \overline{\mathrm{Q}} + \phi_e \frac{(e^E_{t-1} - \bar{e}^E)}{\bar{e}^E} + \phi_z \frac{(z_t - \bar{z})}{\bar{z}},$$

where ϕ_e and ϕ_z are coefficients that determine the sensitivity of the cap to changes in emissions and abatement costs, respectively. This approach ensures that the cap is not static but dynamically adjusts in response to abatement and climate sentiment shocks, similar to how the Taylor rule adjusts interest rates based on economic indicators. By responding to these deviations, the CCR aims to align the realization of the marginal costs of emission reduction with the marginal benefits, akin to how the SCC is calculated.[23] We note that the regulatory will be reacting to realized changes in emissions e^E_{t-1} to adjust the cap level.

We show how the shock sensitivity of the CCR differs from that of the SCC curve. To do this, we use the parameters and shock series we previously estimated, replacing the carbon price equation in our model with the CCR formula.[24] We identify the optimal values for the coefficients ϕ_e and ϕ_z that result in the lowest possible standard deviation of the carbon price. We employ a quasi-Newton method to refine our initial guess. The economy's path is simulated to the second order for each parameter pair in the carbon cap rule until the algorithm reaches convergence. Both estimated coefficients, ϕ_e and ϕ_z, are found to be positive. This indicates that in the event of a positive shock to either emissions or abatement

[23] The concept of the proposed CCR draws parallels with the "target-consistent pricing" approach, a notion championed by among others Stern, Stiglitz, Karlsson, and Taylor (2022). This method pivots around the idea that the Social Cost of Carbon should be formulated in a manner that inherently aligns with the objectives set out in the Paris Agreement. Instead of determining the SCC based on estimated damages from an additional ton of carbon dioxide, this approach works backward: it starts with the goals of the Paris Agreement and then calculates the SCC required to achieve those specific targets. This perspective ensures that pricing is consistent with broader climate objectives and should provide a clear policy and, crucially, price signal for the necessary transition.

[24] Note that we retain the climate sentiment shock, even though one could contend that if a carbon cap rule were in place, climate sentiment might not influence the emissions path. Thus, our counterfactual can be viewed as a conservative scenario where some unexplained (residual) volatility in emissions persists.

costs, the regulator's response, as per the CCR, would be to increase the cap.

	ETS Cap Policy Estimated Column (1)	Social Cost of Carbon Optimal Column (2)	Carbon Cap Rule $\phi_z = 1.0467$ and $\phi_e = 0.4439$ Column (3)
Consumption (Std. Dev.)	0.13 %	0.13 %	.13 %
Output - Industrial Prod (Std. Dev.)	.10 %	.10 %	.10 %
Emissions (Std. Dev.)	3.21 %	2.81 %	2.05 %
Abatement Cost (Std. Dev.)	29.15 %	11.01 %	04.62 %
Marginal Abatement Cost (Std. Dev.)	15.51 %	11.17 %	13 .66 %
Carbon Price (in euros)	7.50	7.52	7.53
Carbon Price (Std. Dev.)	18.91 %	0.17 %	6.31 %

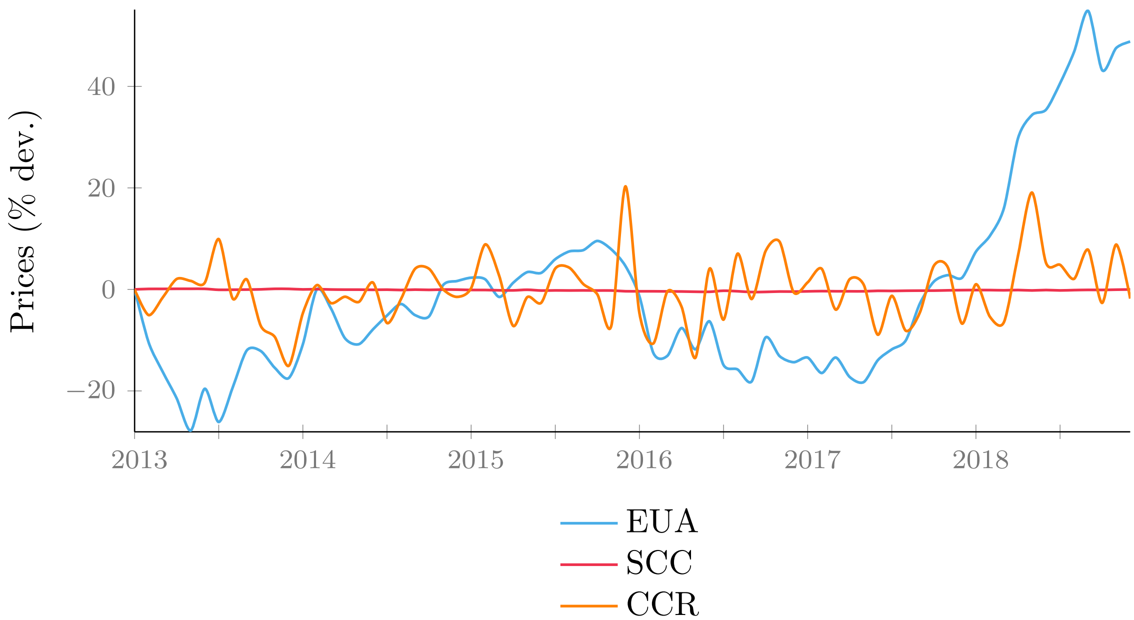
Table 1: Policy Scenarios Estimated Second Moments

Notes: The table reports various moments under three cap policy scenarios. The first column corresponds to the estimated ETS model, the second column corresponds to the Social Cost of Carbon – the optimal case, and the third column corresponds to the Carbon Cap Rule (CCR). The CCR is $\mathrm{Q}_t = \overline{\mathrm{Q}} + \phi_e \frac{(e^E_{t-1} - \bar{e}^E)}{\bar{e}^E} + \phi_z \frac{(z_t - \bar{z})}{\bar{z}}$.

Table 1 presents key statistical moments under three different cap policy scenarios. The first column displays results from the estimated ETS model, the second column represents the SCC (considered the optimal case), and the third column details the outcomes under the CCR. First, regarding macro aggregates, the presence of habits, allows for better matching the standard deviation of consumption with respect to output. Second, our proposed carbon cap rule allows for a reduction of about 34% in price volatility. The CCR demonstrates a significantly stronger response to deviations in steady-state abatement costs as compared to deviations in steady-state emissions. This implies that the rule prioritizes the management of abatement costs over strictly adhering to per-period emission levels. This approach underscores the importance of keeping abatement costs aligned, suggesting that economic feasibility in emission reduction efforts is given precedence over rigid adherence to per-period emission targets.

Considering the CCR's emphasis on managing abatement costs, it's not unexpected to find that the volatility of these costs under the CCR is quite similar to that in the SCC scenario. Moreover, the volatility of the marginal abatement costs under both scenarios is almost identical. Yet, the volatility of emissions under the CCR is about half of what it is under the SCC. In terms of carbon price volatility, the standard deviation under the CCR is higher than that under the SCC. However, it is notably lower compared to the volatility observed in the current ETS model. This suggests that although the CCR does not completely eliminate price volatility, it markedly reduces the extremes of volatility observed in the existing ETS framework, mitigating the unpredictability associated with abatement costs for firms. This effect is observable in Figure 10.

Figure 10: EUA vs SCC vs CCR Variations



Notes: The figure shows the deviations of the estimated EUA futures price, the counterfactual SCC, and the counterfactual CCR in percentage deviations from their respective steady states.

In addition, as demonstrated in Table 2, our proposed carbon cap rule is capable of reducing the business cycle cost related to price volatility and economic uncertainty to approximately 0.03% in comparison to the social cost, as opposed to the 0.22% welfare losses in CE observed in the case of the EU ETS cap concerning the SCC.

	ETS Cap Policy $\lvert\Delta\rvert = 0.0191931$ Column (1)	Social Cost of Carbon $\lvert\Delta\rvert = 0.0191509$ Column (2)	Carbon Cap Rule $\lvert\Delta\rvert = 0.0191574$ Column (3)
Welfare loss (in CE) w.r.t SCC	0.22 %	—	.03 %

Table 2: Business Cycle Welfare Cost

Notes: The table reports welfare business cycle costs of uncertainty for the three cap policy scenarios studied. The first column corresponds to the estimated ETS model, the second column corresponds to the Social Cost of Carbon – the optimal case, and the third column corresponds to the Carbon Cap Rule (CCR). The CCR is $\mathrm{Q}_t = \overline{\mathrm{Q}} + \phi_e \frac{(e^E_{t-1} - \bar{e}^E)}{\bar{e}^E} + \phi_z \frac{(z_t - \bar{z})}{\bar{z}}$.

In summary, our analysis demonstrates that by dynamically adjusting the cap in response to deviations from steady-state values of emissions and abatement costs, the proposed mechanism can effectively reduce carbon price volatility, thereby decreasing overall market uncertainty.

We acknowledge the potential for such a rule to be implemented by a Central Carbon Bank, tasked with overseeing carbon market dynamics (de Perthuis (2011), Pahle and Edenhofer (2021), Blanchard and Tirole (2021)). As a regulatory authority, this entity would manage the supply of EUA allowances, intervening as needed to stabilize prices. While our study does not delve into the specific governance structure of a Carbon Central Bank, we recognize that its primary function of managing the cap could be guided by the principles of the CCR. This approach would ensure a more controlled and predictable carbon market, aligning with broader environmental and economic objectives.

8 Conclusion

Cap and trade systems currently stand as the primary market-based method for regulating greenhouse gas emissions. However, recent years have highlighted significant shortcomings in how these systems react to market fundamental shocks. Some of these shocks are easily observable, like those related to energy dynamics, while others are less well-observable, such as abatement shocks and climate sentiment shocks. In response, we propose a novel approach to estimate less well-observable shocks and we construct a mechanism, the 'Carbon Cap Rule' (CCR), which dynamically adjusts the number of emission permits (the cap) in response to abatement and sentiment shocks, crucial factors influencing emission permit demand. The CCR conceptually parallels the environmental equivalent of the Taylor rule in monetary policy and aligns with practical mechanisms like the Market Stability Reserve in the EU ETS, which also adjusts the cap. Our analysis suggests that the CCR could reduce the volatility of emission permit prices by about 35 percent, resulting in welfare gains of about 85% in CE terms with respect to the SCC scenario. This approach aims to enhance the efficiency and responsiveness of cap and trade systems to market changes.

Drawing on existing empirical research, we employ a panel Vector Autoregressive (VAR) model to examine the response of the EU emission permit price to essential macroeconomic and energy supply factors. Our analysis underscores the pivotal role of energy in influencing emission permit prices. This finding underpins our decision to include energy production as an intermediary input in our macro-model, which is used to assess the impact of less observable factors like abatement and policy changes. We then introduce a new approach for estimating the factors driving the EU ETS. Our innovative methodology allow us to quantify the shocks associated with abatement and climate sentiment/policy changes, enabling us to pinpoint all the critical factors that have been empirically and theoretically affecting the price of emission allowances. We have identified these key factors as abatement shocks, climate sentiment shocks, and energy shocks, each playing a significant role in the dynamics of emission allowance pricing. Subsequently, we contrast the actual emission permit price with a hypothetical situation where the environmental regulator sets the carbon price in line with the Social Cost of Carbon (SCC). The aim of this comparison is to assess the additional volatility present in the EU ETS market compared to the more stable pricing scenario under the SCC. This serves as the baseline for evaluating the effectiveness of the CCR, essentially demonstrating that the proposed mechanism makes the emission permit price in standard cap and trade systems more closely with the SCC.

9 Bibliography

P. Aatola, M. Ollikainen, and A. Toppinen. Price determination in the eu ets market: Theory and econometric analysis with market fundamentals. *Energy Economics*, 36(1):380–395, 2013.

D. Acemoglu, P. Aghion, L. Bursztyn, and D. Hemous. The environment and directed technical change. *American Economic Review*, 102(1):131–66, 2012.

D. Acemoglu, D. Hemous, L. Barrage, P. Aghion, et al. Climate change, directed innovation, and energy transition: The long-run consequences of the shale gas revolution. In *2019 Meeting Papers*, number 1302. Society for Economic Dynamics, 2019.

P. Aghion, A. Dechezleprêtre, D. Hemous, R. Martin, and J. Van Reenen. Carbon taxes, path dependency, and directed technical change: Evidence from the auto industry. *Journal of Political Economy*, 124(1):1–51, 2016.

A. Algaba, D. Ardia, K. Bluteau, S. Borms, and K. Boudt. Econometrics meets sentiment: an overview of methodology and applications. *Journal of Economic Surveys*, 34(3):512–547, 2020.

D. Ardia, K. Bluteau, K. Boudt, and K. Inghelbrecht. Climate change concerns and the performance of green vs. brown stocks. *Management Science*, 2022.

R. Bachmann, D. Baqaee, C. Bayer, M. Kuhn, A. Löschel, B. Moll, A. Peichl, K. Pittel, and M. Schularick. What if? the economic effects for germany of a stop of energy imports from russia. Technical report, ECONtribute Policy Brief, 2022.

M. Barnett, W. Brock, and L. P. Hansen. Pricing uncertainty induced by climate change. *The Review of Financial Studies*, 33(3):1024–1066, 2020.

L. Barrage. Optimal dynamic carbon taxes in a climate–economy model with distortionary fiscal policy. *The Review of Economic Studies*, 87(1):1–39, 2020.

J. A. Batten, G. E. Maddox, and M. R. Young. Does weather, or energy prices, affect carbon prices? *Energy Economics*, 96(1):105–116, 2021.

G. Benmir and J. Roman. Policy Interactions and the Transition to Clean Technology. Working Paper No. 368, Centre for Climate Change Economics and Policy, 2020.

G. Benmir, I. Jaccard, and G. Vermandel. Green asset pricing. Technical report, ECB Working Paper, 2020.

O. Blanchard and J. Tirole. Major future economic challenges. ifo Institute, 2021.

S. Borenstein, J. Bushnell, F. A. Wolak, and M. Zaragoza-Watkins. Expecting the unexpected: Emissions uncertainty and environmental market design. *American Economic Review*, 109(11):3953–77, 2019.

Y. Cai and T. S. Lontzek. The social cost of carbon with economic and climate risks. *Journal of Political Economy*, 127(6):2684–2734, 2019.

R. Calel. Adopt or innovate: Understanding technological responses to cap-and-trade. *American Economic Journal: Economic Policy*, 12(3):170–201, 2020.

J. Y. Campbell and J. H. Cochrane. By force of habit: A consumption-based explanation of aggregate stock market behavior. *Journal of Political Economy*, 107(2):205–251, 1999.

C. de Perthuis. Carbon markets regulation: The case for a co2 central bank. Climate Economics Chair, University Paris-Dauphine, 2011.

P. Deeney, M. Cummins, M. Dowling, and A. F. Smeaton. Influences from the european parliament on eu emissions prices. *Energy Policy*, 88(1):561–572, 2016.

S. Dietz and F. Venmans. Cumulative carbon emissions and economic policy: in search of general principles. *Journal of Environmental Economics and Management*, 96:108–129, 2019.

B. Doda. How to price carbon in good times... and bad! *WIREs Climate Change*, 7(1): 135–144, 2016.

A. Ellerman and I. Wing. Absolute versus intensity-based emission cap. *Climate Policy*, 3 (1):7–20, 2003.

L. G. Epstein and S. E. Zin. Substitution, risk aversion, and the temporal behavior of consumption and asset returns: A theoretical framework. *Econometrica*, 57(4):937–969, 1989.

European Parliament. The role of financial operators in the ets market and the incidence of their activities in determining the allowances' price. European Parliament's Committee on Industry, Research and Energy (ITRE)., 2022.

C. Fischer and M. Springborn. Emissions targets and the real business cycle: Intensity targets versus caps or taxes. *Journal of Environmental Economics and Management*, 62 (3):352–366, 2011.

M. Fowlie. Allocating emissions permits in cap-and-trade programs: Theory and evidence. *University of California, Berkeley*, 2010.

M. Fowlie and J. M. Perloff. Distributing pollution rights in cap-and-trade programs: are outcomes independent of allocation? *Review of Economics and Statistics*, 95(5):1640–1652, 2013.

M. Friedrich, E.-M. Mauer, M. Pahle, and O. Tietjen. From fundamentals to financial assets: the evolution of understanding price formation in the eu ets. 2020.

M. Golosov, J. Hassler, P. Krusell, and A. Tsyvinski. Optimal taxes on fossil fuel in general equilibrium. *Econometrica*, 82(1):41–88, 2014.

L. H. Goulder. Markets for pollution allowances: What are the (new) lessons? *Journal of economic perspectives*, 27(1):87–102, 2013.

G. Grosjean, W. Acworth, C. Flachsland, and R. Marschinski. After monetary policy, climate policy: is delegation the key to eu ets reform? *Climate Policy*, 16(1):1–25, 2016.

G. Heutel. How should environmental policy respond to business cycles? Optimal policy under persistent productivity shocks. *Review of Economic Dynamics*, 15(2):244–264, 2012.

B. Hintermann, S. Peterson, and W. Rickels. Price and market behavior in phase ii of the eu ets: A review of the literature. *Review of Environmental Economics and Policy*, 2016.

M. Hoel and L. Karp. Taxes versus quotas for a stock pollutant. *Resource and Energy Economics*, 24(4):367–384, 2002.

J. D. Jenkins. Political economy constraints on carbon pricing policies: What are the implications for economic efficiency, environmental efficacy, and climate policy design? *Energy Policy*, 69:467–477, 2014.

F. Jotzo and J. Pezzey. Optimal intensity targets for greenhouse gas emissions trading under uncertainty. *Environmental and Resource Economics*, 35(1):259–299, 2007.

D. R. Känzig. The unequal economic consequences of carbon pricing. *Available at SSRN 3786030*, 2021.

L. Karp and C. Traeger. Smart cap. *CESifo*, 2023.

L. S. Karp and C. P. Traeger. Prices versus quantities reassessed. Technical report, CESifo Working Paper, 2018.

N. Koch, G. Grosjean, S. Fuss, and O. Edenhofer. Politics matters: Regulatory events as catalysts for price formation under cap-and-trade. *Journal of Environmental Economics and Management*, 78(1):121–139, 2016.

S. Kollenberg and L. Taschini. Emissions trading systems with cap adjustments. *Journal of Environmental Economics and Management*, 80(1):20–36, 2016.

X. Labandeira, J. M. Labeaga, and X. López-Otero. A meta-analysis on the price elasticity of energy demand. *Energy policy*, 102:549–568, 2017.

R. E. Lucas. Models of business cycles. *(Oxford: Blackwell)*, 1987.

R. Martin, M. Muuls, and U. Wagner. Carbon markets, carbon prices and innovation: Evidence from interviews with managers. London School of Economics, 2011.

H. D. Matthews, N. P. Gillett, P. A. Stott, and K. Zickfeld. The proportionality of global warming to cumulative carbon emissions. *Nature*, 459(7248):829–832, 2009.

W. D. Montgomery. Markets in licenses and efficient pollution control programs. *Journal of economic theory*, 5(3):395–418, 1972.

R. G. Newell and W. A. Pizer. Regulating stock externalities under uncertainty. *Journal of environmental economics and management*, 45(2):416–432, 2003.

R. G. Newell and W. A. Pizer. Indexed regulation. *Journal of environmental economics and management*, 56(2):221–233, 2008.

R. G. Newell, A. B. Jaffe, and R. N. Stavins. The induced innovation hypothesis and energy-saving technological change. *Quarterly Journal of Economics*, 114(3):941–975, 1999.

W. Nordhaus. A question of balance: Weighing the options on global warming policies. Yale university press. *New Haven, CT*, 2008.

W. D. Nordhaus. To slow or not to slow: the economics of the greenhouse effect. *Economic Journal*, 101(407):920–937, 1991.

W. D. Nordhaus. Revisiting the social cost of carbon. *Proceedings of the National Academy of Sciences*, 114(7):1518–1523, 2017.

W. D. Nordhaus and Z. Yang. A regional dynamic general-equilibrium model of alternative climate-change strategies. *American Economic Review*, pages 741–765, 1996.

M. Pahle and O. Edenhofer. Discretionary intervention destabilizes the eu emissions trading system: Evidence and recommendations for a rule-based cap adjustment. ifo Institute, 2021.

A. Pommeret and K. Schubert. Intertemporal emission permits trading under uncertainty and irreversibility. *Environmental and Resource Economics*, 71:73–97, 2018.

W. Rickels, D. Görlich, and S. Peterson. Explaining european emission allowance price dynamics: Evidence from phase ii. *German Economic Review*, 16(2):181–202, 2015.

J. D. Rubin. A model of intertemporal emission trading, banking, and borrowing. *Journal of environmental economics and management*, 31(3):269–286, 1996.

S. Schennach. The economics of pollution permit banking in the context of title iv of the 1990 clean air act amendments. *Journal of Environmental Economics and Management*, 40(1):189–210, 2000.

K. Schubert. Macroeconomics and the environment. *Revue de l'OFCE*, (3):117–132, 2018.

F. Smets and R. Wouters. Shocks and frictions in US business cycles: A bayesian DSGE approach. *American Economic Review*, 97(3):586–606, 2007.

N. Stern. The economics of climate change. *American Economic Review*, 98(2):1–37, 2008.

N. Stern, J. Stiglitz, K. Karlsson, and C. Taylor. A social cost of carbon consistent with a net-zero climate goal. *Roosvelt Institute*, 2022.

L. Taschini. Flexibility premium of emissions permits. *Journal of Economic Dynamics and Control*, 126(1):1–15, 2021.

J. B. Taylor. Discretion versus policy rules in practice. In *Carnegie-Rochester conference series on public policy*, volume 39, pages 195–214. Elsevier, 1993.

M. L. Weitzman. Prices vs. quantities. *The review of economic studies*, 41(4):477–491, 1974.

P. J. Wood and F. Jotzo. Price floors for emissions trading. *Energy Policy*, 39(3):1746–1753, 2011.

Appendix A Model Calibration and Estimation

Table 3: Parameters Value

Parameter	Value	Definition
σ^U	1.5	Risk Aversion
β	0.9986	Discount Factor
α^E	0.33	Elasticity to Capital Input in Energy Production
α^{NE}	0.33	Elasticity to Capital Input in Non-Energy Production
χ	0.02	Share of Energy in the CES
σ	0.20	Substitution Parameter in the CES
δ	0.0083	Depreciation of Energy and Non-Energy Capital
φ^E	0.0055	Emission Intensity in Energy Production
φ^{NE}	0.0002	Emission Intensity in Non-Energy Production
Θ^T	26.29	Dis-utility Sensitivity to Temperature
η	0.0004	Decay Rate of Emissions in the Atmosphere
ζ_1^o	0.50	Climate Transient Parameter
ζ_2^o	0.00125	Climate Transient Parameter
θ_1	0.239	Level of the Abatement Cost Function
θ_2	2.7	Curvature of the Abatement Cost Function
$\frac{\bar{g}}{\bar{y}}$	0.22	Government Spending to Output Ratio

Table 4: Moments matching

Variable	Label	Target	Source
ETS Mean Carbon Price (euros)	τ	7.54	ICE
Cumulative Emission (World, GtC)	X	800	Copernicus (EC)
Monthly Emission Flow (World, GtCO2)	$E^T + E^*$	4.51	Ourworldindata
Share of EU27 in World Emissions (%)	$E^T/(E^T + E^*)$	6.73	Ourworldindata
Share of Emissions from Energy Generation in the EU (%)	E^E/E^T	33.56	OECD
Emission intensity in the EU (kCO2 / euros)	E^T/Y	0.20	OECD
Emission intensity from Energy Generation in the EU (kCO2 / euros)	E^E/Y	0.07	OECD
Abatement level (percentage of energy emissions)	μ	0.20	EDGAR (EC)
Temperature	T^o	1.00	NOAA

Notes: All the values reported in this table are perfectly matched by the model at the steady state.

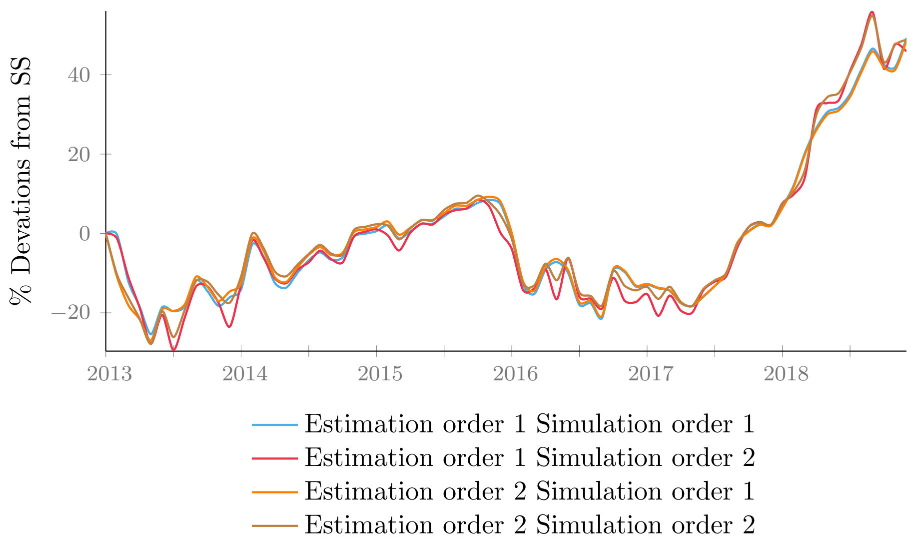
Table 5: Estimated Parameters

		Prior Distributions			Posterior Distributions	
		Distribution	Mean	Std. Dev.	Mean	[0.05 ; 0.95]
Shock processes:						
Std. Dev. Goods Productivity	σ_A	$\mathcal{IG}_2$	0.10	0.05	0.02	[0.01 ; 0.03]
Std. Dev. Energy Productivity	σ_{A_n}	$\mathcal{IG}_2$	0.10	0.05	0.01	[0.01 ; 0.02]
Std. Dev. Energy Price	σ_p	$\mathcal{IG}_2$	0.10	0.05	0.08	[0.05 ; 0.10]
Std. Dev. Climate Sentiment	σ_{CS}	$\mathcal{IG}_2$	0.10	0.05	0.03	[0.01 ; 0.04]
Std. Dev. Consumption	σ_B	$\mathcal{IG}_2$	0.10	0.05	0.07	[0.02 ; 0.12]
Std. Dev. Abatement Cost	σ_Z	$\mathcal{IG}_2$	0.10	0.05	0.18	[0.09 ; 0.30]
AR(1) Goods Productivity	ρ_A	$\mathcal{B}$	0.30	0.10	0.41	[0.08 ; 0.68]
AR(1) Energy Productivity	ρ_{A_n}	$\mathcal{B}$	0.30	0.10	0.29	[0.01 ; 0.43]
AR(1) Energy Price	ρ_p	$\mathcal{B}$	0.30	0.10	0.47	[0.20 ; 0.70]
AR(1) Climate Sentiment	ρ_{CS}	$\mathcal{B}$	0.30	0.10	0.60	[0.21 ; 0.88]
AR(1) Consumption	ρ_C	$\mathcal{B}$	0.30	0.10	0.26	[0.03 ; 0.55]
AR(1) Abatement Cost	ρ_Z	$\mathcal{B}$	0.30	0.10	0.53	[0.14 ; 0.82]
Structural Parameters:						
TFP Trend	$(\gamma^y - 1) \times 100$	$\mathcal{U}$	0.00	0.28	0.13	[-0.14 ; 0.28]
Emissions Trend	$(\gamma^x - 1) \times 100$	$\mathcal{U}$	0.00	0.13	-0.07	[-0.29 ; -0.27]

Notes: $\mathcal{IG}_2$ denotes the Inverse Gamma distribution (type 2), $\mathcal{B}$ the Beta distribution, and $\mathcal{N}$ the Gaussian distribution.

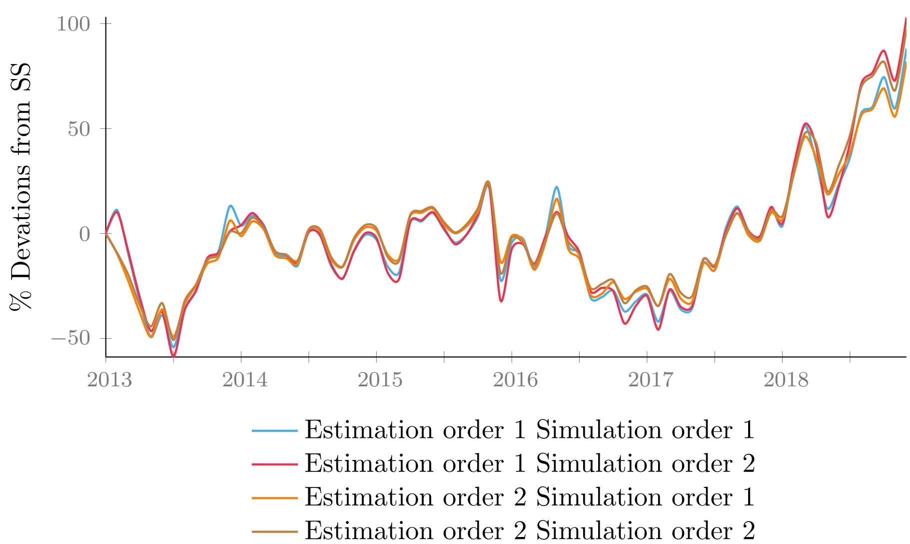
Appendix B Order of Estimation

Figure 11: EUA Futures Pathway



Notes: The figure shows the path of the ETS carbon price (2013 – 2019) when estimated using 1st order and second order estimations as well as when simulating a model using first and second orders.

Figure 12: Abatement Cost Pathway

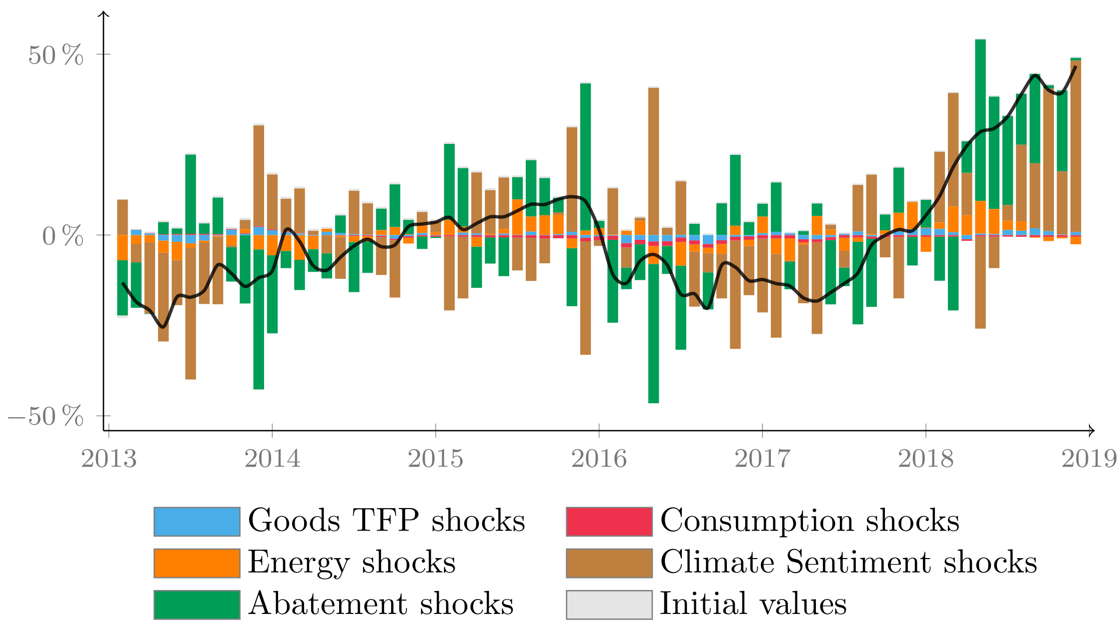


Notes: The figure shows the path of abatement cost carbon price (2013 – 2019) when estimated using 1st order and second order estimations as well as when simulating a model using first and second orders.

Appendix C Case of Production Damages

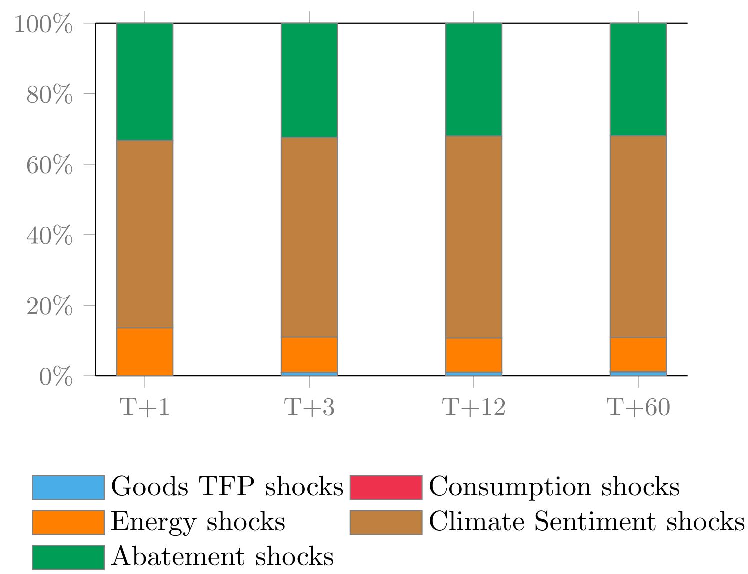
In this section we presents the 2nd order estimation results where climate damages are modeled following a production damages framework à la Nordhaus. We note that the results are sensitively similar to the climate diutility case. Figure 13, figure 14, and figure 15/ presents the shock decomposition, variance decomposition, and the comparison between the SCC and EU ETS.

Figure 13: EUA Futures Price Historical Decomposition



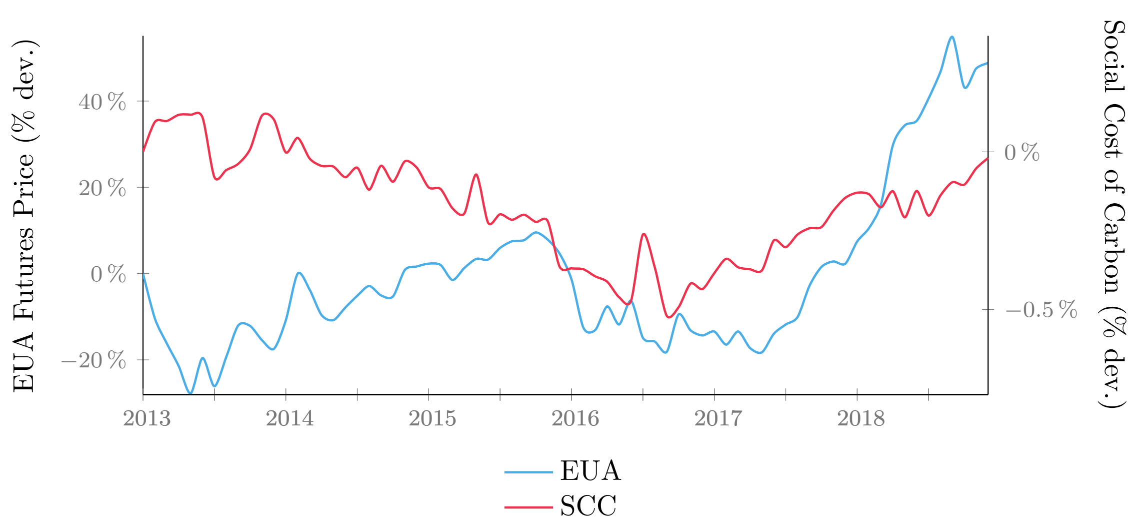
Notes: The figure depicts the path of the EUA futures price (black line) broken down into different drivers over the estimated period (2013 – 2019).

Figure 14: EUA Futures Price Variance Decomposition



Notes: The figure displays the variance decomposition of the EUA futures price based on different horizons: one month, three months, one year, and five years. This represents the theoretical variance decomposition of the permit price, taking into account the estimated variances of shocks.

Figure 15: EUA Futures Price vs SCC Variations



Notes: The figure shows the deviations of the estimated EUA futures price and the SCC in percentage deviations from their respective steady states.

Appendix D Balanced Growth Path

To carry out our structural parameters estimation via Bayesian estimation, we first need to specify the de-trended economy along its balanced growth path.

The growth rate of Γ_t^Y dictates the economy's growth rate on the balanced growth path.[25] This growth rate is denoted by γ^Y, where:

$$\Gamma_t^Y = \gamma^Y \Gamma_{t-1}^Y \tag{19}$$

Variables that are stationary are represented by lowercase letters, while those that are growing are indicated by uppercase letters. For instance, in the expanding economy, the final firm output, the non-energy output (intermediate variable), and non-energy output are denoted by Y_t, Y_t^{NE} and Y_t^{E}, resepctively. To obtain the de-trended output, one must divide the output in the growing economy by the level of growth progress:

$$y_t = \frac{Y_t}{\Gamma_t^Y} \tag{20}$$

$$y_t^{\mathrm{NE}} = \frac{Y_t^{\mathrm{NE}}}{\Gamma_t^Y} \tag{21}$$

$$\tilde{y}_t^{\mathrm{E}} = \frac{\tilde{Y}_t^{\mathrm{E}}}{\Gamma_t^Y \Gamma_t^E} \tag{22}$$

$$y_t^{\mathrm{E}} = \frac{Y_t^{\mathrm{E}}}{\Gamma_t^Y} \tag{23}$$

where $\Gamma_t^Y \Gamma_t^E = 1$ (given that energy production in EU is not growing over the studied period). In the growing economy, emissions from the energy sector are represented by E_t and are defined as follows:

$$E_t^{\mathrm{E}} = (1 - \mu_t)\varphi_{\mathrm{E}} Y_t^{\mathrm{E}} \Gamma_t^X \tag{24}$$

$$E_t^{\mathrm{NE}} = \varphi_{\mathrm{NE}} Y_t^{\mathrm{NE}} \Gamma_t^X \tag{25}$$

where Γ_t^X represents the decoupling of CO_2 emissions relative to the output trend. Consequently, in the de-trended economy, the law of motion for CO_2 emissions is expressed

[25] In our setup both final firms and non-energy firms grow at the identical rate Γ_t^Y.

as:

$$e_t^E = (1-\mu_t)\varphi_{\mathrm{E}} y_t^{\mathrm{E}} \tag{26}$$

$$e_t^{\mathrm{NE}} = \varphi_{\mathrm{NE}} y_t^{\mathrm{NE}} \tag{27}$$

where:

$$e_t^E = \frac{E_t^E}{\Gamma_t^Y \Gamma_t^X} \tag{28}$$

$$e_t^{\mathrm{NE}} = \frac{E_t^{\mathrm{NE}}}{\Gamma_t^Y \Gamma_t^X} \tag{29}$$

The abatement cost in the growing economy is:

$$Z_t = f(\mu_t) Y_t^{\mathrm{E}} \tag{30}$$

In the de-trended economy, the abatement cost is represented as:[26]

$$z_t = f(\mu_t) y_t^{\mathrm{E}} \tag{31}$$

where $z_t = \frac{Z_t}{\Gamma_t^Y}$.

In this context, X_t denotes the cumulative emissions in the atmosphere, while the temperature in the growing economy is represented by T_t^o:

$$X_{t+1} = \eta X_t + E_t^T + E_t^* \tag{32}$$

$$T_{t+1}^o = \zeta_1(\zeta_2 X_t - T_t^o) + T_t^o \tag{33}$$

The de-trended X_t and T_t^o read as follows:

$$\gamma^s x_{t+1} = \eta x_t + e_t^T + e^* \tag{34}$$

$$\gamma^s t_{t+1}^o = \zeta_1(\zeta_2 x_t - t_t^o) + t_t^o \tag{35}$$

[26]Note that μ_t is stationary.

where:

$$x_t = \frac{X_t}{\Gamma_t^Y \Gamma_t^X} \tag{36}$$

$$t_t^o = \frac{T_t^o}{\Gamma_t^Y \Gamma_t^X} \tag{37}$$

with $\gamma^s = \gamma^y \gamma^x$.

In the growing economy, given the aforementioned growth progress, the production functions for both the energy and non-energy sectors are defined as follows:

$$\tilde{Y}_t^{\mathrm{E}} = \varepsilon_t^{A^{\mathrm{E}}} A_t^{\mathrm{E}} (K_t^{\mathrm{E}})^{\alpha_{\mathrm{E}}} (\Gamma_t^Y l_t^{\mathrm{E}})^{1-\alpha_{\mathrm{E}}} \Gamma_t^E \tag{38}$$

$$Y_t^{\mathrm{NE}} = \varepsilon_t^{A^{\mathrm{NE}}} A_t^{\mathrm{NE}} (K_t^{\mathrm{NE}})^{\alpha_{\mathrm{E}}} (\Gamma_t^Y l_t^{\mathrm{NE}})^{1-\alpha_{\mathrm{NE}}} \tag{39}$$

$$Y_t = \left((1-\chi)^{\frac{1}{\sigma}} (Y_t^{\mathrm{NE}})^{\frac{\sigma-1}{\sigma}} + \chi^{\frac{1}{\sigma}} Y_t^{\mathrm{E}\,\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} \tag{40}$$

where, for both energy and non-energy labor $l_t^{\mathrm{E}}, l_t^{\mathrm{NE}}$, the technology shocks $\varepsilon_t^{A^{\mathrm{E}}}, \varepsilon_t^{A^{\mathrm{NE}}}$ as well as the TFP levels A_t^{E} and A_t^{NE}, are all stationary variables. Additionally, in the robustness exercise we consider the climate damage function which incorporates the growth rate Γ_t^y such that $d(T_t^o) = ae^{-b_t (T_t^o)^2} = e^{-\frac{b}{\Gamma_t^2}(T_t^o)^2}$. By embedding the economy's growth rate within the damage function, we can simplify the de-trended form of the damage function without sacrificing generality, especially over the studied period (less than a 7 year horizon).

De-trending the production functions gives the following:

$$\tilde{y}_t^{\mathrm{E}} = \varepsilon_t^{A^{\mathrm{E}}} A_t^{\mathrm{E}} (k_t^{\mathrm{E}})^{\alpha_{\mathrm{E}}} (l_t^{\mathrm{E}})^{1-\alpha_{\mathrm{E}}} \tag{41}$$

$$y_t^{\mathrm{NE}} = \varepsilon_t^{A^{\mathrm{NE}}} A_t^{\mathrm{NE}} (k_t^{\mathrm{NE}})^{\alpha_{\mathrm{E}}} (l_t^{\mathrm{NE}})^{1-\alpha_{\mathrm{NE}}} \tag{42}$$

$$y_t = \left((1-\chi)^{\frac{1}{\sigma}} (y_t^{\mathrm{NE}})^{\frac{\sigma-1}{\sigma}} + \chi^{\frac{1}{\sigma}} (y_t^{\mathrm{E}})^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} \tag{43}$$

The capital-accumulation equations for both the energy and non-energy sectors in the growing economy read as:

$$K_{t+1}^{\mathrm{E}} = (1-\delta) K_t^{\mathrm{E}} + I_t^{\mathrm{E}} \tag{44}$$

$$K_{t+1}^{\mathrm{NE}} = (1-\delta) K_t^{\mathrm{NE}} + I_t^{\mathrm{NE}} \tag{45}$$

In the de-trended economy, we thus have:

$$\gamma^y k^{\mathrm{E}}_{t+1} = (1-\delta)k^{\mathrm{E}}_t + i^{\mathrm{E}}_t \tag{46}$$

$$\gamma^y k^{\mathrm{NE}}_{t+1} = (1-\delta)k^{\mathrm{NE}}_t + i^{\mathrm{NE}}_t \tag{47}$$

with both capital and investment de-trended variables reading as: $k^{\mathrm{NE}}_t = \frac{K^{\mathrm{NE}}_t}{\Gamma^y_t}$ and $i^{\mathrm{NE}}_t = \frac{I^{\mathrm{NE}}_t}{\Gamma^y_t}$, respectively. The same applies for the energy sector.

Moreover, the economy's resource constraint is:

$$y_t = c_t + g_t + f(\mu_t)y^{\mathrm{E}}_t + i^{\mathrm{E}}_t + i^{\mathrm{NE}}_t \tag{48}$$

Finally, in the growing economy, the utility function is as follow:

$$\sum_{t=0}^{\infty} \beta^t \left(\frac{(C_t - hC_{t-1} - \Theta^T_t T^o_t)^{1-\sigma}}{1-\sigma} \right) \tag{49}$$

where C_t is consumption, β the subjective discount factor, and σ the curvature parameter. The de-trended utility function takes the following form:

$$\sum_{t=0}^{\infty} \tilde{\beta}^t \left(\frac{(c_t - \tilde{h}c_{t-1} - \Theta^T t^o_t)^{1-\sigma}}{1-\sigma} \right) \tag{50}$$

where we denote $\tilde{\beta} = \beta(\gamma^y)^{1-\sigma}$, $\tilde{h} = h(\gamma^y)^{-1}$, and $\Theta^T = \Gamma^X_t \Theta^T_t$.

Appendix E The Social Planner Equilibrium: Centralized Economy

The social planner's optimal allocation and plan would aim to maximize the welfare of the society. This is achieved by selecting a sequence of allocations, given the initial conditions for the endogenous state variables, that adhere to the economy's constraints. This equilibrium serves as a benchmark solution, which we use to compare with the allocation derived in the decentralized economy for the carbon policy.

Please note, for ease of presentation, we show both cases of production damages and utility damages within the same framework. We then set the parameter $b = 0$ to eliminate production damages, and similarly we set $\tilde{\Theta}^T = 0$ to revert back to the standard Nordhaus framework with production damages. The planners' problem reads as follows:

$$
\begin{aligned}
\mathcal{L} = E_0 \sum_{t=0}^{\infty} \tilde{\beta}^t \Bigg(& \frac{(c_t - \tilde{h}c_{t-1} - \Theta^T t_t^o)^{1-\sigma^U}}{1-\sigma^U} \\
& + \lambda_t (y_t - c_t - i_t^E - i_t^{\mathrm{NE}} - g_t - f(\mu_t) y_t^E) \\
& + \lambda_t q_t^{\mathrm{NE}} ((1-\delta) k_t^{\mathrm{NE}} + i_t^{\mathrm{NE}} - \gamma^y k_{t+1}^{\mathrm{NE}}) \\
& + \lambda_t q_t^{\mathrm{E}} ((1-\delta) k_t^{\mathrm{E}} + i_t^{\mathrm{E}} - \gamma^y k_{t+1}^{\mathrm{E}}) \\
& + \lambda_t \Psi_t^{\mathrm{NE}} (\varepsilon_t^{A_{\mathrm{NE}}} e^{-b(t_t^o)^2} A^{\mathrm{NE}} (k_t^{\mathrm{NE}})^{\alpha_{\mathrm{NE}}} (l_t^{\mathrm{NE}})^{1-\alpha_{\mathrm{NE}}} - y_t^{\mathrm{NE}}) \\
& + \lambda_t \Psi_t^{\mathrm{E}} (\varepsilon_t^{A_{\mathrm{E}}} A^{\mathrm{E}} (k_t^{\mathrm{E}})^{\alpha_{\mathrm{E}}} (l_t^{\mathrm{E}})^{1-\alpha_{\mathrm{E}}} - y_t^{\mathrm{E}}) \\
& + \lambda_t \Psi_t (((1-\chi)^{\frac{1}{\sigma}} (y_t^{\mathrm{NE}})^{\frac{\sigma-1}{\sigma}} + \chi^{\frac{1}{\sigma}} y_t^{\mathrm{E}\,\frac{\sigma-1}{\sigma}})^{\frac{\sigma}{\sigma-1}} - y_t) \\
& + \lambda_t V_t^X (\gamma^s x_{t+1} - \eta x_t - e_t^E - e_t^{\mathrm{NE}} - e^*) \\
& + \lambda_t V_t^T (\gamma^s t_{t+1}^o - \upsilon_1^o (\upsilon_2^o x_t - t_t^o) - t_t^o) \\
& + \lambda_t V_t^{E^{\mathrm{E}}} (e_t^{\mathrm{E}} - (1-\mu_t) \varphi_{\mathrm{E}} y_t^{\mathrm{E}} \\
& + \lambda_t V_t^{E^{\mathrm{NE}}} (e_t^{\mathrm{NE}} - \varphi_{\mathrm{NE}} y_t^{\mathrm{NE}}) \Bigg)
\end{aligned}
$$

where, as we will demonstrate below, the Social Cost of Carbon SCC_t represents the shadow value associated with the temperature damages $\S_T^t$.

The first order conditions (FOCs) that determine SCC_t are the FOC with respect to CO_2 energy emissions e_t^E. In combination with the FOCs with respect to t_t^o and x_t we can pin down the expression of the SCC. Meanwhile, the FOC with respect to μ_t determine the required level of abatement:

$$V_t^{E^E} = V_t^X \tag{51}$$

$$\gamma^s V_t^T = \tilde{\beta} E_t \{ \Lambda_{t+1} ((1-\zeta_1) V_{t+1}^T + \Theta^T + \frac{\partial y_{t+1}^{\mathrm{NE}}}{\partial t_{t+1}^o} \Psi_{t+1}^{\mathrm{NE}}) \} \tag{52}$$

$$\gamma^s V_t^X = \tilde{\beta} E_t \{ \Lambda_{t+1} (\zeta_1 \zeta_2 V_{t+1}^T + \eta V_{t+1}^X) \tag{53}$$

$$f'(\mu_t) = \varphi_{\mathrm{NE}} V_t^E \tag{54}$$

The remaining of the FOCs will be presented in the decentralized economy.

Appendix F The Decentralized Economy

F.1 Households

Households maximize utility over consumption subject to their budget constraint. They choose consumption expenditures and holdings of government bonds,pay taxes and receive dividends for firms they own.

$$\max_{\{c_t, b_{t+1}\}} E_0 \sum_{t=0}^{\infty} \tilde{\beta}^t \frac{(c_t - \tilde{h}c_{t-1} - \Theta^T t_t^o)^{1-\sigma^U}}{1-\sigma^U}$$

s.t.

$$w_t^{\mathrm{NE}} l_t^{\mathrm{NE}} + w_t^{\mathrm{E}} l_t^{\mathrm{E}} + r_t b_t + \Pi_t^E + \Pi_t^F - t_t = c_t + b_{t+1}$$

From the FOCs, we get:

$$\lambda_t = \varepsilon_t^B \left(c_t - \tilde{h}c_{t-1} - \Theta^T t_t^o\right)^{-\sigma^U} - \varepsilon_{t+1}^B \tilde{\beta}\tilde{h} \left(c_{t+1} - \tilde{h}c_t - \Theta^T t_{t+1}^o\right)^{-\sigma^U} \tag{55}$$

$$\tilde{\beta} r_{t+1} \Lambda_{t+1} = 1 \tag{56}$$

where we note $\Lambda_t = \frac{\lambda_t}{\lambda_{t-1}}$.

F.2 Energy Firms

Energy producers maximize profits by choosing capital investment and labour wages, as well as the investment in abatement as the regulator imposes a carbon price on their level of emissions. The production technology is a Cobb-Douglas, while the abatement investment is a convex function on abatement levels. Capital depreciates and follows a standard law of motion.

The firms' problem reads:

$$\max_{\{y_t^{\mathrm{E}}, i_t^{\mathrm{E}}, \mu_t\}} E_0 \sum_{t=0}^{\infty} \tilde{\beta}^t \Lambda_{t+1} \Pi_t^{\mathrm{E}}$$

where

$$\Pi_t^{\mathrm{E}} = \varepsilon_t^p p_t^{\mathrm{E}} y_t^{\mathrm{E}} - w_t^{\mathrm{E}} l_t^{\mathrm{E}} - i_t^{\mathrm{E}} - f(\mu_t)\, y_t^{\mathrm{E}} - e_t^E \tau_t$$

s.t.

$$y_t^{\mathrm{E}} = \varepsilon_t^{A_{\mathrm{E}}} A^{\mathrm{E}} (k_t^{\mathrm{E}})^{\alpha_{\mathrm{E}}} (l_t^{\mathrm{E}})^{1-\alpha_{\mathrm{E}}}$$
$$e_t^{E} = (1-\mu_t)\varphi_{\mathrm{E}} y_t^{\mathrm{E}}$$
$$\gamma^y k_{t+1}^{\mathrm{E}} = (1-\delta) k_t^{\mathrm{E}} + i_t^{\mathrm{E}}$$

The FOCs with respect to capital, investment, labour, abatement, and energy output read as:

$$q_t^{\mathrm{E}} \gamma^y = \tilde{\beta} \Lambda_{t+1} q_{t+1}^{\mathrm{E}} \left((1-\delta) + \alpha_{\mathrm{E}} \Psi_{t+1}^{\mathrm{E}} \frac{y_{t+1}^{\mathrm{E}}}{k_{t+1}^{\mathrm{E}}} \right) \tag{57}$$
$$q_t^{\mathrm{E}} = 1 \tag{58}$$
$$w_t^{\mathrm{E}} = (1-\alpha_{\mathrm{E}}) \frac{y_t^{\mathrm{E}}}{l_t^{\mathrm{E}}} \tag{59}$$
$$f'(\mu_t) = \varphi_{\mathrm{E}} \tau_t \tag{60}$$
$$\Psi_t^{\mathrm{E}} = p_t^{\mathrm{E}} - (\theta_1 \mu_t^{\theta_2} + \tau_t (1-\mu_t) \varphi_{\mathrm{E}}) \tag{61}$$

where we denote Ψ_t^{E} and q_t^n the Lagrange multipliers associated with production inputs and investment.

F.3 Non-energy final firms

Non-energy producers maximize profits:

$$\Pi_t^F = y_t - w_t^{\mathrm{NE}} l_t^{\mathrm{NE}} - i_t^{\mathrm{NE}} - \varepsilon_t^p p_t^{\mathrm{E}} y_t^{\mathrm{E}}.$$

s.t.

$$y_t = \left((1-\chi)^{\frac{1}{\sigma}} (y_t^{\mathrm{NE}})^{\frac{\sigma-1}{\sigma}} + \chi^{\frac{1}{\sigma}} y_t^{\mathrm{E}\,\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \tag{62}$$
$$y_t^{\mathrm{NE}} = \varepsilon_t^{A_{\mathrm{NE}}} A^{\mathrm{NE}} (k_t^{\mathrm{NE}})^{\alpha_{\mathrm{NE}}} (l_t^{\mathrm{NE}})^{1-\alpha_{\mathrm{NE}}} \tag{63}$$
$$\gamma^y k_{t+1}^{\mathrm{NE}} = (1-\delta) k_t^{\mathrm{NE}} + i_t^{\mathrm{NE}} \tag{64}$$

The FOCs with respect to capital, investment, labour, energy, and non-energy production, yield the factor prices:

$$q_t^{\text{NE}}\gamma^{\text{NE}} = \tilde{\beta}\Lambda_{t+1}q_{t+1}^{\text{NE}}\left((1-\delta) + \Psi_{t+1}^{\text{NE}}\frac{\partial y_{t+1}^{\text{NE}}}{\partial k_{t+1}^{\text{NE}}}\right) \tag{65}$$

$$q_t^{\text{NE}} = 1 \tag{66}$$

$$w_t^{\text{NE}} = \Psi_t^{\text{NE}}\frac{\partial y_{t+1}^{\text{NE}}}{\partial l_{t+1}^{\text{NE}}} \tag{67}$$

$$p_t^{E} = \Psi_t^{\text{NE}}\frac{\partial y_{t+1}^{\text{NE}}}{\partial y_{t+1}^{\text{E}}} \tag{68}$$

$$q_t^{\text{NE}} = q_t\frac{\partial y_t}{\partial y_t^{\text{NE}}} \tag{69}$$

where we denote Ψ_t^{NE}, and q_t^{NE}, and q_t the Lagrange multipliers associated with production inputs, non-energy investment, and total output, respectively.

We can also easily check that $\Psi_t^{\text{NE}} = 1$ as we are in an RBC case.

F.4 Environmental Policy

When the environmental regulator optimally sets the environmental policy, the carbon price is set equal to the social cost of carbon, as demonstrated in the social planner's case:

$$\tau_t = V_t^{E^E} \tag{70}$$

Alternatively, the regulator might choose to set an emission cap as follows:

$$e_t^E = \text{Carbon Cap} \tag{71}$$